TM 54/50-63
LMSC/HREC A783963

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Volume II

NONLINEAR DYNAMIC ANALYSIS

March 1967

Contract NAS8-20387

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FOREWORD

The work described in this report was carried out by Lockheed Missiles & Space Company, Huntsville Research & Engineering Center, for the George C. Marshall Space Flight Center of the National Aeronautics and Space Administration (NASA/MSFC), in accordance with the requirements of Contract NAS8-20387.

This report is Volume II of three volumes which comprise the Final Report under Contract NAS8-20387, as follows:

Volume I	-	"Synthesis of Structural Damping," by C. S. Chang and R. E. Bieber (LMSC/HREC A783975)
Volume II	-	"Nonlinear Dynamic Analysis," by R. O. Hultgren (LMSC/HREC A783963)
Volume III	-	"A Study of Hereditary Springs in Relation to Hysteretic Damping," by G. A. Ramerez (LMSC/HREC A783201)

The work was administered under the direction of the Aero-Astrodynamics Laboratory, NASA/MSFC, with Dr. George F. McDonough as Contracting Officer Representative.

SUMMARY

This memorandum analyzes the lateral plane vibration characteristics of a simple structure possessing hysteresis damping. This damping is taken to exist in an interstage between a nonuniform, flexible beam and a nonuniform, rigid beam.

The equations of motion are developed and the numerical techniques employed in their solution are presented. A user's manual and a sample problem for the developed computer program is also included.

CONTENTS

NOTATION

$[A]$	=	mass matrix
a_{ij}	=	elements of $[A]$ matrix
$[D]$	=	damping matrix
d_{ij}	=	elements of $[D]$ matrix
d	=	deflection
E	=	modulus of elasticity
$F(t)$	=	forcing function
F_r	=	force at hysteresis spring motion reversal
$f_{(x, y)}$	=	arbitrary function
I	=	first moment of inertia of flexible beam cross section
$\bar{I}$	=	first moment of inertia of rigid beam cross section
i, j	=	indices
J	=	second moment of inertia of flexible beam cross section
$\bar{J}$	=	second moment of inertia of rigid beam cross section
K	=	linear spring constant
$[K]$	=	stiffness matrix
K_{ij}	=	functions in Runge-Kutta-Gill integration routine
$K_i(\Delta)$	=	nonlinear interstage spring force-deflection function for i^{th} spring
k_{ij}	=	elements of $[K]$ matrix

L_1	=	length of system flexible beam
L_2	=	length of system rigid beam
M	=	mass of system flexible beam
$\overline{M}$	=	mass of system rigid beam
M_p, M_{p_r}	=	interstage moments
M_i	=	interstage moment due to i^{th} spring
$[MI]$, $[MII]$, $[MIII]$	=	coefficient matrices
$m(x)$	=	mass distribution of flexible beam
$m(\bar{x})$	=	mass distribution of rigid beam
N	=	number of deflection functions
P_p, P_{p_r}	=	interstage lateral force
p	=	index
(Q)	=	generalized force matrix
Q_i	=	generalized force components
q_i	=	arbitrary, unknown functions of time
r_i	=	radius to i^{th} interstage spring
S	=	dummy displacement variable
T	=	kinetic energy functional
t	=	time
Δt	=	time increment
U_1	=	deflection at one end ($x=o$) of flexible beam
U_2	=	deflection at one end ($x=L_1$) of flexible beam

$U(x)$	=	deflection of flexible beam
V	=	potential energy functional
δW_1	=	virtual work associated with flexible beam
δW_2	=	virtual work associated with rigid beam
x	=	position coordinate of flexible beam
$\bar{x}$	=	position coordinate of rigid beam
x_f	=	position coordinate of forcing function
x_o	=	limit of linear force-deflection hysteresis spring relation
α	=	parameter of hysteresis spring force-deflection relation
Δ	=	deflection history of a hysteresis spring
δ	=	linear displacement between ends of the interstage
θ	=	difference between rigid beam slope and flexible beam slope at the interstage
λ	=	slope of rigid beam
Φ_i	=	polar angular coordinate to i^{th} spring of the interstage
ϕ_i	=	i^{th} displacement function

"primed" variables = derivatives of the variables with respect to the length coordinate

"dotted" variables = derivatives of the variables with respect to time

Section 1
INTRODUCTION

The analysis and computer programs presented in this report treat the class of structures which can be characterized by a flexible beam of arbitrary length connected by a nonlinear interstage to a rigid mass. A nonlinear interstage, as used here, denotes an interstage having hysteresis spring damping and nonlinear conservative springs.

In this analysis, the flexible beam is assumed to move in one plane, and the attached rigid body may rotate in relation to the beam end, and it may also be displaced. As assumed for the beam, however, motion is restricted to a single plane. Deflections are assumed to be small so that the beam is always elastic. Beam elastic and mass properties are considered variable with respect to the beam length.

The displacements and rotations at various points on the system are determined by numerically integrating the equations of motion of the system. These equations are derived using Lagrange's equation after determining the system kinetic and potential energy functionals. Hysteresis effects in the interstage are included by monitoring the deflection history of each spring in the interstage. In this monitoring process, the spring force-deflection curve is reversed whenever the deflection velocity changes sign. When the spring force-deflection curve slope decreases with deflection, i.e., a "softening" spring, energy dissipation by hysteresis occurs.

In the following sections, this brief outline is more fully explained. The mathematical model employed is first defined, especially with regard to the interstage configuration. With this established, the kinetic and potential energy functionals are written and the equations of motion presented. The mass, stiffness, damping, and generalized force matrices are then written in their elemental form.

The computer program which performs the necessary lengthy calculations is then briefly explained. First the integration technique is presented and, finally, a more detailed explanation of how the hysteretic effects are accounted for is included. This section ends with a sample problem and a user's manual for the developed program.

The report ends with a section indicating a few of the areas into which this investigation should be expanded. It should be remembered that the analytical and computational tools concerning the hysteretic effects have a far wider application than for the rather simple vehicle model used in this analysis. It is hoped that the mathematical model analyzed here will be only a first step toward the more sophisticated efforts outlined in Section 5, Recommended Additional Developments.

Section 2
ANALYSIS

The mathematical model, Figure 1, which has been used as a developmental tool for the hysteretic analysis consists of a flexible beam, an interstage possessing springs with hysteresis damping, and a rigid mass. The beam is assumed to have mass and stiffness properties which vary with the beam position coordinate. It can move only in a plane. The interstage is assumed to consist of conservative springs which may have nonlinear force-deflection curves and springs with hysteresis which are necessarily nonlinear in character. In the following text the conservative springs will be referred to simply as springs, while the springs with hysteresis will be referred to as dampers.

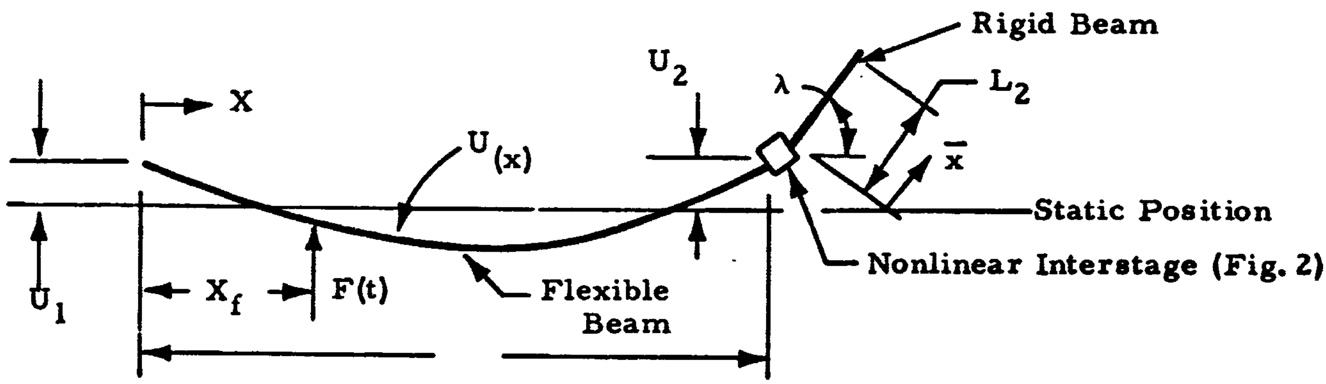


Figure 1

Defining the interstage more specifically, the springs and dampers are assumed to be arbitrarily distributed about the beam center as shown in Figure 2 where r_i and Φ_i are polar coordinates which locate the point where either a

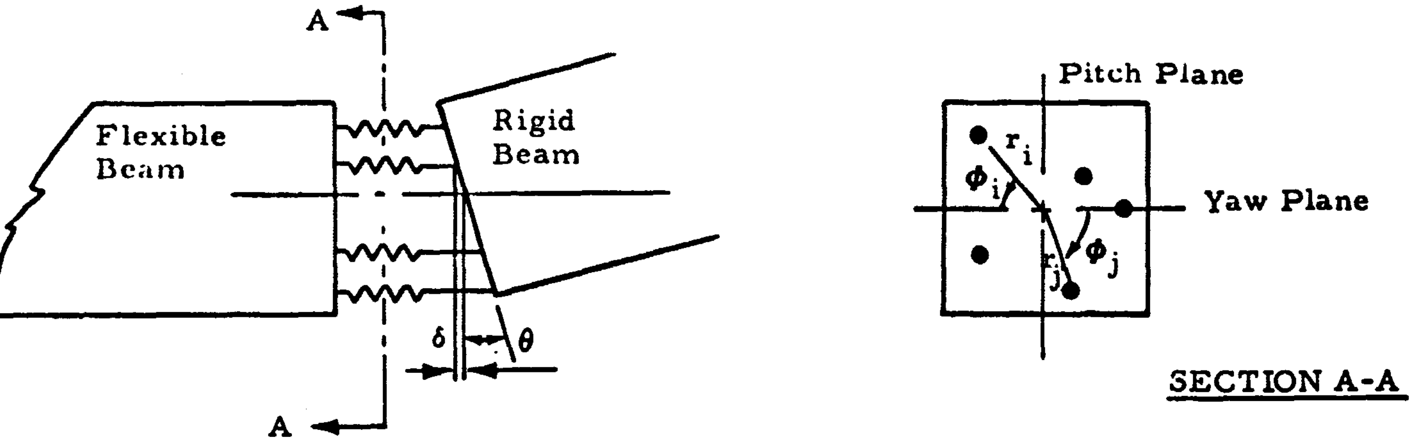


Figure 2

spring or a damper are attached to the flexible beam. The rigid beam is permitted to move only in the plane of the flexible beam motion.

It is now necessary to derive the kinetic and potential energy functionals of this system. Figures 1 and 2 present the nomenclature which will be used in the following equations. The lengths of the flexible and rigid beams are L_1 and L_2, respectively. U_1 and U_2 are the displacements of the flexible beam ends and $U(x)$ is the displacement function of the beam where x is the length coordinate of the flexible beam. Hence,

$$\begin{aligned} U_1 &= U_{(o)} \\ U_2 &= U_{(L_1)} \end{aligned} \tag{1}$$

The displacement of the rigid beam is $\overline{U}(\overline{x})$ where $\overline{x}$ is the length coordinate and λ is the slope of the rigid beam. θ is the difference between the rigid beam slope and the flexible beam slope at $x = L_1$. δ denotes the distance that the rigid beam moves away from the flexible beam, discounting any rotation effects.

The U(x) displacement function may be considered to consist of a rigid body motion defined by the displacements U_1 and U_2 plus a flexible motion defined by the sum of the products of N arbitrary functions and their respective generalized coordinate time varying amplitudes, q_i. Hence,

$$U(x) = U_1 + \frac{U_2 - U_1}{L_1} \; x \; + \sum_{i=1}^{N} q_i \; \phi_i(x) \tag{2}$$

The displacement of the rigid beam is

$$\bar{U}(\bar{x}) = U_2 + \lambda \, \bar{x} \; + \sum_{i=1}^{N} q_i \, \phi_i \, (L_1)$$

$$= U_2 + \sum_{i=1}^{N} q_i \, \phi_i \, (L_1) + \left(\frac{U_2 - U_1}{L_1} + \sum_{i=1}^{N} q_i \, \phi_i'(L) + \theta \right) \bar{x} \tag{3}$$

where the "prime" on ϕ_i denotes differentiation with respect to the spatial length variable x.

It is now possible to derive the kinetic energy of the system using the relation

$$2T = \int_0^{L_1} m(x) \, \dot{U}^2(x)dx + \int_0^{L_2} m(\bar{x}) \, \dot{\bar{U}}^2 \, (\bar{x})d\bar{x} \tag{4}$$

where T is the kinetic energy and $m(x)$ and $m(\bar{x})$ represent the mass distributions in the flexible and rigid beams, respectively. As customary, the dots represent differentiations with respect to time. Substituting the displacement functions (2) and (3), differentiated with respect to time, into the kinetic energy expression (4) yields the desired kinetic energy functional.

$$2T = \int_0^{L_1} m(x) \left[\dot{U}_1 + \frac{\dot{U}_2 - \dot{U}_1}{L_1} x + \sum_{i=1}^{N} \dot{q}_i \phi_i(x) \right]^2 dx \qquad (5)$$

$$+ \int_0^{L_2} m(\bar{x}) \left[\dot{U}_2 + \sum_{i=1}^{N} \dot{q}_i \phi_i(L) + \left(\frac{\dot{U}_2 - \dot{U}_1}{L_1} + \sum_{i=1}^{N} \dot{q}_i \phi_i'(L_1) + \dot{\theta}\,\bar{x} \right) \right]^2 \left(-\frac{\bar{x}}{L_1} \right) d\bar{x}$$

Leaving this equation, it is necessary to determine the potential energy functional and the generalized forces. For this analysis, the potential energy is taken to be stored only in the flexible beam. The spring forces of the interstage will be included in the generalized forces of the equations. The rigid beam, of course, cannot possess flexural potential energy. Hence, the potential energy functional V of the entire system can be written as

$$2V = \int_0^{L_1} EI \left[U''(x) \right]^2 dx$$

$$= \int_0^{L_1} EI \left[\sum_{i=1}^{N} q_i \phi_i''(x) \right]^2 dx \qquad (6)$$

where E is the modulus of elasticity and I is the moment of inertia of the beam cross section. The two "primes" denote two derivatives with regard to the length variable x.

The generalized forces can be written using the concept of virtual work in relation to Figure 3.

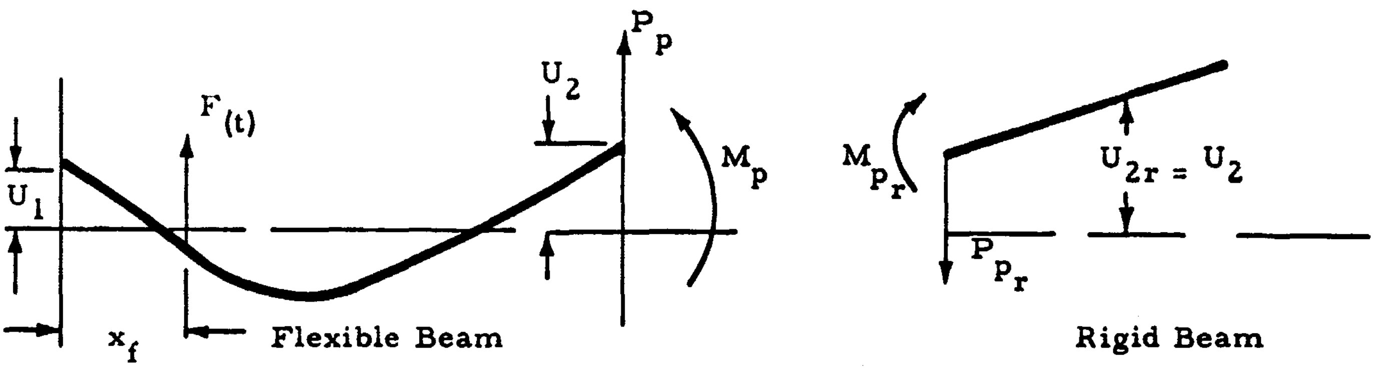


Figure 3

In Figure 3, F(t) represents a time dependent forcing function which is applied at some arbitrary distance, x_f, from the end of the flexible beam. M_p, M_{p_r} are the equal moments which are applied by the interstage springs to the flexible and rigid beams, respectively. P_p and P_{p_r} are equal (but opposite in sign) forces which will be seen to negate in the ultimate generalized force expressions.

The virtual work of the flexible beam can be written as

$$\delta W_1 = F(t)\delta U(x_f) + P_p \delta U(L_1) + M_p \delta U'(L_1) \tag{7}$$

Substituting the displacement function (2) yields

$$\delta W_1 = F(t)\,\delta\left[U_1 + \frac{U_2 - U_1}{L} x_f + \sum_{i=1}^{N} q_i \phi_i(x_f)\right]$$

$$+ P_p \delta\left[U_2 + \sum_{i=1}^{N} q_i \phi_i(L_1)\right]$$

$$+ M_p \delta\left[\frac{U_2 - U_1}{L_1} + \sum_{i=1}^{N} q_i \phi_i'(L_1)\right] \tag{8}$$

Proceeding in the same fashion with the rigid beam yields

$$\delta W_2 = -P_{p_r} \delta U_{2_r} - M_{p_r} \delta\lambda \tag{9}$$

$$\delta W_2 = -P_{p_r} \delta\left[U_2 + \sum_{i=1}^{N} q_i \phi_i(L)\right] - M_{p_r} \delta\left[\frac{U_2 - U_1}{L} + \sum_{i=1}^{N} q_i \phi'(L) + \theta\right]$$

where $P_{p_r} = -P_p$ and $M_p = M_{p_r}$.

The equations of motion can now be derived using Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{S}}\right) + \frac{\partial V}{\partial S} = Q_i \tag{10}$$

where S is a dummy coordinate and Q_i are the components of the generalized force associated with the coordinate S. These equations can be written in the matrix form

$$[A]\,(\ddot{S}) + [D]\,(\dot{S}) + [K]\,(S) = (Q) \tag{11}$$

where (S) represents the unknown generalized coordinate column matrix (12) and $(\dot{S})$ and $(\ddot{S})$ represent the column matrices of the first and second time derivatives of the generalized coordinates, respectively.

$$(S) = \begin{pmatrix} U_1 \\ U_2 \\ \theta \\ q_1 \\ q_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ q_n \end{pmatrix} \tag{12}$$

The mechanical derivation of the elements of the [A], [K], and (Q) matrices is accomplished in a straightforward fashion and only the final form of the matrix elements will be presented here. In the development of these elements, the following notation is used

$$M = \int_0^{L_1} m(x)\, dx \tag{13}$$

$$\overline{M} = \int_0^{L_2} m(\bar{x})\, d\bar{x} \tag{14}$$

$$J = \int_0^{L_1} x\, m(x)\, dx \tag{15}$$

$$\bar{J} = \int_0^{L_2} \bar{x}\, m(\bar{x})\, d\bar{x} \tag{16}$$

$$I = \int_0^{L_1} x^2\, m(x)\, dx \tag{17}$$

$$\bar{I} = \int_0^{L_2} \bar{x}^2\, m(\bar{x})\, d\bar{x} \tag{18}$$

The elements of the upper triangular matrix of the mass matrix [A] are

$$a_{11} = M - 2\frac{J}{L_1} + \frac{I}{L_1} + \frac{\bar{I}}{L_1^2} \tag{19}$$

$$a_{12} = \frac{J}{L_1} - \frac{\bar{J}}{L_1} - \frac{I}{L_1^2} - \frac{\bar{I}}{L_1^2} \tag{20}$$

$$a_{22} = M + \frac{I}{L_1^2} + \frac{\bar{I}}{L_1^2} + 2\frac{\bar{J}}{L_1} \tag{21}$$

$$a_{13} = -\frac{I}{L_1} \tag{22}$$

$$a_{23} = \bar{J} + \frac{\bar{I}}{L_1} \tag{23}$$

$$a_{33} = \bar{I} \tag{24}$$

(for i going from 1 to N)

$$a_{1,\ i+3} = -\frac{\bar{J}}{L_1}\ \phi_i(L_1) - \frac{\bar{I}}{L_1}\ \phi_i'(L_1) + \int_o^{L_1} m(x)\ \left(1 - \frac{x}{L_1}\right)\ \phi_i(x)\ dx \tag{25}$$

$$a_{2,\ i+3} = \left(\bar{M} + \frac{\bar{J}}{L_1}\right)\ \phi_i(L_1) + \left(\bar{J} + \frac{\bar{I}}{L_1}\right)\ \phi_i'(L_1) + \int_o^{L_1} m(x)\ \frac{x}{L}\ \ \phi_i(x)\ dx \tag{26}$$

$$a_{3,\ i+3} = \bar{J}\ \phi_i\ (L_1) + \bar{I}\ \phi_i'\ (L_1) \tag{27}$$

(for p going from 1 to N)

$$a_{3+p,\ 3+p} = \int_o^{L_1} m(x)\ \phi_p^2\ (x)\ dx + \left[\bar{M}\ \ \phi_p\ (L_1) + \bar{J}\ \phi_p'(L_1)\right]\ \phi_p\ (L_1)$$

$$+ \left[\ \bar{J}\ \phi_p\ (L_1) + \bar{I}\ \phi_p'(L_1)\right]\ \ \phi_p'\ (L_1) \tag{28}$$

$$a_{3+p,\ 3+p+i} = \left[\bar{M}\ \ \phi_p\ (L_1) + \bar{J}\ \phi_p'\ (L_1)\right]\ \phi_i\ (L_1)$$

$$+ \left[\bar{J}\ \phi_p\ (L_1) + \bar{I}\ \phi_p'\ (L_1)\right]\ \phi_i'\ \ (L)$$

$$+ \int_o^{L_1} m(x)\ \phi_p(x)\ \phi_i(x)\ dx \tag{29}$$

Only the upper triangular matrix is presented here, since this matrix is symmetric. Note that the full expression (29) is given for $a_{3+p,\ 3+p+i}$. If, however, the input mode functions ϕ_i are orthogonal, the integral appearing in these elements will be zero.

In the generation of the $[K]$ matrix, the first three rows and the first three columns will be zero, since the rigid body displacements do not influence the potential energy of this system. Thus the non-zero elements of this matrix are

(for i going from 1 to N)

$$K_{i+3,\ i+3} = \int_o^{L_1} EI \left[\phi_i''(x)\right]^2 dx \tag{30}$$

$$K_{i+3,\ i+3+p} = \int_o^{L_1} EI\ \phi_i''(x)\ \phi_p''(x)\ dx \tag{31}$$

where (31) will be zero if the input mode functions, ϕ_i, are orthogonal.

The elements of the generalized force matrix are

$$Q_1 = \frac{L_1 - x_f}{L_1} F(t) \tag{32}$$

$$Q_2 = \frac{x_f}{L_1} F(t) \tag{33}$$

$$Q_3 = - M_p \tag{34}$$

(for i going from 1 to N)

$$Q_i = \phi_i (x_f) F(t) \tag{35}$$

Physically, the moment M_p can be written in terms of the individual springs composing the interstage.

$$M_p = \sum_{i=1}^{NSP} M_i$$

$$M_p = \sum_{i=1}^{NSP} K_i (\Delta) \left[\delta + (r_i \sin \Phi_i) \theta\right] \left[r_i \sin \Phi_i\right] \tag{36}$$

$K_i (\Delta)$ is the relation between the spring deflection and force. For a conservative spring, this relation depends only on the deflection at the particular time instant. For a spring with hysteresis, however, $K_i (\Delta)$ depends on the entire deflection (Δ) history of the spring.

The final matrix of equation (11) is the $[D]$ matrix which represents the damping in the flexible beam associated with each of the input deflection functions ϕ_i. Only the elements $d_{i+3,\ i+3}$ are filled since the rigid body deflections do not participate in this form of damping. The numerical values of these elements depend on the analyzed system.

To carry out the solution, the matrix equation (11) is rewritten in the form

$$(\ddot{S}) = -[A]^{-1}[D](\dot{S}) - [A]^{-1}[K](S) + [A]^{-1}(Q) \tag{37}$$

and a numerical integration of (37) using a Runge-Kutta-Gill routine is carried out. This general method is amplified in Section 3, Numerical Techniques.

The foregoing coefficients, of course, represent the system of Figure 1 which, essentially, has a free-free flexible beam. To make the analysis applicable to a system having the flexible beam cantilevered at x=o, both U_1 and U_2 must be suppressed. This may be accomplished by setting all the elements in the first two rows of the $[A]$ matrix, with the exception of a_{11} and a_{22}, equal to zero. a_{11} and a_{22} must be 1 for this case. In the generalized force matrix Q_1 and Q_2 must be zero for the cantilevered flexible beam.

Section 3
NUMERICAL TECHNIQUES

Of the numerical techniques used in the computer programs for the foregoing analysis, only two require some explanation. The first is the integration method which is not unique, but must be included for completeness. Hence, only a brief outline follows with a reference to the basic paper on which this technique is based. The second general numerical technique involves the treatment of the hysteretic effects.

Equation (37) may be rewritten in the form

$$(\ddot{S}) = \left[MI\right](\dot{S}) + \left[MII\right](S) + \left[MIII\right](Q) \tag{38}$$

A time increment for the integration is chosen, based on the forcing function frequency and the natural frequencies of the system. Naturally, if the system is expected to have a large number of oscillations in a short time, the integration time increment should be very small.

The Runge-Kutta-Gill technique (Reference 1) integrates a differential equation of the form

$$\frac{dy}{dx} = f(x, y) \tag{39}$$

based on the fact that if y is known at one x, its value at $x + \Delta x$ may be determined from the relation

$$y_{x+\Delta x} = y_i + 1/6\, K_{i0} + 1/3\left[1-\sqrt{1/2}\right]K_{i1} + 1/3\left[1+\sqrt{1/2}\right]K_{i2} + 1/6\, K_{i3} \tag{40}$$

The K_{ij} depend on the function $f(x, y)$, the previous values of y, and the time increment.

Applying this to equation system (38), if (S) and $(\dot{S})$ are known at zero time, $(\ddot{S})$ can be calculated. Then, at the end of the selected time increment, $(\ddot{S})$ may be calculated using equation (40). With $(\ddot{S})$ known at t=o and t=Δt, the velocity $(\dot{S})$ is determined at t=Δt. By a trapezoidal rule integration

$$(\dot{S})_{\Delta t} = \frac{(\ddot{S})_{t=o} + (\ddot{S})_{t=\Delta t}}{2} \quad \Delta t \tag{41}$$

With $(\dot{S})$ known at t=o and t=Δt, (S) may be determined at t=Δt from the generalization of the trapezoidal rule integration of (41). By continuing this process along the time line, the transient displacements are determined.

Hysteresis springs are springs having their loading force-deflection curves different from their unloading force-deflection curves (see Reference 2 for a more complete description). Graphically, this is shown in Figure 4.

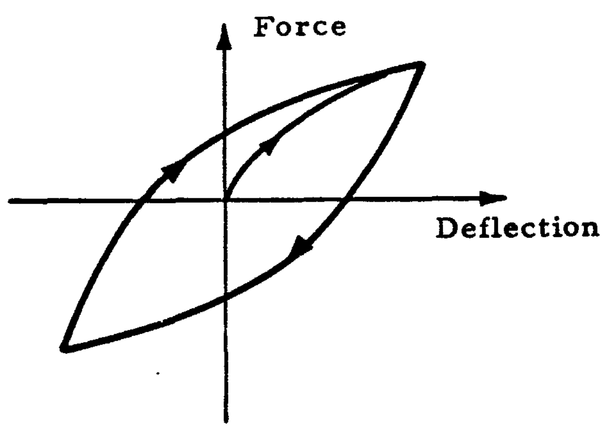


Figure 4

where the arrowheads indicate the direction of the deflection. Figure 4 shows a system oscillating between two fixed values of deflection. In the more typical case, however, a pattern such as that of Figure 5 occurs. Hence, in a transient hysteretic analysis, the entire deflection history must be taken into account. Two general methods exist to implement this accounting.

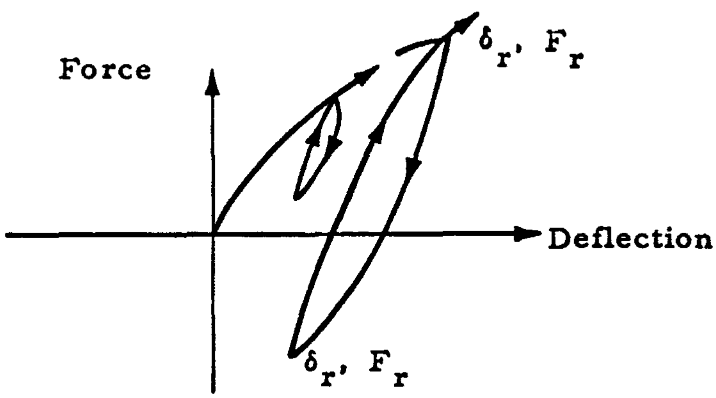


Figure 5

If the force-deflection data for the hysteresis system is available in tabular form to the computer program, the following procedure is used. The integration of the system equations of motion proceeds as previously described using the input force-deflection spring data to determine the moment (36). When the displacement at time $t+\Delta t$ is calculated to have a lesser absolute value than the absolute value of the displacement at time, t, a new hysteresis spring force-deflection curve may be required. The integration time increment is decreased by an arbitrary factor, in this program the factor is 5, and the integration is resumed. When the requirement for a new force-deflection curve is now sensed, it is constructed and the integration continues using the original time increment.

The new curve construction is accomplished by mathematically moving the 0, 0 point of the original force-deflection data to the force and deflection recorded at the time increment just prior to the spring's velocity direction change. The original force-deflection data is also altered by doubling the force portion of the data. The time increment is now restored to its original size and the integration continues using the moment calculated with the new force-deflection curve. For the second and all subsequent reversals of the spring velocity direction, the procedure is exactly the same as outlined previously with the single exception of the existing force-deflection data not being subject to the redoubling of the force data.

The second method to account for hysteresis relies upon the existance of a functional relation between the force and deflection for the spring

$$F = f(\delta) \tag{42}$$

One such relation, presented by Chang and Bieber, (Reference 3) is

$$F = Kd \qquad d \le x_o \qquad \text{(a)}$$

$$F = Kd + K\alpha\ (d - x_o) - K\alpha d\ \ln \frac{d}{x_o} \qquad d > x_o \qquad \text{(b)} \tag{43}$$

where K and α are constants and x_o is a constant whose sign is always the same as the sign of d.

With the functional relation available, the solution procedure is the same as for the tabular force-deflection data technique. However, upon the first reversal of the velocity direction, the mathematical movement of the force-deflection curve is accomplished as

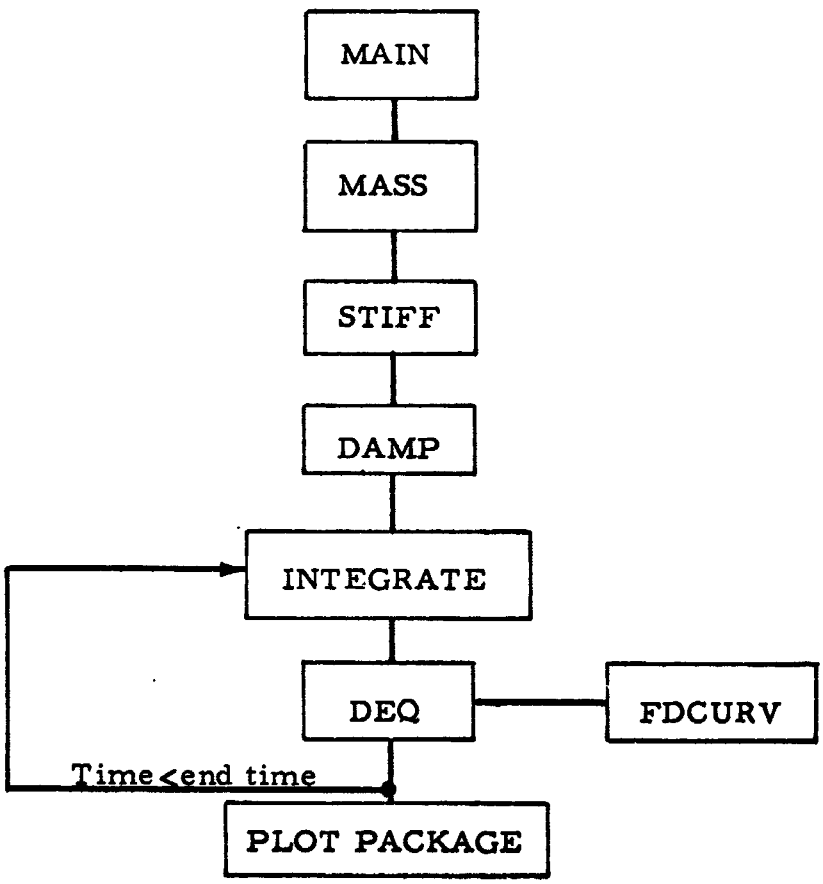
$$F = 2\, f\left(\frac{d - \delta_r}{2}\right) + F_r \tag{44}$$

where F_r, δ_r is the point on the force-deflection curve where the velocity reversal occurred. As can be expected, this latter method is computationally more swift.

Section 4
COMPUTER PROGRAM

Of the two possible methods to account for the hysteretic effects, the use of a functional relation between the force and deflection of the hysteresis spring is computationally much faster than the use of a tabular force-deflection relation. For this reason, only the program using the functional relation is presented here. As it is written, the program uses the relation of equation (43) but modification to any desired relation is an easy matter.

A brief function chart of the program is indicated below.



The MAIN program reads in most of the data and calls the balance of the subroutines in the proper order. The MASS subroutine calculates or reads in the mass matrix while the STIFF subroutine calculates or reads in the stiffness matrix. DAMP calculates the damping matrix associated with the flexible beam of the system. With the matrices assembled, INTEGRATE performs the Runge-Kutta-Gill integration and assures that the hysteresis spring force-deflection curve is correct. Subroutine DEQ is used with INTEGRATE to determine the acceleration column matrix for each succeeding time increment. The PLOT PACKAGE consists of a series of subroutines which produce plots of the data on the SC4020 system.

The manual for the proper input of data follows.

Card Number	Variable Description	Symbol	Format	Columns
1	Length of flexible part of the system.	XLEN	E12.8	1-12
2	Distance from aft end of flexible part of the system to the point of application of the force.	XFF	E12.8	1-12
3	Number of displacement functions of the elastic body which will be put into the program (7 max).	NMODE	I3	1-3
Block 1	A block of values giving the deflection at the top of the elastic body for each input displacement function. The lowest mode displacements must be given first (7 max).	PHIL(I)	6E12.8	1-12 13-24 25-36 37-48 49-60 61-72 For as many cards as necessary
Block 2	A block of values giving the slope at the top of the elastic body for each input displacement function. The lowest mode slopes must be given first (7 max).	PHIPL(I)	6E12.8	(same as Block 1)
Block 3	A block of values giving the deflections at the point of application of the force for each input displacement function. The lowest mode displacements must be given first (7 max).	PHIXF(I)	6E12.8	(same as Block 1)
4	Number of springs in the interstage (10 max).	NNLSP	I3	1-3

Card Number	Variable Description	Symbol	Format	Columns
Block 4	A block of values giving the angles, in degrees, from the assumed horizontal to the radii drawn to the assumed location of the springs. The angles must be positive numbers less than or equal to 90° (10 max).	PHIID(I)	6E12.8	(same as Block 1)
Block 5	A block of values giving the radii to the springs. The radii are positive if below the assumed horizontal thru the interstage center, negative if above. These radii must be in the same order as the angles of Block 4 (10 max).	RADNL(I)	6E12.8	(same as Block 1)
5	Linear spring constant.	CK1	E12.8	1-12
	Nonlinear parameter.	ALPH1	E12.8	13-24
	Parameter defining linear range.	XO1	E12.8	25-36
6	An indicator which defines the existence of dissipative (hysteresis) springs. If it is zero, no dissipative springs exist and the next three items are skipped. If it is not zero, dissipative springs exist and data for the next six items must be available (I3).	KKK	I3	1-3
7	Number of dissipative springs (10 max).	NSPG	I3	1-3
Block 6	A block of values giving the angle, in degrees, from the assumed horizontal to the radii drawn to the assumed location of the springs. The angles must be positive numbers less than or equal to 90° (10 max).	PHID(I)	6E12.8	(same as Block 1)

Card Number	Variable Description	Symbol	Format	Columns
Block 7	A block of values giving the radii to the springs. The radii are positive if below the assumed horizontal through the interstage center, negative if above. These radii must be in the same order as the angles of Block 8 (10 max).	RADIUS(I)	6E12.8	(same as Block 1)
8	Linear spring constant.	CK	E12.8	1-12
	Nonlinear parameter.	ALPH	E12.8	13-24
	Parameter defining linear range.	XO	E12.8	25-36
9	Amplitude of the forcing function.	AMP	E12.8	1-12
	Rotational frequency of the forcing function.	OMEG	E12.8	13-24
10	Time increment size.	DH	E12.8	1-12
	Final time of output data.	TEND	E12.8	13-24
	Starting time of output data.	TSTRT	E12.8	25-36
11	Number of points on system where response is desired (7 max).	NRESP	I3	1-3

The following two items are repeated for each point on the system where the response is desired.

Card Number	Variable Description	Symbol	Format	Columns
12	An indicator which is 1 if point of desired response is on the rigid part of the system, or 0 if the point of desired response is on the flexible part of the system.	K1O(I)	I3	1-3
13	For a point on the rigid part of the system: the distance from the interstage to the point.	XLX(I)	E12.8	1-12

Card Number	Variable Description	Symbol	Format	Columns
13 (cont.)	For a point on the flexible part of the system: the distance from the vehicle aft end to the point followed by the displacements at the point for each input elastic mode (lowest mode first) followed by the slopes at the point for each input elastic mode (lowest mode first).	XLX(I) PHILX(I, J) PHIPLX(I, J)	6E12.8	(same as Block 1)
14	An indicator which is 0 if the response is to be written, and 1 if the response is not to be written. Plots are always available.	KLM	I3	1-3
15	An indicator which is 1 if the mass matrix is input data and 0 if the mass matrix is to be calculated.	K1	I3	1-3
	An indicator which is 1 if the mass matrix is to be written, and is 0 if the matrix is not written.	K2	I3	4-6
	An indicator which is 1 if the flexible part of the system is cantilevered, and is 0 if it is free.	K4	I3	7-9

If the mass matrix is to be calculated skip the next two items.

Card Number	Variable Description	Symbol	Format	Columns
16	Number of columns or rows in the mass matrix (10 max).	K3	I3	1-3

Card Number	Variable Description	Symbol	Format	Columns
Block 8	Elements of the mass matrix. A new card must be begun for each row; elements of the first row are given first. Elements of each row must be presented in the order of increasing columns.	A(I, J)	6E12.8	(same as Block 1)

If the mass matrix is input data, skip the next seven items.

Card Number	Variable Description	Symbol	Format	Columns
17	The number of mass points in the flexible part of the system (100 values max).	NPT1	I3	1-3
Block 9	A block of distances of the mass points from the vehicle aft end (100 values max).	X(I)	6E12.8	(same as Block 1)
Block 10	A block of values of mass per unit length, in an order corresponding to the values of Block 9.	VM(I)	6E12.8	(same as Block 1)

The following block is repeated for each elastic input deflection function; the functions correspond to the order used in previous blocks.

Card Number	Variable Description	Symbol	Format	Columns
Block 11	A block of values of the deflections at each mass point for the particular deflection function. The deflections correspond to the order used in Blocks 9 and 10.	PH1(J, K)	6E12.8	(same as Block 1)
18	The number of mass points on the rigid part of the system (25 max).	NPT2	I3	1-3

Card Number	Variable Description	Symbol	Format	Columns
Block 12	The distances from the inter-stage to the masses of the rigid part of the system.	X2(I)	6E12.8	(same as Block 1)
Block 13	Mass at the rigid body mass points corresponding to the order of the distances of Block 12.	VM2(I)	6E12.8	(same as Block 1)
19	Indicator which is 1 if the stiffness matrix is input data, and is 0 if the stiffness matrix is to be calculated.	L1	I3	1-3
	Indicator which is 1 if the stiffness matrix is to be written, and is 0 if the stiffness matrix is not to be written.	L2	I3	4-6

If the stiffness matrix is to be calculated, skip the next item.

Card Number	Variable Description	Symbol	Format	Columns
Block 14	Elements of the stiffness matrix. A new card must be begun for each row; elements of the first row are given first. Elements of of each row must be presented in the order of increasing columns.	B(I, J)	6E12.8	(same as Block 1)

If the stiffness matrix is input data, skip the next six items.

Card Number	Variable Description	Symbol	Format	Columns
20	Number of points in the EI vs length tabular data (100 pts max).	NPTT	I3	1-3
Block 15	The distances from the vehicle aft end to the points where EI data are given.	XX(I)	6E12.8	(same as Block 1)

Card Number	Variable Description	Symbol	Format	Columns
Block 16	The values of EI at the points of the vehicle defined by the data of Block 15.	EI(I)	6E12.8	(same as Block 1)

The next three items are repeated for each input displacement function.

Card Number	Variable Description	Symbol	Format	Columns
21	Number of values in vehicle slope vs distance data.	NPPHP(J)	I3	1-3
Block 17	The distances from the vehicle aft end to the points where slope data are given.	XPHPP(J, I)	6E12.8	(same as Block 1)
Block 18	The values of the slope of the vehicle at the points defined by the data of Block 17.	PHIPP(J, I)	6E12.8	(same as Block 1)
Block 19	Natural frequencies associated with the input deflection functions, lowest frequency first.	OMEGA(I)	6E12.8	(same as Block 1)
Block 20	Values of system Zeta for each input function.	ZETA(I)	6E12.8	(same as Block 1)
22	Indicator which is 1 if the program is to carry some problem on in time from the point achieved by a previous program run, and is 0 if the calculations are to begin at time equal to zero.	N7	I3	1-3

If the indicator of card 22 is 1, the following data, taken from the printed output of a previous run of this program, must be used as input data. If the indicator of card 22 is 0, none of the following data are necessary.

Card Number	Variable Description	Symbol	Format	Columns
Block 21	Values of QQQ(I) array having "I" in the range of 1 to 9. Values for lowest "I" first.		6E12.8	(same as Block 1)

Card Number	Variable Description	Symbol	Format	Columns
Block 22	Values of QQQ(I) array having "I" in the range of 10 to 20. Values for lowest "I" first.		6E12.8	(same as Block 1)
Block 23	Values of DY(I) array having "I" in the range of 1 to 9. Values for lowest "I" first.		6E12.8	(same as Block 1)
Block 24	Values of DY(I) array having "I" in the range of 10 to 20. Values for lowest "I" first.		6E12.8	(same as Block 1)
Block 25	Values of Y(I) array having "I" in the range of 1 to 9. Values for lowest "I" first.		6E12.8	(same as Block 1)
Block 26	Values of Y(I) array having "I" in the range of 10 to 20. Values for lowest "I" first.		6E12.8	(same as Block 1)
23	Value of H		E12.8	1-12
Block 27	Values of FORCR(I) array. Values for smallest "I" first.		6E12.8	(same as Block 1)
Block 28	Values of ZMAX1(I) array. Values for smallest "I" first.		6E12.8	(same as Block 1)
Block 29	Values of XP(I) array. Values for smallest "I" first.		6E12.8	(same as Block 1)

The data required to generate the data plots follow all numerical data. For each plot, a card for the unknown desired to be on the horizontal axis is followed by a card for the unknown desired to be on the vertical axis, which in turn, is followed by a card which has a 1 in column 1 if another plot is desired or by a blank card which signifies the end of the required plots. The format for the cards representing any of the unknowns capable of being plotted is as follows.

Start in Column 1 of Card	Description (Columns 16 to 72)
YX (1)	Time (seconds)
YX (2)	Forcing Function (pounds)
YX (3)	Theta (degrees)
YX (4)	Conservative spring moment (inch pounds)
YX (5)	Hysteresis spring moment (inch pounds)
YX (6)	Total moment (inch pounds)

For each point of the system where the deflection or rotation is desired, the position in the YX array is determined by the order of the numerical input data of cards 12 and 13. The displacements for N points represented by sets of cards 12 and 13 will be stored in YX (7) to YX (7 + N). The rotations for N points represented by sets of cards 12 and 13 will be stored in YX (7 + N + 1) to YX (7 + N + 1 + N). For example, the plot cards for the displacement and rotation at the point represented by the second set of cards 12 and 13 when 5 sets of cards 12 and 13 are present are

YX (8)	Displacement at Point 2 (inches)
YX (13)	Rotation at Point 2 (degrees)

The descriptive data printed in columns 16 to 72 for the latter type of data are at the discretion of the user of this program.

Two example problems are presented here. The flexible part of the system represents the three lower stages of the Saturn V system. The input displacement functions ϕ_i are the modal shape functions generated by a lumped spring analysis and associated computer program (Reference 3) using the parameters of the previously mentioned system. The ϕ_i are orthogonal functions. The rigid beam is represented by one lumped mass. This mass, in turn, represents the Apollo payload stages. The Saturn V is assumed to be cantilevered, i. e., still on the launching pad.

All of the results and the input data are presented in the Appendix.

Section 5
RECOMMENDED ADDITIONAL DEVELOPMENTS

As stated in Section 1, Introduction, the mathematical model used for this program is quite rudimentary. It has been used here as a developmental model to permit an initial application of the hysteretic experience gained in the present program.

In themselves, the computer programs could be modified to include a flexible second beam instead of the rigid beam now used. They could be modified quite easily to accept a variety of hysteresis system force-deflection functions and forcing functions. It is desired, however, to make these modifications as a part of any of the following options.

When the provision is made for different hysteresis spring force-deflection functions, a mechanical analog can be built for this analysis. An experimental model consisting of a cantilevered flexible beam of accurately known properties, an interstage assembly of unknown properties, and an upper beam fulfilling the rigidity requirements could be built. The lower beam, for example, could be a beam of rectangular cross section. With an accurately controlled forcing function applied to the flexible beam in compliance with the mathematical model, the response history of a few points on the system could be recorded. By comparing these recorded histories with those developed by one of the computer programs for a variety of interstage force-deflection curves, it would be possible to determine the hysteretic properties of various materials and/or fabrication techniques in the interstage assembly of the mechanical analog.

The computer programs could also be modified to include bending in more than the single plane used here. Some complication would be added in the generation of the interstage moments but the balance of the modification would be quite straightforward. With this development, it would then be possible to compare results of this program to the experimental test results generated for the Saturn V system. Once again the program could be used to generate the interstage properties in the manner described above concerning the mechanical analog.

Another option which would require a small modification to the existing program is to generate an interstage hysteresis spring force-deflection relation by the method of Reference 1, using the energy dissipated per cycle in the interstage. This will lead directly to the improved force-deflection relation.

REFERENCES

1. Chang, C. S. and R. E. Bieber, "Synthesis of Structural Damping," LMSC/HREC A783975, Lockheed Missiles and Space Company, Huntsville Research and Engineering Center, Huntsville, Alabama, April 1967.

2. Gill, S., "A Process for the Step-by-Step Integration of Differential Equations in an Automatic Digital Computing Machine," Proceedings of the Cambridge Philosophical Society, Vol. 47, 1951, pp. 96-108.

3. Taylor, A., "The Lumped Spring Computer Program," TM 54/50-22, LMSC/HREC A712616, Lockheed Missiles and Space Company, Huntsville Research and Engineering Center, Huntsville, Alabama, November 1965.

APPENDIX

APPENDIX

The input data for one example problem and the results of two example problems are presented below. In these problems the flexible part of the system represents the empty lower three stages of the Saturn V system.

For the first problem, the forcing function acts for 13 seconds of problem time and is then set equal to zero, permitting the system to vibrate freely after that time. This is accomplished by a small program modification. The input data and results of this problem are presented below.

```
$DATA
  3680.0
    60.0
  2
    1.00          1.00
 .385494E-03  .153208E-02
 .012181193  -.062402129
  6
    27.0        83.0        23.0        27.0        83.0        23.0
    37.0        37.0        37.0       -37.0       -37.0       -37.0
   48000.0      0.20        0.00001
  1
  6
   27.0         83.0        23.0        27.0        83.0        23.0
   37.0         37.0        37.0       -37.0       -37.0       -37.0
   48000.0      0.20        0.00001
  500.0         3.558379
  0.008        20.00       15.00
  5
  0
  60.0         .121811933 -.062402129  .111284E-03-.190479E-03
  0
 1080.0        .19196503  -.33091658   .217731E-03-.175269E-03
  0
 2100.0        .45858617  -.36704829   .280471E-03-.141879E-03
  0
 3680.0        1.0         1.0         .385494E-03 .153208E-02
```

```
  1
  60.0
  1
  1  0  1
  5
  1.00          0.0          0.0          0.0          0.0
  0.0           1.0          0.0          0.0          0.0
  0.0           0.0          .107712 E 6  .1836722E 4  .1960223E 4
  0.0           0.0          .1836722E 4  .2000233E 3  .3342604E 2
  0.0           0.0          .1960223E 4  .3342604E 2  .1378138E 3
  1  0
  0.0           0.0          0.0          0.0          0.0
  0.0           0.0          0.0          0.0          0.0
  0.0           0.0          0.0          0.0          0.0
  0.0           0.0          0.0          .109154 E 4  0.0
  0.0           0.0          0.0          0.0          .1717691E 5
 3.558378      15.4799
     .01            .01
  0
YX(1)           TIME (SEC)
YX(2)           FORCE  (LBS.)
1
YX(1)          TIME (SEC)
YX(3)           THETA (DEGREES)
1
YX(1)           TIME (SEC)
YX(4)           SPRING MOMENT  (INCH LBS)
1
YX(1)           TIME (SEC)
YX(5)          HYSTERISIS MOMENT
1
YX(1)           TIME (SEC)
YX(6)           TOTAL MOMENT (INCH LBS)
1
YX(1)           TIME (SEC)
YX(7)           DISPLACEMENT AT 60
1
YX(1)           TIME (SEC)
YX(8)           DISPLACEMENT AT 1080
1
YX(1)           TIME (SEC)
YX(9)           DISPLACEMENT AT 2100
1
YX(1)           TIME (SEC)
YX(10)           DISPLACEMENT AT 3680
1
YX(1)           TIME (SEC)
YX(11)          RIGID CAPSULE DISPLACEMENT
```

```
1
YX(1)          TIME (SEC)
YX(12)         ROTATION AT 60
1
YX(1)          TIME (SEC)
YX(13)         ROTATION AT 1080
1
YX(1)          TIME (SEC)
YX(14)         ROTATION AT 2100
1
YX(1)          TIME (SEC)
YX(15)         ROTATION AT 3680
1
YX(1)          TIME (SEC)
YX(16)         RIGID CAPSULE DISPLACEMENT
```

For the second problem, the forcing function frequency was set equal to the natural frequency of the system determined from the first problem results. Again, the force was allowed to act for 13 seconds of problem time and then was set equal to zero. The plotted results indicate the response for 20 seconds of problem time.

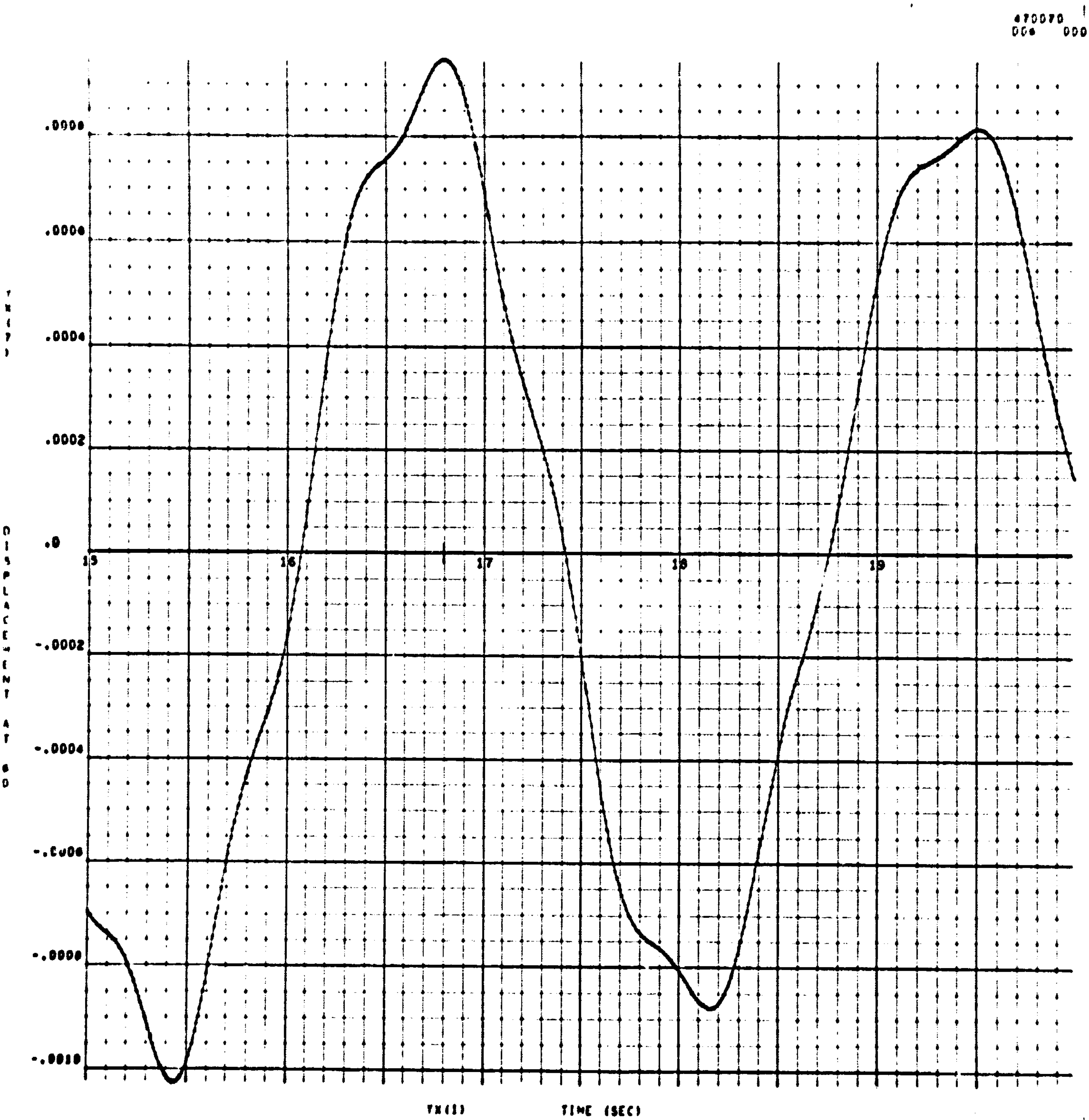


Figure 1a - First Example Problem

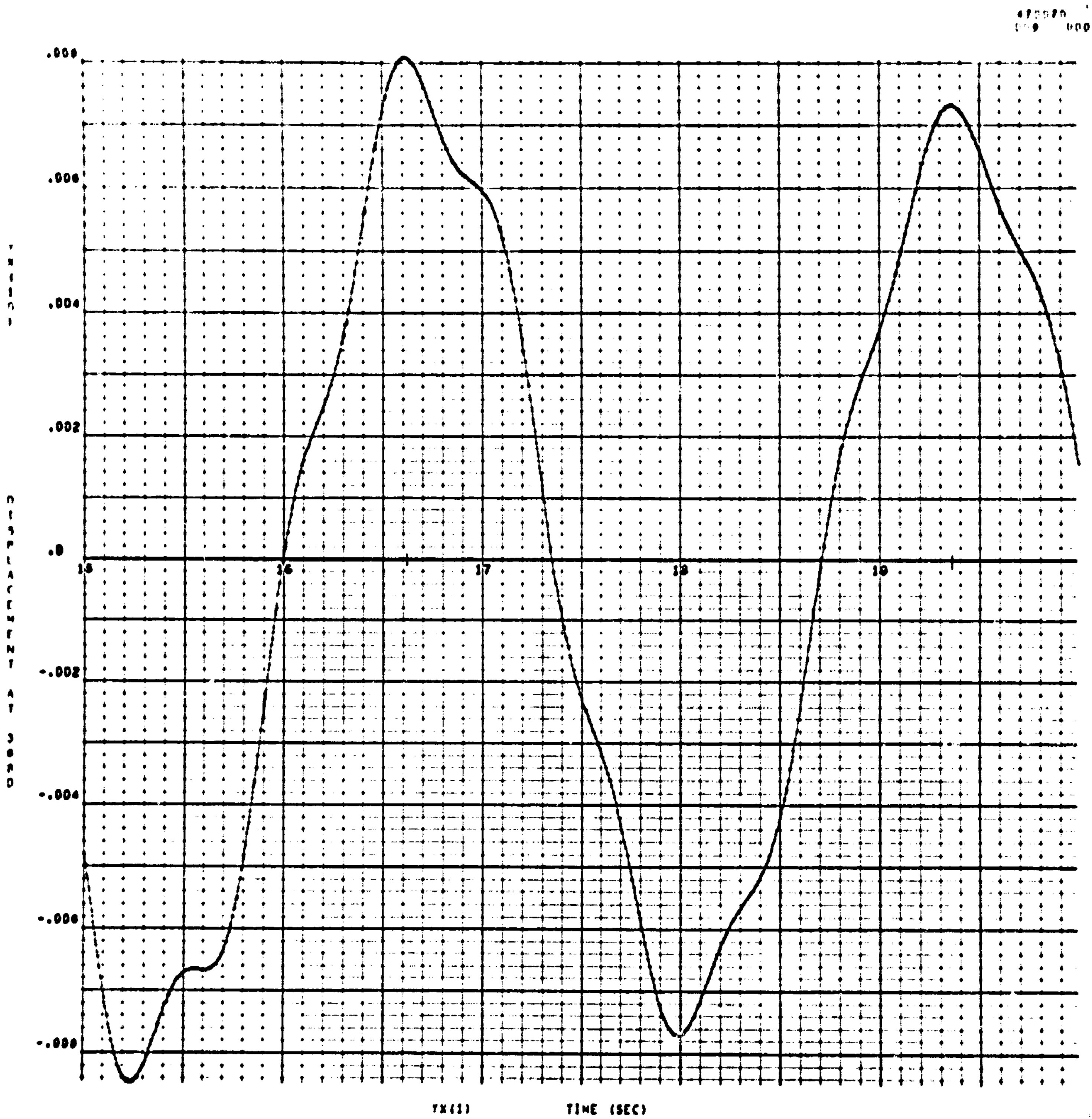


Figure 1b - First Example Problem

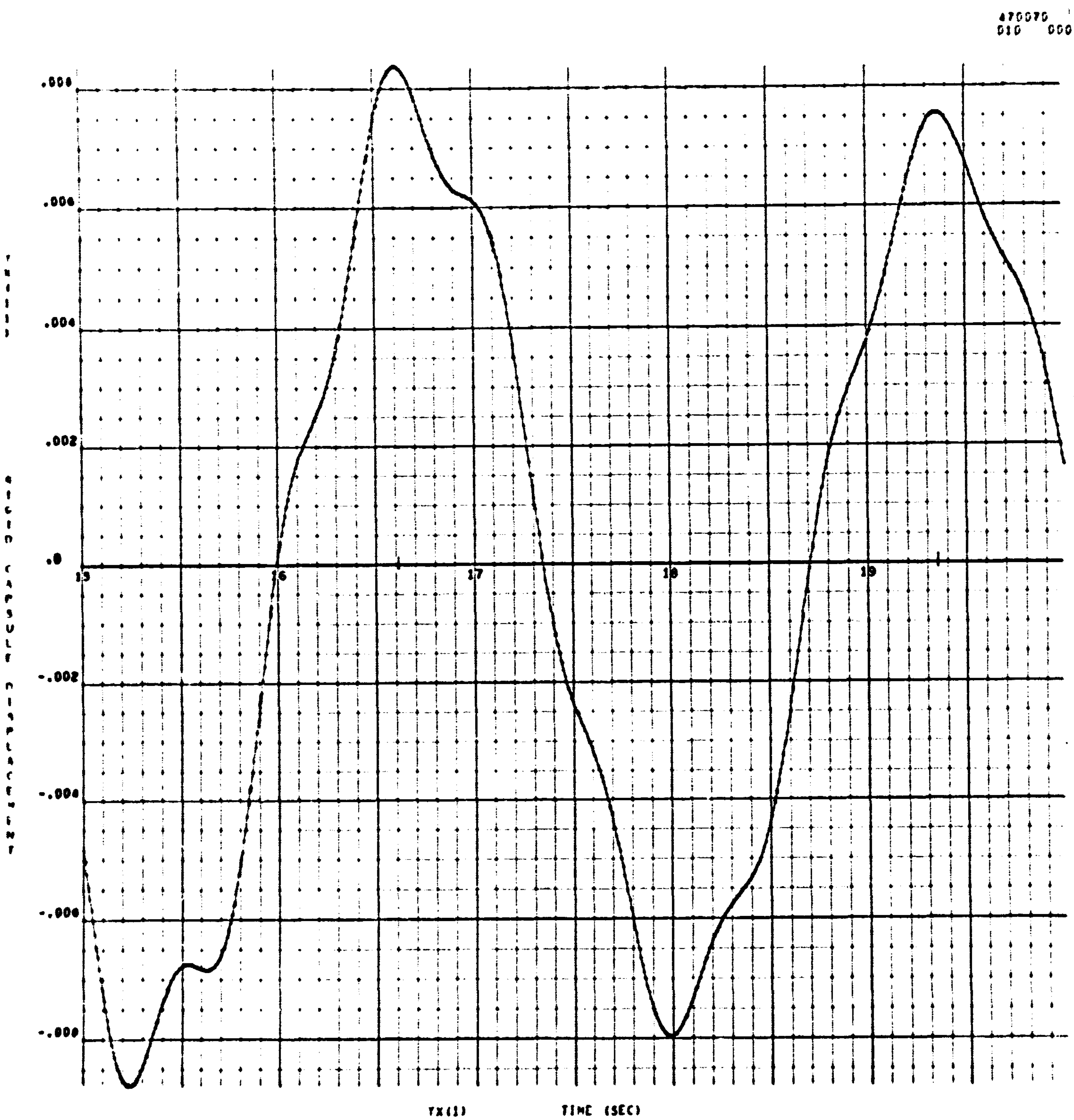


Figure 1c - First Example Problem

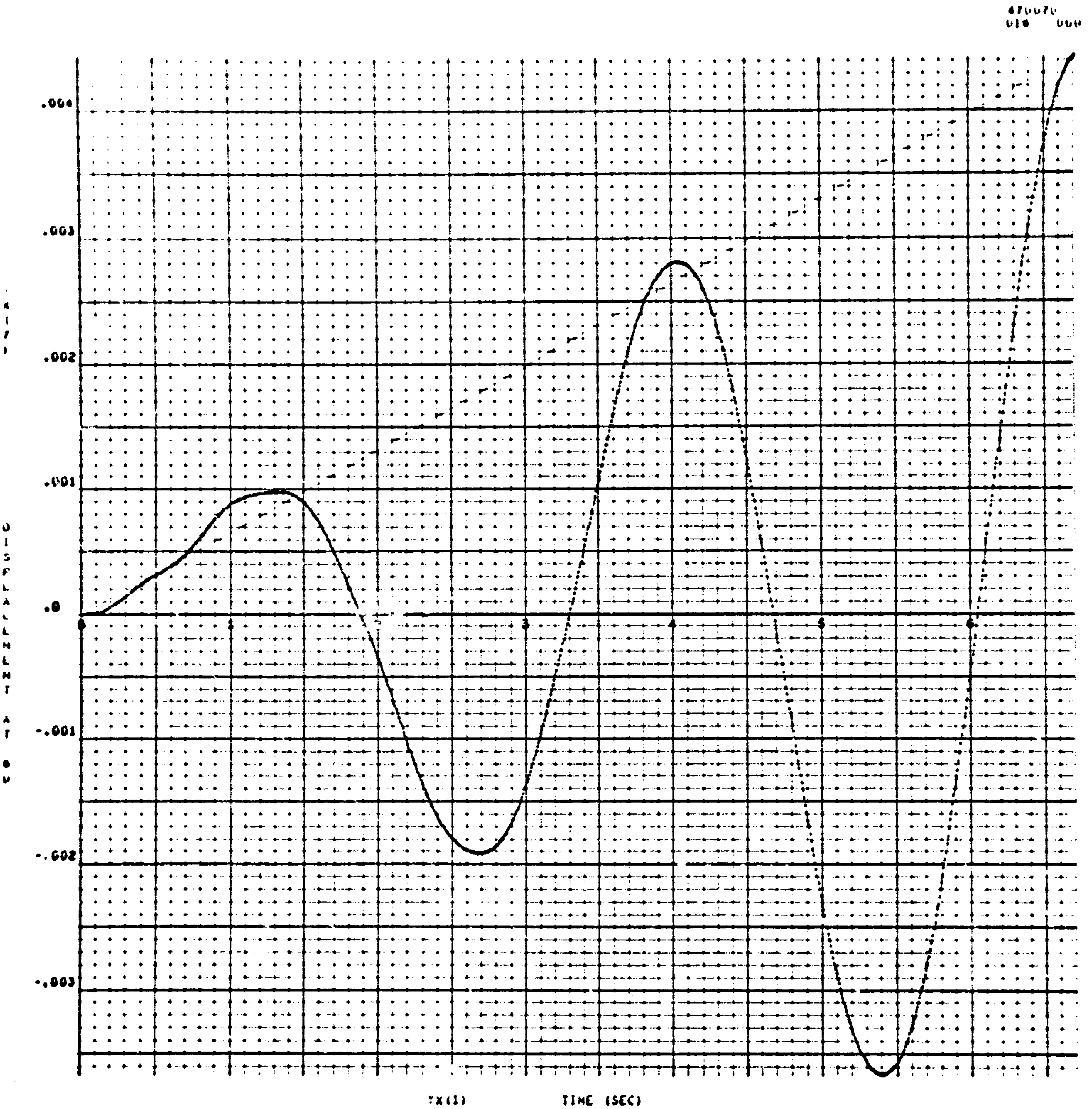


Figure 2a - Second Example Problem

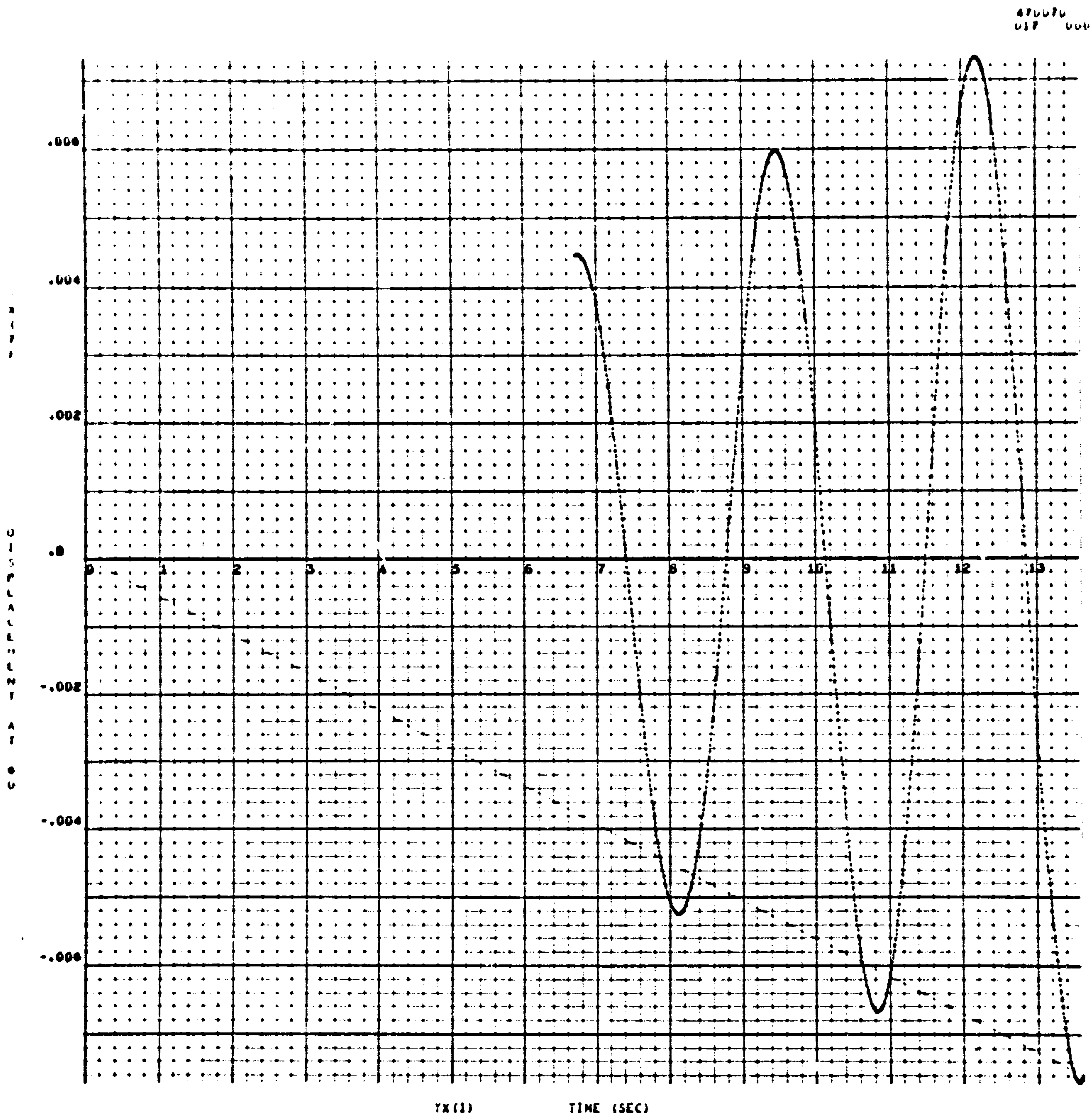


Figure 2b - Second Example Problem

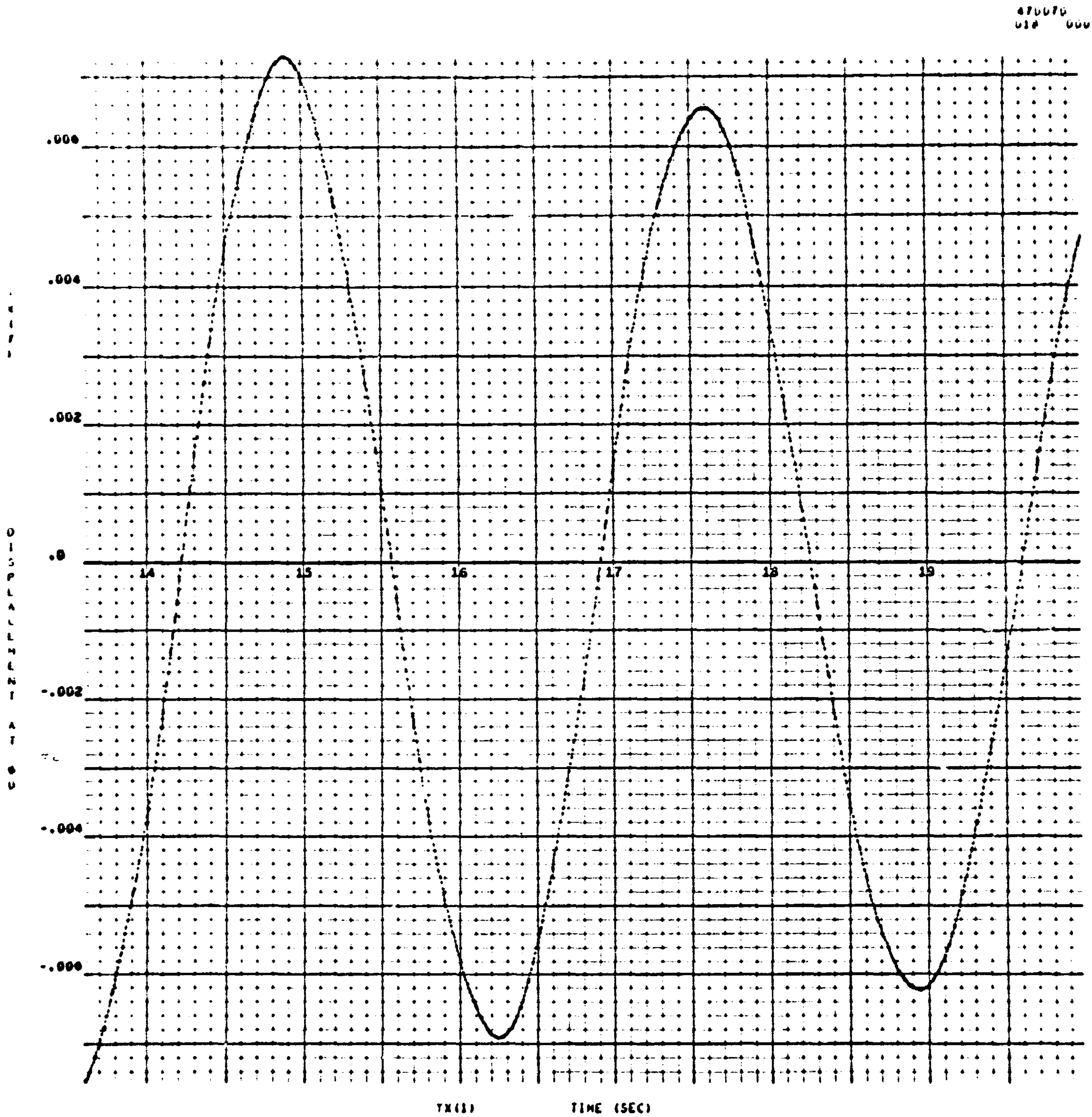


Figure 2c - Second Example Problem

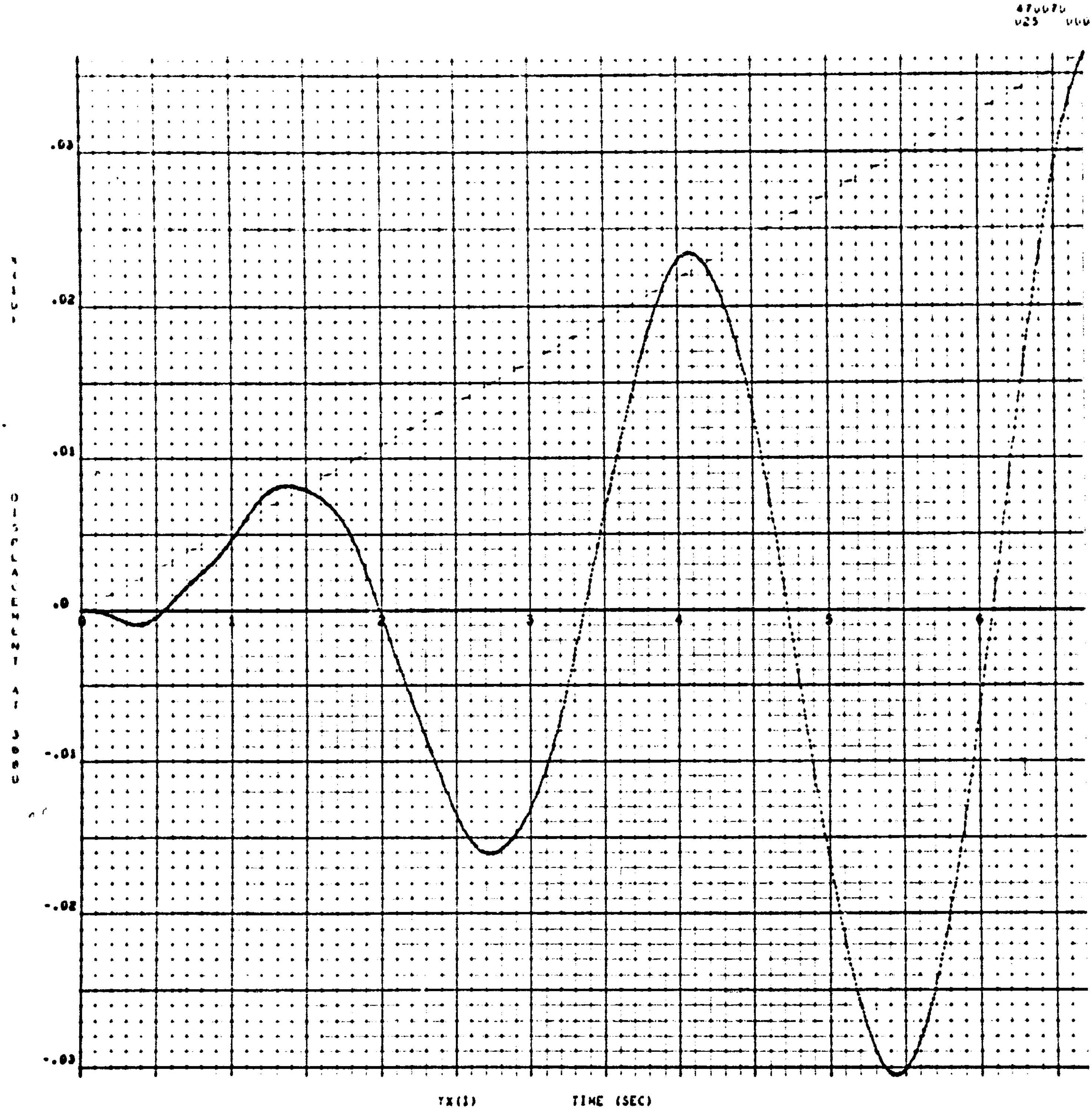


Figure 2d - Second Example Problem

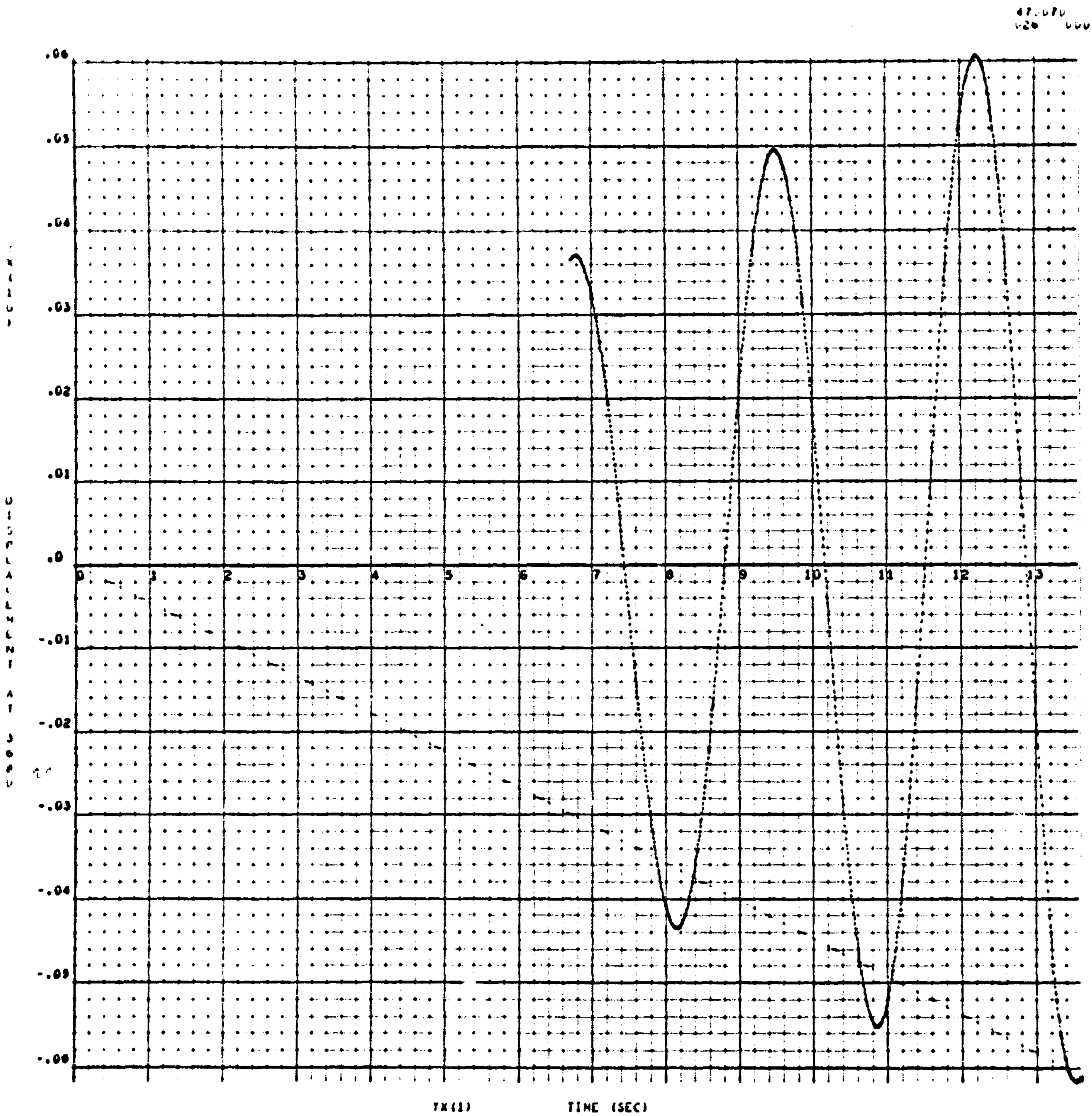


Figure 2e - Second Example Problem

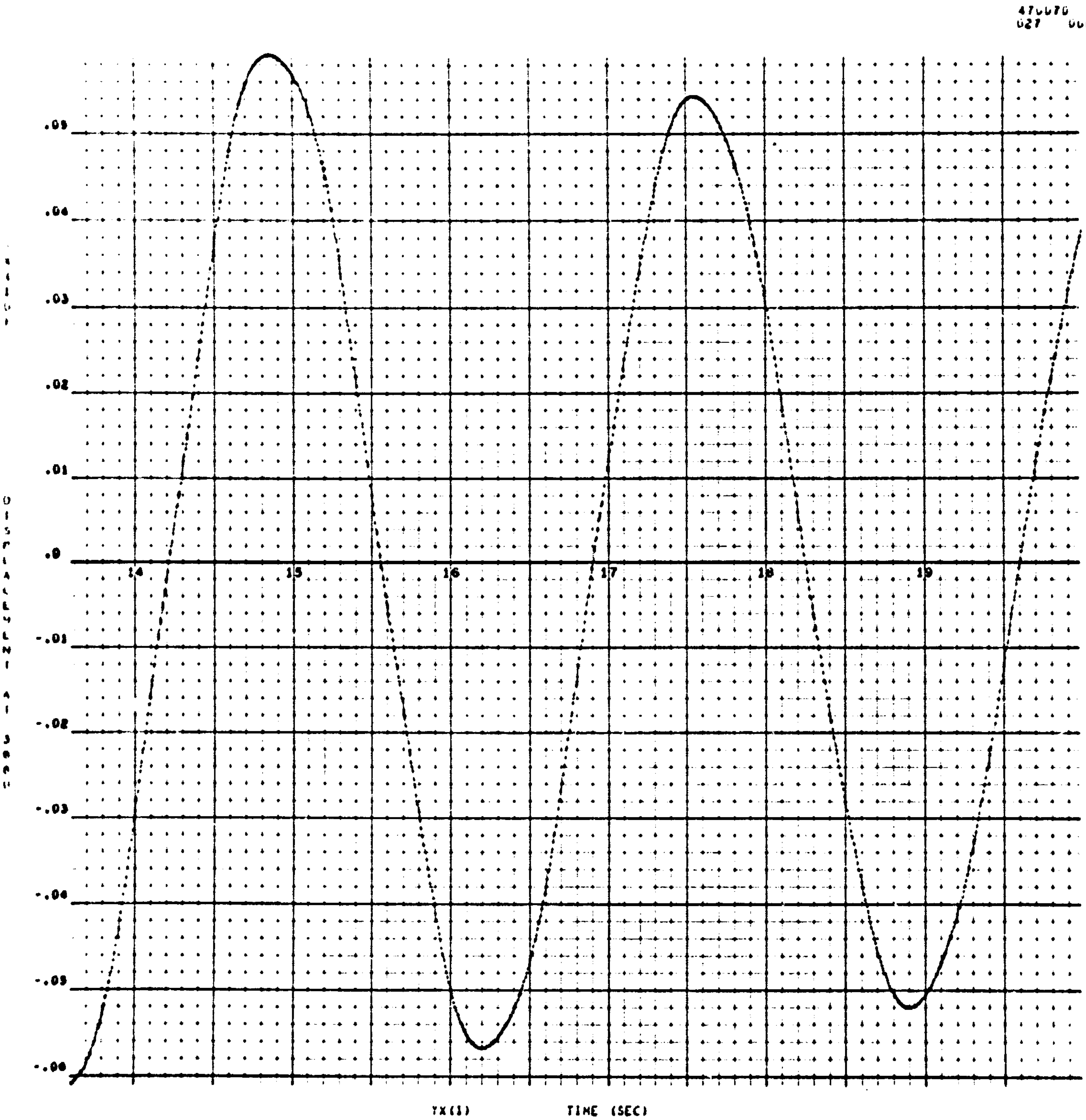


Figure 2f - Second Example Problem

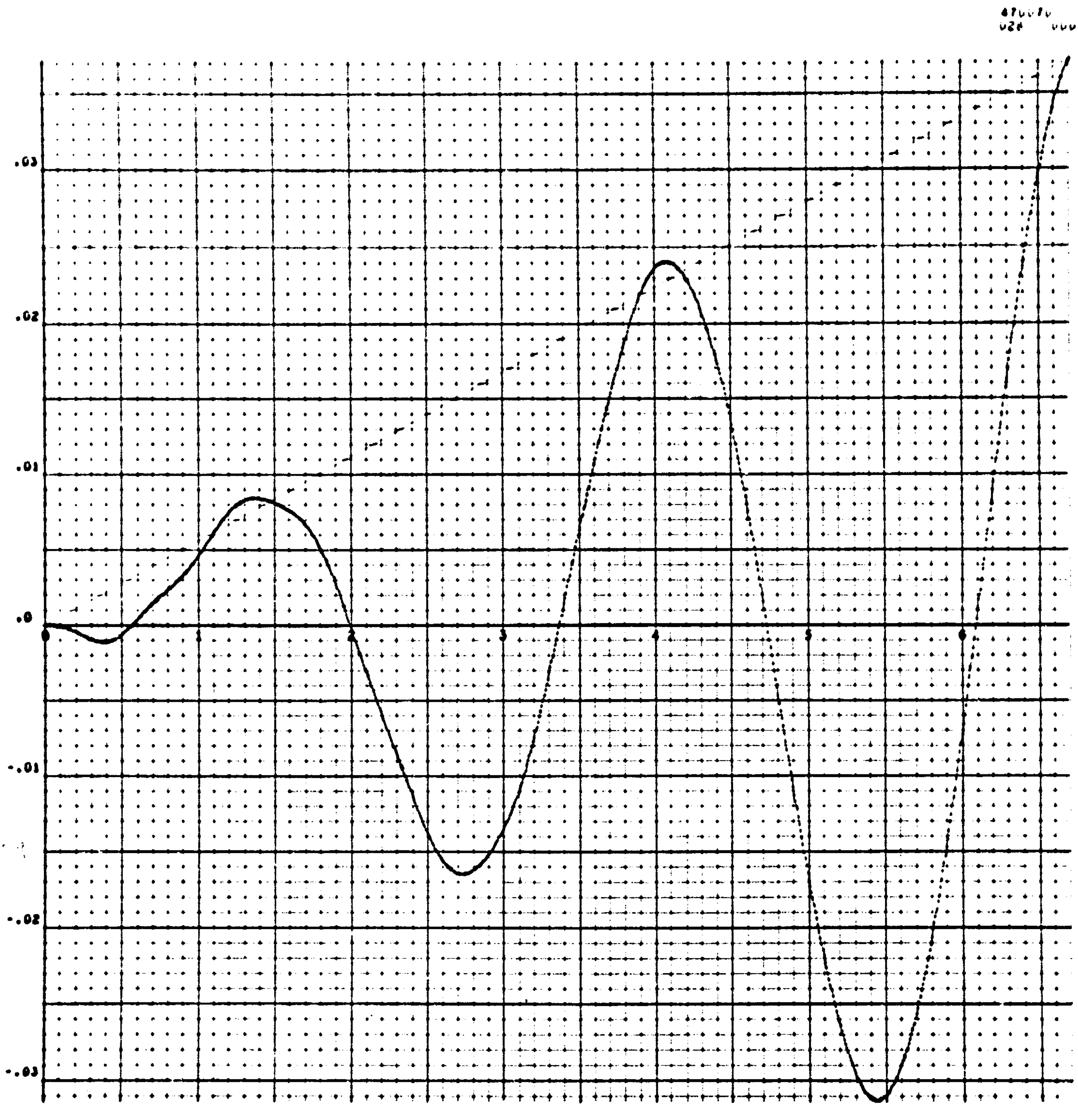


Figure 2g - Second Example Problem

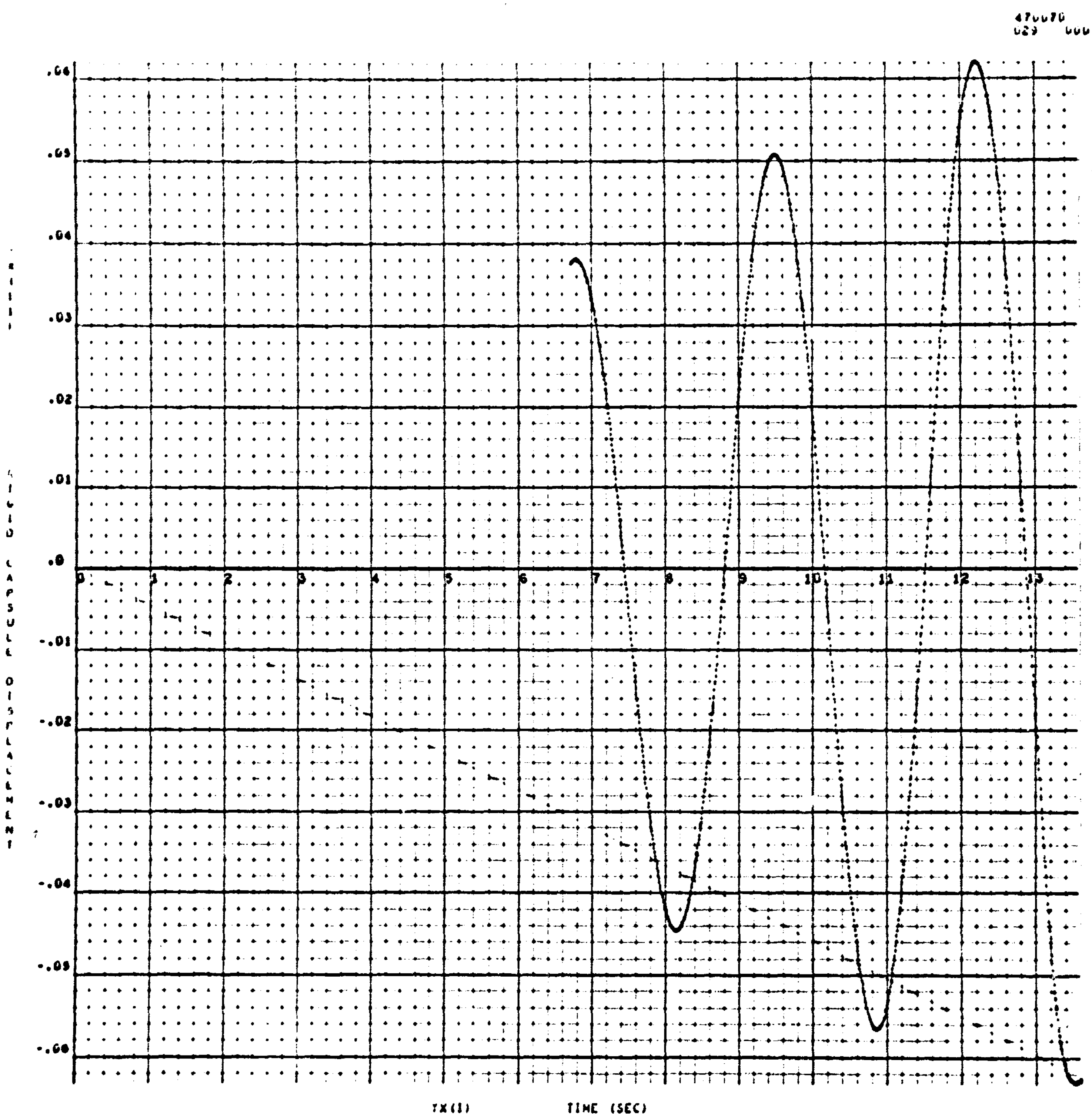


Figure 7h - Second Example Problem

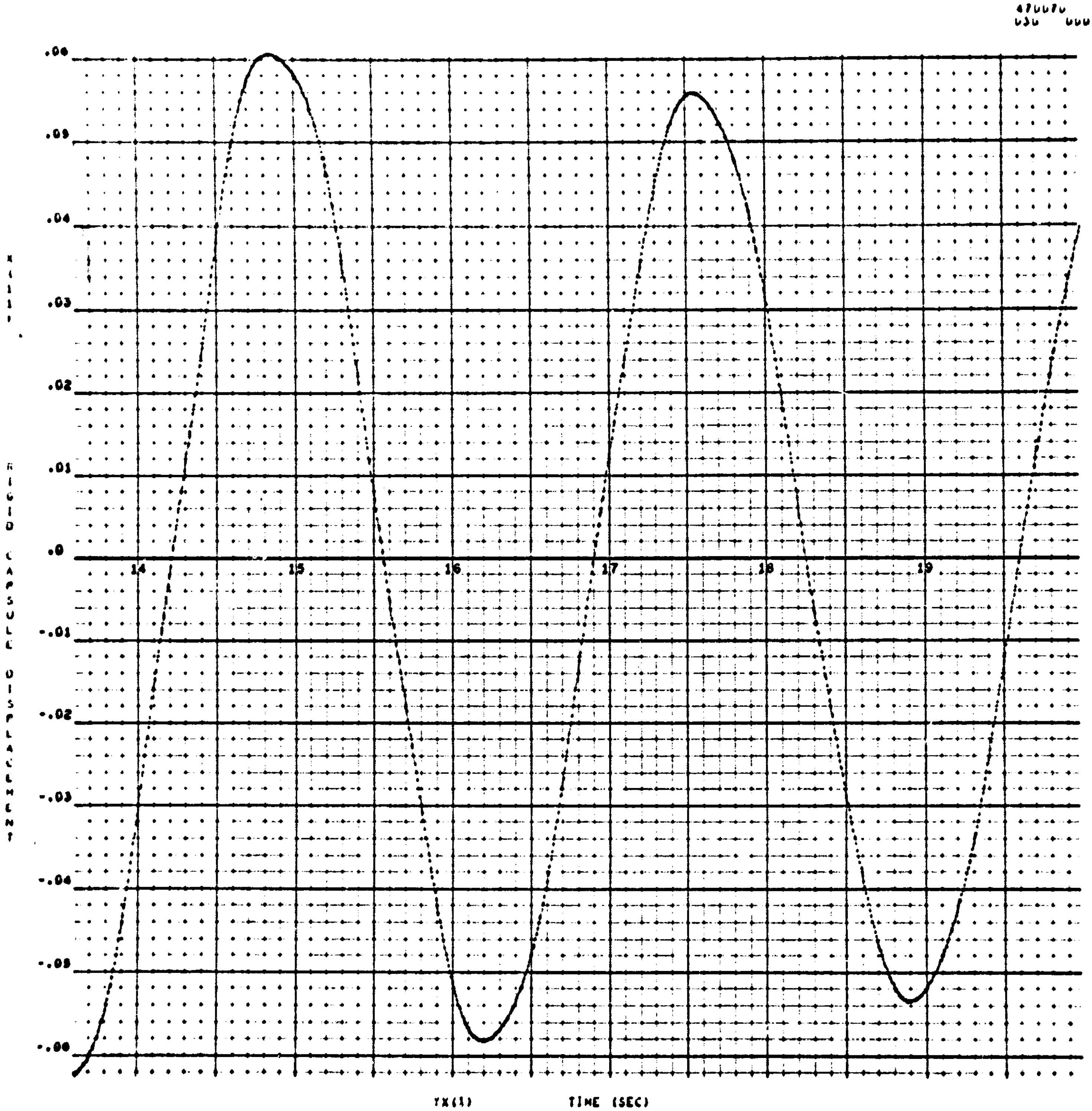


Figure 2i - Second Example Problem