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ABSTRACT

A new mechanism involving normally forbidden processes, namely spin dependent electric dipole and spin dependent magnetic quadrupole transitions, is applied to the photoionization of atoms with a single n s outer electron. If the incident light is circularly polarized, the photo-ionized electrons via these processes are shown to be spin polarized with the direction of light propagation as the axis of the spin quantization. The cross section of photoionization by these transitions is shown to be at its maximum when the propagation vectors of the ejected electrons are in parallel with the quantization axis and is computed for the case of H, Li and Na. This is in contrast to the spin independent multipole transitions where the cross section is zero for electrons ejected in parallel to that direction. The photo-ejected electrons are shown to be 100% spin polarized when they propagate along the quantization axis. However the spin polarization drops rapidly when the electron propagation vector deviates from that axis. This is illustrated by the calculations on H, Li, and Na.

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1. INTRODUCTION

Photoionization, the ejection of valence electrons from the material system interacting with light, is a well known process and has been studied for atomic and molecular systems[1]. Theoretically[2] this process has been treated as an electric dipole interaction between electrons and the incoming light. The thus photoionized electron will propagate mostly along the directions which are perpendicular to that of light, and there will be no electrons propagating in parallel with the direction of light. Since this dipole interaction process is electron spin independent, the spin of the ejected electron will remain the same as that of the valence electron in the atom (the effect due to spin-orbit coupling is considered to be very small).

The forbidden photoionization considered here is the process due to the electron spin dependent part of the interaction between the atoms and light. This interaction is of several order of magnitude smaller than that of the above mentioned electric dipole interaction. The spin contribution to the interaction between the radiation field and the material system has also been emphasized recently by Mizushima[3], Hameka[4] and by Chiu[5] in their study of the higher multipole radiation in molecules. The importance of the spin dependent interaction in the photoionization process is that the electrons ejected via this process will be spin polarized if we use circularly polarized light, due to the angular momentum transfer between the photons and the spins of the electrons. The axis of spin quantization is the axis of light propagation. In contrast to the spin independent process, the

cross section of photoionization via spin dependent transitions is at its maximum when the electrons propagate in parallel with the quantization direction.

There are also spin-independent forbidden processes, namely electric quadrupole and magnetic dipole transitions. For atoms with s valence electrons (e.g. H , Li and Na), the magnetic dipole interaction gives no contributions to photoionization. We will restrict our treatment here to these atoms only. The electrons dissociated via electric quadrupole process also will not propagate along the direction of light. The ejected electrons which propagate in parallel with the quantization axis can therefore come only via the spin-dependent photoionization process, these electrons are 100% spin polarized if the light is circularly polarized. Since the cross section of spin-dependent photoionization is relatively much smaller than that of the allowed transitions, the spin polarization drops rapidly when the propagation vector deviates from the quantization axis. This is illustrated in section 3 for the cases of H , Li and Na .

Polarized electrons are very useful in the study of spin dependent interactions in atomic and nuclear physics. There have been considerable interests and efforts to produce polarized electrons, such as by Mott scattering[6]; by photoionizing polarized atomic beams[7,8] and by elastic resonance scattering from Ne.[9] The forbidden photoionization process described in this work represents a new method although it may not be as practical as the above mentioned methods in producing an intense polarized electron beam.

2. PHOTOIONIZATION BY MULTIPOLE EXCITATION

When an atom of N electrons is placed under the electromagnetic field of light, the interacting Hamiltonian may be written as follows[10]:

$$\mathcal{H}_q = \sum_{j=1}^{N} \left[H_q\left(\underset{\sim}{r}_j\right) + H_q\left(\underset{\sim}{S}_j\right) \right] \tag{2-1}$$

where

$$H_q\left(\underset{\sim}{r}_j\right) = -(e/2mc) \, g_\ell \left[\underset{\sim}{A}_q\left(\underset{\sim}{r}_j\right) \cdot \underset{\sim}{P}_j + \underset{\sim}{P}_j \cdot \underset{\sim}{A}_q\left(\underset{\sim}{r}_j\right) \right] \tag{2-1a}$$

is the interaction between light and the orbital motion of the electron j, and

$$H_q\left(\underset{\sim}{S}_j\right) = -(e/2mc) \, g_s \, \hbar \, \underset{\sim}{S}_j \cdot \nabla \times \underset{\sim}{A}_q\left(\underset{\sim}{r}_j\right) \tag{2-1b}$$

is the interaction between light and the electron spin. e and m refer to the charge and the mass of the electron respectively. c is the velocity of the light; $g_s = 2$ and $g_\ell = 1$; $\underset{\sim}{A}_q\left(\underset{\sim}{r}_j\right)$ is the vector potential at the electron j and q refers to the polarization of the light; $\underset{\sim}{p}_j$ is the linear momentum of the electron j and $\underset{\sim}{S}_j$ is the electron spin. We choose the z-axis to be the axis of light propagation, then

$$\underset{\sim}{A}_q\left(\underset{\sim}{r}_j\right) = \hat{e}_q \, \exp\left(i\kappa z_j\right) \tag{2-2}$$

where κ is related to the frequency of the light, ν, by $\kappa = 2\pi\nu/c$ and $\hat{e}_q$ is the unit vector of the polarization and is defined as follows,

$$\hat{e}_{+1} = -(1/\sqrt{2})\,(\hat{x} + i\hat{y}) \,, \quad \hat{e}_{-1} = (1/\sqrt{2})\,(\hat{x} - i\hat{y}) \quad \text{and} \quad \hat{e}_0 = \hat{z} \tag{2-3}$$

$\hat{e}_{+1}$ and $\hat{e}_{-1}$ represent the left handed and the right handed circular polarizations[11] respectively. $\hat{e}_0$ represents the linear polarization along the z-axis.

Since light wave is propagating along the z-axis, we have the case of $\hat{e}_{+1}$, $\hat{e}_{-1}$ and their admixtures only.

Upon expanding $A_q(\underset{\sim}{r}_j)$ one obtains a multipole expansion of the interacting Hamiltonian[5, 10],

$$H_q(\underset{\sim}{r}_j) = -A_0 \sum_{\ell=1}^{\infty} \frac{(i)^\ell (\kappa)^\ell}{\sqrt{2}(2\ell-1)!!} \left\{ e \left[\frac{4\pi(\ell+1)}{\ell(2\ell+1)} \right]^{\frac{1}{2}} r_j^\ell \, Y_{\ell,q}(\theta_j \; \varphi_j) \right.$$

$$\left. -2i\beta q \left[\frac{4\pi}{\ell+1} \right]^{\frac{1}{2}} C(\ell-1, 1, \ell; 0q) r_j^{\ell-1} Y_{\ell-1,0}(\theta_j \; \varphi_j) L_q(j) \right\} \tag{2-4}$$

$$H_q(\underset{\sim}{S}_j) = -A_0 \sum_{\ell=1}^{\infty} \frac{(i)^\ell (\kappa)^\ell}{\sqrt{2}(2\ell-1)!!} \left\{ -2\kappa\beta \left(\frac{4\pi}{2\ell+1} \right)^{\frac{1}{2}} C(\ell 1\ell; 0q) \right.$$

$$\times r_j^\ell \, Y_{\ell,0}(\theta_j \varphi_j) \, S_q(j)$$

$$\left. -2iq\beta \left[4\pi(\ell+1) \right]^{\frac{1}{2}} C(\ell-1, 1, \ell; 0q) r_j^{\ell-1} Y_{\ell-1,0}(\theta_j \varphi_j) S_q(j) \right\} \tag{2-5}$$

where $q = +1$ or -1; $\beta = e\hbar/2mc$; $L_q(j)$ and $S_q(j)$ are the qth spherical components of the angular momentum operator $\underset{\sim}{L}(j)$ and the electron spin operator $\underset{\sim}{S}(j)$ respectively. The first term inside the curly bracket of both Equations (2-4) and (2-5) are the electric multipoles with parity $(-1)^\ell$ and the second terms are the magnetic multipoles with parity $(-1)^{\ell-1}$.

Let us consider atoms with a single ns valence electron. The initial state Ψ_i is a bound S state and the final state Ψ_f is a continium state with its outer electron at infinity. Both Ψ_i and Ψ_f are approximately

represented here by an antisymmetrized product wave-function of single electron wave function ψ. Since $\mathcal{H}_q$ is a sum of one electron operators, H_q , (valence) its matrix element over Ψ_i and Ψ_f equals to that of a single electron $\wedge$ integral as follows:

$$\left(\Psi_f \mid \mathcal{H}_q \mid \Psi_i \right) = \left(\psi_f \mid H_q(r) + H_q(s) \mid \psi_i \right) \tag{2-6}$$

where $\psi_i = X_{\frac{1}{2}} \phi_{ns}$ or $X_{-\frac{1}{2}} \phi_{ns}$ and $\psi_f = X_{\frac{1}{2}} \phi_k$ or $X_{-\frac{1}{2}} \phi_k$. The spin wave functions $X_{\frac{1}{2}}$ and $X_{-\frac{1}{2}}$ refer to the cases of $m_s = +\frac{1}{2}$ and $m_s = -\frac{1}{2}$ respectively, ϕ_{ns} and ϕ_k are the spatial wave-functions.

We now choose $q = +1$, e.g. the incident light is right-handed circularly polarized. For various initial and final spin states Eq. (2-6) now becomes,

$$H_{\frac{1}{2},\frac{1}{2}} = \left(X_{\frac{1}{2}} \phi_k \mid H_{+1}(r) + H_{+1}(s) \mid X_{\frac{1}{2}} \phi_{ns} \right) = \left(X_{\frac{1}{2}} \phi_k \mid H_{+1}(r) \mid X_{\frac{1}{2}} \phi_{ns} \right)$$

$$H_{\frac{1}{2},-\frac{1}{2}} = \left(X_{\frac{1}{2}} \phi_k \mid H_{+1}(r) + H_{+1}(s) \mid X_{-\frac{1}{2}} \phi_{ns} \right) = \left(X_{\frac{1}{2}} \phi_k \mid H_{+1}(s) \mid X_{-\frac{1}{2}} \phi_{ns} \right)$$

$$H_{-\frac{1}{2},-\frac{1}{2}} = H_{\frac{1}{2},\frac{1}{2}} \quad \text{and} \quad H_{-\frac{1}{2},\frac{1}{2}} = 0 \tag{2-7}$$

The corresponding differential cross sections of photoionization are as follows:

$$\sigma_{\frac{1}{2},\frac{1}{2}}\left(\theta_k\right) = \left(4\pi^2 mk/\hbar^2 \kappa \mid A_0 \mid^2\right) \mid H_{\frac{1}{2},\frac{1}{2}} \mid^2$$

$$\sigma_{\frac{1}{2},-\frac{1}{2}}\left(\theta_k\right) = \left(4\pi^2 mk/\hbar^2 \kappa \mid A_0 \mid^2\right) \mid H_{\frac{1}{2},-\frac{1}{2}} \mid^2 \tag{2-8}$$

$$\sigma_{-\frac{1}{2},-\frac{1}{2}}\left(\theta_k\right) = \sigma_{\frac{1}{2},\frac{1}{2}} \quad \text{and} \quad \sigma_{-\frac{1}{2},\frac{1}{2}} = 0$$

where k is the magnitude of the propagation vector $\underset{\sim}{k}$ of the electrons, and θ_k is the polar angle between $\underset{\sim}{k}$ and the quantization axis (z-axis).

If the atoms are unpolarized at the initial state, the number of atoms with $X_{\frac{1}{2}}$ and $X_{-\frac{1}{2}}$ are equal. The cross section of obtaining photoionized electrons with $m_s = +\frac{1}{2}$ is therefore

$$\sigma_{+\frac{1}{2}}\left(\theta_k\right) = \sigma_{\frac{1}{2},\frac{1}{2}}\left(\theta_k\right) + \sigma_{\frac{1}{2},-\frac{1}{2}}\left(\theta_k\right) \tag{2-9a}$$

and the cross section for obtaining $m_s = -\frac{1}{2}$ electrons is

$$\sigma_{-\frac{1}{2}}\left(\theta_k\right) = \sigma_{-\frac{1}{2},-\frac{1}{2}}\left(\theta_k\right) \tag{2-9b}$$

Notice the cross sections $\sigma_{\frac{1}{2},\frac{1}{2}}$ and $\sigma_{-\frac{1}{2},-\frac{1}{2}}$ are due to spin independent process and the cross section $\sigma_{\frac{1}{2},-\frac{1}{2}}$ is due to spin dependent process. The spin polarization P of the ejected electrons is then,

$$P\left(\theta_k\right) = \frac{\sigma_{\frac{1}{2}}\left(\theta_k\right) - \sigma_{-\frac{1}{2}}\left(\theta_k\right)}{\sigma_{\frac{1}{2}}\left(\theta_k\right) + \sigma_{-\frac{1}{2}}\left(\theta_k\right)} = \frac{\sigma_{\frac{1}{2},-\frac{1}{2}}\left(\theta_k\right)}{2\sigma_{\frac{1}{2},\frac{1}{2}}\left(\theta_k\right) + \sigma_{\frac{1}{2},-\frac{1}{2}}\left(\theta_k\right)} \tag{2-10}$$

The above expression for polarization and the expressions in Equations (2-7), (2-8) and (2-9) are all for right-handed circularly polarized light. If the light is left-handed circularly polarized, a set of similar expressions can be easily obtained by merely interchanging all the subscripts $\frac{1}{2}$ and $-\frac{1}{2}$.

3. TRANSITION MATRIX ELEMENTS AND NUMERICAL RESULTS

By collecting the leading multipole terms of $\ell = 1$ and $\ell = 2$ in Eq. (2-4) and Eq. (2-5) we have the following expression of the interacting Hamiltonian for the case of right-handed circularly polarized light $(q = +1)$:

$$H_{+1}(r) = -A_0 \kappa \left\{ i \, (4\pi/3)^{\frac{1}{2}} \, e \, r \, Y_{11}(\theta\varphi) - (\kappa e/2)(4\pi/15)^{\frac{1}{2}} \, r^2 \, Y_{21}(\theta\varphi) \right.$$

$$\left. + (4\pi)^{\frac{1}{2}} \beta \, Y_{00}(\theta\varphi) L_1 + (2i\beta\kappa/3)(\pi/3)^{\frac{1}{2}} r \, Y_{10}(\theta\varphi) L_1 \right\} \quad (3\text{-}1a)$$

$$H_{+1}(s) = -A_0 \kappa \left\{ 2i \, (4\pi/3)^{\frac{1}{2}} \kappa \beta \, r \, Y_{10}(\theta\varphi) S_1 + 2(4\pi)^{\frac{1}{2}} \beta \, Y_{00}(\theta\varphi) S_1 \right\} \quad (3\text{-}1b)$$

where $L_1 = -\frac{1}{\sqrt{2}}\left(L_x + iL_y\right)$ and $S_1 = -\frac{1}{\sqrt{2}}\left(S_x + iS_y\right)$. The first two terms on the right hand side of Eq. (3-1a) are the electric multipoles (electric dipole and electric quadrupole); the last two terms are the magnetic multipoles (magnetic dipole and magnetic quadrupole). The magnetic multipole terms give no contribution in evaluating the matrix elements over the initial s state and the final continium states. In Eq. (3-1b) the first term on the right hand side is the sum of "spin electric dipole" and "spin magnetic quadrupole";[12] the second term is the "spin magnetic dipole". Due to the orthogonality between the initial and the final states, the matrix element over the spin magnetic dipole term will again be zero.

For the wave-function of the initial bound state, we use the normalized Slater[13] orbitals namely,

$$\phi_{ns} = Y_{00}(\theta\varphi) R_n(r) \quad (3\text{-}2a)$$

and

$$R_n(r) = (2n!)^{-\frac{1}{2}} \left(2\zeta/a_0\right)^{n+\frac{1}{2}} r^{n-1} \exp\left(-\zeta r/a_0\right) \quad (3\text{-}2b)$$

where a_0 is the first Bohr radius, and the value of ζ is determined by Slater's [13] rule. For H, Li and Na they are as follows:

$$\zeta(H) = 1, \quad \zeta(Li) = 1.30 \text{ and } \zeta(Na) = 2.20 \qquad (3-3)$$

For the final continium state, ϕ_k, we use the partial wave expansion of the coulomb wave function for an electron in the attractive field of $+1$ charge.

$$\phi_k = (1/k) \sum_{\ell m} (i)^\ell \, e^{-i\delta_\ell(k)} R_{k\ell}(r) Y_{\ell m}(\underset{\sim}{k}) \, Y_{\ell m}^*(\underset{\sim}{r}) \qquad (3-4a)$$

and

$$R_{k\ell}(r) = (2/\pi)^{\frac{1}{2}} k \cdot e^{\pi/2ka_0} \left((2k)^\ell / (2\ell+1)! \right) r^\ell \, e^{ikr}$$

$$\times \, | \Gamma(\ell + 1 - i/ka_0) | \; F(\ell + 1 - i/ka_0, \; 2\ell + 2, \; -2ikr) \qquad (3-4b)$$

where

$$\delta_\ell(k) = \arg \Gamma\left(\ell + 1 - i/ka_0\right) \qquad (3-5)$$

and $F(a,b,c)$ is the confluent hypergeometric function. ϕ_k and R_{k1} are normalized as follows:

$$\int \phi_k(\underset{\sim}{r}) \, \phi_{k'}(\underset{\sim}{r}) \, d\underset{\sim}{r} = \delta(\underset{\sim}{k} - \underset{\sim}{k'})$$

and

$$\int R_{k\ell}(r) \, R_{k'\ell}(r) \, r^2 dr = \delta(k - k') \qquad (3-6)$$

Substituting Equations (3-1), (3-2) and (3-4) into Eq. (2-7), and integrating over the angular part, the matrix elements are now expressed in terms of radial integrals,

$$H_{\frac{1}{2}, \frac{1}{2}} = \left(A_0 \kappa e / k\sqrt{3}\right) \Bigg\{ e^{-i\delta_1} \, Y_{11}(\underset{\sim}{k}) \int R_{k1}(r) \, R_n(r) \, r^3 dr$$

$$- \left(\kappa / 2\sqrt{5}\right) e^{-i\delta_2} \, Y_{21}(\underset{\sim}{k}) \int R_{k2}(r) \, R_n(r) \, r^4 dr \Bigg\} \qquad (3-7a)$$

$$H_{\frac{1}{2},-\frac{1}{2}} = -\left(2A_0 \kappa^2 \beta/k\sqrt{6}\right) e^{-i\delta_1} Y_{10}(\underset{\sim}{k}) \int R_{k1}(r) R_n(r) r^3 \, dr \qquad (3\text{-}7b)$$

and
$$H_{-\frac{1}{2},-\frac{1}{2}} = H_{\frac{1}{2},\frac{1}{2}}$$

where $\underset{\sim}{k}$ is the propagation vector of the electron. The radial integrals in Equations (3-7a) and (3-7b) are integrated as follows[14] :

$$I(k,\ell,n) \equiv \int R_{k\ell}(r) R_n(r) r^{\ell+2} \, dr$$

$$= (2n!)^{-\frac{1}{2}} \left(2\zeta/a_0\right)^{n+\frac{1}{2}} (2/\pi)^{\frac{1}{2}} k \, e^{\pi/2ka_0} (2k)^\ell \left[(2\ell+1)!\right]^{-1} \left|\Gamma(\ell+1-i/ka_0)\right|$$

$$\times \Gamma(2\ell+n+2) \left[a_0/\left(\zeta - ika_0\right)\right]^{2\ell+n+2}$$

$$\times F\left(\ell + 1 - i/ka_0, \ 2\ell + n + 2, \ 2\ell + 2, \ -2ika_0/\left(\zeta - ika_0\right)\right) \qquad (3\text{-}8)$$

where $F(a, c+n, c, d)$ is the hypergeometric function. Since n here is a simple integer (n=1 for H; n=2 for Li and n=3 for Na) $F(a, c+n, c, d)$ can be evaluated by the following recursion formula[2, 15] :

$$b(1-d) F(a, b+1, c, d) + (b-c) F(a, b-1, c, d)$$

$$= \left[(b-a)(1-d) - (c-b-a)\right] F(a, b, c, d)$$

and
$$F(a, b, b, d) = (1-d)^{-a} \qquad (3\text{-}9)$$

Now we can express the photoionization cross sections $\sigma_{\frac{1}{2},\frac{1}{2}}$ and $\sigma_{\frac{1}{2},-\frac{1}{2}}$ in terms of the functions $I(k, \ell, n)$ as follows:

$$\sigma_{\frac{1}{2},\frac{1}{2}}(\theta_k) = \left(\pi\kappa/2a_0 k\right) \Bigg\{ \left[I(k, 1, n) \sin\theta_k\right]^2$$

$$+ \left(\kappa^2/4\right) \left[I(k, 2, n) \sin\theta_k \cos\theta_k\right]^2$$

$$- \kappa \sin^2\theta_k \cos\theta_k \, I(k,1,n) \, I(k,2,n) \cos\tan^{-1}\left(1/2ka_0\right) \Bigg\} \qquad (3\text{-}10)$$

$$\sigma_{\frac{1}{2},-\frac{1}{2}}\left(\theta_k\right) \;=\; \pi \kappa^3 \left(\alpha^2 a_0/2k\right)\left[\mathrm{Cos}\,\theta_k \; I\,(k,1,n)\right]^2 \qquad (3\text{-}11)$$

where integral $I\,(k,\ell,n)$ has been defined in Eq. (3-8). $\sigma_{\frac{1}{2},\frac{1}{2}}$ is the photo-ionization cross section by electron spin independent process and it is consisted of three terms as shown on the right hand side of Eq. (3-10). The first term is due to electric dipole interaction and is the leading term. The second term is contributed by electric quadrupole interaction and is of the order of α^2 (α is the fine structure constant) times smaller than that of electric dipole. The last term in Eq. (3-10) is due to the interference between the electric dipole and the electric quadrupole. This interference term will equal to zero when one integrates over all the directions of the propagation vector for the ejected electrons. However at a given θ_k it is non-zero, and especially when θ_k is small this interference term is of the same order of magnitude of the leading electric dipole term. This type of interference, namely interference between multipoles of opposite parity, does not occur for transitions between a pair of discret states. It occurs here due to the property of infinite degeneracy of the continium state.

The cross section $\sigma_{\frac{1}{2},-\frac{1}{2}}\left(\theta_k\right)$, which is defined in Eq. (3-11) is due to the "spin electric dipole" and "spin magnetic quadrupole" interaction and is smaller than that of the electric dipole term in Eq. (3-10) by a factor of α^4. At $\theta_k = 0$ and $\pi, \sigma_{\frac{1}{2},-\frac{1}{2}}$ is equal to its maximum while $\sigma_{\frac{1}{2},\frac{1}{2}}$ is equal to zero. From Eq. (2-10), we therefore have,

$$P\left(\theta_k = 0\right) \;=\; 1 \quad \text{and} \quad P\left(\theta_k = \pi\right) \;=\; 1$$

That is to say the ejected electrons with propagation vector parallel to the quantization axis is 100% spin polarized. In Figure 1 we have plotted the
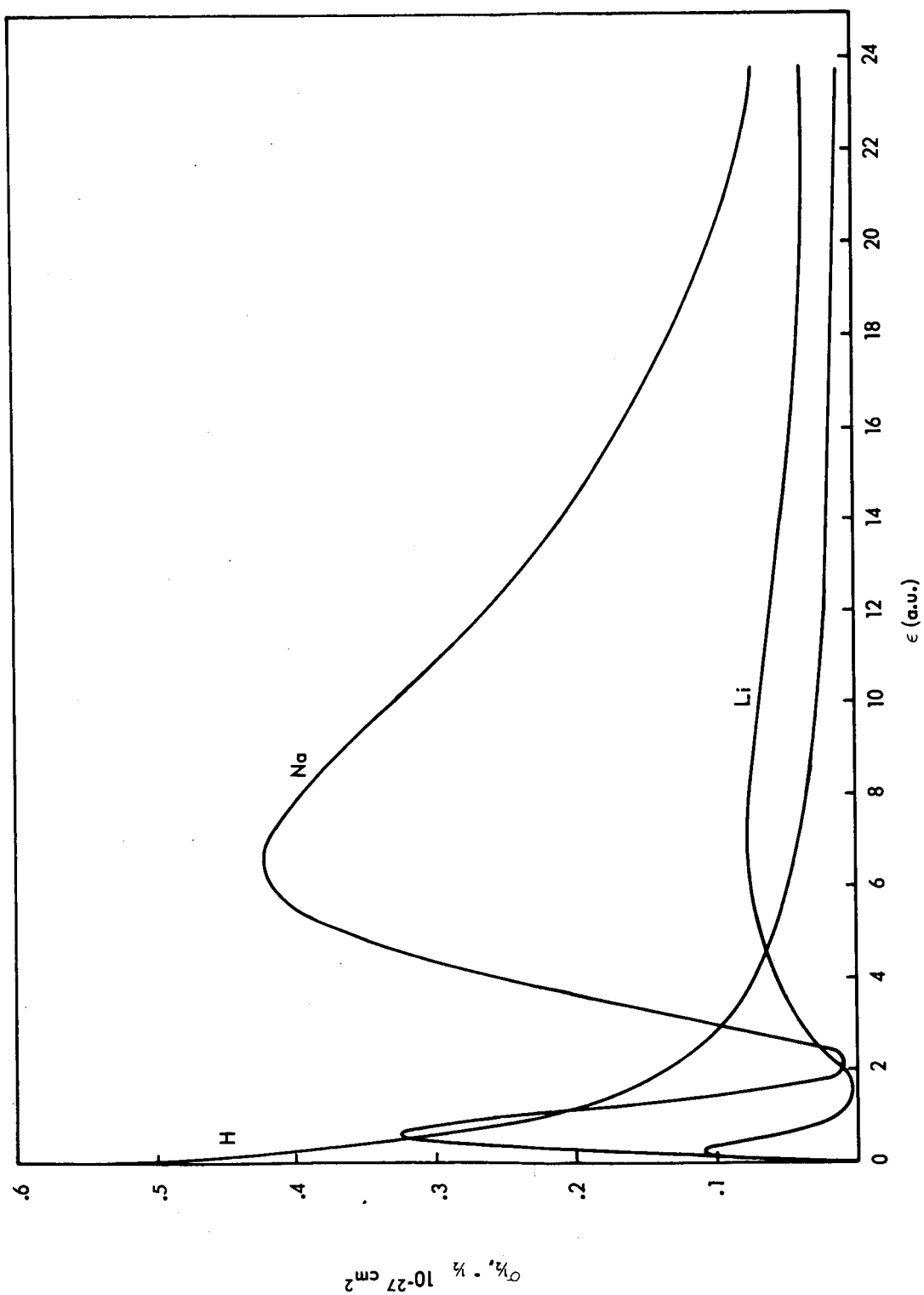
Figure 1. The differential cross section at $\theta_k = 0°$ of the spin-dependent photoionization is plotted against the energy of the emitted electrons (the total cross section is equal to $4\pi/3$ times the value here).

spin dependent cross section $\sigma_{\frac{1}{2},-\frac{1}{2}}\left(\theta_k = 0\right)$ for H, Li and Na as a function of the kinetic energy ϵ of the ejected electron and ϵ is related to the photon energy as follows:

$$\kappa\, a_0 \;=\; \alpha\left(E_i + \epsilon\right)$$

where E_i is the ionization energy of the atoms. Both E_i and ϵ are in atomic units $\left(me^4/\hbar^2\right)$ $\left(E_i(H) = 0.5,\; E_i(Li) = 0.1982,\; \text{and}\; E_i(Na) = 0.1893\right)$ α is the fine structure constant $\left(= e^2/\hbar c\right)$. a_0 is the first Bohr radius $\left(= \hbar^2/me^2\right)$. κ is related to the frequency, ν, of the light by $\kappa = 2\pi\nu/c$. In Figure 2 the average spin polarization $\overline{P}$ over a small solid angle, Ω_k, about the quantization axis is plotted against ϵ for H, Li and Na. It is shown there that $\overline{P}$ drops sharply when θ_k deviates from 0°. The same will also be true when θ_k deviates from 180°. For a given θ_k, $\overline{P}$ increases as ϵ increases, and approaches unity when ϵ is very large (e.g. 100 a.u.) even when the angle θ_k is as far as 1° away from 0°. Such high energy photoionized electrons may not be easily produced in the labora- but are quite common in the upper atmosphere due to the radiation from the sun. The forbidden and spin dependent process discussed here might find its usefulness in understanding some aspects of the photoionizing phenomenon in the upper atmosphere. At lower energies, there is a sharp peak of $\overline{P}$ for the cases of Li and Na. Since it occurs at the energy where the spin dependent photoionization cross section $\sigma_{\frac{1}{2},-\frac{1}{2}}$ is a minimum (see Figure 1), we therefore cannot attach much practical significance to this peak. This peak is absent for the case of H where the wave function is exact, therefore it might have arisen from the inexactness of the Slater wave functions of Li and Na.
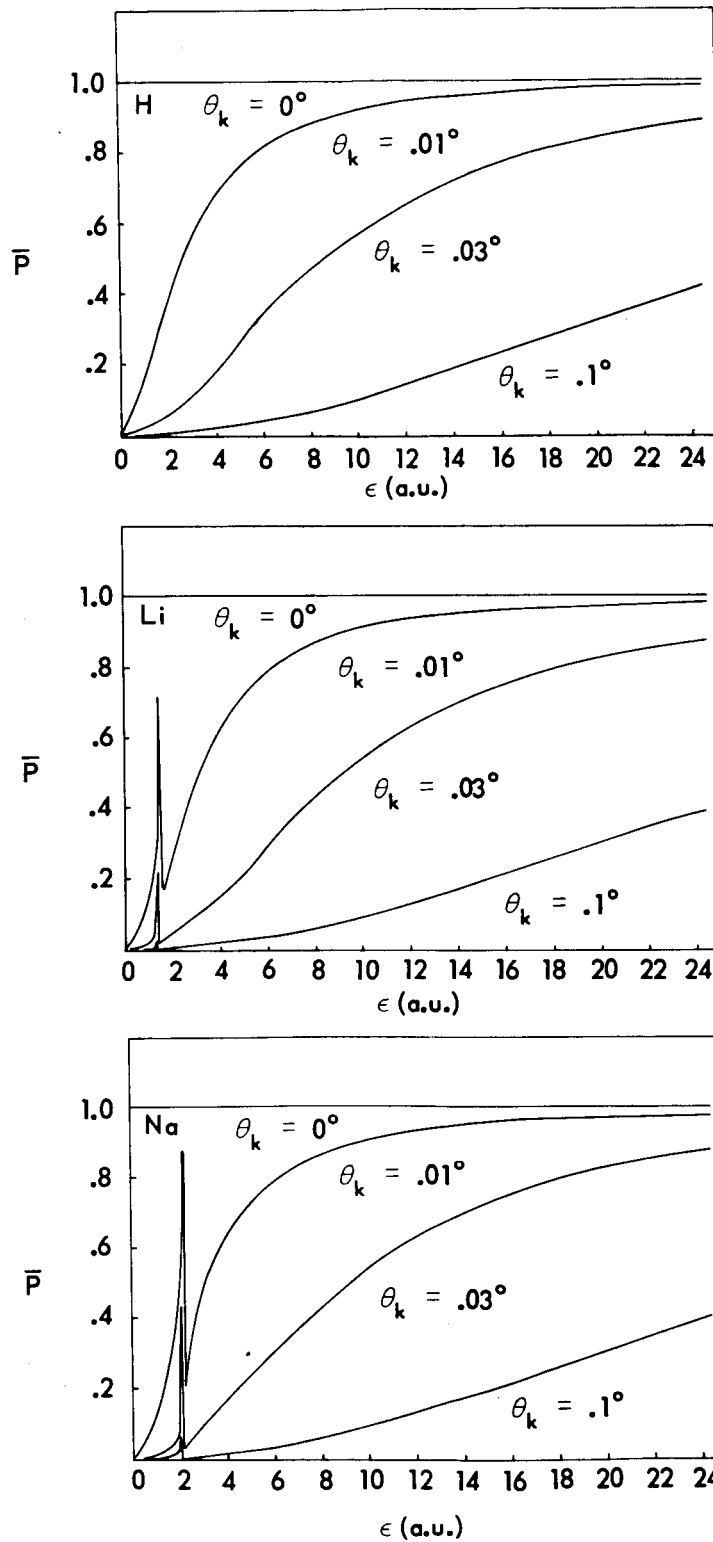
Figure 2. The Spin Polarization Averaged Over θ_k Is Plotted Against
the Energy of the Electrons.

ACKNOWLEDGMENT

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11. The unit vector of polarization, $\hat{e}_q$, defined here follows the convention of the spherical tensor. It is sometimes defined differently elsewhere (by a normalization factor), e.g. $\hat{x} + i\hat{y}$ for right handed and $\hat{x} - i\hat{y}$ for left handed circular polarization.

12. The spin electric dipole comes from the first term (with $\ell = 1$) on the right hand side of Eq. (2-5) and the spin magnetic quadrupole comes from the second term (with $\ell = 2$). These two terms are identical in this case because the quantization axis is coincident with the light propagation axis (instead of being fixed by an external field).

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FIGURE CAPTIONS

Figure 1.   The differential cross section (at $\theta_k = 0°$) of the spin-dependent photoionization is plotted against the energy of the emitted electrons (the total cross section is equal to $4\pi/3$ times the value here).

Figure 2.   The spin polarization averaged over θ_k is plotted against the energy of the electrons.