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FORTRAN PROGRAM FOR COMPUTATION OF WEBER FUNCTIONS AND THEIR FIRST DERIVATIVES

by Antra Priede and Gabriel Allen

Lewis Research Center
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FORTRAN PROGRAM FOR COMPUTATION OF WEBER FUNCTIONS AND THEIR FIRST DERIVATIVES

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SUMMARY

This report contains a description of a FORTRAN IV program for computing $D_\nu(x)$ and $(d/dx)D_\nu(x)$, which are the Weber function of order ν and argument x and its first derivative, respectively. The range of values is $x[0, 30]$ and $\nu[1, 200]$. The computations are performed in double precision and the values are accurate to seven significant figures over most of this region. However, in the range $x[5.5, \sqrt{50}]$ and $(x^2/4) + 1 < \nu < (x^2/4) + 13$, the accuracy is reduced. Included herein are tables of $D_\nu(x)$ and $(d/dx)D_\nu(x)$ with values listed at intervals of 1 in x and 5 in ν; the tables begin with $x = 0$ and $\nu = 4.5$, but they omit part of the region covered by $x[5.5, \sqrt{60}]$.

The program is so written to take advantage of asymptotic forms, recursion relations, and series computations at whatever combinations of ν and x these methods yield the desired accuracy in the shortest machine time.

INTRODUCTION

Many problems of physical and engineering interest are concerned with waves emanating into and out of systems having parabolic geometry (ref. 1). Separation of the appropriate wave equation (either classical or quantum-mechanical) in parabolic coordinates results in a differential equation that can be transformed into Weber's equation. The latter may be written as

$$\frac{d^2y}{dx^2} + \left(\nu + \frac{1}{2} - \frac{1}{4}x^2\right)y = 0 \tag{1}$$

The solutions to this equation are called Weber functions or parabolic cylinder functions. They are commonly denoted by $D_\nu(x)$, where both x and ν may take on general complex values. Physically, x usually signifies a distance from the axis of the parabolic region, while the order ν contains other physical properties intrinsic to the system. Thus, the solutions of greatest interest are those for which both x and ν are real.

For physical applications it is convenient to use, as independent solutions to Weber's equation (1), functions distinguished by their asymptotic behavior. Thus, $D_\nu(x)$ goes to 0 as $x \to \infty$, whereas a second solution, which is linearly independent of $D_\nu(x)$, approaches infinity as $x \to \infty$. There is an important connection between this second solution and the form of the first solution (i.e., of $D_\nu(x)$) in the range $x < 0$.

As indicated in reference 2, Weber's equation is defined for complex arguments z. The asymptotic expansion of $D_\nu(z)$ is valid in the sector of the complex plane given by $|\arg z| < 3\pi/4$. Note that the negative part of the real axis ($\arg z = \pi$) is not in this region of validity. Thus, the use of the solution to Weber's equation which is valid for $x < 0$ will require the second independent solution. This solution may take any of the forms $D_{-\nu-1}(iz)$, $D_{-\nu-1}(-iz)$, or $D_\nu(-z)$. These three forms are not identical, but there are two features common to each of them: first, the region of validity includes $\arg z = \pi$; and second, the forms are linearly independent of $D_\nu(z)$.

For $5\pi/4 > \arg z > \pi/4$,

$$D_\nu(z) \sim e^{\nu\pi i} D_\nu(-z) + \frac{\sqrt{2\pi}}{\Gamma(-\nu)} e^{(\nu+1)\pi i/2} D_{-\nu-1}(-iz)$$

and for $-\pi/4 > \arg z > -(5\pi/4)$,

$$D_\nu(z) \sim e^{-\nu\pi i} D_\nu(-z) + \frac{\sqrt{2\pi}}{\Gamma(-\nu)} e^{-(\nu+1)\pi i/2} D_{-\nu-1}(iz)$$

where the symbol $\sim$ means approaches asymptotically.

Because of these properties, one should not substitute negative values of the argument into $D_\nu(x)$ before carefully examining reference 2 in order to obtain the proper combination of second solutions which have the behavior desired.

This particular program was developed in order to solve a quantum-mechanical problem in solid-state physics (ref. 3) by using solutions of Weber's equation for a dense distribution of ν in the interval [1, 200] at a fixed value of x (about 12.5). The most complete tables and discussion of $D_\nu(x)$ which are currently available (ref. 4) cover the range $\nu[-5.5, 4.5]$ and $x[0, 5]$. The use of these tables for the range of orders required would necessitate repeated recursion (see eq. (14)) followed by interpolation. Furthermore, the desired value of x lay outside the covered range. For these reasons, it was necessary to devise a method for computing $D_\nu(x)$ for the desired values of ν and x. A FORTRAN IV program which could accomplish this task was written for the Lewis 7094II - 7044 direct-couple system. This report describes a modified program which can be used to evaluate $D_\nu(x)$ and $(d/dx)D_\nu(x)$ in the range $x[0, 30]$ and $\nu[1, 200]$. Computations are made only for that solution to equation (1) which goes to zero as $x \to \infty$.

ANALYSIS OF COMPUTATIONAL PROCEDURES FOR THE WEBER FUNCTION

In what follows, the notation of reference 4 will be used. The relations between this notation and the definitions in the introduction are

$$a = -\nu - \frac{1}{2} \tag{2}$$

$$U(a, x) = D_\nu(x) \tag{3a}$$

In this connection, the following relation will be used in order to compute $(d/dx)U(a, x)$:

$$\frac{d}{dx}U(a, x) = -\frac{x}{2}U(a, x) - \left(a + \frac{1}{2}\right)U(a+1, x) \tag{3b}$$

Weber's equation in this notation becomes

$$\frac{d^2y}{dx^2} - \left(\frac{1}{4}x^2 + a\right)y = 0 \tag{4}$$

Reasonable procedures for computing values of $U(a, x)$ and its derivative with an accuracy of seven significant figures are presented herein. The range covered is $x[0, 30]$ and $a[-1.5, -200.5]$. Thus, the properties described in this section are only those which are relevant to this objective. As might be expected, over the range of a and x desired, several different methods of computation must be used to obtain the required accuracy efficiently. Discussion of this matter will be facilitated by reference to figure 1 where that portion of the a, x-plane covering this range has been divided into various regions. In each region, a distinct method of computation for $U(a, x)$ is used. Following a general description, the different regions will be discussed individually.

General Description of Methods of Computation of $U(a, x)$

For small values of $|a|$ and x, direct computation of $U(a, x)$ by series gives satisfactory results. The region containing such combinations of a and x is denoted by S

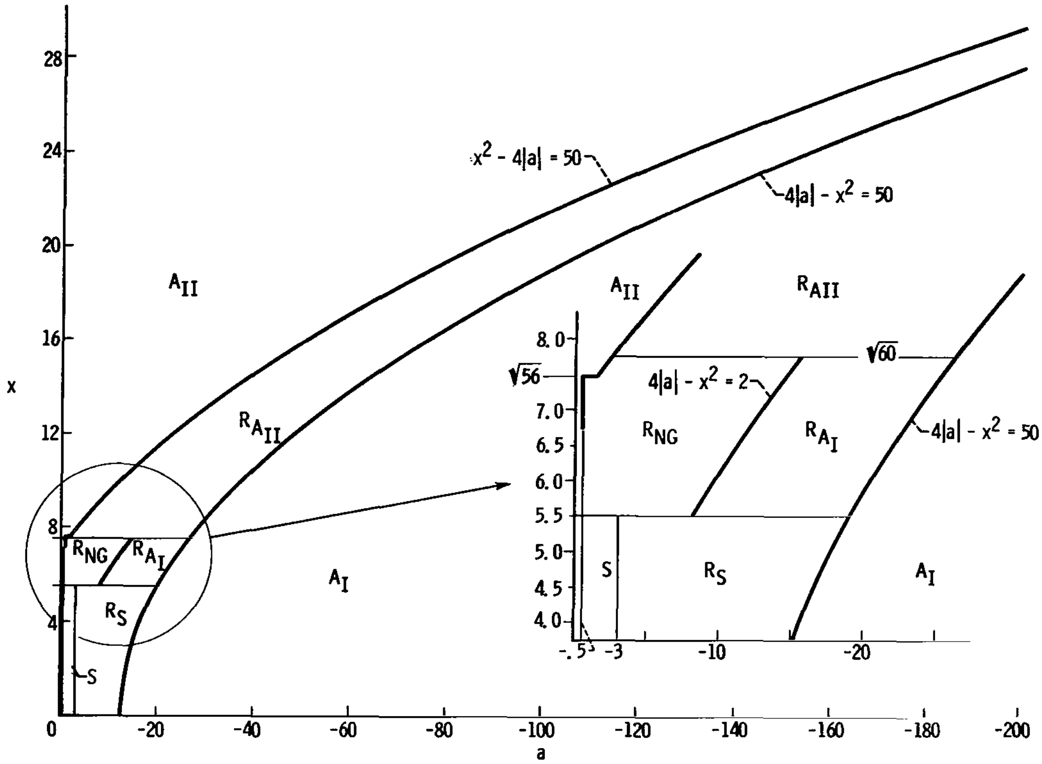


Figure 1. - Computational regions of U(a, x).

on the figure. The rapidity of convergence of the series decreases as $|a|$ increases, so that more terms must be used to calculate a given U(a, x) to a given accuracy. For these values of a, the recursion relation for Weber functions can be used to compute U(a, x) with the same accuracy as the series computation but in a shorter time. The region where this condition prevails is denoted by R_S. For certain combinations of $|a|$ and x, asymptotic expressions exist that can be used to obtain accurate values for U(a, x) in a shorter time than is required by the other procedures. Region A_I denotes the case in which $4|a| - x^2 \gg 1$ and region A_{II} denotes the case in which $x^2 - 4|a| \gg 1$. Unfortunately, there exists a region for which all the aforementioned techniques, singly or in combination, fail to give reliable accuracy. This region is denoted by R_{NG} and has been omitted from the program. The individual regions will be discussed next.

Region S - Computation of U(a, x) by Series

Region S is defined by the boundaries a[-0.5, -3], $0 \le x \le 5.5$. The even and odd solutions (denoted by y_1 and y_2, respectively) to Weber's equation (eq. (4)) can be expressed in terms of hypergeometric functions (ref. 4) as

$$y_1 = \exp\left(-\frac{x^2}{4}\right) {}_1F_1\left(\frac{a}{2} + \frac{1}{4};\ \frac{1}{2};\ \frac{x^2}{2}\right) \tag{5}$$

$$y_2 = x \exp\left(-\frac{x^2}{4}\right) {}_1F_1\left(\frac{a}{2} + \frac{3}{4};\ \frac{3}{2};\ \frac{x^2}{2}\right) \tag{6}$$

Then, $U(a,x)$ may be written as

$$U(a, x) = \frac{\sqrt{\pi}}{2^{[(a/2) + (1/4)]}\,\Gamma\left(\frac{a}{2} + \frac{3}{4}\right)}\, y_1 - \frac{\sqrt{\pi}}{2^{[(a/2) - (1/4)]}\,\Gamma\left(\frac{a}{2} + \frac{1}{4}\right)}\, y_2 \tag{7}$$

The series expressions for y_1 and y_2 can be conveniently written in the form

$$y_1(x) = \sum_{n=0}^{\infty} A_{2n}(x) \tag{8}$$

where

$$\left.\begin{aligned} A_0(x) &= 1 \\ A_2(x) &= a\,\frac{x^2}{2!} \end{aligned}\right\} \tag{9}$$

and

$$A_{2n+2}(x) = \frac{x^2}{(2n + 1)(2n + 2)}\left[aA_{2n}(x) + \frac{1}{4}\,x^2 A_{2n-2}(x)\right] \tag{10}$$

when $n \geq 1$, and

$$y_2(x) = \sum_{n=0}^{\infty} A_{2n+1}(x) \tag{11}$$

where

$$\left.\begin{aligned} A_1(x) &= x \\ A_3(x) &= a\,\frac{x^3}{3!} \end{aligned}\right\} \tag{12}$$

and

$$A_{2n+3}(x) = \frac{x^2}{(2n+3)(2n+2)}\left[aA_{2n+1}(x) + \frac{1}{4}x^2A_{2n-1}(x)\right] \tag{13}$$

when $n \geq 1$.

Region R_S - Extension of Series Computations by Use of Recursion Relations

This region is defined by $-(50 + x^2)/4 \leq a \leq -3.0,\ 0 \leq x \leq 5.5$. The recursion relation used may be written

$$U(a-1, x) = xU(a, x) + \left(a + \frac{1}{2}\right)U(a+1, x) \tag{14}$$

and is called the backward recursion relation. From it, a Weber function of order $a - 1$ is obtained from two Weber functions of higher order, that is, a and $a + 1$. The use of the recursion formula still requires that two values of $U(a, x)$ be computed by series, that is, $U(a, x)$ and $U(a+1, x)$.

It was found that, starting with 8-figure accuracy, equation (14) can be used 30 successive times with a loss of accuracy only in the last place. If the corresponding "forward" recursion relation (which can be used to obtain higher orders of $U(a, x)$ starting with lower orders) be used, loss of accuracy occurs after the very first use of the recursion formula. However, using double precision, the forward recursion relation may be used about 15 times and still maintain 6-figure accuracy.

Region A_I - Asymptotic Solutions for $|a|$ Large and x Moderate

Region A_I is defined as that region for which $4|a| - x^2 > 50$. In this region a function which will be called asymptotic I is used for computing $U(a, x)$. This function may be written as

$$U(a,x) \sim 2\sqrt{\frac{\Gamma\left(\frac{1}{2} - a\right)}{\sqrt{2\pi}Y}} e^{v_r} \cos\left(\frac{\pi}{4} + \frac{\pi a}{2} + \theta + v_i\right) \tag{15}$$

where

$$\theta = \frac{1}{2}\int_0^x Y\,dx = \frac{xY}{4} + |a|\,\sin^{-1}\left[\frac{x}{2(|a|)^{1/2}}\right]$$

$$Y = \sqrt{|4|a| - x^2|} \tag{16}$$

$$v_r = -\frac{d_6}{Y^6} + \frac{d_{12}}{Y^{12}} - \ldots$$

$$v_i = \frac{d_3}{Y^3} - \frac{d_9}{Y^9} + \ldots$$

$$d_3 = \frac{1}{a}\left(\frac{x^3}{48} + \frac{1}{2}ax\right) \tag{17}$$

$$d_6 = \frac{3}{4}x^2 - 2a \tag{18}$$

$$d_9 = \frac{1}{a^3}\left(-\frac{7}{5760}x^9 - \frac{7}{320}ax^7 - \frac{49}{320}a^2x^5 + \frac{31}{12}a^3x^3 - 19a^4x\right) \tag{19}$$

$$d_{12} = \frac{153}{8}x^4 - 186ax^2 + 80a^2 \tag{20}$$

This asymptotic approximation truncating v_r and v_i after two terms gives 7-figure accuracy if 16 figures are used in the calculations. It should be noted that the form of asymptotic I differs from that in reference 4 (p. 690) because of a slightly different definition of v_i.

Region A_{II} - Asymptotic Solutions for Large x

At the opposite extreme, where $x^2 - 4|a| >> 1$, a different approximation exists which gives satisfactory results. Region A_{II} is defined as that region for which $x^2 - 4|a| > 50$. The asymptotic expression used to compute $U(a, x)$ in this region will be called asymptotic II and can be written as

$$U(a, x) \sim \sqrt{\frac{\Gamma\left(\frac{1}{2} - a\right)}{\sqrt{2\pi}\, Y}} \exp[-\theta + v(a, x)] \tag{21}$$

where

$$\theta = \frac{1}{2} \int_{2|a|}^{x} Y\, dx = \frac{1}{4} xY + a \ln\left[\frac{x + Y}{2(|a|)^{1/2}}\right]$$

$$v(a, x) = \sum_{s=1} (-1)^s \frac{d_{3s}}{Y^{3s}}$$

and Y, d_3, d_6, d_9, and d_{12} are given by equations (16) to (20). The accuracy of asymptotic II is the same as that of asymptotic I and the same note applies also.

Region $R_{A_{II}}$ - Asymptotic II Plus Recursion

Region $R_{A_{II}}$ in figure 1 (p. 4) is a region in which none of the conditions prevail which would enable the use of any of the preceding four methods of computation to give accurate results. It is essentially the region between A_I and A_{II} but above $x = \sqrt{60}$. In $R_{A_{II}}$, the values of x are too large to enable series computations to be made accurately. On the other hand, they are not so large that $x^2 - 4|a| > 50$. Neither is a so large that $4|a| - x^2 > 50$. Fortunately, the use of the recursion relation (14) applied to starting values of $U(a, x)$ in region A_{II} can carry computations through region $R_{A_{II}}$ with acceptable accuracy (7 significant figures).

Region R_{A_I} - Asymptotic I Plus Forward Recursion

This region is bounded by $x = [5.5, \sqrt{60}]$, $-[(x^2 + 2)/4] \geq a \geq -[(x^2 + 50)/4]$.

Forward recursion from A_I will still give fairly accurate results (5 figures) up to 15 recursions. Therefore, (a, x) combinations in R_{A_I} can be computed by locating the point in A_I and then recurring in the forward direction. The smaller the number of recursions, the greater the accuracy of the results.

Region R_{NG}

This region is bounded on the bottom by $x = 5.5$, on the right by $a = -[(x^2 + 2)/4]$, by $a = -0.50$ between $x = 5.5$ and $\sqrt{56}$, by $x = \sqrt{56}$ between $a = -0.50$ and $a = -1.50$, by $a = [(x^2 - 50)/4]$ between $x = \sqrt{56}$ and $x = \sqrt{60}$, and by $x = \sqrt{60}$ between $a = -2.5$ and $a = -15.5$. For $x > 5.5$, the accuracy of the series begins to decrease rapidly, and by $x = 8$, $a = -0.6$, only the magnitude of $U(a,x)$ can be relied on from the series computations. As explained in the preceding section, the accuracy of forward recursion from region A_I is also not very great after many recursions have been made. If values of $U(a,x)$ for (a, x) combinations in R_{NG} are nevertheless required, the fact that the accuracy decreases as x increases and as $|a|$ decreases should serve as a guide.

PROGRAM DESCRIPTION

The subroutine WEBER computes the Weber function and its derivative for any point in the region a[-1.5, -200.5], x[0, 30]. The subroutine is referenced by the statement CALL WEBER (XD, AD, V, UP, ISI, IFLAG). The first four arguments of the calling vector are double precision and the last two are integers. These arguments are defined as follows:

INPUT

AD	a coordinate
XD	x coordinate

OUTPUT

V	$U(a,x)$
UP	$U'(a,x)$
ISI	exponent of scale factor 10^{ISI} of U and U'
IFLAG	indicator showing whether scaling was used or calculation was omitted
	IFLAG = 0, no calculation made
	IFLAG = 1, no scale factor used
	IFLAG > 1, scale factor is 10^{ISI}

The structure of the subroutine WEBER is based on locating a specified a, x-combination in one of the regions of figure 1. The methods of computing $U(a,x)$ in each region are those described in the previous section. To compute $U'(a,x)$ both $U(a,x)$ and

U(a+1, x) must be available (eq. (3a)). In regions S, A_I, or A_{II}, U(a+1, x) must be computed separately, but in regions R_{A_I}, $R_{A_{II}}$, and R_S, the value comes as a byproduct of recursion.

An outline of the sectional breakup of WEBER follows. (Refer to the flow charts and listings in appendixes A and B, respectively, for a more detailed description.)

BLOCK 1 - Constants Defined

Constants used in WEBER as well as in other subroutines through COMMON are computed and given literal names.

BLOCK 2 - Region of (a, x) Determined

The process by which any input point (a, x) is located in a region of figure 1(p. 4) is as follows. The case when $5.5 < x < \sqrt{60}$ is checked to see if the point (a, x) is in the special regions R_{A_I} or R_{NG}. In the event that (a, x) is in R_{NG}, that is, $(4|a| - x^2 < 2$, IFLAG is set to 0 and no calculation is made. When $2 < (4|a| - x^2) < 50$, the point lies in R_{A_I} and two base points in region A_I are picked at which U is evaluated for use in forward recursion to obtain U(a, x). Program flow is transferred to BLOCK 6. For points in the other regions, there is the following locating process. If $|a| \geq [(x^2 + 50)/4]$, (a, x) is in region A_I. If $|a| \leq [(x^2 - 50)/4]$, (a, x) is in region A_{II}. The procedure for the remaining possibility where $[(x^2 + 50)/4] < |a| < [(x^2 - 50)/4]$ is as follows: if $x \geq - \sqrt{60}$, then (a, x) is in region R_A; if $x \leq 5.5$, (a, x) is in S if $|a| \leq 3$, otherwise it is in R_S.

BLOCK 3 - Series Evaluation in Region S

The subroutine SERIES is called in BLOCK 3 for two adjacent points in region S (the same x coordinate but one unit apart in a). The two values of U are stored for use in the recursion relation or in the evaluation of U'.

BLOCK 4 - Evaluation by Asymptotic II in Region A_{II}

The Weber function is evaluated for two adjacent points in region A_{II}. As in BLOCK 3 these values are stored for use in either the recursion relation or for evaluating the derivative. The logic within the DO-LOOP chooses the correct gamma function subroutine for the magnitude of a and keeps the same scale factor for both U values when $a < -50.0$.

For example, if $a = -45.0$ and $x = 17.6$, find $U(a,x)$ and $U'(a,x)$. Since $R = 129.76$ and $x^2 > |a|$, the program locates (a, x) in A_{II}. Then

$$a_1 = a + 1.0 = -44.0$$

$$a_2 = a_1 - 1.0 = -45.0$$

The DO-LOOP in BLOCK 4 computes U for these two values of a by using asymptotic II. The results are

$$U(-44.0, 17.6) = 1.0484715 \times 10^{19}$$

$$U(-45.0, 17.6) = 1.5357027 \times 10^{20}$$

Coming out of the loop, a is set equal to a_2, which is the original a coordinate. This condition signals the program to use the two computed values to evaluate $U'(a, x)$. The result is

$$U'(-45.0, 17.6) = -\frac{1}{2}(17.6)\, U(-45.0, 17.6) - (-44.5)\, U(-44.0, 17.6) = -8.8484851 \times 10^{20}$$

BLOCK 5 - Evaluation in Regions R_A and R_S

Values of $U(a_2, x_1)$ and $U(a_2+1, x_1)$ coming from either BLOCK 3 or BLOCK 4 are used in the recursion equation to generate the next value $U(a_2-1, x_1)$. When moving along the line $x = x_1$ in the negative direction in a, $U(a, x)$ is evaluated at every point $U(a_2-n, x_1)$, $n = 1, 2 \ldots$, until $U(a_1, x_1)$ is found. Then $U'(a, x)$ is computed.

For example, if $a \neq -40.00$, $x = 12.00$, and $R = |(12.0)^2 - 4|-40.0||= 16.0$, find $U(a, x)$ and $U'(a, x)$ when (a, x) lies in R_A.

It is noted that the point $(-40.0, 12.0)$ is closer to region A_I than to region A_{II} on the line $x = 12.0$. Nevertheless, A_{II} base points will be used because backward recursion is the accurate one. The two base points for recursion are $(-22.0, 12.0)$ and $(-23.0, 12.0)$.

From BLOCK 4 the following results are obtained:

$$U(-22.0, 12.0) = 6.0415211 \times 10^6$$

$$U(-23.0, 12.0) = 5.9467925\times10^{7}$$

Applying equation (14) repeatedly gives

$$U(-39.0, 12.0) = 3.5549969\times10^{22}$$

$$U(-40.0, 12.0) = 2.2109229\times10^{23}$$

Using equation (3a) gives

$$U'(-40.0, 12.0) = -6.0\ U(-39.0, 12.0) - (-39.5)\ U(-39.0, 12.0)$$

$$= 7.7670203\times10^{22}$$

BLOCK 6 - Evaluation by Asymptotic I in Region A_I

In this DO-LOOP as in BLOCK 4, U is evaluated at two adjacent points. These points (a, x) and (a+1, x) are used in computing U'. The same type of coding as in BLOCK 4 is used for finding the gamma function and keeping the scale factor consistent. In case the calculations are made to provide base points for recursion, the following forward recursion relation is used until U is evaluated for the desired a:

$$U(a, x) = \frac{U(a-2, x) - xU(a-1, x)}{a - \frac{1}{2}}$$

The derivative is evaluated by the relation

$$U'(a, x) = \frac{1}{2}xU(a, x) - U(a-1, x)$$

Subroutines

Of the seven subroutines, five are used for evaluating the gamma function. Subroutines GAMMA, GAMMAF, GAMMAN, STRGAM, and STRSCL have been developed to evaluate the gamma function in the range $-2 < z < 200$. The two approximations used are (1) the series expansion to 26 terms for $1/\Gamma(z)$ and (2) Stirling's approximation

$$\Gamma(z) \sim z^{(z-1/2)} e^{-z} (2\pi)^{(1/2)} \left(1 + \frac{1}{12z} + \frac{1}{288z^2} - \frac{139}{51840z^3} - \frac{571}{2488320z^4} + \ldots\right) \tag{22}$$

Subroutine GAMMA is written to evaluate $\Gamma(z)$ by computing

$$\sum_{k=1}^{26} C_k z^k \quad \text{for} \quad 1 \leq z \leq 2$$

and then taking the reciprocal. The C_k are displayed in the listing for GAMMA. The maximum error of $\Gamma(z)$ in this range of argument is 2×10^{-8} when 16-place numbers are used. GAMMA is called by two subroutines; either GAMMAF, which monitors the argument $z \geq 1$, or GAMMAN, which handles nonintegral arguments $z < 1$. GAMMAF reduces the positive argument to within the range of GAMMA and then uses the relation

$$\Gamma(z + 1) = z\Gamma(z) \tag{23}$$

GAMMAN, on the other hand, increases the argument until it falls within the range used in GAMMA and uses the relation (eq. (23)) in the negative direction; that is,

$$\Gamma(z) = \frac{\Gamma(z + 1)}{z} \tag{24}$$

STRGAM and STRSCL are altered forms of Stirling's approximation (eq. (20)). Since $\Gamma(z)$, where $z = 1/2 - a$, appears in both asymptotic I and II in the form $\sqrt{\Gamma(z)}/(2\pi)^{1/4}$, equation (20) can be rewritten to compute this factor directly by using

$$\frac{\sqrt{\Gamma(z)}}{(2\pi)^{1/4}} \cong \left(\frac{z}{e}\right)^{z/2} z^{-1/4} \left(1 + \frac{1}{12z} + \frac{1}{288z^2} - \frac{139}{51840z^3} - \frac{571}{2488320z^4}\right)^{1/2} \tag{25}$$

STRGAM computes this factor for $|a| > 10.0$ with at least 8-figure accuracy when 16 figures are used.

STRSCL computes the same factor for $|a| > 50.0$ with the added feature of a scale factor, since the value of the altered gamma function becomes too large for the computer. Scaling is done on equation (25) as follows: Let

$$y = \frac{z}{2} \log_{10} \frac{z}{e}$$

and

$$ISI = [y]$$

greatest integer function. Then ISI, a power of 10, becomes the scale factor and $10.0^{(y-ISI)}$ replaces $(z/e)^{z/2}$ in equation (25). The ISI is carried back to WEBER in the calling vector STRSCL.

Scaling of the gamma function is meshed with scaling of the factor $\exp[-\theta + \nu(a, x)]$ in asymptotic II. Whenever the argument $[-\theta + \nu(a, x)]$ approaches the value -88, below which the exponential would be set to zero, a scale factor is factored from the exponential and combined with the scale factor of gamma. The quantity

$$\exp\left[-\theta + \nu(a, x)\right] \text{ is written } 10^{\left\{\left[-\theta + (a, x)\right] \log_{10} e\right\}}$$

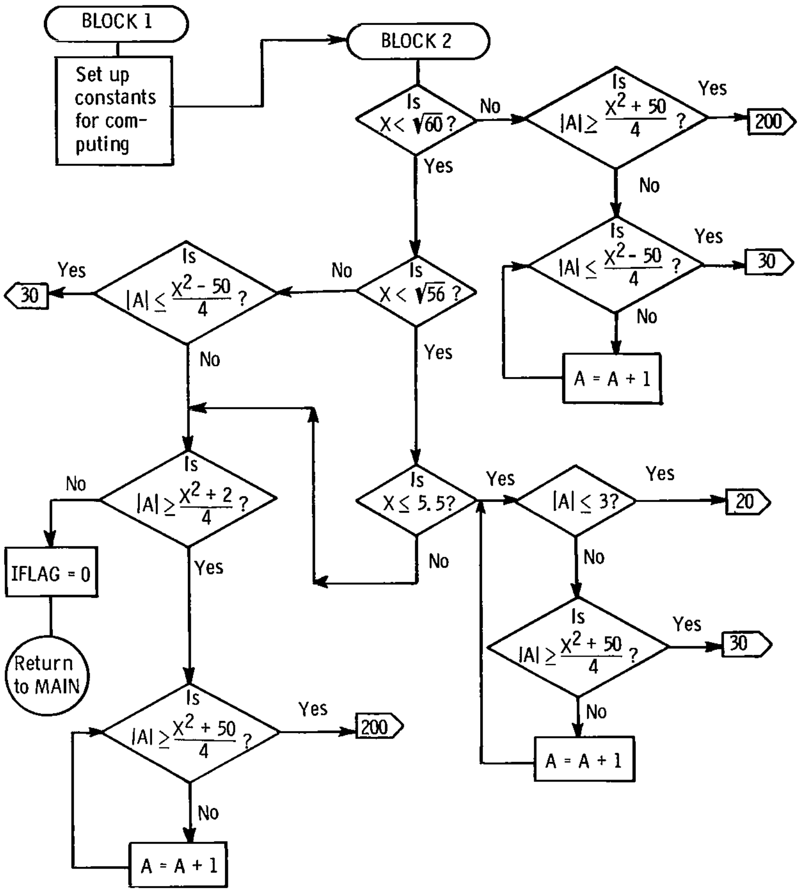
The exponent of 10 is split into the largest integer and a remainder. The integer is combined with S coming from STRGAM to give a scale factor for U, and the value of 10 raised to the remainder replaces $\exp[-\theta + \nu(a, x)]$ in the computation of U.

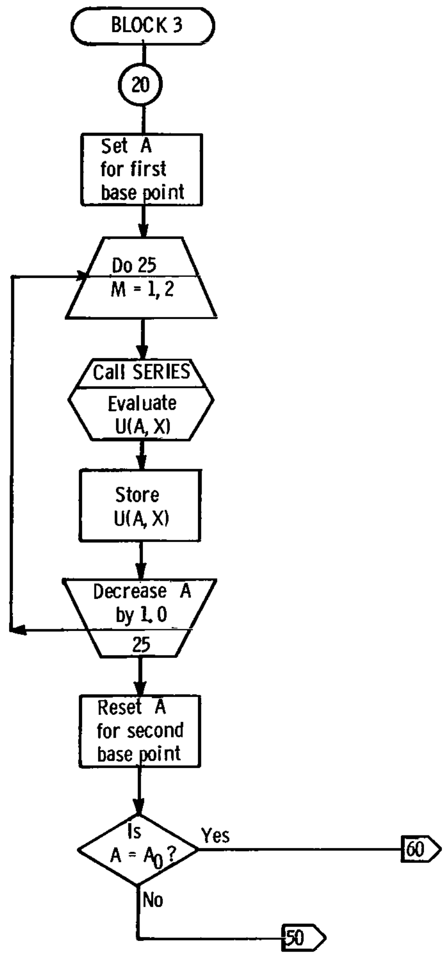
The other two subroutines are SERIES, which computes $U(a, x)$, equations (7) to (13) and DFIND, which evaluates the coefficients d_{3i}, equations (17) to (20).

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, October 5, 1966,
125-23-02-10-22.

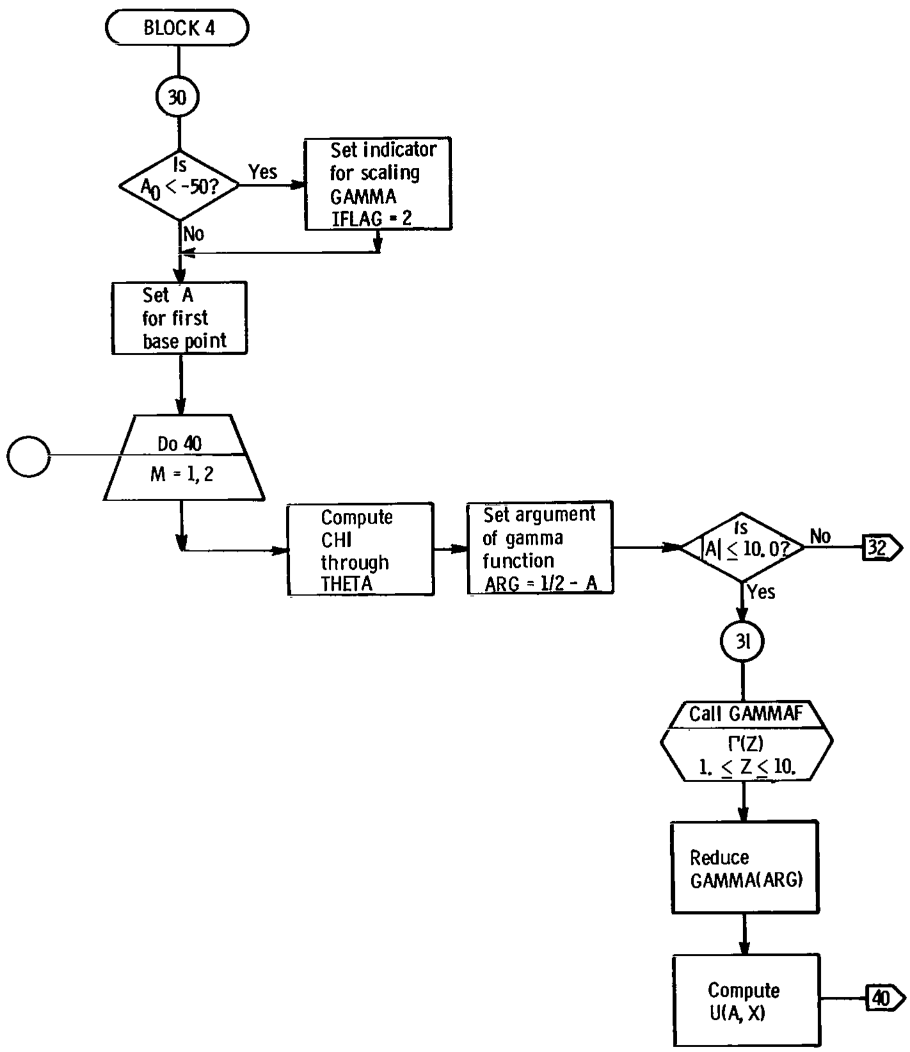
APPENDIX A

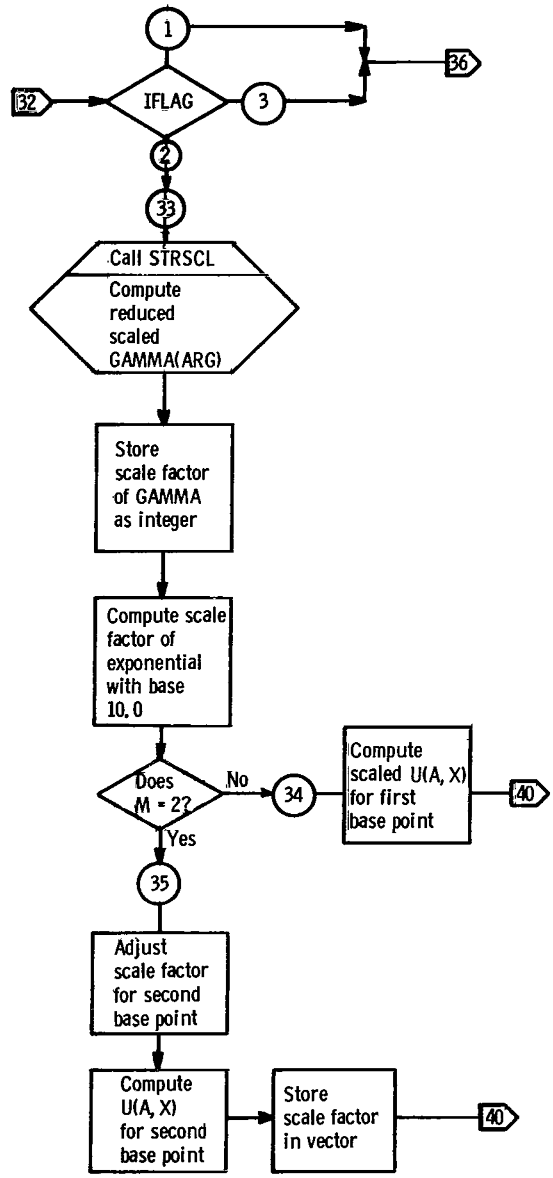
FLOW CHARTS FOR COMPUTATION OF WEBER FUNCTIONS AND FIRST DERIVATIVES

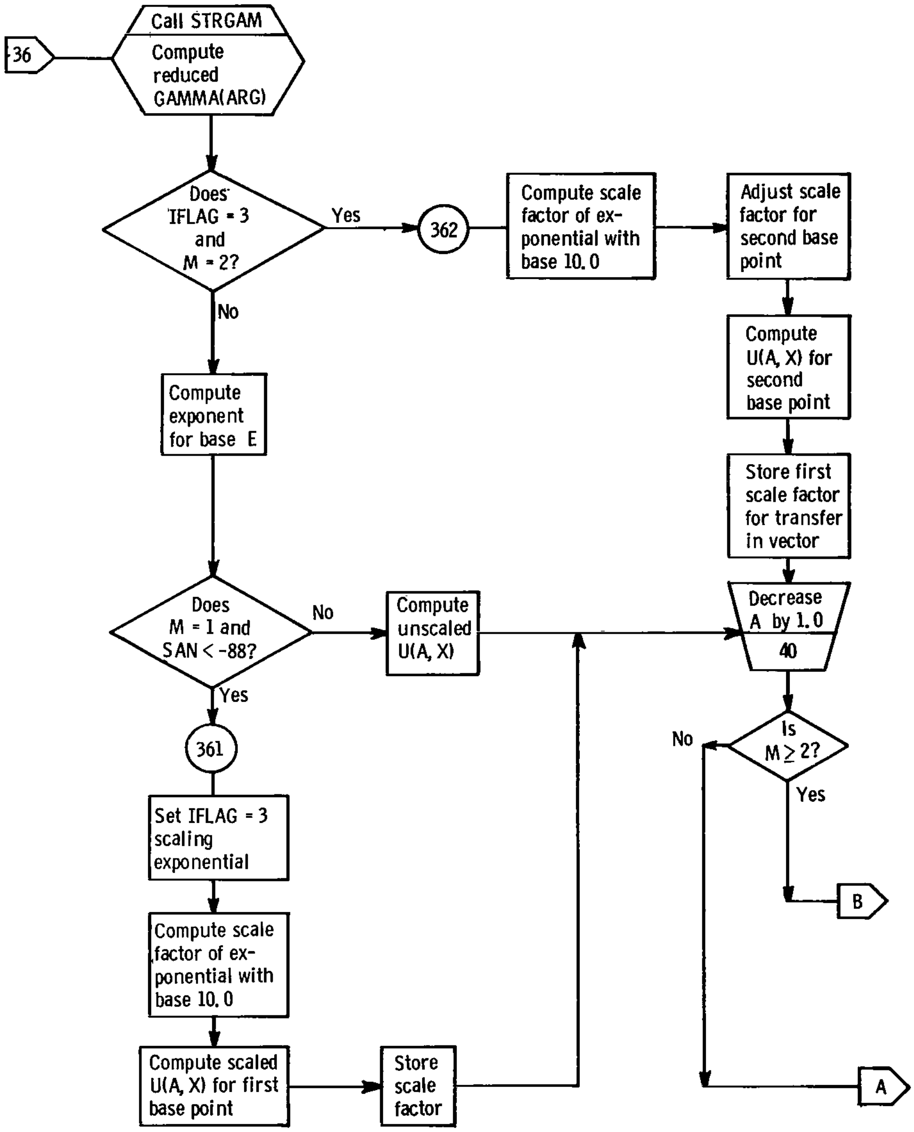


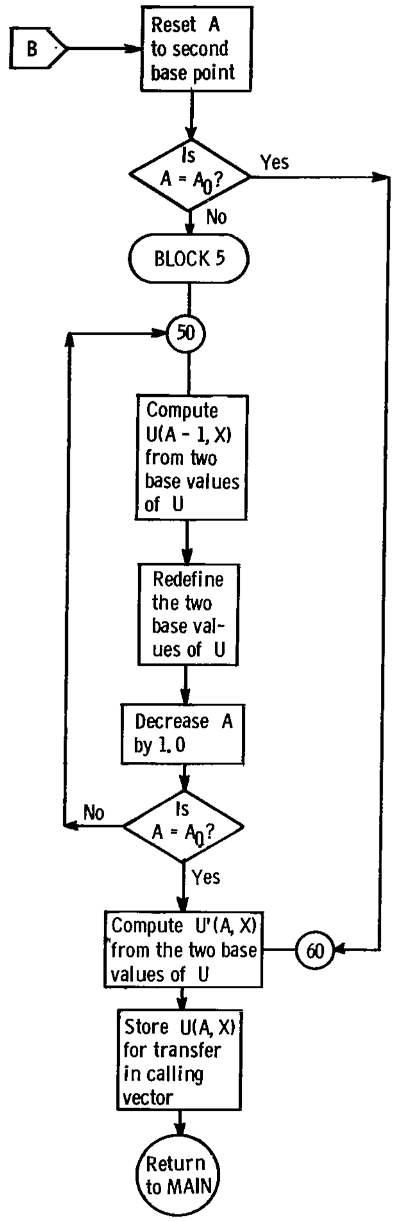


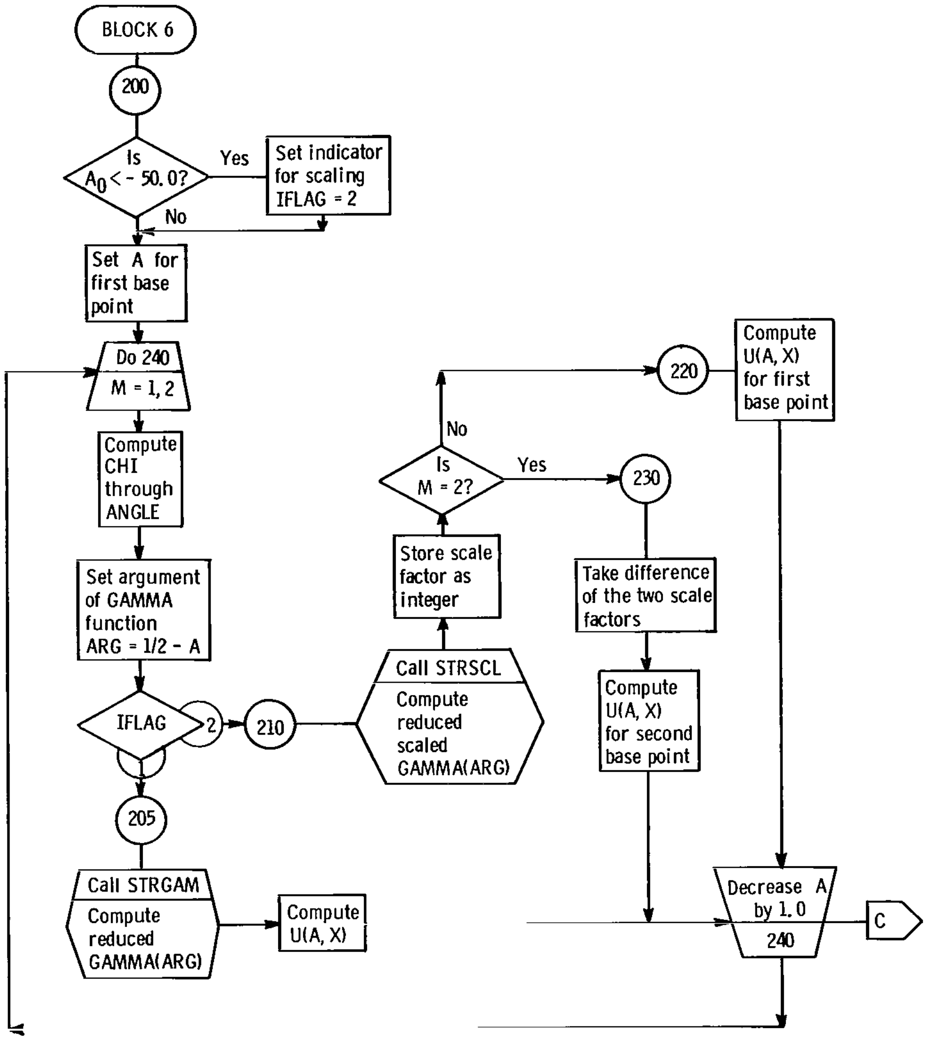
BLOCK 3
20
Set A for first base point
Do 25 M = 1, 2
Call SERIES
Evaluate U(A, X)
Store U(A, X)
Decrease A by 1.0
25
Reset A for second base point
Is A = A0 ?
Yes
60
No
50


BLOCK 4
30
Is $A_0 < -50$?
Yes
Set indicator for scaling GAMMA IFLAG = 2
No
Set A for first base point
Do 40
M = 1, 2
Compute CHI through THETA
Set argument of gamma function ARG = 1/2 - A
Is $|A| \leq 10.0$?
No
32
Yes
31
Call GAMMAF
$\Gamma(Z)$
$1. \leq Z \leq 10.$
Reduce GAMMA(ARG)
Compute U(A, X)
40

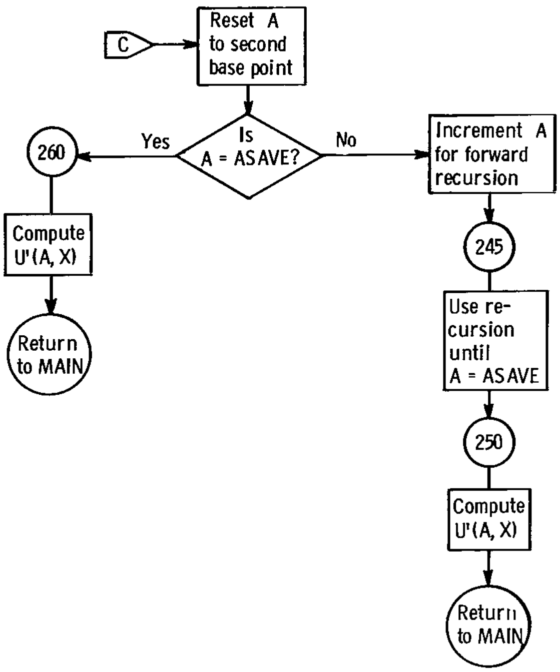

1
36
32
IFLAG
3
2
33
Call STRSCL
Compute reduced scaled GAMMA(ARG)
Store scale factor of GAMMA as integer
Compute scale factor of exponential with base 10.0
Does M = 2?
No
34
Compute scaled U(A, X) for first base point
40
Yes
35
Adjust scale factor for second base point
Compute U(A, X) for second base point
Store scale factor in vector
40


36
Call STRGAM
Compute reduced GAMMA(ARG)
Does IFLAG = 3 and M = 2?
Yes
No
362
Compute scale factor of exponential with base 10.0
Adjust scale factor for second base point
Compute U(A, X) for second base point
Store first scale factor for transfer in vector
Compute exponent for base E
Does M = 1 and SAN < -88?
No
Yes
Compute unscaled U(A, X)
Decrease A by 1.0
40
Is M ≥ 2?
No
Yes
B
361
Set IFLAG = 3 scaling exponential
Compute scale factor of exponential with base 10.0
Compute scaled U(A, X) for first base point
Store scale factor
A


B
Reset A to second base point
Is A = A_0?
Yes
No
BLOCK 5
50
Compute U(A - 1, X) from two base values of U
Redefine the two base values of U
Decrease A by 1.0
Is A = A_0?
No
Yes
Compute U'(A, X) from the two base values of U
60
Store U(A, X) for transfer in calling vector
Return to MAIN



BLOCK 6
200
Is $A_0 < -50.0$?
Yes
No
Set indicator for scaling IFLAG = 2
Set A for first base point
Do 240 M = 1, 2
Compute CHI through ANGLE
Set argument of GAMMA function ARG = 1/2 - A
IFLAG
2
1
205
Call STRGAM
Compute reduced GAMMA(ARG)
Compute U(A, X)
210
Call STRSCL
Compute reduced scaled GAMMA(ARG)
Store scale factor as integer
Is M = 2?
No
Yes
220
Compute U(A, X) for first base point
230
Take difference of the two scale factors
Compute U(A, X) for second base point
Decrease A by 1.0
240
C


C
Reset A to second base point
Is A = ASAVE?
Yes
No
260
Compute U'(A, X)
Return to MAIN
Increment A for forward recursion
245
Use recursion until A = ASAVE
250
Compute U'(A, X)
Return to MAIN

APPENDIX B

PROGRAM LISTINGS FOR COMPUTATION OF WEBER FUNCTIONS AND FIRST DERIVATIVES

```
        MAIN     DEBUG

       DOUBLEPRECISION A,X,U,UP,S,AMIN,AMAX,DA,XMAX,DX,SAVE,DABSU,DABSUP
       AMIN = -5.D0
       AMAX = -200.D0
       DA = -5.D0
       XMAX = 30.D0
       DX = 1.D0
       A = AMIN
    25 WRITE (6,1) A,A
     1 FORMAT (1H1 6X 1HX 7X 2HU( F6.1, 3H,X) 10X 2HU( F6.1,3H,X)/1HJ)
       X = 0.D0
       ILJA = 0
    50 CALL WEBER (X,A,U,UP,ISI,ISCALE)
       IF (ISCALE.EQ.0) GO TO 120
       IS = 0
       JS = 0
       DABSU = DABS(U)
       IF (DABSU) 60,175,60
    60 IF (DABSU.LT.1.D0) GO TO 300
       IF (DABSU.LT.10.D0) GO TO 100
C   NUMBER GT 10.0
   200 U = U*1.D-1
       IS = IS+1
       DABSU = DABS(U)
       IF (DABSU -10.D0) 100,200,200
C   NUMBER LT 1.0
   300 U = U*1.D1
       IS = IS-1
       DABSU = DABS(U)
       IF (DABSU-1.D0) 300,100,100
   100 DABSUP = DABS(UP)
       IF (DABSUP.LT.1.D0) GO TO 400
       IF (DABSUP.LT.10.D0) GO TO 700
C   NUMBER GT 10.0
   500 UP = UP*1.D-1
       JS = JS+1
       DABSUP = DABS(UP)
       IF (DABSUP-10.D0) 700,500,500
   400 UP = UP*1.D+1
       JS = JS-1
       DABSUP = DABS(UP)
       IF (DABSUP-1.D0) 400,700,700
   700 IF (ISCALE.EQ.1) GO TO 75
       IS = IS+ISI
       JS = JS+ISI
    75 XX = X
       UU = U
       UUP = UP
       IF (ILJA.EQ.0) GO TO 118
       IS = IS-IS1
       JS = JS-JS1
       WRITE (6,2) XX,UU,IS,UUP,JS
     2 FORMAT (5X F4.1, 3X F10.7, 1X 1H( I4,1H) 3X F10.7,1X 1H( I4,1H))
       GO TO 120
   118 WRITE (6,3) IS,JS
     3 FORMAT (24X I4, 16X I4)
       JS1 = JS
       IS1 = IS
       ILJA = 1
       JS = 0
       IS = 0
       WRITE (6,2) XX,UU,IS,UUP,JS
   120 X = X+DX
       IF (X-XMAX) 50,50,125
   125 IF (A-AMAX) 250,250,150
   150 A = A+DA
       GO TO 25
```

```
  175 XX = X
      UU = U
      UUP = UP
      WRITE (6,2) XX,UU,IS,UUP,JS
      GO TO 125
  250 STOP
      END


      SUBROUTINE WEBER (XD,AD,V,UP,ISI,IFLAG)
C  FOR THE REGION A (-1.5,-200.0), X (0.0,30.0)
C    THE CALLING VECTOR
C  XD - THE X COORDINATE
C  AD - THE A COORDINATE
C  V - U(A,X)
C  UP - DERIVATIVE OF U(A,X)
C  ISI - SCALE FACTOR
C  IFLAG = 2 OR 3 WHEN 10.0**S IF USED AS A SCALE FACTOR OF V AND UP
C  IFLAG = 0 WHEN NO CALCULATION MADE
      COMMON CA3, CB2,CC9,CC7,CC5,CC3,CC1,CD4,CD2,CD0
      DOUBLE PRECISION    A,ASAVE,ABSA,ARG,ANS,ANGLE,CHI,CHID
     *,CA3,CB2,CC9,CC7,CC5,CC3,CC1,CD4,CD2,CD0,D3,D6,D9,D12
     * ,GAM,PI,PI2,PI24,R,S,THETA,U,UP,V,VAX,VI,VRP,X,X2,XD,AD
     * ,SAM,TAM,DLOGE,RP
      DIMENSION IS(2),U(3)
      IFLAG = 1
CCC BLOCK 1 - SETUP DATA
      X = XD
      A = AD
      PI = 3.141592653589793
      PI2 = 6.283185307179586
      PI24 = PI2**.25
C  COMMON CONSTANTS
      CA3 = 1.D0/24.D0
      CB2 = 75.D-2
      CC9 = 7.D0/5760.D0
      CC7 = 7.D0/320.D0
      CC5 = 49.D0/320.D0
      CC3 = 31.D0/12.D0
      CC1 = 19.D0
      CD4 = 153.D0/8.D0
      CD2 = 186.D0
      CD0 = 80.D0
      DLOGE = .4342944819032518 3D0
      ASAVE = A
      X2 = X*X
CCC BLOCK 2 - DETERMINE REGION OF (A,X) AND EVALUATION METHOD
      ABSA = DABS(A)
      IF (X2 -60.D0) 2,4,4
  2   IF (X2 -56.D0) 3,5,5
  3   IF (X-5.5D0) 6,6,7
  4   IF (ABSA .GE.(50.D0+X2)/4.D0) GO TO 200
  11  IF (ABSA .LE.(X2-50.D0)/4.D0) GO TO 30
      A = A +1.D0
      ABSA = DABS(A)
      GO TO 11
  5   IF(ABSA.LE.(X2-50.D0)/4.D0) GO TO 30
  13  IF(ABSA.GE.(50.D0 +X2)/4.D0) GO TO 200
      IF (ABSA .GE.(X2+2.D0)/4.D0) GO TO 12
      IFLAG = 0
      WRITE(6,16)
  16  FORMAT (1H0 2X 23HPOINT IS IN BAD REGION   / )
      RETURN
  12  A = A-1.D0
      ABSA = DABS(A)
      GO TO 13
  6   IF (ABSA.GE.(50.D0+X2)/4.D0) GO TO 200
  9   IF(ABSA.LE. 3.D0) GO TO 20
      A = A+1.D0
      ABSA = DABS(A)
      GO TO 9
  7   IF(ABSA .GE.(50.D0+X2)/4.D0) GO TO 200
      IF (ABSA .GE.(2.D0+X2)/4.D0) GO TO 8
      IFLAG = 0
      WRITE (6,17)
  17  FORMAT (1H0 2X 23HPOINT IS IN BAD REGION   / )
      RETURN
  8   A = A-1.D0
      ABSA = DABS(A)
      GO TO 7
```

```
CCC BLOCK 3 - SERIES EVALUATION IN REGION S
   20 A = A+1.DO
      DO 25 M=1,2
      CALL SERIES (X,A,ANS)
      U(M) = ANS
   25 A = A-1.DO
      A = A+1.DO
      IF (A-ASAVE) 60,60,50
CCC BLOCK 4 - ASYMPTOTIC II IN REGION AII
   30 IF (ASAVE.LT.-50.DO) IFLAG = 2
      IS(1) = 0
      IS(2) = 0
      A = A+1.DO
      DO 40 M=1,2
      ABSA = DABS(A)
      CHI = DSQRT(X**2-4.DO*ABSA)
      CHID = 1.DO/CHI**3
      CALL DFIND(X,A,D3,D6,D9,D12)
      VAX = -.5DO*DLOG(CHI)+CHID*(-D3+CHID*(D6+CHID*(-D9+CHID*D12)))
      THETA = .25DO*X*CHI+A*DLOG((X+CHI)/(2.DO*DSQRT(ABSA)))
      ARG = .5DO-A
      IF (ABSA-10.DO) 31,31,32
   31 CALL GAMMAF (ARG,GAM)
C  POLYNOMIAL APPROXIMATION OF THE GAMMA FUNCTION
      GAM = DSQRT(GAM)/PI24
      U(M) = GAM*DEXP(-THETA+VAX)
      GO TO 40
   32 GO TO (36,33,36),IFLAG
   33 CALL STRSCL (ARG,GAM,S)
C  SCALED, REDUCED STERLINGS APPROXIMATION OF GAMMA FUNCTION
      IS(M) = S
      SAM = (-THETA+VAX)*DLOGE
      IL = IDINT(SAM)
      TAM = SAM-DBLE(FLOAT(IL))
      IS(M) = IS(M)+IL
      IF (M-2) 34,35,34
   34 U(M) = GAM*10.DO**TAM
      GO TO 40
   35 N = IS(2) - IS(1)
      TAM = DBLE(FLOAT(N))+TAM
      U(M) = GAM*10.DO**TAM
      ISI = IS(1)
      GO TO 40
   36 CALL STRGAM (ARG,GAM)
      IF (IFLAG.EQ.3.AND.M.EQ.2) GO TO 362
      SAM = -THETA+VAX
      IF (M.EQ.1.AND.SAM.LT.-88.DO) GO TO 361
      U(M) = GAM*DEXP(SAM)
      GO TO 40
  361 IFLAG = 3
      SAM = (-THETA+VAX)*DLOGE
      IL = IDINT(SAM)
      TAM = SAM-DBLE(FLOAT(IL))
      U(M)= 10.DO**TAM*GAM
      IS(M) = IL
      GO TO 40
  362 SAM = (-THETA+VAX)*DLOGE
      IL = IDINT(SAM)
      TAM = SAM-DBLE(FLOAT(IL))
      IS(M) = IL
      IM = IS(2)-IS(1)
      U(M) = GAM*10.DO**TAM*10.DO**IM
      ISI = IS(1)
   40 A = A-1.DO
      A = A+1.DO
      IF (A-ASAVE) 60,60,50
CCC BLOCK 5 - RECURSION EQUATION FOR REGIONS RA AND RS
   50 U(3) = (A+.5DO)*U(1)+X*U(2)
      U(1) = U(2)
      U(2) = U(3)
      A = A-1.DO
      DEBUG A,U(2)
      IF (A-ASAVE) 60,60,50
   60 UP = -.5DO*X*U(2) - (A+.5DO)*U(1)
      V = U(2)
      RETURN
```

```
CCC BLOCK 6 - ASYMPTOTIC I IN REGION AI
  200 IF (A.LT.-50.D0) IFLAG = 2
      A = A+1.D0
      DO 240 M=1,2
      ABSA = DABS(A)
      CHI = DSQRT(4.D0*ABSA-X*X)
      CHID = 1.D0/CHI**3
      CALL DFIND(X,A,D3,D6,D9,D12)
      VRP = CHID*CHID*(-D6+D12*CHID*CHID)
      VI = CHID*(D3-D9*CHID*CHID)
      THETA = .25D0*X*CHI+ABSA*DATAN(X/CHI)
      ANGLE = (THETA+VI+PI*(.25D0+.5D0*A))
      ANGLE = DMOD(ANGLE,PI2)
      ARG = .5D0-A
      GO TO (205,210), IFLAG
  205 CALL STRGAM(ARG,GAM)
      U(M) = 2.D0*GAM*DEXP(VRP)/DSQRT(CHI)*DCOS(ANGLE)
      GO TO 240
  210 CALL STRSCL (ARG,GAM,S)
      IS(M) = S
      IF (M-2) 220,230,220
  220 U(M) = 2.D0*GAM*DEXP(VRP)/DSQRT(CHI)*DCOS(ANGLE)
      GO TO 240
  230 N = IS(2)-IS(1)
      U(M) = 2.D0*GAM*10.D0**N*DEXP(VRP)/DSQRT(CHI)*DCOS(ANGLE)
      ISI = IS(1)
  240 A = A-1.D0
      A = A+1.D0
      IF (A.EQ.ASAVE) GO TO 260
C  FOREWARD RECURSION FOR REGION RAI
      A = A+1.D0
  245 U(3) = (U(2) - X*U(1))/(A+.5D0)
      A = A+1.D0
      IF (A.EQ.ASAVE) GO TO 250
      U(2) = U(1)
      U(1) = U(3)
      GO TO 245
  250 UP = .5D0*X*U(3) - U(1)
      V = U(3)
      RETURN
  260 UP = -.5D0*X*U(2)-(A+.5D0)*U(1)
      V = U(2)
      RETURN
      END
```

```
      SUBROUTINE SERIES (X,A,ANS)
C  COMPUTES U(A,X) BY EVALUATING SERIES Y1 AND Y2
      DIMENSION CZERO(1),C(1000)
      DOUBLE PRECISION A,ARG,ANS,C,CZERO,CYONE,CYTWO,ERROR,FN,GAM
     * , PI2,TWOA,TWO4,X,X2,YONE,YTWO,CARG,SONE,STWO
      PI2 = 1.772453850905516
      TWO4 = 1.189207115D0
      ERROR = 1.D-10
      SONE = 0.D0
      STWO = 0.D0
      X2 = X*X
      CZERO = 1.D0
      C(1) = X
      C(2) = A*X2/2.D0
      C(3) = A*X2*X/6.D0
      YONE = CZERO+C(2)
      YTWO = C(1)+C(3)
      DO 100 NT = 2,900,2
      FN = DBLE(FLOAT(NT))
      C(NT+2) = X2/((FN+1.D0)*(FN+2.D0))*(A*C(NT)+.25D0*X2*C(NT-2))
      C(NT+3) = X2/((FN+2.D0)*(FN+3.D0))*(A*C(NT+1)+.25D0*X2*C(NT-1))
      YONE = YONE+C(NT+2)
      YTWO = YTWO+C(NT+3)
      IF (DABS(YONE-SONE) - ERROR) 99,99,101
   99 IF (DABS(YTWO-STWO) - ERROR) 110,110,101
```

```
101 SONE = YONE
    STWO = YTWO
100 CONTINUE
110 ARG = .5D0*A
    TWOA = 2.D0**ARG
    ARG = .75D0+.5D0*A
    CARG = DBLE(FLOAT(IDINT(ARG)))
    IF (ARG-CARG) 130,120,130
120 CYONE = 0.D0
    GO TO 140
130 CALL GAMMAN(ARG,GAM)
    CYONE = PI2*YONE/(TWO4*TWOA*GAM)
140 ARG = .25D0+.5D0*A
    CARG = DBLE(FLOAT(IDINT(ARG)))
    IF (ARG-CARG) 160,150,160
150 CYTWO = 0.D0
    GO TO 170
160 CALL GAMMAN (ARG,GAM)
    CYTWO = PI2*TWO4*YTWO/(TWOA*GAM)
170 ANS = CYONE-CYTWO
    RETURN
    END
```

```
      SUBROUTINE DFIND(X,A,D3,D6,D9,D12)
      COMMON CA3, CB2,CC9,CC7,CC5,CC3,CC1,CD4,CD2,CD0
      DOUBLE PRECISION X,A,D3,D6,D9,D12,A2,X2,CA3,CB2,CC9,CC7,CC5,CC3
     * , CC1,CD4,CD2,CD0
      A2 = A*A
      X2 = X*X
      D3 = .5D0*X*(CA3*X2+A)/A
      D6 = CB2*X2-2.D0*A
      D9 = ((((-CC9*X2-CC7*A)*X2-CC5*A2)*X2+CC3*A2*A)*X2- CC1 *A2*A2)*X
     * /(A2*A)
      D12 = X2*(CD4*X2-CD2*A)+CD0*A2
      RETURN
      END
```

```
      DOUBLE PRECISION FUNCTION GAMMA(X)
      DOUBLE PRECISION Z,C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,
     A C15,C16,C17,C18,C19,C20,C21,C22,C23,C24,C25,C26,RECIPR,X
      DATA C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15,C16,C17,
     A C18,C19,C20,C21,C22,C23,C24,C25,C26     /1.000000000000000000D0,
     A 0.5772156649015329D0,-0.6558780715202538D0,-0.0420026350340952D0,
     B 0.1665386113822915D0,-0.0421977345555443D0,-0.0096219715278770D0,
     C 0.0072189432466630D0,-0.0011651675918591D0,-.0002152416741149D0,
     D 0.0001280502823882D0,-0.0000201348547807D0,-0.0000012504934821D0,
     E 0.0000011330272320D0,-0.0000002056338417D0,0.0000000061160950D0,
     F 0.0000000050020075D0,-0.0000000011812746D0,0.0000000001043427D0,
     G 0.0000000000077823D0,-0.0000000000036968D0,0.0000000000005100D0,
     H -0.0000000000000206D0,-0.0000000000000054D0,0.0000000000000014D0,
     I 0.0000000000000001D0/
      Z = X
      IF (Z.LT.1.D0.OR.Z.GT.2.D0) STOP
      RECIPR = Z*(C1+Z*(C2+Z*(C3+Z*(C4+Z*(C5+Z*(C6+Z*(C7+Z*(C8+Z*(C9+
     A         Z*(C10+Z*(C11+Z*(C12+Z*(C13+Z*(C14+Z*(C15+Z*(C16+Z*(C17+
     B         Z*(C18+Z*(C19+Z*(C20+Z*(C21+Z*(C22+Z*(C23+Z*(C24+Z*(C25+
     C         Z*C26)))))))))))))))))))))))))
      GAMMA = 1.0D0/RECIPR
      RETURN
      END
```

```
      SUBROUTINE GAMMAF (X,GAM)
C   FOR POSITIVE VALUES OF X GREATER OR EQUAL TO 1.0
      DOUBLE PRECISION X,Z,ARG,GAM,GAMMA
      Z = X
      ARG = Z
   10 IF (ARG-2.D0) 20,20,15
   15 ARG = ARG-1.D0
      GO TO 10
   20 GAM = GAMMA(ARG)
   30 IF (ARG-Z) 35,40,35
   35 GAM = GAM*ARG
      ARG = ARG+1.D0
      GO TO 30
   40 RETURN
      END
```

```
      SUBROUTINE GAMMAN (X,GAM)
C   FOR NON-INTEGRAL VALUES OF Z RANGE (-33.0,1.0)
      DOUBLE PRECISION X,Z,ARG,GAM,COEF,GAMMA
      Z = X
      IF (Z) 10,5,5
    5 ARG = Z+1.D0
      GAM = GAMMA(ARG)/Z
      RETURN
   10 COEF = 1.D0
   20 COEF = Z*COEF
      Z = Z+1.D0
      IF (Z-1.D0) 20,20,30
   30 GAM = GAMMA(Z)/COEF
      RETURN
      END
```

```
      SUBROUTINE STRGAM (Z,ANS)
C   COMPUTES SQRT(GAMMA(Z))/(2.*PI)**.25
      DOUBLE PRECISION Z,ANS,A,B,C,D,E,SERIES,Z1,POW
      DATA A,B,C,D/83333333333333333.D-18,34722222222222222.D-19,
     A 26813271604938272.D-19,22947209362139918.D-20/
      DATA E /2.7182818284590452/
      POW = Z/2.0D0
      Z1 = 1.0D0/Z
      SERIES = 1.0D0+Z1*(A+Z1*(B-Z1*(C+Z1*D)))
   80 ANS = (Z/E)**POW/Z**.25D0*DSQRT(SERIES)
      RETURN
      END
```

```
      SUBROUTINE STRSCL (Z,R,S)
C   COMPUTES SQRT(GAMMA(Z))/(2.*PI)**.25 WITH SCALING FACTOR S AS
C   A POWER OF 10.0
      DOUBLE PRECISION Z,Z1,A,B,C,D,E,SERIES,S,P,ZE,R
      DATA A,B,C,D/83333333333333333.D-18,34722222222222222.D-19,
     * 26813271604938272.D-19,22947209362139918.D-20/
      DATA E /2.7182818284590452/
      Z1 = 1.0D0/Z
      SERIES = 1.0D0+Z1*(A+Z1*(B-Z1*(C+Z1*D)))
      S = 0.D0
      ZE = DLOG10(Z/E)
      P = Z/2.D0*ZE
   10 S = S+1.D0
      P = P-1.D0
      IF (P-1.D0) 20,10,10
   20 R = 10.D0**P*DSQRT(SERIES)/Z**.25D0
      RETURN
      END
```

APPENDIX C

TABLES OF THE WEBER FUNCTION AND ITS FIRST DERIVATIVE

The following tables list $D_\nu(x)$ (U(a, x) column heading) and $(d/dx)D_\nu(x)$ (U'(a, x) column heading) at intervals of 1 in x and 5 in ν. The true exponent is obtained by adding the number at the top of each exponent column to the exponent of each individual entry.

X	U(-5.0,X)		U'(-5.0,X)	
		0		0
0.	3.0521837	(0)	6.8415762	(0)
1.0	5.7992601	(-1)	-9.4555887	(0)
2.0	-4.1865842	(0)	3.3663954	(0)
3.0	3.2021291	(0)	6.8170127	(0)
4.0	5.0331933	(0)	-2.3677349	(0)
5.0	1.8799768	(0)	-2.6935859	(0)
POINT IS IN BAD REGION				
POINT IS IN BAD REGION				
8.0	1.1456761	(-3)	-3.8993322	(-3)
9.0	2.8559484	(-5)	-1.1357113	(-4)
10.0	4.0491466	(-7)	-1.8355745	(-6)
11.0	3.3102931	(-9)	-1.6811175	(-8)
12.0	1.5756262	(-11)	-8.8479091	(-11)
13.0	4.3974120	(-14)	-2.7028308	(-13)
14.0	7.2347562	(-17)	-4.8274995	(-16)
15.0	7.0459403	(-20)	-5.0696986	(-19)
16.0	4.0754359	(-23)	-3.1441223	(-22)
17.0	1.4037342	(-26)	-1.1555568	(-25)
18.0	2.8854817	(-30)	-2.5240025	(-29)
19.0	3.5461783	(-34)	-3.2840529	(-33)
20.0	2.6095865	(-38)	-2.6095865	(-37)
21.0	0.	(0)	-0.	(0)

X	U(-10.0,X)		U'(-10.0,X)	
		2		3
0.	-3.7799709	(0)	1.1960745	(0)
1.0	3.7675627	(0)	-1.1949964	(0)
2.0	-4.1101075	(0)	1.0804553	(0)
3.0	5.0326142	(0)	-7.1888978	(-1)
4.0	-6.0616342	(0)	-7.8046901	(-2)
5.0	4.0529365	(0)	1.1308804	(0)
6.0	6.5793091	(0)	-4.3712358	(-1)
POINT IS IN BAD REGION				
8.0	2.0577253	(-1)	-5.3459455	(-2)
9.0	1.0686247	(-2)	-3.5316828	(-3)
10.0	2.8258753	(-4)	-1.1171334	(-4)
11.0	3.9826571	(-6)	-1.8185412	(-6)
12.0	3.0790628	(-8)	-1.5874445	(-8)
13.0	1.3318242	(-10)	-7.6294212	(-11)
14.0	3.2690072	(-13)	-2.0559890	(-13)
15.0	4.6018429	(-16)	-3.1480659	(-16)
16.0	3.7458023	(-19)	-2.7663513	(-19)
17.0	1.7744241	(-22)	-1.4060121	(-22)
18.0	4.9172834	(-26)	-4.1588659	(-26)
19.0	8.0055994	(-30)	-7.1951735	(-30)
20.0	7.6841260	(-34)	-7.3110480	(-34)
21.0	4.3612678	(-38)	-4.5793312	(-38)
22.0	0.	(0)	-0.	(0)

X	U(-15.0,X)		U'(-15.0,X)	
		5		5
0.	-1.8569056	(0)	-7.1937571	(0)
1.0	2.6317424	(0)	6.6710645	(-1)
2.0	-2.2158064	(0)	5.5473151	(0)
3.0	1.2027618	(0)	-8.7388111	(0)
4.0	-4.0493298	(-1)	9.3030182	(0)
5.0	3.9777795	(-1)	-8.7929832	(0)
6.0	-1.7039120	(0)	6.7417567	(0)
7.0	3.9126172	(0)	8.8378327	(-1)
8.0	1.8565878	(0)	-2.7401312	(0)
9.0	2.5128416	(-1)	-6.2181926	(-1)
10.0	1.4010081	(-2)	-4.5933533	(-2)
11.0	3.6757464	(-4)	-1.4671970	(-3)
12.0	4.8632681	(-6)	-2.2624029	(-5)
13.0	3.3847003	(-8)	-1.7866440	(-7)
14.0	1.2746504	(-10)	-7.4970285	(-10)
15.0	2.6501545	(-13)	-1.7139929	(-12)
16.0	3.0877971	(-16)	-2.1739371	(-15)
17.0	2.0394185	(-19)	-1.5506080	(-18)
18.0	7.7049507	(-23)	-6.2856161	(-22)
19.0	1.6772477	(-26)	-1.4602201	(-25)
20.0	2.1162772	(-30)	-1.9573032	(-29)
21.0	1.5553965	(-34)	-1.5222734	(-33)
22.0	6.6866087	(-39)	-6.9015622	(-38)
23.0	1.6873243	(-43)	-1.8311915	(-42)
24.0	2.5068728	(-48)	-2.8530711	(-47)
25.0	2.1985814	(-53)	-2.6178340	(-52)
26.0	1.1408229	(-58)	-1.4181251	(-57)
27.0	3.5093471	(-64)	-4.5455334	(-63)
28.0	6.4110844	(-70)	-8.6376010	(-69)
29.0	6.9664549	(-76)	-9.7472657	(-75)
30.0	4.5089107	(-82)	-6.5420684	(-81)

X	U(-20.0,X)		U'(-20.0,X)	
		8		8
0.	2.1958943	(0)	-9.8218701	(0)
1.0	1.5923444	(0)	1.1911176	(1)
2.0	-3.0617271	(0)	3.1032601	(0)
3.0	6.0073639	(-1)	-1.3230133	(1)
4.0	2.3168492	(0)	9.3812065	(0)
5.0	-3.4021392	(0)	-9.8485444	(-1)
6.0	3.4226960	(0)	-3.5141685	(0)
7.0	-3.8053587	(0)	2.2964628	(0)
8.0	4.1397769	(0)	4.9263846	(0)
9.0	3.0034166	(0)	-3.8819360	(0)
10.0	4.4734779	(-1)	-1.0933758	(0)
11.0	2.4655423	(-2)	-8.1996025	(-2)
12.0	6.0068592	(-4)	-2.4566335	(-3)
13.0	7.0638259	(-6)	-3.3821269	(-5)
14.0	4.2257338	(-8)	-2.3006107	(-7)
15.0	1.3315317	(-10)	-8.0847007	(-10)
16.0	2.2651354	(-13)	-1.5126931	(-12)
17.0	2.1187084	(-16)	-1.5400234	(-15)
18.0	1.1050414	(-19)	-8.6710802	(-19)
19.0	3.2495421	(-23)	-2.7345237	(-22)
20.0	5.4359105	(-27)	-4.8789141	(-26)
21.0	5.2107553	(-31)	-4.9652966	(-30)
22.0	2.8796842	(-35)	-2.9018525	(-34)
23.0	9.2220098	(-40)	-9.7940708	(-39)
24.0	1.7188296	(-44)	-1.9181515	(-43)
25.0	1.8715017	(-49)	-2.1888112	(-48)
26.0	1.1942745	(-54)	-1.4603953	(-53)
27.0	4.4792135	(-60)	-5.7147984	(-59)
28.0	9.8983054	(-66)	-1.3151223	(-64)
29.0	1.2916168	(-71)	-1.7839975	(-70)
30.0	9.9716315	(-78)	-1.4295406	(-76)

X	U(-25.0,X)	U'(-25.0,X)
	11	12
0.	4.9624999 (0)	2.4814978 (0)
1.0	-3.4077701 (0)	3.0608274 (0)
2.0	-6.8212472 (0)	-9.5437977 (-1)
3.0	1.0666866 (0)	-3.3879421 (0)
4.0	7.2127171 (0)	6.1653653 (-1)
5.0	-3.1322989 (0)	2.9606545 (0)
6.0	-4.4556876 (0)	-2.6048033 (0)
7.0	7.9038132 (0)	9.5945013 (-1)
8.0	-8.9489181 (0)	-4.8909757 (-1)
9.0	8.4193877 (0)	1.5803454 (0)
10.0	7.5532734 (0)	-9.4899257 (-1)
11.0	1.1015971 (0)	-2.7629839 (-1)
12.0	5.5346743 (-2)	-1.9054864 (-2)
13.0	1.1764943 (-3)	-4.9924685 (-4)
14.0	1.1690888 (-5)	-5.8102166 (-6)
15.0	5.7623740 (-8)	-3.2551544 (-8)
16.0	1.4651922 (-10)	-9.2241396 (-11)
17.0	1.9760996 (-13)	-1.3671270 (-13)
18.0	1.4431875 (-16)	-1.0857263 (-16)
19.0	5.7988961 (-20)	-4.7051433 (-20)
20.0	1.2981864 (-23)	-1.1285611 (-23)
21.0	1.6357127 (-27)	-1.5152768 (-27)
22.0	1.1697092 (-31)	-1.1494105 (-31)
23.0	4.7804623 (-36)	-4.9634815 (-36)
24.0	1.1231359 (-40)	-1.2280173 (-40)
25.0	1.5245440 (-45)	-1.7502021 (-45)
26.0	1.2007856 (-50)	-1.4436445 (-50)
27.0	5.5085137 (-56)	-6.9194309 (-56)
28.0	1.4766186 (-61)	-1.9339440 (-61)
29.0	2.3196273 (-67)	-3.1616939 (-67)
30.0	2.1408761 (-73)	-3.0316617 (-73)

X	U(-30.0,X)	U'(-30.0,X)
	15	16
0.	-1.8741512 (0)	1.0265861 (0)
1.0	-2.6549715 (0)	-4.1377011 (-2)
2.0	-1.6888622 (0)	-1.1172806 (0)
3.0	7.1614698 (-1)	-1.3719444 (0)
4.0	2.6877589 (0)	-2.8397018 (-1)
5.0	1.3161185 (0)	1.2134438 (0)
6.0	-2.5285032 (0)	6.3943835 (-1)
7.0	-6.1379063 (-1)	-1.2500496 (0)
8.0	2.7854999 (0)	6.1357149 (-1)
9.0	-3.4310635 (0)	-2.6135907 (-1)
10.0	3.3641100 (0)	6.1121024 (-1)
11.0	2.7673977 (0)	-3.7889651 (-1)
12.0	3.5327418 (-1)	-9.3959173 (-2)
13.0	1.5025105 (-2)	-5.4441360 (-3)
14.0	2.6344511 (-4)	-1.1716193 (-4)
15.0	2.1140856 (-6)	-1.0978477 (-6)
16.0	8.2658277 (-9)	-4.8674681 (-9)
17.0	1.6417056 (-11)	-1.0752479 (-11)
18.0	1.7063306 (-14)	-1.2260073 (-14)
19.0	9.4893221 (-18)	-7.4027610 (-18)
20.0	2.8725435 (-21)	-2.4135242 (-21)
21.0	4.7981538 (-25)	-4.3138946 (-25)
22.0	4.4714994 (-29)	-4.2789777 (-29)
23.0	2.3461594 (-33)	-2.3789724 (-33)
24.0	6.9838371 (-38)	-7.4749995 (-38)
25.0	1.1870327 (-42)	-1.3366895 (-42)
26.0	1.1583997 (-47)	-1.3684327 (-47)
27.0	6.5215251 (-53)	-8.0612992 (-53)
28.0	2.1268380 (-58)	-2.7447113 (-58)
29.0	4.0326576 (-64)	-5.4222241 (-64)
30.0	4.4597567 (-70)	-6.2362686 (-70)

X	U(-35.0,X)	U'(-35.0,X)
	19	19
0.	-1.0833050 (0)	-6.4092458 (0)
1.0	-6.1395058 (-1)	-8.2945639 (0)
2.0	7.3659544 (-3)	-8.9984686 (0)
3.0	7.5629253 (-1)	-7.7848532 (0)
4.0	1.4351793 (0)	-3.6455996 (0)
5.0	1.4926757 (0)	3.2569963 (0)
6.0	2.3177205 (-1)	8.3386467 (0)
7.0	-1.5905633 (0)	2.8825511 (0)
8.0	-4.0913528 (-1)	-7.5962487 (0)
9.0	1.7734731 (0)	2.7592266 (0)
10.0	-2.0908619 (0)	-1.4002044 (-1)
11.0	2.3048476 (0)	2.6729187 (0)
12.0	1.3627638 (0)	-2.1892107 (0)
13.0	1.3848458 (-1)	-3.9966021 (-1)
14.0	4.6781050 (-3)	-1.8051870 (-2)
15.0	6.4433721 (-5)	-3.0250016 (-4)
16.0	4.0129146 (-7)	-2.1880155 (-6)
17.0	1.2036196 (-9)	-7.4134721 (-9)
18.0	1.8141595 (-12)	-1.2391771 (-11)
19.0	1.4168036 (-15)	-1.0591406 (-14)
20.0	5.8666711 (-19)	-4.7522310 (-18)
21.0	1.3112900 (-22)	-1.1420431 (-21)
22.0	1.6048534 (-26)	-1.4933805 (-25)
23.0	1.0880621 (-30)	-1.0761968 (-29)
24.0	4.1260692 (-35)	-4.3190473 (-34)
25.0	8.8225764 (-40)	-9.7375023 (-39)
26.0	1.0710593 (-44)	-1.2424293 (-43)
27.0	7.4259299 (-50)	-9.0280807 (-49)
28.0	2.9554518 (-55)	-3.7564544 (-54)
29.0	6.7821395 (-61)	-8.9923259 (-60)
30.0	9.0090456 (-67)	-1.2435851 (-65)

X	U(-40.0,X)	U'(-40.0,X)
	22	23
0.	9.0065926 (0)	-5.6964918 (0)
1.0	8.6992992 (0)	-5.8805156 (0)
2.0	8.7845715 (0)	-5.8240818 (0)
3.0	9.6220399 (0)	-5.2909337 (0)
4.0	1.1323639 (1)	-3.9091195 (0)
5.0	1.3140763 (1)	-1.1281611 (0)
6.0	1.2283640 (1)	3.2472417 (0)
7.0	3.7451150 (0)	7.0948651 (0)
8.0	-1.1518383 (1)	4.2444201 (0)
9.0	-8.8182141 (0)	-5.5502557 (0)
10.0	1.6212810 (1)	-3.7468258 (-1)
11.0	-1.6155325 (1)	2.3276019 (0)
12.0	2.2109229 (1)	7.7670202 (-1)
13.0	8.2941046 (0)	-1.6039685 (0)
14.0	6.2376245 (-1)	-1.9788606 (-1)
15.0	1.5881339 (-2)	-6.5751489 (-3)
16.0	1.6487175 (-4)	-8.2098670 (-5)
17.0	7.7008089 (-7)	-4.4228202 (-7)
18.0	1.7208765 (-9)	-1.1111879 (-9)
19.0	1.9190248 (-12)	-1.3692999 (-12)
20.0	1.1011391 (-15)	-8.5748106 (-16)
21.0	3.3276742 (-19)	-2.8014325 (-19)
22.0	5.3937824 (-23)	-4.8725922 (-23)
23.0	4.7583361 (-27)	-4.5849766 (-27)
24.0	2.3122123 (-31)	-2.3646398 (-31)
25.0	6.2508326 (-36)	-6.7563342 (-36)
26.0	9.4805360 (-41)	-1.0791613 (-40)
27.0	8.1249849 (-46)	-9.7097681 (-46)
28.0	3.9589921 (-51)	-4.9536349 (-51)
29.0	1.1026676 (-56)	-1.4411008 (-56)
30.0	1.7637519 (-62)	-2.4025282 (-62)

X	U(-45.0,X)	U'(-45.0,X)
	27	27
0.	1.0283592 (0)	6.8986560 (0)
1.0	1.3596847 (0)	3.4940344 (0)
2.0	1.4623417 (0)	-1.3923169 (-1)
3.0	1.3976819 (0)	-3.0298377 (0)
4.0	1.2972201 (0)	-4.6591374 (0)
5.0	1.2890117 (0)	-4.8716425 (0)
6.0	1.4272773 (0)	-3.4039985 (0)
7.0	1.5735181 (0)	3.6004787 (-1)
8.0	1.1783641 (0)	6.0521923 (0)
9.0	-4.7099249 (-1)	8.0480465 (0)
10.0	-1.7205196 (0)	-2.1600330 (0)
11.0	1.3766985 (0)	-5.0241227 (0)
12.0	-1.0102552 (0)	5.6317064 (0)
13.0	2.5443538 (0)	-1.0507478 (0)
14.0	5.7741230 (-1)	-1.3400708 (0)
15.0	3.0481711 (-2)	-1.0685702 (-1)
16.0	5.6097215 (-4)	-2.5014861 (-3)
17.0	4.2410890 (-6)	-2.2460139 (-5)
18.0	1.4429446 (-8)	-8.7460197 (-8)
19.0	2.3425040 (-11)	-1.5878690 (-10)
20.0	1.8901902 (-14)	-1.4102948 (-13)
21.0	7.8126687 (-18)	-6.3420152 (-17)
22.0	1.6927233 (-21)	-1.4817603 (-20)
23.0	1.9578555 (-25)	-1.8351981 (-24)
24.0	1.2268165 (-29)	-1.2243644 (-28)
25.0	4.2154914 (-34)	-4.4580800 (-33)
26.0	8.0239375 (-39)	-8.9560226 (-38)
27.0	8.5334324 (-44)	-1.0018143 (-42)
28.0	5.1079458 (-49)	-6.2885502 (-48)
29.0	1.7318531 (-54)	-2.2300783 (-53)
30.0	3.3444069 (-60)	-4.4939419 (-59)

X	U(-50.0,X)	U'(-50.0,X)
	31	32
0.	-1.5558194 (0)	1.1001579 (0)
1.0	-7.1261448 (-3)	1.5539003 (0)
2.0	1.4967401 (0)	1.1403240 (0)
3.0	2.2000074 (0)	2.3487673 (-1)
4.0	2.0564914 (0)	-6.1121667 (-1)
5.0	1.5298513 (0)	-1.1115303 (0)
6.0	1.1244797 (0)	-1.2916277 (0)
7.0	1.1580953 (0)	-1.2610801 (0)
8.0	1.7432210 (0)	-9.7615173 (-1)
9.0	2.4826110 (0)	-1.7295799 (-1)
10.0	1.5982669 (0)	1.0439183 (0)
11.0	-2.1587107 (0)	7.6012257 (-1)
12.0	-2.4195505 (-1)	-1.1319531 (0)
13.0	8.5763234 (-1)	9.6887016 (-1)
14.0	3.1791695 (0)	-3.5233672 (-1)
15.0	4.3000416 (-1)	-1.1805804 (-1)
16.0	1.5350546 (-2)	-5.9471125 (-3)
17.0	1.9760228 (-4)	-9.5020409 (-5)
18.0	1.0575711 (-6)	-5.9630836 (-7)
19.0	2.5571809 (-9)	-1.6371635 (-9)
20.0	2.9510315 (-12)	-2.1012482 (-12)
21.0	1.6899844 (-15)	-1.3190652 (-15)
22.0	4.9447371 (-19)	-4.1854970 (-19)
23.0	7.5606650 (-23)	-6.8831422 (-23)
24.0	6.1509286 (-27)	-5.9830596 (-27)
25.0	2.7017083 (-31)	-2.7927626 (-31)
26.0	6.4850633 (-36)	-7.0920015 (-36)
27.0	8.5938532 (-41)	-9.9047772 (-41)
28.0	6.3418883 (-46)	-7.6780835 (-46)
29.0	2.6256486 (-51)	-3.3297268 (-51)
30.0	6.1382324 (-57)	-8.1332351 (-57)

X	U(-55.0,X)	U'(-55.0,X)
	35	36
0.	-3.0312102 (0)	-2.2480520 (0)
1.0	-4.0429030 (0)	1.0645264 (0)
2.0	-5.7928758 (-1)	3.1356477 (0)
3.0	3.3679785 (0)	1.9809633 (0)
4.0	4.2410118 (0)	-7.4390292 (-1)
5.0	2.2964008 (0)	-2.6323972 (0)
6.0	-2.5187638 (-1)	-3.0359684 (0)
7.0	-1.9345973 (0)	-2.7078851 (0)
8.0	-2.4452225 (0)	-2.4921353 (0)
9.0	-1.6188034 (0)	-2.6743404 (0)
10.0	1.2173437 (0)	-2.6447533 (0)
11.0	5.0338852 (0)	-6.8354977 (-1)
12.0	1.3577904 (0)	2.3761128 (0)
13.0	-5.4801198 (0)	-1.0825909 (0)
14.0	6.3250470 (0)	1.1112163 (0)
15.0	4.0287138 (0)	-7.0511566 (-1)
16.0	3.2380931 (-1)	-1.0342298 (-1)
17.0	7.6144602 (-3)	-3.2508981 (-3)
18.0	6.6823465 (-5)	-3.4632084 (-5)
19.0	2.4737526 (-7)	-1.4849999 (-7)
20.0	4.1636350 (-10)	-2.8158062 (-10)
21.0	3.3525015 (-13)	-2.5076570 (-13)
22.0	1.3398144 (-16)	-1.0939984 (-16)
23.0	2.7328794 (-20)	-2.4120719 (-20)
24.0	2.9079545 (-24)	-2.7530961 (-24)
25.0	1.6427406 (-28)	-1.6580177 (-28)
26.0	4.9981788 (-33)	-5.3507752 (-33)
27.0	8.2893465 (-38)	-9.3727024 (-38)
28.0	7.5698932 (-43)	-9.0074713 (-43)
29.0	3.8395049 (-48)	-4.7929342 (-48)
30.0	1.0897249 (-53)	-1.4232242 (-53)

X	U(-60.0,X)	U'(-60.0,X)
	39	40
0.	7.4303740 (0)	-5.7556429 (0)
1.0	-6.5498150 (0)	-6.3639948 (0)
2.0	-9.1232682 (0)	4.0692888 (0)
3.0	3.2940353 (0)	7.6659752 (0)
4.0	1.0690721 (1)	-3.9544761 (-2)
5.0	4.5733024 (0)	-7.1687600 (0)
6.0	-5.4335510 (0)	-6.7923079 (0)
7.0	-1.0612679 (1)	-2.3264053 (0)
8.0	-1.1154765 (1)	1.3833340 (0)
9.0	-1.0820338 (1)	2.6867304 (0)
10.0	-1.1752947 (1)	1.4578823 (0)
11.0	-1.1638556 (1)	-2.5728723 (0)
12.0	-1.2848317 (0)	-6.4521703 (0)
13.0	1.4226375 (1)	3.8141584 (-1)
14.0	-1.3080283 (1)	2.8802921 (0)
15.0	1.9781022 (1)	-2.0066543 (-1)
16.0	4.9206260 (0)	-1.1590396 (0)
17.0	2.3523240 (-1)	-8.6038326 (-2)
18.0	3.5767580 (-3)	-1.6756152 (-3)
19.0	2.0973628 (-5)	-1.1695436 (-5)
20.0	5.2694353 (-8)	-3.3649679 (-8)
21.0	6.0667558 (-11)	-4.3317877 (-11)
22.0	3.3542165 (-14)	-2.6346791 (-14)
23.0	9.2187147 (-18)	-7.8722633 (-18)
24.0	1.2933471 (-21)	-1.1899574 (-21)
25.0	9.4589336 (-26)	-9.3104064 (-26)
26.0	3.6680642 (-30)	-3.8404512 (-30)
27.0	7.6488262 (-35)	-8.4780775 (-35)
28.0	8.6780961 (-40)	-1.0142557 (-39)
29.0	5.4108177 (-45)	-6.6454003 (-45)
30.0	1.8699605 (-50)	-2.4062445 (-50)

X	U(-65.0,X)		U'(-65.0,X)	
		44		45
0.	2.2477224 (	0)	1.8121986 (	0)
1.0	1.7494952 (	0)	-2.1382603 (	0)
2.0	-2.9265717 (	0)	-1.0188632 (	0)
3.0	-9.4912874 (	-1)	2.4259866 (	0)
4.0	3.0987539 (	0)	7.1333867 (	-1)
5.0	1.2665507 (	0)	-2.3012474 (	0)
6.0	-2.5746434 (	0)	-1.5475418 (	0)
7.0	-3.0608147 (	0)	9.8222766 (	-1)
8.0	-1.0099566 (	0)	2.2789620 (	0)
9.0	9.1606771 (	-1)	2.2549689 (	0)
10.0	1.7045736 (	0)	2.0029318 (	0)
11.0	1.1981359 (	0)	2.0793799 (	0)
12.0	-1.1721669 (	0)	1.9913812 (	0)
13.0	-4.1208606 (	0)	1.1150825 (	-1)
14.0	1.0495022 (	0)	-1.7460099 (	0)
15.0	7.8732734 (	-1)	1.5592748 (	0)
16.0	4.7333598 (	0)	-5.6083162 (	-1)
17.0	5.5622474 (	-1)	-1.6329067 (	-1)
18.0	1.5846853 (	-2)	-6.5464455 (	-3)
19.0	1.5374162 (	-4)	-7.8645691 (	-5)
20.0	5.9294097 (	-7)	-3.5492062 (	-7)
21.0	9.9539819 (	-10)	-6.7526054 (	-10)
22.0	7.7244776 (	-13)	-5.8179122 (	-13)
23.0	2.8925367 (	-16)	-2.3842973 (	-16)
24.0	5.3978875 (	-20)	-4.8180784 (	-20)
25.0	5.1474808 (	-24)	-4.9346336 (	-24)
26.0	2.5591716 (	-28)	-2.6178051 (	-28)
27.0	6.7429665 (	-33)	-7.3207365 (	-33)
28.0	9.5446867 (	-38)	-1.0949786 (	-37)
29.0	7.3420285 (	-43)	-8.8668468 (	-43)
30.0	3.0993297 (	-48)	-3.9276166 (	-48)

X	U(-70.0,X)		U'(-70.0,X)	
		48		49
0.	-8.2550175 (	0)	6.9067312 (	0)
1.0	1.1237285 (	1)	2.6769304 (	0)
2.0	-2.3291853 (	0)	-9.5391145 (	0)
3.0	-9.5953097 (	0)	5.6054814 (	0)
4.0	9.5954323 (	0)	5.6526650 (	0)
5.0	4.9124843 (	0)	-8.6936119 (	0)
6.0	-1.1233594 (	1)	-3.4895772 (	0)
7.0	-6.1182044 (	0)	8.0554454 (	0)
8.0	7.7497735 (	0)	7.1811961 (	0)
9.0	1.2711297 (	1)	-1.7410197 (	-1)
10.0	1.0572020 (	1)	-5.0889205 (	0)
11.0	8.8319100 (	0)	-6.3638855 (	0)
12.0	1.1162106 (	1)	-4.8625917 (	0)
13.0	1.4665924 (	1)	6.8482406 (	-1)
14.0	2.3655702 (	0)	7.1699440 (	0)
15.0	-1.6201410 (	1)	-2.6846520 (	0)
16.0	1.9820932 (	1)	2.5065236 (	0)
17.0	9.3224490 (	0)	-1.8839025 (	0)
18.0	5.6275266 (	-1)	-1.9688247 (	-1)
19.0	9.5756634 (	-3)	-4.4155179 (	-3)
20.0	5.8692558 (	-5)	-3.2621210 (	-5)
21.0	1.4702940 (	-7)	-9.4218634 (	-8)
22.0	1.6281892 (	-10)	-1.1714066 (	-10)
23.0	8.4110583 (	-14)	-6.6746116 (	-14)
24.0	2.1081710 (	-17)	-1.8219858 (	-17)
25.0	2.6417330 (	-21)	-2.4629008 (	-21)
26.0	1.6945574 (	-25)	-1.6915917 (	-25)
27.0	5.6714013 (	-30)	-6.0257041 (	-30)
28.0	1.0060330 (	-34)	-1.1320511 (	-34)
29.0	9.5836275 (	-40)	-1.1374293 (	-39)
30.0	4.9577462 (	-45)	-6.1842970 (	-45)

X	U(-75.0,X)		U'(-75.0,X)	
		53		54
0.	-3.6296600 (	0)	-3.1434126 (	0)
1.0	8.3002517 (	-2)	4.4411821 (	0)
2.0	3.6295400 (	0)	-3.1422187 (	0)
3.0	-5.1605433 (	0)	-3.0054021 (	-1)
4.0	2.7593023 (	0)	3.7197407 (	0)
5.0	2.5345821 (	0)	-3.8061259 (	0)
6.0	-5.2422722 (	0)	-6.3856767 (	-1)
7.0	1.3156587 (	-1)	4.2505531 (	0)
8.0	5.4487531 (	0)	1.1243811 (	-1)
9.0	1.8230922 (	0)	-3.8776686 (	0)
10.0	-3.3097519 (	0)	-3.2730158 (	0)
11.0	-5.4932781 (	0)	-1.3435583 (	0)
12.0	-5.9710953 (	0)	-6.0911416 (	-1)
13.0	-5.6939296 (	0)	-1.5899248 (	0)
14.0	-1.7415699 (	0)	-3.3059393 (	0)
15.0	6.9622246 (	0)	-8.1427246 (	-1)
16.0	-4.4199582 (	0)	2.2558730 (	0)
17.0	9.4437430 (	0)	-6.1622412 (	-1)
18.0	1.5254898 (	0)	-4.1853375 (	-1)
19.0	4.9524848 (	-2)	-2.0052589 (	-2)
20.0	5.0437725 (	-4)	-2.5702759 (	-4)
21.0	1.9387819 (	-6)	-1.1651559 (	-6)
22.0	3.1232240 (	-9)	-2.1366118 (	-9)
23.0	2.2573236 (	-12)	-1.7191609 (	-12)
24.0	7.6811508 (	-16)	-6.4134955 (	-16)
25.0	1.2755624 (	-19)	-1.1546418 (	-19)
26.0	1.0629319 (	-23)	-1.0342007 (	-23)
27.0	4.5443522 (	-28)	-4.7204214 (	-28)
28.0	1.0149762 (	-32)	-1.1193954 (	-32)
29.0	1.2021995 (	-37)	-1.4013309 (	-37)
30.0	7.6476125 (	-43)	-9.3854223 (	-43)

X	U(-80.0,X)		U'(-80.0,X)	
		58		59
0.	1.8876979 (	0)	-1.6884248 (	0)
1.0	-2.5521180 (	0)	7.0541673 (	-1)
2.0	2.6176231 (	0)	5.0348138 (	-1)
3.0	-1.9073493 (	0)	-1.6719139 (	0)
4.0	3.6159548 (	-1)	2.3364379 (	0)
5.0	1.6164765 (	0)	-1.8820302 (	0)
6.0	-2.7498657 (	0)	4.4919842 (	-2)
7.0	1.3299702 (	0)	2.0138140 (	0)
8.0	2.0399653 (	0)	-1.5576586 (	0)
9.0	-2.0148285 (	0)	-1.5854937 (	0)
10.0	-2.5716650 (	0)	1.0381491 (	0)
11.0	-3.4075700 (	-1)	2.1058715 (	0)
12.0	1.3546188 (	0)	1.8542651 (	0)
13.0	1.7242896 (	0)	1.6790735 (	0)
14.0	4.9661109 (	-1)	1.8666079 (	0)
15.0	-2.8342616 (	0)	1.0723310 (	0)
16.0	-1.6230050 (	0)	-1.4803582 (	0)
17.0	3.1829774 (	0)	1.0759750 (	0)
18.0	2.9039545 (	0)	-5.1248435 (	-1)
19.0	2.0595110 (	-1)	-7.0106529 (	-2)
20.0	3.7003524 (	-3)	-1.6985212 (	-3)
21.0	2.2593072 (	-5)	-1.2615902 (	-5)
22.0	5.4158300 (	-8)	-3.5033705 (	-8)
23.0	5.5651733 (	-11)	-4.0528891 (	-11)
24.0	2.6019637 (	-14)	-2.0936252 (	-14)
25.0	5.7797875 (	-18)	-5.0704518 (	-18)
26.0	6.3034385 (	-22)	-5.9694978 (	-22)
27.0	3.4634394 (	-26)	-3.5135471 (	-26)
28.0	9.7889968 (	-31)	-1.0572452 (	-30)
29.0	1.4477803 (	-35)	-1.6563211 (	-35)
30.0	1.1366253 (	-40)	-1.3716060 (	-40)

X	U(-85.0,X)		U'(-85.0,X)	
		63		64
0.	1.1490153	(0)	1.0593489	(0)
1.0	-8.8534378	(-1)	-1.2558482	(0)
2.0	5.4177603	(-1)	1.4089328	(0)
3.0	-7.7566081	(-2)	-1.4864936	(0)
4.0	-5.2750921	(-1)	1.4016722	(0)
5.0	1.2015020	(0)	-1.0107224	(0)
6.0	-1.6562892	(0)	1.9160855	(-1)
7.0	1.3291470	(0)	8.9107503	(-1)
8.0	1.6772865	(-1)	-1.4149627	(0)
9.0	-1.6943190	(0)	3.1340357	(-1)
10.0	4.8001969	(-1)	1.3229288	(0)
11.0	1.8137982	(0)	8.2434472	(-3)
12.0	1.1313634	(0)	-1.0342921	(0)
13.0	3.0658793	(-1)	-1.2445087	(0)
14.0	3.3165020	(-1)	-1.1905917	(0)
15.0	1.4618605	(0)	-8.2172480	(-1)
16.0	2.0448153	(0)	5.0591381	(-1)
17.0	-2.2945970	(0)	4.0484279	(-1)
18.0	3.2271903	(0)	-6.7384110	(-2)
19.0	6.5551412	(-1)	-1.7234239	(-1)
20.0	2.2667811	(-2)	-9.1262002	(-3)
21.0	2.2978753	(-4)	-1.1775479	(-4)
22.0	8.4231349	(-7)	-5.1165364	(-7)
23.0	1.2536707	(-9)	-8.6924617	(-10)
24.0	8.1633649	(-13)	-6.3113534	(-13)
25.0	2.4506667	(-16)	-2.0792382	(-16)
26.0	3.5263613	(-20)	-3.2454943	(-20)
27.0	2.5064315	(-24)	-2.4803671	(-24)
28.0	9.0129462	(-29)	-9.5239854	(-29)
29.0	1.6719654	(-33)	-1.8759980	(-33)
30.0	1.6261577	(-38)	-1.9284333	(-38)

X	U(-90.0,X)		U'(-90.0,X)	
		67		68
0.	-8.1095808	(0)	7.6934832	(0)
1.0	7.6334316	(0)	-8.1178873	(0)
2.0	-7.3767011	(0)	8.3219378	(0)
3.0	7.5932639	(0)	-8.1389232	(0)
4.0	-8.4800299	(0)	7.3349930	(0)
5.0	1.0008134	(1)	-5.4977354	(0)
6.0	-1.1552082	(1)	2.0405033	(0)
7.0	1.1297887	(1)	3.2966835	(0)
8.0	-6.1975854	(0)	-8.8920017	(0)
9.0	-4.8052074	(0)	9.3789275	(0)
10.0	1.2426586	(1)	4.9972556	(-1)
11.0	1.6932195	(-2)	-9.8212481	(0)
12.0	-1.2403733	(1)	-2.9688748	(0)
13.0	-1.1522941	(1)	4.7379250	(0)
14.0	-8.4465696	(0)	7.0809538	(0)
15.0	-1.0687144	(1)	5.7667835	(0)
16.0	-1.5608692	(1)	-6.1073910	(-1)
17.0	1.6125385	(0)	-7.2072073	(0)
18.0	8.4611273	(0)	5.7955644	(0)
19.0	1.4658093	(1)	-2.3613278	(0)
20.0	1.1239728	(0)	-3.7972654	(-1)
21.0	2.0077608	(-2)	-9.2801839	(-3)
22.0	1.1639405	(-4)	-6.5804025	(-5)
23.0	2.5646115	(-7)	-1.6840719	(-7)
24.0	2.3618654	(-10)	-1.7485284	(-10)
25.0	9.6928185	(-14)	-7.9345700	(-14)
26.0	1.8566064	(-17)	-1.6577556	(-17)
27.0	1.7191568	(-21)	-1.6574389	(-21)
28.0	7.9105568	(-26)	-8.1703750	(-26)
29.0	1.8494341	(-30)	-2.0336089	(-30)
30.0	2.2373990	(-35)	-2.6058097	(-35)

X	U(-95.0,X)		U'(-95.0,X)	
		72		73
0.	-6.5819672	(0)	-6.4153525	(0)
1.0	8.3145995	(0)	4.0877947	(0)
2.0	-9.1896328	(0)	-1.5821923	(0)
3.0	9.3444597	(0)	-5.8222589	(-1)
4.0	-9.1662859	(0)	2.0202675	(0)
5.0	9.0853244	(0)	-2.5038930	(0)
6.0	-9.3327731	(0)	1.8383637	(0)
7.0	9.6316974	(0)	2.4474566	(-1)
8.0	-8.7504281	(0)	-3.8282670	(0)
9.0	4.3862844	(0)	7.6639020	(0)
10.0	4.4181293	(0)	-7.5415589	(0)
11.0	-1.0194394	(1)	-8.3509685	(-1)
12.0	-8.8506942	(-1)	8.0232711	(0)
13.0	9.6557069	(0)	3.5159260	(0)
14.0	1.0709772	(1)	-2.0831494	(0)
15.0	1.0147229	(1)	-3.5098743	(0)
16.0	1.2197236	(1)	-8.6119777	(-1)
17.0	7.6343327	(0)	5.2704467	(0)
18.0	-1.4722080	(1)	8.3004632	(-1)
19.0	1.8918748	(1)	1.2723518	(-1)
20.0	4.3009829	(0)	-1.1181233	(0)
21.0	1.4760150	(-1)	-5.9968981	(-2)
22.0	1.4125761	(-3)	-7.3457980	(-4)
23.0	4.7297859	(-6)	-2.9222666	(-6)
24.0	6.2708044	(-9)	-4.4272633	(-9)
25.0	3.5635490	(-12)	-2.8068581	(-12)
26.0	9.1753327	(-16)	-7.9328036	(-16)
27.0	1.1153659	(-19)	-1.0461168	(-19)
28.0	6.6081090	(-24)	-6.6638128	(-24)
29.0	1.9570056	(-28)	-2.1070554	(-28)
30.0	2.9574426	(-33)	-3.3804936	(-33)

X	U(-100.0,X)		U'(-100.0,X)	
		77		78
0.	6.0979530	(0)	-6.0979910	(0)
1.0	-1.8349623	(0)	8.4211276	(0)
2.0	-2.8161430	(0)	-8.1337771	(0)
3.0	6.3858919	(0)	5.8047751	(0)
4.0	-8.2608104	(0)	-2.7165246	(0)
5.0	8.7639332	(0)	5.3126889	(-2)
6.0	-8.6820974	(0)	1.5256895	(0)
7.0	8.7064905	(0)	-1.7657935	(0)
8.0	-8.9925756	(0)	4.7203347	(-1)
9.0	8.6749059	(0)	2.5418765	(0)
10.0	-5.4511979	(0)	-6.4991146	(0)
11.0	-2.6654004	(0)	7.5550205	(0)
12.0	9.6340237	(0)	-2.8241749	(-1)
13.0	9.9234064	(-2)	-7.5173010	(0)
14.0	-9.0135731	(0)	-3.4476432	(0)
15.0	-1.0534242	(1)	7.5202336	(-1)
16.0	-1.1011194	(1)	9.1998399	(-1)
17.0	-1.0491577	(1)	-3.0157836	(0)
18.0	4.6364132	(0)	-5.2729886	(0)
19.0	4.4507841	(0)	4.7430734	(0)
20.0	1.1694803	(1)	-1.8425959	(0)
21.0	8.8691424	(-1)	-3.0355605	(-1)
22.0	1.4838366	(-2)	-6.9829361	(-3)
23.0	7.7962301	(-5)	-4.4946417	(-5)
24.0	1.5191981	(-7)	-1.0178589	(-7)
25.0	1.2130161	(-10)	-9.1639832	(-11)
26.0	4.2440448	(-14)	-3.5451283	(-14)
27.0	6.8299062	(-18)	-6.2219356	(-18)
28.0	5.2449629	(-22)	-5.1579751	(-22)
29.0	1.9783549	(-26)	-2.0837379	(-26)
30.0	3.7515524	(-31)	-4.2055598	(-31)

X	U(-105.0,X)		U'(-105.0,X)	
		82		83
0.	6.4060920	(0)	6.5643282	(0)
1.0	-9.0601171	(0)	3.0285459	(-1)
2.0	6.1522305	(0)	-6.8106472	(0)
3.0	3.2692787	(-3)	9.2332282	(0)
4.0	-5.6878862	(0)	-7.2033302	(0)
5.0	8.7070268	(0)	2.9569385	(0)
6.0	-9.2020477	(0)	1.0477533	(0)
7.0	8.5760516	(0)	-3.5668726	(0)
8.0	-8.2000681	(0)	4.4064317	(0)
9.0	8.7402196	(0)	-3.5497059	(0)
10.0	-9.6730336	(0)	5.9233531	(-1)
11.0	8.3758323	(0)	4.5179491	(0)
12.0	-7.0162250	(-1)	-8.3396264	(0)
13.0	-9.7335036	(0)	2.6525144	(0)
14.0	1.9796413	(0)	7.8001745	(0)
15.0	9.9889714	(0)	3.2125900	(0)
16.0	1.1459140	(1)	1.2505387	(-1)
17.0	1.1832909	(1)	1.5793024	(0)
18.0	5.0759546	(0)	5.9673693	(0)
19.0	-1.4706693	(1)	2.8338816	(-1)
20.0	1.8707037	(1)	2.0809471	(-1)
21.0	4.1681927	(0)	-1.1073579	(0)
22.0	1.3237151	(-1)	-5.5037334	(-2)
23.0	1.1365187	(-3)	-6.0477037	(-4)
24.0	3.3362675	(-6)	-2.1085017	(-6)
25.0	3.8058969	(-9)	-2.7474038	(-9)
26.0	1.8314821	(-12)	-1.4743651	(-12)
27.0	3.9379643	(-16)	-3.4781865	(-16)
28.0	3.9482812	(-20)	-3.7814838	(-20)
29.0	1.9078794	(-24)	-1.9638484	(-24)
30.0	4.5617062	(-29)	-5.0112752	(-29)

X	U(-110.0,X)		U'(-110.0,X)	
		87		88
0.	-7.5852453	(0)	7.9555135	(0)
1.0	-2.9041758	(0)	-1.0825293	(1)
2.0	1.0386738	(1)	2.9021312	(0)
3.0	-8.0615004	(0)	7.4305016	(0)
4.0	-7.6609208	(-1)	-1.1119516	(1)
5.0	8.4512879	(0)	6.9928151	(0)
6.0	-1.0957527	(1)	1.4023016	(-1)
7.0	9.3640940	(0)	-5.7890148	(0)
8.0	-6.6851211	(0)	8.6533338	(0)
9.0	5.2125115	(0)	-9.4777707	(0)
10.0	-6.0318369	(0)	8.9546648	(0)
11.0	9.1203016	(0)	-6.4218481	(0)
12.0	-1.1843447	(1)	1.1833536	(-1)
13.0	6.6435685	(0)	8.3488880	(0)
14.0	9.0766836	(0)	-6.6076161	(0)
15.0	-5.8653878	(0)	-8.3867494	(0)
16.0	-1.2682196	(1)	-2.8588468	(0)
17.0	-1.3903605	(1)	-1.1551063	(0)
18.0	-1.2094975	(1)	-4.8429975	(0)
19.0	6.4578288	(0)	-6.6663280	(0)
20.0	6.0945109	(0)	6.0473829	(0)
21.0	1.4217287	(1)	-2.3613129	(0)
22.0	9.7712313	(-1)	-3.4531772	(-1)
23.0	1.4461238	(-2)	-6.9978358	(-3)
24.0	6.5897178	(-5)	-3.8987378	(-5)
25.0	1.0950347	(-7)	-7.5194931	(-8)
26.0	7.3473978	(-11)	-5.6835153	(-11)
27.0	2.1323055	(-14)	-1.8223085	(-14)
28.0	2.8132708	(-18)	-2.6202682	(-18)
29.0	1.7525132	(-22)	-1.7609940	(-22)
30.0	5.3104590	(-27)	-5.7120464	(-27)

X	U(-115.0,X)		U'(-115.0,X)	
		93		94
0.	-1.0067901	(0)	-1.0796672	(0)
1.0	1.2434478	(0)	-7.4457191	(-1)
2.0	2.9862750	(-1)	1.4898776	(0)
3.0	-1.4178536	(0)	-2.0491639	(-1)
4.0	7.5259740	(-1)	-1.2887483	(0)
5.0	7.0221078	(-1)	1.3160310	(0)
6.0	-1.4420573	(0)	-1.8527802	(-1)
7.0	1.1477367	(0)	-9.2106240	(-1)
8.0	-4.0812638	(-1)	1.4131698	(0)
9.0	-2.0855075	(-1)	-1.4407238	(0)
10.0	4.8829363	(-1)	1.3600412	(0)
11.0	-3.8484878	(-1)	-1.3702531	(0)
12.0	-1.8902020	(-1)	1.3795343	(0)
13.0	1.1816650	(0)	-9.1302661	(-1)
14.0	-1.5331523	(0)	-4.6746605	(-1)
15.0	-5.1445611	(-1)	1.2275574	(0)
16.0	1.2708279	(0)	8.5872667	(-1)
17.0	1.7905786	(0)	2.3374880	(-1)
18.0	1.8571838	(0)	3.1973475	(-1)
19.0	8.7570438	(-1)	9.5305284	(-1)
20.0	-2.3254318	(0)	1.2764623	(-1)
21.0	2.9864305	(0)	-2.3595786	(-2)
22.0	5.7384806	(-1)	-1.6039600	(-1)
23.0	1.5790883	(-2)	-6.8011935	(-3)
24.0	1.1596723	(-4)	-6.3602004	(-5)
25.0	2.8721067	(-7)	-1.8658633	(-7)
26.0	2.7290905	(-10)	-2.0216080	(-10)
27.0	1.0811554	(-13)	-8.9197073	(-14)
28.0	1.8932700	(-17)	-1.7120439	(-17)
29.0	1.5307774	(-21)	-1.4997594	(-21)
30.0	5.9109273	(-26)	-6.2194542	(-26)

X	U(-120.0,X)		U'(-120.0,X)	
		98		99
0.	1.4905076	(0)	-1.6327763	(0)
1.0	1.4228764	(0)	1.7036473	(0)
2.0	-1.6520205	(0)	1.4355887	(0)
3.0	-1.1235354	(0)	-1.9485072	(0)
4.0	1.9769834	(0)	-8.4083624	(-1)
5.0	2.1496918	(-1)	2.2669767	(0)
6.0	-2.0583310	(0)	-6.5350943	(-1)
7.0	1.5942170	(0)	-1.5198636	(0)
8.0	8.5787756	(-2)	2.2263145	(0)
9.0	-1.4386085	(0)	-1.6739949	(0)
10.0	2.0441249	(0)	8.8276139	(-1)
11.0	-2.2082095	(0)	-4.8784016	(-1)
12.0	2.1803490	(0)	6.8772318	(-1)
13.0	-1.7037970	(0)	-1.4300199	(0)
14.0	3.0165712	(-2)	2.0251122	(0)
15.0	2.3030590	(0)	-7.0369253	(-1)
16.0	-6.4182817	(-1)	-1.8494329	(0)
17.0	-2.4490420	(0)	-7.1738140	(-1)
18.0	-2.7798449	(0)	-1.7460336	(-1)
19.0	-2.6065668	(0)	-8.1616389	(-1)
20.0	7.6138263	(-1)	-1.4275040	(0)
21.0	1.8426655	(0)	1.1424592	(0)
22.0	2.5150577	(0)	-4.6640727	(-1)
23.0	1.4465090	(-1)	-5.3622369	(-2)
24.0	1.7972441	(-3)	-9.0175875	(-4)
25.0	6.8194017	(-6)	-4.1628415	(-6)
26.0	9.3424258	(-9)	-6.6004294	(-9)
27.0	5.1166548	(-12)	-4.0643167	(-12)
28.0	1.2005533	(-15)	-1.0520869	(-15)
29.0	1.2691731	(-19)	-1.2107686	(-19)
30.0	6.2818515	(-24)	-6.4592614	(-24)

X	U(-125.0,X)		U'(-125.0,X)	
		103		104
0.	2.4500217	(0)	2.7392185	(0)
1.0	-1.9697123	(0)	3.1859563	(0)
2.0	-3.1327630	(0)	-1.6669981	(0)
3.0	1.1339957	(0)	-3.6455356	(0)
4.0	3.3514248	(0)	1.0847514	(0)
5.0	-1.1396948	(0)	3.6166105	(0)
6.0	-3.1141044	(0)	-1.7928049	(0)
7.0	2.5241722	(0)	-2.6567048	(0)
8.0	1.1191026	(0)	3.5574655	(0)
9.0	-3.4162396	(0)	-1.2333964	(0)
10.0	3.4326233	(0)	-1.2760549	(0)
11.0	-2.5817038	(0)	2.5943182	(0)
12.0	2.1499226	(0)	-2.9202280	(0)
13.0	-2.6531799	(0)	2.5216061	(0)
14.0	3.7546313	(0)	-9.8488116	(-1)
15.0	-3.2724113	(0)	-1.9497508	(0)
16.0	-1.6849924	(0)	2.9527979	(0)
17.0	3.2052072	(0)	2.0935763	(0)
18.0	4.4909258	(0)	1.8920158	(-1)
19.0	4.7614474	(0)	2.0853388	(-1)
20.0	3.3043219	(0)	2.0285366	(0)
21.0	-5.3347213	(0)	8.7736249	(-1)
22.0	7.2121274	(0)	-3.2102984	(-1)
23.0	1.0748133	(0)	-3.2299946	(-1)
24.0	2.4168475	(-2)	-1.0889159	(-2)
25.0	1.4536064	(-4)	-8.2658224	(-5)
26.0	2.9319964	(-7)	-1.9660142	(-7)
27.0	2.2520198	(-10)	-1.7170214	(-10)
28.0	7.1545907	(-14)	-6.0634068	(-14)
29.0	9.9686919	(-18)	-9.2461755	(-18)
30.0	6.3646194	(-22)	-6.3883063	(-22)

X	U(-130.0,X)		U'(-130.0,X)	
		108		109
0.	-4.4526682	(0)	5.0768417	(0)
1.0	-5.8439981	(0)	-2.6812524	(0)
2.0	1.4019815	(-2)	-7.1658711	(0)
3.0	5.9743493	(0)	-2.3440163	(0)
4.0	3.6071141	(0)	5.8627517	(0)
5.0	-4.5070613	(0)	5.0132957	(0)
6.0	-4.8341773	(0)	-4.6349753	(0)
7.0	4.5596607	(0)	-4.9543155	(0)
8.0	3.4412405	(0)	5.8998162	(0)
9.0	-6.5044428	(0)	9.5802703	(-1)
10.0	3.1311210	(0)	-5.9990712	(0)
11.0	1.3649766	(0)	6.5818805	(0)
12.0	-3.9893747	(0)	-5.3797873	(0)
13.0	4.6526319	(0)	4.8414426	(0)
14.0	-3.3760559	(0)	-5.6160569	(0)
15.0	-9.4184226	(-1)	6.1760563	(0)
16.0	6.9366397	(0)	-2.2085747	(0)
17.0	-2.9358308	(0)	-5.4312885	(0)
18.0	-7.9383301	(0)	-9.1123691	(-1)
19.0	-8.2564359	(0)	1.1342306	(0)
20.0	-8.9762678	(0)	-8.3344170	(-1)
21.0	-1.2395223	(0)	-4.4696937	(0)
22.0	8.5671839	(0)	2.8779675	(0)
23.0	6.1468509	(0)	-1.3124757	(0)
24.0	2.7657542	(-1)	-1.0880648	(-1)
25.0	2.7535261	(-3)	-1.4419676	(-3)
26.0	8.3841060	(-6)	-5.3037692	(-6)
27.0	9.1805918	(-9)	-6.6942962	(-9)
28.0	3.9954705	(-12)	-3.2668347	(-12)
29.0	7.4018052	(-16)	-6.6637993	(-16)
30.0	6.1376633	(-20)	-6.0062823	(-20)

X	U(-135.0,X)		U'(-135.0,X)	
		113		115
0.	-8.9125224	(0)	-1.0355451	(0)
1.0	2.0783148	(0)	-1.4437661	(0)
2.0	1.1463785	(1)	-6.1275482	(-1)
3.0	1.0566394	(1)	8.0317644	(-1)
4.0	-1.1801803	(0)	1.4471764	(0)
5.0	-1.2148565	(1)	4.4016833	(-1)
6.0	-7.2596079	(0)	-1.1869970	(0)
7.0	9.3521832	(0)	-9.8496388	(-1)
8.0	8.9709362	(0)	1.0283314	(0)
9.0	-1.0993429	(1)	7.6735799	(-1)
10.0	-2.4419319	(0)	-1.3678957	(0)
11.0	1.2113411	(1)	5.9500079	(-1)
12.0	-1.3280185	(1)	2.9897701	(-1)
13.0	1.1483712	(1)	-7.4265960	(-1)
14.0	-1.1453913	(1)	7.6155449	(-1)
15.0	1.3874271	(1)	-3.4612701	(-1)
16.0	-1.3034634	(1)	-5.9444644	(-1)
17.0	-2.5537931	(0)	1.1913433	(0)
18.0	1.4716581	(1)	4.3834015	(-1)
19.0	1.5517067	(1)	-3.8818128	(-1)
20.0	1.6284373	(1)	-3.9284892	(-1)
21.0	1.7584356	(1)	4.0873743	(-1)
22.0	-1.4723657	(1)	5.9444770	(-1)
23.0	2.4494407	(1)	-2.2389191	(-1)
24.0	2.6200843	(0)	-8.5808949	(-2)
25.0	4.5766621	(-2)	-2.1727273	(-3)
26.0	2.1686065	(-4)	-1.2845169	(-5)
27.0	3.4502884	(-7)	-2.3957833	(-8)
28.0	2.0841680	(-10)	-1.6395607	(-11)
29.0	5.1832773	(-14)	-4.5210059	(-15)
30.0	5.6236070	(-18)	-5.3582426	(-19)

X	U(-140.0,X)		U'(-140.0,X)	
		119		120
0.	1.9577239	(0)	-2.3164175	(0)
1.0	2.7667261	(0)	-1.5589259	(-1)
2.0	2.1013166	(0)	2.1347528	(0)
3.0	1.6215987	(-1)	3.2571555	(0)
4.0	-2.0137945	(0)	2.2491096	(0)
5.0	-2.7515862	(0)	-6.0354322	(-1)
6.0	-8.9784682	(-1)	-3.0541807	(0)
7.0	2.1571456	(0)	-2.0737876	(0)
8.0	2.2891372	(0)	1.8997171	(0)
9.0	-1.6113987	(0)	2.6091784	(0)
10.0	-2.1202597	(0)	-2.1368907	(0)
11.0	2.6819432	(0)	-1.2645158	(0)
12.0	-6.8521230	(-1)	2.9589479	(0)
13.0	-1.1482353	(0)	-2.7729199	(0)
14.0	1.9400705	(0)	2.2899841	(0)
15.0	-1.7903712	(0)	-2.3737679	(0)
16.0	3.8341639	(-1)	2.7930349	(0)
17.0	2.4406764	(0)	-1.8442562	(0)
18.0	-2.5885595	(0)	-1.7457241	(0)
19.0	-3.3372087	(0)	9.0927511	(-1)
20.0	-2.6615508	(0)	1.6871353	(0)
21.0	-3.7312449	(0)	8.6319337	(-1)
22.0	-2.7212002	(0)	-1.6355449	(0)
23.0	5.1750743	(0)	8.0065553	(-1)
24.0	1.9695730	(0)	-4.8858128	(-1)
25.0	6.5658424	(-2)	-2.7617292	(-2)
26.0	5.0295888	(-4)	-2.7624324	(-4)
27.0	1.1890184	(-6)	-7.8211207	(-7)
28.0	1.0118351	(-9)	-7.6340018	(-10)
29.0	3.4144520	(-13)	-2.8792240	(-13)
30.0	4.8862761	(-17)	-4.5262677	(-17)

X	U(-145.0,X)	U'(-145.0,X)
	124	125
0.	4.7035436 (0)	5.6638334 (0)
1.0	1.6927566 (0)	7.7432050 (0)
2.0	-1.9168352 (0)	7.6576985 (0)
3.0	-5.1573687 (0)	5.0670547 (0)
4.0	-6.6944268 (0)	2.7483903 (-1)
5.0	-5.1315030 (0)	-5.1232243 (0)
6.0	-1.6248672 (-1)	-7.8803920 (0)
7.0	5.5255932 (0)	-4.5632348 (0)
8.0	6.0031976 (0)	3.7496468 (0)
9.0	-1.5892683 (0)	7.5058197 (0)
10.0	-6.8808767 (0)	-1.2519103 (0)
11.0	2.2417908 (0)	-7.1602697 (0)
12.0	5.2744928 (0)	5.0373662 (0)
13.0	-7.1921514 (0)	9.1471807 (-1)
14.0	5.7707508 (0)	-4.4877050 (0)
15.0	-4.9056523 (0)	5.3595406 (0)
16.0	6.1259376 (0)	-4.1747261 (0)
17.0	-7.9033115 (0)	-6.0108822 (-2)
18.0	3.2601893 (0)	5.9967326 (0)
19.0	8.1914797 (0)	-1.6023756 (0)
20.0	4.4794501 (0)	-5.1438528 (0)
21.0	4.5096881 (0)	-4.9004915 (0)
22.0	1.0184085 (1)	-9.6883970 (-1)
23.0	-2.5292583 (0)	4.2281476 (0)
24.0	1.0926593 (1)	-1.5645270 (0)
25.0	7.9528081 (-1)	-2.8574391 (-1)
26.0	1.0347336 (-2)	-5.2013366 (-3)
27.0	3.7336771 (-5)	-2.3115300 (-5)
28.0	4.5532568 (-8)	-3.2822848 (-8)
29.0	2.1098398 (-11)	-1.7158265 (-11)
30.0	4.0177882 (-15)	-3.6122466 (-15)

X	U(-150.0,X)	U'(-150.0,X)
	130	131
0.	-1.2321811 (0)	1.5091117 (0)
1.0	-1.5596705 (0)	9.5288709 (-1)
2.0	-1.7328518 (0)	2.5557499 (-1)
3.0	-1.6888688 (0)	-5.5385550 (-1)
4.0	-1.3318730 (0)	-1.3802412 (0)
5.0	-5.7704760 (-1)	-1.9953121 (0)
6.0	5.2082985 (-1)	-2.0081046 (0)
7.0	1.5507950 (0)	-1.0246979 (0)
8.0	1.6544393 (0)	7.9950178 (-1)
9.0	1.7128573 (-1)	2.0490958 (0)
10.0	-1.6888373 (0)	7.6820334 (-1)
11.0	-8.1123084 (-1)	-1.8124668 (0)
12.0	1.7973527 (0)	-5.3431978 (-1)
13.0	-3.6013266 (-1)	1.9283715 (0)
14.0	-1.0892239 (0)	-1.5954027 (0)
15.0	1.6967761 (0)	9.5300548 (-1)
16.0	-1.7881555 (0)	-8.4019431 (-1)
17.0	1.3682158 (0)	1.3542139 (0)
18.0	2.7049587 (-1)	-1.7423623 (0)
19.0	-2.1913699 (0)	6.3547364 (-2)
20.0	-8.9804860 (-1)	1.4877458 (0)
21.0	3.4517611 (-2)	1.5315654 (0)
22.0	-1.0359983 (0)	1.2912399 (0)
23.0	-2.8485408 (0)	-3.9439388 (-1)
24.0	3.7949046 (0)	1.1146837 (-1)
25.0	7.8651420 (-1)	-2.2586488 (-1)
26.0	1.8626752 (-2)	-8.4133775 (-3)
27.0	1.0603581 (-4)	-6.1283328 (-5)
28.0	1.8903336 (-7)	-1.2961254 (-7)
29.0	1.2190167 (-10)	-9.5342506 (-11)
30.0	3.1192525 (-14)	-2.7167590 (-14)

X	U(-155.0,X)	U'(-155.0,X)
	135	136
0.	-3.5094770 (0)	-4.3692750 (0)
1.0	-3.0661839 (0)	-4.8583802 (0)
2.0	-2.4947268 (0)	-5.3364405 (0)
3.0	-1.6871252 (0)	-5.7930917 (0)
4.0	-5.2169462 (-1)	-6.1054400 (0)
5.0	1.0830339 (0)	-5.9710316 (0)
6.0	3.0189035 (0)	-4.8718351 (0)
7.0	4.7064361 (0)	-2.2379221 (0)
8.0	4.8470707 (0)	1.8742674 (0)
9.0	1.9500017 (0)	5.5221286 (0)
10.0	-3.2209715 (0)	4.6315112 (0)
11.0	-4.8702493 (0)	-2.1648232 (0)
12.0	1.7284833 (0)	-5.4658602 (0)
13.0	4.5492360 (0)	3.0446158 (0)
14.0	-4.8566387 (0)	2.5555726 (0)
15.0	2.1740806 (0)	-5.0760543 (0)
16.0	-4.8060787 (-1)	5.3883375 (0)
17.0	1.0816130 (0)	-5.1866549 (0)
18.0	-4.1032681 (0)	3.7187753 (0)
19.0	5.9310043 (0)	1.3998408 (0)
20.0	2.4800413 (0)	-4.3890534 (0)
21.0	-2.5366136 (0)	-4.2147636 (0)
22.0	-2.6650409 (0)	-3.9552213 (0)
23.0	4.3879591 (0)	-3.1479081 (0)
24.0	3.6436870 (0)	3.0630589 (0)
25.0	6.0283377 (0)	-1.1865761 (0)
26.0	2.8823148 (-1)	-1.1381743 (-1)
27.0	2.6989940 (-3)	-1.4406962 (-3)
28.0	7.2012672 (-6)	-4.6707866 (-6)
29.0	6.5622243 (-9)	-4.9199684 (-9)
30.0	2.2807447 (-12)	-1.9202607 (-12)

X	U(-160.0,X)	U'(-160.0,X)
	141	142
0.	1.0838096 (0)	-1.3709261 (0)
1.0	9.9473087 (-1)	-1.4747854 (0)
2.0	9.2443699 (-1)	-1.5452727 (0)
3.0	9.0149199 (-1)	-1.5651394 (0)
4.0	9.5291947 (-1)	-1.5146269 (0)
5.0	1.0950305 (0)	-1.3564326 (0)
6.0	1.3139059 (0)	-1.0215156 (0)
7.0	1.5253244 (0)	-4.1685721 (-1)
8.0	1.5186342 (0)	4.9599214 (-1)
9.0	9.6209103 (-1)	1.4905426 (0)
10.0	-3.0505899 (-1)	1.8237581 (0)
11.0	-1.5333795 (0)	5.7645069 (-1)
12.0	-9.0832233 (-1)	-1.5130617 (0)
13.0	1.3750585 (0)	-9.9720268 (-1)
14.0	4.8141631 (-1)	1.6959272 (0)
15.0	-1.6096886 (0)	-5.8476391 (-1)
16.0	1.6958213 (0)	-3.8477325 (-1)
17.0	-1.6463026 (0)	6.3267732 (-1)
18.0	1.8194589 (0)	-1.5582816 (-1)
19.0	-1.4426308 (0)	-1.0198808 (0)
20.0	-9.3602731 (-1)	1.3288626 (0)
21.0	1.2047211 (0)	1.1785572 (0)
22.0	1.8425773 (0)	7.4195142 (-1)
23.0	1.1389537 (0)	1.1100439 (0)
24.0	-2.4289630 (0)	4.4661226 (-1)
25.0	3.2352028 (0)	-2.1763321 (-1)
26.0	3.7384345 (-1)	-1.2315633 (-1)
27.0	6.0883873 (-3)	-2.9582483 (-3)
28.0	2.5013712 (-5)	-1.5243040 (-5)
29.0	3.2780595 (-8)	-2.3468583 (-8)
30.0	1.5662324 (-11)	-1.2716353 (-11)

X	U(-165.0,X)	U'(-165.0,X)
	146	147
0.	3.6199646 (0)	4.6499394 (0)
1.0	4.4702469 (0)	3.2079518 (0)
2.0	4.9632133 (0)	1.6480346 (0)
3.0	5.1335870 (0)	2.4022231 (-1)
4.0	5.1127411 (0)	-7.9257118 (-1)
5.0	5.0647241 (0)	-1.2999676 (0)
6.0	5.1064446 (0)	-1.1678158 (0)
7.0	5.2145904 (0)	-2.6481106 (-1)
8.0	5.1010185 (0)	1.5276087 (0)
9.0	4.0945526 (0)	4.0325062 (0)
10.0	1.3331372 (0)	6.1122657 (0)
11.0	-3.0024066 (0)	5.1865745 (0)
12.0	-5.4162049 (0)	-6.3367972 (-1)
13.0	-6.8090938 (-1)	-6.0614086 (0)
14.0	5.5821049 (0)	-3.2734914 (-1)
15.0	-1.8811564 (0)	5.5877448 (0)
16.0	-2.6737139 (0)	-5.1640934 (0)
17.0	4.6294163 (0)	3.5523613 (0)
18.0	-4.6595550 (0)	-3.5645066 (0)
19.0	2.2150910 (0)	5.0508029 (0)
20.0	4.1700556 (0)	-3.9637950 (0)
21.0	-4.6186243 (0)	-3.6595827 (0)
22.0	-7.1045938 (0)	-3.8534808 (-1)
23.0	-7.5690501 (0)	-7.6726945 (-1)
24.0	-2.6231571 (0)	-3.7803298 (0)
25.0	9.0624576 (0)	1.9199379 (0)
26.0	3.9105033 (0)	-9.8123766 (-1)
27.0	1.1998259 (-1)	-5.1971070 (-2)
28.0	7.8625028 (-4)	-4.4626364 (-4)
29.0	1.5125027 (-6)	-1.0292396 (-6)
30.0	1.0070264 (-9)	-7.8622460 (-10)

X	U(-170.0,X)	U'(-170.0,X)
	152	153
0.	-1.3045521 (0)	1.7009315 (0)
1.0	-5.7448332 (-1)	2.2850944 (0)
2.0	2.4526655 (-1)	2.3807146 (0)
3.0	9.5857545 (-1)	2.0511924 (0)
4.0	1.4510305 (0)	1.4913189 (0)
5.0	1.7170248 (0)	9.2332313 (-1)
6.0	1.8252354 (0)	5.1792870 (-1)
7.0	1.8557171 (0)	3.7721788 (-1)
8.0	1.8369747 (0)	5.5849798 (-1)
9.0	1.6884895 (0)	1.0789483 (0)
10.0	1.1847859 (0)	1.8199670 (0)
11.0	6.2922468 (-2)	2.2893424 (0)
12.0	-1.4486952 (0)	1.5231619 (0)
13.0	-1.8351276 (0)	-8.4717752 (-1)
14.0	4.5478594 (-1)	-2.1514277 (0)
15.0	1.8879136 (0)	8.2703906 (-1)
16.0	-1.6266671 (0)	1.3251941 (0)
17.0	4.8342369 (-1)	-2.0383928 (0)
18.0	2.0766226 (-2)	2.0461312 (0)
19.0	5.0432689 (-1)	-1.9376920 (0)
20.0	-1.9362912 (0)	1.0363869 (0)
21.0	1.6743516 (0)	1.3322527 (0)
22.0	2.4092716 (0)	-4.9816453 (-1)
23.0	2.2211834 (0)	-9.1254628 (-1)
24.0	2.9458644 (0)	9.3629754 (-2)
25.0	-9.9491311 (-1)	1.2109134 (0)
26.0	3.0936452 (0)	-4.5893451 (-1)
27.0	2.0272668 (-1)	-7.5805694 (-2)
28.0	2.2159466 (-3)	-1.1581192 (-3)
29.0	6.4115102 (-6)	-4.1236364 (-6)
30.0	6.0411012 (-9)	-4.5205771 (-9)

X	U(-175.0,X)	U'(-175.0,X)
	157	158
0.	-5.0611643 (0)	-6.6953047 (0)
1.0	-7.1032417 (0)	-1.1912439 (0)
2.0	-6.2409003 (0)	4.6495262 (0)
3.0	-3.1502608 (0)	8.4805882 (0)
4.0	6.5552098 (-1)	9.3750987 (0)
5.0	3.8368928 (0)	7.9509903 (0)
6.0	5.8251878 (0)	5.5694688 (0)
7.0	6.7630696 (0)	3.4704261 (0)
8.0	7.0787779 (0)	2.4093723 (0)
9.0	7.0448676 (0)	2.7445680 (0)
10.0	6.4417918 (0)	4.5614973 (0)
11.0	4.3739341 (0)	7.3421084 (0)
12.0	-2.3875377 (-1)	8.9341302 (0)
13.0	-6.1922987 (0)	5.2061059 (0)
14.0	-6.5858113 (0)	-4.6377358 (0)
15.0	3.7188927 (0)	-7.5723858 (0)
16.0	5.7739353 (0)	5.8752753 (0)
17.0	-8.0876911 (0)	1.2027033 (0)
18.0	6.6921601 (0)	-4.8429083 (0)
19.0	-6.7078970 (0)	4.9062806 (0)
20.0	8.6917956 (0)	-1.3955458 (0)
21.0	-5.8647935 (0)	-5.7038973 (0)
22.0	-8.2080278 (0)	3.6213145 (0)
23.0	-3.5150136 (0)	6.2245935 (0)
24.0	-5.8890620 (0)	5.1390405 (0)
25.0	-1.1972914 (1)	-1.7388733 (0)
26.0	1.6084351 (1)	2.3840556 (-1)
27.0	2.8540587 (0)	-8.7011508 (-1)
28.0	5.5358711 (-2)	-2.6224513 (-2)
29.0	2.4812726 (-4)	-1.4977502 (-4)
30.0	3.3679521 (-7)	-2.4061851 (-7)

X	U(-180.0,X)	U'(-180.0,X)
	163	164
0.	2.1093725 (0)	-2.8300256 (0)
1.0	-1.8358382 (-1)	-3.9933048 (0)
2.0	-2.3210574 (0)	-2.5163161 (0)
3.0	-2.9744170 (0)	4.3729784 (-1)
4.0	-1.9919601 (0)	2.9752653 (0)
5.0	-2.0125816 (-1)	3.9581051 (0)
6.0	1.4418808 (0)	3.4730045 (0)
7.0	2.4644752 (0)	2.2979910 (0)
8.0	2.9084942 (0)	1.1918276 (0)
9.0	3.0384632 (0)	5.8659525 (-1)
10.0	3.0516185 (0)	6.5812592 (-1)
11.0	2.8870023 (0)	1.4617085 (0)
12.0	2.1118020 (0)	2.8137857 (0)
13.0	1.2063666 (-1)	3.7408184 (0)
14.0	-2.5985454 (0)	2.1918117 (0)
15.0	-2.6219791 (0)	-2.1888996 (0)
16.0	2.1147120 (0)	-2.7660770 (0)
17.0	1.5226356 (0)	3.1486624 (0)
18.0	-3.2405069 (0)	-1.2252560 (0)
19.0	3.5412923 (0)	2.4411742 (-1)
20.0	-3.5106097 (0)	-9.1586411 (-1)
21.0	1.8774232 (0)	2.7480463 (0)
22.0	3.0405082 (0)	-1.9135524 (0)
23.0	-5.8992414 (-1)	-2.8470761 (0)
24.0	-1.4467850 (0)	-2.5443903 (0)
25.0	2.3160198 (0)	-2.1030526 (0)
26.0	3.0791568 (0)	1.8026561 (0)
27.0	3.2093484 (0)	-7.0718791 (-1)
28.0	1.2079527 (-1)	-5.0697985 (-2)
29.0	8.7010496 (-4)	-4.8850704 (-4)
30.0	1.7371055 (-6)	-1.1793747 (-6)

X	U(-185.0,X)	U'(-185.0,X)
	168	170
0.	9.4254711 (0)	1.2820050 (0)
1.0	1.2930548 (1)	-4.4244822 (-1)
2.0	4.0778279 (0)	-1.7239541 (0)
3.0	-8.2642270 (0)	-1.4210437 (0)
4.0	-1.3400775 (1)	2.9944720 (-2)
5.0	-8.7335206 (0)	1.3663191 (0)
6.0	6.2554229 (-1)	1.7886673 (0)
7.0	8.5964724 (0)	1.3787553 (0)
8.0	1.2676881 (1)	6.5329745 (-1)
9.0	1.3717696 (1)	4.2884535 (-2)
10.0	1.3658694 (1)	-2.6715336 (-1)
11.0	1.3812894 (1)	-2.3063520 (-1)
12.0	1.3990276 (1)	1.8471417 (-1)
13.0	1.1787046 (1)	9.5219969 (-1)
14.0	3.1108213 (0)	1.6395278 (0)
15.0	-1.0882743 (1)	1.1017471 (0)
16.0	-1.1701248 (1)	-1.0027543 (0)
17.0	1.1275139 (1)	-1.0621888 (0)
18.0	2.1921415 (0)	1.5543801 (0)
19.0	-1.0007762 (1)	-1.1871793 (0)
20.0	1.0976056 (1)	1.1005270 (0)
21.0	-4.2761393 (0)	-1.3989433 (0)
22.0	-1.2895161 (1)	9.2672728 (-1)
23.0	9.0924100 (0)	1.1535492 (0)
24.0	1.7180407 (1)	5.9301311 (-1)
25.0	1.3787797 (1)	8.8061615 (-1)
26.0	-1.8343459 (1)	6.1765108 (-1)
27.0	2.6692718 (1)	-2.8080939 (-1)
28.0	2.2580056 (0)	-8.0964116 (-2)
29.0	2.7394755 (-2)	-1.4135910 (-3)
30.0	8.2456020 (-5)	-5.2898207 (-6)

X	U(-190.0,X)	U'(-190.0,X)
	174	175
0.	-4.5069206 (0)	6.2123721 (0)
1.0	2.6537336 (0)	7.9860075 (0)
2.0	6.3725089 (0)	-4.8155760 (-1)
3.0	2.2259291 (0)	-8.2108985 (0)
4.0	-4.3665653 (0)	-6.3969361 (0)
5.0	-6.2847381 (0)	1.8224918 (0)
6.0	-2.6307432 (0)	7.9241956 (0)
7.0	2.5729388 (0)	7.9316087 (0)
8.0	5.8219569 (0)	3.8618211 (0)
9.0	6.5350663 (0)	-6.7566772 (-1)
10.0	5.9388792 (0)	-3.7014109 (0)
11.0	5.3782407 (0)	-4.9519596 (0)
12.0	5.5811504 (0)	-4.6336295 (0)
13.0	6.4670344 (0)	-2.4974980 (0)
14.0	6.6690601 (0)	1.9527910 (0)
15.0	3.2493773 (0)	7.1206943 (0)
16.0	-4.4561887 (0)	6.1440547 (0)
17.0	-5.7741635 (0)	-4.6477699 (0)
18.0	6.0964994 (0)	-4.2241884 (0)
19.0	-1.4177819 (0)	7.3398720 (0)
20.0	-9.6858931 (-1)	-7.2332424 (0)
21.0	-6.6764866 (-1)	7.0443546 (0)
22.0	6.3492585 (0)	-4.2988494 (0)
23.0	-6.1234421 (0)	-4.6021056 (0)
24.0	-8.9923973 (0)	8.2049658 (-1)
25.0	-9.6657717 (0)	9.0036500 (-1)
26.0	-7.0547147 (0)	-4.0091680 (0)
27.0	1.3610078 (1)	1.3702099 (0)
28.0	3.5065223 (0)	-1.0053659 (0)
29.0	7.6555914 (-2)	-3.5713688 (-2)
30.0	3.5800001 (-4)	-2.1548341 (-4)

X	U(-195.0,X)	U'(-195.0,X)
	180	181
0.	-2.3020100 (0)	-3.2145874 (0)
1.0	-2.6701618 (0)	2.6014877 (0)
2.0	1.3163795 (0)	4.1537625 (0)
3.0	3.2086871 (0)	-8.3758074 (-1)
4.0	3.1944062 (-1)	-4.5009359 (0)
5.0	-2.9498066 (0)	-1.9781894 (0)
6.0	-2.5826141 (0)	2.7879598 (0)
7.0	3.7167102 (-1)	4.4448394 (0)
8.0	2.7927438 (0)	2.4181913 (0)
9.0	3.3030083 (0)	-7.0467407 (-1)
10.0	2.5222488 (0)	-2.9105170 (0)
11.0	1.5484420 (0)	-3.8776818 (0)
12.0	1.0625446 (0)	-4.1059512 (0)
13.0	1.3323015 (0)	-3.9457794 (0)
14.0	2.3551409 (0)	-3.1252407 (0)
15.0	3.4740869 (0)	-8.2418600 (-1)
16.0	2.6274835 (0)	2.8139198 (0)
17.0	-1.6933041 (0)	3.5856400 (0)
18.0	-3.0815109 (0)	-2.2369815 (0)
19.0	3.4420774 (0)	-1.6464645 (0)
20.0	-2.0748563 (0)	3.2094528 (0)
21.0	1.9392801 (0)	-3.2247612 (0)
22.0	-3.5328225 (0)	1.8564517 (0)
23.0	3.5585575 (0)	1.9597879 (0)
24.0	3.6092143 (0)	-1.9600396 (0)
25.0	2.3370083 (0)	-2.6455715 (0)
26.0	4.7831321 (0)	-1.2022906 (0)
27.0	1.0621889 (0)	2.3062718 (0)
28.0	4.3228575 (0)	-8.5289662 (-1)
29.0	1.8712500 (-1)	-7.7048879 (-2)
30.0	1.4112923 (-3)	-7.8979215 (-4)

X	U(-200.0,X)	U'(-200.0,X)
	186	187
0.	1.2538578 (0)	-1.7732255 (0)
1.0	-1.2568110 (0)	-1.7691228 (0)
2.0	-1.2723948 (0)	1.7465854 (0)
3.0	1.1734661 (0)	1.8790820 (0)
4.0	1.4543866 (0)	-1.4417000 (0)
5.0	-7.6183155 (-1)	-2.2508266 (0)
6.0	-1.7664834 (0)	4.2992016 (-1)
7.0	-3.5740170 (-1)	2.4191726 (0)
8.0	1.3827137 (0)	1.5863005 (0)
9.0	1.7775078 (0)	-5.3040910 (-1)
10.0	1.0322007 (0)	-2.0037527 (0)
11.0	6.2263448 (-2)	-2.4055720 (0)
12.0	-5.8515450 (-1)	-2.2661662 (0)
13.0	-7.9838275 (-1)	-2.1408087 (0)
14.0	-5.5868941 (-1)	-2.2351390 (0)
15.0	2.4391444 (-1)	-2.2900898 (0)
16.0	1.4862239 (0)	-1.4748704 (0)
17.0	1.8453005 (0)	8.2520684 (-1)
18.0	-5.2335498 (-1)	2.1262000 (0)
19.0	-1.7435762 (0)	-1.1535499 (0)
20.0	2.0402108 (0)	-5.2800487 (-1)
21.0	-1.8211655 (0)	1.1064063 (0)
22.0	2.1128785 (0)	-6.4490662 (-1)
23.0	-2.0126891 (0)	-9.6534933 (-1)
24.0	-1.1922458 (0)	1.5848084 (0)
25.0	6.8645296 (-1)	1.6589496 (0)
26.0	2.7531652 (-1)	1.5695891 (0)
27.0	-3.0751331 (0)	3.7641032 (-1)
28.0	3.8786398 (0)	-2.7595972 (-1)
29.0	3.9219450 (-1)	-1.3703444 (-1)
30.0	5.0064759 (-3)	-2.5737245 (-3)

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