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COMPUTER PROGRAM FOR FINITE-DIFFERENCE SOLUTIONS OF SHELLS OF REVOLUTION UNDER ASYMMETRIC LOADS

by Harry G. Schaeffer

Langley Research Center
Langley Station, Hampton, Va.

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SUMMARY

A general computer program written in FORTRAN IV symbolic language is presented which determines the linear asymmetric bending behavior of an arbitrarily loaded elastic thin shell of revolution. The variables are separated by representing the loads, displacements, and stresses by Fourier series expansions in the circumferential coordinate. The resulting set of equations is solved numerically by using finite-difference approximations in the meridional direction. A three-layered cross section which is symmetric about the middle surface is allowed. The boundary conditions are taken in a general form which allows the program to handle elastic restraints by specifying a linear combination of edge forces and displacements. The program is described in detail and sample calculations are included.

INTRODUCTION

The linear behavior of an arbitrary shell of revolution loaded asymmetrically cannot, in general, be solved exactly. Numerical techniques for solving the governing equations for this general class of problems on high-speed digital computers can, however, be employed and several formulations of the numerical solutions of the applicable shell equations exist in the literature (see, for example, refs. 1 and 2). The program described herein is a mechanization of a finite-difference method substantially as presented in reference 1, the analytical formulation being identical. Several features have been incorporated in order to make the program as flexible as possible.

The present report includes sections dealing with analysis, programing techniques, and the computer program. Sample problems include the analysis of a cylinder and a sphere. In the analysis, the shell material is assumed to be isotropic, and Poisson's ratio and the modulus of elasticity are assumed to be constant with respect to the directions tangent to the middle surface. The stiffnesses and the Fourier coefficients for the surface loads and temperature distribution may have smooth variations in the meridional coordinate. The shell may also be of symmetric sandwich construction through the thickness provided transverse shear deformations are neglected. Elastic constraints at the boundaries are admissible.

The section describing the computer program is intended to be a user's document and contains all the information necessary to prepare input data and subprograms. The program is written in FORTRAN IV language for operation in the IBSYS-IBJOB operating system (version 13). The program and internal storage requirements utilize approximately 25 000 words of memory.

The program is organized into 25 subprograms to facilitate modifications for specific shell shapes and load distributions. A maximum of 500 equal increments along the shell meridian is allowed. The program output consists of a problem description together with nondimensional displacements, rotations, and moment and force resultants in tabular form.

SYMBOLS

a	reference (or characteristic) length
b	nondimensional membrane stiffness
d	nondimensional bending stiffness
E	Young's modulus of elasticity
E_o	reference modulus of elasticity
$e_{\xi}^{(n)},e_{\theta}^{(n)},e_{\xi\theta}^{(n)}$	Fourier coefficients for membrane strains
F	membrane force
F_{ξ}	Fourier coefficient for transverse shear
$\hat{f}_{\xi}^{(n)}$	Fourier coefficient for effective (boundary) transverse shear
h	shell thickness
h_o	reference thickness
$k_{\xi}^{(n)},k_{\theta}^{(n)},k_{\xi\theta}^{(n)}$	Fourier coefficients for bending distortion

L last station

$M_\xi, M_\theta, M_{\xi\theta}, M_{\theta\xi}$ bending moments

$\overline{M}_{\xi\theta}$ modified twisting moment (see eq. (12))

$m_T^{(n)}$ thermal moment coefficient

$m_\xi^{(n)}, m_\theta^{(n)}, m_{\xi\theta}^{(n)}$ Fourier coefficients for bending moments

N total number of shell stations

$N_\xi, N_\theta, N_{\xi\theta}, N_{\theta\xi}$ membrane forces per unit length

$\overline{N}_{\xi\theta}$ modified membrane shear (see eq. (11))

$\hat{N}_{\xi\theta}$ effective (boundary) membrane shear

n Fourier index

O first station

$p^{(n)}, p_\xi^{(n)}, p_\theta^{(n)}$ Fourier coefficients for loads

Q_ξ, Q_θ transverse shear

$\hat{Q}_\xi$ effective (boundary) transverse shear

q, q_ξ, q_θ distributed loads in normal, meridional, and circumferential directions, respectively

R_s, R_θ principal radii of curvature

r radial distance from axis of symmetry to shell middle surface

S total arc length of shell meridian

s meridional shell coordinate

T temperature

$T_1^{(n)}(\xi)$ midplane temperature variation

$\Delta T_1^{(n)}(\xi)$ temperature gradient per unit thickness normal to the middle surface

$T^{(n)}(\xi,\zeta)$ Fourier coefficient for temperature

t cover-plate thickness

$t_T^{(n)}$ thermal force coefficient

$t_\xi^{(n)}, t_\theta^{(n)}, t_{\xi\theta}^{(n)}$ Fourier coefficients for membrane forces

$\hat{t}_{\xi\theta}^{(n)}$ Fourier coefficient for effective (boundary) membrane shear

U_ξ, U_θ meridional and circumferential displacements

$u_\xi^{(n)}, u_\theta^{(n)}$ Fourier coefficients for meridional and circumferential displacements

W normal displacement

$w^{(n)}$ Fourier coefficient for normal displacement

α coefficient of thermal expansion

$\gamma = \frac{\rho'}{\rho}$

Δ meridional increment, $\frac{S}{a(N - 2)}$

$\delta = \frac{h}{t}$

$\epsilon_\xi, \epsilon_\theta, \epsilon_{\xi\theta}$ membrane strains

ζ coordinate normal to middle surface of shell, positive outward, whose origin is at the middle surface

$\eta = \frac{h}{h_o}$ nondimensional thickness

θ circumferential coordinate

$\kappa_\xi, \kappa_\theta, \kappa_{\xi\theta}$ bending distortions

$\lambda = \frac{h_o}{a}$

ν Poisson's ratio

$\xi = \frac{s}{a}$

$\rho = \frac{r}{a}$

$\sigma_\xi, \sigma_\theta, \sigma_{\xi\theta}$ stress components

σ_o reference stress

Φ_θ, Φ_ξ rotations

$\varphi_\theta^{(n)}, \varphi_\xi^{(n)}$ Fourier components of rotations

ϕ angle between shell center line and normal to shell middle surface

ϕ_o maximum value of ϕ

$\omega_\theta, \omega_\xi$ nondimensional curvatures

Subscripts:

H horizontal component

L last station

max maximum

O first station

V vertical component

Matrices:

A, B, C, E, F, G, H, J, Ω, Λ, P 4×4 matrices

e,f,g,l,x,y,z 1×4 matrices

A prime indicates a derivative with respect to ξ.

FORTRAN names for symbols are as follows:

Symbol	FORTRAN NAME
a	CHAR
b	B
d	D
N	NMAX
n	N
γ	GAM
λ	LAM
ν	NU
ρ	R
ω_θ	OMT
ω_ξ	OMXI
Δ	DEL

ANALYTICAL FORMULATION

The theory and the method of numerical solution are described in detail in reference 1. They are summarized in sufficient detail in the text and appendixes so that the program can be used without the use of referenced material.

Shell Geometry

The shell geometry and coordinate system for the middle surface of a general shell of revolution are shown in figure 1. Any point in the shell may be located by specifying the orthogonal coordinates (ξ, θ, ζ) where $\xi = s/a$ is a nondimensional meridional coordinate, s is the meridional shell coordinate, a is a reference dimension of the shell, θ is the circumferential coordinate, and ζ is a coordinate normal to and originating at the middle surface, positive outward. If the shape of the middle surface is given by $\rho = \rho(\xi)$ where $\rho = r/a$ and r is the distance OP, the nondimensional principal curvatures can be written as

$$\omega_\theta = \frac{a}{R_\theta} = \frac{\left[1 - (\rho')^2\right]^{1/2}}{\rho} \tag{1}$$

$$\omega_\xi = \frac{a}{R_s} = -\frac{(\gamma' + \gamma^2)}{\omega_\theta} \tag{2}$$

where the following expressions have been used:

$$R_\theta = r\left[1 - \left(\frac{dr}{ds}\right)^2\right]^{-1/2} \tag{3}$$

$$R_s = -\frac{\left[1 - \left(\frac{dr}{ds}\right)^2\right]^{1/2}}{\left(\frac{d^2r}{ds^2}\right)} \tag{4}$$

$$\gamma = \frac{\rho'}{\rho} \tag{5}$$

and a prime denotes differentiation with respect to ξ. Finally, the curvatures are related by the Codazzi equation

$$\omega_\theta' = \gamma\left(\omega_\xi - \omega_\theta\right) \tag{6}$$

and the equation

$$\frac{\rho''}{\rho} = -\omega_\xi \omega_\theta \tag{7}$$

Equilibrium Equations

The membrane forces per unit length, transverse force per unit length, moment per unit length, and load per unit area are shown in the positive sense in figure 2. The displacements and rotations are shown in the positive sense by figure 3; the effective boundary force and moment are shown in the positive sense by figure 4. The analysis presented in reference 1 is based on the equilibrium equations presented in reference 3. The equilibrium equations from reference 1 are written in the following form:

$$\omega_\xi\left[\frac{\partial}{\partial\xi}\left(\rho M_\xi\right) + \frac{\partial}{\partial\theta}\left(\overline{M}_{\xi\theta}\right) - \rho' M_\theta\right] + a\left[\frac{\partial}{\partial\xi}\left(\rho N_\xi\right) + \frac{\partial}{\partial\theta}\left(\overline{N}_{\xi\theta}\right) - \rho' N_\theta\right]$$
$$+ \frac{1}{2}\left(\omega_\xi - \omega_\theta\right)\frac{\partial}{\partial\theta}\left(\overline{M}_{\xi\theta}\right) + a^2\rho q\xi = 0 \tag{8}$$

$$a\left[\frac{\partial}{\partial\theta}\left(N_\theta\right) + \frac{\partial}{\partial\xi}\left(\rho\overline{N}_{\xi\theta}\right) + \rho'\overline{N}_{\xi\theta}\right] + \omega_\theta\left[\frac{\partial}{\partial\theta}\left(M_\theta\right) + \frac{\partial}{\partial\xi}\left(\rho\overline{M}_{\xi\theta}\right) + \rho'\overline{M}_{\xi\theta}\right]$$
$$+ \frac{\rho}{2}\frac{\partial}{\partial\xi}\left(\omega_\theta - \omega_\xi\right)\overline{M}_{\xi\theta} + a^2\rho q\theta = 0 \tag{9}$$

$$\frac{\partial}{\partial\xi}\left[\frac{\partial}{\partial\xi}\left(\rho M_\xi\right) + \frac{\partial}{\partial\theta}\left(\overline{M}_{\xi\theta}\right) - \rho' M_\theta\right] + \frac{1}{\rho}\frac{\partial}{\partial\theta}\left[\frac{\partial}{\partial\theta}\left(M_\theta\right) + \frac{\partial}{\partial\xi}\left(\rho\overline{M}_{\xi\theta}\right) + \rho'\overline{M}_{\xi\theta}\right]$$
$$-a\rho\left(\omega_\xi N_\xi + \omega_\theta N_\theta\right) + a^2\rho q = 0 \tag{10}$$

where the transverse shear forces Q_ξ and Q_θ have been eliminated by using the moment equilibrium equations. In reference 3 the inplane shear forces $N_{\theta\xi}$ and $N_{\xi\theta}$ as well as the twisting couples $M_{\xi\theta}$ and $M_{\theta\xi}$ are combined to provide the following modified variables:

$$\overline{N}_{\xi\theta} = \frac{1}{2}\left(N_{\xi\theta} + N_{\theta\xi}\right) + \frac{1}{4}\left(\frac{1}{R_\theta} - \frac{1}{R_s}\right)\left(M_{\xi\theta} - M_{\theta\xi}\right) \tag{11}$$

and

$$\overline{M}_{\xi\theta} = \frac{1}{2}\left(M_{\xi\theta} + M_{\theta\xi}\right) \tag{12}$$

which appear in equations (8), (9), and (10).

Reduction to a Set of Four Ordinary Differential Equations

It is shown in appendix A and reference 1 that the equilibrium equations (8), (9), and (10) may be reduced to a set of four second-order ordinary differential equations.

This set is obtained by expanding the unknowns in appropriate Fourier series and considering the strain displacement and stress-strain relations. The resulting set, written in matrix form, is given by equation (A28) and is as follows:

$$Ez^{(n)''} + Fz^{(n)'} + Gz^{(n)} = e \tag{13}$$

where

$$z^{(n)} = \begin{Bmatrix} u_\xi^{(n)} \\ u_\theta^{(n)} \\ w^{(n)} \\ m_\xi^{(n)} \end{Bmatrix} \tag{14}$$

and the coefficient matrices E, F, G, and e are defined in appendix A of reference 1. These matrices contain the shell stiffnesses and are programed to handle the symmetric layered construction shown in figure 5.

Boundary Conditions

The set of differential equations which is given by equation (13) is subject to a set of boundary conditions at the edges of the shell, or to a set of finiteness conditions at points where the radius ρ becomes zero. The boundary conditions may be any linear combination of z and z' at the edge, as is pointed out in reference 1. A set of linear relations which is appropriate to the shell problem can be represented by the following matrix equation:

$$\Omega y^{(n)} + \Lambda z^{(n)} = l \tag{15}$$

where the components of the $y^{(n)}$ vector are the Fourier coefficients of the quantities N_ξ, $\hat{N}_{\xi\theta}$, $\hat{Q}_\xi$, and Φ_ξ, respectively, and where $\hat{N}_{\xi\theta}$ and $\hat{Q}_\xi$ are effective membrane and transverse shear force intensities, respectively, on the shell boundaries. The elements of the Ω and Λ matrices are specified for a particular boundary condition which is to be represented. Examples are given in a later section to illustrate the specification of these elements for edge conditions which are representative of those which may be found in actual shells.

The conditions which are to be specified at a shell edge which corresponds to a shell closure or a pole (i.e., at points where ρ becomes zero) are those which are presented in references 4 and 5. These conditions are as follows:

$$\left.\begin{array}{ll} u_\xi = u_\theta = w' = m_\xi' = 0 & (n = 0,\ 2) \\ u_\xi + u_\theta = u_\xi' = w = m_\xi = 0 & (n = 1) \\ u_\xi = u_\theta = w = m_\xi = 0 & (n \geqq 3) \end{array}\right\} \tag{16}$$

COMPUTATIONAL TECHNIQUES

The set of equations (13) subject to an appropriate set of boundary or finiteness conditions is to be solved numerically. The general numerical procedure which is used in the present report is very similar to that presented in reference 1. Only the manner of handling the boundary conditions is different. The shell meridian showing the finite-difference stations used in the present analysis is shown in figure 6. The shell ends O and L are halfway between stations 1 and 2 and stations N - 1 and N, respectively. The stations associated with points 1 and N lie off the shell and are used to evaluate the conditions which exist at the half-stations associated with each end of the shell. In reference 1 the need for off-shell points is eliminated by using backward and forward differences at the shell ends; also, half-stations are not used at the ends. The distance between stations is equal to Δ where

$$\Delta = \frac{S}{a(N - 2)}$$

and S is the total arc length of the shell.

At the ends of the shell, vectors z_O, z_O', z_L, and z_L' are given by the following relations:

$$\left.\begin{array}{l} z_O = \frac{1}{2}\left(z_1 + z_2\right) \\ z_O' = \frac{1}{\Delta}\left(z_2 - z_1\right) \\ z_L = \frac{1}{2}\left(z_N + z_{N-1}\right) \\ z_L' = \frac{1}{\Delta}\left(z_N - z_{N-1}\right) \end{array}\right\} \tag{17}$$

where the subscripts O and L refer to the first and last shell edges, respectively.

In the shell interior $2 \leqq i \leqq N - 1$, the following central difference approximations of the derivatives are used:

$$\left.\begin{aligned} z_i' &= \frac{(z_{i+1} - z_{i-1})}{2\Delta} \\ z_i'' &= \frac{(z_{i+1} - 2z_i + z_{i-1})}{\Delta^2} \end{aligned}\right\} \tag{18}$$

where Δ is the arc length increment. With these approximations, equation (13) becomes a set of linear algebraic equations which can be represented by the following matrix form:

$$A_i z_{i+1} + B_i z_i + C_i z_{i-1} = g_i \qquad (i = 2, 3, \ldots, N - 2, N - 1) \tag{19}$$

where N is the total number of stations and

$$\left.\begin{aligned} A_i &= \frac{2E_i}{\Delta} + F_i \\ B_i &= -\frac{4E_i}{\Delta} + 2\Delta G_i \\ C_i &= \frac{2E_i}{\Delta} - F_i \\ g_i &= 2\Delta e_i \end{aligned}\right\} \tag{20}$$

It is shown in reference 1 that the vector $y^{(n)}$ which was defined in connection with the boundary conditions (see eq. (15)) can be represented in terms of $z^{(n)'}$ and $z^{(n)}$ by the following relation:

$$y^{(n)} = Hz^{(n)'} + Jz^{(n)} + f \tag{21}$$

where the nonzero elements of the H, J, and f matrices are given in appendix A of reference 1.

By using equation (21), the boundary conditions can be written entirely in terms of z and z'. At the first shell edge, equation (15) becomes

$$\Omega_O\left(H_O z_O' + J_O z_O + f_O\right) + \Lambda_O z_O = l_O \tag{22}$$

where the Fourier index has been dropped for convenience. By substituting the applicable relations of equations (17) into equation (22) the following relation between z_1 and z_2 is found:

$$A_O z_2 + B_O z_1 = g_O \tag{23}$$

where

$$\left.\begin{aligned} A_O &= \left[\Omega_O\left(\frac{J_O}{2} + \frac{H_O}{\Delta}\right) + \frac{\Lambda_O}{2}\right] \\ B_O &= \left[\Omega_O\left(\frac{J_O}{2} - \frac{H_O}{\Delta}\right) + \frac{\Lambda_O}{2}\right] \\ g_O &= l_O - \Omega_O f_O \end{aligned}\right\} \tag{24}$$

Because of the character of the system of equations to be solved a general result for z_i in terms of z_{i+1} can be written as follows:

$$z_i = -P_i z_{i+1} + x_i \tag{25}$$

It would appear to be expedient to determine P_1 and x_1 directly by solving the matrix equation (23) and equating coefficients of equation (25); however, the matrix B_O is singular for the case where displacements and rotation are prescribed. The solution is thus obtained by solving first for z_1 from the equilibrium equation written at station 2 as follows:

$$z_1 = C_2^{-1}\left(g_1 - A_2 z_3 - B_2 z_3\right) \tag{26}$$

By substituting equation (26) into equation (23), z_2 is determined as follows:

$$z_2 = \left(B_O C_2^{-1} B_2 - A_O\right)^{-1}\left(-B_O C_2^{-1} A_2 z_3 + B_O C_2^{-1} g_2 - g_O\right) \tag{27}$$

The matrices P_2 and x_2 are seen by inspection to be as follows:

$$\left.\begin{aligned} P_2 &= \left(B_O C_2^{-1} B_2 - A_O\right)^{-1} B_O C_2^{-1} A_2 \\ x_2 &= \left(B_O C_2^{-1} B_2 - A_O\right)^{-1}\left(B_O C_2^{-1} g_2 - g_O\right) \end{aligned}\right\} \tag{28}$$

The general expressions for the P_i and x_i matrices, which are valid for $2 \leqq i \leqq N - 1$, are found by substituting equation (25) into equation (19) and are

$$\left.\begin{aligned} P_i &= \left(B_i - C_i P_{i-1}\right)^{-1} A_i \\ x_i &= \left(B_i - C_i P_{i-1}\right)^{-1}\left(g_i - C_i x_{i-1}\right) \end{aligned}\right\} \tag{29}$$

It is seen that, after calculating P_2 and x_2 from equation (28), P_i and x_i can be calculated at each interior shell station by successively applying equation (29).

In order to calculate the solution vector at each station, z_N must be determined. The vector z_N is found by simultaneously solving the boundary equation (15) and equation (25) for $i = N - 1$. The result is as follows:

$$z_N = \left\{\left[\Omega_L\left(\frac{H_L}{\Delta} + \frac{J_L}{2}\right) + \frac{\Lambda_L}{2}\right] - \left[\Omega_L\left(-\frac{H_L}{\Delta} + \frac{J_L}{2}\right) + \frac{\Lambda_L}{2}\right]P_{N-1}\right\}^{-1}$$
$$\left\{l_L - \Omega_L f_L\left[\Omega_L\left(-\frac{H_L}{\Delta} + \frac{J_L}{2}\right) + \frac{\Lambda_L}{2}\right]x_{N-1}\right\} \tag{30}$$

The solution can now be determined at each station by successive application of equation (25). After z_i $(i = 1, \ldots, N)$ have been calculated all the other shell quantities of interest can be determined. The quantities which are calculated and presented as program output are the nondimensional Fourier coefficients t_ξ, $t_{\xi\theta}$, f_ξ, φ_ξ, m_θ, $m_{\xi\theta}$, and t_θ. The quantity f_ξ is the transverse shear and is given by the relation

$$f_\xi = \gamma\left(m_\xi - m_\theta\right) + m_\xi' + \frac{nm_{\xi\theta}}{\zeta} \tag{31}$$

The remaining quantities are defined in terms of z_i in reference 1.

COMPUTER PROGRAM

The set of equations and the associated boundary conditions presented in reference 1 and described in other sections of this report have been written in FORTRAN IV language for operation in an IBSYS-IBJOB operating system (version 13). In deriving the set of equations the unknowns are expressed as suitable Fourier expansions; thus, the program calculates a set of Fourier coefficients for each value of the Fourier index. These coefficients may then be summed by using the appropriate Fourier series to give the total value of the variable as a function of the circumferential coordinate.

In developing any computer program a basic decision must be made regarding its organization. On the one hand, many of the important parameters characterizing the problem to be solved may be predefined, limiting to a great degree the burden imposed on the user of providing input information but at the same time limiting the flexibility of the program. On the other hand, the program may be kept general by requiring that the user prepare subprograms which define the important parameters as mathematical functions. The latter choice has been made herein. The user must prepare compatible subprograms which define the geometrical properties of the middle surface and which specify the loads, temperature, and stiffnesses along the shell meridian. In addition, input data must be provided for the edge conditions and for the user-prepared subprograms. A detailed description of input requirements is presented in a later section.

Program Organization

The flow diagram for the main program is presented in figure 7. It can be seen that the main program causes input data to be read and then calls for a number of subroutines in a logical order. As an aid in reading the flow chart, a list of subroutines and their description is presented in table 1. As diagramed in figure 7, the calculations and logical operations are carried out sequentially in the following order:

I. Read input data and program control words
   (1) Calculate geometric parameters of reference surface if the control word called IND3 $\neq 0$.
   (2) List problem description and input data.

II. Calculate the P and x matrix and vector, respectively, at the shell station $i = 2$. This step utilizes subprograms called HFJ, FORCE, ABCG, and INIT.

III. Calculate and store P and x at each shell station through $i = N - 1$. This step utilizes EFG, FORCE, and PANDX.

IV. Calculate and store the vector z_N. This step utilizes HFJ and FINAL.

V. Calculate and store the vector z_i associated with each shell station starting at $i = N$ and proceeding to $i = 1$. This step utilizes EQ73.

VI. Calculate output quantities t_ξ, t_θ, $t_{\xi\theta}$, f_ξ, m_θ, $m_{\xi\theta}$, φ_ξ and print results. This step utilizes STRESS AND OUTPUT.

A complete list of the FORTRAN IV source deck for the main program, SHELLS, and all the subroutines is presented in appendix B. A glossary of the FORTRAN names of the variables is given in table 2.

Input Data

The format for the required and optional input data cards is now given. These cards must be in the sequence given.

1. Shell properties and program control indicators: (FORMAT 5F6.3, 6I4)
   NU, TKN, CHAR, N, EALSIG, NO, IND1, IND2, IND3, IND4, IND5

where

NU Poisson's ratio (floating point)

TKN reference thickness to be selected by the user and is used internally to nondimensionalize the results

CHAR reference shell dimension to be selected by user and is used internally to nondimensionalize the results

N Fourier index (floating point)

EALSIG $\frac{E\alpha}{\sigma_o}$, thermal coefficient used only if thermal stresses are calculated

NO problem number (fixed point)

IND1 an integer value for option of printing geometrical properties of reference surface as a part of the output:

= 0 geometrical data are printed

≠ 0 geometrical data not printed

IND2 an integer indicating the number of words to be read into an array called CONST which is described later. The number of words is equal to the numerical value of IND2 (IND2 $\leqq$ 100)

IND3 the call by the main program for the calculation of shell geometric parameters is conditional depending on the numerical value of this indicator as follows:

= 0 the shell geometric data which have been previously calculated and stored are to be used. This allows several load distributions to be applied to the same shell configuration without the necessity of recalculating the geometric parameters.

≠ 0 the shell geometric parameters are to be calculated. The value of IND3 must be other than zero for the first problem of a sequence of problems and for any succeeding problem in a sequence when a change of geometry is desired. When IND3 ≠ 0, additional input cards are required as described under item 4.

IND4 boundary condition matrix control:

= 0 indicates that only diagonal elements of Ω and Λ matrices are to be read. The off-diagonal terms are automatically set to zero.

≠ 0 indicates that entire Ω and Λ matrices are to be read.

IND5 an integer value of 0, 1, 2, or 3:

= 0 no poles

= 1 pole at $\xi = 0$

= 2 poles at $\xi = 0$, ξ_{max}

= 3 pole at $\xi = \xi_{max}$

2. Problem description; 72 characters of alphanumeric information in order to describe briefly the problem being run.

3. Variable parametric data: Data card or cards appear only if the indicator IND2 ≠ 0. If IND2 ≠ 0, IND2 numbers are to be provided by using a maximum of 25 cards having FORMAT 4E16.8. These data are stored in the CONST array and are available in COMMON location BL14.

4. Geometric data: The following card or cards appear only if the indicator IND3 ≠ 0. If IND3 ≠ 0, the following data are to be presented:

a. FORMAT (2I4)
NMAX, FREQ

where

NMAX total number of stations (NMAX ≦ 502)

FREQ integer which controls the frequency for printing numerical results. Results are printed at every FREQth station.

b. Cards are inserted as required for calculation of shell geometric properties. The format of such cards is to be specified by the INPUT subroutine.

5. Boundary Condition Cards (FORMAT 4E16.8): Two indicators, IND4 and IND5, are used to specify first the need for and second the format of the data which specify the boundary condition at each end of the shell. For the case where a pole exists on the shell, as indicated by an appropriate value of IND5, the appropriate finiteness conditions are generated within the program and no cards are to be supplied for these conditions. If, however, the value of IND5 indicates that no pole is present, boundary condition cards must be supplied in one of two formats depending on the value of IND4.

a. IND4 = 0 signifies that only the diagonal elements of Ω and Λ are required. The diagonal elements of the boundary condition matrices and the vector l are to appear in the following order:

(1) Ω_{11}, Ω_{22}, Ω_{33}, Ω_{44}

(2) Λ_{11}, Λ_{22}, Λ_{33}, Λ_{44}

(3) l_1, l_2, l_3, l_4

b. IND4 ≠ 0 signifies that the entire arrays Ω and Λ are required. This allows the specification of elastic boundary conditions or the specification of a force or displacement in a direction other than an intrinsic coordinate direction. The elements of these arrays and the l-vector are to appear in the following order:

(1) Ω_{11}, Ω_{21}, Ω_{31}, Ω_{41}

(2) Ω_{12}, Ω_{22}, Ω_{32}, Ω_{42}

(3) Ω_{13}, Ω_{23}, Ω_{33}, Ω_{43}

(4) Ω_{14}, Ω_{24}, Ω_{34}, Ω_{44}

(5) Λ_{11}, Λ_{21}, Λ_{31}, Λ_{41}

(6) Λ_{12}, Λ_{22}, Λ_{32}, Λ_{42}

(7) Λ_{13}, Λ_{23}, Λ_{33}, Λ_{43}

(8) Λ_{14}, Λ_{24}, Λ_{34}, Λ_{44}

(9) l_1, l_2, l_3, l_4

It should be noted that the cards described must be in the indicated order with those cards associated with the edge $\xi = 0$ preceding those associated with the edge $\xi = \xi_{max}$. In addition, if IND4 ≠ 0, the full arrays Ω and Λ are required at each edge unless, of course, the need of either or both sets of cards is negated by an appropriate value of IND5. If IND5 = 2, indicating a pole at each end of the shell, no boundary condition cards are to be supplied; all the necessary conditions are generated by the program.

6. Punch control card (FORMAT 11I4): The option of punching any or all of the 11 output variables is controlled by this card. If it is desired to punch data, a one is punched in the appropriate column of the punch control card as follows:

Variable	Column
t_ξ	4
t_θ	8
$t_{\xi\theta}$	12
f_ξ	16
m_ξ	20
m_θ	24
$m_{\xi\theta}$	28
u_ξ	32
u_θ	36
w	40
φ_ξ	44

If no punching is desired, a blank card must appear.

User-Prepared Subprograms

In addition to preparing the input data cards described in the previous section, the user must prepare a group of subroutines and functions which specify the geometry and the load distribution. The structure of each individual subprogram is described subsequently in detail.

In order to allow the use of parameters in the subprograms which may change from problem to problem in a sequence of problems a block of data called CONST(100), an array containing 100 elements, has been set aside. This array can be referenced in any subprogram by a COMMON statement. The locations of this array and others in COMMON which are required in preparing the subprograms are given in table 3. A description of the subprograms follows. Examples are presented in a later section.

Geometry of the middle surface SUBROUTINE INPUT (NMAX).- The purpose of this subprogram is to calculate values for the following variables at the edges O and L and at each interior shell station; R, GAM, OMT, OMXI, and DEOMX where

$$\begin{aligned}
&\text{R(K)} && \rightarrow \rho_K \\
&\text{GAM(K)} && \rightarrow \gamma_K \\
&\text{OMT(K)} && \rightarrow (\omega_\theta)_K \\
&\text{OMXI(K)} && \rightarrow (\omega_\xi)_K \\
&\text{DEOMX(K)} && \rightarrow (\omega_\xi')_K
\end{aligned}$$

and where K is the shell station. In addition, the arc length increment DEL must be calculated as follows

$$\text{DEL} = \frac{S}{a(\text{NMAX}-2)}$$

where S is the maximum arc length. This information is transmitted to the main program by a COMMON statement. The location of these variables in COMMON is given in table 3.

Any data required to calculate the preceding geometric quantities can be read by this subprogram or be transmitted from the main program through the CONST array. The quantity NMAX, the number of shell stations, is referenced through the argument list.

Shell wall thickness and ratio of wall thickness to cover-plate thickness.- The shell wall considered in this program is as shown in figure 5 and discussed in appendix C. It is composed of three layers and is symmetric about the middle surface. It is assumed that the core is rigid normal to the middle surface but that it offers no resistance to extension or bending tangent to the middle surface. Four FUNCTION subprograms are to be specified which describe the shell wall construction so that the bending and membrane stiffnesses and the thermal loads may be calculated. The quantities to be specified by FUNCTION subprograms at station K are as follows:

$$\frac{h}{h_o} \rightarrow \text{FUNCTION HHT(K,DEL)}$$

$$\left(\frac{h}{h_o}\right)' \rightarrow \text{FUNCTION DHHT(K,DEL)}$$

$$\frac{h}{t} \rightarrow \text{FUNCTION HRA(K,DEL)}$$

$$\left(\frac{h}{t}\right)' \rightarrow \text{FUNCTION DHRA(K,DEL)}$$

where K is an index indicating the shell station and DEL is the arc length instrument Δ. For an isotropic shell of uniform thickness these quantities have the following values:

$$\frac{h}{h_o} = 1$$

$$\left(\frac{h}{h_o}\right)' = 0$$

$$\frac{h}{t} = 2$$

$$\left(\frac{h}{t}\right)' = 0$$

Variable parametric data can be transmitted to these functions by reading data into COMMON location BL14 (CONST(100)) in the main program and then transmitting the data to the subprograms by using a suitable COMMON statement.

Temperature distribution.- The temperature, for any Fourier index, is assumed to be given in the following form:

$$T^{(n)} = T_1^{(n)} + \Delta T_1^{(n)} \zeta \tag{32}$$

where $T_1^{(n)}$ is the temperature of the reference surface and $\Delta T_1^{(n)}$ is the temperature difference between inner and outer surfaces per unit thickness. This specification for this temperature distribution results in thermal forces and moments as presented in appendix D. The quantities $T_1^{(n)}$ and $\Delta T_1^{(n)}$, and their derivatives with respect to ξ, are to be specified at the station K by functions as follows:

$T_1^{(n)}$ → FUNCTION TEMP(K,DEL)

$\Delta T_1^{(n)}$ → FUNCTION DELT(K,DEL)

$T_1^{(n)'}$ → FUNCTION DTEMP(K,DEL)

$\Delta T_1^{(n)'}$ → FUNCTION DDELT(K,DEL)

where K and DEL are the same as defined previously.

Surface loads.- The surface loads $p^{(n)}$, $p_\xi^{(n)}$, and $p_\theta^{(n)}$ are specified at the station K by FUNCTION subprograms as follows:

p → FUNCTION P(K,DEL)

p_ξ → FUNCTION PX(K,DEL)

p_θ → FUNCTION PT(K,DEL)

where K and DEL are as defined in the preceding section.

Program Output

The output for this program is divided into three sections. The first section is devoted to a statement of the problem being run. It contains much of the input information and serves as a check on the accuracy of such information. The entire problem is summarized and the user may easily check the input data for errors. The data presented in this section should be scrutinized before proceeding to an evaluation of the program calculations.

The second section presents a listing of shell geometrical data and is printed at the user's option. This output option is controlled by IND1 where geometrical data are printed if IND1 = 0. The third section presents the data calculated by the program in nondimensional form. The relation between the nondimensional quantities and physical quantities for the output variables is given in table 4. Also given in table 4 is the title of the column heading under which the data appear on the output list and the location of each of the variables in storage at output time. The form of the output is presented in table 5.

Errors

The results should be checked to make sure that necessary conditions are satisfied. For instance, a simple check can usually be made to determine that an arbitrary portion of the shell is in a state of overall equilibrium. If the answers appear unreasonable, further checks should be made.

Inconsistency of input data.- It is possible that the boundary conditions are incorrectly specified or that incorrect input functions specifying geometry, loads, or temperature are in use. Careful checking of output data describing shell parameters will help to eliminate many problems of this type.

Numerical error.- There are two types of numerical error which are of importance. The first is the numerical error which is inherent in the size of the increment. The meridional increment must be sufficiently small so that a number of shell stations exist in any boundary layer associated with boundary condition effects. Another type of error, inherent in digital calculations, is the loss of significant digits due to truncation error as the number of shell stations increases.

Estimation of Increment Size

The practical problem of round-off error which occurs in the floating point arithmetic operations on a digital computer requires that attention be given to the estimation of a minimum increment size. Because of the complexity of the general problem it is not possible to write down an equation which relates minimum increment size to the geometric properties of the shell.

A primary source of round-off error can occur in the calculation of the A, B, and C matrices, as defined by equations (20). It is seen that the elements of B, for instance, are formed by addition of appropriate elements of the E and G matrices (see eq. (13)). Furthermore, since the first term of the sum involves the element of E times the large number $\frac{1}{\Delta}$ and the second term involves the element of G times the small number Δ, it is conceivable that for very small Δ the second term would have no significance in the sum. Thus, for a homogeneous conical shell of constant thickness under axisymmetric loading, for example, it can be shown that the first element of the first row of B is of the following order:

$$0\left(\frac{B_{11}}{b}\right) = 0\left(\frac{1}{\Delta}\right) + 0(\Delta) \tag{33}$$

and since $\Delta \ll 1$ the quantity $\frac{B_{11}}{b}$ is of order $0\left(\frac{1}{\Delta}\right)$. In order for the second term on the right-hand side of equation (33) to have j significant figures in a sum where the first term has m significant figures, Δ can be estimated as follows:

$$\Delta^2 \cong 10^{j-m}$$

Where $m = 8$ and j is desired to be 3, Δ is found to be of the order of $\Delta \cong 3 \times 10^{-3}$. Thus, for a very short cone, that is, a cone having a maximum nondimensional meridional length which is only one order of magnitude greater than the order of the minimum value of Δ calculated, only a relatively few stations can be considered.

In order to check this rationale a very short truncated cone was considered, having a nose half-angle of $\frac{\pi}{3}$ radians and a nondimensional length approximately 20 times the calculated minimum increment length. The nondimensional radius to one edge was taken to be 1, whereas the radius to the other edge was specified to be 0.94545. This gave a nondimensional arc length of 0.0628. The edge having the smaller radius was free of forces; at the other edge the following conditions were prescribed:

$$W_R = 1$$

$$W_A = 0$$

$$\varphi_\xi = 0$$

$$u_\theta = 0$$

where W_R and W_A are the radial and axial components of displacement, respectively. Since no axial forces are applied to the system, the axial component of the edge forces at the edge $\rho = 1$ should be zero. The axial-force resultant calculated for each increment is plotted against the increment size in figure 8.

It can be seen that the residual axial force increases rapidly with decrease in increment size. For this particular case the largest increment which is the same order as the minimum increment length calculated in this example gives the smallest axial-force resultant.

The message here is that, for some problems, increasing the number of stations does not increase the accuracy of the solution. In fact, for specific cases, a relatively large error may result from using a large number of stations and thereby a small increment size.

General Considerations

Suggested procedure.- It is suggested that when the INPUT subroutine is fairly complicated it be debugged separately until the user is convinced that the geometry is being correctly calculated. The program can then be easily converted to a subroutine. Simple problems should be considered first in order to gain familiarity with the potentialities of the program.

Program running times.- On the average, a problem having 502 stations and having nonsimple geometry takes approximately 30 seconds on an IBM 7040/7094 II Direct Coupled System for one Fourier component.

Program limitations and restrictions.- Most of these limitations have been mentioned in various places throughout the text; however, they are summarized here for convenience.

a. Young's modulus, Poisson's ratio, and the thermal expansion parameter $\frac{E\alpha}{\sigma_o}$ are constant with respect to tangents to the middle surface.

b. Three layers, symmetric with respect to the middle surface, are allowed.

c. Only load and temperature distributions which can be represented as a Fourier series with respect to the circumferential coordinate can be considered.

EXAMPLE PROBLEMS

The example problems are typical of those which the user of the program might encounter in actual practice. The problems are meant to illustrate the preparation of input data, input subroutines, and boundary conditions.

Cylindrical Shell With Edge Moments

The problem considered here is that of a cylindrical shell of uniform thickness h_o, subjected to the end moments $M_\xi = 10^{-2} \frac{Eh_o^2}{1 - \nu^2} \cos n\theta$ with $U_\xi = U_\theta = W = 0$ at each boundary. The calculations were made for $\frac{a}{h_o} = 50$, $\nu = 0.3$, $\frac{S}{a} = 1$, where a is taken to be the radius of the cylinder and S is the total arc length; 500 increments were used.

Nondimensional parameters.- It is seen from table 4 that the relation between the physical and the nondimensional moment is given by

$$M_\xi = \frac{\sigma_o h_o{}^3}{a} m_\xi \cos n\theta$$

The value of m_ξ is taken to be unity at the boundary for each Fourier index to give the following expression for σ_o:

$$\sigma_o = \frac{10^{-2} aE}{h_o\left(1 - \nu^2\right)}$$

In general, the reference stress σ_o is determined by an examination of the relationship between the dimensional and the dimensionless forcing function for a specific problem. The forcing function may be a surface pressure, a prescribed nonzero edge force or displacement, or a prescribed temperature variation.

Boundary conditions.- For each value of the Fourier index the boundary conditions at $\frac{S}{a} = 0, 1$ are $m_\xi = 1$, $u_\xi = u_\theta = w = 0$. The boundary condition matrices Ω, Λ, and l are identical at each end and are as follows:

$$\Omega = \left[0\right]$$

$$\Lambda = \left[I\right]$$

$$l = \left[0,0,0,1\right]$$

where I is the unit matrix.

User-prepared subprograms.-

a. INPUT: At each station the following relations hold: $\rho = \omega_\theta = a$; $\rho' = \omega_\xi = \omega_\xi{}' = 0$. Also, the arc length increment is given by $\Delta = \frac{S}{a(N - 2)}$. A suitable INPUT subroutine is as follows:

```
      SUBROUTINE INPUT (NMAX)
      COMMON R(502)/BL1/GAM(502),OMT(502),OMXI(502),DEOMX(502)
     1/BL3/NU,LAM,N,EALSIG,CHAR,DEL
      READ(5,1)SMAX
    1 FORMAT(F10.5)
      DEL=SMAX/FLOAT(NMAX-2)
      DO 2 I=1,NMAX
      R(I) = 1.
      OMT(I)=1.
```

```
      OMXI(I)=0.
      DEOMX(I)=0.
    2 GAM(I)=0.
      RETURN
      END
```

b. Thickness, thickness variation, ratio of total thickness to cover-plate thickness, and variation of ratio of total thickness to cover-plate thickness: A suitable set of functions to define these variables for a shell of uniform thickness h_o is as follows:

```
      FUNCTION HHT(K,DEL)
      HHT=1.
      RETURN
      END
      FUNCTION DHHT(K,DEL)
      DHHT=0.
      RETURN
      END
      FUNCTION HRA(K,DEL)
      HRA=2.
      RETURN
      END
      FUNCTION DHRA(K,DEL)
      DHRA=0.
      RETURN
      END
```

c. Surface loads: For the problem considered, $p^{(n)} = p_\xi^{(n)} = p_\theta^{(n)} = 0$. A suitable set of functions specifying these variables is as follows:

```
      FUNCTION P(K,DEL)
      P=0.
      RETURN
      END
```

```
FUNCTION PT(K,DEL)
PT=0.
RETURN
END
FUNCTION PX(K,DEL)
PX=0.
RETURN
END
```

d. Temperature: The temperature is uniform and is equal to a reference temperature level at which no thermal strains are produced. A suitable set of functions which specify the temperature variables is as follows:

```
FUNCTION TEMP(K,DEL)
TEMP=0.
RETURN
END
FUNCTION DTEMP(K,DEL)
DTEMP=0.
RETURN
END
FUNCTION DELT(K,DEL)
DELT=0.
RETURN
END
FUNCTION DDELT(K,DEL)
DDELT=0.
RETURN
END
```

Results.- The results of this calculation were compared with those presented in reference 1 for this shell configuration. The agreement was excellent.

Closed Spherical Shell Segment

Membrane solution.- The problem shown in figure 9 is a closed spherical shell of uniform thickness h_o, subjected to a uniform hydrostatic external pressure q over the surface, and having the edge conditions $U_\xi = \hat{Q}_\xi = M_\xi = N_{\xi\theta} = 0$ which correspond to the membrane solution. The calculations were made for $\frac{a}{h_o} = 1000$, $\nu = 0.3$, $\phi_o = \frac{\pi}{6}$, and $n = 0$, where a is the shell radius; 50 increments were used. For this case, since the shell is closed at $\xi = 0$, the indicator IND5 is set equal to 1 and the finiteness conditions which are associated with the Fourier coefficient $n = 0$ are generated within the program. The results are in generally good agreement with the membrane solution for the problem. The stress resultants N_ξ and N_θ are each found to be equal to $\frac{qa}{2}$, in agreement with the membrane theory; however, a small residual moment on the order of $10^{-5}qh_o^2$ is calculated, whereas the membrane theory is based on the assumption that the moment is identically equal to zero.

Elastic constraint at the boundary.- In the previous problem the edge conditions corresponded to those associated with the membrane solution. The present calculation is identical except for the boundary conditions. For this case the boundary conditions are taken in a more complex form in order to illustrate the specification of nondiagonal boundary condition matrices. The boundary conditions are taken to be

$$W_V = 0$$

$$M_\xi = 0$$

$$F_H + kW_H = 0$$

$$U_\theta = 0$$

where the components W_V, W_H, and F_H are defined in figure 9, and k is a proportionality constant.

The horizontal and vertical components are related to shell variables as follows:

$$F_H = \hat{Q}_\xi \sin \phi_o + N_\xi \cos \phi_o$$

$$W_H = W \sin \phi_o + U_\xi \cos \phi_o$$

$$W_V = W \cos \phi_o - U_\xi \sin \phi_o$$

The boundary conditions in terms of the nondimensional shell variables are as follows:

$$-u_\xi \sin \phi_o + w \cos \phi_o = 0$$

$$u_\theta = 0$$

$$\hat{f}_\xi \sin \phi_o + t_\xi \cos \phi_o + \bar{k}\left(u_\xi \cos \phi_o + w \sin \phi_o\right) = 0$$

$$m_\xi = 0$$

where

$$\bar{k} = \frac{a}{h_o} \frac{k}{E_o}$$

The boundary condition matrices for this case are as follows:

$$\Omega_L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \cos \phi_o & 0 & \sin \phi_o & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Lambda_L = \begin{bmatrix} -\sin \phi_o & 0 & \cos \phi_o & 0 \\ 0 & 1 & 0 & 0 \\ \bar{k} \cos \phi_o & 0 & \bar{k} \sin \phi_o & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$l_L = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

The results N_θ and M_ξ for the calculation of the sphere with hydrostatic pressure for these boundary conditions for $\bar{k} = 0.1$ are presented in figures 10 and 11, respectively. These results show that the membrane results are reasonably valid through $\frac{s}{S} = 0.8$; beyond that point a large boundary-layer effect is present, especially for N_θ and M_ξ. An output listing for this problem for 50 equal increments is presented in appendix E.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., January 5, 1967,
124-08-06-11-23.

APPENDIX A

DETAILS OF ANALYTIC FORMULATION

The detailed derivation of the set of equilibrium equations (13) is presented in reference 1, and is repeated here to give a complete presentation of the basis for the computer program. In proceeding from equations (8), (9), and (10) it is necessary to define strain displacement relations, stress-strain relations, and to separate variables. The derivation of equations (13) is given in the following sections.

Strain Displacement Relations

The displacements and rotations are shown in the positive sense in figure 3. The relation between rotations and displacements are as follows:

$$\left.\begin{aligned} \Phi_\xi &= \frac{1}{a}\left(-\frac{\partial W}{\partial \xi} + \omega_\xi U_\xi\right) \\ \Phi_\theta &= \frac{1}{a}\left(-\frac{1}{\rho}\frac{\partial W}{\partial \theta} + \omega_\theta U_\theta\right) \end{aligned}\right\} \tag{A1}$$

The membrane strains are given by the following equations:

$$\left.\begin{aligned} \epsilon_\xi &= \frac{1}{a}\left(\frac{\partial U_\xi}{\partial \xi} + \omega_\xi W\right) \\ \epsilon_\theta &= \frac{1}{a}\left(\frac{1}{\rho}\frac{\partial U_\theta}{\partial \theta} + \gamma U_\xi + \omega_\theta W\right) \\ \epsilon_{\xi\theta} &= \frac{1}{2a}\left(\frac{1}{\rho}\frac{\partial U_\xi}{\partial \theta} + \frac{\partial U_\theta}{\partial \xi} - \gamma U_\theta\right) \end{aligned}\right\} \tag{A2}$$

The bending distortions are given by

$$\left.\begin{aligned} \kappa_\xi &= \frac{1}{a}\frac{\partial \Phi_\xi}{\partial \xi} \\ \kappa_\theta &= \frac{1}{a}\left(\frac{1}{\rho}\frac{\partial \Phi_\theta}{\partial \theta} + \gamma \Phi_\xi\right) \\ \kappa_{\xi\theta} &= \frac{1}{2a}\left[\frac{1}{\rho}\frac{\partial \Phi_\xi}{\partial \theta} + \frac{\partial \Phi_\theta}{\partial \xi} - \gamma \Phi_\theta + \frac{1}{2a}\left(\omega_\xi - \omega_\theta\right)\left(\frac{1}{\rho}\frac{\partial U_\xi}{\partial \theta} - \frac{\partial U_\theta}{\partial \xi} - \gamma U_\theta\right)\right] \end{aligned}\right\} \tag{A3}$$

APPENDIX A

Stress-Strain Relations

By using the usual Kirchhoff hypothesis the stress-strain relations are written as follows:

$$\left.\begin{aligned}
\epsilon_\xi + \zeta\kappa_\xi &= \frac{(\sigma_\xi - \nu\sigma_\theta)}{E} + \alpha T \\
\epsilon_\theta + \zeta\kappa_\theta &= \frac{(\sigma_\theta - \nu\sigma_\xi)}{E} + \alpha T \\
\epsilon_{\xi\theta} + \zeta\kappa_{\xi\theta} &= \frac{(1 + \nu)}{E}\sigma_{\xi\theta}
\end{aligned}\right\} \tag{A4}$$

where ζ is the normal distance from the middle surface and T the temperature change may vary with ζ as well as with ξ and θ. The forces and moments are assumed to be related to the stresses by the following integrals:

$$\left.\begin{aligned}
N_\xi &= \int \sigma_\xi \, d\zeta & M_\xi &= \int \zeta\sigma_\xi \, d\zeta \\
N_\theta &= \int \sigma_\theta \, d\zeta & M_\theta &= \int \zeta\sigma_\theta \, d\zeta \\
\overline{N}_{\xi\theta} &= \int \sigma_{\xi\theta} \, d\zeta & \overline{M}_{\xi\theta} &= \int \zeta\sigma_{\xi\theta} \, d\zeta
\end{aligned}\right\} \tag{A5}$$

where the integrations extend through the total shell thickness.

Equations (A4) and (A5) are combined to give the following relations:

$$\left.\begin{aligned}
\epsilon_\xi &= \frac{N_\xi - \nu N_\theta}{\int E \, d\zeta} + \frac{\int E\alpha T \, d\zeta}{\int E \, d\zeta} \\
\epsilon_\theta &= \frac{N_\theta - \nu N_\xi}{\int E \, d\zeta} + \frac{\int E\alpha T \, d\zeta}{\int E \, d\zeta} \\
\epsilon_{\xi\theta} &= \frac{(1 + \nu)\overline{N}_{\xi\theta}}{\int E \, d\zeta}
\end{aligned}\right\} \tag{A6}$$

and

$$\left.\begin{aligned}
\kappa_\xi &= \frac{M_\xi - \nu M_\theta}{\int \zeta^2 E\, d\zeta} + \frac{\int \zeta E \alpha T\, d\zeta}{\int \zeta^2 E\, d\zeta} \\
\kappa_\theta &= \frac{M_\theta - \nu M_\xi}{\int \zeta^2 E\, d\zeta} + \frac{\int \zeta E \alpha T\, d\zeta}{\int \zeta^2 E\, d\zeta} \\
\kappa_{\xi\theta} &= \frac{(1 + \nu)\overline{M}_{\xi\theta}}{\int \zeta^2 E\, d\zeta}
\end{aligned}\right\} \tag{A7}$$

where it has been assumed that the middle surface is defined so that the following relation is satisfied:

$$\int \zeta E\, d\zeta = 0 \tag{A8}$$

Fourier Series Expansion

The complete set of field equations for the 17 unknown quantities N_ξ, N_θ, $\overline{N}_{\xi\theta}$, M_ξ, M_θ, $\overline{M}_{\xi\theta}$, U_ξ, U_θ, W, φ_ξ, φ_θ, ϵ_ξ, ϵ_θ, $\epsilon_{\xi\theta}$, κ_ξ, κ_θ, and $\kappa_{\xi\theta}$ are given by equations (8) to (10), (A1), (A2), (A3), (A6), and (A7). It is assumed that the surface loads and temperature distribution can be expanded into a Fourier series in terms of the circumferential coordinate. The loads and temperature distribution are given by the following equations:

$$\left.\begin{aligned}
q &= \frac{\sigma_o h_o}{a} \sum_{n=0}^{\infty} p^{(n)}(\xi)\cos n\theta \\
q_\xi &= \frac{\sigma_o h_o}{a} \sum_{n=0}^{\infty} p_\xi^{(n)}(\xi)\cos n\theta \\
q_\theta &= \frac{\sigma_o h_o}{a} \sum_{n=1}^{\infty} p_\theta^{(n)}(\xi)\sin n\theta
\end{aligned}\right\} \tag{A9}$$

$$T = \sum_{n=0}^{\infty} T^{(n)}(\xi,\zeta)\cos n\theta \tag{A10}$$

where σ_o is a reference stress level and h_o is a reference thickness.

The independent variables are thus expanded in the following nondimensional form:

$$\left.\begin{aligned}
N_\xi &= \sigma_o h_o \sum_{n=0}^{\infty} t_\xi^{(n)} \cos n\theta \\
N_\theta &= \sigma_o h_o \sum_{n=0}^{\infty} t_\theta^{(n)} \cos n\theta \\
\overline{N}_{\xi\theta} &= \sigma_o h_o \sum_{n=1}^{\infty} t_{\xi\theta}^{(n)} \sin n\theta
\end{aligned}\right\} \tag{A11}$$

$$\left.\begin{aligned}
M_\xi &= \frac{\sigma_o {h_o}^3}{a} \sum_{n=0}^{\infty} m_\xi^{(n)} \cos n\theta \\
M_\theta &= \frac{\sigma_o {h_o}^3}{a} \sum_{n=0}^{\infty} m_\theta^{(n)} \cos n\theta \\
\overline{M}_{\xi\theta} &= \frac{\sigma_o {h_o}^3}{a} \sum_{n=1}^{\infty} m_{\xi\theta}^{(n)} \sin n\theta
\end{aligned}\right\} \tag{A12}$$

$$\left.\begin{aligned}
U_\xi &= \frac{a\sigma_o}{E_o} \sum_{n=0}^{\infty} u_\xi^{(n)} \cos n\theta \\
U_\theta &= \frac{a\sigma_o}{E_o} \sum_{n=1}^{\infty} u_\theta^{(n)} \sin n\theta \\
W &= \frac{a\sigma_o}{E_o} \sum_{n=0}^{\infty} w^{(n)} \cos n\theta
\end{aligned}\right\} \tag{A13}$$

$$\left.\begin{aligned}
\epsilon_\xi &= \frac{\sigma_o}{E_o} \sum_{n=0}^{\infty} e_\xi^{(n)} \cos n\theta \\
\epsilon_\theta &= \frac{\sigma_o}{E_o} \sum_{n=0}^{\infty} e_\theta^{(n)} \cos n\theta
\end{aligned}\right\} \tag{A14}$$

(Equations continued on next page)

$$\left.\epsilon_{\xi\theta} = \frac{\sigma_o}{E_o} \sum_{n=1}^{\infty} e_{\xi\theta}^{(n)} \sin n\theta \right\} \tag{A14}$$

$$\left.\begin{aligned} \Phi_\xi &= \frac{\sigma_o}{E_o} \sum_{n=0}^{\infty} \varphi_\xi^{(n)} \cos n\theta \\ \Phi_\theta &= \frac{\sigma_o}{E_o} \sum_{n=1}^{\infty} \varphi_\theta^{(n)} \sin n\theta \end{aligned}\right\} \tag{A15}$$

$$\left.\begin{aligned} \kappa_\xi &= \frac{\sigma_o}{aE_o} \sum_{n=0}^{\infty} k_\xi^{(n)} \cos n\theta \\ \kappa_\theta &= \frac{\sigma_o}{aE_o} \sum_{n=0}^{\infty} k_\theta^{(n)} \cos n\theta \\ \kappa_{\xi\theta} &= \frac{\sigma_o}{aE_o} \sum_{n=1}^{\infty} k_{\xi\theta}^{(n)} \sin n\theta \end{aligned}\right\} \tag{A16}$$

By using equations (A11), (A12), and (A9), the equilibrium equations (8), (9), and (10) can be decoupled for each Fourier index n. Omitting the superscript (n), the set of partial differential equilibrium equations (8) to (10) become the following set of coupled ordinary differential equations which govern the Fourier coefficients:

$$\left.\begin{aligned} & t_\xi' + \gamma\left(t_\xi - t_\theta\right) + \left(\frac{n}{\rho}\right)t_{\xi\theta} + \lambda^2\left[\omega_\xi m_\xi' + \gamma\omega_\xi\left(m_\xi - m_\theta\right) + \left(\frac{n}{2\rho}\right)\left(3\omega_\xi - \omega_\theta\right)m_{\xi\theta}\right] + p_\xi = 0 \\ & t_{\xi\theta}' + 2\gamma t_{\xi\theta} - \left(\frac{n}{\rho}\right)t_\theta + \lambda^2\left\{-\left(\frac{n}{\rho}\right)\omega_\theta m_\theta + \frac{1}{2}\left(3\omega_\theta - \omega_\xi\right)m_{\xi\theta}' \right. \\ & \left. + \frac{1}{2}\left[\gamma\left(3\omega_\theta + \omega_\xi\right) - \omega_\xi'\right] m_{\xi\theta}\right\} + p_\theta = 0 \\ & -\omega_\xi t_\xi - \omega_\theta t_\theta + \lambda^2\left\{m_\xi'' + 2\lambda m_\xi' - \omega_\xi\omega_\theta m_\xi + \left[\omega_\xi\omega_\theta - \left(\frac{n}{\rho}\right)^2\right]m_\theta \right. \\ & \left. - \gamma m_\theta' + \left(\frac{2n}{\rho}\right)m_{\xi\theta}' + \left(\frac{2\gamma n}{\rho}\right)m_{\xi\theta}\right\} + p = 0 \end{aligned}\right\} \tag{A17}$$

where

$$\lambda = \frac{h_o}{a} \tag{A18}$$

Similarly, using equations (A13), (A14), (A15), and (A16), equations (A1), (A2), and (A3) yield the following additional equations for each Fourier coefficient:

$$\left.\begin{aligned} \varphi_\xi &= -w' + \omega_\xi u_\xi \\ \varphi_\theta &= \left(\frac{n}{\rho}\right)w + \omega_\theta u_\theta \end{aligned}\right\} \tag{A19}$$

$$\left.\begin{aligned} e_\xi &= u_\xi' + \omega_\xi w \\ e_\theta &= \left(\frac{n}{\rho}\right)u_\theta + \gamma u_\xi + \omega_\theta w \\ e_{\xi\theta} &= \frac{1}{2}\left[u_\theta' - \gamma u_\theta - \left(\frac{n}{\rho}\right)u_\xi\right] \end{aligned}\right\} \tag{A20}$$

$$\left.\begin{aligned} k_\xi &= \varphi_\xi' \\ k_\theta &= \left(\frac{n}{\rho}\right)\varphi_\theta + \gamma\varphi_\xi \\ k_{\xi\theta} &= \frac{1}{2}\left[-\left(\frac{n}{\rho}\right)\varphi_\xi + \varphi_\theta' - \gamma\varphi_\theta + \frac{1}{2}\left(\omega_\theta - \omega_\xi\right)\left(\frac{n}{\rho}u_\xi + u_\theta' + \gamma u_\theta\right)\right] \end{aligned}\right\} \tag{A21}$$

The Fourier coefficients for forces and moments are found to be

$$\left.\begin{aligned} t_\xi &= b\left(e_\xi + \nu e_\theta\right) - t_T^{(n)} \\ t_\theta &= b\left(e_\theta + \nu e_\xi\right) - t_T^{(n)} \\ t_{\xi\theta} &= b(1 - \nu)e_{\xi\theta} \end{aligned}\right\} \tag{A22}$$

and

$$\left.\begin{aligned} m_\xi &= d\left(k_\xi + \nu k_\theta\right) - m_T^{(n)} \\ m_\theta &= d\left(k_\theta + \nu k_\xi\right) - m_T^{(n)} \\ m_{\xi\theta} &= d(1 - \nu)k_{\xi\theta} \end{aligned}\right\} \tag{A23}$$

$$b = \frac{\int E \, d\zeta}{E_o h_o \left(1 - \nu^2\right)} \tag{A24}$$

$$d = \frac{\int \zeta^2 E \, d\zeta}{E_o h_o{}^3 \left(1 - \nu^2\right)} \tag{A25}$$

$$t_T^{(n)} = \frac{\int E\alpha T^{(n)} d\zeta}{\sigma_o h_o (1 - \nu)} \tag{A26}$$

$$m_T^{(n)} = \frac{a \int \xi E\alpha T^{(n)} d\zeta}{\sigma_o h_o{}^3 (1 - \nu)} \tag{A27}$$

The set of 17 field equations for each Fourier number n for the Fourier coefficients t_ξ, t_θ, $t_{\xi\theta}$, m_ξ, m_θ, $m_{\xi\theta}$, u_ξ, u_θ, w, φ_ξ, φ_θ, e_ξ, e_θ, $e_{\xi\theta}$, k_ξ, k_θ, $k_{\xi\theta}$ is given by the 17 equations (A17) and (A19) to (A23).

Reduction to Set of Second-Order Differential Equations

The set of 17 field equations constitutes an eighth-order system. By combining equations to eliminate variables, the 17 equations can be reduced to the following set of 4 second-order differential equations in terms of the variables u_ξ, u_θ, w, and m_ξ:

$$Ez'' + Fz' + Gz = e \tag{A28}$$

where the relation

$$m_\theta = \nu m_\xi + d\left(1 - \nu^2\right) k_\theta - (1 - \nu) m_T \tag{A29}$$

has been used and where

$$z = \begin{Bmatrix} u_\xi \\ u_\theta \\ w \\ m_\xi \end{Bmatrix} \tag{A30}$$

and

$$\left.\begin{aligned}
E &= \begin{bmatrix} E_{11} & 0 & 0 & 0 \\ 0 & E_{22} & E_{23} & 0 \\ 0 & E_{32} & E_{33} & E_{34} \\ 0 & 0 & E_{43} & 0 \end{bmatrix} \\
F &= \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ F_{21} & F_{22} & F_{23} & 0 \\ F_{31} & F_{32} & F_{33} & F_{34} \\ F_{41} & 0 & F_{43} & 0 \end{bmatrix} \\
G &= \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} \\
e &= \begin{Bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{Bmatrix}
\end{aligned}\right\} \tag{A31}$$

The nonzero elements of the E, F, G, and e matrices are defined in appendix A of reference 1.

Boundary Conditions

In the Sander theory the expressions for virtual work per unit length at boundaries $\xi = 0$, ξ_{max} are

$$W_B = \pm\left(N_\xi U_\xi + \hat{N}_{\xi\theta} U_\theta + \hat{Q}_\xi W + M_\xi \Phi_\xi\right) \tag{A32}$$

where W_B is the virtual work per unit length and

$$\hat{N}_{\xi\theta} = \overline{N}_{\xi\theta} + \left[\left(\frac{3}{2R_\theta}\right) - \left(\frac{1}{2R_s}\right)\right]\overline{M}_{\xi\theta} \tag{A33}$$

and

$$\hat{Q}_\xi = \left(\frac{1}{a\rho}\right)\left[\left(\frac{\partial}{\partial\xi}\right)\left(\rho M_\xi\right) + 2\left(\frac{\partial \overline{M}_{\xi\theta}}{\partial\theta}\right) - \rho' M_\theta\right] \tag{A34}$$

Expanding $\hat{N}_{\xi\theta}$ and $\hat{Q}_\xi$ in Fourier series and normalizing gives

$$\left.\begin{aligned} \hat{t}_{\xi\theta} &= t_{\xi\theta} + \left(\frac{\lambda^2}{2}\right)\left(3\omega_\theta - \omega_\xi\right)m_{\xi\theta} \\ \hat{f}_\xi &= \lambda^2\left[m_\xi' + \gamma\, m_\xi - m_\theta + \left(\frac{2n}{\rho}\right)m_{\xi\theta}\right] \end{aligned}\right\} \tag{A35}$$

The forces defined by equations (A33) and (A34) are effective inplane force and transverse shears per unit length, respectively. The forces and moments acting on the boundary are shown in figure 4. This form of the virtual work of the forces indicates that either the effective force (moment) or related displacement (rotation) may be prescribed or, more generally, the force and displacements may be linearly related. The linear relation provides a means of handling elastic restraints. A convenient boundary condition representation expressed in nondimensional form can thus be written for the nth Fourier coefficient as follows:

$$\Omega y + \Lambda z = l \tag{A36}$$

where y is the force vector

$$y = \begin{bmatrix} t_\xi \\ \hat{t}_{\xi\theta} \\ \hat{f}_\xi \\ \varphi_\xi \end{bmatrix} \tag{A37}$$

The vector y is related to z as follows

$$y = Hz' + Jz + f \tag{A38}$$

The nonzero coefficients of H, f, and J are presented in appendix A of reference 1.

APPENDIX B

FORTRAN IV SOURCE DECK

The listing of the source decks for the main program is on the following pages:

APPENDIX B

```
C       MAIN PROGRAM
C       THIS PROGRAM CONSISTS OF THE MAIN PROGRAM TOGETHER WITH THE FOLLOWING
C       SUBROUTINES
C       MATINV
C       EFG
C       OUTPUT
C       STRESS
C       EQ73
C       INIT
C       FINAL
C       FORCE
C       PANOX
C       ABCG
C       HFJ
C       KLT
C       BDB
C       POLE
C       IN ADDITION TO THE ABOVE THE USER MUST SUPPLY THE FOLLOWING SUB
C       ROUTINES AND FUNCTIONS.
C       SUBROUTINE INPUT--CALCULATES THE SHELL GEOMETRY.
C       FUNCTION P-SPECIFIES THE NORMAL PRESSURE DISTRIBUTION.
C       FUNCTION PX-SPECIFIES THE MERIDIONAL PRESSURE DISTRIBUTION.
C       FUNCTION PT-SPECIFIES THE CIRCUMFERENTIAL PRESSURE DISTRIBUTION
C       FUNCTION TEMP-SPECIFIES THE MERIDIONAL TEMPERATURE DISTRIBUTION
C       FUNCTION DTEMP- SPECIFIES THE DERIVATIVE OF THE TEMPERATURE
C       FUNCTION DELT- SPECIFIES THE DISTRIBUTION OF THE TEMPERATURE
C                     VARIATION THROUGH THE THICKNESS.
C       FUNCTION DDELT- SPECIFIES RATE OF CHANGE OF DELT
C       FUNCTION HHT-SPECIFIES THE SHELL THICKNESS DISTRIBUTION
C       FUNCTION DHHT-SPECIFIES THE DERIVATIVE OF THE THICKNESS.
C       FUNCTION HRA-SPECIFIES THE DISTRIBUTION OF THE RATIO OF THE
C                   TOTAL THICKNESS- TO -COVER PLATE THICKNESS.
C       FUNCTION DHRA- THE DERIVATIVE OF THE ABOVE RATIO.
        INTEGER FREQ
        REAL NU,LAM,N,JAY
        COMMON R(502)/BL1/GAM(502),GMT(502),OMXI(502),DEOMX(502)
       1/BL2/E(4,4),G(4,4),F(4,4)
       2/BL3/NU,LAM,N,FALSIG,CHAR,DEL
       3/BL4/CEF(4)
       4/BL5/H(4,4),FF(4),JAY(4,4)
       5/BL6/A(4,4),B(4,4),C(4,4),SMAG(4)
       6/BL7/PEE(4,4,502),X(4,502)
       7/BL9/OMEG1(4,4),CAPL1(4,4),EL1(4)
       8/BL11/OMEGL(4,4),CAPLL(4,4),ELL(4)
       9/BL13/AK(3,4),ALL(3,4),STHER(3)/BL14/CONST(100)
        DIMENSION Z(4,502),IPIVOT(4),INDEX(4,2)
        EQUIVALENCE(X(1,1),Z(1,1))
      1 FORMAT(5F6.3,6I4)
      2 FORMAT(1H1,16H PROBLEM NUMBER I6///)
     21 FORMAT (4E16.8)
     31 FORMAT(72H
       1                )
     50 FORMAT(2I4)
     60 FORMAT(1H0,55X10HINPUT DATA///)
    100 FORMAT(52H THE FOLLOWING CONSTANTS APPEAR IN COMMON BLOCK BL14///(
       11X1P8F16.8))
```

```
164 FORMAT(1X20H REFERENCE LENGTH    =1PE16.7//1X20H POISSONS RATIO     =
   11PE16.7//1X20H TEMP COEFFICIENT   =1PE16.7// 1X20H REF. THICKNESS
   2 =1PE16.7//1X20H FOURIER INDEX      =1PE16.7//1X20H NUMBER OF STATI
   3ONS=I5///)
165 FORMAT(34H THE INDICATORS ARE SET AS FOLLOWS//7H  IND1=I4,3X7H  IN
   1D2=I4,3X7H  IND3=I4,3X7H  IND4=I4,3X7H  IND5=I4//)
 12 READ(5,1) NU,TKN,CHAR,N,EALSIG,NO,IND1,IND2,IND3,IND4,IND5
    WRITE(6,2) NO
    READ(5,31)
    WRITE(6,31)
    IF(IND2.EQ.0)GO TO 70
    READ(5,21)(CONST(I),I=1,IND2)
 70 IF(IND3.NE.0)GO TO 3
    GO TO 4
  3 READ(5,50)NMAX,FREQ
    CALL INPUT(NMAX)
  4 LAM=TKN/CHAR
    WRITE (6,60)
    WRITE(6,164)CHAR,NU,EALSIG,      TKN,N,NMAX
    WRITE(6,165)IND1,IND2,IND3,IND4,IND5
    IF(IND2.NE.0)
   1WRITE(6,100)(CONST(I),I=1,IND2)
    CALL HFJ(1,IND5,NMAX,2.)
    CALL EFG(2,IND5,NMAX)
    CALL FORCE(2,IND5,NMAX)
    CALL ABCG
    CALL INIT(IND5,IND4)
    NMAX1=NMAX-1
    DO 6 K=3,NMAX1
    CALL EFG(K,IND5,NMAX)
    CALL FORCE(K,IND5,NMAX)
    CALL ABCG
  6 CALL PANDX(K)
    CALL HFJ(NMAX,IND5,NMAX,2.)
    CALL FINAL(NMAX,IND5,IND4)
    DO 7 L=2,NMAX1
    K=NMAX+1-L
  7 CALL EQ73(K)
    IF(IND5.EQ.0.OR.IND5.EQ.3) GO TO 14
    CALL EQ73(1)
    GO TO 15
 14 CALL EFG(2,IND5,NMAX)
    CALL FORCE(2,IND5,NMAX)
    CALL ABCG
    DO 8 I=1,4
    S1=0.
    S2=0.
    DO 9 J=1,4
    S1=S1+A(I,J)*Z(J,3)
  9 S2=S2+B(I,J)*Z(J,2)
  8 SMAG(I)=SMAG(I)-S1-S2
    CALL MATINV(C,4,SMAG,1,DETERM,IPIVOT,INDEX,4,ISCALE)
    DO 10 I=1,4
 10 Z(I,1)=SMAG(I)
 15 CALL STRESS(FREQ,NMAX,N,IND5)
    CALL OUTPUT(FREQ,NMAX,IND1,DEL,NO)
    GO TO 12
    END
```

```
      SUBROUTINE HFJ(K,IND5,NMAX,YAH)
C SUBROUTINE HFJ    THIS SUBROUTINE CALCULATES THE ELEMENTS OF THE H,F,
C AND J MATRICES,AS DEFINED IN APPENDIX A OF REFERENCE(1),AT THE STATION
C SPECIFIED BY THE INDEX K.
      COMMON R(502)/BL1/GAM(502),OMT(502),OMXI(502),DEOMX(502)/BL3/NU,
     1LAM,N,EALSIG,CHAR,DEL/BL5/H(4,4),FF(4),JAY(4,4)
      REALNU,LAM,N,JAY,L2
      CALL BDB(K,DEL,NU,B,DB,D,DD)
      L2=LAM**2
      D1=(1.-NU)
      OX=OMXI(K)
      REG=0.
      IF(YAH.EQ.2.) REG=1.
      EAL=EALSIG
      T=TEMP(K,DEL)
      DLT=DELT(K,DEL)
      H1=HHT(K,DEL)
      HRB=HRA(K,DEL)
      FF(1)=-2.*H1*EAL/(D1*HRB)*T
      FF(2)=0.
      FF(3)=L2*CHAR*EAL*DLT*H1**3/3.*(1.5/HRB-3./HRB**2+2./HRB**3)
      FF(4)=0.
      IF(IND5)1,1,2
    2 IF(((IND5-2).LE.0).AND.(K.EQ.1)) GO TO 8
      IF(((IND5-2).GT.0).AND.(K.EQ.NMAX)) GO TO 8
      GA=GAM(K)
      FF(3)=FF(3)*GA
    1 RA=R(K)
      OT=OMT(K)
      ENR=N/RA
      OXT=3.*OMXI(K)-OMT(K)
      OTX=3.*OMT(K)-OMXI(K)
      DL=D*L2*D1*ENR
      H(1,1)=B
      H(1,2)=0.
      H(1,3)=0.
      H(1,4)=0.
      H(2,1)=0.
      H(2,2)=B*D1/2.+L2*D*D1/8.*OTX**2*REG
      H(2,3)=DL/2.*OTX*REG
      H(2,4)=0.
      H(3,1)=0.
      H(3,2)=DL*OTX*YAH/4.
      ENR2=ENR**2
      H(3,3)=L2*D*D1*(YAH*ENR2+(1.+NU)*GA**2)
      GA2=GA**2
      H(3,4)=L2
      H(4,1)=0.
      H(4,2)=0.
      H(4,3)=-1.
      H(4,4)=0.
      JAY(1,1)=NU*GA*B
      JAY(1,2)=NU*B*ENR
      JAY(1,3)=B*(OX+NU*OT)
      JAY(1,4)=0.
      JAY(2,1)=-B*D1*ENR/2.-DL/8.*OXT*OTX*REG
      JAY(2,2)=-GA*H(2,2)
      JAY(2,3)=-GA*H(2,3)
      JAY(2,4)=0.
      JAY(3,1)=-L2*D*D1*((1.+NU)*GA2*OX+ENR2/4.*OXT*YAH)
      JAY(3,2)=-GA*DL/2.*(     2.*OT*(1.+NU)+OTX/2.*YAH)
      JAY(3,3)=-L2*D*D1*(1.+NU+YAH)*GA*ENR2
      JAY(3,4)=L2*D1*GA
```

```
      JAY(4,1)=OX
      JAY(4,2)=0.
      JAY(4,3)=0.
      JAY(4,4)=0.
      GO TO 5
    8 DO 6I=1,4
      DO 6J=1,4
      H(I,J)=0.
    6 JAY(I,J)=0.
      H(1,1)=B*(1.+NU)
      H(1,2)=N*B*NU
      H(2,1)=B*D1*(   -N)/2.
      AWB=D1*(-3.*(1.+NU)+N**2*(3.+NU))
      AW=AWB*D
      C1=AW/(-2.+NU*(N**2-1.))
      ATHETA=1.5*D*N*OX*D1*(3.+NU)
      AXI=1.5*D*OX*D1*(2.*(1.+NU)-N)
      H(3,4)=L2*(2.-NU+2.*AWB)
      H(4,3)=-1.
      JAY(1,3)=B*(1.+NU)*OX
      JAY(4,1)=OX
      DH=DHHT(M,DEL)
      DTOH=DHRA(M,DEL)
      DDLT=DDELT(M,DEL)
      DTMOM=CHAR*EALSIG/3./D1*((1.5/HRB-3./HRB**2+2./HRB**3)*(H1**3*DDLT
     1+3.*DH*H1**2*DLT)+DLT*H1**3*DTOH/HRB**2*(-1.5+6./HRB-6./
     2HRB**2))
      FF(3)=DTMOM*(2.*AWB/(-2.+NU*(N**2-1.))+2.-NU)
    5 RETURN
      END
```

```
      SUBROUTINE EFG(K,IND5,NMAX)
C SUBROUTINE EFG    THIS SUBROUTINE CALCULATES THE ELEMENTS OF THE E,F,AND G
C MATRICES,AS DEFINED IN APPENDIX A OF REFERENCE (1),AT THE STATION SPECIFIED
C BY THE INDEX K.
      COMMON R(502)/BL1/GAM(502),OMT(502),OMXI(502),DEOMX(502)/BL2/E(4,4
     1),G(4,4),F(4,4)/BL3/NU,LAM,N,EALSIG,CHAR,DEL
      REAL   NU,LAM,N,LAM2
      CALL BDB(K,DEL,NU,B,DB,D,DD)
      E(1,1)=B
      E(1,2)=0.
      E(1,3)=0.
      E(1,4)=0.
      E(2,1)=0.
      D1=(1.-NU)
      LAM2=LAM**2
      RA=R(K)
      GA=GAM(K)
      OX=OMXI(K)
      OT=OMT(K)
      DEX=DEOMX(K)
      GA2=GA**2
      REX=(3.*OT-OX)
      RXE=(3.*OX-OT)
      OTX=OT*OX
      DNLR=LAM2*D*N*D1/(2.*RA)
      DDNLR=DNLR*DD/D
      E(2,2)=B*D1/2.+LAM2*D*D1*REX**2/8.
      E(2,3)=DNLR*REX
      E(2,4)=0.
      E(3,1)=0.
      E(3,2)=E(2,3)
      RAN=(N/RA)**2
      E(3,3)=LAM2*D*D1*(2.*RAN+(1.+NU)*GA2)
      E(3,4)=LAM2
      E(4,1)=0.
      E(4,2)=0.
      E(4,3)=-D
      E(4,4)=0.
      F(1,1)=GA*B+DB
      F(1,2)=(1.+NU)*B*N/(2.*RA)+DNLR*REX*RXE/4.
      F(1,3)=B*(OX+NU*OT)+LAM2*D*D1*((1.+NU)*GA2*OX+RAN*RXE/2.)
      F(1,4)=LAM2*OX
      F(2,1)=-F(1,2)
      F(2,2)=(D1/2.)*(GA*B+DB)-(LAM2*D*D1*REX/8.)*(2.*DEX-GA*(5.*OX
     1-3.*OT))+LAM2*DD*D1*REX**2/8.
      F(2,3)=DNLR*(2.*(1.+NU)*GA*OT-DEX+3.*GA*(OX-OT))+DDNLR*REX
      F(2,4)=0.
      F(3,1)=-F(1,3)
      F(3,2)=DNLR*(3.*GA*OX-GA*OT*(5.+2.*NU)-DEX)+DDNLR*REX
      F(3,3)=-LAM2*D*D1*((1.+NU)*(2.*GA*OX*OT+GA**3)+2.*GA*RAN)
     1+LAM2*DD*D1*((1.+NU)*GA2+2.*RAN)
      F(3,4)=LAM2*GA*(2.-NU)
      F(4,1)=D*OX
      F(4,2)=0.
      F(4,3)=-D*NU*GA
      F(4,4)=0.
      G(1,1)=NU*DB*GA-NU*B*OTX-B*GA2-D1*B*RAN/2.-LAM2*D*D1*((1.+NU)*GA2*
     1OX**2+RXE**2*RAN/8.)
      G(1,2)=NU*N*DB/RA-(3.-NU)/(2.*RA)*GA*B*N-DNLR*2.*GA*(REX*RXE/8.
     1+(1.+NU)*OTX)
      G(1,3)=B*(DEX+GA*(OX-OT))+DB*(OX+NU*OT)-LAM2*D*D1*GA*RAN*(RXE/2.+(
     11.+NU)*OX)
      G(1,4)=LAM2*D1*GA*OX
```

```
 G(2,1)=-B*GA*N*(3.-NU)/(2.*RA)-D1*N*DB/(2.*RA)+DNLR*2.*(-1.*(1.+
1NU)*GA*OTX+GA/8.*(6.*OTX-7.*OX**2-3.*OT**2)-DEX/4.*(5.*OT-3.*OX))
2-DDNLR/4.*REX*RXE
 G(2,2)=-GA*F(2,2)+D1/2.*B*OTX-B*RAN-LAM2*D*D1*((1.+NU)*OT**2*RAN
1-OTX/8.*REX**2)
 G(2,3)=-B*N*(OT+NU*OX)/RA+DNLR*(GA*DEX-2.*GA2*OX-2.*(1.+NU)*OT
1*RAN+REX*(GA2+OTX))-DDNLR*REX*GA
 G(2,4)=-NU*LAM2*OT*N/RA
 G(3,1)=-B*GA*(OT+NU*OX)+LAM2*D*D1*(GA*(1.+NU)*(-GA*DEX+GA2*OX
1-OX*RAN+2.*OTX*OX)+RAN/2.*(GA*OX-GA*OT-3.*DEX))
2-LAM2*DD*D1*((1.+NU)*GA2*OX+RAN/2.*RXE)
 G(3,2)=-B*N*(OT+NU*OX)/RA+DNLR*(2.*(1.+NU)*(OTX*OT-GA2*OX+2.*GA2
1*OT-OT*RAN)+GA*DEX+3.*GA2*(OT-OX)+OTX*REX)-DDNLR*(2.*(1.+NU)*GA
2*OT+GA*REX)
 G(3,3)=-B*(OX**2+2.*NU*OTX+OT**2)+LAM2*D*D1*RAN*((1.+NU)*(OTX-RAN
1+2.*GA2)+2.*(GA2+OTX))-LAM2*DD*D1*RAN*(3.+NU)*GA
 G(3,4)=-LAM2*(D1*OTX+NU*RAN)
 G(4,1)=D*(DEX+NU*GA*OX)
 G(4,2)=D*NU*N*OT/RA
 G(4,3)=D*NU*RAN
 G(4,4)=-1.
 RETURN
 END
```

```
      SUBROUTINE FORCE(K,IND5,NMAX)
C SUBROUTINE FORCE  THIS SUBROUTINE CALCULATES THE ELEMENTS OF THE LOWER CASE
C E-VECTOR AS DEFINED IN APPENDIX A OF REFERENCE(1),AT THE STATION SPECIFIED
C BY THE INDEX K.
      COMMON R(502)/BL1/GAM(502),OMT(502),OMXI(502),DEOMX(502)/BL3/NU,
     1LAM,N,EALSIG,CHAR,DEL/BL4/CEE(4)
      REAL NU,LAM,N,L2
      RA=R(K)
      GA=GAM(K)
      OX=OMXI(K)
      OT=OMT(K)
      T=TEMP(K,DEL)
      DT=DTEMP(K,DEL)
      DELT1=DELT(K,DEL)
      DLT1=DDELT(K,DEL)
      PX1=PX(K,DEL)
      PT1=PT(K,DEL)
      P1=P(K,DEL)
      H=HHT(K,DEL)
      DH=DHHT(K,DEL)
      HRB=HRA(K,DEL)
      DHRB=DHRA(K,DEL)
      D1=1.-NU
      L2=LAM**2
      EAL=EALSIG
      CEE(4)=CHAR*EAL*DELT1*H**3*
     1                          (3./(2.*HRB)-3./HRB**2+2./HRB**3)/3./D1
      TSUBT=2.*H*EAL/(D1*HRB)*T
      CEE(1)=-PX1+2.*EAL*(H*DT+T*DH)/(D1*HRB)-TSUBT*DHRB/HRB
      CEE(2)=-PT1-(N/RA)*TSUBT-L2*D1*(N/RA)*OT*CEE(4)
      CEE(3)=-P1-(OX+OT)*TSUBT-L2*D1*CHAR*EALSIG*((1.5/HRB-3./HRB**2
     1+2./HRB**3)*(GA*H**3*DLT1+(3.*H**2*GA*DH+H**3*(OX*OT-(N/RA)**2)))
     2*DELT1+DELT1*H**3*DHRB
     3                       /HRB**2*(-1.5+6./HRB-6./HRB**2))/3./D1
      RETURN
      END
```

```
      SUBROUTINEABCG
C     SUBROUTINE ABCG-- THIS SUBROUTINE CALCULATES THE A,B,C, AND LOWER
C     CASE G MATRICES  USING THE CURRENT VALUES OF THE E,F,G, AND LOWER CASE
C     E MATRICES.
      COMMON/BL4/CEE(4)/BL2/E(4,4),G(4,4),F(4,4)/BL6/A(4,4),B(4,4),
     1C(4,4),SMAG(4)/BL3/NU,LAM,N,EALSIG,CHAR,DEL
      REAL N,NU,LAM
      D2=2./DEL
      D4=4./DEL
      DX=2.*DEL
      DO1I=1,4
      SMAG(I)=DX*CEE(I)
      DO1J=1,4
      B(I,J)=-D4*E(I,J)+DX*G(I,J)
      C(I,J)=D2*E(I,J)-F(I,J)
    1 A(I,J)=D2*E(I,J)+F(I,J)
      RETURN
      END
```

```
      SUBROUTINE INIT(IND5,IND4)
C     CALCULATION OF THE P-MATRIX AND THE X-VECTOR AT THE FIRST STATION.
      COMMON/BL5/H(4,4),FF(4),JAY(4,4)/BL7/PEE(4,4,502),X(4,502)/BL9/
     1OMEG1(4,4),CAPL1(4,4),EL1(4)
     2/BL3/NU,LAM,N,EALSIG,CHAR,DEL
     3/BL2/E(4,4),G(4,4),F(4,4)
     4/BL4/CEE(4)
     5/BL6/A(4,4),B(4,4),C(4,4),SMAG(4)
      DIMENSION IPIVOT(4),INDEX(4,2),A0(4,4),B0(4,4),A3(4,4),A4(4,4),G0(
     14)
      REAL JAY,N,NU,LAM
   21 FORMAT(4E16.8)
  166 FORMAT(1H0,46H THE BOUNDARY CONDITIONS AT S=0 ARE AS FOLLOWS//)
  167 FORMAT(21X5HOMEGA49X5HLAMDA34X3HELL///)
  169 FORMAT(1P4E12.4,6X1P4E12.4,6X1PE12.4)
      DO 8 I=1,4
      G0(I)=0.
      EL1(I)=0.
      DO 8 J=1,4
      CAPL1(I,J)=0.
    8 OMEG1(I,J)=0.
      IF(IND5) 9,9,10
   10 IF(IND5-2)11,11,9
   11 CALL POLE(N,DEL,F,G)
      WRITE(6,166)
      WRITE(6,40)
   40 FORMAT(1H0, 52H THE CONDITIONS FOR A SHELL POLE HAVE BEEN GENERATE
     1D///)
      GO TO 16
    9 IF(IND4) 13,13,14
   13 READ(5,21) OMEG1(1,1),OMEG1(2,2),OMEG1(3,3),OMEG1(4,4)
      READ(5,21) CAPL1(1,1),CAPL1(2,2),CAPL1(3,3),CAPL1(4,4)
      READ(5,21) EL1
      GO TO 15
   14 READ(5,21)OMEG1,CAPL1,EL1
   15 WRITE(6,166)
      WRITE(6,167)
      DO 168 I=1,4
  168 WRITE(6,169)OMEG1(I,1),OMEG1(I,2),OMEG1(I,3),OMEG1(I,4),
     1                CAPL1(I,1),CAPL1(I,2),CAPL1(I,3),CAPL1(I,4),
     2                EL1(I)
      DO 1 I=1,4
      DO 1 J=1,4
      B0(I,J)=H(I,J)/DEL+JAY(I,J)/2.
    1 A0(I,J)=JAY(I,J)/2.-H(I,J)/DEL
      DO 2 I=1,4
      DO 2 J=1,4
      S1=0.
      S2=0.
      DO 3 L=1,4
      S1=S1+OMEG1(I,L)*A0(L,J)
    3 S2=S2+OMEG1(I,L)*B0(L,J)
      A3(I,J)=S1
    2 A4(I,J)=S2
      DO 5 I=1,4
      S1=0.
      DO 4 J=1,4
      S1=S1+OMEG1(I,J)*FF(J)
      B0(I,J)=A3(I,J)+CAPL1(I,J)/2.
    4 A0(I,J)=A4(I,J)+CAPL1(I,J)/2.
    5 G0(I)=EL1(I)-S1
      CALL MATINV (C,4,CEE,0,DETERM,IPIVOT,INDEX,4,ISCALE)
      DO 50 I=1,4
```

```
      DO 50 J=1,4
      S1=0.
      S2=0.
      S3=0.
      DO 51 K=1,4
      S1=S1+C(I,K)*B(K,J)
      S2=S2+C(I,K)*A(K,J)
   51 S3=S3+B0(I,K)*C(K,J)
      A3(I,J)=S1
      A4(I,J)=S2
   50 E(I,J)=S3
      DO 52 I=1,4
      DO 52 J=1,4
      S1=0.
      S2=0.
      DO 53 K=1,4
      S1=S1+B0(I,K)*A3(K,J)
   53 S2=S2+B0(I,K)*A4(K,J)
      F(I,J)=S1-A0(I,J)
   52 G(I,J)=S2
   16 CALL MATINV(F,4,G,4,DETERM,IPIVOT,INDEX,4,ISCALE)
      IF(IND5.EQ.0.OR.IND5.EQ.3) GO TO 59
      DO 60 I=1,4
      X(I,1)=0.
      DO 60 J=1,4
   60 PEE(I,J,1)=G(I,J)
      CALL PANDX(2)
      RETURN
   59 DO 54 I=1,4
      S1=0.
      DO 61 J=1,4
      PEE(I,J,2)=G(I,J)
   61 S1=E(I,J)*SMAG(J)+S1
   54 G0(I)=S1-G0(I)
      DO 56 I=1,4
      S1=0.
      DO 57 J=1,4
   57 S1=S1+F(I,J)*G0(J)
   56 X(I,2)=S1
      RETURN
      END
```

```
      SUBROUTINE PANDX(K)
C SUBROUTINE PANDX  THIS SUBROUTINE CALCULATES THE P MATRIX AND THE X-VECTOR
C AT THE STATION SPECIFIED BY THE INDEX K USING EQUATION (29) OF THE TEXT.
      COMMON/BL6/A(4,4),B(4,4),C(4,4),SMAG(4)/BL7/PEE(4,4,502),X(4,502)
      DIMENSIONP1(4,4),IPIVOT(4),INDEX(4,2),X1(4),X2(4)
      DO 1 I=1,4
      DO 1 J=1,4
      SUM=0.
      DO 2 L=1,4
    2 SUM=SUM+C(I,L)*PEE(L,J,K-1)
    1 P1(I,J)=B(I,J)-SUM
      CALLMATINV(P1,4,X2,0,DETERM,IPIVOT,INDEX,4,ISCALE)
      DO 4 I=1,4
      SUM=0.
      DO 3 J=1,4
    3 SUM=SUM+C(I,J)*X(J,K-1)
    4 X1(I)=SMAG(I)-SUM
      DO 5 I=1,4
      DO 5 J=1,4
      SUM=0.
      DO 6 L=1,4
    6 SUM=SUM+P1(I,L)*A(L,J)
    5 PEE(I,J,K)=SUM
      DO 7 I=1,4
      SUM=0.
      DO 8 J=1,4
    8 SUM=SUM+P1(I,J)*X1(J)
    7 X(I,K)=SUM
      RETURN
      END
```

```
      SUBROUTINE EQ73(K)
C SUBROUTINE EQ73   THIS SUBROUTINE CALCULATES THE SOLUTION VECTOR AT
C THE STATION(K),GIVEN THE SOLUTION AT K+1.
      COMMON/BL7/PEE(4,4,502),X(4,502)
      DIMENSION Z(4,502)
      EQUIVALENCE (X(1,1),Z(1,1))
      DO 1 I=1,4
      SUM=0.
      DO 2 J=1,4
    2 SUM=SUM+PEE(I,J,K)*Z(J,K+1)
    1 Z(I,K)=X(I,K)-SUM
      RETURN
      END
```

```
      SUBROUTINE  FINAL(NMAX,IND5,IND4)
C     CALCULATION OF SOLUTION VECTOR ASSOCIATED WITH LAST STATION
      COMMON/BL3/NU,LAM,N,EALSIG,CHAR,DEL/BL5/H(4,4),FF(4),JAY(4,4)/BL7/
     1PEE(4,4,502),X(   4,502)/BL11/OMEGL(4,4),CAPLL(4,4),ELL(4)
      DIMENSION IPIVOT(4),INDEX(4,2),A1(4,4),A2(4,4),A3(4,4),PSI(4,4),
     1GM(4,4),ETA(4),B(4,4)
      REAL JAY,N,NU,LAM
   21 FORMAT(4E16.8)
  170 FORMAT(1H0,49H THE BOUNDARY CONDITIONS AT S=SMAX ARE AS FOLLOWS//)
  169 FORMAT(1P4E12.4,6X1P4E12.4,6X1PE12.4)
  167 FORMAT(21X5HOMEGA49X5HLAMDA34X3HELL///)
      DO 8 I=1,4
      ELL(I)=0.
      DO 8 J=1,4
      CAPLL(I,J)=0.
    8 OMEGL(I,J)=0.
      IF(IND5) 9,9,10
   10 IF(IND5-2) 9,11,11
   11 CALL POLE (N,DEL,PSI,GM)
      WRITE(6,170)
      WRITE(6,40)
   40 FORMAT(1H0, 52H THE CONDITIONS FOR A SHELL POLE HAVE BEEN GENERATE
     1D///)
      GO TO 16
    9 IF(IND4) 13,13,14
   13 READ(5,21) OMEGL(1,1),OMEGL(2,2),OMEGL(3,3),OMEGL(4,4)
      READ(5,21) CAPLL(1,1),CAPLL(2,2),CAPLL(3,3),CAPLL(4,4)
      READ (5,21) ELL
      GO TO 15
   14 READ(5,21) OMEGL,CAPLL,ELL
   15 WRITE(6,170)
      WRITE(6,167)
      DO 171 I=1,4
  171 WRITE(6,169)OMEGL(I,1),OMEGL(I,2),OMEGL(I,3),OMEGL(I,4),
     1            CAPLL(I,1),CAPLL(I,2),CAPLL(I,3),CAPLL(I,4),
     2            ELL(I)
      DO 1 I=1,4
      DO 1J=1,4
      A1(I,J)=JAY(I,J)/2.+H(I,J)/DEL
    1 A2(I,J)=JAY(I,J)/2.-H(I,J)/DEL
      DO 2 I=1,4
      DO 2 J=1,4
```

```
      S2=0.
      S3=0.
      DO 3 L=1,4
      S2=OMEGL(I,L)*A1(L,J)+S2
    3 S3=OMEGL(I,L)*A2(L,J)+S3
      PSI(I,J)=S3+CAPLL(I,J)/2.
    2 GM(I,J)=S2+CAPLL(I,J)/2.
   16 DO 4 I=1,4
      DO 4 J=1,4
      S1=0.
      DO 5 L=1,4
    5 S1=S1+PSI(I,L)*PEE(L,J,NMAX-1)
    4 B(I,J)=GM(I,J)-S1
      DO 6 I=1,4
      S1=0.
      S2=0.
      DO 7 J=1,4
      S1=S1+PSI(I,J)*X(J,NMAX-1)
    7 S2=S2+OMEGL(I,J)*FF(J)
    6 ETA(I)=ELL(I)-S1-S2
      CALL MATINV(B,4,ETA,1,DETERM,IPIVOT,INDEX,4,ISCALE)
      DO 12 I=1,4
   12 X(I,NMAX)=ETA(I)
      RETURN
      END
```

```
      SUBROUTINE STRESS(FREQ,NMAX,N,IND5)
C     SUBROUTINE STRESS-- THIS SUBROUTINE CALCULATES THE SECONDARY QUANTITIES
C     N XI, N XI THETA, Q XI, PHI, M THETA, M XI THETA, M XI THETA, AND
C     N THETA AT EACH STATION ALONG THE SHELL.
      COMMON R(502)/BL1/GAM(502),OMT(502),OMXI(502),DEOMX(502)/BL3/NU,
     1LAM,N,EALSIG,CHAR,DEL/BL7/PEE(4,4,502),X(4,502)/BL5/H(4,4),FF(4),
     2JAY(4,4)/BL13/AK(3,4),ALL(3,4),STHER(3)
      DIMENSION Z(4,502),Y(4),DZ(4)
      EQUIVALENCE                      (Z(1,1),X(1,1))
      INTEGER FREQ
      REAL N,NU,JAY,LAM
      KOUNT=0
      DO 9 I=1,NMAX
      IF((I.EQ.1).OR.(I.EQ.NMAX)) GO TO 1
      IF(I-2) 13,13,14
   14 KOUNT=KOUNT+1
      IF(KOUNT-FREQ)9,12,12
   12 KOUNT=0
   13 DO 3 L=1,4
      Y(L)=Z(L,I)
    3 DZ(L)=(Z(L,I+1)-Z(L,I-1))/2./DEL
      GO TO 2
    1 IF(I-1)4,4,5
    4 K=2
      GO TO 6
    5 K=NMAX
    6 DO 11 L=1,4
      Y(L)=.5*(Z(L,K)+Z(L,K-1))
   11 DZ(L)=(Z(L,K)-Z(L,K-1))/DEL
    2 CALL HFJ(I,IND5,NMAX,1.)
      CALL KLT(I,IND5,NMAX)
      DO 7 L=1,4
      SUM1=0.
      SUM2=0.
      DO8M=1,4
      SUM1=SUM1+ H(L,M)*DZ(M)
    8 SUM2=SUM2+JAY(L,M)*Y(M)
    7 PEE(L,2,I)=SUM1+SUM2+FF(L)
      DO 9 L=1,3
      SUM3=0.
      SUM4=0.
      DO 10 M=1,4
      SUM3=SUM3+AK(L,M)*DZ(M)
   10 SUM4=SUM4+ALL(L,M)*Y(M)
      PEE(L,1,I)=SUM3+SUM4+STHER(L)
    9 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE KLT(K,IND5,NMAX)
C SUBROUTINE KLT    THIS SUBROUTINE CALCULATES THE ELEMENTS OF THE MATRICES
C WHICH ALLOW THE CALCULATION OF THE QUANTITIES M-THETA,N-THETA,AND M-XI THETA
C AT THE STATION SPECIFIED BY THE INDEX K.
      COMMON R(502)/BL1/GAM(502),OMT(502),OMXI(502),DEOMX(502)/BL3/NU,
     1LAM,N,EALSIG,CHAR,DEL/BL13/AK(3,4),ALL(3,4),STHER(3)
      REAL NU,LAM,N
      CALL BDB(K,DEL,NU,B,DB,D,DD)
      D1=D*(1.-NU)
      D2=D*(1.-NU**2)
      OX=OMXI(K)
      EAL=EALSIG
      H=HHT(K,DEL)
      HRB=HRA(K,DEL)
      TEMPER=TEMP(K,DEL)
      DELTAT=DELT(K,DEL)
      STHER(1)=-CHAR*EAL*DELTAT *H**3*(1./(2.*HRB )-1./HRB**2
     1+2./(3.*HRB**3))
      STHER(2)=-2.*H*EAL*TEMPER /((1.-NU)*HRB)
      STHER(3)=0.
      IF(IND5)1,1,2
    2 IF(((IND5-2).LE.0).AND.(K.EQ.1)) GO TO 8
      IF(((IND5-2).GT.0).AND.(K.EQ.NMAX)) GO TO 8
    1 RA=R(K)
      GA=GAM(K)
      OT=OMT(K)
      REX=3.*OT-OX
      RXE=3.*OX-OT
      RAN=N/RA
      RAN2=RAN**2
      AK(1,1)=0.
      AK(1,2)=0.
      AK(1,3)=-GA*D2
      AK(1,4)=0.
      AK(2,1)=B*NU
      AK(2,2)=0.
      AK(2,3)=0.
      AK(2,4)=0.
      AK(3,1)=0.
      AK(3,2)=D1*REX/4.
      AK(3,3)=RAN*D1
      AK(3,4)=0.
```

```
      ALL(1,1)=GA*OX*D2
      ALL(1,2)=D2*RAN*OT
      ALL(1,3)=D2*RAN2
      ALL(1,4)=NU
      ALL(2,1)=B*GA
      ALL(2,2)=B*RAN
      ALL(2,3)=B*(OT+NU*OX)
      ALL(2,4)=0.
      ALL(3,1)=-D1*RAN*RXE/4.
      ALL(3,2)=-GA*D1*REX/4.
      ALL(3,3)=-GA*RAN*D1
      ALL(3,4)=0.
      GO TO 6
    8 DO 3 I=1,3
      DO 3 J=1,4
      AK(I,J)=0.
    3 ALL(I,J)=0.
      C1=(N**2/2.-1.)/(1.+NU-N**2*NU/2.)
      AK(1,1)=D2*OX*((1.+NU)*C1+1.)
      AK(1,2)=D2*OX*N*(NU*C1+1.)
      ALL(1,4)=(NU-(1.-NU**2)*C1)
      AK(2,1)=B*(1.+NU)
      AK(2,2)=B*N
      ALL(2,3)=B*(1.+NU)*OX
      C2=2.+2.*NU-NU*N**2
      AK(3,1)=D1*N*(1./C2-OX/2.)
      ALL(3,4)=-N*(1.-NU)/C2
    6 RETURN
      END
```

```
      SUBROUTINE BDB(K,DEL,NU,B,DB,D,DD)
C     SUBROUTINE BDB-- THIS SUBROUTINE CALCULATES THE BENDING STIFFNESS
C     ,D, THE MEMBRANE STIFFNESS,B, AND THE DERIVATIVES OF D AND B,DD AND
C     DB, RESPECTIVELY, FOR A SHELL COMPOSED OF A CORE HAVING NO STIFFNESS
C     AND TWO SYMMETRICAL COVER PLATES
      REAL NU,N,LAM
      HRB=HRA(K,DEL)
      DHRB=DHRA(K,DEL)
      H=HHT(K,DEL)
      DH=DHHT(K,DEL)
      D2=1.-NU**2
      B=2.*H/(D2*HRB)
      D=H**3*(3./(2.*HRB)-3./HRB**2+2./HRB**3)/(D2*3.)
      DB=2.*DH/(D2*HRB)-    B*DHRB/HRB
      DD=3.*DH*D/H+H**3*DHRB/(D2*HRB**2)*(-.5+2./HRB-2./HRB**2)
      DD=DD/3.
      RETURN
      END
```

```
      SUBROUTINE POLE(N,DEL,A1,A2)
C SUBROUTINE POLE-  THIS SUBROUTINE CALCULATES THE FINITENESS CONDITIONS FOR
C A CLOSED SHELL
      DIMENSION A1(4,4),A2(4,4)
      REAL N
      DO 1 I=1,4
      DO 1 J=1,4
      A1(I,J)=0.
    1 A2(I,J)=0.
      IF(N.EQ.0.) GO TO 2
      IF((N.EQ.1.).OR.(N.EQ.-1.)) GO TO 3
      A1(1,1)=.5
      A1(2,2)=.5
      A2(1,1)=.5
      A2(2,2)=.5
      IF(N.NE.2.) GO TO 4
      A2(3,3)=1./DEL
      A1(3,3)=-1./DEL
      A1(4,4)=-1./DEL
      A2(4,4)=1./DEL
      RETURN
    4 A1(4,4)=.5
      A2(4,4)=.5
      A1(3,3)=.5
      A2(3,3)=.5
      RETURN
    2 A1(1,1)=.5
      A1(2,2)=.5
      A1(3,3)=-1./DEL
      A1(4,4)=-1./DEL
      A2(1,1)=.5
      A2(2,2)=.5
      A2(3,3)=1./DEL
      A2(4,4)=1./DEL
      RETURN
    3 A1(2,1)=.5
      A1(2,2)=.5
      A1(1,1)=-1./DEL
      A1(3,3)=.5
      A1(4,4)=.5
      A2(2,1)=.5
      A2(2,2)=.5
      A2(1,1)=1./DEL
      A2(3,3)=.5
      A2(4,4)=.5
      RETURN
      END
```

```
      SUBROUTINE OUTPUT(FREQ,NMAX,IND1,DEL,NO)
C SUBROUTINE OUTPUT      THIS SUBROUTINE CONTROLS PROGRAM PRINTING AND
C PUNCHING.GEOMETRIC DATA IS PRINTED IF IND1 IS NOT EQUAL TO ZERO.ANY
C OR ALL OF THE ELEVEN OUTPUT QUANTITIES CAN BE PUNCHED BY SUITABLE
C SPECIFICATION OF THE FIELDS OF THE NOL CARD
      COMMON R(502)/BL1/GAM(502),OMT(502),OMXI(502),DEOMX(502)/BL7/PEE(
     14,4,502),X(4,502)
      DIMENSION NOL(11),JJ(11),KK(11),ESS(502),YORD(502)
      EQUIVALENCE  (X(1,1),ESS(1)),(X(1,127),YORD(1))
      INTEGER FREQ
      IF(IND1.NE.0)GO TO 51
      WRITE(6,11)
   11 FORMAT(1H1,16H        R/RB     16H        Z/RB     16H        S/RB
    1  16H  OMEGA THETA   16H    OMEGA XI     16H   DEOMEGA XI    16H
    2 GAMMA       ///)
   12 FORMAT(1P7E16.8)
      ZED=0.
      S=0.
      DO 8 I=1,NMAX
      IF(I-1)8,8,9
    9 DEM=DEL
      IF((I.EQ.2).OR.(I.EQ.NMAX))DEM=.5*DEL
      S=S+DEM
      ARGU=DEM**2-(R(I)-R(I-1))**2
      IF(ARGU.LE.0.) GO TO 8
      ZED=SQRT(ARGU)+ZED
    8 WRITE(6,12)R(I),ZED,S,OMT(I),OMXI(I),DEOMX(I),GAM(I)
   51 WRITE (6,2)
    2 FORMAT(1H1,     1X5H  S  11H     N XI    11H  N THETA  11H     N XT
     111H     Q XI    11H     M XI    11H  M THETA  11H     M XT   11H    U X
     2I    11H  U THETA  11H       W     11H   PHI XI  ///)
    7 FORMAT(1XF6.3,1P11E11.4)
      KOUNT=0
      DO 33 I=1,NMAX
      IF((I.EQ.1).OR.(I.EQ.NMAX)) GO TO 103
      IF(I-2) 14,14,15
   15 KOUNT=KOUNT+1
      IF(KOUNT-FREQ) 33,16,16
   16 KOUNT=0
   14 PEE(4,3,I)=X(4,I)
      PEE(1,3,I)=X(1,I)
      PEE(2,3,I)=X(2,I)
      PEE(3,3,I)=X(3,I)
      GO TO 33
  103 IF(I-1) 104,104,105
  104 K=2
      J=1
      GO TO 106
  105 K=NMAX
      J=NMAX
  106 PEE(4,3,J)=.5*(X(4,K)+X(4,K-1))
      PEE(1,3,J)=.5*(X(1,K)+X(1,K-1))
      PEE(2,3,J)=.5*(X(2,K)+X(2,K-1))
      PEE(3,3,J)=.5*(X(3,K)+X(3,K-1))
   33 CONTINUE
      NOPTS=(NMAX-2)/FREQ+2
      SMAX=DEL*FLOAT(NMAX-2)
      WRITE(6,53)SMAX
   53 FORMAT(9H SMAX/RB=1PE16.7)
      ESS(1)=0.
      DO 20 I=2,NMAX
      DEM=DEL
      IF((I.EQ.2).OR.(I.EQ.NMAX)) DEM=DEL/2.
```

```
   20 ESS(I)= ESS(I-1)+DEM /SMAX
      KOUNT=0
      DO 3 I=1,NMAX
      IF((I.EQ.1).OR.(I.EQ.2).OR.(I.EQ.NMAX)) GO TO 17
      KOUNT=KOUNT+1
      IF(KOUNT-FREQ) 3,18,18
   18 KOUNT=0
   17 WRITE(6,7)ESS(I),PEE(1,2,I),PEE(2,1,I),PEE(2,2,I),PEE(3,2,I),
     1PEE(4,3,I),PEE(1,1,I),PEE(3,1,I),PEE(1,3,I),PEE(2,3,I),PEE(3,3,I),
     2PEE(4,2,I)
    3 CONTINUE
      READ (5,29) (NOL(I),I=1,11)
   29 FORMAT (11I4)
      M=0
      DO 30 I=1,11
   30 M=NOL(I)+M
      IF(M.FQ.0) GO TO 13
      JJ(1) =1
      JJ(2) =2
      JJ(3) =2
      JJ(4) =3
      JJ(5) =4
      JJ(6) =1
      JJ(7) =3
      JJ(8) =1
      JJ(9) =2
      JJ(10)=3
      JJ(11)=4
      KK(1) =2
      KK(2) =1
      KK(3) =2
      KK(4) =2
      KK(5) =3
      KK(6) =1
      KK(7) =1
      KK(8) =3
      KK(9) =3
      KK(10)=3
      KK(11)=2
      DO 50 L=1,11
      IF (NOL(L).EQ.1) GO TO 31
      GO TO 50
   31 NOL(L)=0
      J=JJ(L)
      K=KK(L)
      KOUNT=0
      DO 19 I=1,NMAX
      M=I
      IF((I.EQ.1).OR.(I.EQ.2)) GO TO 19
      M=(NMAX-2)/FREQ+2
      IF(I.EQ.NMAX) GO TO 19
      KOUNT=KOUNT+1
      IF(KOUNT-FREQ)19,21,21
   21 KOUNT=0
      M=(I-2)/FREQ+2
   19 YORD(M)=PEE(J,K,I)
      N1=(NMAX-2)/FREQ+2
  101 FORMAT(5E12.5,4X2I4)
      DO 102 I=1,N1,5
  102 PUNCH 101,YORD(I),YORD(I+1),YORD(I+2),YORD( I+3),YORD(I+4),NO,L
   50 CONTINUE
   13 RETURN
      END
```

```
C      MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS
C
       SUBROUTINE MATINV(A,N,B,M,DETERM,IPIVOT,INDEX,NMAX,ISCALE)
C
       DIMENSION IPIVOT(N),A(NMAX,N),B(NMAX,M),INDEX(NMAX,2)
       EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM), (AMAX, T, SWAP)
C
C      INITIALIZATION
C
     5 ISCALE=0
     6 R1=10.0**18
     7 R2=1.0/R1
    10 DETERM=1.0
    15 DO 20 J=1,N
    20 IPIVOT(J)=0
    30 DO 550 I=1,N
C
C      SEARCH FOR PIVOT ELEMENT
C
    40 AMAX=0.0
    45 DO 105 J=1,N
    50 IF (IPIVOT(J)-1) 60, 105, 60
    60 DO 100 K=1,N
    70 IF (IPIVOT(K)-1) 80, 100, 740
    80 IF (ABS(AMAX)-ABS(A(J,K)))85,100,100
    85 IROW=J
    90 ICOLUM=K
    95 AMAX=A(J,K)
   100 CONTINUE
   105 CONTINUE
       IF (AMAX) 110,106,110
   106 DETERM=0.0
       ISCALE=0
       GO TO 740
   110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C
C      INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
   130 IF (IROW-ICOLUM) 140, 260, 140
   140 DETERM=-DETERM
   150 DO 200 L=1,N
   160 SWAP=A(IROW,L)
   170 A(IROW,L)=A(ICOLUM,L)
   200 A(ICOLUM,L)=SWAP
   205 IF(M) 260, 260, 210
   210 DO 250 L=1, M
   220 SWAP=B(IROW,L)
   230 B(IROW,L)=B(ICOLUM,L)
   250 B(ICOLUM,L)=SWAP
   260 INDEX(I,1)=IROW
   270 INDEX(I,2)=ICOLUM
   310 PIVOT=A(ICOLUM,ICOLUM)
C
C      SCALE THE DETERMINANT
C
  1000 PIVOTI=PIVOT
  1005 IF(ABS(DETERM)-R1)1030,1010,1010
  1010 DETERM=DETERM/R1
       ISCALE=ISCALE+1
       IF(ABS(DETERM)-R1)1060,1020,1020
```

```
 1020 DETERM=DETERM/R1
      ISCALE=ISCALE+1
      GO TO 1060
 1030 IF(ABS(DETERM)-R2)1040,1040,1060
 1040 DETERM=DETERM*R1
      ISCALE=ISCALE-1
      IF(ABS(DETERM)-R2)1050,1050,1060
 1050 DETERM=DETERM*R1
      ISCALE=ISCALE-1
 1060 IF(ABS(PIVOTI)-R1)1090,1070,1070
 1070 PIVOTI=PIVOTI/R1
      ISCALE=ISCALE+1
      IF(ABS(PIVOTI)-R1)320,1080,1080
 1080 PIVOTI=PIVOTI/R1
      ISCALE=ISCALE+1
      GO TO 320
 1090 IF(ABS(PIVOTI)-R2)2000,2000,320
 2000 PIVOTI=PIVOTI*R1
      ISCALE=ISCALE-1
      IF(ABS(PIVOTI)-R2)2010,2010,320
 2010 PIVOTI=PIVOTI*R1
      ISCALE=ISCALE-1
  320 DETERM=DETERM*PIVOTI
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
  330 A(ICOLUM,ICOLUM)=1.0
  340 DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT
  355 IF(M) 380, 380, 360
  360 DO 370 L=1,M
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT
C
C     REDUCE NON-PIVOT ROWS
C
  380 DO 550 L1=1,N
  390 IF(L1-ICOLUM) 400, 550, 400
  400 T=A(L1,ICOLUM)
  420 A(L1,ICOLUM)=0.0
  430 DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
  455 IF(M) 550, 550, 460
  460 DO 500 L=1,M
  500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
  550 CONTINUE
C
C     INTERCHANGE COLUMNS
C
  600 DO 710 I=1,N
  610 L=N+1-I
  620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
  630 JROW=INDEX(L,1)
  640 JCOLUM=INDEX(L,2)
  650 DO 705 K=1,N
  660 SWAP=A(K,JROW)
  670 A(K,JROW)=A(K,JCOLUM)
  700 A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
  740 RETURN
      END
```

APPENDIX C

MEMBRANE AND BENDING STIFFENESSES

The membrane and bending stiffnesses of the shell are defined by equations (A24) and (A25), respectively. The membrane and bending stiffnesses of a sandwich shell such as that shown in sectional view by figure 5 can be used in the program. For such a sandwich, shear deformations in the core are neglected as well as core compressibility through the thickness, yet the core is assumed to have no bending or membrane stiffness. The face sheets are equal and assumed to carry only membrane forces. These assumptions lead to the expression:

$$b = \frac{2Et}{E_o h_o \left(1 - \nu^2\right)} \qquad \text{(C1)}$$

Let

$$\left.\begin{aligned} \eta &= \frac{h}{h_o} \\ \delta &= \frac{h}{t} \end{aligned}\right\} \qquad \text{(C2)}$$

then equation (C1) becomes

$$b = \frac{2E}{E_o\left(1 - \nu^2\right)} \frac{\eta}{\delta} \qquad \text{(C3)}$$

similarly,

$$d = \frac{1}{3h_o^{3}\left(1 - \nu^2\right)}\left(\frac{3}{2} h^2 t - 3ht^2 + 2t^3\right)\frac{E}{E_o}$$

or

$$d = \frac{1}{3\left(1 - \nu^2\right)} \eta^3 \left(\frac{3}{2\delta} - \frac{3}{\delta^2} + \frac{2}{\delta^3}\right)\frac{E}{E_o} \qquad \text{(C4)}$$

The derivatives of b and d are also required and can be obtained from equations (C3) and (C4)

$$b' = \frac{2E}{E_o\left(1 - \nu^2\right)\delta}\left(\eta' - \frac{\eta}{\delta}\,\delta'\right) \qquad \text{(C5)}$$

$$d' = \frac{3}{\eta}\,\eta' d + \frac{\eta^3 \delta'}{\left(1 - \nu^2\right)\delta^2}\left(-\frac{1}{2} + \frac{2}{\delta} - \frac{2}{\delta^2}\right)\frac{E}{E_o} \qquad \text{(C6)}$$

Note that for the isotropic case b and d reduce to the well-known relations

$$b = \frac{1}{\left(1 - \nu^2\right)} \frac{h}{h_o} \frac{E}{E_o} \tag{C7}$$

$$d = \frac{1}{12\left(1 - \nu^2\right)} \frac{E}{E_o} \left(\frac{h}{h_o}\right)^3 \tag{C8}$$

APPENDIX D

THERMAL FORCES AND MOMENTS

Using the definition of thermal force and moment given by equations (A26) and (A27) with the assumed form of the temperature distribution equation (31) gives the following equations:

$$t_T^{(n)} = \frac{2E\alpha T_1^{(n)}}{\sigma_o(1 - \nu)} \frac{\eta}{\delta} \tag{D1}$$

$$m_T^{(n)} = \frac{aE\alpha\Delta T_1}{3\sigma_o(1 - \nu)} \frac{\eta^3}{\delta}\left(\frac{3}{2} - \frac{3}{\delta} + \frac{2}{\delta^2}\right) \tag{D2}$$

Differentiating equations (D1) and (D2) gives

$$t_T^{(n)'} = \frac{2E\alpha}{\sigma_o(1 - \nu)\delta}\left(\eta T_1' + T_1\eta'\right) - \frac{\delta'}{\delta} t_T^{(n)} \tag{D3}$$

$$m_T^{(n)'} = \frac{aE\alpha}{3\sigma_o(1 - \nu)}\left[\left(\frac{3}{2\delta} - \frac{3}{\delta^2} + \frac{2}{\delta^3}\right)\left(\Delta T_1'\eta^3 + 3\eta^2\eta'\Delta T_1\right) + \Delta T_1 \frac{\eta^3}{\delta^2}\left(-\frac{3}{2} + \frac{6}{\delta} - \frac{6}{\delta^2}\right)\delta'\right] \tag{D4}$$

APPENDIX E

OUTPUT LISTING FOR CLOSED SPHERICAL SHELL SEGMENT

The following is the output list for the problem of a spherical segment subjected to a hydrostatic pressure and having a horizontal elastic boundary restraint. The problem is described in the text in the section entitled "Example Problems."

PROBLEM NUMBER 1

SPHERICAL CAP WITH HORIZONTAL EDGE RESTRAINT SUBJECT TO HYDRO PRESSURE

INPUT DATA

REFERENCE LENGTH = 1.0000000E 03

POISSONS RATIO = 3.0000000E-01

TEMP COEFFICIENT = 0.0000000E-39

REF. THICKNESS = 1.0000000E 00

FOURIER INDEX = 0.0000000E-39

NUMBER OF STATIONS= 52

THE INDICATORS ARE SET AS FOLLOWS

IND1= 0 IND2= 0 IND3= 1 IND4= 1 IND5= 1

THE BOUNDARY CONDITIONS AT S=0 ARE AS FOLLOWS

THE CONDITIONS FOR A SHELL POLE HAVE BEEN GENERATED

THE BOUNDARY CONDITIONS AT S=SMAX ARE AS FOLLOWS

OMEGA				LAMDA				ELL
-0.0000E-39	-0.0000E-39	-0.0000E-39	-0.0000E-39	-5.0000E-01	-0.0000E-39	8.6603E-01	-0.0000E-39	-0.0000E-39
-0.0000E-39	-0.0000E-39	-0.0000E-39	-0.0000E-39	0.0000E-39	1.0000E 00	0.0000E-39	-0.0000E-39	-0.0000E-39
8.6603E-01	-0.0000E-39	5.0000E-01	-0.0000E-39	8.6603E-02	-0.0000E-39	5.0000E-02	-0.0000E-39	-0.0000E-39
-0.0000E-39	-0.0000E-39	-0.0000E-39	-0.0000E-39	-0.0000E-39	-0.0000E-39	-0.0000E-39	1.0000E 00	-0.0000E-39

R/RB	Z/RB	S/RB	OMEGA THETA	OMEGA XI	DEOMEGA XI	GAMMA
0.00000000E-39	0.00000000E-39	0.00000000E-39	1.00000000E 00	1.00000000E 00	0.00000000E-39	0.00000000E-39
5.23596370E-03	1.58508020E-05	5.23598770E-03	1.00000000E 00	1.00000000E 00	0.00000000E-39	1.90984190E 02
1.57073170E-02	1.29997290E-04	1.57079630E-02	1.00000000E 00	1.00000000E 00	0.00000000E-39	6.36567420E 01
2.61769480E-02	3.51582440E-04	2.61799390E-02	1.00000000E 00	1.00000000E 00	0.00000000E-39	3.81884600E 01
3.66437080E-02	6.82035750E-04	3.66519140E-02	1.00000000E 00	1.00000000E 00	0.00000000E-39	2.72714870E 01
4.71064500E-02	1.12168680E-03	4.71238890E-02	1.00000000E 00	1.00000000E 00	0.00000000E-39	2.12049490E 01
5.75640270E-02	1.67065710E-03	5.75958650E-02	1.00000000E 00	1.00000000E 00	0.00000000E-39	1.73431540E 01
6.80152890E-02	2.32896940E-03	6.80678400E-02	1.00000000E 00	1.00000000E 00	0.00000000E-39	1.46685290E 01
7.84590950E-02	3.09655760E-03	7.85398160E-02	1.00000000E 00	1.00000000E 00	0.00000000E-39	1.27062050E 01
8.88942960E-02	3.97340650E-03	8.90117910E-02	1.00000000E 00	1.00000000E 00	0.00000000E-39	1.12047800E 01
9.93197490E-02	4.95940750E-03	9.94837670E-02	1.00000000E 00	1.00000000E 00	0.00000000E-39	1.00187080E 01
1.09734310E-01	6.05447100E-03	1.09955740E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	9.05788660E 00
1.20136840E-01	7.25850890E-03	1.20427720E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	8.26355480E 00
1.30526190E-01	8.57137240E-03	1.30899690E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	7.59575420E 00
1.40901230E-01	9.99292840E-03	1.41371670E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	7.02636630E 00
1.51260820E-01	1.15230340E-02	1.51843640E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	6.53502940E 00
1.61603820E-01	1.31615020E-02	1.62315620E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	6.10663600E 00
1.71929100E-01	1.49081790E-02	1.72787590E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	5.72974170E 00
1.82235520E-01	1.67628660E-02	1.83259570E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	5.39551720E 00
1.92521970E-01	1.87253640E-02	1.93731540E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	5.09704260E 00
2.02787290E-01	2.07954630E-02	2.04203520E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	4.82881740E 00
2.13030380E-01	2.29729340E-02	2.14675500E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	4.58641420E 00
2.23250110E-01	2.52575300E-02	2.25147470E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	4.36622930E 00
2.33445360E-01	2.76490130E-02	2.35619450E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	4.16529980E 00
2.43615010E-01	3.01471200E-02	2.46091420E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	3.98116690E 00
2.53757940E-01	3.27515710E-02	2.56563400E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	3.81177330E 00
2.63873050E-01	3.54620880E-02	2.67035370E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	3.65538440E 00
2.73959220E-01	3.82783790E-02	2.77507340E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	3.51052730E 00
2.84015340E-01	4.12001210E-02	2.87979320E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	3.37594350E 00
2.94040320E-01	4.42270220E-02	2.98451290E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	3.25055090E 00
3.04033060E-01	4.73586980E-02	3.08923260E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	3.13341410E 00
3.13992450E-01	5.05948640E-02	3.19395240E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	3.02372070E 00
3.23917410E-01	5.39351300E-02	3.29867210E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	2.92076100E 00
3.33806850E-01	5.73791490E-02	3.40339180E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	2.82391290E 00
3.43659690E-01	6.09265230E-02	3.50811160E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	2.73262840E 00
3.53474840E-01	6.45768910E-02	3.61283130E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	2.64642330E 00
3.63251230E-01	6.83298160E-02	3.71755100E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	2.56486740E 00
3.72987780E-01	7.21849260E-02	3.82227080E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	2.48757820E 00
3.82683430E-01	7.61417650E-02	3.92699050E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	2.41421360E 00
3.92337110E-01	8.01999340E-02	4.03171030E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	2.34446720E 00
4.01947770E-01	8.43589570E-02	4.13643000E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	2.27806360E 00
4.11514350E-01	8.86183880E-02	4.24114970E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	2.21475450E 00
4.21035810E-01	9.29777650E-02	4.34586950E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	2.15431560E 00
4.30511090E-01	9.74366070E-02	4.45058920E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	2.09654360E 00
4.39939160E-01	1.01994430E-01	4.55530890E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	2.04125400E 00
4.49318990E-01	1.06650720E-01	4.66002870E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	1.98827870E 00
4.58649550E-01	1.11404980E-01	4.76474840E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	1.93746450E 00
4.67929810E-01	1.16256690E-01	4.86946810E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	1.88867140E 00
4.77158760E-01	1.21205300E-01	4.97418790E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	1.84177090E 00
4.86335370E-01	1.26250300E-01	5.07890760E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	1.79664540E 00
4.95458660E-01	1.31391110E-01	5.18362730E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	1.75318670E 00
4.99999990E-01	1.33997220E-01	5.23598720E-01	1.00000000E 00	1.00000000E 00	0.00000000E-39	1.73205080E 00

S	N XI	N THETA	N XT	Q XI	M XI	M THETA	M XT	U XI	U THETA	W	PHI XI
SMAX/RB= 5.2359877E-01											
0.000	-5.0000E-01	-5.0000E-01	0.0000E-39	-0.0000E-39	1.9635E-02	1.9635E-02	0.0000E-39	0.0000E-39	-0.0000E-39	-5.2879E 00	0.0000E-39
0.010	-5.0011E-01	-5.0008E-01	-0.0000E-39	-3.6883E-07	1.9635E-02	1.9696E-02	-0.0000E-39	2.5855E-02	-0.0000E-39	-5.2879E 00	8.6744E-04
0.030	-5.0003E-01	-5.0001E-01	-0.0000E-39	-8.5912E-07	1.2155E-02	1.5403E-02	-0.0000E-39	7.7562E-02	0.0000E-39	-5.2873E 00	2.2163E-03
0.050	-5.0001E-01	-5.0000E-01	-0.0000E-39	-6.7961E-07	5.9719E-03	1.0938E-02	-0.0000E-39	1.2926E-01	0.0000E-39	-5.2863E 00	2.8742E-03
0.070	-5.0000E-01	-5.0000E-01	-0.0000E-39	-4.5585E-07	1.8935E-03	7.4838E-03	-0.0000E-39	1.8095E-01	-0.0000E-39	-5.2847E 00	3.0431E-03
0.090	-4.9999E-01	-5.0000E-01	-0.0000E-39	-2.7285E-07	-3.8242E-04	5.0591E-03	-0.0000E-39	2.3261E-01	-0.0000E-39	-5.2825E 00	2.9279E-03
0.110	-4.9999E-01	-5.0001E-01	-0.0000E-39	-1.4675E-07	-1.4043E-03	3.4513E-03	-0.0000E-39	2.8425E-01	-0.0000E-39	-5.2799E 00	2.6795E-03
0.130	-4.9998E-01	-5.0001E-01	-0.0000E-39	-6.8667E-08	-1.6923E-03	2.4263E-03	-0.0000E-39	3.3586E-01	-0.0000E-39	-5.2767E 00	2.4002E-03
0.150	-4.9998E-01	-5.0002E-01	0.0000E-39	-2.2707E-08	-1.5771E-03	1.7912E-03	0.0000E-39	3.8744E-01	-0.0000E-39	-5.2729E 00	2.1385E-03
0.170	-4.9998E-01	-5.0002E-01	0.0000E-39	1.2151E-09	-1.2715E-03	1.4140E-03	0.0000E-39	4.3897E-01	-0.0000E-39	-5.2686E 00	1.9228E-03
0.190	-4.9998E-01	-5.0002E-01	0.0000E-39	7.7308E-09	-9.2149E-04	1.1872E-03	0.0000E-39	4.9045E-01	-0.0000E-39	-5.2637E 00	1.7531E-03
0.210	-4.9998E-01	-5.0002E-01	0.0000E-39	4.9003E-09	-6.6710E-04	1.0277E-03	0.0000E-39	5.4188E-01	-0.0000E-39	-5.2583E 00	1.6266E-03
0.230	-4.9997E-01	-5.0003E-01	0.0000E-39	-5.3300E-09	-4.9734E-04	9.1359E-04	0.0000E-39	5.9325E-01	-0.0000E-39	-5.2524E 00	1.5434E-03
0.250	-4.9997E-01	-5.0003E-01	-0.0000E-39	-1.9341E-08	-5.3453E-04	7.6781E-04	-0.0000E-39	6.4456E-01	-0.0000E-39	-5.2460E 00	1.4664E-03
0.270	-4.9997E-01	-5.0003E-01	-0.0000E-39	-2.5722E-08	-6.9523E-04	5.9698E-04	-0.0000E-39	6.9580E-01	-0.0000E-39	-5.2390E 00	1.3758E-03
0.290	-4.9997E-01	-5.0003E-01	-0.0000E-39	-2.3194E-08	-8.8310E-04	4.2761E-04	-0.0000E-39	7.4695E-01	-0.0000E-39	-5.2314E 00	1.2717E-03
0.310	-4.9997E-01	-5.0003E-01	-0.0000E-39	-1.0825E-08	-1.0016E-03	2.8829E-04	-0.0000E-39	7.9803E-01	-0.0000E-39	-5.2233E 00	1.1570E-03
0.330	-4.9997E-01	-5.0003E-01	0.0000E-39	1.0299E-08	-9.4485E-04	2.1598E-04	0.0000E-39	8.4902E-01	-0.0000E-39	-5.2147E 00	1.0460E-03
0.350	-4.9996E-01	-5.0003E-01	0.0000E-39	3.6780E-08	-6.4659E-04	2.3962E-04	0.0000E-39	8.9992E-01	-0.0000E-39	-5.2056E 00	9.6437E-04
0.370	-4.9996E-01	-5.0003E-01	0.0000E-39	8.6276E-08	-7.4387E-05	3.7242E-04	0.0000E-39	9.5072E-01	-0.0000E-39	-5.1959E 00	9.2933E-04
0.390	-4.9996E-01	-5.0003E-01	0.0000E-39	1.6799E-07	1.2081E-03	7.6398E-04	0.0000E-39	1.0014E 00	-0.0000E-39	-5.1857E 00	9.9790E-04
0.410	-4.9995E-01	-5.0003E-01	0.0000E-39	2.7539E-07	3.3991E-03	1.5018E-03	0.0000E-39	1.0520E 00	-0.0000E-39	-5.1749E 00	1.2613E-03
0.430	-4.9995E-01	-5.0003E-01	0.0000E-39	4.0124E-07	6.7935E-03	2.7062E-03	0.0000E-39	1.1025E 00	-0.0000E-39	-5.1637E 00	1.8364E-03
0.450	-4.9994E-01	-5.0005E-01	0.0000E-39	5.0389E-07	1.1429E-02	4.4244E-03	0.0000E-39	1.1528E 00	-0.0000E-39	-5.1519E 00	2.8686E-03
0.470	-4.9994E-01	-5.0007E-01	0.0000E-39	4.8082E-07	1.6736E-02	6.4969E-03	0.0000E-39	1.2030E 00	-0.0000E-39	-5.1396E 00	4.4493E-03
0.490	-4.9993E-01	-5.0011E-01	0.0000E-39	1.4090E-07	2.0645E-02	8.2689E-03	0.0000E-39	1.2531E 00	-0.0000E-39	-5.1268E 00	6.5333E-03
0.510	-4.9994E-01	-5.0018E-01	-0.0000E-39	-7.7039E-07	1.8699E-02	8.2632E-03	-0.0000E-39	1.3031E 00	-0.0000E-39	-5.1135E 00	8.7113E-03
0.530	-4.9994E-01	-5.0027E-01	-0.0000E-39	-2.4711E-06	3.7115E-03	4.0103E-03	-0.0000E-39	1.3529E 00	-0.0000E-39	-5.0997E 00	9.9024E-03
0.550	-4.9996E-01	-5.0036E-01	-0.0000E-39	-5.1343E-06	-3.3035E-02	-7.6211E-03	-0.0000E-39	1.4026E 00	-0.0000E-39	-5.0854E 00	8.1375E-03
0.570	-4.9999E-01	-5.0041E-01	-0.0000E-39	-8.7291E-06	-1.0202E-01	-3.0505E-02	-0.0000E-39	1.4521E 00	-0.0000E-39	-5.0705E 00	3.7716E-04
0.590	-5.0004E-01	-5.0035E-01	-0.0000E-39	-1.2598E-05	-2.1099E-01	-6.7849E-02	-0.0000E-39	1.5014E 00	-0.0000E-39	-5.0549E 00	-1.7435E-02
0.610	-5.0010E-01	-5.0004E-01	-0.0000E-39	-1.5010E-05	-3.5648E-01	-1.1943E-01	-0.0000E-39	1.5506E 00	-0.0000E-39	-5.0387E 00	-4.9564E-02
0.630	-5.0014E-01	-4.9934E-01	-0.0000E-39	-1.2651E-05	-5.1033E-01	-1.7706E-01	-0.0000E-39	1.5996E 00	-0.0000E-39	-5.0214E 00	-9.8425E-02
0.650	-5.0012E-01	-4.9805E-01	0.0000E-39	-2.1785E-07	-6.0105E-01	-2.1815E-01	0.0000E-39	1.6485E 00	-0.0000E-39	-5.0031E 00	-1.6078E-01
0.670	-4.9998E-01	-4.9607E-01	0.0000E-39	2.9503E-05	-4.9225E-01	-1.9814E-01	0.0000E-39	1.6971E 00	-0.0000E-39	-4.9835E 00	-2.2159E-01
0.690	-4.9960E-01	-4.9353E-01	0.0000E-39	8.4371E-05	3.3679E-02	-4.4108E-02	0.0000E-39	1.7455E 00	-0.0000E-39	-4.9629E 00	-2.4582E-01
0.710	-4.9888E-01	-4.9102E-01	0.0000E-39	1.6956E-04	1.2705E 00	3.4491E-01	0.0000E-39	1.7937E 00	-0.0000E-39	-4.9418E 00	-1.6955E-01
0.730	-4.9771E-01	-4.8995E-01	0.0000E-39	2.8097E-04	3.5352E 00	1.0824E 00	0.0000E-39	1.8416E 00	-0.0000E-39	-4.9218E 00	1.0550E-01
0.750	-4.9609E-01	-4.9292E-01	0.0000E-39	3.9447E-04	7.0273E 00	2.2503E 00	0.0000E-39	1.8894E 00	-0.0000E-39	-4.9055E 00	7.0634E-01
0.770	-4.9428E-01	-5.0393E-01	0.0000E-39	4.5118E-04	1.1555E 01	3.8104E 00	0.0000E-39	1.9371E 00	-0.0000E-39	-4.8970E 00	1.7597E 00
0.790	-4.9295E-01	-5.2823E-01	0.0000E-39	3.4093E-04	1.6096E 01	5.4596E 00	0.0000E-39	1.9848E 00	-0.0000E-39	-4.9018E 00	3.3223E 00
0.810	-4.9346E-01	-5.7110E-01	-0.0000E-39	-1.0912E-04	1.8188E 01	6.4259E 00	-0.0000E-39	2.0327E 00	-0.0000E-39	-4.9250E 00	5.2524E 00
0.830	-4.9807E-01	-6.3512E-01	-0.0000E-39	-1.1280E-03	1.3265E 01	5.2378E 00	-0.0000E-39	2.0812E 00	-0.0000E-39	-4.9692E 00	7.0088E 00
0.850	-5.0999E-01	-7.1486E-01	-0.0000E-39	-2.9543E-03	-5.7989E 00	-4.4895E-01	-0.0000E-39	2.1303E 00	-0.0000E-39	-5.0282E 00	7.3877E 00
0.870	-5.3296E-01	-7.8867E-01	-0.0000E-39	-5.7190E-03	-4.8376E 01	-1.3789E 01	-0.0000E-39	2.1803E 00	-0.0000E-39	-5.0793E 00	4.2523E 00
0.890	-5.7000E-01	-8.0773E-01	-0.0000E-39	-9.2224E-03	-1.2410E 02	-3.8159E 01	-0.0000E-39	2.2305E 00	-0.0000E-39	-5.0716E 00	-5.6051E 00
0.910	-6.2075E-01	-6.8469E-01	-0.0000E-39	-1.2579E-02	-2.3795E 02	-7.5616E 01	-0.0000E-39	2.2796E 00	-0.0000E-39	-4.9152E 00	-2.6208E 01
0.930	-6.7703E-01	-2.8737E-01	-0.0000E-39	-1.3741E-02	-3.8097E 02	-1.2394E 02	-0.0000E-39	2.3248E 00	-0.0000E-39	-4.4750E 00	-6.1333E 01
0.950	-7.1624E-01	5.5159E-01	-0.0000E-39	-8.9860E-03	-5.1558E 02	-1.7188E 02	-0.0000E-39	2.3610E 00	-0.0000E-39	-3.5820E 00	-1.1209E 02
0.970	-6.9351E-01	1.9923E 00	0.0000E-39	7.3936E-03	-5.5591E 02	-1.9261E 02	0.0000E-39	2.3813E 00	-0.0000E-39	-2.0780E 00	-1.7254E 02
0.990	-5.3477E-01	4.0893E 00	0.0000E-39	4.2745E-02	-3.4706E 02	-1.3671E 02	0.0000E-39	2.3775E 00	-0.0000E-39	8.1551E-02	-2.2306E 02
1.000	-3.5297E-01	5.3445E 00	0.0000E-39	6.6343E-02	7.6294E-06	-3.4973E 01	0.0000E-39	2.3601E 00	0.0000E-39	1.3626E 00	-2.4230E 02

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2. Kalnins, A.: Analysis of Shells of Revolution Subjected to Symmetrical and Nonsymmetrical Loads. Trans. ASME, Ser. E: J. Appl. Mech., vol. 31, no. 3, Sept. 1964, pp. 467-476.
3. Sanders, J. Lyell, Jr.: An Improved First-Approximation Theory for Thin Shells. NASA TR R-24, 1959.
4. Greenbaum, Gilbert A.: Comments on "Numerical Analysis of Unsymmetrical Bending of Shells of Revolution." AIAA J. (Tech. Notes), vol. 2, no. 3, Mar. 1964, pp. 590-591.
5. Budiansky, Bernard; and Radkowski, Peter P.: Reply by Authors to G. A. Greenbaum. AIAA J. (Tech. Notes), vol. 2, no. 3, Mar 1964, p. 592.

TABLE 1.- DESCRIPTION OF SUBROUTINES

FORTRAN name	Description
HFJ	Calculates the elements of the H, J, and f matrices
FORCE	Calculates the forcing function due to surface loads and/or temperature distribution
ABCG	Calculates the elements of the A, B, C, and g matrices
INIT	Calculates the initial P and x matrices
EFG	Calculates the elements of the E, F, and G matrices
PANDX	Calculates the P and x matrices at each station
FINAL	Calculates the vector z associated with station NMAX
EQ73	Calculates the solution vector z at each shell station
STRESS	Calculates explicitly the moments, forces, and displacements at each shell station
OUTPUT	Controls listing of calculated data
MATINV	Performs matrix inversion and matrix multiplication
KLT	Calculates the elements of the matrices necessary for calculation of the output quantities m_θ, t_θ, and $m_{\xi\theta}$
BDB	Calculates the membrane and bending stiffnesses at each shell station
POLE	Sets up the finiteness conditions at a pole in the shell geometry

TABLE 2.- GLOSSARY OF FORTRAN VARIABLES

Variable	Subroutine where variable is calculated or defined	Description
A(4,4)	ABCG	Matrix of coefficients A_i as defined by equations (20)
B	BDB	Membrane stiffness
B(4,4)	ABCG	Matrix of coefficients B_i as defined by equations (20)
C(4,4)	ABCG	Matrix of coefficients C_i as defined by equations (20)
CAPL1(4,4)	SHELLS	Matrix $\left[\Lambda\right]_1$, see equation (15)
CAPLL(4,4)	SHELLS	Matrix $\left[\Lambda\right]_N$, see equation (15)
CEE(4)	FORCE	Load vector e_i as defined by equation (13)
CHAR	SHELLS	a, characteristic shell dimension
CONST(100)	SHELLS	Unassigned storage
D	BDB	Bending stiffness
DB	BDB	Derivative of membrane stiffness
DD	BDB	Derivative of bending stiffness
DEL	SHELLS	$\Delta\xi$, arc length increment
DEOMX(502)	INPUT	$d\omega_\xi/d\xi$
DH	BDB	η'
DHRB	BDB	δ'
E(4,4)	EFG	Matrix $\left[E\right]_i$, see equation (13)
EALSIG	SHELLS	$E\alpha/\sigma_o$, see equation (D1)
EL1(4)	SHELLS	Vector $\left\{l\right\}_1$, see equation (15)
ELL(4)	SHELLS	Vector $\left\{l\right\}_N$, see equation (15)
F(4,4)	EFG	Matrix $\left[F\right]_i$, see equation (13)
FF(4)	HFJ	Vector $\left\{f\right\}$, see equation (21)
G(4,4)	EFG	Matrix $\left[G\right]_i$, see equation (13)

TABLE 2.- GLOSSARY OF FORTRAN VARIABLES – Concluded

Variable	Subroutine where variable is calculated or defined	Description
GAM(502)	INPUT	γ
H	BDB	η
H(4,4)	HFJ	Matrix $[H]_i$, see equation (21)
HRB	BDB	δ
IND1	SHELLS	Logical control for output of geometrical data
IND2	SHELLS	Logical control of data into CONST
IND3	SHELLS	Logical control for calculation of geometrical data
IND4	SHELLS	Logical control for boundary condition matrices
IND5	SHELLS	Logical control for closed shell
JAY(4,4)	HFJ	Matrix $[J]_i$, see equation (21)
LAM	SHELLS	λ
N	SHELLS	n (Fourier coefficient number)
NMAX	SHELLS	N, total number of shell stations
NU	SHELLS	ν
OMEG1(4,4)	SHELLS	Matrix $[\Omega]_1$, see equation (15)
OMEGL(4,4)	SHELLS	Matrix $[\Omega]_N$, see equation (15)
OMT(502)	INPUT	ω_θ
OMXI(502)	INPUT	ω_ξ
PEE(4,4,502)	INIT,PANDX	Matrix $[P]_i$, see equation (25)
R(502)	INPUT	ρ
SMAG(4)	ABCG	Vector $\{g\}_i$ defined by equation (20)
SMAX	OUTPUT	Total nondimensional arc length
X(4,502)	INIT,PANDX	Vector $\{x\}_i$, see equation (25)

TABLE 3.- COMMON LOCATION OF VARIABLES REQUIRED FOR USER-PREPARED SUBPROGRAMS

Variable name	COMMON location
CONST(100)	BL14
R(502)	Blank COMMON
GAM(502), OMT(502), OMXI(502), DEOMX(502)	BL1[a]
NU, LAM, N, EALSIG, CHAR, DEL	BL3[a]

[a]Variables in this COMMON block must be in order specified.

TABLE 4.- RELATIONSHIP BETWEEN NONDIMENSIONAL PROGRAM OUTPUT AND PHYSICAL QUANTITIES

Relationship	Storage location at output time (a)	Nondimensional output variable name
$t_{\xi}^{(n)} = \frac{N_{\xi}^{(n)}}{\sigma_o h_o}$	PEE(1,2,K)	N XI
$t_{\theta}^{(n)} = \frac{N_{\theta}^{(n)}}{\sigma_o h_o}$	PEE(2,1,K)	N THETA
$t_{\xi\theta}^{(n)} = \frac{\overline{N}_{\xi\theta}^{(n)}}{\sigma_o h_o}$	PEE(2,2,K)	N XT
$\hat{f}_{\xi}^{(n)} = \frac{Q_{\xi}^{(n)}}{\sigma_o h_o}$	PEE(3,2,K)	Q XI
$m_{\xi}^{(n)} = M_{\xi}^{(n)} \frac{a}{\sigma_o h_o{}^3}$	PEE(4,3,K)	M XI
$m_{\theta}^{(n)} = M_{\theta}^{(n)} \frac{a}{\sigma_o h_o{}^3}$	PEE(1,1,K)	M THETA
$m_{\xi\theta}^{(n)} = \overline{M}_{\xi\theta}^{(n)} \frac{a}{\sigma_o h_o{}^3}$	PEE(3,1,K)	M XT
$u_{\xi}^{(n)} = U_{\xi}^{(n)} \frac{E_o}{a\sigma_o}$	PEE(1,3,K)	U XI
$u_{\theta}^{(n)} = U_{\theta}^{(n)} \frac{E_o}{a\sigma_o}$	PEE(2,3,K)	U THETA
$w^{(n)} = W^{(n)} \frac{E_o}{a\sigma_o}$	PEE(3,3,K)	W
$\varphi_{\xi}^{(n)} = \Phi_{\xi}^{(n)} \frac{E_o}{\sigma_o}$	PEE(4,2,K)	PHI XI

[a]Where K is an index specifying the shell station; K = 1, 2, . . ., NMAX.

TABLE 5.- OUTPUT FORMAT

PROBLEM NUMBER (I4)

(PROBLEM DESCRIPTION)

INPUT DATA

REFERENCE LENGTH = (1PE16.7)

POISSON'S RATIO = (1PE16.7)

TEMP PARAMETER = (1PE16.7)

REFERENCE THICK = (1PE16.7)

FOURIER INDEX = (1PE16.7)

NUMBER OF STATIONS = (I4)

THE INDICATORS ARE SET AS FOLLOWS

IND1 = (I4) IND2 = (I4) IND3 = (I4) IND4 = (I4) IND5 = (I4)

THE FOLLOWING CONSTANTS APPEAR IN COMMON BLOCK BL14

(Data are printed columnwise according to FORMAT 1X1P4E16.8)

THE BOUNDARY CONDITIONS AT S = 0 ARE AS FOLLOWS

OMEGA LAMDA ELL

(Matrix elements are printed by rows using FORMAT 1P12.4)

THE BOUNDARY CONDITIONS AT S = SMAX ARE AS FOLLOWS

OMEGA LAMDA ELL

(Matrix elements are printed by rows using FORMAT 1P12.4)

R/RB	Z/RB	S/RB	OMEGA THETA	OMEGA XI	DEOMEGA XI	GAMMA
(1PE16.8)	(1PE16.8)	(1PE16.8)	(1PE16.8)	(1PE16.8)	(1PE16.8)	(1PE16.8)

S	N XI	N THETA	N XT	Q XI	M XI	M THETA	M XT	U XT	U THETA	W	PHX XI
(1PE11.4)	(1PE11.4)	(1PE11.4)	(1PE11.4)	(1PE11.4)	(1PE11.4)	(1PE11.4)	(1PE11.4)	(1PE11.4)	(1PE11.4)	(1PE11.4)	(1PE11.4)

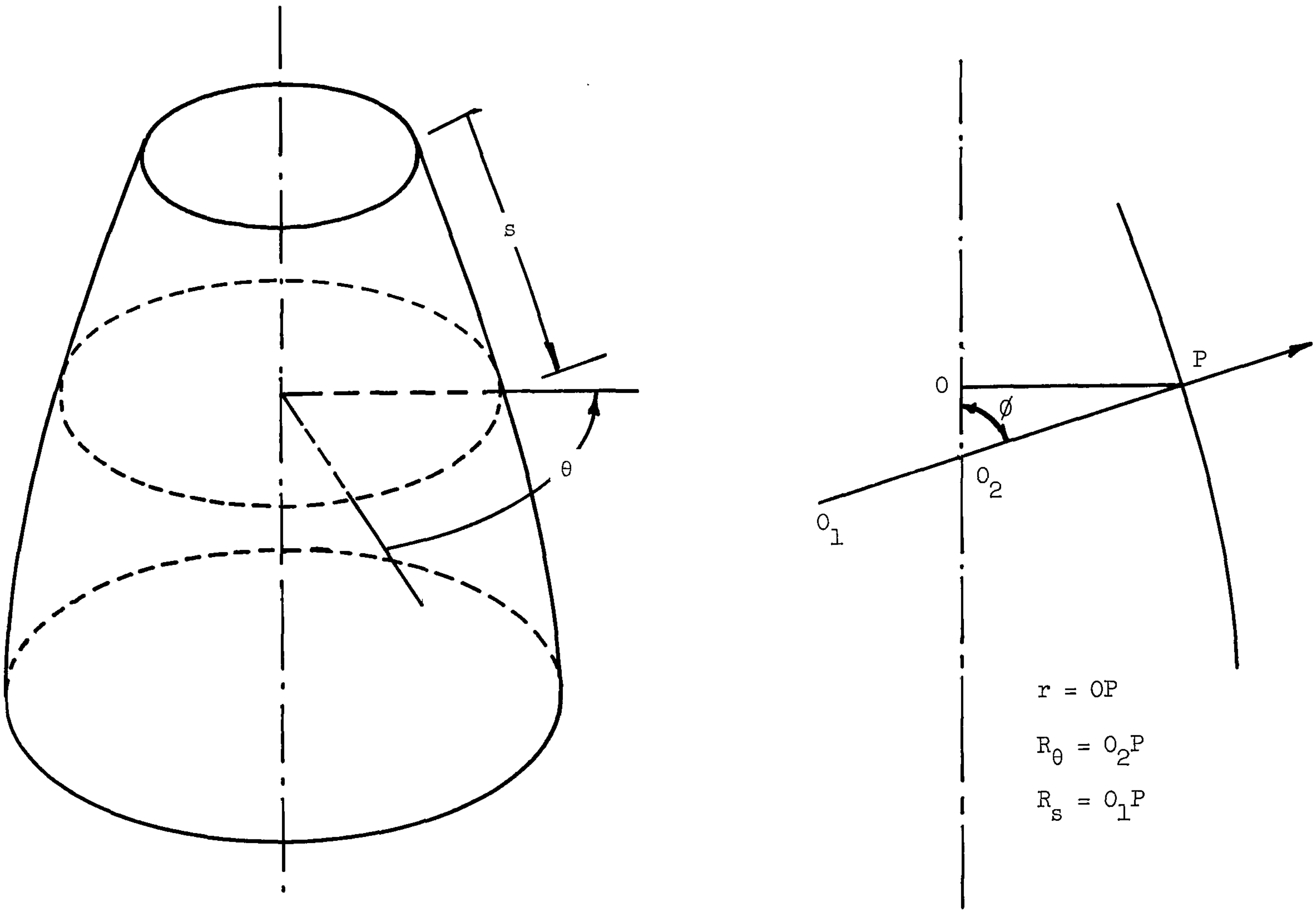
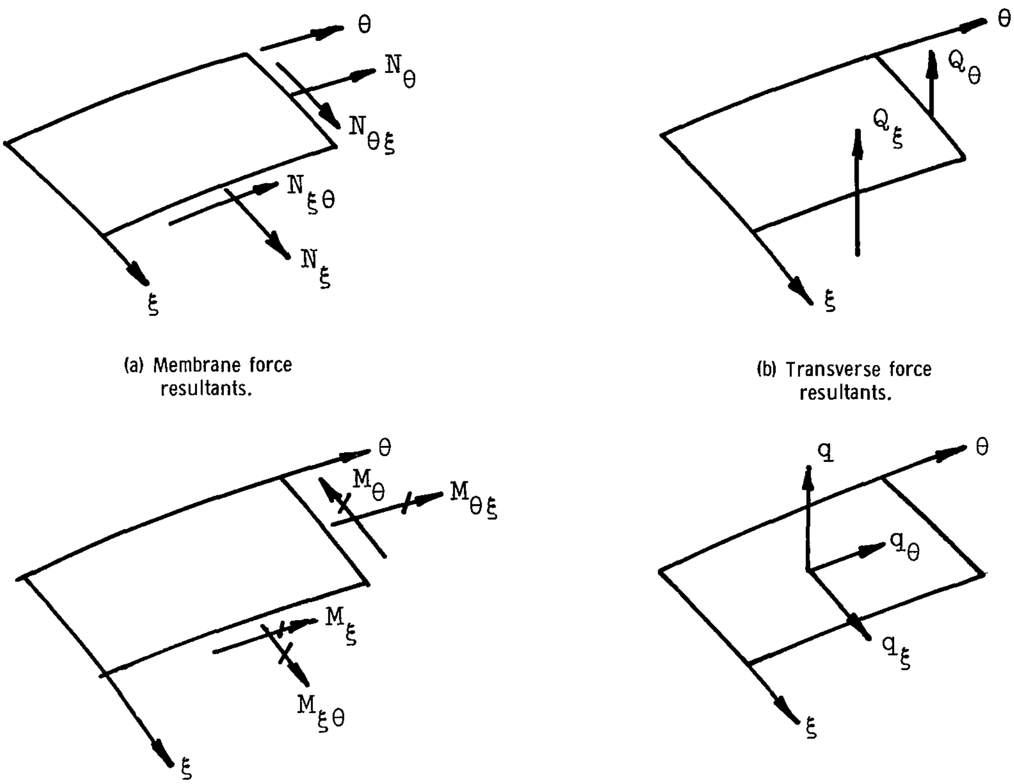


Figure 1.- Surface geometry and coordinates.



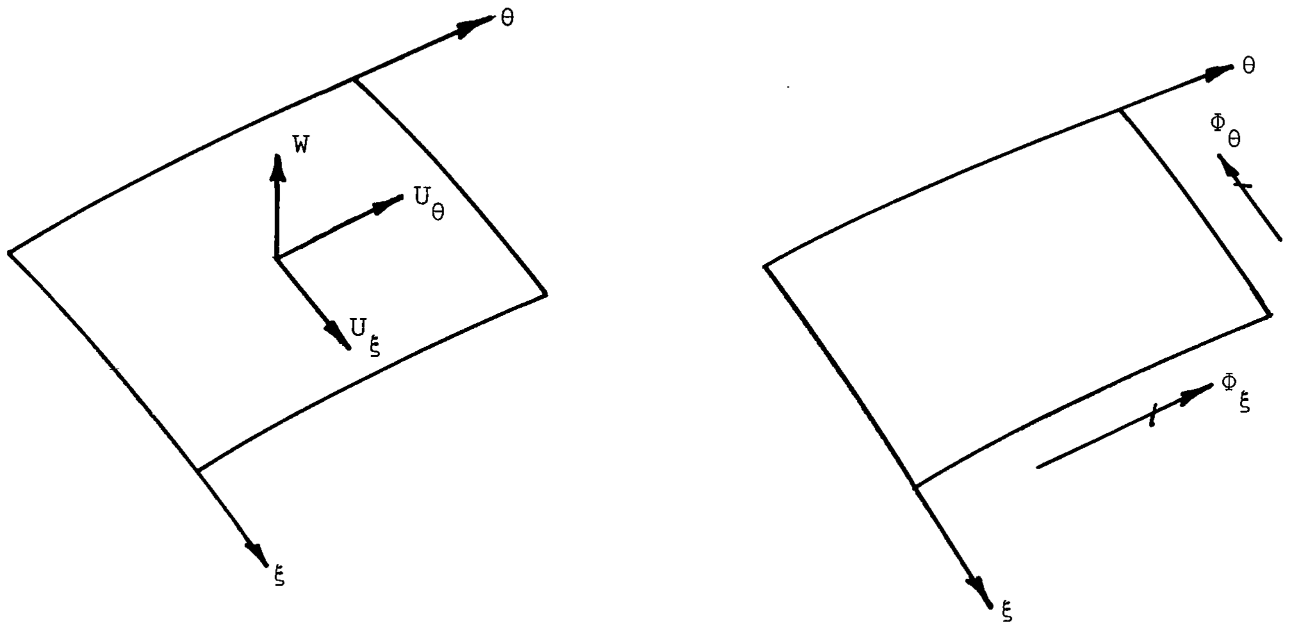
(a) Membrane force resultants.

(b) Transverse force resultants.

(c) Moment resultants.

(d) Loads per unit area.

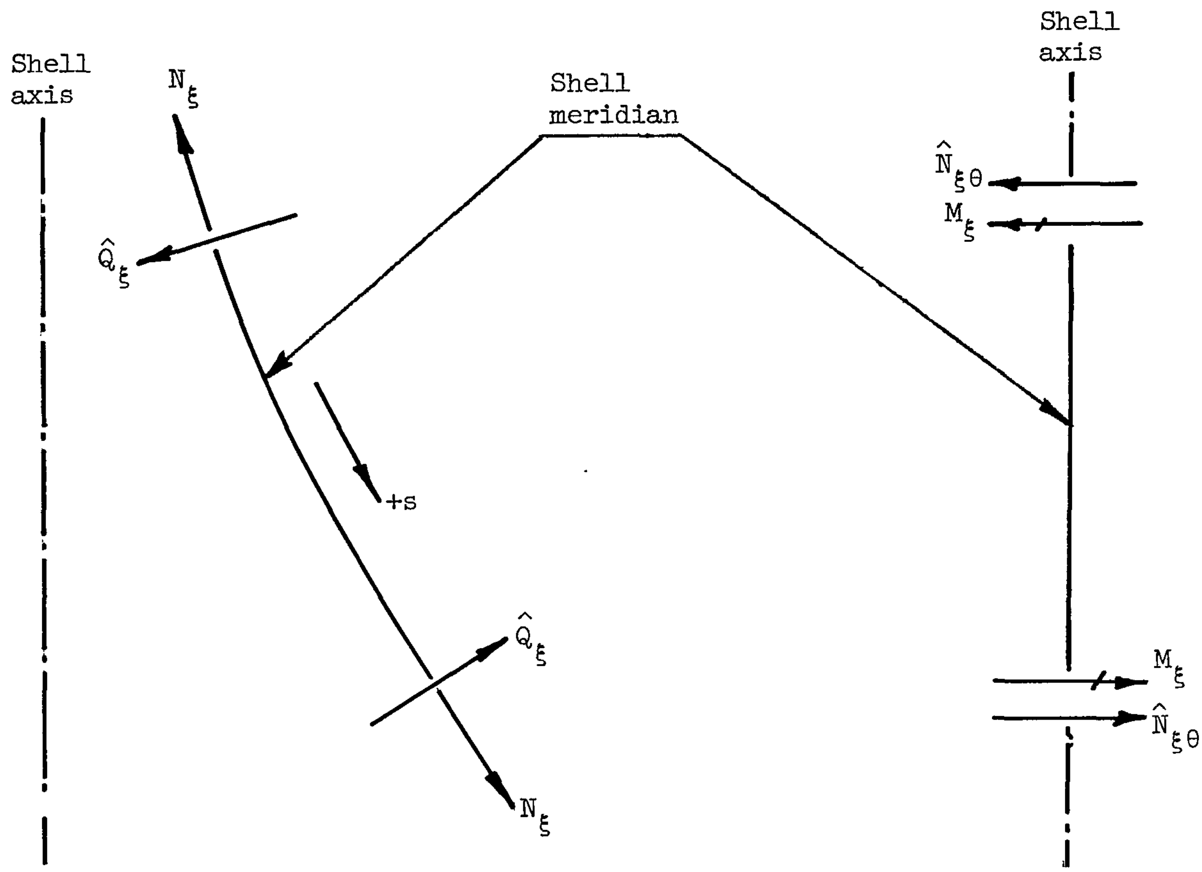
Figure 2.- Positive sense of forces, moments, and loads on shell segment.



(a) Displacements.

(b) Rotations.

Figure 3.- Shell element displacements and rotations.



(a) Effective transverse shear and meridional force.

(b) Effective membrane shear force and moment.

Figure 4.- Effective force and moment at shell boundary.

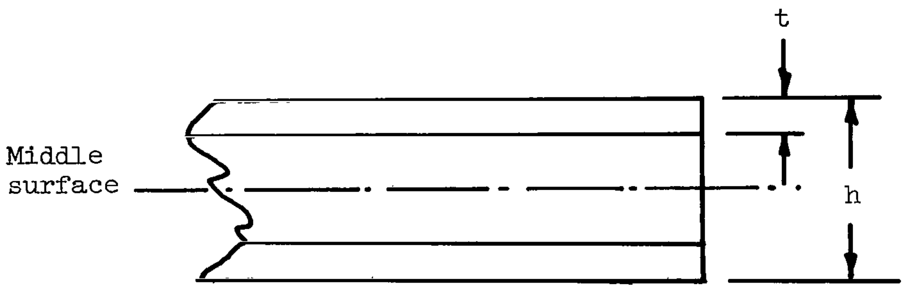


Figure 5.- Cross-section detail of layered shell.

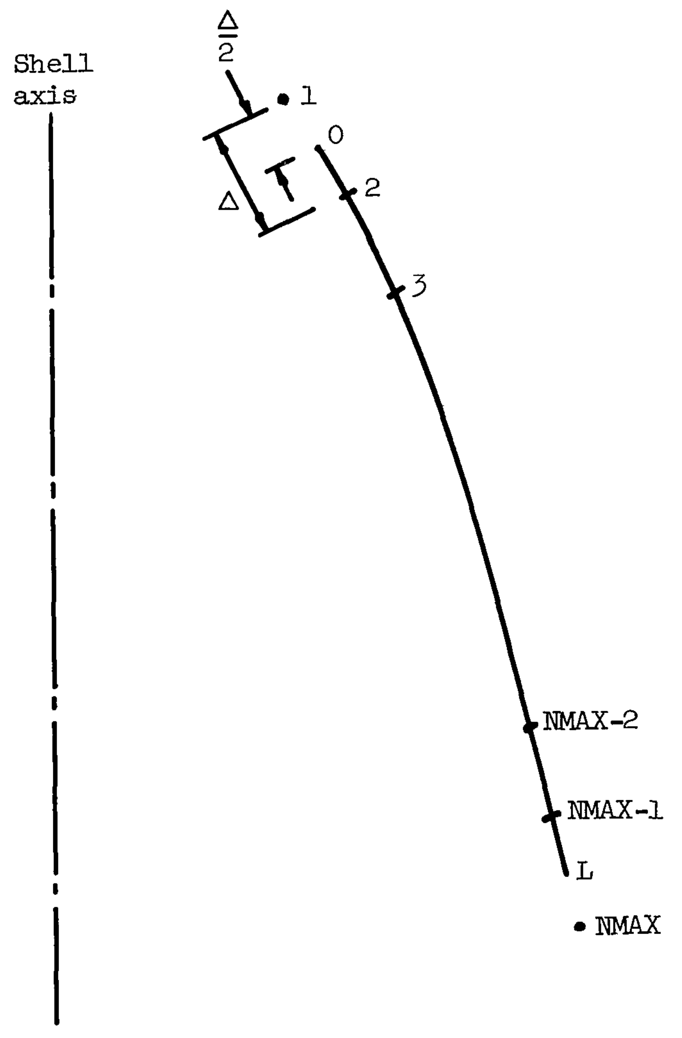


Figure 6.- Segment of shell meridian showing station locations at shell edges.

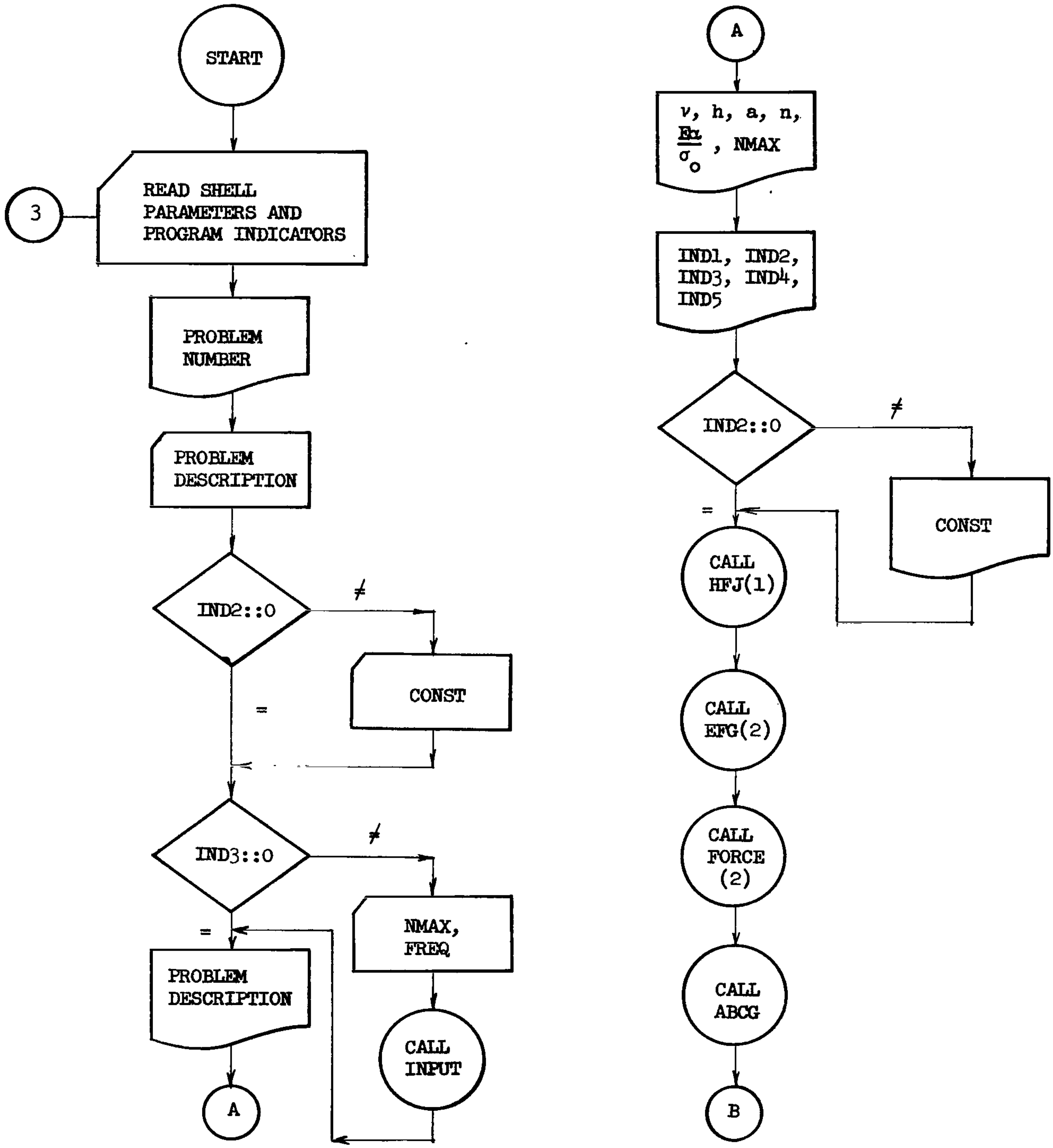


Figure 7.- Flow diagram for main program.

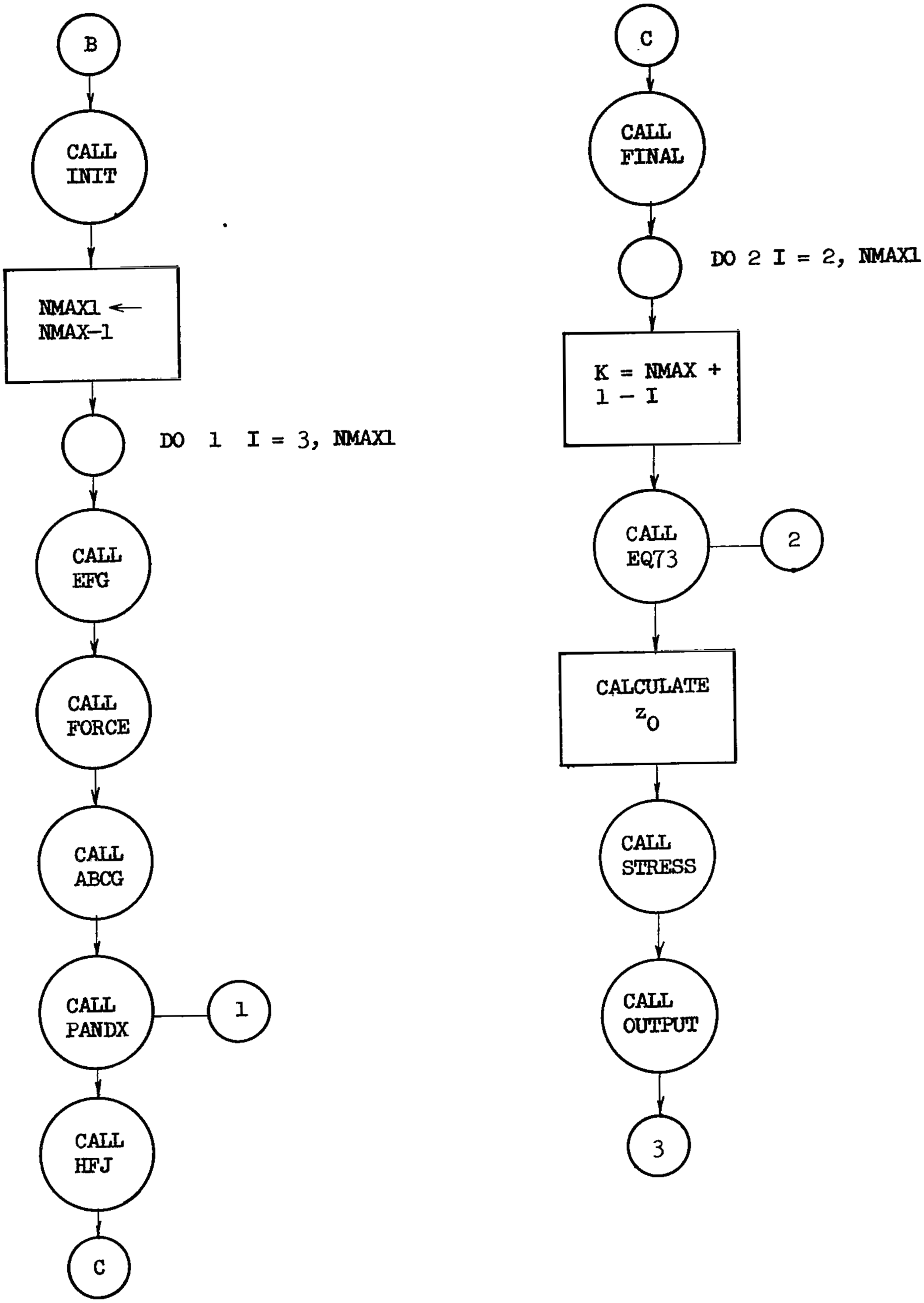


Figure 7.- Concluded.

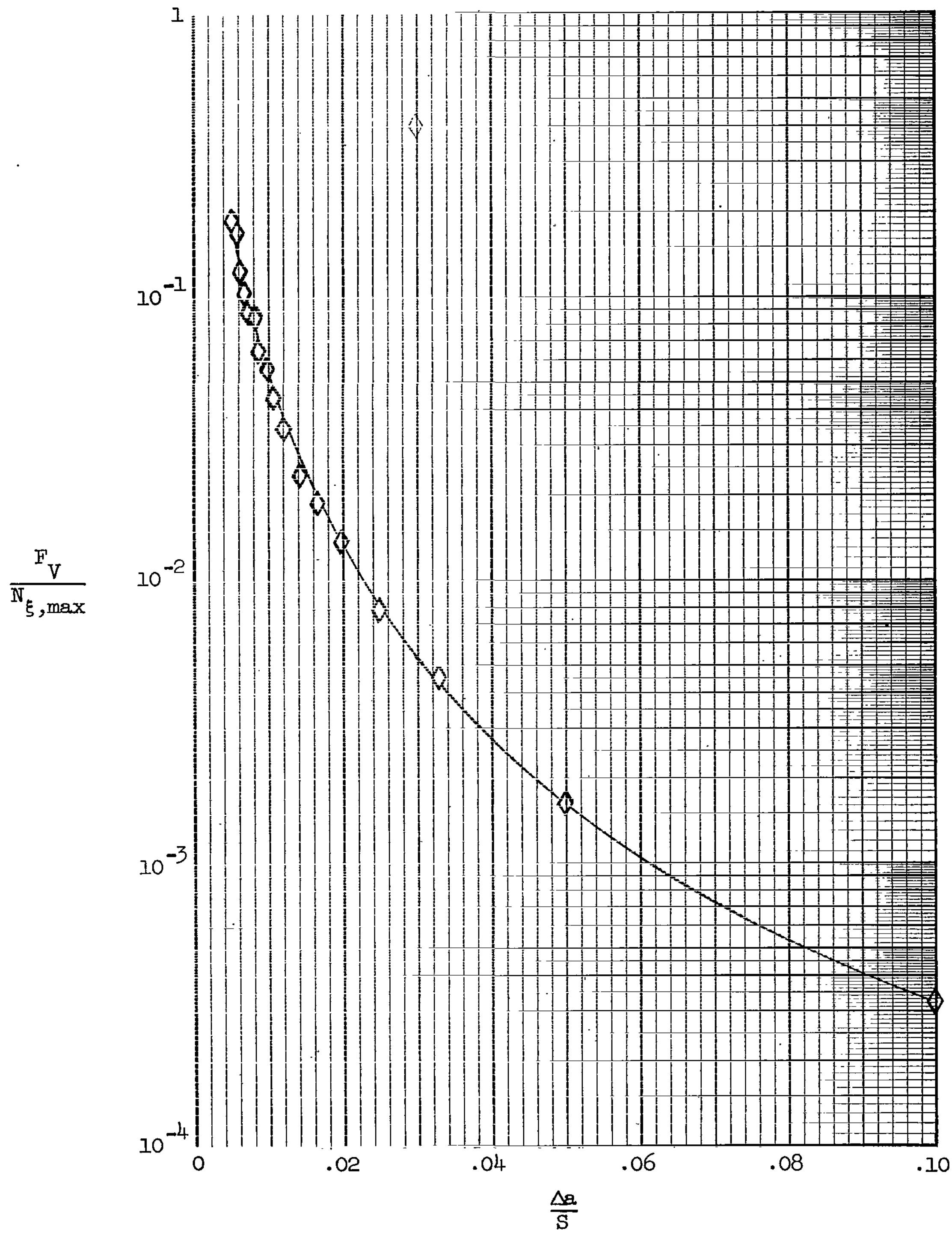


Figure 8.- Variation of axial force F_V with increment size for short cone (S/a = 0.0628) having nose half-angle of $\pi/3$ radians. Symbols represent results of numerical calculation; $N_{\xi,max}$ is the value of meridional stress calculated for $\Delta/(S/a) = 0.1$.

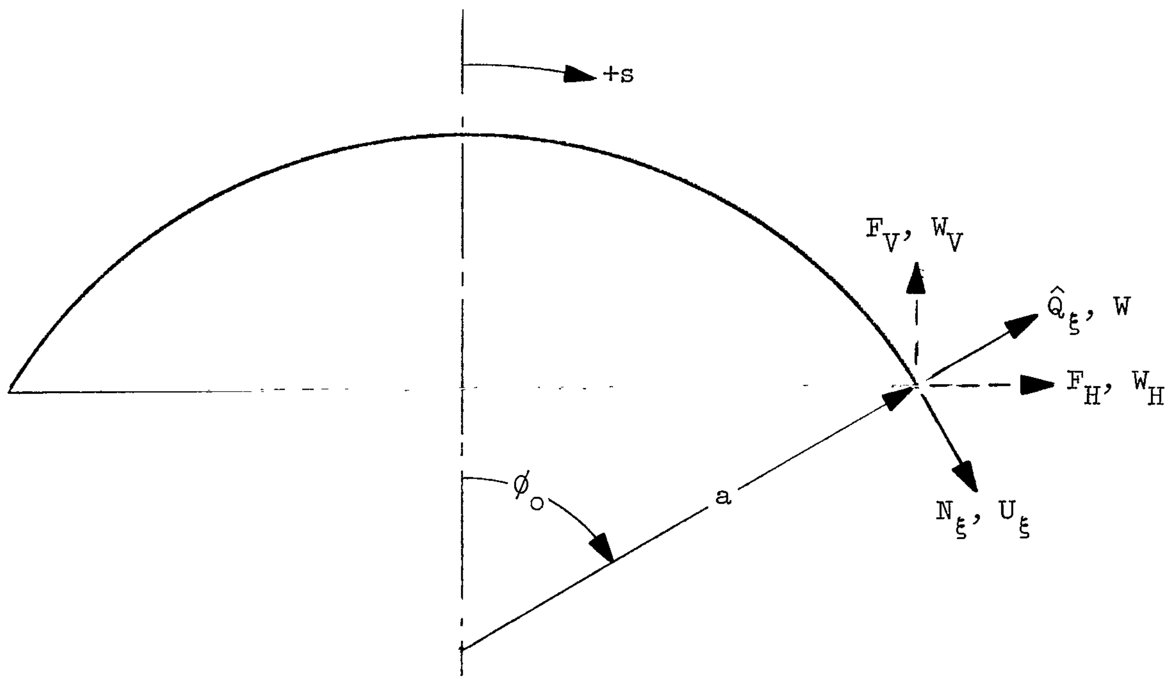


Figure 9.- Coordinate system for spherical segment.

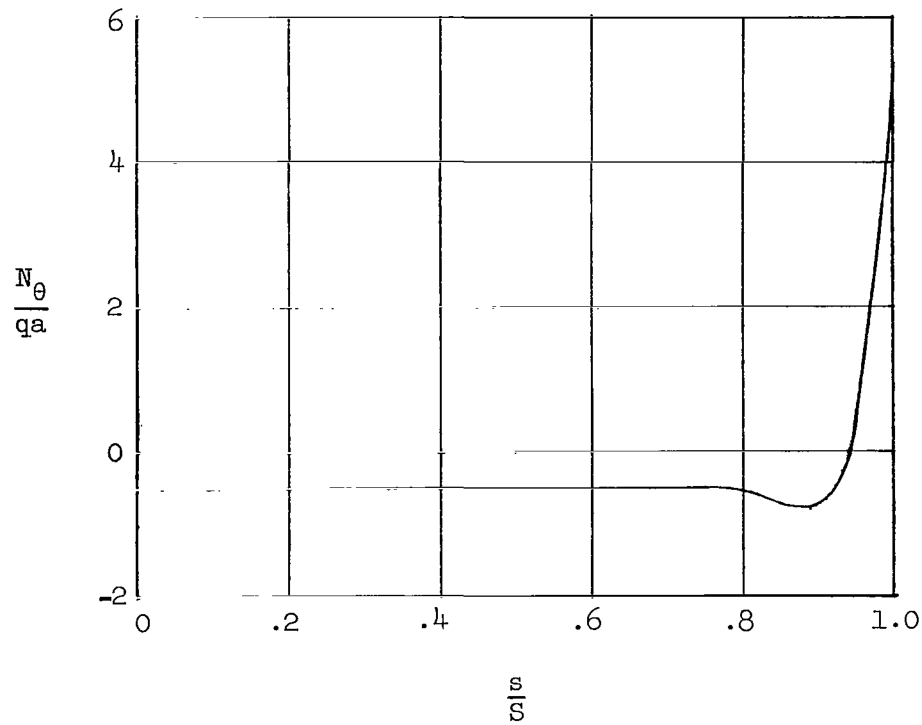


Figure 10.- Variation of N_θ with arc length for hydrostatically loaded spherical segment having horizontal elastic restraint at shell edge. $\Phi_0 = \frac{\pi}{6}, \frac{a}{h_0}\frac{k}{E_0} = 0.1$ where k is a proportionality constant relating horizontal edge force and edge displacement.

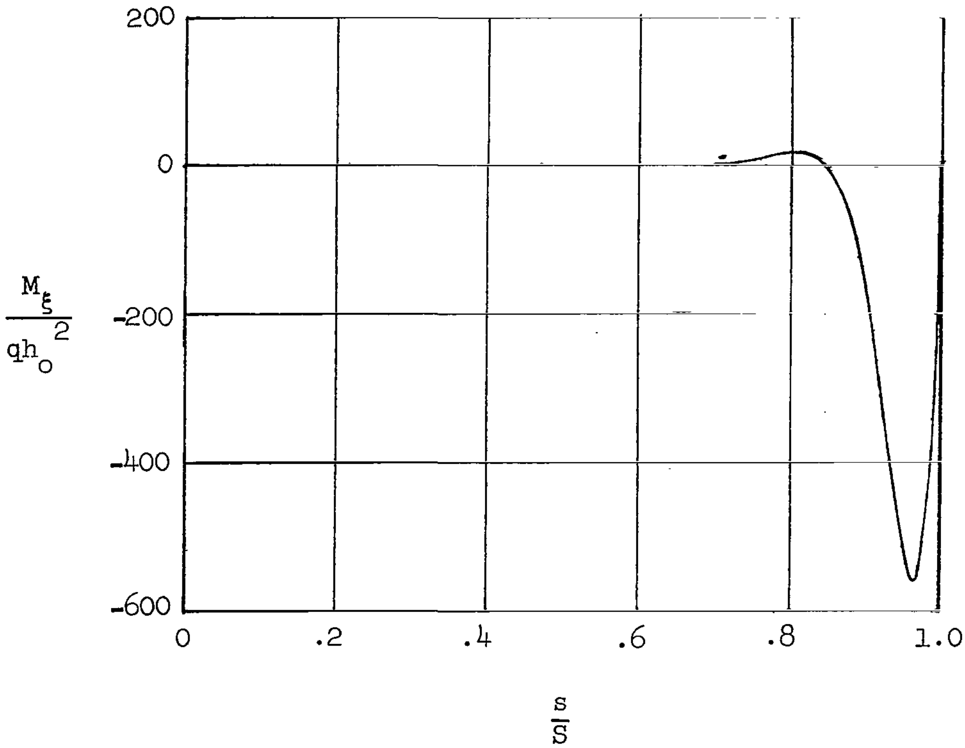


Figure 11.- Variation of M_ξ with arc length for hydrostatically loaded spherical segment having horizontal elastic restraint at shell edge. $\Phi_0 = \frac{\pi}{6}$; $\frac{a}{h_0}\frac{k}{E_0} = 0.1$ where k is a proportionality constant relating horizontal edge force and edge displacement.

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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