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THEORETICAL STUDY OF NATURAL CONVECTION FLOWS IN CLOSED-END CYLINDRICAL VESSELS

20 March 1967

UNIVERSITY OF DENVER

DENVER RESEARCH INSTITUTE

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I. ABSTRACT

This study is concerned with the analytical solutions of natural convection flows in closed-end cylindrical vessels to obtain exact solutions of temperature and velocity distribution in the laminar flow region under steady state condition.

The temperature and velocity distributions, in general power series of displacements, are substituted into the three basic equations of continuity, momentum and energy. The relationships between coefficients of all powers are obtained through initial and boundary conditions and recurrence formulae.

For constant wall temperature conditions the general solution of temperature and velocity may be expressed as a function of displacement, geometry ratio and Rayleigh number. Prandtl number does not enter as an independent parameter.

II. INTRODUCTION

Natural convection flows in closed-end cylindrical vessels or tubes has been of considerable interest in many practical applications such as the stratification problem of a partially filled liquid propellant cylindrical tank and cooling problem of turbine blades. Theoretical studies are usually based on three fundamental equations; namely continuity, momentum, and energy equations. Theoretically speaking, with initial and boundary conditions one should be able to solve for the temperature and velocity distributions from these equations. Unfortunately, no close-form transient solutions are available up to this date except some numerical solutions by finite difference method.[1] Steady state approximate solutions have been obtained by many investigators[2,3,4] mostly through integrated equations of the continuity, momentum, and energy rather than solving from the differential equations. In this report, the steady state solutions are obtained directly from the differential equations in the form of more exact power series solutions. An interesting result from this report which differs from that reported in the references (2, 3), is that the Prandtl number does not enter as an independent variable but rather, is included in the Rayleigh number.

III. BASIC EQUATIONS AND BOUNDARY CONDITIONS

Geometrical Configuration: Consider the configuration under study for a cylindrical vessel partially filled with liquid as shown in Figure 1. The side wall temperature is kept as a constant, Tw, the bottom of the vessel is insulated, the cold liquid at a temperature, T_1, is fed through the center of the bottom. The level of the liquid inside the vessel is maintained at a constant level. This means that the amount of incoming liquid is completely vaporized on the top surface. Other cases will simply change the boundary conditions.

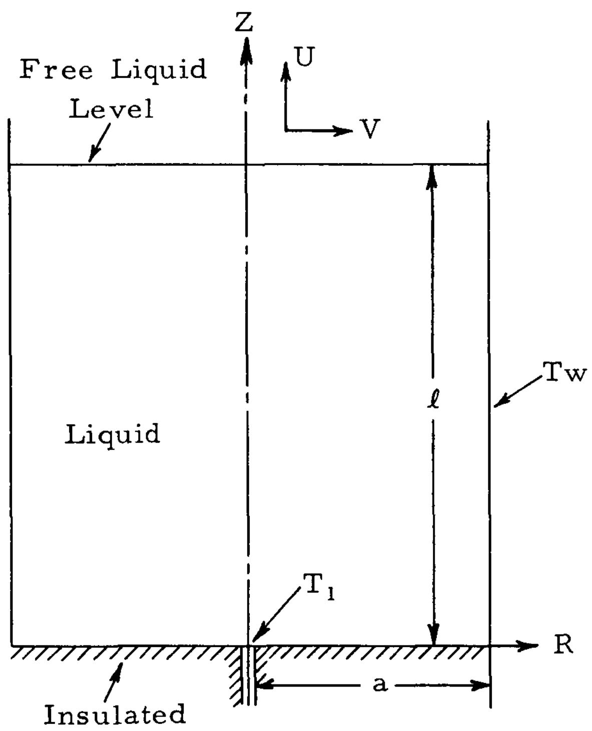


Figure 1. Cylindrical Vessel Partially Filled with Liquid

Differential Equations: Three differential equations of continuity, momentum, and energy may be written respectively for any cylindrical differential liquid element as a result of the steady axisymmetrical flow in the following:

$$\frac{\partial}{\partial R}(RV) + \frac{\partial}{\partial Z}(RU) = 0$$

$$U \frac{\partial U}{\partial Z} + V \frac{\partial U}{\partial R} = \beta f(T - Tw) + \nu \left[\frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial U}{\partial R} \right) + \frac{\partial^2 U}{\partial Z^2} \right] \tag{2}$$

$$U \frac{\partial T}{\partial Z} + V \frac{\partial T}{\partial R} = \alpha \left[\frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial T}{\partial R} \right) + \frac{\partial^2 T}{\partial Z^2} \right] \tag{3}$$

where

- U, V - Velocity components of the liquid in the direction of Z and R axis.
- β - The coefficient of the volumetric expansion.
- f - The axial body force per unit mass or the gravitational field.
- ν - The kinematic viscosity of the liquid.
- α - The thermal diffusivity of the liquid.
- T - The temperature of the liquid.
- Tw - The wall surface temperature.

Boundary Conditions: Since the velocities on the wall surface are zero and the thermal boundary conditions are specified, the boundary conditions are as follows:

$$U(Z, a) = U(0, R) = U(\ell, R) = V(Z, a) = V(0, R) = 0 \tag{4}$$

$$T(Z, a) = Tw, \left. \frac{\partial T}{\partial Z} \right|_{Z=0} = 0, \left. \frac{\partial T}{\partial Z} \right|_{Z=\ell} = 0 \tag{5}$$

where

- ℓ - The liquid level in the vessel.
- a - The inside radius of the vessel.

Normalized Differential Equations: Let velocity components, the temperature of the liquid and the coordinates of the system be normalized according to the following dimensionless quantities:

$$u = \frac{a^2}{\alpha \ell} U, \ v = \frac{a}{\alpha} V, \ t = \frac{\beta f a^4}{\alpha \ell \nu} (Tw - T), \ z = \frac{Z}{\ell}, \ r = \frac{R}{a} \tag{6}$$

The normalized differential equations may be obtained by the substitution of equation (6) into equations (1) through (3) as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial z} = 0 \tag{7}$$

$$\frac{1}{P_r}\left[u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r}\right] = -t + \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{a^2}{\ell^2}\frac{\partial^2 u}{\partial z^2}\right] \tag{8}$$

$$v\frac{\partial t}{\partial r} + u\frac{\partial t}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right) + \frac{a^2}{\ell^2}\frac{\partial^2 t}{\partial z^2} \tag{9}$$

where P_r is the Prandtl number.

Normalized Boundary Conditions: The boundary conditions, as given by equations (4) and (5) may be normalized similarly into:

$$u(z, 1) = u(0, r) = u(1, r) = v(z, 1) = v(0, r) = 0 \tag{10}$$

$$t(z, 1) = 0, \;\; t(0, 0) = R_a \frac{a}{\ell}, \;\; \left.\frac{\partial t}{\partial z}\right|_{z=0} = \left.\frac{\partial t}{\partial z}\right|_{z=1} = 0 \tag{11}$$

where $R_a = \dfrac{\beta f a^3 (Tw - T_1)}{\nu \alpha}$, is the Rayleigh number.

IV. POWER SERIES SOLUTIONS

Power series solutions of the velocity components and the temperature may be assumed as a function of z and r with arbitrary coefficients. All arbitrary coefficients may be evaluated by recurrence formulae resulting from satisfying differential equations and boundary conditions.

Let

$$u = -\left(z^4 - \frac{5}{2} z^3 + \frac{3}{2} z^2\right)\left[(1 - r^2)\sum_{n=0}^{\infty} a_n r^{2n}\right] \tag{12}$$

$$t = \left(\frac{z^3}{3} - \frac{z^2}{2} + 1\right)\left[(1 - r^2)\sum_{m=0}^{\infty} b_m r^{2m}\right] \tag{13}$$

Substituting equation (12) into continuity equation (7)

$$v = \left(2z^3 - \frac{15}{4} z^2 + \frac{3}{2} z\right)\left[\sum_{n=0}^{\infty} \frac{a_n}{n+1} r^{2n+1} - \sum_{n=0}^{\infty} \frac{a_n}{n+2} r^{2n+3}\right] \tag{14}$$

It is obvious that u and v satisfy the boundary conditions

$$u(0, r) = u(1, r) = u(z, 1) = v(0, r) = 0$$

To satisfy $v(z, 1) = 0$, one relationship between all coefficients is obtained, namely

$$\sum_{n=0}^{\infty} \frac{a_n}{(n+1)(n+2)} = 0 \tag{15}$$

It is also obvious that t satisfies the thermal boundary conditions as given in equations (11)

$$t(z, 1) = 0, \quad \left.\frac{\partial t}{\partial z}\right|_{z=0} = \left.\frac{\partial t}{\partial z}\right|_{z=1} = 0$$

To satisfy the condition $t(0, 0) = 0$, this gives

$$b_0 = R_a \frac{a}{\ell}$$

Recurrence formulae:

From equations (12) through (14),

$$u\frac{\partial u}{\partial z} = \left(4z^7 - \frac{35}{2}z^6 + \frac{111}{4}z^5 - \frac{75}{4}z^4 + \frac{9}{2}z^3\right)\left[(1-r^2)\sum_{n=0}^{\infty} a_n r^{2n}\right]^2$$

$$v\frac{\partial u}{\partial r} = -\left(4z^7 - \frac{35}{2}z^6 + \frac{111}{4}z^5 - \frac{75}{4}z^4 + \frac{9}{2}z^3\right)\left[\sum_{n=0}^{\infty}\frac{a_n}{n+1}r^{2n+1} - \sum_{n=0}^{\infty}\frac{a_n}{n+2}r^{2n+3}\right]\left[\sum_{n=0}^{\infty} n a_n r^{2n-1} - \sum_{n=0}^{\infty}(n+1)a_n r^{2n+1}\right]$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = -4\left(z^4 - \frac{5}{2}z^3 + \frac{3}{2}z^2\right)\left[\sum_{n=0}^{\infty} n^2 a_n r^{2n-2} - \sum_{n=0}^{\infty}(n+1)^2 a_n r^{2n}\right]$$

$$\frac{a^2}{\ell^2}\frac{\partial^2 u}{\partial z^2} = -\frac{a^2}{\ell^2}(12z^2 - 15z + 3)\left[(1-r^2)\sum_{n=0}^{\infty} a_n r^{2n}\right]$$

By substitutions of above four equations and equation (13) into the momentum equation (8) the following equation is obtained:

$$\begin{aligned}
\frac{1}{P_r}&\left(4z^7 - \frac{35}{2}z^6 + \frac{111}{4}z^5 - \frac{75}{4}z^4 + \frac{9}{2}z^3\right)\Bigg\{\left[(1-r^2)\sum_{n=0}^{\infty} a_n r^{2n}\right]^2 \\
&- \left[\sum_{n=0}^{\infty}\frac{a_n}{n+1}r^{2n+1} - \sum_{n=0}^{\infty}\frac{a_n}{n+2}r^{2n+3}\right]\left[\sum_{n=0}^{\infty} n a_n r^{2n-1} - \sum_{n=0}^{\infty}(n+1)a_n r^{2n+1}\right]\Bigg\} \\
&= -\left(\frac{z^3}{3} - \frac{z^2}{2} + 1\right)(1-r^2)\sum_{m=0}^{\infty} b_m r^{2m} \\
&\quad - 4\left(z^4 - \frac{5}{2}z^3 + \frac{3}{2}z^2\right)\left[\sum_{n=0}^{\infty} n^2 a_n r^{2n-2} - \sum_{n=0}^{\infty}(n+1)^2 a_n r^{2n}\right] \\
&\quad - \frac{a^2}{\ell^2}(12z^2 - 15z + 3)(1-r^2)\sum_{n=0}^{\infty} a_n r^{2n} \qquad (17)
\end{aligned}$$

Multiplying both sides by dz and integrating from 0 to 1 and rearranging, a recurrence formulae for a_n's is obtained by equating coefficients of like powers of r^{2n} as follows:

$$a_{n+1} = a_n - \frac{5}{3(n+1)^2}\left[\frac{11}{6}(b_n - b_{n-1}) - \frac{a^2}{\ell^2}(a_n - a_{n-1})\right] \tag{18}$$

Note that the Prandtl number dropped because the left hand side of equation (17) vanishes.

Similarly, from equations (12) through (14),

$$v\,\frac{\partial t}{\partial r} = 2\left(\frac{2}{3}z^7 - \frac{9}{4}z^6 + \frac{19}{8}z^5 - \frac{5}{4}z^4 - \frac{15}{4}z^3 + \frac{3}{2}z^2\right)\left[\sum_{n=0}^{\infty}\frac{a_n}{n+1}r^{2n+1}\right.$$

$$\left.-\sum_{n=0}^{\infty}\frac{a_n}{n+2}r^{2n+3}\right]\left[\sum_{m=0}^{\infty} m b_m r^{2m-1} - \sum_{m=0}^{\infty}(m+1)b_m r^{2m+1}\right]$$

$$u\,\frac{\partial t}{\partial z} = -\left[z^6 - \frac{7}{2}z^5 + 4z^4 - \frac{3}{2}z^3\right]\left[\sum_{n=0}^{\infty} a_n r^{2n} - \sum_{n=0}^{\infty} a_n r^{2n+2}\right]$$

$$\cdot\left[\sum_{m=0}^{\infty} b_m r^{2m} - \sum_{m=0}^{\infty} b_m r^{2m+2}\right]$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial t}{\partial r}\right) = 4\left(\frac{z^3}{3} - \frac{z^2}{2} + 1\right)\left[\sum_{m=0}^{\infty} m^2 b_m r^{2m-2} - \sum_{m=0}^{\infty}(m+1)^2 b_m r^{2m}\right]$$

$$\frac{a^2}{\ell^2}\frac{\partial^2 t}{\partial z^2} = \frac{a^2}{\ell^2}(2z-1)\left[\sum_{m=0}^{\infty} b_m r^{2m} - \sum_{m=0}^{\infty} b_m r^{2m+2}\right]$$

By substitutions of the above four equations into the energy equation (9) and multiplying both sides by dz and integrating from 0 to 1, a recurrence formula for b_m's is obtained by equating coefficients of like powers of r^{2n} as follows:

$$-1.0596\left[\sum_{n=0}^{\infty}\frac{a_n}{n+1}r^{2n+1}-\sum_{n=0}^{\infty}\frac{a_n}{n+2}r^{2n+3}\right]\left[\sum_{m=0}^{\infty}mb_m r^{2m-1}\right.$$

$$\left.-\sum_{m=0}^{\infty}(m+1)b_m r^{2m+1}\right]$$

$$+0.1405\left[\sum_{n=0}^{\infty}a_n r^{2n}-\sum_{n=0}^{\infty}a_n r^{2n+2}\right]\left[\sum_{m=0}^{\infty}b_m r^{2m}-\sum_{m=0}^{\infty}b_m r^{2m+2}\right]$$

$$=\frac{11}{3}\left[\sum_{m=0}^{\infty}m^2 b_m r^{2m-2}-\sum_{m=0}^{\infty}(m+1)^2 b_m r^{2m}\right]$$

or,

$$-1.0596\left[\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{m}{n+1}a_n b_m r^{2m+2n}-\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{m}{n+2}a_n b_m r^{2m+2n+2}\right.$$

$$\left.-\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(m+1)}{n+1}a_n b_m r^{2m+2n+2}+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{m+1}{n+2}a_n b_m r^{2m+2n+4}\right]$$

$$+0.1405\left[\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}a_n b_m r^{2m+2n}-\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}a_n b_m r^{2m+2n+2}\right.$$

$$\left.-\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}a_n b_m r^{2m+2n+2}+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}a_n b_m r^{2m+2n+4}\right]$$

$$=\frac{11}{3}\left[\sum_{m=0}^{\infty}m^2 b_m r^{2m-2}-\sum_{m=0}^{\infty}(m+1)^2 b_m r^{2m}\right]$$

All exponents of r's may be changed into r^{2m}, the above equation becomes,

$$\sum_{m=0}^{\infty} b_{m+1} r^{2m} = \sum_{m=0}^{\infty} b_m r^{2m} - \sum_{m=0}^{\infty} \frac{3}{11(m+1)^2} \left\{ \sum_{n=0}^{m} \left[\frac{1.0596(m-n)}{n+1} - 0.1405 \right] a_n b_{m-2} - \sum_{n=0}^{m-1} \left[1.0596 \left(\frac{m-n-1}{n+1} + \frac{m-n}{n+1} \right) + .281 \right] a_n b_{m-n-1} + \sum_{n=0}^{m-2} \left[1.0596 \left(\frac{m-n-1}{n+2} \right) - 0.1405 \right] a_n b_{m-n-2} \right\} r^{2m}$$

Therefore,

$$b_{m+1} = b_m - \frac{3}{11(m+1)^2} \left\{ \sum_{n=0}^{m} \left[\frac{1.0596(m-n)}{n+1} - .1405 \right] a_n b_{m-n} - \sum_{n=0}^{m-1} \left[1.0596 \left(\frac{m-n-1}{n+2} + \frac{m-n}{n+1} \right) + .281 \right] a_n b_{m-n-1} + \sum_{n=0}^{m-2} \left[1.0596 \left(\frac{m-n-1}{n+2} \right) - .1405 \right] a_n b_{m-n-2} \right\} \tag{19}$$

Determination of all Arbitrary Coefficients a_n's and b_m's: From the recurrence formulae as given by equations (18) and (19), all arbitrary coefficients, a_n's b_m's, may be expressed in terms of a_0 and b_0. Where b_0 is determined by the Rayleigh number $R_a \frac{a}{\ell}$, as given by equation (16). Values of a_0 may be obtained from the equation (15) by finding the root of the equation. All calculations of a_n's, b_m's, and root finding can be achieved easily by the use of high-speed digital computer. In this report all calculations are programmed in Algol language and performed by Burrough's B5500 digital computer. A computer program is attached in Appendix A.

V. DISCUSSION OF RESULTS

Coefficients a_n's and b_m's: All calculations were based on a geometry ratio of $\frac{a}{\ell}$ = 1 and 4. Values of b_0 were chosen from 5 stepping 10 until 55. Values of a_0 corresponding to various b_0, as a result of root finding by computer, are tabulated in Table 1. Three roots or a_0's for the case of $\frac{a}{\ell}$ = 1 are obtained while only one root for the case of $\frac{a}{\ell}$ = 4 is obtained. Other roots may be obtained by further finer increments of tried a_0's, but this involves more computer time.

Table 1. Values of a_0's

a/ℓ	1	1	1	4
b_0	5	15	25	55
a_0	4.77	14.84	25.16	23.86

The first ten values of a_n's and b_m's corresponding to various values of b_0, are tabulated in Table 2. For the case of $\frac{a}{\ell}$ = 1, actually 20 terms of a_n's and b_m's are calculated. For the case of $\frac{a}{\ell}$ = 4, only ten terms are calculated.

From Table 2 it is obvious that for b_0 ranging from 5 to 55, all coefficients a_n's and b_m's tend to converge to a constant value; thus, a_n's and b_m's higher than a_9 and b_9 may be assumed to be a_9 and b_9. For b_0 larger than 55, a larger number of summation terms has to be considered to converge to a constant coefficient. This involves a longer computer time.

In the root finding process of a_0 in order to satisfy the equation (15) values of a_0's starting from 0 step 1 until 200 are tried. When the total summation is close to the allowable value which is 0.01, the increment 1 is further subdivided into 64 equal divisions until finally the summation is $\leq$ .01.

Table 2. First Ten Values of a_n's and b_m's

a/ℓ	1	1	1	4
b_0	5	15	25	55
a_0	4.76	14.84	25.16	23.86
a_1	- 2.57	- 6.25	- 9.31	14.87
a_2	- 6.34	-21.55	-42.06	-38.49
a_3	- 7.18	-23.86	-41.35	-65.27
a_4	- 7.12	-22.89	-37.91	-75.59
a_5	- 7.07	-22.34	-36.01	-74.13
a_6	- 7.05	-22.12	-35.49	-71.75
a_7	- 7.04	-22.03	-35.22	-70.31
a_8	- 7.04	-22.00	-35.08	-69.50
a_9	- 7.04	-22.00	-35.00	-69.14

Dimensionless Velocity and Temperature: Once all coefficients a_n's and b_m's are determined for b_0 ranging from 5 to 55, the dimensionless velocity and temperature for the case that the coefficients a_n's converge before a_9 may be written as follows:

$$u = -(z^4 - \frac{5}{2} z^3 + \frac{3}{2} z^2)\left[(1-r^2)\left(\sum_{n=0}^{9} a_n r^{2n} + a_9 \sum_{n=10}^{\infty} r^{2n}\right)\right] \tag{20}$$

But the last term of equation (20) is a series of geometric progression, namely,

$$\sum_{n=10}^{\infty} r^{2n} = r^{20}[1 + r^2 + r^4 + \ldots +] = r^{20} \frac{(1-r^{2n})}{1-r^2}$$

for $r < 1$, as $n \to \infty$, $r^{2n} = 0$

$$\text{or} \quad \sum_{n=10}^{\infty} r^{2n} = \frac{r^{20}}{1-r^2} \tag{21}$$

Equation (20) becomes

$$u = -\left(z^4 - \frac{5}{2} z^3 + \frac{3}{2} z^2\right)\left[(1 - r^2)\sum_{n=0}^{9} a_n r^{2n} + a_9 r^{20}\right] \tag{22}$$

Similarly,

$$t = +\left(\frac{z^3}{3} - \frac{z^2}{2} + 1\right)\left[(1 - r^2)\sum_{m=0}^{9} b_m r^{2m} + b_9 r^{20}\right] \tag{23}$$

where values of a_n's and b_m's are given in Table 2. For the case of slow converging coefficients a_n's and b_m's, more terms are to be considered.

Sample velocity and temperature profiles are plotted by a calcomp digital plotter and are shown in Figures 1 through 12 of Appendix B. The plotter uses a magnetic tape which is prepared by a computer program that is included in the computer program of all calculations as shown in Appendix A. Figures 1 through 9 of Appendix B are for $\frac{a}{\ell} = 1$, and $b_0 = 5$, 15, and 25, or B(0) = 5, 15, and 25, respectively, while Figures 10 through 12 are for $\frac{a}{\ell} = 4$ and $b_0 = 55$. Actual values of the dimensionless velocities u/a_0 or U/A(0), v/a_0 or V/A(0), and the dimensionless temperature t/b_0 or T/B(0) should be multiplied by the scale factor as shown in each figure.

It is interesting to see that for higher Rayleigh numbers or higher b_0, the slope of the temperature close to the side wall is less steep than that for lower Rayleigh numbers as evidenced by the comparison of Figures 3, 6, 9 and 12 of Appendix B. The patterns of velocities for same $\frac{a}{\ell}$ does not show any fluctuations for $b_0 = 5$, 15 and 25. However, for $\frac{a}{\ell} = 4$ the velocity patterns for $b_0 = 55$ have larger fluctuation than those of lower Rayleigh numbers for $\frac{a}{\ell} = 1$ as shown by Figures 1, 2, 4, 5, 7, 8, 10 and 11 of Appendix B.

VI. CONCLUSION

More exact power series solutions are obtained for evaluating the velocity and temperature distributions for natural convection flow in an end-closed cylindrical vessel partially filled with liquid. Once a computer program is written, laborious calculations may be performed by a high-speed digital computer. The power series solutions can be applied to problems with various different boundary conditions.

The Rayleigh number seems to be the only important parameter in this study, as it should be in most natural convection study. The geometry factor definitely enters into the picture.

VII. REFERENCES

1. Barakat, Hussein Zaky and John A. Clark, "Analytical and Experimental Study of the Transient Laminar Natural Convection Flow in Partially filled Liquid Containers." Proceedings of Third International Heat Transfer, 1966.

2. Lightfill, M. J., "Theoretical Considerations on Free Convection in Tubes." Quart. Journal of Mech. and Applied Math., Vol VI, 1953, pp. 398-439.

3. Ostrach, Simon and P. R. Thornton, "On the Stagnation of Natural Convection Flows in Closed-End Tubes," Transactions of the ASME, 1956, pp. 363-366.

4. Schmidt, E. H. W., "Heat Transmission by Natural Convection at High Centrifugal Acceleration in Water-Cooled, Gas-Turbine Blades." Proceedings of the General Discussion on Heat Transfer, The Institution of Mechanical Engineers and ASME, London, England, Sept. 1951, pp. 361-363.

APPENDIX A

```
            BEGIN
FILE IN          CARD(2,10);
FILE OUT         LINE 4(2,15);
FORMAT IN        FMI (I4,F12.6,F12.6),
                 FMI1 (3(F12.4,X4));

FORMAT           FMI(11A6);

FORMAT OUT       FMO (X30,"CONVECTION FLOWS IN CLOSED-END CYLINDERS"/
                 X40,"C.W.CHIANG"//),
                 FMO3 (10(X4," A[I]",X4)),
                 FMO4 (10(F12.4    )/),
                 FMO5 (10(X3,"SB[I]",X4)),
                 FMO6 (10(F12.4    )/),
                 FMO7 (10(X3,"SAA[I]",X3)),
                 FMO8 (10(F12.4    )/),
                 FMO11 (10(X3,"  B[I]",X3)),
                 FMO12 (10(F12.4)/);

 PROCEDURE DATELINE (PROGRAM);
 VALUE PROGRAM;
 ALPHA PROGRAM;
 BEGIN OWN BOOLEAN USED;
   FORMAT HD(A4,I3,", ",A4,X2,"TIME:",I5,X10,"OUTPUT FROM PROGRAM ",
   A6,X10,"UNIVERSITY OF DENVER COMPUTING CENTER" ///),
                 LAYT (/ / "EXECUTION TIME =", I5, X03, "I/O TIME =", I5,
" SECONDS  ",A4,I3,", ", A4, X03, "TIME:", I7            / / /);

    LABEL GOTIT;
    ALPHA MO, MINS, FEB, HRS, YR, DAY;
    USED ← USED AND PROGRAM . [18 : 6] = 0;
    DAY ← TIME (0);
    YR ← DAY . [18 : 12] + "1900";
    DAY ← DAY . [42 : 6] + 10 × DAY . [36 : 6] + 100 × DAY . [30 : 6];
    FEB ← IF YR . [42 : 6] MOD 4 = 0 THEN "(FEB." ELSE "&FEB.";
    FOR MO ← "←JAN.", FEB, "←MAR.", "<APR.", "← MAY", "<JUNE", "←JULY"
    , "←AUG.", "<SEPT", "←OCT.", "<NOV.", "←DEC." DO
    BEGIN IF DAY ≤ MO . [18 : 06] THEN GO TO GOTIT;
       DAY ← DAY - MO . [18 : 6];
    END;
    GOTIT: MINS ← TIME (1) / 3600;
    HRS ← 100 × (MINS DIV 60) + MINS MOD 60;
    IF USED THEN WRITE (LINE, LAYT, TIME (2) / 60, TIME (3) / 60, MO,
    DAY, YR, HRS) ELSE WRITE (LINE, HD,MO,DAY,YR,HRS,PROGRAM);
    USED ← TRUE;
 END OF DATELINE;

PROCEDURE        DRAWGRAPH(C,N3,N4,N,NAME,B); ARRAY C[*,*,*];
                 ALPHA ARRAY NAME[*,*]; INTEGER N,N3,N4,B;
            BEGIN
```

```
    INTEGER        I,J,PX,PY,N2;
    REAL           MAX1,MAX2,MIN,PX1,PX2,PY1,PY2,PY3,PY4,S1,S2,P;
    ARRAY          X,Y[0:90];
    ALPHA ARRAY    N1[0:10];  ALPHA A1,A2;
    LABEL          L1,L2;
                   MAX1 ← C[1,1,1];
                   MAX2 ← MIN ← C[1,1,2];
                   FOR I ← 1 STEP 1 UNTIL N3 DO
                   FOR J ← 1 STEP 1 UNTIL N4 DO
              BEGIN
                   IF C[I,J,1] > MAX1 THEN MAX1 ← C[I,J,1];
                   IF C[I,J,2] > MAX2 THEN MAX2 ← C[I,J,2];
                   IF C[I,J,2] < MIN THEN MIN ← C[I,J,2];
              END;
                   IF MIN < 0 THEN GO TO L1;
                   FILL X[*] WITH 0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,
                     0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,0,0.75,0.75,
                     0.75,1.5,1.5,1.5,2.25,2.25,2.25,3,3,3,3.75,3.75,3.75,
                     4.5,4.5,4.5,5.25,5.25,5.25,6,6,6,6.75,6.75,6.75,7.5,
                     7.5;

                   FILL Y[*] WITH 5.5,5.5,5.5,4.95,4.95,4.95,4.4,4.4,4.4,
                     3.85,3.85,3.85,3.3,3.3,3.3,2.75,2.75,2.75,2.2,2.2,2.2,
                     1.65,1.65,1.65,1.1,1.1,1.1,0.55,0.55,0.55,0,0,0.1,0,0,
                     0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,
                     0,0.1,0,0,0.1;

                   PLOT(2,2,-3);
                   LYNE(X,Y,59,1);
                   PX ← 9; PX1 ← 1; PX2 ← 0.75;
                   PY ← -2; PY1 ← 0; PY2 ← 0.55; PY3 ← 0; PY4 ← 5.6;
                  N2 ← 5;
                   S1 ← 7.5 / MAX1;
                   S2 ← 5.5 / MAX2;
                   GO TO L2;
L1:                FILL X[*] WITH 0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,0,
                     0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,
                     0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,0,0,0.1,
                     0,0,0.55,0.55,0.55,1.1,1.1,1.1,1.65,1.65,1.65,2.2,2.2,
                     2.2,2.75,2.75,2.75,3.3,3.3,3.3,3.85,3.85,3.85,4.4,4.4,
                     4.4,4.95,4.95,4.95,5.5,5.5;

                   FILL Y[*] WITH 3.8,3.8,3.8,3.42,3.42,3.42,3.04,3.04,3.04,
                     2.66,2.66,2.66,2.28,2.28,2.28,1.9,1.9,1.9,1.52,1.52,
                     1.52,1.14,1.14,1.14,0.76,0.76,0.76,0.38,0.38,0.38,
                     -0.38,-0.38,-0.38,-0.76,-0.76,-0.76,-1.14,-1.14,-1.14,
                     -1.52,-1.52,-1.52,-1.9,-1.9,-1.9,-2.28,-2.28,-2.28,
                     -2.66,-2.66,-2.66,-3.04,-3.04,-3.04,-3.42,-3.42,-3.42,
                     -3.8,-3.8,-3.7,-3.8,-3.8,-3.7,-3.8,-3.8,-3.7,-3.8,-3.8,
                     -3.7,-3.8,-3.8,-3.7,-3.8,-3.8,-3.7,-3.8,-3.8,-3.7,-3.8,
                     -3.8,-3.7,-3.8,-3.8,-3.7,-3.8,-3.8,-3.7;

                   PLOT(2,5,-3);
                   LYNE(X,Y,86,1);
X[1] ← Y[1] ← Y[2] ← 0; X[2] ← 5.5;
```

```
                DASHLINE(X,Y,2,1);
                PX ← 7; PX1 ← 0; PX2 ← 0.55;
                PY ← -5; PY1 ← -2.8; PY2 ← 0.76; PY3 ← -3.8; PY4 ← 3.9;
                N2 ← 6;
                S1 ← 3.8 / MAX2; S2 ← -3.8 /MIN;
                IF S1 < S2 THEN S2 ← S1;
                S1 ● 5.5 / MAX1;
L2:             FOR I ← 0 STEP 1 UNTIL 7 DO N1[I] ← NAME[N,I];
                SYMBOL(-0.36,PY1,0.14,N1,90,46);
                FOR I ← 0 STEP 1 UNTIL 10 DO BEGIN
                  N1[0] ← NAME[N2,I]; P ← PY2 × I + PY3;
                  SYMBOL(-0.1,P,0.1,N1,90,6); END;
                FOR I ← 0 STEP 1 UNTIL 7 DO N1[I] ← NAME[4,I];
                SYMBOL(PX1,PY3-0.5,0.14,N1,0,46);
                FOR I ← 0 STEP 1 UNTIL 10 DO BEGIN
                  N1[0] ← NAME[7,I]; P ← PX2 × I;
                  SYMBOL(P,PY3-0.2,0.1,N1,0,6); END;
                FILL N1[*] WITH "   SCA","LE FAC","TOR  =";

                CONVERT(S2,2,A1,A2);
                N1[3] ← A1;  N1[4] ← A2;
                SYMBOL(0,PY4+0.2,0.14,N1,0,27);
                CONVERT(B,0,A1,A2);
                FILL N1[*] WITH "     B","[0] VA","LUE  =";

                N1[3] ← A1;  N1[4] ← A2;
                SYMBOL(0.6,PY4,0.14,N1,0,27);
                FOR I ← 1 STEP 1 UNTIL N3 DO
            BEGIN
                N1[0] ← NAME[8,I-1];
                FOR J ← 1 STEP 1 UNTIL N4 DO
            BEGIN
                X[J] ← C[I,J,1] × S1;
                Y[J] ← C[I,J,2] × S2;
            END;
                NAMELINE(X,Y,N4,1,N1,6,FALSE);
            END;
                PLOT(PX,PY,-3);
            END OF DRAWGRAPH;

PROCEDURE       UVALUE(A,B,NAME); ARRAY A,B[*]; ALPHA ARRAY NAME[*,*];
            BEGIN
  REAL          Z,ZFACTOR,R,SUM,BVALUE;
  INTEGER       I,J,K,N;
  ARRAY         C[0:5,0:26,0:2];
                BVALUE ← B[0];
                Z ← 0; N ← 9;
                FOR I ← 1 STEP 1 UNTIL 5 DO
            BEGIN
                Z ← Z + 0.2; R ← -0.1;
                ZFACTOR ← -1 × (Z*4 -5/2×Z*3 + 3/2×Z*2);
                FOR J ← 1 STEP 1 UNTIL 26 DO
            BEGIN
                R ← IF R < 0.5 THEN R + 0.1 ELSE R + 0.025;  SUM ← 0;
                FOR K ← 0 STEP 1 UNTIL N DO
```

```
                  SUM ← SUM + (A[K] × (R*(2×K) - R*(2×K+2))/A[0]);
                C[I,J,1] ← R;
                C[I,J,2] ← ZFACTOR × (SUM + (A[N] × R*(2×N+2)×(1-R*2))/
                  A[0]);
            END;
            END;
                N ← 1; I ← 5; J ← 26; DRAWGRAPH(C,I,J,N,NAME,BVALUE);
            END OF UVALUE;

PROCEDURE       VVALUE(A,B,NAME); ARRAY A,B[*]; ALPHA ARRAY NAME[*,*];
            BEGIN
  REAL          Z,ZFACTOR,R,SUM1,SUM2,BVALUE;
  INTEGER       I,J,K,N;
  ARRAY         C[0:5,0:11,0:2];
                BVALUE ← B[0];
                Z ← 0; N ← 9;
                FOR I ← 1 STEP 1 UNTIL 5 DO
            BEGIN
                Z ← Z + 0.2; R ← -0.1;
                ZFACTOR ← (2×Z*3 - 15/4×Z*2 + 3/2×Z);
                FOR J ← 1 STEP 1 UNTIL 11 DO
            BEGIN
                C[I,J,1] ← R ← R+0.1; SUM1 ← SUM2 ← 0;
                FOR K ← 0 STEP 1 UNTIL N DO
            BEGIN
                SUM1 ← SUM1 + (A[K] / (K+1) × R*(2×K+1))/A[0];
                SUM2 ← SUM2 + (A[K] / (K+2) × R*(2×K+3))/A[0];
            END;
                C[I,J,2] ← ZFACTOR × (SUM1-SUM2+(A[N]/(N+1)×R*(2×N+3)×
                  (1-R*2))/A[0]);
            END;
            END;
                N ← 2; I ← 5; J ← 11; DRAWGRAPH(C,I,J,N,NAME,BVALUE);
            END OF VVALUE;

PROCEDURE       TVALUE(A,NAME); ARRAY A[*]; ALPHA ARRAY NAME[*,*];
            BEGIN
  REAL          Z,ZFACTOR,R,SUM,BVALUE;
  INTEGER       I,J,K,N;
  ARRAY         C[0:5,0:26,0:2];
                BVALUE ← A[0];
                Z ← 0; N ← 9;
                FOR I ← 1 STEP 1 UNTIL 5 DO
            BEGIN
                Z ● Z + 0.2; R ← -0.1;
                ZFACTOR ← (Z*3/3 - Z*2/2 + 1);
                FOR J ← 1 STEP 1 UNTIL 26 DO
            BEGIN
                R ● IF R < 0.5 THEN R + 0.1 ELSE R + 0.025;  SUM ← 0;
                FOR K ← 0 STEP 1 UNTIL N DO
                  SUM ← SUM + (A[K] × (R*(2×K) - R*(2×K+2))/A[0]);
                C[I,J,1] ← R;
                C[I,J,2] ← ZFACTOR × (SUM + (A[N] × R*(2×N+2)×(1-R*2)/
                  A[0]));
            END;
```

```
          END;
               N ← 3; I ← 5; J ← 26; DRAWGRAPH(C,I,J,N,NAME,BVALUE);
          END OF TVALUE;

INTEGER        I,J,M,IMAX,D,N;
REAL           AI,BI,      AIMAX,BIMAX,DELTAI,DEL1,TEMP,MAXAI,DELTBI,
               DEL2,K1,RTI,RT,L;
BOOLEAN        BOOL1,BOOL2;
ALPHA ARRAY    NAME[0:8,0:10];
               DATELINE("CONVEC");
               FOR I ← 1 STEP 1 UNTIL 8 DO
               READ(CARD,FMT,FOR J ← 0 STEP 1 UNTIL 10 DO NAME[I,J]);
               READ (CARD,FMI,IMAX,RTI   );
          BEGIN
ARRAY          A[0:IMAX+1],B[0:IMAX+1],B3[0:IMAX+1],SAA[0:IMAX+1],
               SB[0:IMAX+1], AA[0:IMAX+1],B1[0:IMAX+1],B2[0:IMAX+1];
               WRITE (LINE,FMO);
               READ (CARD,FMI1,BI,DEL2   ,BIMAX);
               READ (CARD,FMI1,AI, DEL1,MAXAI);
               DELTBI←DEL2;FOR B[0]←BI STEP DELTBI UNTIL BIMAX DO
          BEGIN
               BOOL1←BOOL2←FALSE; DELTAI←DEL1;AIMAX←MAXAI;
                         FOR A[0]←AI STEP DELTAI UNTIL AIMAX DO
          BEGIN
               RT ←RTI;K1←(RT )*2;SB[0]←B[0];AA[0]←A[0]/2;SAA[0]←AA[0];
               A[1]← A[0]-3.0556 ×B[0]+ 1.6667×K1×A[0];
               A[2]←A[1]-0.7639×(B[1]-B[0])+0.4167×K1×(A[1]-A[0]);
               A[3]←A[2]-0.3395×(B[2]-B[1])+0.1852×(A[2]-A[1]);
               AA[1]←A[1]/6;AA[2]←A[2]/12;AA[3]←A[3]/20;
               SAA[1]←SAA[0]+AA[1];SAA[2]←SAA[1]+AA[2];SAA[3]←SAA[2]
               +AA[3];B[1]←B[0]+0.0383×A[0]×B[0];
               B[2]←B[1]-0.0627×A[0]×B[1]+0.0914×A[0]×B[0];
               B[3]←B[2]-0.05996×A[0]×B[2]-0.0118×A[1]×B[1]+0.08879×
               A[0]×B[1]+0.02430×A[1]×B[0]-0.0118×A[0]×B[0];
               SB[1]←B[0]+B[1];SB[2]←SB[1]+B[2];SB[3]←SB[2]+B[3];
               FOR I←3 STEP 1 UNTIL IMAX+1 DO
          BEGIN
               D←I*2;M←I-1;N←I-2;L←I-3;
               A[I]←A[I-1]-(3.0556×(B[I-1]-B[I-2])-1.6667×K1×(A[I-1]
                -A[I-2]))/D;
               FOR J←0 STEP 1 UNTIL M DO
               B1[I]←(1.0596×(I-J-1)/(J+1)-0.1405)×A[J]×B[I-J-1];
               FOR J←0 STEP 1 UNTIL N DO
               B2[I]←(1.0596×((I-J-2)/(J+2)+(I-J-1)/(J+1))+0.281)
                ×A[J]×B[I-J-2];
               FOR J←0 STEP 1 UNTIL L DO
               B3[I]←(1.0596×(I-J-2)/(J+2)-0.1405)×A[J]×B[I-J-3];
               B[I]←B[I-1]-3×(B1[I]-B2[I]+B3[I])/(11×D);
               AA[I]←A[I]/((I+1)×(I+2));
               SB[I]←SB[I-1]+ B[I];SAA[I]←SAA[I-1]+ AA[I];
          END;
               IF A[0]=AI THEN TEMP←SAA[IMAX+1];
               IF SIGN(TEMP)≠SIGN(SAA[IMAX+1]) AND NOT BOOL1 THEN
          BEGIN
               BOOL1←BOOL2; BOOL2←TRUE;
```

```
        A[0]←A[0]−DELTAI;
        AIMAX←A[0]+DELTAI;
        DELTAI←DELTAI×0.125;
    END
    ELSE TEMP←SAA[IMAX+1];
        IF ABS(SAA[IMAX+1])≤ 0.0100 THEN
    BEGIN
        UVALUE(A,B,NAME);
        VVALUE(A,B,NAME);
        TVALUE(B,NAME);
        WRITE (LINE,FM03);
        WRITE (LINE,FM04,FOR I←0 STEP 1 UNTIL (IMAX+1) DO
         [ A[I]]);
        WRITE (LINE,FM05);
        WRITE (LINE,FM06,FOR I←0 STEP 1 UNTIL (IMAX+1) DO
         [SB[I]]);
        WRITE (LINE,FM07);
        WRITE (LINE,FM08,FOR I←0 STEP 1 UNTIL (IMAX+1) DO
        [SAA[I]]);
        WRITE (LINE,FM011);
        WRITE (LINE,FM012,FOR I←0 STEP 1 UNTIL (IMAX+1) DO
        [B[I]]);
                              AIMAX←MAXAI;DELTAI←DEL1;
    END;
    END;
    END;
    END;

        DATELINE(0);
    END OF PROGRAM.

ARCTAN IS SEGMENT NUMBER 0050,PRT ADDRESS IS 0170
COS IS SEGMENT NUMBER 0051,PRT ADDRESS IS 0146
EXP IS SEGMENT NUMBER 0052,PRT ADDRESS IS 0154
LN IS SEGMENT NUMBER 0053,PRT ADDRESS IS 0153
SIN IS SEGMENT NUMBER 0054,PRT ADDRESS IS 0147
SQRT IS SEGMENT NUMBER 0055,PRT ADDRESS IS 0167
OUTPUT(W) IS SEGMENT NUMBER 0056,PRT ADDRESS IS 0113
BLOCK CONTROL IS SEGMENT NUMBER 0057,PRT ADDRESS IS 0005
INPUT(W) IS SEGMENT NUMBER 0058,PRT ADDRESS IS 0137
X TO THE I IS SEGMENT NUMBER 0059,PRT ADDRESS IS 0155
GO TO SOLVER  IS SEGMENT NUMBER 0060,PRT ADDRESS IS 0115
ALGOL WRITE   IS SEGMENT NUMBER 0061,PRT ADDRESS IS 0014
ALGOL READ    IS SEGMENT NUMBER 0062,PRT ADDRESS IS 0015
ALGOL SELECT  IS SEGMENT NUMBER 0063,PRT ADDRESS IS 0016
```

APPENDIX B

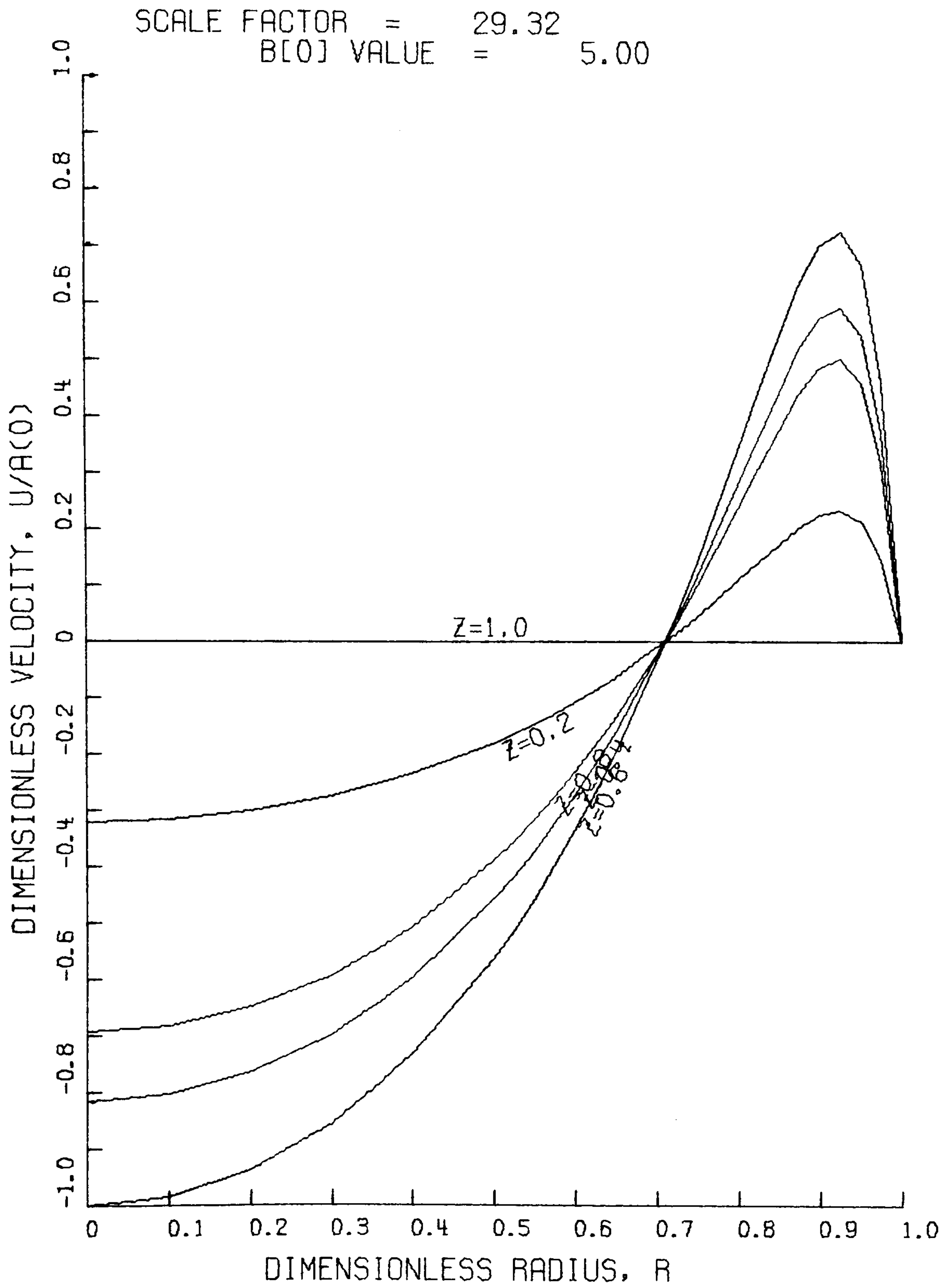
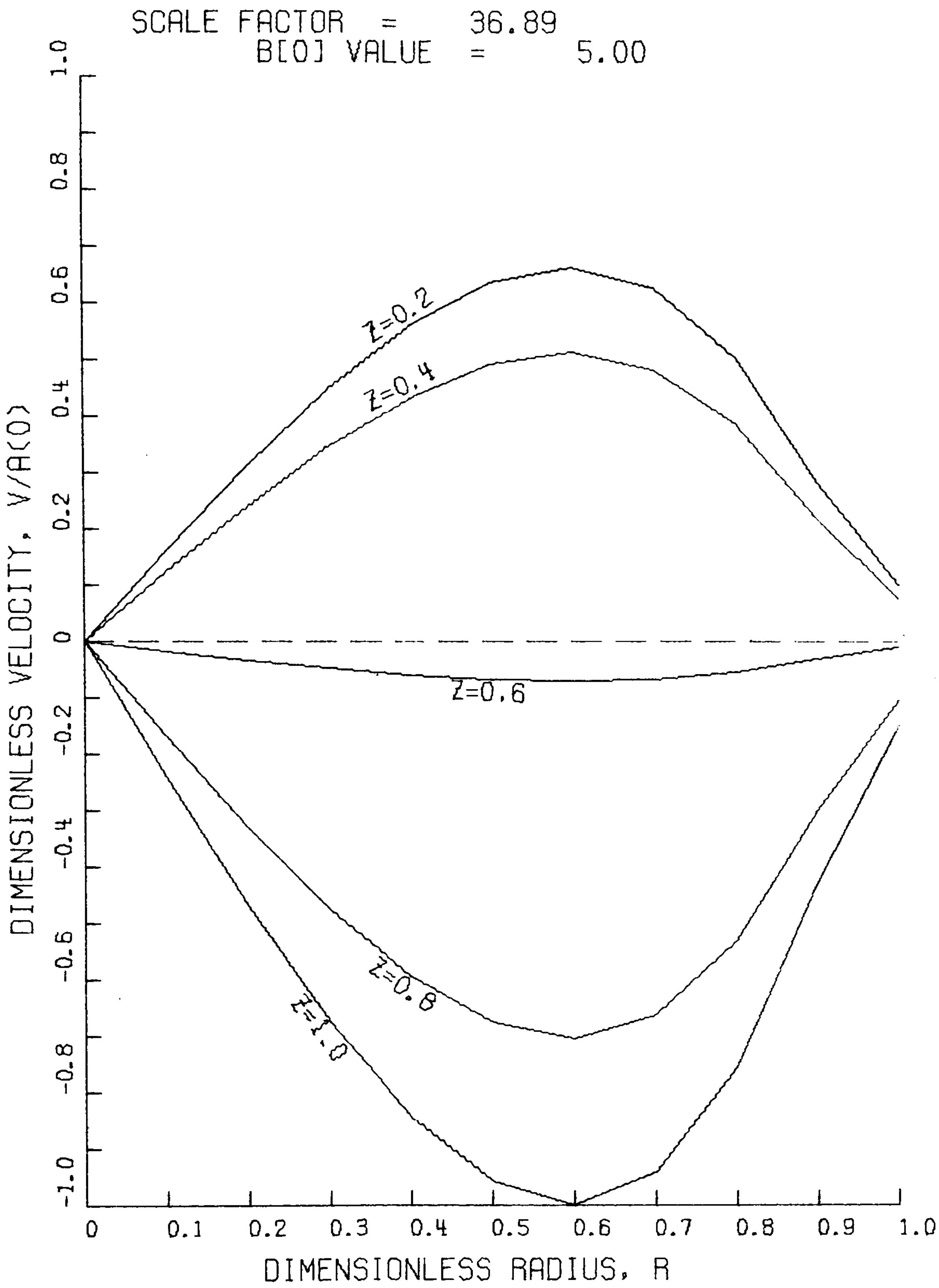

SCALE FACTOR = 29.32
B[0] VALUE = 5.00
DIMENSIONLESS VELOCITY, U/A(0)
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
Z=1.0
Z=0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
DIMENSIONLESS RADIUS, R

Figure 1



SCALE FACTOR = 36.89
B[O] VALUE = 5.00
DIMENSIONLESS VELOCITY, V/A(O)
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
Z=0.2
Z=0.4
Z=0.6
Z=0.8
Z=1.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
DIMENSIONLESS RADIUS, R

Figure 2

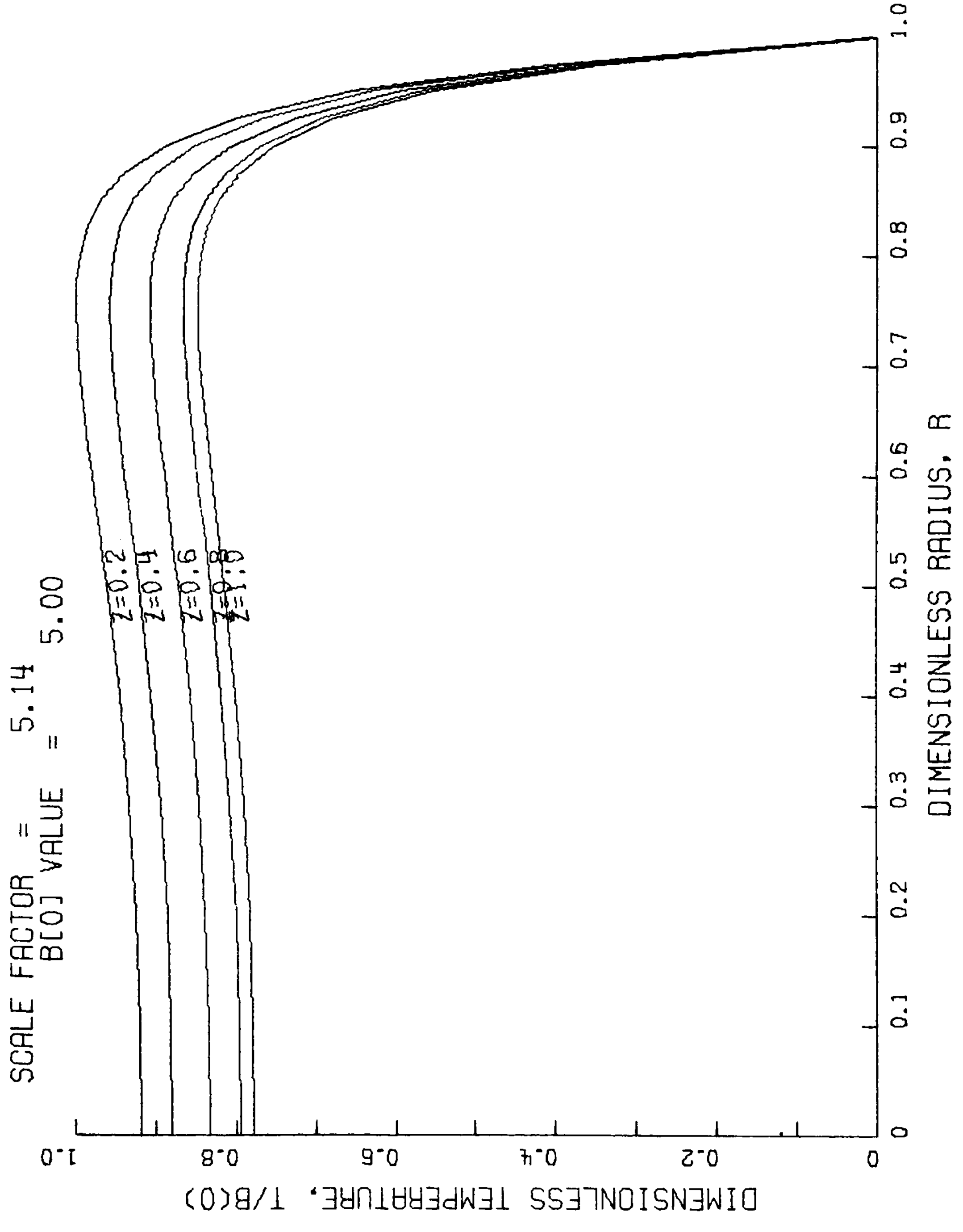
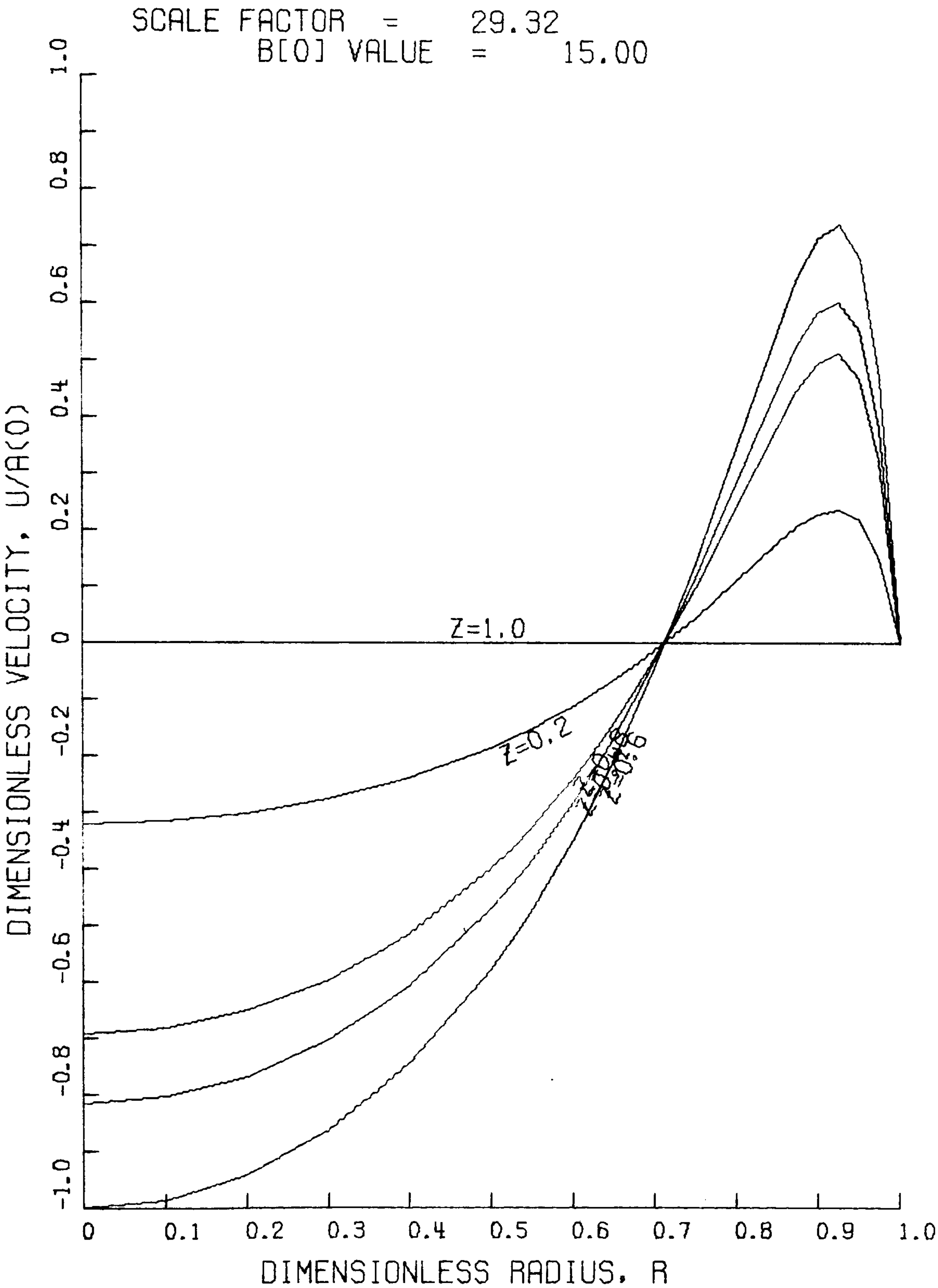
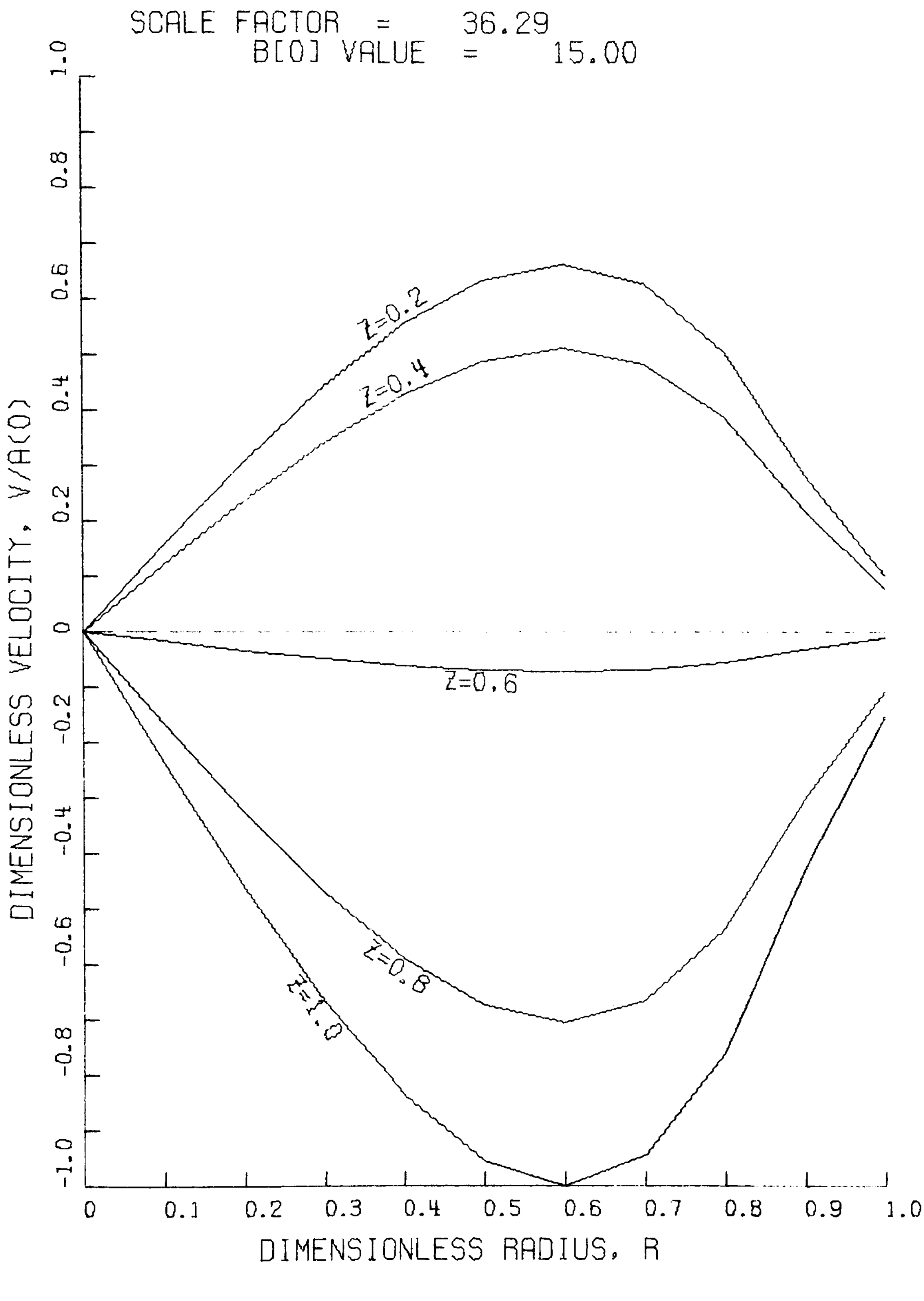

SCALE FACTOR = 5.14
B[O] VALUE = 5.00
Z=0.2
Z=0.4
Z=0.6
Z=0.8
Z=1.0
DIMENSIONLESS RADIUS, R
DIMENSIONLESS TEMPERATURE, T/B(O)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0
0.2
0.4
0.6
0.8
1.0

Figure 3



SCALE FACTOR = 29.32
B[O] VALUE = 15.00
DIMENSIONLESS VELOCITY, U/A(O)
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
Z=1.0
Z=0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
DIMENSIONLESS RADIUS, R

Figure 4



SCALE FACTOR = 36.29
B[O] VALUE = 15.00
Z=0.2
Z=0.4
Z=0.6
Z=0.8
Z=1.0
DIMENSIONLESS VELOCITY, V/A(O)
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
DIMENSIONLESS RADIUS, R

Figure 5

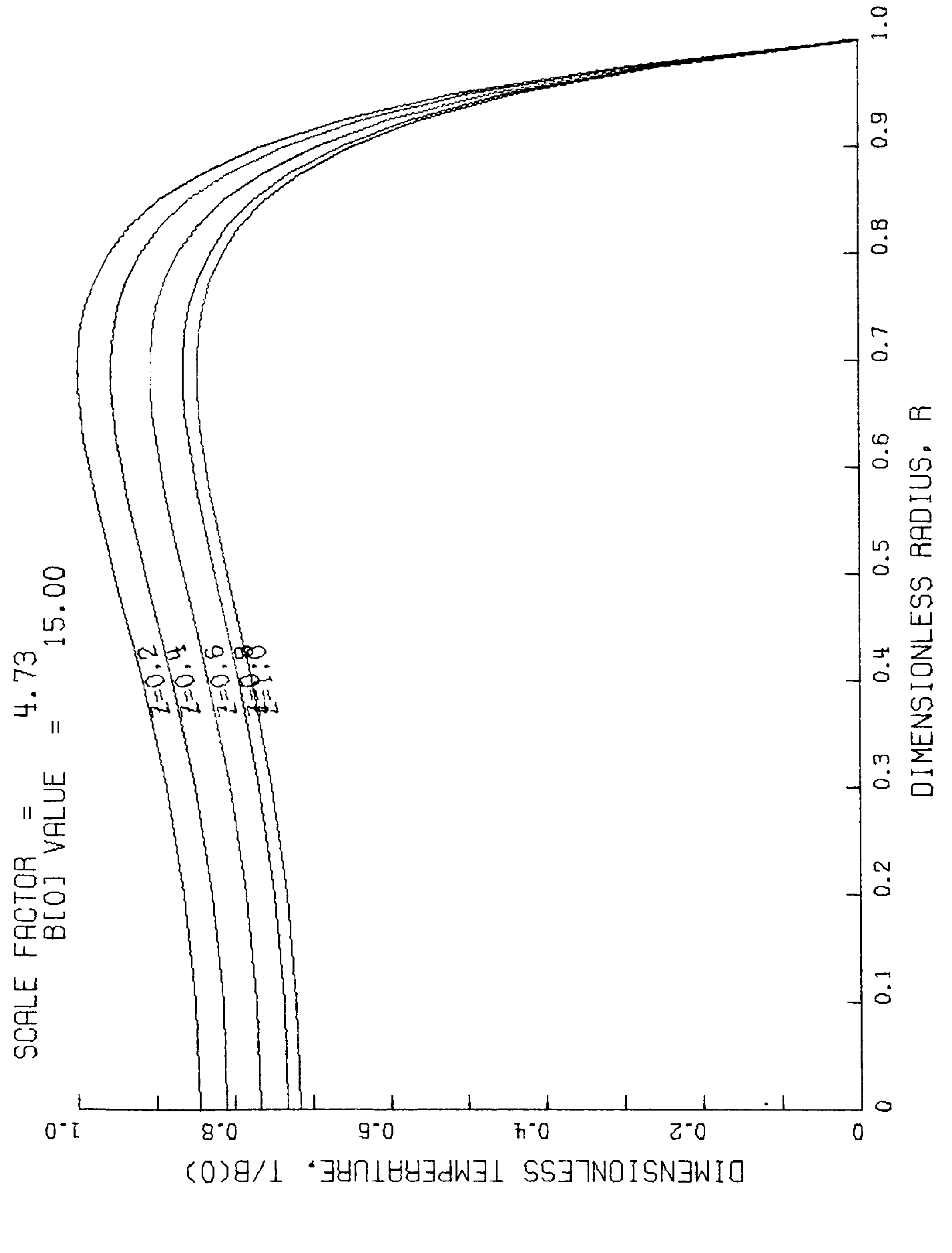

SCALE FACTOR = 4.73
B[O] VALUE = 15.00
Z=0.2
Z=0.4
Z=0.6
Z=0.8
Z=1.0
DIMENSIONLESS TEMPERATURE, T/B(O)
0
0.2
0.4
0.6
0.8
1.0
DIMENSIONLESS RADIUS, R
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Figure 6

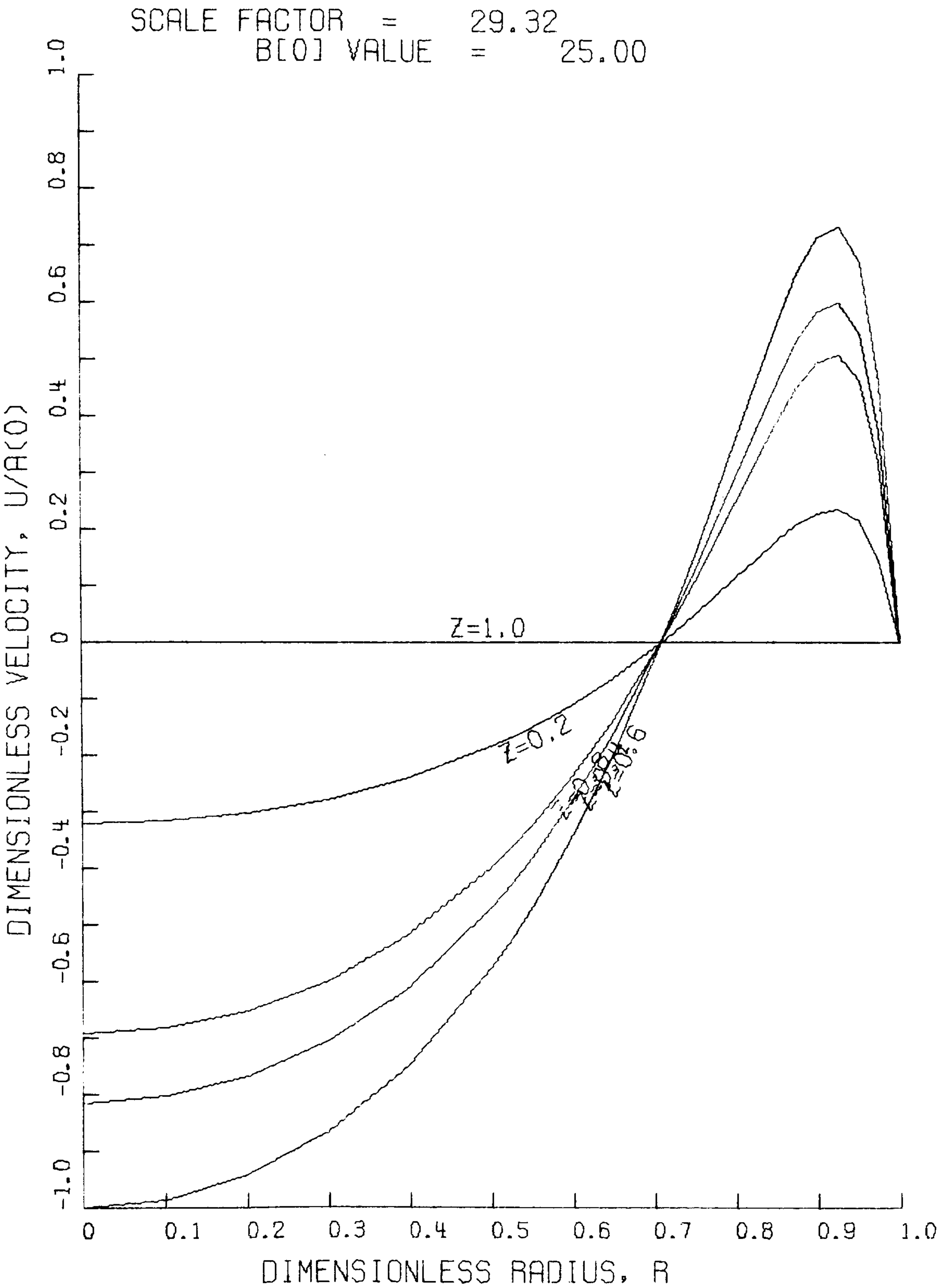

SCALE FACTOR = 29.32
B[0] VALUE = 25.00
DIMENSIONLESS VELOCITY, U/A(0)
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
Z=1.0
Z=0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
DIMENSIONLESS RADIUS, R

Figure 7

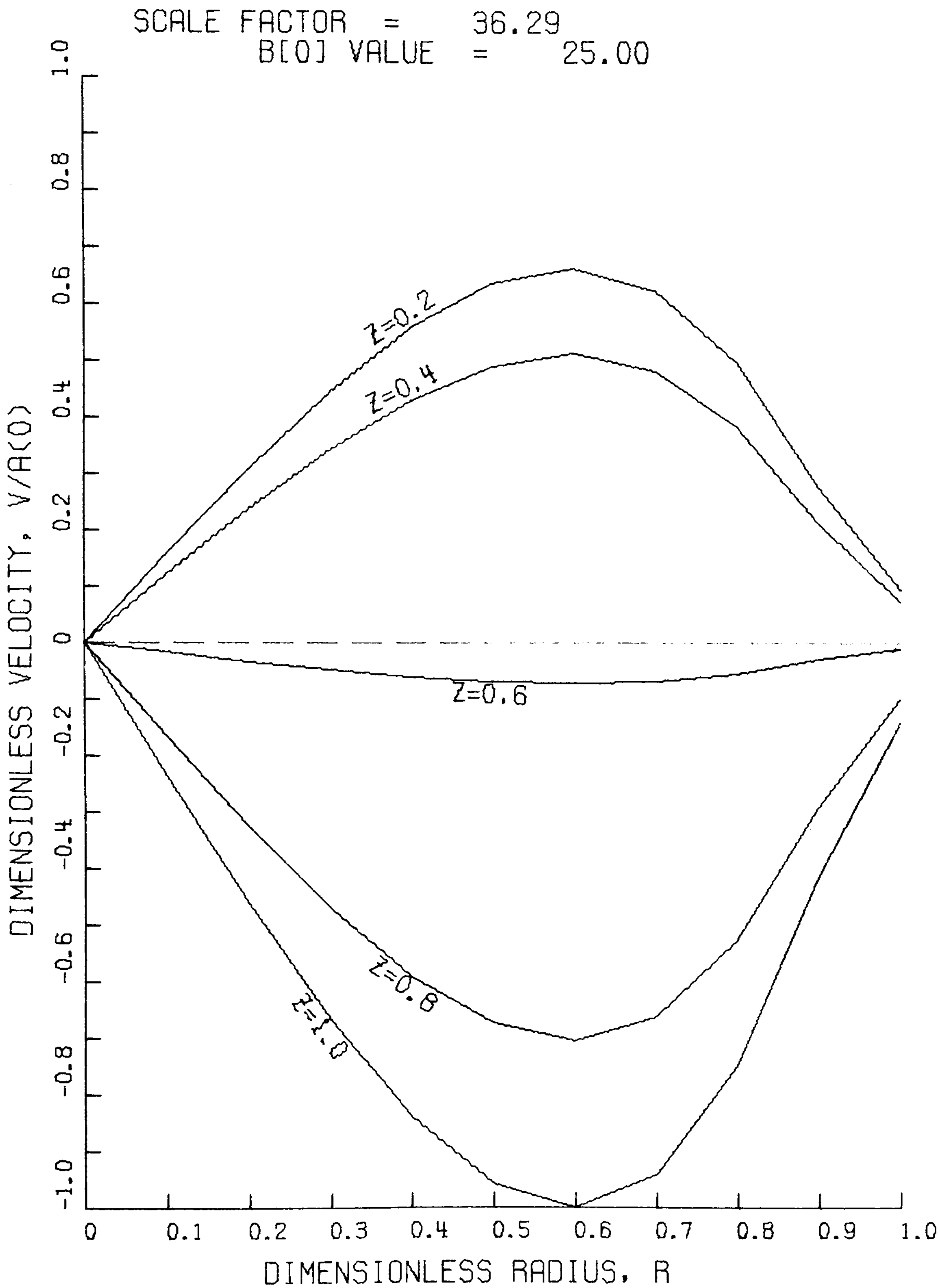

SCALE FACTOR = 36.29
B[O] VALUE = 25.00
Z=0.2
Z=0.4
Z=0.6
Z=0.8
Z=1.0
DIMENSIONLESS VELOCITY, V/A(O)
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
DIMENSIONLESS RADIUS, R

Figure 8

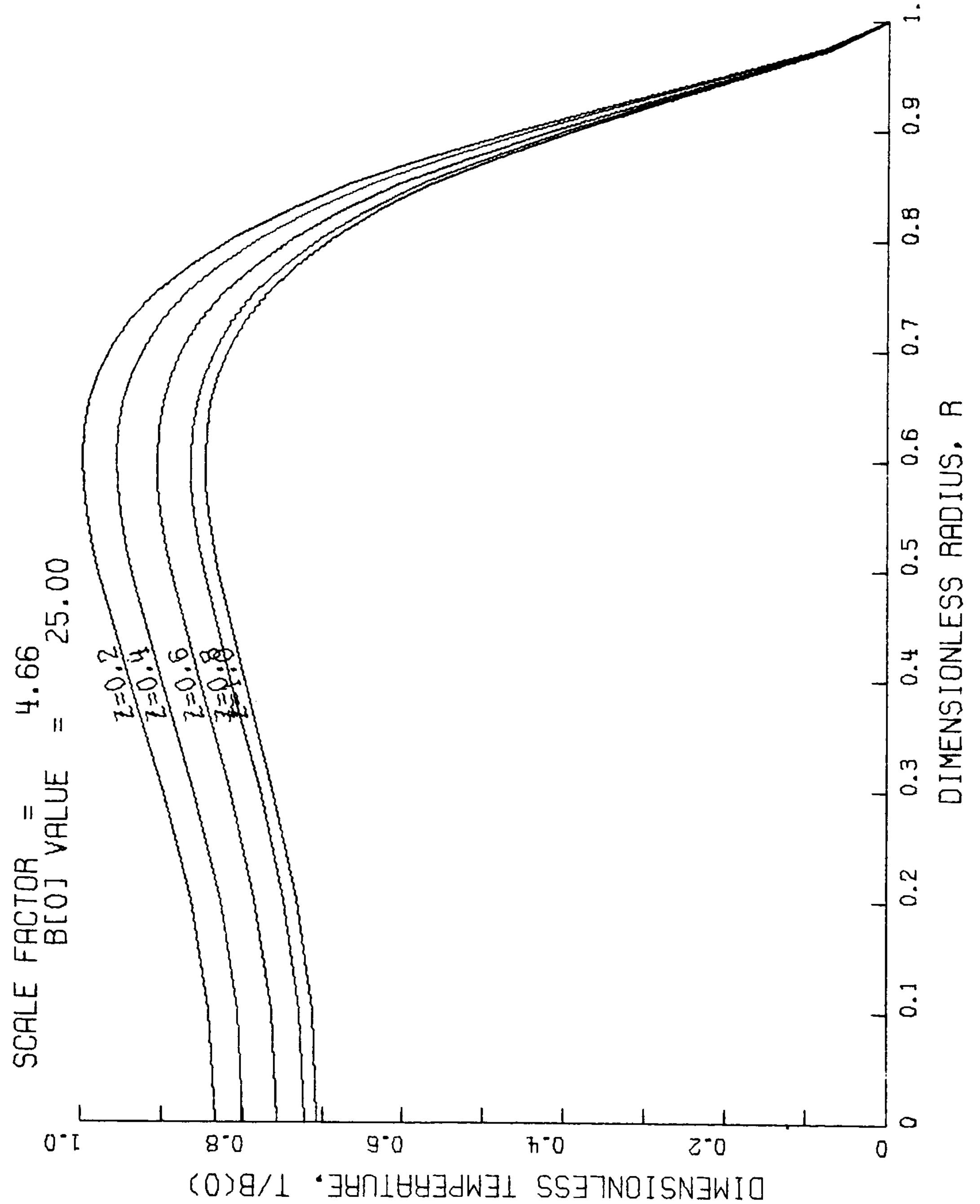

SCALE FACTOR = 4.66
B[O] VALUE = 25.00
Z=0.2
Z=0.4
Z=0.6
Z=0.8
DIMENSIONLESS RADIUS, R
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
DIMENSIONLESS TEMPERATURE, T/B(O)
0
0.2
0.4
0.6
0.8
1.0

Figure 9

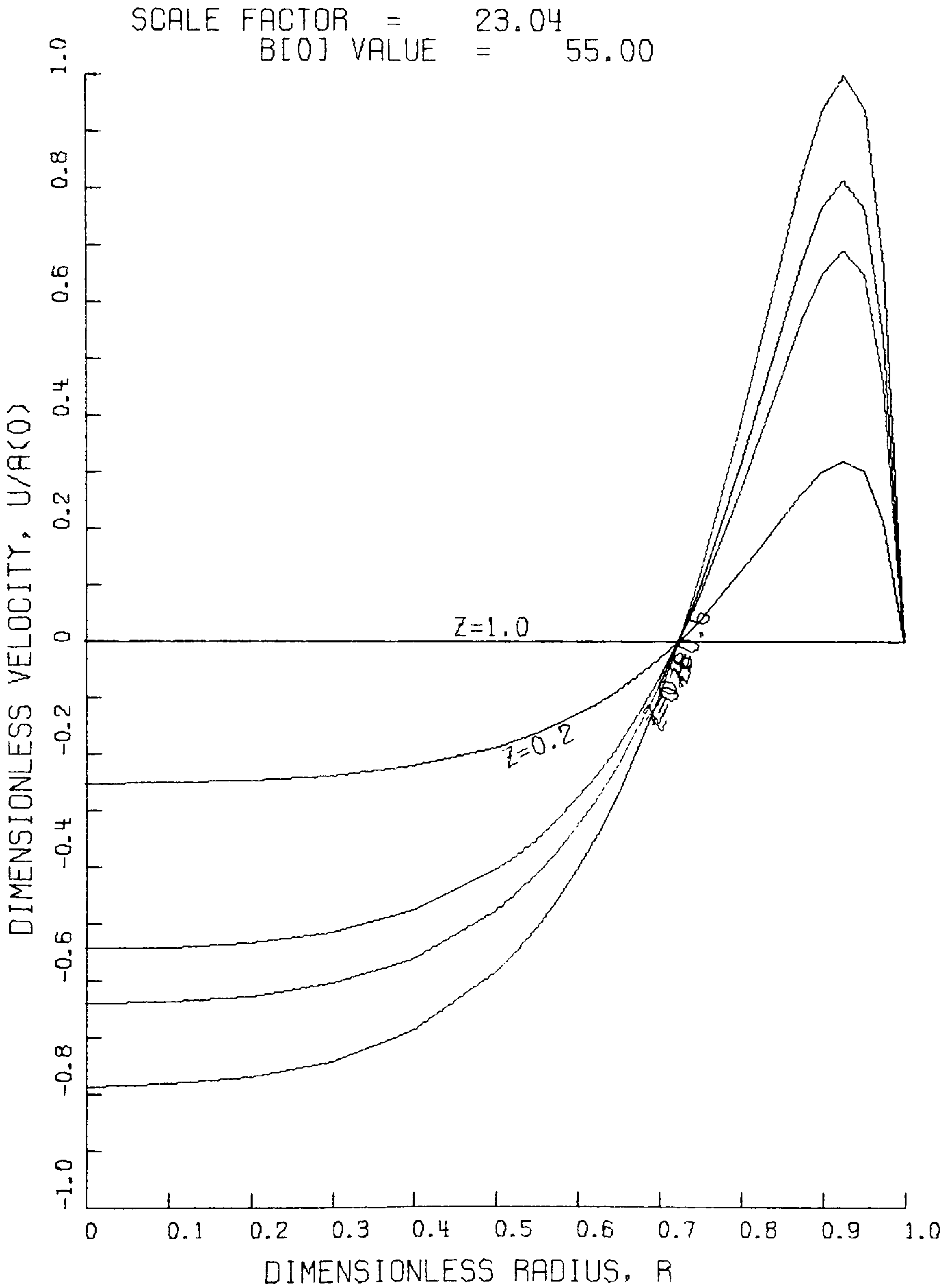

SCALE FACTOR = 23.04
B[0] VALUE = 55.00
Z=1.0
Z=0.2
DIMENSIONLESS VELOCITY, U/A(0)
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
DIMENSIONLESS RADIUS, R

Figure 10

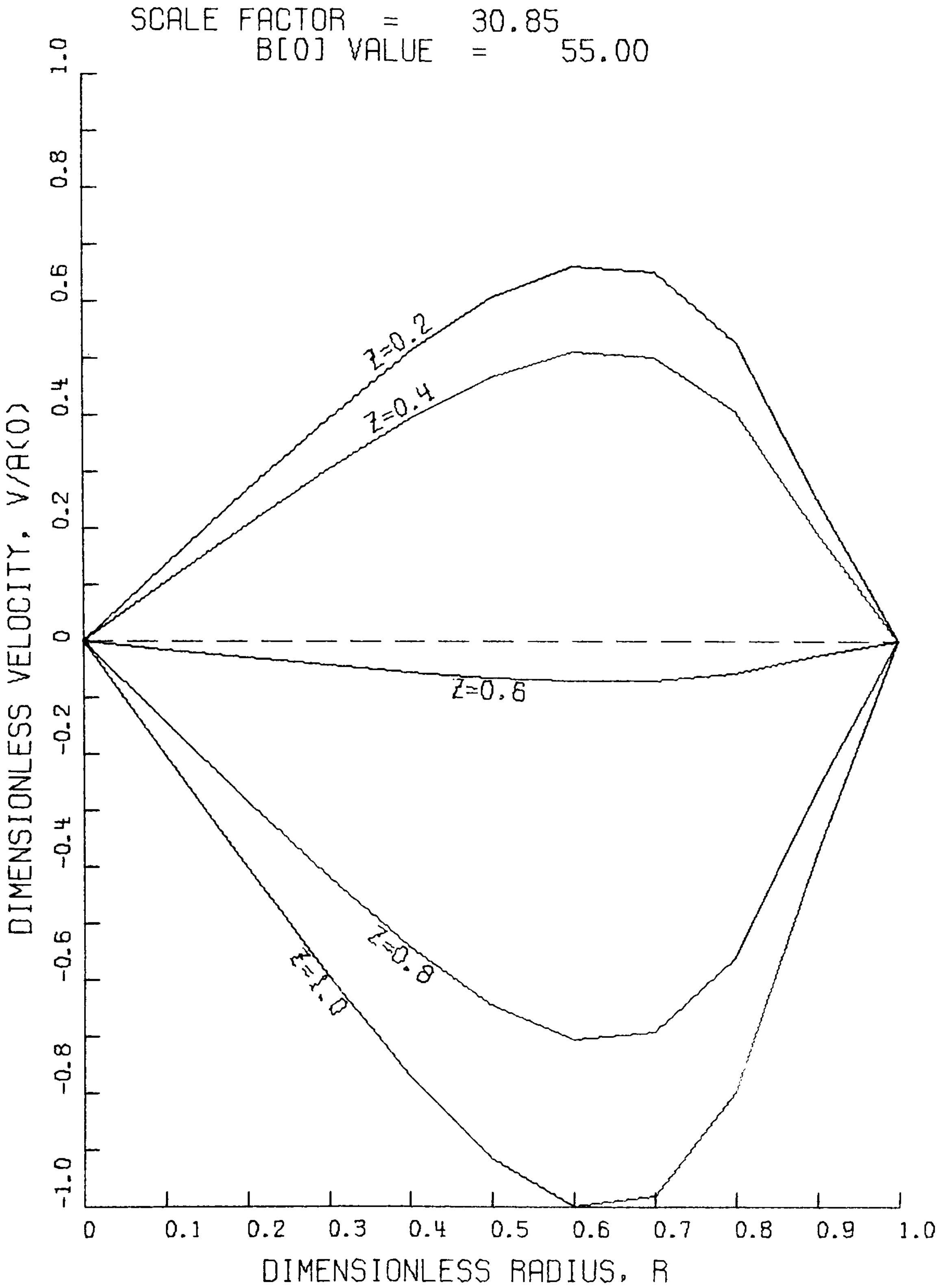

SCALE FACTOR = 30.85
B[0] VALUE = 55.00
Z=0.2
Z=0.4
Z=0.6
Z=0.8
Z=1.0
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
DIMENSIONLESS VELOCITY, V/A(O)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
DIMENSIONLESS RADIUS, R

Figure 11

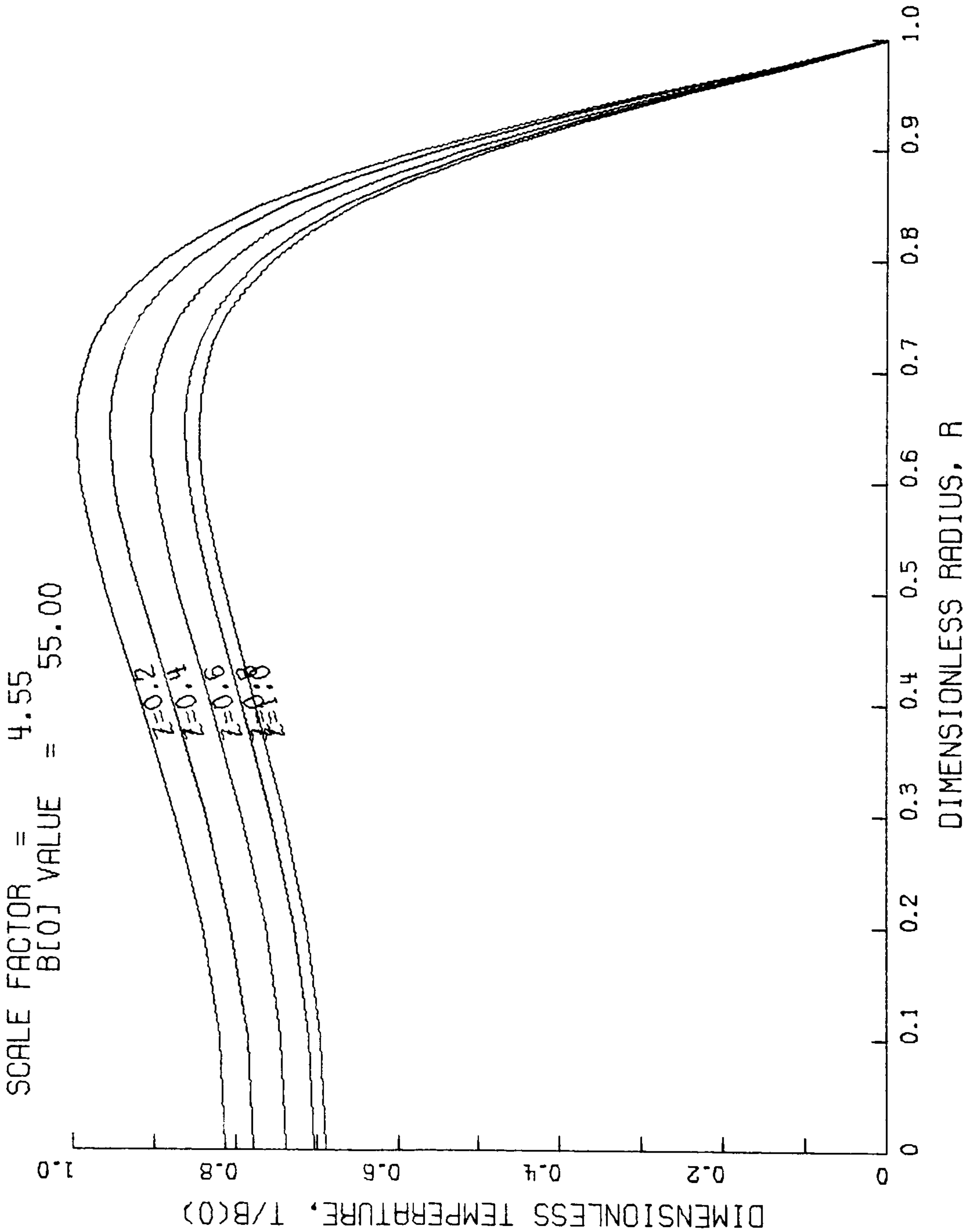

SCALE FACTOR = 4.55
B[O] VALUE = 55.00
Z=0.2
Z=0.4
Z=0.6
Z=0.8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
DIMENSIONLESS RADIUS, R
0
0.2
0.4
0.6
0.8
1.0
DIMENSIONLESS TEMPERATURE, T/B(O)

Figure 12