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CALCULATION OF CAUCHY NUMBERS

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SUMMARY

This report presents a method of calculating Cauchy numbers which has certain advantages.

INTRODUCTION

A Cauchy number $N_{-p,j,q}$ is the constant term of the expansion of $x^{-p}(x+1/x)^{j}(x-1/x)^{q}$ where p, j, q are integers. Since the direct method is difficult for even moderate values of p, j, q, alternative methods for calculating $N_{-p,j,q}$ were developed. These alternate or indirect methods are based on calculating tables for $N_{-p,o,q}$ and then using the recurssion formula

$$N_{-p,\ j+1,\ q} = N_{-p+1,\ j,\ q} + N_{-p\ -\ 1,\ j,\ q}$$

to calculate $N_{-p,j,q}$ for j greater than 0. A computer technique for directly calculating a Cauchy number and eliminating the intermediate construction of tables has been developed.

DEVELOPMENT

This method for calculating a Cauchy number is based upon a program for the "literal" multiplication of polynomials as developed by the author, see Reference 1. In this method, $(x + 1/x)$ is represented as follows:

20000000+01 + 10000000+01 + 51505050+08 + 10000000+01 + 49505050+08

The floating point representation of two is the number of terms in the series. One is the numerical coefficient and 51505050+08 represents x to the first

power. Similarly 49505050+08 represents x to -1 power. The series package can be used with four unknowns raised to -49 to +49 power. The scheme is normalized to 50505050+08 which represents $x^0\ y^0\ z^0\ w^0$. This representation enables one to "algebraically" multiply series in a computer.

In the routine for calculating a Cauchy number, p, j, q are inputs. The $(x + 1/x)$ series is raised to the j^{th} power, the $(x - 1/x)$ series to the q^{th} power. The product of these is multiplied by x to the $-p^{th}$ power and the coefficient of the constant term or in our notation the coefficient of that term represented by 50505050+08 is the Cauchy number $N_{-p,j,q}$.

This representation allows one to perform the indicated expansion and extract the coefficient of the constant term which is the desired Cauchy number.

CONCLUSION

The method has the advantage of calculating Cauchy numbers directly and obviates the use of intermediate tables.

REFERENCE

Charnow, Milton L. — "Computer Program For the Algebraic Manipulation of Series", Goddard Report X-542-63-324.