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TECHNICAL MEMORANDUM

AUTHOR(S): B. L. Amstadter

TITLE: RELIABILITIES OF REDUNDANT ELECTRICAL GENERATOR SYSTEMS

ABSTRACT

This report summarizes the objectives, conclusions, and methodology of a reliability study of various redundant configurations of electrical generating systems. Two general redundancy types are considered: active-parallel and sequential. Applicable equations are defined, and the numbers of units required to provide 35 kw of power with various degrees of reliability are calculated. Several relatively equivalent options are presented for achieving each level of reliability.

APPROVED:
DEPARTMENT HEAD M. A. Patterson

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SNAP-8 Reliability

NOTE: This document is considered preliminary and is subject to revision as analysis progresses and additional data are acquired. The general reader may encounter internal reference not available to him.

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RELIABILITIES

OF

REDUNDANT ELECTRICAL GENERATOR SYSTEMS

I. OBJECTIVE

The objective of this study was to determine and compare the reliabilities of various redundant configurations of SNAP-8 and SNAP-2 electrical generating systems. A number of assumptions were made to facilitate the statistical calculations; these assumptions are listed in the discussion.

II. CONCLUSION

The number of SNAP-2 systems required to provide equivalent power at a reliability equal to that of a SNAP-8 system is such that this approach becomes impractical, even if switching and other auxiliary functions were 100% reliable.

III. SUMMARY

There are two basic redundant configurations: active-parallel and sequential. In an active-parallel system, all units are operating at once, and the total output capability exceeds the system requirements. As units fail, the available output drops until it equals the overall system requirement. The failure of one additional unit then results in a system output below requirements, and the system is considered to have failed.

In a sequential system, only enough units to meet system output requirements are initially operated. A number of spares are available which are used to replace the original units as they fail. After the last spare has been used, the next failure results in an output below system requirements, and a system failure has occurred. If perfect failure-sensing and switching devices are used, the reliability of a sequential configuration will be somewhat higher than that of an active parallel system having the same total number of units. This results from the fact that the extra units in a sequential system are not subject to failure during their standby period, whereas in an active system, all units are experiencing the failure rate.

A more complex analysis stems from the fact that extra units in a sequential system do experience some smaller failure rate when in a standby condition. This failure rate is the result of meteoroids and other mission conditions which affect both operating and nonoperating units. Consequently, the gain in reliability of a sequential system over an active system is less than that which would otherwise be obtained.

The reliabilities of a number of redundant configurations have been computed. These include both active-parallel and sequential configurations. The results are summarized below.

A. BASIC CRITERIA

System (EGS)	SNAP-8	SNAP-2
Operating Reliability (10,000 hr)	.800	.800
Standby Reliability (10,000 hr)	.960*	.960*
Output	35 kw	3 kw or 5 kw

*Reliability of .960 = failure rate (FR) of .0408

B. OPTIONS

1. System Reliability = .80 -- One SNAP-8 System = .800

Equivalent SNAP-2 Systems

Power	3 kw (36 total)		5 kw (35 total)	
Configuration	Number	Reliability	Number	Reliability
Active Parallel	16	.798	9 10	.739 .879
Sequential (no stand-by FR)	15 16	.719 .866	9	.793
Sequential (standby FR = .0408)	15 16	.706 .853	9	.783

2. System Reliability = .96 -- Two SNAP-8 Systems in Active Parallel-Redundancy = .960

Equivalent SNAP-2 Systems

Power	3 kw (36 total)		5 kw (35 total)	
Configuration	Number	Reliability	Number	Reliability
Active Parallel	18 19	.948 .977	11	.950
Sequential (no standby FR)	17 18	.945 .980	10 11	.926 .978
Sequential (standby FR = .0408	17 18	.935 .974	10 11	.917 .973

3. System Reliability = .98 -- Two SNAP-8 Systems in Sequential Redundancy = .979 (No Standby Failure Rate)

Equivalent SNAP-2 Systems

Power	3 kw (36 total)		5 kw (35 total)	
Configuration	Number	Reliability	Number	Reliability
Active Parallel	19	.977	12	.981
Sequential (no standby FR)	18	.980	11	.978
Sequential (standby FR = .0408	18	.973	11	.972

4. System Reliability = .975 -- Two SNAP-8 Systems in Sequential Redundancy = .975 (Standby FR = .0408)

Equivalent SNAP-2 Systems

Power	3 kw (36 total		5 kw (35 total)	
Configuration	Number	Reliability	Number	Reliability
Active Parallel	19	.977	12	.981
Sequential (no standby FR)	18	.980	11	.978
Sequential (standby FR = .0408	18	.973	11	.972

IV. DISCUSSION

A. SCOPE OF REDUNDANCY STUDY

There are numerous technical problems associated with any redundant system, particularly one as complex as an EGS. The nature and scope of these problems, however, were not the subject of this study. A major program would be necessary to determine, investigate, and resolve even the most immediate problems.

Another possible consideration is the application of redundancy to subsystems or components, either instead of redundancy on an EGS level, or as a supplement to it. For example, an EGS consisting of one reactor and two power conversion systems could be evaluated; or the possible use of two L/C pump-motor assemblies could be investigated. Again, this study did not undertake the analysis of these possibilities; but was limited to redundancy considerations at the EGS level.

B. ASSUMPTIONS IN REDUNDANCY ANALYSIS

1. The reliability of the detection and switching functions is perfect, i.e., 1.00. That is, these functions do not contribute to the overall system failure rate which is determined only by the reliability of the EGS (SNAP-8 or SNAP-2) elements and the redundancy configuration. This assumption may be made because the relative contribution of detection and switching devices to the overall system failure rate is small compared to the contribution of the EGS's (perhaps an order of magnitude smaller). Secondly, since the relative reliabilities of two alternate systems are being compared; whatever small failure rates may exist for these functions, to some degree, tend to balance out.

2. The standby systems are subject to the same constant failure rate from meteoroids as the operating systems throughout the entire mission life. The reliability of one unit with respect to meteoroids is .96 for the mission life, corresponding to a

failure rate of .0408 per 10,000 hours (4.08×10^{-6} per hr). Vacuum, temperature, radiation, and other space environments are assumed to have no effect on the reliability or longevity of nonoperating systems. (Any effects would be considerably less than those resulting from meteoroids, and can be considered to be included in the .0408 standby failure rate.)

3. The 10,000 hour functional reliability for any single system is .8333 from the time that the particular system starts, not from the start of the mission. That is, the functional failure rate for an individual EGS is .1824 per 10,000 hours (18.24×10^{-6} per hour) commencing from the start of that unit's functional operation. Thus, the total operational reliability, which is the product of the meteoroid reliability x the functional reliability, = .800.

4. Subjection to launch environments does not affect the standby failure rate. Launch dynamics result in an increase in the operating random failure rate from some lower level to 18.24×10^{-6} per hour. This assumption, although a simplification of actual phenomena, is valid because a random failure rate is the result of sudden stress accumulations occurring randomly. The relationship of these stresses to the 'strength' of the system determines the system failure rate. The effects of launch dynamics are to lower the system 'strength' and increase the probability of random stress accumulation exceeding system capability during functional operation.

5. All systems are assumed to operate within their design lives after debugging and before wearout. The calculations are based on a constant failure rate (exponential reliability function, $R = e^{-\lambda t}$) for a single EGS unit.

6. Failure of one unit will not affect the reliability or longevity of any of the other units, either operating or non-operating. That is, there are no interactions between redundant units. It is assumed that, even in an active-parallel system, the amount of derating resulting from additional units sharing the load is not enough to significantly affect the failure rate of an individual unit.

7. The mission will not be affected by the startup time required to attain electrical power output. Either auxiliary power will be available, or, in the case of SNAP-2, the mission will be capable of proceeding for short durations at reduced power levels.

8. Redundancy is accomplished by switching in a complete EGS, rather than by redundancy of selected subsystems. No component of a failed system will be used in another system. (Not necessarily the best approach.)

V. CALCULATIONS

A. ACTIVE-PARALLEL SYSTEM

The Binomial expansion is applicable = $(R + Q)^n$

where R = reliability of one unit

Q = probability of failure of one unit = 1-R

n = number of units in parallel

$$(R + Q)^n = R^n + nR^{n-1}Q + \frac{n(n-1)R^{n-2}Q^2}{2!} + \frac{n(n-1)(n-2)R^{n-3}Q^3}{3!} + \ldots \qquad (1)$$

1st term is probability of 0 failures
2nd term is probability of exactly 1 failure
3rd term is probability of exactly 2 failures, etc.

System reliability, R_s, is the sum of the first k + 1 terms where k is the allowable number of failures. That is, R_s = probability of 0 failures + probability of 1 failure + probability of 2 failures + . . . probability of k failures.

For example; if there are 8 units in the system and 3 failures are permitted, n = 8, k = 3, k + 1 = 4

$$R_s = P(0 \text{ failures}) + P(1 \text{ failure}) + P(2 \text{ failures}) + P(3 \text{ failures})$$

R for 1 unit = .80

Q for 1 unit = 1 - R = .2

1. <u>35 kw Unit (1 Unit is Required to Provide 35 kw)</u>

n = 2 $R_s = .8^2 + 2(.8)(.2)$

$= .640 + .320 = \underline{.960}$

2. <u>3 kw Unit (12 Units are Required to Provide 36 kw)</u>

a. $n = 15$

$$R_s = .8^{15} + 15(.8)^{14}(.2) + \frac{15(14)(.8)^{13}(.2)^2}{2} + \frac{15(14)(13)(.8)^{12}(.2)^3}{6}$$

$$= .0352 + .1319 + .2309 + .2501 = \underline{.648}$$

b. $n = 16$

$$R_s = .8^{16} + 16(.8)^{15}(.2) + \frac{16(15)(.8)^{14}(.2)^2}{2} + \frac{16(15)(14)(.8)^{13}(.2)^3}{6} + \frac{16(15)(14)(13)(.8)^{12}(.2)^4}{24}$$

$$= .0281 + .1126 + .2111 + .2463 + .2001 = \underline{.798}$$

c. $n = 17$

$$R_s = .8^{17} + 17(.8)^{16}(.2) + \frac{17(16)(.8)^{15}(.2)^2}{2} + \frac{17(16)(15)(.8)^{14}(.2)^3}{6} + \frac{17(16)(15)(14)(.8)^{13}(.2)^4}{24} + \frac{17(16)(15)(14)(13)(.8)^{12}(.2)^5}{120}$$

$$= .0225 + .0956 + .1912 + .2391 + .2092 + .1360 = \underline{.894}$$

d. $n = 18$

$$R_s = .8^{18} + 18(.8)^{17}(.2) + \frac{18(17)(.8)^{16}(.2)^2}{2} + \frac{18(17)(16)(.8)^{15}(.2)^3}{6} + \frac{18(17)(16)(15)(.8)^{14}(.2)^4}{24} + \frac{18(17)(16)(15)(14)(.8)^{13}(.2)^5}{120} + \frac{18(17)(16)(15)(14)(13)(.8)^{12}(.2)^6}{720}$$

$$= .0180 + .0811 + .1723 + .2297 + .2153 + .1507 + .0314 = \underline{.950}$$

e. $n = 19$

$$R_s = .8^{19} + 19(.8)^{18}(.2) + \frac{19(18)(.8)^{17}(.2)^2}{2} + \frac{19(18)(17)(.8)^{16}(.2)^3}{6} + \frac{19(18)(17)(16)(.8)^{15}(.2)^4}{24} + \frac{19(18)(17)(16)(15)(.8)^{14}(.2)^5}{120} + \frac{19(18)(17)(16)(15)(14)(.8)^{13}(.2)^6}{720} + \frac{19(18)(17)(16)(15)(14)(13)(.8)^{12}(.2)^7}{5040}$$

$$= .0144 + .0685 + .1540 + .2182 + .2182 + .1637 + .0955 + .0443 = \underline{.977}$$

3. <u>5 kw Unit (7 Units are required to provide 35 kw)</u>

a. n = 9

$$R_s = .8^9 + 9(.8)^8(.2) + \frac{9(8)(.8)^7(.2)^2}{2}$$

$$= .1343 + .3022 + .3022 = \underline{.739}$$

b. n = 10

$$R_s = .8^{10} + 10(.8)^9(.2) + \frac{10(9)(.8)^8(.2)^2}{2} + \frac{10(9)(8)(.8)^7(.2)^3}{6}$$

$$= .1074 + .2684 + .3020 + .2013 = \underline{.879}$$

c. n = 11

$$R_s = .8^{11} + 11(.8)^{10}(.2) + \frac{11(10)(.8)^9(.2)^2}{2} + \frac{11(10)(9)(.8)^8(.2)^3}{6} + \frac{11(10)(9)(8)(.8)^7(.2)^4}{24}$$

$$= .0859 + .2362 + .2953 + .2215 + .1107 = \underline{.950}$$

d. n = 12

$$R_s = .8^{12} + 12(.8)^{11}(.2) + \frac{12(11)(.8)^{10}(.2)^2}{2} + \frac{12(11)(10)(.8)^9(.2)^3}{6} + \frac{12(11)(10)(9)(.8)^8(.2)^4}{24} + \frac{12(11)(10)(9)(8)(.8)^7(.2)^5}{120}$$

$$= .0687 + .2062 + .2835 + .2362 + .1329 + .0532 = \underline{.981}$$

B. SEQUENTIAL SYSTEM - NO STANDBY FAILURE RATE

The Poisson distribution is applicable ($e^{-x} \cdot e^{x} = 1$),

since $e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \cdots$

$$e^{-x} \cdot e^{x} = x^{-x} \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)$$

Let $x = \lambda t$ = average number of failures in time t. This results in the equation:

$$e^{-\lambda t}\left[1 + \lambda t + \frac{(\lambda t)^2}{2!} + \frac{(\lambda t)^3}{3!} + \frac{(\lambda t)^4}{4!} + \cdots\right] = 1, \qquad (2)$$

where $R = e^{-\lambda t}$ = the probability of 0 failures

$Q_1 = \lambda t(e^{-\lambda t})$ = the probability of exactly 1 failure

$Q_2 = \frac{(\lambda t)^2}{2!}(e^{-\lambda t})$ = the probability of exactly 2 failures

$Q_3 = \frac{(\lambda t)^3}{3!}(e^{-\lambda t})$ = the probability of exactly 3 failures

etc.

Again, system reliability is the sum of the first k + 1 terms where k is the allowable number of failures:

R_s = probability of 0 failures + probability of 1 failure + . . .

probability of k failures

The failure rate, λt, however, is the failure rate of the operating system, not the failure rate of an individual unit. If there are m units operating, and each unit has a failure rate $\lambda_u t$, the system failure rate $\lambda t = m \cdot \lambda_u t$.

R for 1 unit = .80; λt = .223

1. <u>35 kw Unit; One Unit is Required to Provide 35 kw</u>

$$n = 2,\ R_s = e^{-.223}(1 + .223) = .979$$

2. 3 kw Unit (12 Units are Required to Provide 36 kw)

a. n = 15

$$R_s = e^{-2.6777}\left(1 + 2.6777 + \frac{2.6777^2}{2} + \frac{2.6777^3}{6}\right)$$

$$= .06872\ (1 + 2.6777 + 3.5850 + 3.1998) \qquad = \underline{.719}$$

b. n = 16

$$R_s = e^{-2.6777}\left(1 + 2.6777 + \frac{2.6777^2}{2} + \frac{2.6777^3}{6} + \frac{2.6777^4}{24}\right)$$

$$= .06872\ (1 + 2.6777 + 3.5850 + 3.1998 + 2.1420) \qquad = \underline{.866}$$

c. n = 17

$$R_s = e^{-2.6777}\left(1 + 2.6777 + \frac{2.6777^2}{2} + \frac{2.6777^3}{6} + \frac{2.6777^4}{24} + \frac{2.6777^5}{120}\right)$$

$$= .06872\ (1 + 2.6777 + 3.5850 + 3.1998 + 2.1420 + 1.1471) \qquad = \underline{.945}$$

d. n = 18

$$R_s = e^{-2.6777}\left(1 + 2.6777 + \frac{2.6777^2}{2} + \frac{2.6777^3}{6} + \frac{2.6777^4}{24} + \frac{2.6777^5}{120} + \frac{2.6777^6}{720}\right)$$

$$= .06872\ (1 + 2.6777 + 3.5850 + 3.1998 + 2.1420 + 1.1471 + .5119) \qquad = \underline{.980}$$

3. 5 kw Unit (7 Units are Required to Provide 35 kw)

a. n = 9

$$R_s = e^{-1.5620}\left(1 + 1.5620 + \frac{1.5620^2}{2}\right)$$

$$= .20972\ (1 + 1.5620 + 1.2200) \qquad = \underline{.793}$$

b. n = 10

$$R_s = e^{-1.5620}\left(1 + 1.5620 + \frac{1.5620^2}{2} + \frac{1.5620^3}{6}\right)$$

$$= .20972\ (1 + 1.5620 + 1.2200 + .6352) \qquad = \underline{.926}$$

c. n = 11

$$R_s = e^{-1.5620}\left(1 + 1.5620 + \frac{1.5620^2}{2} + \frac{1.5620^3}{6} + \frac{1.5620^4}{24}\right)$$

$$= .20972\ (1 + 1.5620 + 1.2200 + .6352 + .2480) \qquad = \underline{.978}$$

C. SEQUENTIAL SYSTEM - OPERATIONAL FAILURE RATE = $\lambda_1 t$, STANDBY FAILURE RATE = $\lambda_2 t$

The reliability of a system with one unit in operation and a second unit on standby is given by:

$$R_s = e^{-\lambda_1 t} \left[1 + \frac{\lambda_1}{\lambda_2} \left(1 - e^{-\lambda_2 t} \right) \right] \qquad (3)$$

When the number of units exceeds 2, the exact formula becomes quite complex; and the following approximation, which is accurate to three decimal places, may be used:

$$R_6 \approx R_5 + \frac{R_2 - R_3}{R_1 - R_3} \left(R_4 - R_5 \right) \qquad (4)$$

R_1 = reliability of 2 unit sequential system with no standby failure rate. (From Equation 2)

R_2 = reliability of 2 unit sequential system with standby failure rate (From Equation 3)

R_3 = reliability of 2 unit active-parallel system (From Equation 1)

R_4 = reliability of n unit sequential system with no standby failure rate. (From Equation 2)

R_5 = reliability of n unit active-parallel system (From Equation 1)

R_6 = reliability of n unit sequential system with standby failure rate = $\lambda_2 t$. (This is the unknown quantity)

As long as $\lambda_2 t$ (standby FR) is less than $\lambda_1 t$ (operational FR), R_2 will be between R_1 and R_3, and R_6 will be between R_4 and R_5.

1. 35 kw unit; one unit is required to provide 35 kw

$R_{operating} = .800 \quad (\lambda_1 t = .223)$

$R_{standby} = .960 \quad (\lambda_2 t = .0408)$

$$R_s = .800 \left[1 + \frac{.223}{.0408}(1 - .960)\right]$$

$$= .800 \left[1 + .219\right] = \underline{.975}$$

2. 3 kw unit; 12 units are required to provide 36 kw (from Equation 4, for n = 18)

$R_1 = .979$

$R_2 = .975$

$R_3 = .960$

$R_4 = .980$

$R_5 = .948$

$R_6 = .948 + \frac{.015}{.019}(.032) = \underline{.973}$

3. 5 kw unit; 7 units are required to provide 36 kw (from Equation 4 for n = 11)

$R_1 = .979$

$R_2 = .975$

$R_3 = .960$

$R_4 = .978$

$R_5 = .950$

$R_6 = .950 + \frac{.015}{.019}(.028) = \underline{.972}$