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ANATOMY AND DC CHARACTERISTICS OF THE ARTERIAL SYSTEM
WITH AN INTRODUCTION TO ITS AC CHARACTERISTICS 6

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SUMMARY

The program under the referenced contract is devoted to a study of the dynamics of internal systems in the mammal, understanding of the human system being always the ultimate objective. One of the basic internal systems of interest is the cardiovascular system and its dynamic regulation. In earlier reports under this contract, a variety of cardiovascular topics have been discussed. They have all tended to return attention to a primary focal problem: "In a quantitative sense, what is the average anatomy, geometry, and topology of the vascular system?" This report provides an updated version for H. Green's tabulation of arterial parameters (1). It is based on more data than the earlier table.

The table is shown to be consistent, in its geometric properties, with the DC resistance characteristics of the arterial bed, and the internal volume available to blood. Such a table should be quite useful for beginning discussion of many cardiovascular topics. Two are very briefly illustrated.

One important application of the internal geometry of the arterial system is in the development of transmission dynamics of pulsatile (AC) flow. The connection of the DC characteristics to an earlier treatment of the AC characteristics (14) is sketched.

A second important application of the resistance properties of the arterial system is to characterize the actual clinically observed system in both normal and pathological operation and, ultimately, to determine the detailed nature of pathological deformation. In spite of a considerable clinical variation in resistance, some data (22) are assembled to indicate that the homeostatic regulation of the normal system in quiescent operation, as seen by the heart, appears to be at a constant mechanical power level. Hypertensives, as a pathological deformation, appear to require higher power levels.

Future reports in the cardiovascular system from this program will depend on reading equally valid vantage points.

INTRODUCTION TO THE RELATION BETWEEN THE DC AND AC CHARACTERISTICS

A complete technical description of the cardiovascular system involves its physics - general anatomy and geometry of the arterial tree, mean pressure and flow relations, transmission line theory, pulsatile flow, flow characteristics of the microcirculation; physiological-physics - arterial control functions of the heart, determinants of stroke volume and heart rate, characteristics of the independent circulations, chemo-electric exchanges in the capillary (i.e., exchange of metabolites and electrolytes), electro-physiological control; medical aspects - preventive maintenance, diagnosis, management, and repair.

To provide a physical foundation for such a technical description, this report treats three topics that relate to the physics of the arterial system and touches on the actual regulated state of the terminal resistance system that is found in clinical observation. The three topics are the quantitative anatomy, geometry and topology of the arterial system; the calculation of average peripheral resistance; and some introductory remarks to the pulsatile flow characteristics of the system.

The anatomy of the human arterial tree is pictured in atlases - typically, Gray, Morris, Grant, Cunningham, Adachi. However, references with quantitative data are quite sparse. Most sources (e.g., (23) or (24)) reference Green (1), who based his arterial model on Mall's (2) 1887 measurements of branch sizes and numbers in the mesenteric artery of a 6 kg dog. Green's table is shown as Table 1.

An elementary physical model which may serve as a background for this table is an elongated central chamber, the aorta, representing a storage chamber for blood, whose elastic walls provide sufficient tension to support a high central pressure; a peristaltic heart pump, capable of producing pulses of flow against the high central back pressure, whose stroke volume and rate are jointly modified by regulatory action to provide a fairly constant mean pressure and saturated oxygen tension for the blood in the aorta. Physically, the heart pump may be regarded essentially as a constant pressure source (100 mm Hg). Its pulsatile nature (80-120 mm Hg) is analogous to a fluctuating DC voltage generator.

Starting from the single aorta, the arterial tree branches and distributes blood to the order of a billion capillary subscribers to the central service. They are served by two major system parameters, a high central pressure for mechanical purposes, and a high oxygen tension for chemical purposes.

TABLE 1. ESTIMATE OF THE GEOMETRIC CHARACTERISTICS
OF THE ARTERIAL SYSTEM IN A 13 KG DOG

(Source: Green (1), from Mall (2))

Name	Level No.	Bore - mm	No. of Tubes	Total area - cm ²	Length - cm	Velocity - cm/sec	Pressure drop - mm Hg	Cumulative drop - mm Hg
aorta	1	10	1	0.8	40	50	1.1	1.1
large arteries	2	3	40	3.0	20	13.4	1.7	2.8
main branches	3	1	600	5.0	10	8	4.6	7.4
secondary	4	0.6	1,800	5.0	4	8	4.8	12.2
tertiary	5	0.14	76,000	11.7	1.4	3.4	13.4	25.6
terminal arteries	6	0.05	10 ⁶	19.6	0.1	2	4.5	30.1
terminal branches	7	0.03	13 x 10 ⁶	91	0.15	.44	4.0	34.1
arterioles	8	0.02	40 x 10 ⁶	125	0.2	.32	8.6	42.7
capillaries	9	0.008	1.2 x 10 ⁹	600	0.1	.07	5.6	48.3
								60 mm (up to the venous arterioles)

The capillary subscribers are not fixed in number. There is a dynamic twinkling, both in the local microcirculation and the systemic circulations (i.e., the capillaries open and close, in time), knowledge of which dates back at least to Krogh (THE ANATOMY AND PHYSIOLOGY OF CAPILLARIES). In a study of temperature regulation (13) a spectrum of effects was uncovered in the energetics of the whole body system, with large amplitude metabolic cycles demonstrating periods of the order of 100 seconds, 400 seconds, 1500 seconds, 5000 seconds, and 12,000 seconds. It was hypothesized and it is in process of demonstration that the 100-second cycle represents local dynamics in the microcirculation (at the level of the capillaries) while the 400-second cycle indicates vasomotor dynamics of the blood flow among several of the systemic circulations (at the level of the arterioles). Thus, subject to these dynamics, the acting resistance bed, especially the capillary bed, is really represented by an average geometry that conforms to particular operative states of the entire animal.

A dominant anatomical impression appears to be a subdivision into levels, approximately associated with size; and a reduction in average blood velocity or proliferation in area as one descends from aorta to capillary. McDonald (3) states that the velocity in a branch is regarded as having perhaps 0.8 of the velocity in the parent trunk. Rushmer (4) portrays a common view, that the mean velocity drops from a value in the tens of cm/sec in the aorta to a few hundredths of a cm/sec in the capillaries. Physically, diminution in velocity implies proliferation in the cross-sectional area. All of this is what Green indicates in his table.

Such gross facts - the number of tubes from anatomical casts, geometric subdivision of tubes into levels, proliferation of cross-section indicated by the association of velocity with level, pressure or pressure drop at various levels - appear to be available experimentally, and are generally regarded as adequate description of anatomical and hemotological characteristics. They do not furnish a satisfactory description for many substantial physical questions, particularly when quantitative modelling is required. The following pulsating (AC) flow problem is sketchily drawn as an illustration. (A second problem, the actual operative system resistance, clinically found, is discussed at the end.)

1. The observations indicate that the high mean pressure does not fall until one reaches anatomical subdivisions near the terminal levels in the arterial tree. The measurements in Landis (5), Rappaport, Bloch, Irwin (6), Wiederhielm (7), and Intaglietta, Zweifach (8) indicate that major drops in the mean pressure only take place in tubes below the 100 micron diameter level (1000 micron = 1 mm), with significant contribution from tubes below the 30 micron level. Furthermore (as a relation of length and diameter of vessels suggests) the length associated with these last levels is quite small, a few mm. Since there is a pulse velocity, there are transit times in the pressure pulse getting out to these levels. However, they are not large. Remington and Wood (9) furnished direct experimental data on the pressure pulse in man from the aorta extending peripherally out more than two feet into the radial artery. Thus the arrival of pressure and flow are little delayed in time or magnitude of pulsation.

2. However from the aortic 'windchamber,' or 'windkessel,' the terms used by Hales and by Weber for the central storage system with elastic walls and with remote terminal resistances, one might expect a resonant response, namely that the transformation of the pulse-like flow from the heart, into pressure, would take place with a considerable oscillatory character. Thus, unless a source for sufficient damping loss can be proposed, one would expect a highly throbbing character in the response of the aorta to the periodic pulsing. However the system response is not highly throbbing. It is the characteristic lub-a-dub response that is traditionally taught. An elementary possible source of damping, visco-elastic loss in the walls, is ruled out by excellent simultaneous measurements of Patel et al (10), which showed the essential in-phase nature of pressure and diameter throughout the entire length of the aorta. (Earlier Womersley (1957) discussed the data of Lawton.)

3. One can estimate the wave characteristics in the elastic wall (this has been known for a hundred years) from its elastic characteristics, the so-called Moens-Kortweg wave and, thus, the resonant frequency that might be associated with this wave in the aorta. It is about 4 cps and, since the study of Hamilton and Dow (11), it is regarded that some evidence exists for near-resonant or standing waves in the dicrotic wave that appears as one goes down-stream in the system.

4. However, since the time of Otto Frank (12), it appears clear that the system response (the driving point characteristic) is very nearly resistance-capacitative in nature. The source of this resistance must be accounted for in order to trust any further estimate of line impedances. One may note in Figure 1 that the flow pulse from the heart, if summed in time, charges up the central aortic volume, which decays as a near constant efflux through the many subscriber arterioles. The resultant volume change into the aorta forms a triangular wave. The wall elasticity transforms the net input volume change into pressure. Further, an earlier report (13) showed that the characteristic response of a pulse of flow transforming into a triangular saw-tooth of pressure held at every point in the arterial tree for which experimental data could be found. This is also illustrated in Figure 1.

5. At most, the pulsatile character is a mild oscillation around the flow pulse and pressure saw-tooth at values near the dicrotic wave frequency. A dynamic physical analysis of the pulse wave accounting for this response characteristic of non-resonance but with no significant loss in mean pressure was presented in (14). (An earlier study by Karreman (15) illustrated, with a few lumped elements, the type of network that might lead to the dicrotic wave response.) Its general features are of interest. To simplify the analysis, while maintaining the fundamental characteristic of no damping loss of the pulse with no resonant throbbing, a constant cross-sectional area from level to level was assumed, except at the capillary level. (A high mean velocity of 50 cm/sec was assumed throughout the system, dropping precipitously to a mean of 0.07 cm/sec in capillaries.) This is crudely consistent with data in the physiology texts of Rushmer, Best and Taylor, and Guyton. The levels in the system, which may be viewed as a sequence of diminishing steps in diameter, were replaced by a continuous equivalent branching.

This represented the system as an ever-branching, tapered tube model. One may then treat the wave that spreads out and reflects through such a system.

Fortunately, this model in its mathematical details seemed to have the right characteristics and the apparent discrepancies fell into place. There is little or no damping throughout most of the system. There is little or no drop in mean pressure. There is the appearance of a dicrotic wave (not the incisura in the input pressure, which has other causality.) There is rational damping of the higher frequency components (of which the incisura is an example), yet there is no resonant throbbing. The dicrotic wave is formed from out-of-phase lags of the harmonic components higher than the fundamental. The basic ingredient from which these properties emerge, put somewhat paradoxically, is that while there is little damping in the outgoing or incoming wave, there is no reinforcing amplitude left in the returning wave. It has been damped at the remote terminal resistance end. The description is similar to Rayleigh's explanation of acoustic damping near a porous wall and, in fact, the model may be referred to as a tapered porous wall model of the arterial system.

In subsequent discussion with research workers, various modeling questions were raised involving the physics, physiology, and pathology of the system. To insure that the background for these questions was adequate, it was necessary to clarify various details about the real system. For example a question of the choice of mean entrance velocity and its subsequent near constancy indicated that projecting a model of the hemodynamic events required that it be as faithful as possible to all known anatomical details and based on the best possible model of anatomy and peripheral DC resistance. Only then could any pulsating or varying results be seriously considered. This report is designed to perform the function of providing a representative anatomical description.

THE ANATOMY AND DC CHARACTERISTICS

From literature search, it was realized that apparently only Green (1) had attempted to assemble an anatomical description into a coordinated whole, that his data simply scaled Mall's 1887 measurements on one mesenteric artery, and that very few other investigators had ever been similarly involved.

Based on considerable search, data have been assembled for a more certain anatomical summary from the work of three investigators, Patel (16), Mall's post-1887 studies (17), and Suwa (18).

Patel et al reported on geometric data in the dog aorta and large arteries. Most significantly, the data were derived from the averages of many dogs, both living and dead, corrected to the living state; whereas data in atlases are generally based on post-mortem sources. Averaged for 23 kg dogs, they indicated an aorta entrance area of 3.2 cm^2 and about 29 'major' arteries whose total entrance area is about 3.4 cm^2 . It may be surmised that the cross-sectional area of the cardiovascular system does not change much from the aorta to the major arteries. The same conclusion was stressed by Mall (17). "Thoma made many measurements of arteries and their branches and tabulated Bencke's measurements of the aorta with its branches. These measurements show that the area of all of the branches of the aorta equals about the area of the ascending aorta being a little less before the thirtieth year of age and a little greater thereafter ..."

From Altman (20), the cardiac output for a 23 kg dog may be estimated as nearly 50 cc/sec at rest (or 200 cc/sec at high activity). The mean velocity in the aorta is thus about 15-20 cm/sec at rest (or about 60-70 cm/sec at high activity). This also seems to be the correct magnitude of mean velocity in the ascending aorta in other species including humans. (For example, in the human at rest, a cardiac output of 120 cc/sec with an internal diameter of 2.7 cm for the aorta gives about 20 cm/sec). The present geometric concern is with cross-sectional area, not velocity, which depends on activity level. However, it is necessary to point out the need for consistency.

Thus it appears that the first two lines of Green's table are inconsistent regarding velocity and area. The burden exists to propose a more nearly correct table. The source data are assembled in Figure 2 based on:

1. The Patel data - aorta and main artery sizes in living 23 kg dogs.
2. The post-1887 Mall data - based on cast counts from 5 circulations in a 6 kg dog - intestine, stomach, adrenal, spleen, liver.

3. The 1963 Suwa data - based on cast counts from 7 circulations in humans - kidney, intestine, femoral, pancreas, heart, cortex, basal ganglia. (It is noteworthy that Suwa's geometric studies stem from his interests as a pathologist of the cardiovascular system.)

Figure 2 exhibits level area versus level diameter range. From Patel's data on the aorta, the values chosen were a maximum entrance diameter of 20 mm; a minimum exit diameter of 7 mm, and an entrance area of 3.2 cm². For major arteries, a maximum diameter of 11 mm was estimated, a probable minimum exit of 2 mm, and a similar area of 3.4 cm².

Mall's data, taken on a 6 kg dog, can be scaled up to Patel's 23 kg dog data by increasing cross-sectional area by a factor of 4. This may be specifically justified by comparing cross-sectional area data on those arteries that are reported in common.

Unfortunately, while Mall's data provide the most extensive arterial count found, it only covered 30% of the high pressure arterial circulation. In utilizing his data, it was preferred to regard each circulation (including the one Green used) as an independent fractional estimate of the entire circulation. This would be valid if mean velocity and diameter were uniformly associated throughout the system. Thus Mall's data are scaled, for each major artery that he lists, to fit the Patel data. The scatter is large, but one can reasonably infer that the cross-sectional area doesn't change much until arterial diameters of the order of 1/2 mm are reached, then an approximate uniform proliferation rate per level for arterial sizes down to about 20-30 micron, and then a large proliferation in area down to capillary sizes of the order of 8 micron.

Suwa's results can be summarized algebraically. Let d_0 = an entrance diameter and $d_{1,2..}$ = branching diameters. If the area after every branching were constant

$$d_0^2 = d_1^2 + d_2^2 + \dots$$

Instead Suwa showed that at every branching

$$d_0^{2.7} = d_1^{2.7} + d_2^{2.7} + \dots$$

This result was also obtained earlier by Groat (19). One can then estimate that the area at every level is nearly proportional to $1/d^{0.7}$. This line is plotted in the figure. The approximate agreement may be noted. Thus, Patel's, Mall's, and Suwa's data are consistent and they now permit the necessary amendment of Green's table. The Suwa 2.7 power law derived from humans in the 25 to 1000 micron diameter range seems to agree with data derived by Mall from a dog. For larger size tubes, the area doesn't change much. For the one or two levels below 25 micron the area proliferates more rapidly. Thus one finds three anatomical regions.

A substantive question is: What constitutes a level? In the present view, Suwa's data destroy the idea that levels intrinsically exist as specifically sized entities, since he showed an essentially continuous distribution of diameters. While the basis for anatomical levels of subdivision in the tree has not been reviewed here, clearly they have been associated with size, distinctness, uniqueness, and function. The aorta and major arterial branches form two distinct levels. The system ends in capillaries which are said to be fairly uniform in diameter and near 10 micron in mammals. Yet one may note that Mall lists capillaries down to 3 micron, as does Wiedemann (23), and it is common in the microcirculation literature to find capillaries described up to 20 micron.

These three levels appear certain. Obviously, the anatomist successfully follows secondary branches from the major arterial branches. However, as one aorta becomes 20 main branches, with many variants even in a given species, and becomes hundreds of branches in the next few levels, the difficulty increases. At the microcirculation end, the distinctions among arterioles, terminal arterioles, small arteries also become difficult. Thus it is five levels of division that appear certain and up to thirteen levels have been discussed in the literature.

Yet Suwa shows, in plots of segment length versus diameter in different circulations, that the diameter range is essentially continuously distributed over his experimental range of 20 to 4000 micron, and Patel's data, covering diameters from 2 to 20 mm, is also consistent with an essentially continuous diameter variable. What then is a level?

It is the present opinion that the concept of level is an intrinsic concomitant of the act of branching. A level may be associated with a tube as long as one can retain the view of a 'main' tube. As smaller tubes are shed from view, one can continue to pursue the main tube. However, ultimately there arises a bifurcation into nearly equal tubes. Then the concept of that level ends. What seems really intrinsic to the concept of 'level' is the number of significant branchings that take place before an equal bifurcation occurs.

On this basis one can assign some kind of estimate to the number of levels. Taking some common numbers regarding capillaries, say one capillary segment of about 100 micron length associated with a region of about 30 micron diameter, this represents about 3×10^{11} capillary segments in a 23 kg dog (i.e., one computes 3×10^{11} segments of tissue 30 micron in diameter and 100 micron long in a 23 kg dog if near water density is assumed. One way to assess whether the capillary number is reasonable is to compute the internal volume as 8 micron capillaries. A common view is that in complete dilation - extreme shock - the open capillaries will hold substantially all the blood. A 23 kg dog has about 2200 cc of blood, from Altman (20) who presents an average of about 95 cc/kg for dogs. 3×10^{11} capillaries of 8 micron diameter and 100 micron length has a volume of about 1600 cc).

$$N^m = 3 \times 10^{11}$$

N = no. branchings per level

m = no. of levels.

This leads to the estimate in column (a) of the following table:

No. branchings per level	Estimated number of levels		
	(a)	(b)	(c)
2	38	32	30
4	19	16	15
6	15	12-1/2	12
8	13	11	10
10	11-1/2	10	9
15	10	8-1/2	8
20	9	7-1/2	7
30	8	6-1/2	6
40	7	6	6

This overestimates the number of levels because the terminal arteriolar resistor serves more than one capillary. One can estimate a lesser number of first branch capillaries from the cardiac output and the nominal velocity. Assuming total flows in the range 50-200 cc/sec (resting to active), average capillary velocity of about 0.07 cm/sec, and an average capillary diameter of 8 micron, the number of series-parallel capillary systems that are served range from about 1.4 to 5.6×10^9 (i.e., numbers agreeing with Green's estimate). These differ from the geometric number by about a factor of 60 to 240. The discrepancy is accounted for by the fact that on the average not all of the capillaries are open at any one time and, in fact, only a small percentage are open. (An estimate of the number open varies in different circulations from about a factor of two in rest to maximum activity in muscles, to about a factor of 200 in the skin circulation from highly vasoconstricted to highly vasodilated. The present average estimate is valid to within an order of magnitude.) Then, approximately

$$N^m = 1.4 - 5.6 \times 10^9$$

These lead to estimates (c) and (b), also shown in the table.

Suwa's length to diameter data provided assurance that the number of branchings is certainly at least 5 (for the same diameter, he shows a scatter

of length to diameter on the average of about 5 to 1). More to the point, he showed that 8 branchings are common and 20 to 30 are not excluded. Thus, the number of levels is probably in the 7-12 range. (Therefore, in addition to any anatomical reasons, geometry also helps create a 'reason' for finding anatomical levels.)

One may choose, arbitrarily at this time, either a branching or a level number. At present it appears that the greatest coherence lies near 10-11 levels, in which the diameter range associated with each level is divided in approximately equal ratios. From Patel's data, it is estimated that a diameter range of about 2-3 to 1 is appropriate to a level. It is this type of subdivision that has also quite plausibly been derived from Mall's data.

With these data, one may now attempt to construct a model of the total resistance in the system.

For each level, Poiseuille flow gives

$$\Delta p_i = \frac{128}{\pi} \mu q \frac{\ell}{d^4} \left[\frac{1}{\left(1 + \frac{6}{d}\right)^2} \right]$$

The factor, $1/(1 + 6/d)^2$, the Fahraeus-Lindqvist correction, is a viscosity correction for red blood cell flow in small tubes - (d in micron). References and discussion justifying this correction are given in (1), (18), and Ruch and Fulton.

$$q = \frac{Q}{N}$$

$$\Delta p_i = 32\mu \frac{4Q}{\pi N d^2} \frac{\ell}{d^2} \left[\frac{1}{\left(1 + \frac{6}{d}\right)^2} \right]$$

$$A = \frac{\pi}{4} N d^2$$

$$\Delta p_i = 32\mu Q \frac{\ell}{A d^2} \left[\frac{1}{\left(1 + \frac{6}{d}\right)^2} \right]$$

Summed over levels

$$\Delta p = 32\mu Q \left[\frac{\ell_1}{A_1 d_1^2} \frac{1}{\left(1 + \frac{6}{d_1}\right)^2} + \frac{\ell_2}{A_2 d_2^2} \frac{1}{\left(1 + \frac{6}{d_2}\right)^2} \dots \right]$$

$$\frac{\Delta p}{Q} = R = 32\mu \left[\frac{\ell_1}{A_1 d_1^2} \frac{1}{\left(1 + \frac{6}{d_1}\right)^2} \dots \right]$$

$$\frac{R}{32\mu} = \frac{\ell_1}{A_1 d_1^2} \frac{1}{\left(1 + \frac{6}{d_1}\right)^2} + \dots$$

Δp = pressure drop

q = flow in a single tube

Q = total flow

N = number of parallel tubes in a level

ℓ = tube length at a level

d = tube diameter at a level

μ = viscosity

R = resistance ($= \Delta p/Q$)

Suwa also indicated that for any level in the 20-4000 micron range the ratio ℓ/d tends to be constant. Actually, he showed that an arterial segment will likely branch within 3 diameters and probably will have had appreciable side branchings within 15 diameters. Patel's data on the aorta indicated a 20 to 1 length to entrance diameter ratio. From this, from the lack of much tapering in small tube sizes below the 100 micron level as seen in Bloch's data (21), and from the extremes of Suwa's data, one may estimate that a level may extend to the order of 20 to 30 diameters before a nearly equal bifurcation. This is consistent with the estimate of number of branchings per level that about 7-10 branchings would indicate about 9-12 levels, and check reasonably the findings by Mall of 10-11 levels. Thus $\ell/d = 25 \pm 5$ is not an unreasonable guess. Then a resistance function may be constructed solely from geometric factors as follows:

$$\frac{R}{32\mu (\ell/d)} = \frac{1}{A_1 d_1 \left(1 + \frac{6}{d_1}\right)^2} + \frac{1}{A_2 d_2 \left(1 + \frac{6}{d_2}\right)^2} + \dots$$

If one assumes, for the 23 kg dog,

$$\Delta p = 110 \text{ mm Hg } (= .15 \times 10^6 \text{ dynes/cm}^2)$$

$$Q = 50 \text{ cm}^3/\text{sec (at rest)}$$

$$\ell/d = 25$$

$$\mu = .035 \text{ poise (gr/cm sec)}$$

$$105/\text{cm}^3 = \frac{1}{A_1 d_1 \left(1 + \frac{6}{d_1}\right)^2} + \frac{1}{A_2 d_2 \left(1 + \frac{6}{d_1}\right)^2} + \dots$$

In Table 2, derived from Figure 2, an estimate is made of the $1/Ad$ sums of $109/\text{cm}^3$ which is the correct order of magnitude.

Notes on the construction of the table

- (a) The number of levels - 11 - was assigned a priori.
- (b) The entrance and exit diameter and area for the aorta - level 1 - were taken from (16), as were the entrance diameter range and total area for level 2.
- (c) The average entrance area for all levels, 2 and beyond, were estimated from the prior exit area by the Groat-Suwa bifurcation rule $d_0^n = 2d_1^n$, from which $A_1 = 2^{-2/n} A_2$. If $n = 2$, $A_1 = A_2/2$; $n = 2.7$, $A_1 = A_2/1.67$; $n = \text{large}$, $A_1 = A_2$. The average entrance diameter was then computed from the average entrance area.
- (d) The total diameter range in Figure 2 was divided into 11 overlapping ranges, taking into account the known ranges of level 1 and 2 from (16) and the likely range of the last capillary level (i.e., diameter ranges of about 4 to 1 were selected).
These diameter ranges were assumed to be both the entrance diameter range, and each lower bound was considered to be the exit diameter. The exit area was computed from this diameter.
- (e) The total area was then estimated from Figure 2, essentially for the value of average entrance diameter. (From Suwa, one estimates $A_0 \propto 1/\bar{D}_0^{0.7}$).
- (f) For the viscosity correction $6/D_0$ in the resistance column, D_0 is in micron. This column represents the contributions at each level to the resistance function $R/32\mu(l/d)$. $\mu = 0.035$ poise, $l/d = 25$.
- (g) The capillary area is simply a fictitious value.
- (h) For a meaningful completion of the table with some consistency for capillaries, it is necessary to make use of the experimental data in small sized tubes, which is limited. At present it might be based on seeking consistency among data on the frog and mammals from Landis, Wiederhielm, Rappaport, and Intaglietta. (The frog data are not pertinent. They are simply given as a parallel guide to qualify the mammalian data).

TABLE 2. ESTIMATED ARTERIAL TREE GEOMETRY AND RESISTANCE FOR A 23 KG DOG

Level No. (a) - L	Diameter - cm		Area - cm ²			No. Tubes	Groat- Suwa Expon. - n (c)	$\frac{1/A \bar{D}_O}{(1+6/D_O)^2}$ - 1/cm ³ (f)	Level Volume (k) - cm ³
	Ent. D _O	Exit D _I	Av. Ent. $\bar{D}_O$	Total A _O	Av. Ent. A _I	Exit A ₂			
1	2.0(b)	0.7(b)	2.0	3.2	3.2(b)	0.35(b)	1	.2	47
2	1.0-.18(b)	.18(d)	.47(c)	3.4(b)	.17(c)	.026(d)	20	.6	31
3	.4-.1(d)	.1	.13	3.4(e)	.013	.008	260	2.3	34
4	.2-.04	.04	.08	4.0	.005	1.3 x 10 ⁻³	800	3.1	20
5	.08-.02	.02	.031	5	7.6 x 10 ⁻⁴	3.0 x 10 ⁻⁴	7 x 10 ³	6.4	10
6	.04-.01	.01	.016	6	1.9 x 10 ⁻⁴	8.0 x 10 ⁻⁵	3 x 10 ⁴	10.1	6
7	.02-.004	.004	.008	10	4.8 x 10 ⁻⁵	1.3 x 10 ⁻⁵	2 x 10 ⁵	11.8	5
8	.01-.0025	.0025	.0032	16	7.6 x 10 ⁻⁶	5.0 x 10 ⁻⁶	2 x 10 ⁶	17.3	4
9	.006-.0015	.0015	.0020	25	3.0 x 10 ⁻⁶	1.8 x 10 ⁻⁶	8 x 10 ⁶	16.5	4
10	.003-.0008	.0008	.0012	35	1.1 x 10 ⁻⁶	5.0 x 10 ⁻⁷	3 x 10 ⁷	16.5	3
11(j)	.0015-.0004	.0008	.0008	80(g)	5.0 x 10 ⁻⁷	5.0 x 10 ⁻⁷	2 x 10 ⁸	8.0(h)	- 185
Letter legends generally refer to the entire column following								93	165 350
								16(h)	
								109 cm ⁻³ (i)	

Frog mesentery

[Name or size tube (micron)
Mean pressure (mm Hg)]

Landis (Summary table)				Cap. (art. side) 11		Cap. (ven. side) 9		mm Hg
Wiederhielm	260	126	21	26 (cap ent)	24 (cap middle)	22 (cap ex)		micron
	30	22	14	14	9	7		mm Hg
Intaglietta	17 (cap ent)			17 $\left(\begin{smallmatrix} 1/3 \\ \text{way down} \\ 15 \end{smallmatrix} \right)$	19 $\left(\begin{smallmatrix} 2/3 \\ \text{way down} \\ 13 \end{smallmatrix} \right)$	23 (ex)		micron
	18					6		mm Hg

Summary: Central pressure 30 mm Hg; cap. ent. 16 ± 2 ; cap. ex. 7 ± 2 ;
cap. drop 9 ± 3 .

Mammal

Landis (summary table - human)		modal value in caps. 13		av. limits for the deviation around mode 8-22	
Landis (rat, g.p., human - summary)		cap. (art. side) 27		cap. (ven. side) 12-1/2	
Intaglietta (rat)	16 (cap. ent.) 24	7 (art. side) 24	7 (ven. side) 23	13 (cap. ex.) 20	

Summary: Central pressure 70-100 mm Hg; cap. ent. 24 ± 4 ; cap. ex. 16 ± 4 ;
cap. drop 8 ± 4 . Thus, an apt estimate (quiescent) is 8 mm Hg drop
across capillaries and 16 mm Hg pressure at the capillary exit into
the venous side.

- (i) The estimate may be compared with the following standard: Gregg in Best and Taylor, 1961, states "The carotid blood pressure (mean) of the un-anesthetized, basal dog approximates 110 mm Hg." From Altman (20) the basal flow is about 50 cc/sec. The computed resistance function was 105 cm⁻³.
- (j) One may obtain a more consistent view of the capillaries by assuming approximate parameters, such as Q = 50 cc/sec, V = 0.07 cm/sec, $l/d = 100/8$, $d = 0.0008$ cm, $\Delta p = 8$ mm Hg, N = no. of capillary channels open, m = no. of layers of capillary resistances in the series-parallel bed, then

$$A = Q/V = 710 \text{ cm}^2$$

$$N = \frac{4A}{\pi d^2} = \frac{710}{.785 \times .0008^2} = 1.4 \times 10^9$$

$$m = \frac{\Delta p}{32\mu V} \frac{d}{1} d \left(1 + \frac{6}{d}\right)^2 = \frac{\frac{8}{750} \times 10^6 \times .0008 \times 8 \times 1.75^2}{32 \times .035 \times .07 \times 100} = 27$$

$$mN = 3.7 \times 10^{10}$$

$$\text{Total no. (est.)} = 33 \times 10^{10}$$

Thus 10 percent (i.e., 3.7/33) of the capillaries are normally open (accuracy of calculation is indeterminate).

- (k) A final consistency check is from the probable volumes in the arterial tree. From Altman (20), the blood volume for dogs averages about 95 cc/kg.; or 2200 cc for a 23 kg dog. From Gregg in Best and Taylor; the aorta volume is about 2 percent of the blood volume (50 cc); the arterial volume 8 percent (175 cc); the arterioles 1 percent (20 cc); and the capillaries (quiescent operation) about 5 percent (100 cc). Total 16 percent (350 cc).

The volume may be computed as follows: In the $n = 2$ regime, the volume is pyramidal, $V = 1/3 (A_{ent} + A_{ex}) L = 1/3 (L/D) D_{ent} (A_{ent} + A_{ex}) = 8.3 D_{ent} (A_{ent} + A_{ex}) \approx 9.2 D_{ent} A_{ent}$ (if exit not certain). In the $n = 2.7$ regime, really n approximately 3 regime, there is a near tendency to preserve volume from level to level. Thus $V = A_{ent} L = 25 D_{ent} A_{ent}$.

For the capillaries $Vol = (mN) \pi/4 D^2 L$. Based on the $mN = 3.7 \times 10^{10}$ segments, 100 micron long, 8 micron is diameter, $Vol = 185 \text{ cc}$.

If the long logical process is followed by which the estimate is made, then one can only marvel at the nominal validity of the estimate. All that is proposed is that an estimate or adjustment of peripheral resistance can be made with the available data on a purely geometric-anatomical basis.

In summary: the effective cross-sectional area in the tree has not changed much out to internal diameters of the order of 1 mm. In the size range of human to medium-sized dog, this may be out as far as the first four levels in the tree - namely, the aorta, the main arteries, and the main and secondary branches.

The approximate 'length' of every level is about 25 times the diameter. Thus there has been little change in cross-sectional area out to within a few cm of the arterial termini. Furthermore, there is little mean pressure change out to that level.

The highest resistance region is the last less than 30 micron diameter, approximate 0.6 mm length.

The concept of an anatomical level has a strong geometric foundation, based on the average number of branchings before an equal bifurcation takes place, and on the length to diameter ratio before branching. All of these concepts are self-consistent with the number of subscriber elements and internal volumes.

There is this reason to believe that anatomy, geometry, topology, and flow resistance of the arterial tree can be validly modeled, albeit approximate, and that it is not one level that dominates the resistance picture.

To provide some flavor of how this summary paper may be used, two applications will be briefly commented upon.

SOME FURTHER COMMENTS ON THE PULSATILE (AC) FLOW

This more precise modelling of the DC characteristics permits a review of the previous modelling of pulsatile events in the tree (13). It appears that the model is reasonably valid for the following reasons:

Since the cross-sectional area does not change until one is well out near the periphery of the system, up to that point, the system may be viewed as a tapered porous tube model (that is, through any running length, there is an approximate uniform number of branchings which can be replaced by an effective tapering area with an effective equivalent branching area that distributes into side tubes). This region may be regarded as the long transmission line or wave propagation zone, which, by hydrodynamic reasoning, is fairly undamped at the higher frequencies.

The region in which the area changes markedly is no longer concerned with such traveling waves. It was treated (13) as if it consisted only of 'terminal resistors' of the order of 30 micron in diameter. It now appears that there really is a modest terminal impedance region below a few hundred micron diameter. That is, in addition to flow resistance, there still remains some wall elasticity and some storage capacity. This will make some moderate changes in the frequency response of the long line, but not much. Thus the earlier result presented much of the dominant character of the transmission line. These terminal impedances are where the resonant energy is soaked up in an out-of-phase return of the higher frequency components.

SOME COMMENTS ON PERIPHERAL RESISTANCE FOUND CLINICALLY

Whereas such a table as presented here may give the impression that resistance is determined by a fixed geometry for the system, experimental data shows that there is considerable variability in both the pressure and flow that go to make up the resistance in both clinically normal and abnormal individuals. This raises significant questions about the regulation of pressure and flow in the arterial system.

Two sources of data have been examined. A Russian paper (22) states, "It is now the accepted belief ... that the main hydrodynamic mechanism causing a rise in arterial pressure in hypertension is increased resistance to the blood flow in the arterioles, resulting from spasm of the latter. An indication of this mechanism in hypertension is the increased peripheral resistance accompanying the disease." Data are offered on 111 hypertensive patients - see Figure 3 - presented as mean pressure versus resistance, and mean flow versus resistance, as well as a curve based on normals. They state that normal healthy patients show a mean pressure that varies with the one fourth root of resistance or flow that varies inversely as the $3/4$ power of resistance.

Such a result for normals was somewhat surprising, although not based on any real knowledge about clinical hypertension. A casual impression, confirmed by inquiry among some cardiologists and cardiovascular physiologists, was that the basis for identifying 'hypertension' was a loss in pressure regulation to values higher than normal. Yet these data indicated (as do the studies of Master, in Rushmer (4)) normals with mean pressure in a range 80 to 150 mm Hg, which is a wide range compared to an elementary idea of a near constant mean pressure. Also, evoking an exotic law like the fourth root of resistance is hardly indicative of understanding of a process.

It is clear that hypertension was indicated as pressure above this normal level, but equally surprising was the wide resistance range covered by normal and still-functioning systems, "from 900 to 7900 dyne sec/cm⁵, averaging 2745 $\pm$ 122 ..." In considering these data carefully, a simple orienting hypothesis suggested itself.

Power from its electrical analogue is proportional in the flow case to P_m^2/R , or Q^2R . Consider plots of lines of constant power consumption shown in the lower graphs.

What emerges is that the normal curve is likely associated with a given power level requirement from the heart rather than with pressure. A hypertensive must work at a higher basal power rate. From these data, one would

infer that hypertension is not determined by pressure but by power. In fact, in an extreme hypertensive, the power level appears to be about 2-3 times the normal, and apparently about a 20 percent higher power level than normal is a discriminable level of hypertension.

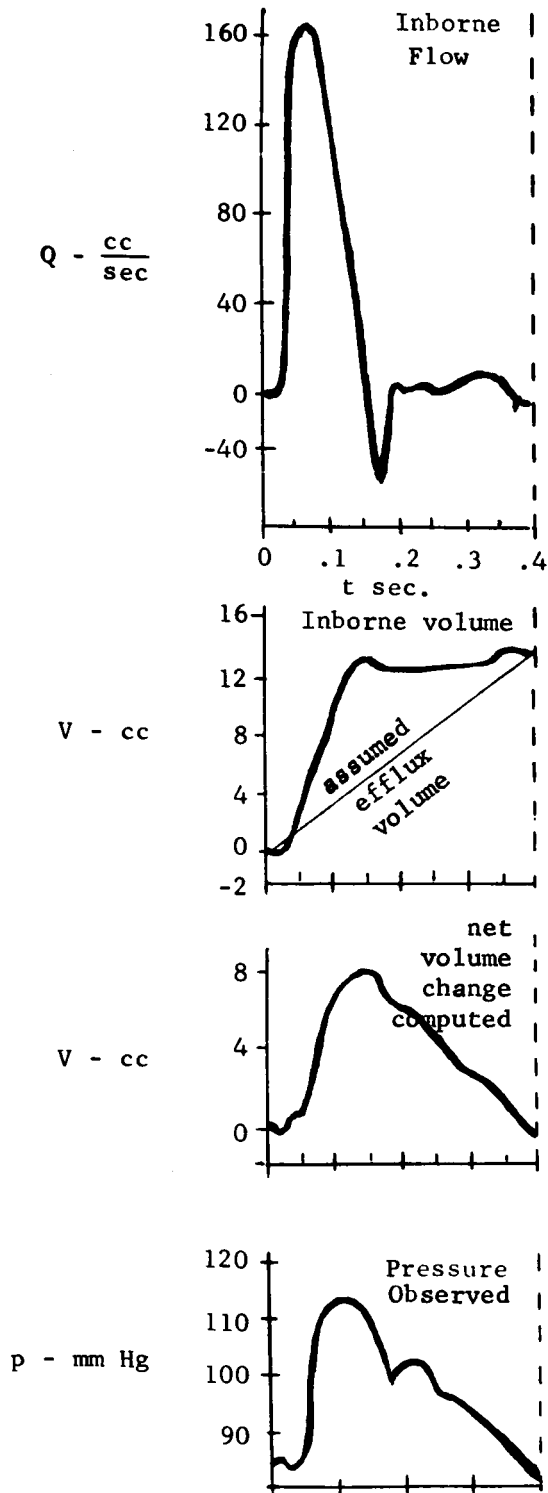
Thus, it would appear that the characterisitic 'homeostatic' regulation of the normal system (at rest) is a near constant power level. These systems that are imperfectly regulated deviated from this power level. However the actual functioning resistance range among humans may be quite large, as large as 1000-5000 dyne sec/cm⁵ (i.e., in the units of the table, 30-170 cm⁻³).

BIBLIOGRAPHY

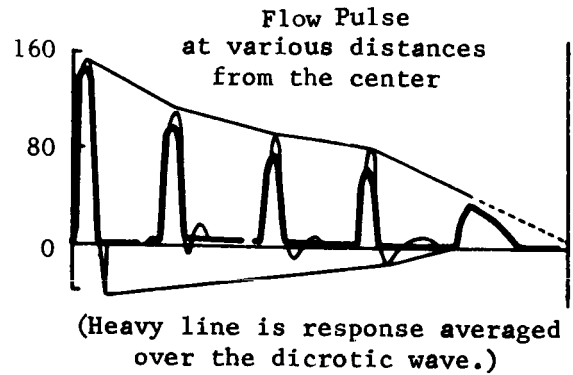
1. Green, H., "Circulatory System: Physical Principles," in O. Glasser, MEDICAL PHYSICS, vol. 2, Yearbook Pub. Co., 1950.
2. Mall, F., König. Sächs. Gesell. d. Wiss., Abh. d. Math. Cl. 14, 1888.
3. McDonald, D., BLOOD FLOW IN ARTERIES, Arnold, 1960.
4. Rushmer, R., CARDIOVASCULAR DYNAMICS, Saunders, 1961.
5. Landis, E., Physiol. Rev. 14, 404, 1934.
6. Rappaport, M., Bloch, E., Irwin, J., "A Manometer for Measuring Dynamic Pressures in the Microvascular System," J. Appl. Physiol. 14, 651, 1959.
7. Wiederhielm, C., Woodbury, J., Kirk, S., Rushmer, R., Am. J. Physiol. 207, 173, 1964.
8. Intaglietta, M., Zweifach, B., Cir. Res. 19, 199, 1966.
9. Remington, J., Wood, B., "Formation of Peripheral Pulse Contour in Man," J. Appl. Physiol., 9, 433, 1957.
10. Patel, D., Mallos, A., Fry, D., "Aortic Mechanics in the Living Dog," J. Appl. Physiol., 16, 293, 1961.
11. Hamilton, W., Dow, P., Am. J. Physiol. 125, 48, 60, 1939.
12. Frank, O., Z. Biol., 37, 483, 1899.
13. Iberall, A., Cardon, S., NASA Contractor Report CR-219, May 1965, Clearinghouse Fed. Sci. Techn. Info., Springfield, Va.
14. Iberall, A., NASA Contractor Report CR-129, October 1964, Office Tech. Services, Dept. Comm., Washington, D.C.
15. Karreman, G., "The Resonance of the Arterial System," Bull. Math. Biophys. 16, 159, 1954.
16. Patel, D., DeFreitas, F., Greenfield, J., Fry, D., "Relationship of Radius to Pressure Along the Aorta in Living Dogs," J. Appl. Physiol., 18, 1111, 1963.

17. Mall, F., Am. J. Anat. 5, 247, 1905-1906.
18. Suwa, N., Niwa, T., Fukasuwa, H., "Estimation of Intravascular Blood Gradient by Mathematical Analysis of Arterial Casts," Tohoku J. Exper. Med. 79, 169, 1963.
19. Groat, R., "Relation of Volumetric Rate of Blood Flow to Arterial Diameter," Fed. Proc. 7, 45, 1948.
20. Altman, P., HANDBOOK OF CIRCULATION, Saunders, 1959.
21. Bloch, E., "A Quantitative Study of the Hemodynamics in the Living Microvascular System," Am. J. Anat., 110, 125, 1962.
22. Karpman, V., Abrikosova, M., Glezer, G., "Hydrodynamic Mechanisms of Arterial Pressure in Hypertensive Disease," Fed. Proc. (Trans. Suppl.) 22, 212, 1963.
23. Weideman, M., Circ. Res. 10, 686, 1962; 12, 375, 1963.
24. Attinger, E., Anné, A., "Simulation of the Cardiovascular System," Ann. N.Y. Acad. Sci. 128, 810, 1966.

Windkessel Model
at Entrance



Flow-Pressure Elsewhere



Pressure Pulse
at various distances
from the center

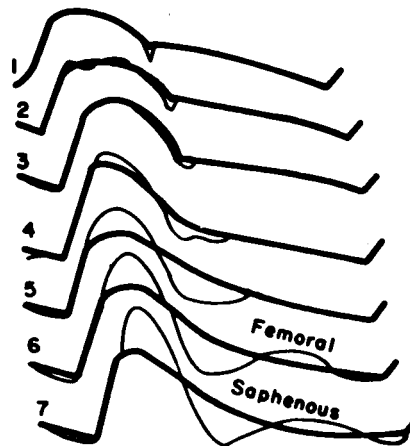


Figure 1. Illustrating the Correlation between the Pulse of Flow and the Saw-tooth of Pressure

ESTIMATE OF CROSS-SECTIONAL AREA, LEVELS, AND
MEAN DIAMETER FOR A 23 Kg DOG'S ARTERIAL TREE
(Data from Patel, Mall, Suwa)

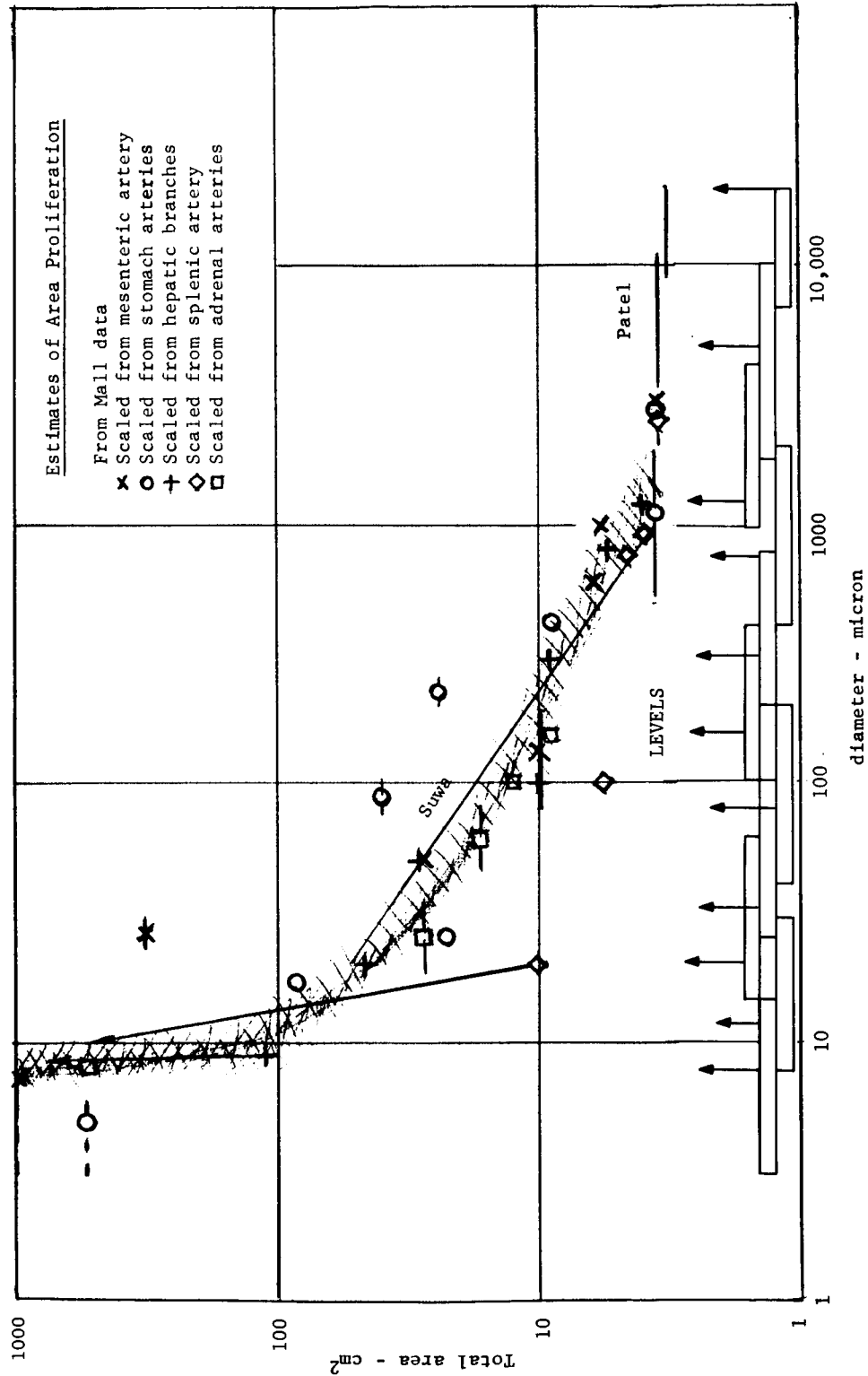


Figure 2. Association of Total Cross-Sectional Area and Diameter from Experimental Sources

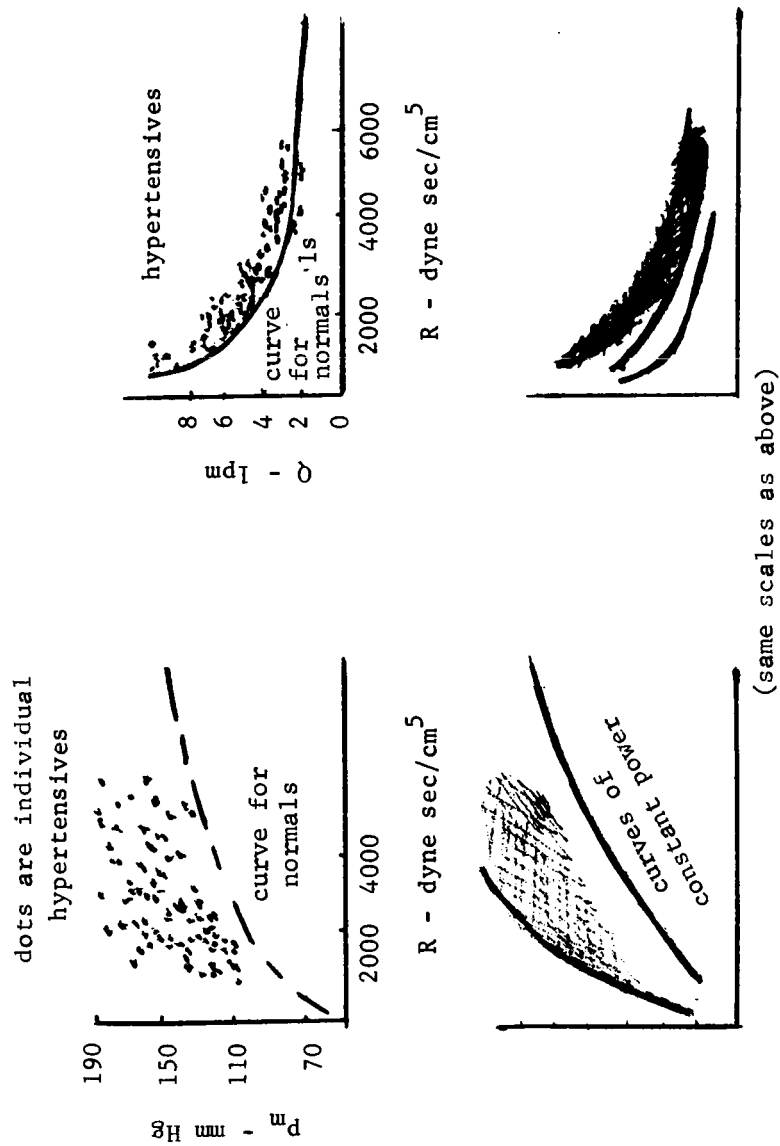


Figure 3. Correlating Mean Pressure and Flow in Normal and Hypertensive (quiescent) Humans by Power Levels (data from (22))