COPY NO. 19

ROTATIONAL APPARENT MASS BY ELECTRICAL ANALOGY

NOR 66-364
December 8, 1966

Prepared Under NASA Ames Contract No. NAS 2-2671

by

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for

National Aeronautics and Space Administration
Ames Research Center
Moffett Field, California

FOREWORD

This report was prepared by the Norair Division of Northrop Corporation, Hawthorne, California, on NASA Ames Research Center Contract NAS 2-2671, Revision 1, "Rotational Apparent Mass by Electrical Analogy." The study presented represents an effort by members of the Research and Technology Section of Northrop Norair.

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SUMMARY

An investigation was conducted to study methods for the determination of rotational apparent masses by electrical analogy techniques. The change in resistance in a $W = Z^2$ flow field due to the presence of a body was found to work for a certain class of shapes. A method of measuring the summation of the resistance changes between four points was found to work for a larger class of shapes.

SYMBOLS

m_{33}	apparent mass
ρ	density
V	velocity
ϕ	potential function
W	complex potential function $\phi + i \Psi$
Z	physical plane $x + i y$
R	distance from origin in physical plane
θ	angle between x axis and R
Ψ	stream function
Ω	electrical resistance
$R_{AB, CD}$	resistance between AB shorted together and CD shorted together.
I_T	moment of inertia of test section
I_B	moment of inertia of body
V_{33}	virtual mass $I_B + m_{33}$

INTRODUCTION

The estimation of aerodynamic stability derivatives for a body can be made by the measurement of the added apparent masses due to the fluid flow about the body. These apparent masses are derived from the kinetic energy imparted to the fluid by the body as it moves through the fluid. It has been shown (Reference 1) that the apparent mass due to a fluid flow can be easily measured using the electrical analogy. An electrical flow of the type desired is created in a medium, such as Teledeltos paper, and electrical resistances recorded. The body shape is then cut out of the medium and again the electrical resistances recorded. The increase in electrical resistances caused by cutting out the body shape will give the apparent mass. This method was used in Reference 1 to determine the translational apparent masses for two-dimensional shapes. Aerodynamic stability derivatives were then calculated from these measured apparent masses, using the equations from Reference 2.

In order to determine certain rotary derivatives, the apparent mass due to the solid body rotation of a two-dimensional shape is needed. The purpose of this report is to study methods by which the apparent mass due to rigid body rotation can be determined by electrical analogy in two-dimensional flow.

THEORY

Consider a body moving with unit velocity in a fluid with a frame of reference fixed to the body. The apparent mass for this body is the summation of the kinetic energy in the fluid with respect to moving axes fixed in the body:

$$m = \frac{1}{2} \rho \int V^2 \; dt \tag{1}$$

This volume integral is shown in Reference 2 to be equal to the surface integral:

$$m = \frac{1}{2} \rho \int \varphi \; \frac{\partial \varphi}{\partial n} \; ds \tag{2}$$

The flow of an electrical current is analogous to that of a potential fluid flow. The velocity potential ϕ becomes the voltage V. At the boundary of the electrical field, which is sufficiently far from the body, the voltage can be measured on all conducting edges and since $\frac{\partial \varphi}{\partial n}$ is zero on all cut edges then Equation (2) can be evaluated. For a unit current flow it is the measured electrical resistance of the flow. This gives the total kinetic energy of the flow in the entire field. When a body is cut out of the field a new resistance value can be measured. The difference between the resistance values before and after cutting the body out is then the kinetic energy imparted to the fluid by the body. This is the method used to determine apparent masses for translational flows in Reference 1.

In order to obtain the apparent mass due to rigid body rotation the change in resistance due to cutting a body out of a rotating electrical field could be used. It is difficult to induce in a simple manner a rigidly rotating electrical field. The simple resistance flow of electricity in a medium is an irrotational flow obeying Laplace's equation, but a rigidly rotating field is rotational and does not obey Laplace's equation. However, to determine the apparent mass of a body in this type of flow it is only necessary to measure the kinetic energy induced by the body and not the total kinetic energy. This induced flow is irrotational and does obey Laplace's equation. Therefore it seems possible to induce a flow field or a combination of flow fields which will cause the body to impart the same kinetic energy as that due to rigid body rotation.

Now consider the flow

$$W_1 = Z^2$$

which can be written

$$W_1 = R^2 \cos 2\theta + i R^2 \sin 2\theta$$

Since

$$W_1 = \varphi_1 + i \Psi_1$$

then the potential function is

$$\varphi_1 = R^2 \cos 2\theta$$

and the stream function is

$$\Psi_1 = R^2 \sin 2\theta$$

If this flow field is rotated 45 degrees the resulting flow function is:

$$W_2 = R^2 \sin 2\theta + i R^2 \cos 2\theta$$

and

$$\varphi_2 = R^2 \sin 2\theta$$

$$\Psi_2 = R^2 \cos 2\theta$$

The sum of the absolute values of the stream function and potential function of the two flows are

$$\sqrt{\Psi_1^{\,2} + \Psi_2^{\,2}} = R^2$$

According to Reference 3 this is the value of the stream function needed to simulate the flow induced by rigid body rotation. This analysis suggests that by the proper addition of the apparent masses of a body in a $W = Z^2$ flow and a $W = Z^2$ flow rotated by 45 degrees, the rotational apparent mass can be determined.

TESTS

Following the suggestion produced by the above analysis, a $W = Z^2$ electrical flow was designed. This was made by cutting a 12-inch diameter octagon out of Teledeltos paper as shown in Figure 1a. Alternate edges of the octagon were painted with silver paint to make them infinitely conducting compared to the general field. A calibration strip was prepared. This consisted of a rectangular sheet with silver paint at each edge (Figure 1b). A Wheatstone bridge was hooked up using the two Teledeltos sheets as arms (Figure 2). With this arrangement the resistance ratio of the $W = Z^2$ field to the calibration strip could be measured.

The first measurements were made using a circle as the rotating shape. The $W = Z^2$ resistance of the octagonal sheet was measured with and without a circle cut out. The percentage change in resistance thus obtained was multiplied by the moment of inertia of the octagonal sheet. The resulting number for various size circles was equal to the moment of inertia of the circles. The next figures measured were right angle crosses. These were measured in one position then rotated forty-five degrees and measured again. By adding the answers resulting from the two positions the theoretical rotational apparent mass was obtained. This method therefore seemed to be a promising approach to the problem. It was further pursued and it was found that it gave good answers for combination circle and fin bodies. The equation used to get the rotational apparent mass is:

$$\frac{m_{33}}{\rho} = \left(\frac{\Delta \Omega}{\Omega}\right) I_T - I_B$$

The first term on the right side of this equation is the virtual mass of the body. When the actual rotational mass or moment of inertia is subtracted out the result is the added apparent mass.

Next ellipses and single straight lines were measured. Here the method seemed to break down. The answers obtained were much smaller than the theoretical values. After further testing, using many techniques to try and obtain the correct answer, it became apparent that these asymetrical shapes distorted the $W = Z^2$ field. In other words it prevented an equal current flow in each of the four branches of the octagonal sheet. To bypass this difficulty the octagonal sheet with its four contacts was studied as a simple resistance network. The equivalent network consists of four points connected by six resistances as shown in Figure 3. The problem now was to determine the values of the equivalent resistances. This was accomplished by taking measurements with various sides shorted together. The following equations were used to determine the equivalent resistances.

$$\frac{2}{R_{AB}} = \frac{1}{R_{A, BCD}} + \frac{1}{R_{B, ACD}} - \frac{1}{R_{AB, CD}}$$

$$\frac{2}{R_{BC}} = \frac{1}{R_{B, ACD}} + \frac{1}{R_{C, ABD}} - \frac{1}{R_{BC, AD}}$$

$$\frac{2}{R_{CD}} = \frac{1}{R_{C, ABD}} + \frac{1}{R_{D, ABC}} - \frac{1}{R_{CD, AB}}$$

$$\frac{2}{R_{AD}} = \frac{1}{R_{D, ABC}} + \frac{1}{R_{A, BCD}} - \frac{1}{R_{AD, BC}}$$

The commas in the subscript in the above equations indicate how the points in the network were grouped by shorting them together. By taking the resistance measurements indicated by these equations the reciprocal of the four equivalent resistances representing the four branches of the $W = Z^2$ flow was calculated. The values of these four resistances were added to get a value with no shape cut out. The same values were obtained with the shape cut out and the percent change of these values was multiplied by the moment of inertia of the octagonal sheet. This method now seemed able to give answers for shapes without a quadrilateral symmetry such as a horizontal line. The results using this method for various configurations are shown in Figure 4.

With the method thus far described there was still some difficulty in obtaining correct answers for shapes with no symmetry (Figure 4). One problem seems to be involved with how the actual moment of inertia of the body is determined. Since the right answer can be obtained for some shapes it is important that further work be devoted to make the method work for all shapes.

One more method was studied and the results indicated the possibility of success. This method was to get the percent change in resistance in each of the four branches measured one at a time. The summation of the change in these resistances was empirically correlated with the virtual mass of various bodies. A good correlation was obtained as shown in Figure 5.

It was decided that the apparent mass of both the M-2 and HL-10 cross sections be measured by this technique. A cross section of the M-2 was taken at fuselage station 230 which includes the vertical fin and a cross section of the HL-10 at the base of, and including the vertical fin. The changes in resistance for the M-2 and HL-10 were measured and a virtual mass determined for both (Figure 5). By subtracting the moment of inertia of the cross sections of the M-2 and HL-10 from the virtual mass, apparent mass $\left(\frac{m_{33}}{\rho}\right)$ were obtained. They are 22.4 and 15.4 for the M-2 and the HL-10, respectively. The model scales are 1/20 and 1/30 for the M-2 and HL-10, respectively.

There was not enough time on this program for complete investigation of other techniques that might have been fruitful. A method which has been used is to measure the voltage around a body and express this voltage with a Fourier series. The coefficients of the Fourier series can be related to the transformation function required to get all the apparent masses. This method has been applied to arbitrary shapes and its feasibility proven. However it does not seem capable of being used for arbitary three-dimensional shapes. It is believed that a method based on this same concept but which measures the voltage on a unit circle surrounding the body can be used in a similar manner to get all of the coefficients of the complete flow field. This method may be useful for three-dimensional shapes.

Another method would be to measure the body's resistance when placed in the flow field due to $w = z$, $w = z^2$, $w = z^3$ and so forth to the highest power required for accuracy. This could be used to determine coefficients of a Laurents series expansion for the transformation of the body into a circle. It seems that this method would also apply to 3-dimensional bodies by assuming them to be psuedo two-dimensional bodies.

Further investigation of these concepts should prove useful to the overall method for determining aerodynamic forces for arbitrary three-dimensional bodies.

CONCLUSIONS

A study of methods to determine the rotational apparent mass for arbitrary two-dimensional shapes by electric analogy has resulted in the following:

1. The rotational apparent mass for shapes with quadrilateral symmetry can be obtained using a $W = Z^2$ electric field.

2. The change in summation of the resistances between the four sides may correlate a larger class of shapes for rotational flow.

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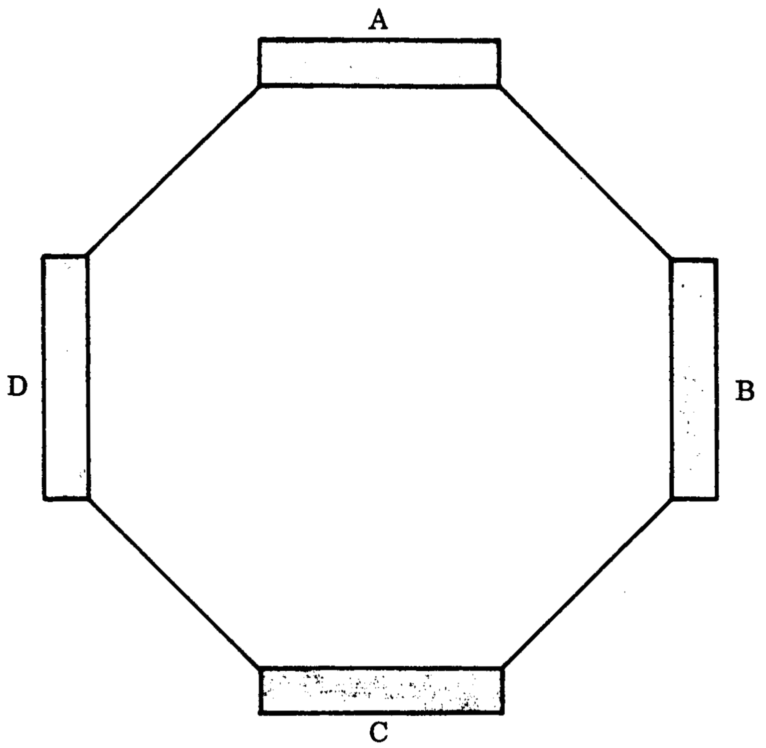

FIGURE 1a. $W=Z^2$ FLOW PATTERN

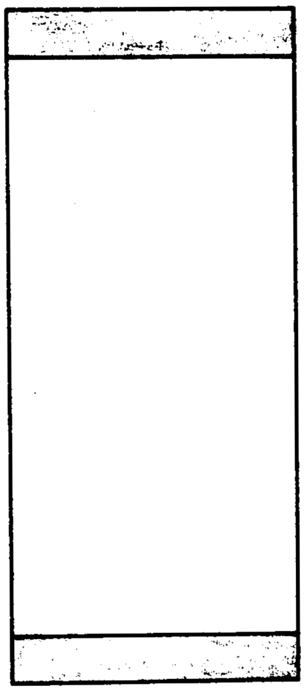

FIGURE 1b. CALIBRATION STRIP

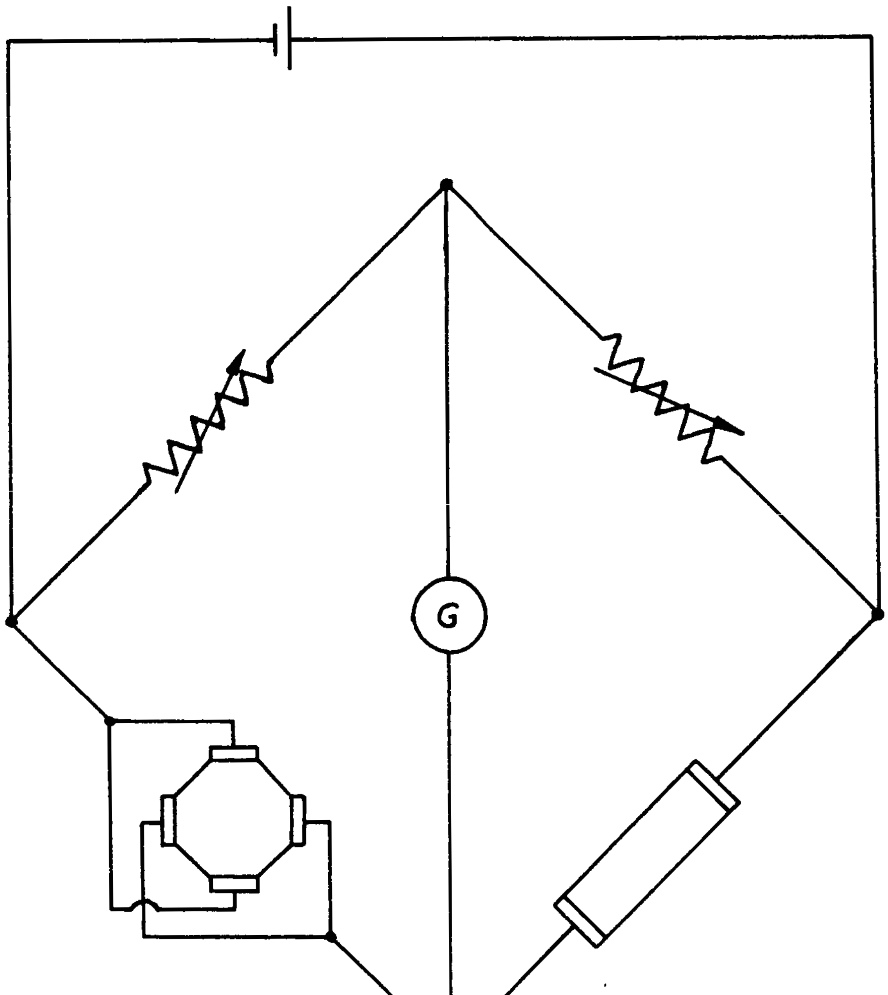

FIGURE 2. CIRCUIT SCHEMATIC

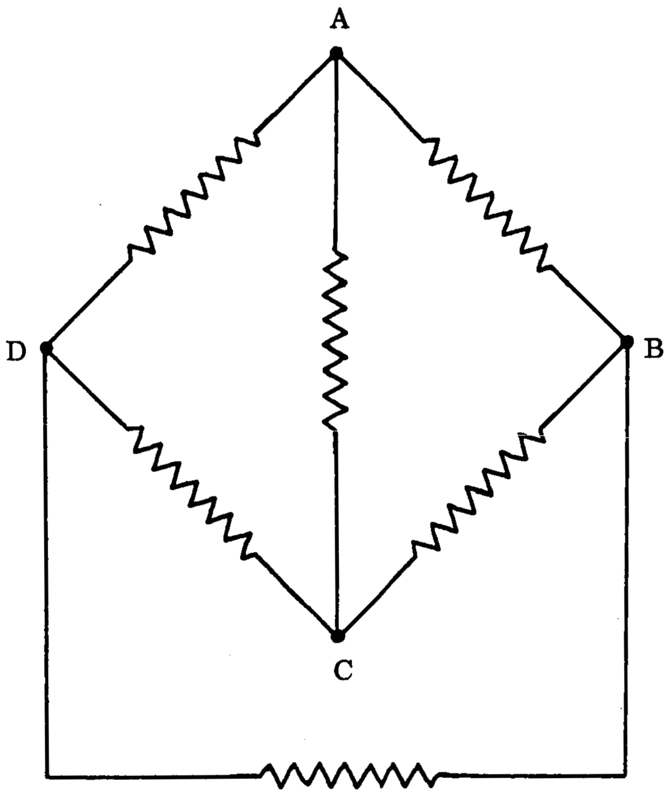

FIGURE 3. EQUIVALENT NETWORK

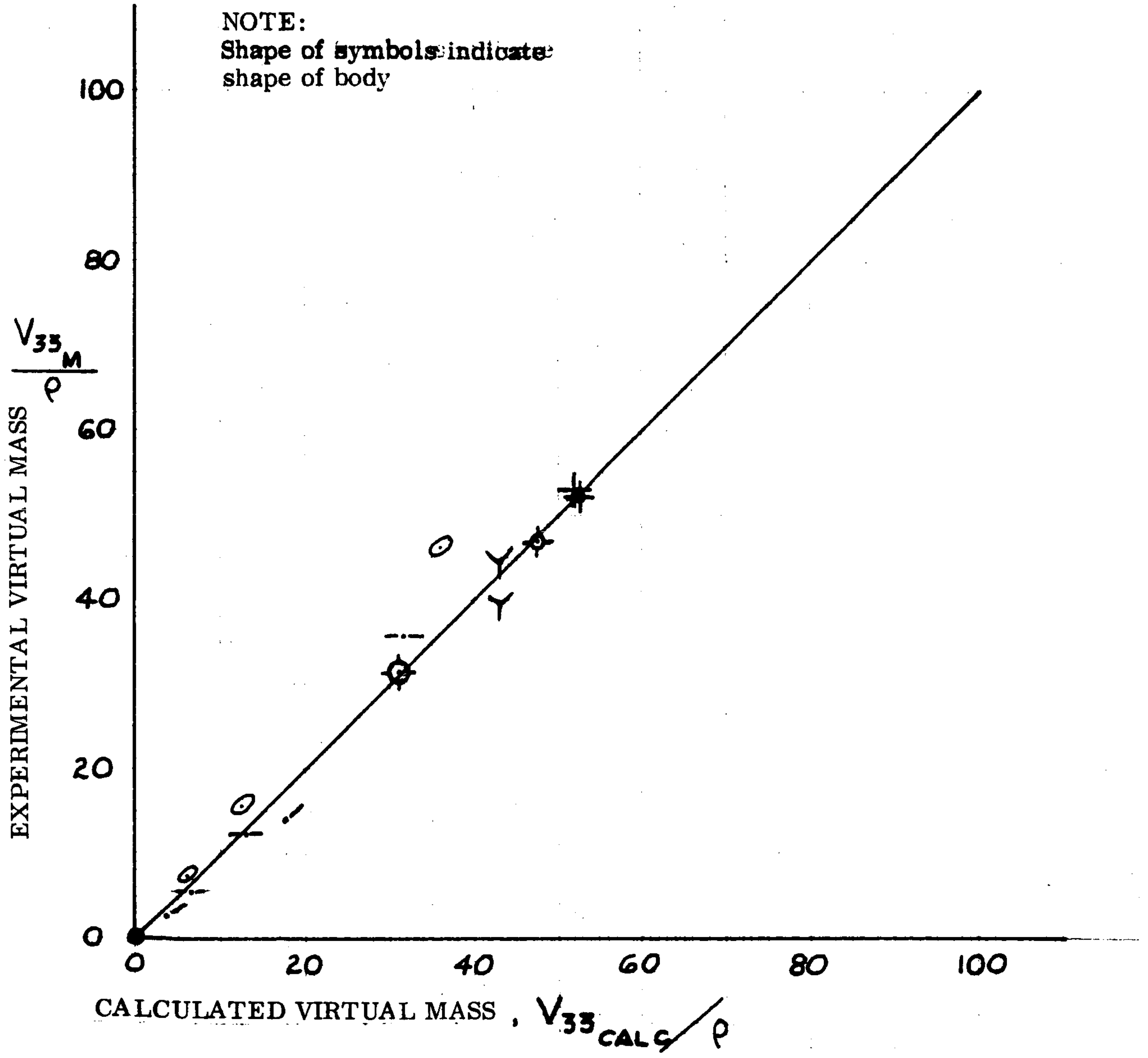

FIGURE 4. CORRELATION BETWEEN EXPERIMENTAL AND CALCULATED VIRTUAL MASS.

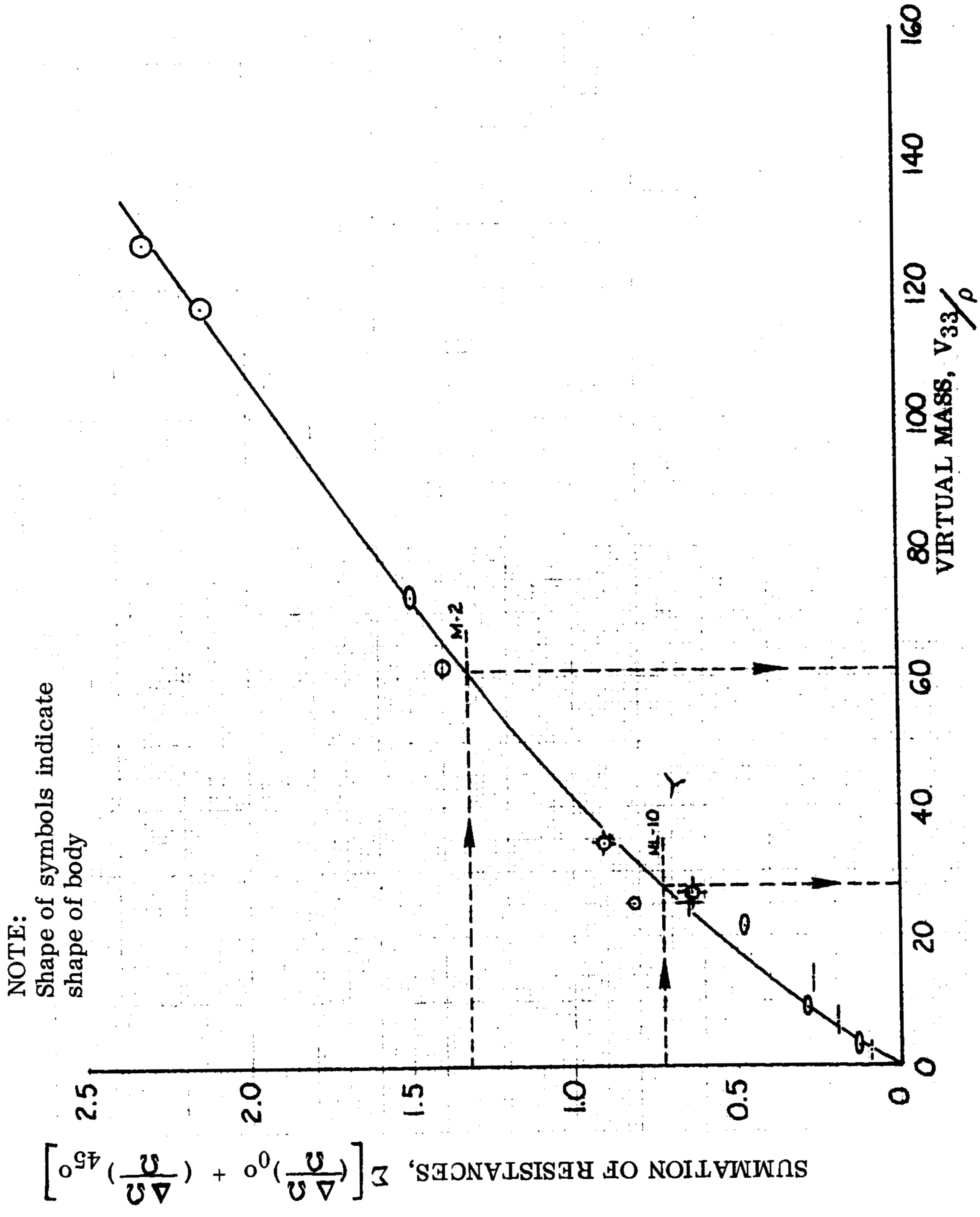

FIGURE 5. VIRTUAL MASS VS SUMMATION OF RESISTANCES