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National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No.NAS-5-12487

ST - PP - 10572

WAVE OF NONEQUILIBRIUM IONIZATION IN A GAS

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6 MARCH 1967

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From the 5th Symposium
on Ionization Events in Gases
YOUGOSLAVIA, 1965 *

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As is well known, there is a nonlinear relationship between the current and the electric field in a gas.

In the fields below the "breakdown" level, gas may be in either current or currentless states. The boundary between these states may shift toward the side of the currentless state at the expense of any energy transfer mechanism. This process is analogous to the propagation of the flame front at slow combustion. We shall consider the simplest situation: the propagation of a plane ionization wave in a uniform electric field.

We shall assume for model of current state a feebly ionized plasma, in which electrons are in thermodynamic equilibrium with the temperature, much higher than that of the gas. Because of gas'high specific heat, we shall neglect the variation of its state behind the ionization front.

The front propagation is determined by energy transfer processes. In a dense plasma the basic energy transfer mechanism is the electronic conduction, for the radiation is locked, while the rate of ambipolar diffusion is small. The temperature of electrons in the plasma (T) is much less than the ionization potential (I), For the above described model the energy balance equation has the form:

$$I\frac{\partial n}{\partial t} = \sigma E^2 - \frac{m}{M}\frac{nT}{\tau} + \frac{\partial}{\partial x}\left(\chi\frac{\partial T}{\partial x}\right). \tag{1}$$

Here σ is the conductance, χ is the electronic heat conductivity, $\chi \sim \frac{Tn\tau}{m}$,

τ is the electron's path length time, m and M are respectively the mass of the electron and the atom, n and T are the density and temperature of electrons linked by the

In a uniform stationary current state

$$\sigma E^2 = \frac{m}{M}\frac{nT}{\tau}. \tag{2}$$

The temperature T_∞ is determined from this relation.

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* VOLNA NERAVNOVESNOY IONIZATSII V GAZE

** The precise source could not be ascertained. (Not available at Aerospace Technology Division, Library of Congress)

We shall introduce a dimensionless temperature $T/T_\infty = \theta$, and a dimensionless coordinate $\xi = x/L$, where

$$L = \left(\frac{IM}{T_\infty m}\right)^{1/2} \lambda \qquad (3)$$

(λ is the length of the free path of the electron), and a dimensionless velocity $\tilde{\nu} = \nu/u$, where

$$u = \left(\frac{T_\infty}{M}\right)^{1/2} \left(\frac{T_\infty}{I}\right)^{3/2}. \qquad (4)$$

Since the problem is spatially homogenous, we shall seek the solution in the form

$$\theta \approx \theta(\xi - \tilde{\nu} u t). \qquad (5)$$

The propagation velocity of the front of $\tilde{\nu}$ is determined from the solution of (1) with the corresponding boundary conditions. In dimensionless denotations equation (1) has the form:

$$\varepsilon \theta_{\xi\xi} \theta^3 = \tilde{\nu} \theta_\xi - \theta^2(\theta - 1) - \theta_\xi^2 \theta \quad (\varepsilon \equiv T_\infty / I \ll 1). \qquad (6)$$

Introducing the phase variable $y = \theta \theta_\xi$, Eq.(6) takes the form:

$$y_\theta = -\frac{1}{\varepsilon y \theta^2} (y - y_1(\theta))(y - y_2(\theta)), \qquad (7)$$

where $y_{1,2} = -\frac{\tilde{\nu}}{2} \pm \sqrt{\frac{\tilde{\nu}_2}{4} - \theta^3(1-\theta)}$ are the roots of the equation

$$y^2 + \tilde{\nu} y + \theta^3(1 - \theta) = 0$$

Because of the smallness of ε everywhere, besides the small neighborhoods (of the order ε) of zero lines $y = y(0)$ and $y = y(0)$, we have $|y_\theta| \gg 1$. The course $y(\theta)$ of the solutions of Eq.(7) is easy to represent in the phase plane (y, θ).

We represented in Fig.1a the integral curves for the velocity $\tilde{\nu} > \nu_{kp} \equiv 3/4$, and in Fig.1b -- for $\tilde{\nu} < \nu_{kp}$. The integral curve satisfying the boundary condition $\theta = 1$, $\theta_\xi = 0$ as $x \to +\infty$, must originate from the point $\theta = 1$, $y = 0$ (point A). The second boundary condition must be set in the region $T = 0$ (zero value of energy flux in currentless state).

As may be seen from Fig.1a, there exists for velocities $\tilde{\nu} < \nu_{kp}$ a unique integral curve AB, linking the current (A) and the currentless (B) states.

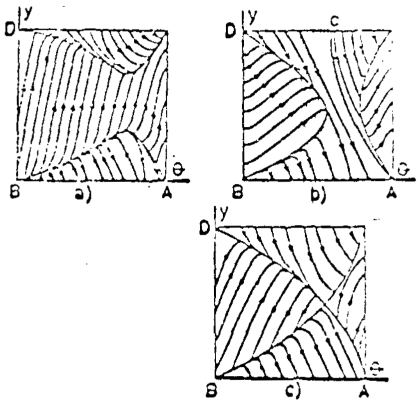

Fig.1. Integral curves for Eq.(7) in the phase plane.
a) $\nu > \nu_{kp}$, b) $\nu < \nu_{kp}$, c) $\nu = \nu_{kp}$

In the neighborhood of the point B ($\theta \to 0$, $y \to 0$) Eq.(7) takes the form:

$$y \equiv \theta\, \theta_\xi = \theta^3/\tilde{\nu}$$

whence $\theta \sim \tilde{\nu}/\xi$.

For finite values of ξ this solution does not vanish. Physically heat conductivity does not play any essential role in this solution and the transition from currentless to current state is materialized at the expense of electron heating in the electric field. By virtue of the above it may be seen that the solution described is unstable relative to local density decrease of electrons. Let us consider the velocities $\tilde{\nu} < \nu_{kp}$ (Fig.1b).

At the same time the integral curves originating from the point A, correspond to $y \to \infty$ at $\theta \to 0$ (curve AC). At T = 0 the continuity of energy flux is not realized on these curves. The curve for which the energy flux is continuous at $\theta = 0$ passes through the point D. Therefore, the solution satisfying all the assumed conditions exists only at $\tilde{\nu} = \nu_{kp}$ (curve AED in Fig.1c). This solution is stable relative to the small variation of boundary conditions as $\theta \to 0$.

Note that factually, the initial equations are not valid for low densities of electrons. At low temperatures the ionization by electrons becomes little effective, and their equilibrium density drops sharply. This is why the condition for energy flux continuity must necessarily be set not at $\theta = 0$, but at a temperature corresponding to the discontinuity on the curve n(T), which somewhat decreases the wave velocity.

Apparently, the wave of nonequilibrium ionization was observed experimentally [1].

***** T H E E N D *****

REFERENCE

1. Yu. M. VOLKOV. Teplofizika vysokikh temperatur, 4, 3, 1965. (High Temparture Thermophysics)

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Translated by ANDRE L. BRICHANT
on 4 March 1967 (Job No.GB-802)