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NOTE ON EXPRESSIONS FOR SECOND-ORDER SHORT-PERIODIC PERTURBATIONS

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TABLE OF CONTENTS

BIOGRAPHICAL NOTE

Dr. Kozai received his degree from the Tokyo University in 1958.

Kozai has been associated with the Tokyo Astronomical Observatory since 1952, and has held concurrent positions as staff astronomer with that observatory and consultant to the Smithsonian Astrophysical Observatory since 1958.

He has specialized in celestial mechanics, his research at SAO being primarily in the determination of zonal coefficients in the earth's gravitational potential by using precisely reduced Baker-Nunn observations. He is also interested in the seasonal variations of the earth's potential.

ABSTRACT

Expressions for second-order short-periodic perturbations for satellite motion are derived by assuming that $J_4 = -1.4\,J_2^2$. The expressions consist of terms for which amplitudes are larger than 10^{-7} when the parameter of the ellipse is equal to the equatorial radius of the earth and the eccentricity is 0.15. The expressions for the J_3 term are also given.

NOTE ON EXPRESSIONS FOR SECOND-ORDER SHORT-PERIODIC PERTURBATIONS

Yoshihide Kozai

My paper (1962) on the second-order theory of satellite motions contains very complicated expressions valid for any eccentricity and inclination. However, these expressions can be simplified by taking into consideration the following facts:

A. We cannot expect the accuracy of observations to become better than 10^{-7} (0.02 sec).

B. We can assume the eccentricity to be small, since, otherwise, the semimajor axis would be large and, therefore, the second-order effects would be negligibly small.

C. We can combine J_2^2 terms with J_4 terms by using the relation $J_4 = -1.4\ J_2^2$ for the earth, since they have common arguments. For example, instead of writing $J_2^2 \cos \alpha + J_4 \cos \alpha$ we can write $-0.4\ J_2^2$ $-0.4\ J_2^2 \cos \alpha$.

In this note simplified expressions for short-periodic perturbations are given. The expressions consist of terms for which amplitudes are larger than 10^{-7} when $p = a(1 - e^2) = 1$ and $e = 0.15$. When the eccentricity is larger than 0.15, p is larger than 1.2, and, therefore, the amplitudes are reduced at least by a factor of one-half because of a divisor p^4 in the expressions.

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In deriving the expressions it is assumed (Kozai, 1964) that

$$J_4 = -1.407\, J_2^2 \quad (J_4 = -1.649 \times 10^{-6}) \quad .$$

In addition to J_2^2 terms, J_3 terms are also given.

Kepler's third law is written as

$$a_0 = \left(\frac{GM}{n^2}\right)^{1/3} \left[1 + \frac{J_2}{4p_0^2} \sqrt{1 - e_0^2}\left(1 - 3\theta_0^2\right)\right.$$

$$\left. - \frac{J_2^2}{16p_0^4}\left(11 - 126\,\theta_0^2 + 126\,\theta_0^4\right)\right] \quad , \qquad (1)$$

where

$$\theta_0 = \cos i_0 \quad , \qquad p_0 = a_0\left(1 - e_0^2\right) \quad .$$

The quantities with suffix zero express mean elements that are constant, or at least cannot be changed by perturbations due to the earth's figure. The semimajor axis a_0, defined in equation (1), is less by $J_2^2 (1 - \theta^2)^2 \, (GM/n^2)^{1/3}/32p^4$ than that given as equation (7.3) in my previous paper (1962). The last term of equation (1) with factor J_2^2 cannot have a value greater than 15×10^{-7}.

The short-periodic perturbations are given for the radius r, the argument of latitude $L = v + \omega$, the inclination i, and the longitude of the ascending node as follows:

$$r = \frac{p'}{1 + e' \cos v'}$$

$$+ \frac{J_2}{4p'} \left\{ (1 - 3\theta'^2) \left[1 + \frac{e'}{1 + \sqrt{1 - e'^2}} \cos v' - \frac{1}{\sqrt{1 - e'^2}} \frac{r'}{a'} \right] \right.$$

$$\left. + (1 - \theta'^2) \cos 2(v' + \omega^*) \right\}$$

$$+ \frac{J_3}{32p^2} \sin i \left\{ 2(1 - 5\theta^2) \left[24e \sin\omega \right. \right. \qquad [5]$$

$$+ 3 \sin(v + \omega) \qquad [5]$$

$$\left. - 8e \sin(2v + \omega) \right] \qquad [2]$$

$$\left. + 5 \sin^2 i \sin 3(v + \omega) \right\} , \qquad [4] \qquad (2)$$

$$L = v' + \omega^* - \frac{J_2}{8p'^2} \left[4e' \left(\frac{1 - 3\theta'^2}{1 + \sqrt{1 - e'^2}} + 1 - 6\theta'^2 \right) \sin v' \right.$$

$$+ (1 - 3\theta'^2) \left(1 - \sqrt{1 - e'^2} \right) \sin 2v' - 2e' (2 - 5\theta'^2) \sin(v' + 2\omega^*)$$

$$\left. - (1 - 7\theta'^2) \sin 2(v' + \omega^*) + 2e'\theta'^2 \sin(3v' + 2\omega^*) \right]$$

$$+\frac{J_3}{96p^3 \sin i}\left[36(4 - 35\theta^2 + 35\theta^4) \cos (v+\omega) \right. \qquad [50]$$

$$+ 63e \sin^2 i\,(1 - 5\theta^2) \cos \omega \qquad [2]$$

$$- 3e\,(19 - 48\theta^2 + 5\theta^4) \cos (2v+\omega) \qquad [4]$$

$$\left. - 10 \sin^2 i\,(1 - 7\theta^2) \cos 3\,(v+\omega)\right] \qquad [3]$$

$$+\frac{J_2^2}{64p^4}\left[(17 - 672\theta^2 + 1205\theta^4)(v - M) \right. \qquad [25]$$

$$+ 3\,(31 - 710\theta^2 + 1289\theta^4)\, e \sin v \qquad [50]$$

$$- (101 - 884\theta^2 + 822\theta^4)\, e \sin (v+2\omega) \qquad [3]$$

$$+ (12 - 19\theta^2 - 18\theta^4) \sin 2(v+\omega) \qquad [5]$$

$$- (4 - 199\theta^2 + 464\theta^4)\, e \sin (3v+2\omega) \qquad [8]$$

$$\left. + (5 - 27\theta^2 + 40\theta^4) \sin 4(v+\omega)\right] , \qquad [4] \qquad (3)$$

$$i = i' + \frac{J_2}{8p'^2} \sin 2i' \left[3e' \cos (v' + 2\omega^*) \right.$$

$$\left. + 3 \cos 2\,(v' + \omega^*) + e' \cos (3v' + 2\omega^*)\right]$$

$$+ \frac{J_3}{16p^3} \theta \{- 3 (1 - 5\theta^2) [2 \sin (v+\omega) \qquad [40]$$

$$+ e \sin (2v + \omega)] \qquad [3]$$

$$+ 5 \sin^2 i [3e \sin (2v + 3\omega) \qquad [2]$$

$$+ 2 \sin 3 (v + \omega)]\} \qquad [6]$$

$$+ \frac{J_4}{16p^4} \sin 2i [(1 + 2\theta^2) \cos 2 (v + \omega)] \qquad [2]$$

$$- 8(1 - 5\theta^2) e \cos (3v + 2\omega)] \quad , \qquad [2] \qquad (4)$$

$$\Omega = \Omega' + \frac{J_2}{4p'^2} \theta'[- 6(v' - M') - 6e' \sin v' + 3e' \sin (v' + 2\omega^*)$$

$$+ 3 \sin 2(v' + \omega^*) + e' \sin (3v' + 2 \omega^*)]$$

$$+ \frac{J_3}{16p^3} \frac{\theta}{\sin i} \{3(11 - 15\theta^2) [2 \cos (v+\omega) \qquad [40]$$

$$+ e \cos (2v +\omega)] \qquad [3]$$

$$- 10 \sin^2 i \cos 3 (v + \omega)\} \qquad [6]$$

$$+ \frac{J_2^2}{16p^4} \theta [33 (1 - 3\theta^2) (v - M) \qquad [15]$$

$$+ (142 - 407\theta^2) e \sin v \qquad [30]$$

$$- 3\,(25 - 28\theta^2)\ e\ \sin(v + 2\omega) \qquad [2]$$

$$- (20 - 27\theta^2)\ \sin 2\,(v + \omega) \qquad [5]$$

$$- (49 - 116\theta^2)\ e\ \sin(3v + 2\omega) \qquad [8]$$

$$+ (6 - 11\theta^2)\ \sin 4\,(v + \omega)] \quad , \qquad [4] \qquad (5)$$

where

$$\omega^* = \omega' - \frac{3J_2}{4p'^2}\,(1 - 5\theta'^2)\,(v' - M') \quad . \qquad (6)$$

The bracketed number given on the right side of each second-order term is the estimated maximum amplitude in units of 10^{-7}.

In the expressions of the first-order perturbations with factor J_2, mean orbital elements free from short-periodic terms should be used as primed quantities. These quantities change because of long-periodic perturbations and should be known with four significant figures in order for the position of the satellite to be computed to seven significant figures. Therefore, it does not matter whether any perturbations less than 10^{-4} are included or not in order to compute the primed quantities.

In the expressions of the second-order perturbations with factors J_3 and J_4, it is not necessary to know all the quantities with more than two significant figures.

Equation (2) is derived from equation (5.7) in my previous paper (1962), and the J_4 and J_2^2 terms are cancelled out and neglected.

In addition to J_2^2 and J_3 terms given here, it may be necessary to compute J_5, J_6, ..., terms in order to know the positions of the satellite with an accuracy of 10^{-7}.

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