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Quark Model of Leptons

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Abstract

A model is proposed in which leptons are deeply bound states of certain combinations of quarks and quirks (R-conjugates of quarks), as well as of their antiparticles. The mass splittings of the leptons are estimated using the static model with unitary-symmetric meson exchange forces and are found in rough agreement with experiment, provided one uses the quark mass differences implied by the quark model of hadrons. A new charged spin-1/2 lepton, $\lambda^{\pm}$, is predicted with mass $\gtrsim M_{pion}$. Weak interactions of known leptons and $\lambda^{\pm}$ are examined from the viewpoint of the model. The electromagnetic decay $\mu \rightarrow e + \gamma$ is strictly forbidden. Rough dynamical arguments are presented to explain why leptons should be devoid of strong interactions.

Quark Model of Leptons

I. Introduction

In the quark (Ref. 1) or ace[1] model of hadrons, the mesons and baryons respectively correspond to the configurations $Q\overline{Q}$ and QQQ, where the spin-1/2 quarks $Q = (Q_1,Q_2,Q_3)$ have the quantum numbers $B = (1/3,1/3,1/3)$, $I_z = (1/2,-1/2,0)$, and $Y = (1/3,1/3,-2/3)$. The purpose of this Report is to propose a quark model of leptons and to discuss, qualitatively, their weak interactions from the viewpoint of this model.

II. Discussion

Let C_B, C_{I_z}, and C_Y be the operators which change the indicated quantum numbers into their negatives and leave the remaining quantum numbers fixed. Then

$$R = C_{I_z} C_Y$$

$$C = C_B R$$

where R is the operator introduced by Gell-Mann (Ref. 3), and C effects the usual particle-antiparticle conjugation of hadrons:

$$C: \quad Q \rightarrow \overline{Q}$$

In addition to Q and $\overline{Q}$, we introduce a triplet of *quirks* q and their antiparticles $\overline{q}$:[2]

$$R: \quad Q \rightarrow q$$

$$C: \quad q \rightarrow \overline{q}$$

The quantum numbers of the q_i are the same as those of the $\overline{Q}_i$, except for $B(q_i) = +1/3$. The discrete operators 1, C, R, and CR form a group (the four-group), which is transitive on the set $\{Q,\overline{Q},q,\overline{q}\}$. Note that superpositions of particles from any two distinct triplets are not allowed by the baryon number and the electric charge superselection rules.

[1] In unpublished CERN Reports 8182/TH401 and 8419/TH412, 1964, by G. Zweig, and in Ref. 2.

[2] The operator C_{I_z} merely interchanges Q_1 and Q_2 and thus does not lead to new particles.

We denote the baryons, leptons, and mesons respectively as B, L, and M, and make the following particle assignments:[3]

$$|B\rangle \sim |QQQ\rangle + |qqq\rangle \quad (N = +3)$$
$$|L\rangle \sim |QQ\bar{q}\rangle + |\bar{Q}qq\rangle \quad (N = +1)$$
$$|M\rangle \sim |Q\bar{Q}\rangle + |q\bar{q}\rangle \quad (N = 0)$$
$$|\bar{L}\rangle \sim |\bar{Q}\bar{Q}q\rangle + |Q\bar{q}\bar{q}\rangle \quad (N = -1)$$
$$|\bar{B}\rangle \sim |\bar{Q}\bar{Q}\bar{Q}\rangle + |\bar{q}\bar{q}\bar{q}\rangle \quad (N = -3)$$

All particle states are assumed to be eigenstates of quark number $N = N_Q = N_q = 3B$, electric charge, mass, and spin but may be mixtures of other quantum numbers. Bound states $|QQ\rangle$, $|Qq\rangle$, etc., of fractional baryon number and/or electric charge may or may not exist, depending on the nature of two-body forces. Transitions between different members of the set $\{B,L,M,\bar{L},\bar{B}\}$ are forbidden by the assumed conservation of N. The purely leptonic, energetically very favored baryon decay $B \rightarrow 3L$ is in principle allowed by the N-superselection rule. A brief discussion of its assumed non-occurrence at normal matter densities[4] is given later in this Report.

To understand qualitatively the possibility of a unified description of both baryons and leptons as bound states of three basic particles, let us roughly estimate the masses and the binding energies involved, disregarding for the moment the symmetry aspects of the problem. Experimentally, it is known that quarks, if they exist, have masses $M_Q \sim 7 - 10$ GeV for reasonable assumed cross sections (Ref. 4). Because of the high quark masses and the presumably low relative quark velocities manifested by the approximate $SU(6)$ symmetry of bound states, the non-relativistic tightbinding model of baryons is considered realistic (Ref. 5). If its extension to the leptons is assumed valid, then $M_X = 3M_Q - E_X$, where E_X denotes the binding energy of $X = B$ or L. We note that $E_B \sim E_L \sim 3M_Q \sim 30$ GeV. On the other hand, $\Delta E \equiv E_B - E_L \sim 1$ GeV so that $\Delta E/E_X \sim 1/30$. Thus the change in the binding energy between the baryon and the lepton configurations is proportionally quite small and could perhaps be explained by N-dependent interquark potentials. Of course, it is quite difficult to see how the quark masses and the energies binding the quarks could conspire in such fashion as to give bound states of essentially, or exactly, zero mass.

Let $\phi(x)$ and $\chi(x)$ respectively be the free Q- and q-field operators transforming as 3 and 3* under $SU(3)$. We assume that ϕ satisfies the Dirac equation $(i\gamma \cdot \partial - M)\, \phi(x) = 0$, where M is a matrix in the $SU(3)$-space. Since the quarks are assumed to be sharp in $B{\sim}\lambda_0$, $I_z{\sim}\lambda_3$, $Y{\sim}\lambda_8$, and mass,[5] it follows that the most general form of M is

$$M = (3/2)^{1/2}\, m_0\lambda_0 - m_3\lambda_3 - 3^{1/2}\, m_8\lambda_8$$

Under the R-conjugation, $\phi_i \leftrightarrow \chi_i$ and $\lambda_i \rightarrow -\lambda_i^T = +\lambda_i$ for $i = 2,5,7$ and $= -\lambda_i$ for $i = 1,3,4,6,8$. Moreover, R: $\lambda_0 \rightarrow \lambda_0$ since the quark number $N \propto \int d^3x\, (\phi^\dagger\lambda_0\phi + \chi^\dagger\lambda_0\chi)$ must not be affected by R. Thus

$$R: M \rightarrow M_R = (3/2)^{1/2}\, m_0\lambda_0 + m_3\lambda_3 + 3^{1/2}\, m_8\lambda_8$$

Putting $q_{1,2,3} \equiv Q_{4,5,6}$

$$\left.\begin{array}{ll} M_1 = m_0 - m_3 - m_8, & M_4 = m_0 + m_3 + m_8 \\ M_2 = m_0 + m_3 - m_8, & M_5 = m_0 - m_3 + m_8 \\ M_3 = m_0 + 2m_8, & M_6 = m_0 - 2m_8 \end{array}\right\} \quad (1)$$

Since $|m_8| >> |m_3|$ (Footnote 1 and Ref. 7), no two of the Q_i are mass-degenerate. Furthermore,

$$M_1 - M_2,\ M_1 - M_3 < 0 \text{ (Ref. 8), so that } m_3, m_8 > 0$$

and hence

$$M_3 > M_4 > M_5 > M_2 > M_1 > M_6$$

We assume that forces between quarks are due to exchanges of various types of mesons, the latter being bound states of quark-antiquark pairs. One evidently has a bootstrap situation for the mesons with the quarks treated as *elementary*. Without solving the bootstrap, we can make some qualitative statements about the structure of baryons and leptons. The experimental rms charge radius of the proton $r_c \sim 0.8\ f$ (Ref. 9), gives a rough estimate of the size of baryons. Since $1/r_c \sim m_\pi$, it is

[3] The notation $|QQQ\rangle + |qqq\rangle$ is purely symbolic and does not imply equal coefficients for the two states.

[4] The possibility of such decay under abnormal conditions has obvious astrophysical implications.

[5] The λ-matrices are defined in Ref. 6.

clear that pion exchange forces between quarks are dominant. The vector-meson forces are expected to be equally important in the leptonic case mainly because of the relatively tighter structure of leptons manifested by their lower energy. Unfortunately, no experimental information on the charge structure of electrons or muons appears to be available. The main mechanism responsible for the lower lepton masses is assumed to be the strong attraction between the single antiquirk (in the case of a $QQ\bar{q}$ configuration) and each of the two quarks. In order to minimize the potential energy, we further assume that, on the average, $QQ\bar{q}$ has the linear spatial structure $Q - \bar{q} - Q$; in this manner the vector repulsion between the two quarks is minimized. For the quarks to crowd the antiquirk, their space wave function must be symmetric. Their spin function is thus antisymmetric, implying zero spin for the QQ-system, and hence the spin projection of the lepton is determined[6] entirely by the antiquirk.

Consider the configurations $Q - \bar{q} - Q$ and $q - \bar{Q} - q$. In the static limit the vector mesons φ, ρ^0, and ω respectively couple with strengths $(\pm 1, \pm 1, \pm 1)f_N$, $(\pm 1, \mp 1, 0)f_I$, and $(\pm 1/\sqrt{3}, \pm 1/\sqrt{3}, \mp 2/\sqrt{3})f_Y$ to the N-, I_z-, and Y-charges of the triplets Q (upper sign) and $\bar{Q}$ (lower sign), and similarly for q and $\bar{q}$. The φ-exchange forces are the same for all combinations of quarks and are henceforth ignored; their sole effect is to provide the bulk of the baryon-lepton mass splitting. Neglecting the mass and coupling constant differences within the vector octet, we have

$$\left.\begin{aligned} V(Q_i\bar{q}_j) &= \frac{f^2}{4\pi}\,\frac{e^{-mr}}{r}\cdot\begin{cases} -2/3 & i \neq j \\ +4/3 & i = j \end{cases} \\ V(Q_iQ_j) &= \frac{1}{2}\,e^{-mr}\,V(Q_i\bar{q}_j) \end{aligned}\right\} \quad (2)$$

If we could ignore the quark mass differences, then the lowest-energy configurations would be those with all quark indices distinct. For the hadrons, we know that the quark mass differences rather than the potential energies dominate the mass splittings (Ref. 8). For the leptons, we do not know. If we try $|V| > m_8$ or $|V| \sim m_8$, then we get nothing sensible. On the other hand, assuming $|V| < m_8$ leads to the following lowest-energy states with $N = +1$:

$$\left.\begin{aligned} a^- &= Q_2\bar{q}_3Q_2 \quad && (I_z = -1, Y = 0, Q = -1) \\ a^0 &= Q_1\bar{q}_3Q_2 && (I_z = 0, Y = 0, Q = 0) \\ a^+ &= Q_1\bar{q}_3Q_1 && (I_z = 1, Y = 0, Q = 1) \\ b^+ &= q_3\bar{Q}_2q_3 && (I_z = 1/2, Y = 1, Q = 1) \\ b^0 &= q_3\bar{Q}_1q_3 && (I_z = -1/2, Y = 1, Q = 0) \end{aligned}\right\} \quad (3)$$

The masses of these states, from Eq. (1) and (2), are

$$\left.\begin{aligned} m(a^-) &\simeq m_l + 2m_3 - 4m_8 + 2V_0 \\ m(a^0) &\simeq m_l - 4m_8 - V_0 \\ m(a^+) &\simeq m_l - 2m_3 - 4m_8 + 2V_0 \\ m(b^+) &\simeq m_l + m_3 - 5m_8 + 2V_0 \\ m(b^0) &\simeq m_l - m_3 - 5m_8 + 2V_0 \end{aligned}\right\} \quad (4)$$

where $V_0 = (2/3)\,(f^2/4\pi)\,(e^{-mr}/2r)$ and m_l is some central leptonic mass.[7] Let us disregard the state a^+ for a moment. Making the identifications[8]

$$\left.\begin{aligned} \mu^- &= a^- \\ \nu' &= a^0 \cos\theta_\nu + b^0 \sin\theta_\nu \\ e^+ &\simeq b^+ \\ \bar{\nu} &= -a^0 \sin\theta_\nu + b^0 \cos\theta_\nu \end{aligned}\right\} \quad (5)$$

and substituting experimental masses into Eq. (4), we find

$$\left.\begin{aligned} m_3 &\sim 0.3\ \text{MeV} \\ m_8 &\sim 100\ \text{MeV} \\ V_0 &\sim 35\ \text{MeV} \end{aligned}\right\} \quad (6)$$

According to the estimates made with the baryon quark model, $m_3 \sim 1$ MeV, $M_3 - M_1 \simeq 3m_8 \sim 200$ MeV, and $V_0 \sim 40$ MeV (Ref. 8). The numbers in both cases roughly agree. In view of the approximate nature of our arguments, this is quite satisfactory. Let us accept this quark model of leptons and investigate its implications.

[6]Assuming, for the moment, charge independence for the QQ- and qq-systems. As will be seen, this assumption may be dropped, since it turns out that the two quarks or quirks are the same, with one exception, for each leptonic state.

[7]The mass is a very sensitive function of the average Q-$\bar{q}$ or q-$\bar{Q}$ distance r, and hence very little can be said about the masses of higher leptonic states without doing more sophisticated calculations.

[8]Allowances are made for the possibility of mixing of (virtual) particles having the same electric charge but different values of hypercharge.

First we note that $\mu^-,\nu',e^+,\bar{\nu}$, and $\mu^+,\bar{\nu}',e^-,\nu$ respectively, have $N = +1$ and -1. Thus $\mu \to e + \gamma$ is forbidden by conservation of N. While the $e - \nu$ mass difference can be ascribed to electromagnetic effects, this is not the case for $\mu - \nu'$ and $\mu - e$ as seen from Eq. (4). According to our model, the muon-electron mass difference has the same origin as the ΔY-proportional mass splittings of hadrons.

The one-handedness of neutrinos is easily understood in our model. Since $\bar{\nu}$ and ν' are mixtures of a^0 and b^0, the only quantum number distinguishing them is the spin projection or helicity. By definition, the states ν' and $\bar{\nu}$ are orthogonal and are taken to have opposite helicities. Since for massless particles helicity is the same as chirality, it follows that ν' and $\bar{\nu}$ are eigenstates with opposite chirality eigenvalues. Thus $\nu' = 1/2(1 + \gamma_5)\nu'$, where the plus sign is determined by experiment.[9] While we have treated the neutrinos as massless, this simplification is not necessary, since it is known experimentally (Ref. 10) that $m(\nu) < 250$ eV and $m(\nu') < 2.5$ MeV. In fact, it is quite difficult to see how one could get *exactly zero* for the neutrino masses in any dynamical calculation. At least, one would hope that such calculations would reveal some mechanism *driving* the masses toward zero.

We consider now the state a^+. Since a^+ and b^+ have the same charge, they are expected to mix[8] just as a^0 and b^0 do. One of the mixtures should be identified with the positron while the other with some as yet undiscovered leptonic state of positive charge, call it λ^+. Thus

$$\left.\begin{aligned} e^+ &= a^+ \sin\theta_e + b^+ \cos\theta_e \\ \lambda^+ &= a^+ \cos\theta_e - b^+ \sin\theta_e \end{aligned}\right\} \qquad (7)$$

where θ_e is estimated below. The $a^+ - b^+$ mixing may push the λ^+ above the μ^- and, hopefully, even above the $\pi^\pm$.

Consider leptonic baryon decays. From Eq. (3) it is clear that the weak leptonic currents[10]

$$\left.\begin{aligned} J^l_\alpha(\Delta I_z = 1,\Delta Y = 0) &= (a^0\, a^-)_\alpha - (\overline{a^0}\,\overline{a^+})_\alpha + (\overline{b^0}\,\overline{b^+})_\alpha \\ J^l_\alpha(\Delta I_z = 1/2,\Delta Y = 1) &= (b^0\, a^-)_\alpha - (\overline{a^0}\,\overline{b^+})_\alpha \\ J^l_\alpha(\Delta I_z = 3/2,\Delta Y = -1) &= (\overline{b^0}\,\overline{a^+})_\alpha \end{aligned}\right\} \qquad (8)$$

where $(xy)_\alpha \equiv \overline{\psi}_x\, \gamma_\alpha\, (1 + \gamma_5)\psi_y$, have the indicated quantum numbers.[11] Let $J^h_\alpha(1,0)$, $J^h_\alpha(1/2,1)$, and $J^h_\alpha(3/2, -1)$ be the corresponding weak hadronic currents. The interaction Lagrangian is taken to be formally I_z- and Y-conserving:

$$L = G\cdot 2^{-1/2} \left\{ J^l_\alpha(1,0)^\dagger J^h_\alpha(1,0) + J^l_\alpha(1/2,1)^\dagger J^h_\alpha(1/2,1) + J^l_\alpha(3/2,-1)^\dagger J^h_\alpha(3/2,-1) \right\} + \text{h.c.}^{12} \qquad (9)$$

It is easy to verify that L may be written in the conventional form[13]

$$L = G\cdot 2^{-1/2}\,(\nu e^- + \nu'\mu^-)_\alpha{}^\dagger \cdot \left\{ J^h_\alpha(1,0)\cos\theta_V + J^h_\alpha(1/2,1)\sin\theta_V \right\} + \text{h.c.} \qquad (10)$$

provided λ^+ is ignored and θ_e is set equal to zero. We identify θ_V with the Cabibbo angle (Ref. 11). The $\Delta I_z = 3/2$, $\Delta Y = -1$ term in Eq. (9) accounts for rare decays such as $K^0 \to \pi^+ + e^- + \bar{\nu}$ (Ref. 12 and 13). The amplitude ratio

$$\frac{(K^0 \to \pi^+ + e^- + \bar{\nu})}{(K^0 \to \pi^- + e^+ + \nu)} = \frac{\tan\theta_e}{\tan\theta_V}$$

has at present an experimental upper limit (Ref. 14) of 0.25. Using (Ref. 11) $|\theta_V| = 0.26$, we find $|\theta_e| < 0.067$.

[9]The ν' and $\bar{\nu}$ neutrinos may be regarded as the left- and right-handed components, respectively, of a massless spinor field with $N = 1$.

[10]The minus signs in Eq. (8) ensure that the Lagrangian L, Eq. (9), has the *conventional* form given by Eq. (10).

[11]The question why currents with $|\Delta Q| \neq 1$ do not appear experimentally remains mysterious.

[12]h. c., hermitian conjugate

[13]Note that CP: $\nu \leftrightarrow \bar{\nu}$, $\nu' \leftrightarrow \bar{\nu}'$; P: $\nu \leftrightarrow \bar{\nu}'$, $\nu' \leftrightarrow \bar{\nu}$, since N does not change while helicity flips; so that C: $\nu \leftrightarrow \nu'$, $\bar{\nu} \leftrightarrow \bar{\nu}'$.

The purely leptonic interaction Lagrangian is

$$L' = G \cdot 2^{-1/2} \left\{ J_a^l(1,0)^\dagger J_a^l(1,0) + J_a^l(1/2,1)^\dagger J_a^l(1/2,1) + J_a^l(3/2,-1)^\dagger J_a^l(3/2,-1) \right\}$$

$$= L_{\mu e} + L_{\lambda\mu} + L_{\lambda e} + \cdots$$

where

$$L_{\mu e} = G \cdot 2^{-1/2} (c_e + c_\nu s_\nu s_e) (\nu'\mu^-)_a^\dagger (\nu e^-)_a + \text{h.c.}$$

$$L_{\lambda\mu} = G \cdot 2^{-1/2} (c_\nu s_\nu c_e - s_e) (\nu'\mu^-)_a^\dagger (\nu\lambda^-)_a + \text{h.c.}$$

$$L_{\lambda e} = G \cdot 2^{-1/2} c_\nu s_\nu (c_e^2 - s_e^2) (\nu e^-)_a^\dagger (\nu\lambda^-)_a + \text{h.c.}$$

with $c_\nu \equiv \cos\theta_\nu$, $s_\nu \equiv \sin\theta_\nu$, etc.[14] The μ-decay amplitude is thus modified by the factor $f = c_e + c_\nu s_\nu s_e$, where $0.986 \lesssim f \lesssim 1.014$.

Because the existence of λ is a crucial test of our model of leptons, we now discuss some of the various processes in which λ would be expected to participate. If λ were lighter than π, then the decay $\pi^+ \to \lambda^+ + \nu$ doubtless would have been seen, since it is expected to dominate $\pi^+ \to e^+ + \nu$. We therefore hope that realistic mass calculations on the basis of our model will yield $m(\lambda^\pm) > m(\pi^\pm)$. Cosmic-ray muons are mostly the decay products of pions; hence we do not expect to see many λs in cosmic rays. An obvious place to look for λs is in photoproduction experiments. Since λ^- can decay weakly into an electron and a ν-$\bar{\nu}$ pair, it might easily be confused with μ^-. Among the known 0^- mesons only $K^\pm$ may decay into $\lambda^\pm$ and $\nu(\bar{\nu})$, according to Eq. (9). If the λ-μ mass difference is ignored, one finds[14]

$\Gamma(K^+ \to \lambda^+ + \nu)/\Gamma(K^+ \to \mu^+ + \nu') = \sin^2\theta_e \lesssim 4.5 \times 10^{-3}$

Now[15]

$\Gamma(K^+ \to \mu^+ + \nu')/\Gamma(K^+ \to \pi^0 + \mu^+ + \nu') \simeq 22$. Thus $K^+ \to \lambda^+ + \nu$ is at least ten times rarer than the $K^+ \to \pi^0 + \mu^+ + \nu'$ decay. The three-body process $K_2^0 \to \pi^- + \lambda^+ + \nu$ is expected to be $\sim 1/\sin^2\theta_\nu \simeq 16$ times as frequent as $K_2^0 \to \pi^- + \mu^+ + \nu'$ in the approximation $m_\lambda = m_\mu$. The rate of the decay $K^+ \to \pi^0 + \lambda^+ + \nu$ is down by $\sin^2\theta_e$ compared to its muonic counterpart.

[14]For the record, expressions for the leptonic currents follow:

$$J_a^l(1,0) = c_\nu(\nu'\mu^-)_a + (c_\nu c_e + s_\nu s_e)(\nu e^-)_a + (s_\nu c_e - c_\nu s_e)(\nu\lambda^-)_a,$$

$$J_a^l(1/2,1) = s_\nu(\nu'\mu^-)_a + s_\nu c_e(\nu e^-)_a - s_\nu s_e(\nu\lambda^-)_a,$$

$$J_a^l(3/2,-1) = c_\nu s_e(\nu e^-)_a + c_\nu c_e(\lambda\nu^-)_a.$$

[15]From data summarized in Ref. 15.

Assuming comparable coupling constants for $\lambda^- \to \pi^- + \nu$ and $\pi^- \to \mu^- + \bar{\nu}'$, we find

$$\frac{\Gamma(\lambda^- \to \pi^- + \nu)}{\Gamma(\pi^- \to \mu^- + \bar{\nu}')} \simeq \sin^2(\theta_\nu - \theta_e)\left(\frac{m_\lambda}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_\lambda^2}{m_\pi^2 - m_\mu^2}\right)^2$$

This ratio will be quite small if $m_\lambda \simeq m_\pi$. On the whole, it appears conceivable that λ, if it exists, could have escaped detection, particularly if it is only slightly heavier than the pion.

Of all possible $B \to 3L$ ("superweak") decays, only $p \to \varepsilon^+ + \nu_1 + \nu_2$ and $p \to \varepsilon_1^+ + \varepsilon_2^+ + \mu^-$, where $\varepsilon^+ = e^+$ or λ^+ and $\nu = \nu'$ or $\bar{\nu}$, need be considered, inasmuch as the $p \to \mu^+ + \cdots$ mode is a five-body process. In analogy with the weak interactions, we assume that the superweak interactions are also of the current $\times$ current type. The superweak currents have $|\Delta N| = 2$ and $|\Delta Q| = 0,1$, and 2. Whatever be the mechanism responsible for the absence of $|\Delta Q| = 0$ and 2 weak currents, we assume (on the basis of the independence of N and Q quantum numbers) that this mechanism is also operative in the $|\Delta N| = 2$ case. The processes $p \to \varepsilon_1^+ + \varepsilon_2^+ + \mu^-$ are thus tentatively excluded. For a crude estimate of the coupling constant involved in the $p \to \varepsilon^+ + \nu_1 + \nu_2$ decay, we neglect lepton masses and compare the experimental lower limit on the proton lifetime (Ref. 16). $\tau_p > 4 \times 10^{23}$ yr, with the neutron β-decay lifetime of $\sim 10^3$ sec:

$$\frac{\tau(p \to 3l)}{\tau(n \to p + 2l)} \approx \left(\frac{G_w}{G_{sw}}\right)^2 \left(\frac{m_p - m_n}{m_p}\right)^5$$

$$\sim \frac{4 \times 10^{23}\ \text{yr}}{10^3\ \text{sec}}$$

Thus we must have $G_{sw} \lesssim 10^{-21} G_w$! A more useful comparison is afforded by assuming that the weak and superweak interactions are mediated by vector mesons W and X, respectively. Then $g_{sw} \sim 10^{-10} g_w$ for $m_W \sim m_X$. It is interesting to note that g_{sw}, although very small ($\sim 10^{-11}$), is still some 10^{10} times larger than the dimensionless gravitational coupling constant. Although several plausible arguments can be given in favor of the smallness of g_{sw}, we feel that it is premature to do so at present.

In conclusion, we briefly discuss some of the major remaining problems or difficulties of our model. First, there exists a whole class of configurations of fractional electric charge (such as Q, $Q\overline{q}$, $QQ\overline{Q}$, etc.) each of which, if bound at all, must have mass $\gtrsim$ 10 GeV in conformity with experiment, thus possibly raising the mass limit on quarks. It is very difficult to see how this can happen, especially for $Q\overline{q}$, unless there exists an interaction depending on the triality quantum number T (Ref. 17), attractive between constituents leading to a bound state with $T = 0$ and repulsive for those leading to $T = 1$ or 2. Second, the status of quirk admixtures to hadron states is not clear; the R-conjugation properties of hadrons certainly are intimately connected with the amount of this admixture. Third and last, the most important question, why leptons are devoid of strong interactions, must be answered in the framework of our model. Here one would like to show that, e.g., the $\mu^- - \pi^0$ coupling constant is of the order of a typical dimensionless weak coupling constant, or even smaller. An approximation scheme for the $\mu\mu\pi$-vertex function Γ_1 is shown graphically in Fig. 1. Here d, a mesonic state of fractional electric charge, simulates the two spin-correlated Qs into which (plus $\overline{q}$) the muon virtually dissociates. Assuming PS-coupling of π^0 to fermions, one can show that, in this model,[16]

$$g_{\mu\mu\pi} \simeq (4\pi)^{-2} \int_{4m_q^2}^{\infty} ds\, s^{-2}\, Re\, \Gamma_2(s) \int_{-t_+(s)}^{-t_-(s)} dt[\Gamma_3(t)]^2$$

where

$s = p_\pi^2$, $t = p_d^2$, and $t_\pm(s) \simeq [(s/4 - m_d^2)^{1/2} \pm (s/4)^{1/2}]^2$,

with the approximation $m_q >> m_\pi$, m_μ. We note[17] that although Γ_2 may be quite large at $s = m_\pi^2$, it appears in the integrand with $s \geq 4m_q^2 >> m_\pi^2$ only. Similarly, Γ_3 is integrated[18] over large negative values of t, far away from the mass-shell value $t = m_d$. Assuming that the vertex functions are reasonably damped when off shell,[19] the integral can be shown to be proportional to sufficiently many powers of $(m/m_q)^2 \sim 10^{-4}$, where $m \sim m_\pi \sim m_\mu$, to make $g_{\mu\mu\pi}$ compatible with experiment. As an example, taking

$\Gamma_2(s) = g_{qq\pi} [(m_\pi^2 - a)^2 + b^2] [(s - a)^2 + b^2]^{-1}$ (with $a, b \sim m^2$) and a similar expression for $\Gamma_3(t)$, one finds that $g_{\mu\mu\pi} \sim (m/m_q)^{12}$! This, of course, is more than enough; less spectacularly damped vertex functions should suffice. Physically, a small value of lepton-meson coupling might be understood as a saturation or rigidity property of the very deeply-bound leptonic systems. The leptons are supposedly so saturated that they have essentially no response to low-energy meson probes. A quantitative analysis of this problem is in progress.

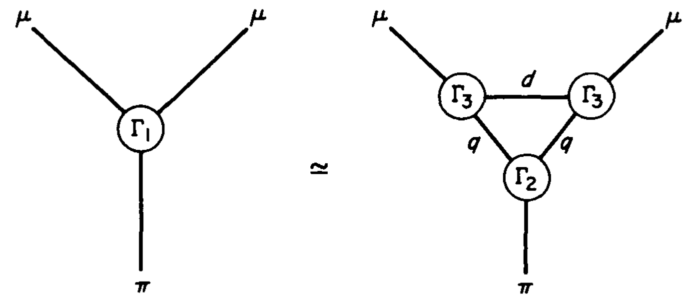

Fig. 1. An approximation for the $\mu\mu\pi$-vertex function

[16]The contribution of a graph involving the $dd\pi$ vertex is ignored; it is of the same order as the one considered.

[17]$g_{QQ\pi}/4\pi \gtrsim 100$ in Schrödinger models.

[18]Only when $s \to \infty$ does $t_- \to 0$; but then the rest of the integrand in s is strongly damped.

[19]As discussed by M. Ida in Ref. 18 for the πN-vertex function, the LSZ inequality (Ref. 19) cannot be satisfied for $\Gamma_{NN\pi}$ (with π off shell) unless $\Gamma_{NN\pi}$ is strongly suppressed for s above $4m_N^2$ by a pole in $\Gamma_{NN\pi}(s)$ with $m_\pi^2 < s < 4m_N^2$ or by a *pseudoresonance* with $9m_\pi^2 < s < 4m_N^2$.

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