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REPORT 4: INSTABILITY OF PERIODIC MOTIONS NEAR COLLINEAR POINTS OF LIBRATION IN THE RESTRICTED PROBLEM OF THREE BODIES

by

G. N. Duboshin

Astronomicheskii Zhurnal, 15, No. 3, 209-216 (1938)

Translated from the Russian

February 1967

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This article examines the problem of periodic motions near collinear points of libration in the plane restricted problem of three bodies. With the aid of the Lyapunov methods, we demonstrate that around each collinear point of libration, a continuous family of periodic motions exists, dependent on one arbitrary constant. We examine the stability of the indicated periodic motions and show that they are unstable. Finally, we establish the orbital instability of the periodic motions under study.

We shall proceed from differential equations, determining the movement of a particle (zero mass) near any collinear point of libration. To be definite, we shall examine point L_2. The indicated equations can be written as follows:

$$\left.\begin{aligned} \frac{d^2\xi}{dt^2} - 2n\frac{d\eta}{dt} &= \frac{\partial\Omega}{\partial\xi}, \\ \frac{d^2\eta}{dt^2} + 2n\frac{d\xi}{dt} &= \frac{\partial\Omega}{\partial\eta}, \end{aligned}\right\} \tag{1}$$

where ξ and η --particle coordinates in a system of coordinates with its beginning at point L_2, $n^2 = 1 + \mu$ and Ω is determined by formula

$$2\Omega = \rho_1{}^2 + \frac{2}{\rho_1} + \mu\left(\rho_2{}^2 + \frac{2}{\rho_2}\right), \tag{2}$$

where

$$\left.\begin{aligned} \rho_1{}^2 &= \left(\xi + a - \frac{\mu}{1+\mu}\right)^2 + \eta^2, \\ \rho_2{}^2 &= \left(\xi + a + \frac{1}{1+\mu}\right)^2 + \eta^2. \end{aligned}\right\} \tag{3}$$

Finally, $|a|$ is the distance of the point of libration to the center of gravity of two finite masses which are equal to 1 and μ, respectively. Since function Ω is a holomorphic function of ξ and η, at least in the direct proximity of the point of libration, it can be expanded into Taylor's series, converging absolutely for all values of ξ and η, the moduli of which do not exceed known limits. This series can be written in the form

$$\begin{aligned} \Omega = \frac{1}{2}p\xi^2 - \frac{1}{2}q\eta^2 - \alpha\xi^3 + \frac{3}{2}\alpha\xi\eta^2 + \beta\xi^4 - 3\beta\xi^2\eta^2 \\ + \frac{3}{8}\beta\eta^4 - \gamma\xi^5 + 5\gamma\xi^3\eta^2 - \frac{15}{8}\xi\eta^4 + \ldots, \end{aligned} \tag{4}$$

where

$$\left.\begin{aligned} p &= 1 + \mu + 4f, \quad q = -1 - \mu + 2f, \\ 2f &= \frac{1}{\rho^3_{10}} + \frac{\mu}{\rho^3_{20}}, \quad \alpha = \frac{1}{\rho^4_{10}} + \frac{\mu}{\rho^4_{10}}, \\ \beta &= \frac{1}{\rho^5_{10}} + \frac{\mu}{\rho^5_{20}}, \quad \gamma = \frac{1}{\rho^6_{10}} + \frac{\mu}{\rho^6_{20}}, \end{aligned}\right\} \tag{5}$$

whereby

$$\rho_{10} = \left| a - \frac{\mu}{1+\mu} \right|, \quad \rho_{20} = \left| a + \frac{1}{1+\mu} \right|. \tag{6}$$

If we retain only the terms of the second degree in our expansion of function Ω, Equation (1) will be transformed into linear equations with constant coefficients. The determining equation of this linear system, as is well known, has the form

$$x^4 - (p - q - 4n^2)x^2 - pq = 0. \tag{7}$$

This equation has the following roots:

$$\lambda i, \ -\lambda i, \ \lambda_1, \ -\lambda_1, \quad \left(i = \sqrt{-1}\right), \tag{8}$$

where

$$\left.\begin{aligned} \lambda_2 &= 1 + \mu - f + \sqrt{9f^2 - 4f(1+\mu)}, \\ \lambda_1^2 &= -1 - \mu + f + \sqrt{9f^2 - 4f(1+\mu)}. \end{aligned}\right\} \tag{9}$$

λ^2 always differs from zero and is positive. λ_1^2 becomes zero only with $\mu \neq 0$. Therefore, with all values of μ other than zero, Equation (7) possesses two purely imaginary and two real roots, one of which is positive and the other negative. The presence of one positive root of Equation (7) causes instability of collinear libration points in the Lyapunov sense, which is a well-known fact.

Let us now examine the matter of the existence of periodic motions near collinear points. This problem is solved without difficulty, with the aid of Lyapunov's theorem alone, proof for which can be found in his well known essay[1]. This theorem can be formulated as follows: if the determining equation of a system of disturbed motion equations has two simple purely imaginary roots, whole multiples of which are not equation roots, and if the latter does not possess zero roots, any time it is possible to find periodic series of a certain type which would formally satisfy the system of equations of disturbed motion, the latter, in fact, has a periodic solution presented by such series.

Thus, to find a periodic solution, it is necessary to set up series of which all terms are periodic functions. Therefore, in the general case, it is impossible to establish the existence of periodic solutions a priori, for it is virtually impossible to determine all terms of an infinite series.

Fortunately, there is at least one case whereby the existence of a periodic solution can be established a priori. This will be a case when equations of disturbed motion have a holomorphic integral, independent of time, of a definite structure. For example, if equations of disturbed motion have canonical form

$$\frac{dx_s}{dt} = -\frac{\partial H}{\partial y_s}, \quad \frac{dy_s}{dt} = \frac{\partial H}{\partial x_s}, \quad (s = 1, 2, \ldots, n),$$

where H is a time-independent holomorphic function of variables x_s and y_s in which the terms of the lowest order generate a ground form, the equations have integral H= const., and if the determining equation has a pair of simple purely imaginary roots λi and $-\lambda i$, not possessing roots of the type $m\lambda i$, where $m=0, \pm 1, \pm 2, \pm 3, \ldots$, the disturbed motion equations will, without doubt, have a periodic solution represented by periodic series of known form.

Let us see what can be said of the existence of periodic motions near collinear libration points in the restricted problem of three bodies. As we have seen, the determining equation in this problem always has two simple purely imaginary roots and does not have (if $\mu \neq 0$) zero roots or roots of type $m\lambda i$, where m is a whole number. It is further known that system (1) has an integral independent of time. To become convinced that this integral has the required form, it is sufficient to reduce Equation (1) to canonical form, assuming

$$\xi_1 = \frac{d}{dt} - \eta \cdot \eta, \quad \eta_1 = \frac{d\eta}{dt} + \eta \cdot \xi .$$

Then Equation (1) will be replaced by the following system:

$$\left.\begin{aligned} \frac{d\xi}{dt} &= -\frac{\partial H}{\partial \xi_1}, \quad \frac{d\xi_1}{dt} = \frac{\partial H}{\partial \xi}, \\ \frac{d\eta}{dt} &= -\frac{\partial H}{\partial \eta_1}, \quad \frac{d\eta_1}{dt} = \frac{\partial H}{\partial \eta}, \end{aligned}\right\} \tag{10}$$

where

$$H = \Omega - \frac{n^2}{2}(\xi^2 + \eta^2) + n(\xi\eta_1 - \eta\xi_1) - \frac{1}{2}(\xi_1^2 + \eta_1^2). \tag{11}$$

Since H is independent of time, Equation (10) has a holomorphic integral in which the terms of the lowest order assume ground form H_2, of type

$$H_2 = 2f\xi^2 - f\eta^2 + n(\xi\eta_1 - \eta\xi_1) - \frac{1}{2}(\xi_1^2 + \eta_1^2). \tag{12}$$

Therefore, system (10) indubitably has a periodic solution, and consequently, such a solution will exist for system (1).

Thus, we have established the existence of periodic motions near collinear points of libration, and it remains for us merely to find these motions. As Lyapunov[2] points out, for this we must proceed as follows. Assuming c to stand for arbitrary constant and T for series

$$T = \frac{2\pi}{\lambda}\left(1 + h_2c^2 + h_3c^3 + \dots\right) \tag{13}$$

with indefinite coefficients h_2, h_3, . . . , we shall insert in Equation (1) a new independent variable τ to replace t by substituting

$$\tau = \frac{2\pi(t - t_0)}{T}. \tag{14}$$

Constants h_2, h_3, . . . shall be selected in such a manner that transformed equation

$$\left.\begin{aligned} \frac{d^2\xi}{d\tau^2} - 2n\frac{T}{2\pi}\cdot\frac{d\eta}{d\tau} &= \frac{T^2}{4\pi^2}\cdot\frac{\partial\Omega}{\partial\xi}, \\ \frac{d^2\eta}{d\tau^2} + 2n\frac{T}{2\pi}\cdot\frac{d\xi}{d\tau} &= \frac{T^2}{4\pi^2}\cdot\frac{\partial\Omega}{\partial\eta}, \end{aligned}\right\} \tag{15}$$

will be satisfied by series

$$\left.\begin{aligned} \xi = x_0 = c x_1 + c^2 x_2 + c^3 x_3 + \ldots, \\ \eta = y_0 = c y_1 + c^2 y_2 + c^3 y_3 + \ldots, \end{aligned}\right\} \tag{16}$$

in which all coefficients, x_1, y_1, x_2, $y_2, \ldots$, will be periodic functions of τ with common period 2π. Since the existence of a periodic solution in our problem is known a priori, such series will indeed be found and will be absolutely convergent at all values of τ and values of c satisfying condition

$$|c| \leqslant \bar{c} . \tag{17}$$

where $\bar{c}$ is a certain positive value differing from zero. Series (13) will also converge absolutely with $|c| \leqslant \bar{c}$, and value T will be the period of the found motion (in respect to time). We have thus obtained a continuous series of periodic motions depending on two arbitrary constants, t_0 and c. Constant t_0 has no significant effect of the nature of the motion, since we can assume that the family of periodic motions depends solely on arbitrary constant c.

Determining in practice functions x_1, y_1, x_2, $y_2, \ldots$, we shall obtain for them expressions of form

$$\left.\begin{aligned} x_1 &= A_1 \left[\cos\tau + \sin\tau\right], \\ y_1 &= B_1 \left[\cos\tau - \sin\tau\right], \\ x_2 &= A_2 \sin 2\tau + C_2, \\ y_2 &= B_2 \cos 2\tau \\ x_3 &= A_3 \left[\cos 3\tau - \sin 3\tau\right], \\ y_3 &= B_3 \left[\cos 3\tau + \sin 3\tau\right], \\ &\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \tag{18}$$

where A_1, B_1, A_2, B_2, $C_2, \ldots$ are constants dependent only on μ. For example,

$$A_1 = \frac{2n}{\lambda}, \quad B_1 = 1 + \frac{p}{\lambda^2} .$$

We shall not write out expressions for the remaining coefficients, in view of their awkwardness.

Let us now turn to an investigation of the stability of the periodic motions found by us. In the absolute sense, this problem is solved immediately and in the negative. Indeed, let us examine any of the found periodic motions. It corresponds to a certain value of constant c, and consequently, a certain period T. Since the family of periodic solutions is continuous in respect to c, there exists an innumerable quantity of other periodic motions orbitally as close as desired to that under examination. The periods of these orbitally close motions will differ from the period of the motion in question, but they will differ in a degree as small as desired. Let us now picture two particles located at the initial moment in two periodic orbits as close as desired, and possessing initial velocity coordinate and component values as close as desired. Let us say that T and T_1 are corresponding periods and let us assume, to be more definite, that $T < T_1$. It is obvious that after time interval T, the first particle will arrive at its initial state at that time when the second has not yet been able to reach that state. In other words, after time interval T, the second particle will lag somewhat behind the first and this gap will continue to grow with time. After a sufficiently large number of revolutions, our particles will be separated a distance comparable to the dimensions of their orbits and thus will cease being as close to one another as desired. This means that each periodic motion obtained by us is unstable in the Lyapunov sense.

But instability of this type is not particularly interesting to us. Indeed, the behavior of the other motions in our problem is still an unknown, that is, motions not belonging to our periodic family. It may occur that trajectories of all remaining motions, as close as desired to periodic according to initial data, will always be as close as desired. This will mean that the periodic motion is orbitally stable. On the other hand, it may occur that motions will be found in our problem as close to periodic as desired according to initial data, the trajectories of which will not remain as close as desired to periodic orbit, which will signify that this periodic motion is also orbitally unstable.

To elucidate this important matter of orbital stability of periodic motions near collinear points of libration we should examine Equation (15), in which τ is an independent variable. Indeed, in respect to τ, all periodic motions of the family possess common period 2π, as a consequence of which instability effect disappears (an effect occuring due to difference in periods of periodic orbits as close together as desired). We shall examine the problem of stability of any of solutions (16) of system (15). For this, we shall first insert new variables x and y in place of ξ and η, assuming

$$\xi = x + x_0, \qquad \eta = y + y_0. \tag{19}$$

The transformed equations will have the form

$$\left.\begin{aligned}\frac{d^2x}{d\tau^2} - 2n\frac{T}{2\pi}\cdot\frac{dy}{d\tau} &= \frac{T^2}{4\pi^2}\cdot\frac{\partial R}{\partial x},\\ \frac{d^2y}{d\tau^2} + 2n\frac{T}{2\pi}\cdot\frac{dx}{d\tau} &= \frac{T^2}{4\pi^2}\cdot\frac{\partial R}{\partial y},\end{aligned}\right\} \tag{20}$$

where

$$R(x,\ y) = \Omega\,(x + x_0,\ y + y_0). \tag{21}$$

Since all periodic orbits lie in an area where function Ω is holomorphic, function $R(x, y)$ will also be a holomorphic function, at least for values of x and y, the moduli of which are sufficiently small. Therefore, function $R(x, y)$ can be represented in the form of a series with coefficients periodic in relation to τ. This series will be written as follows:

$$R = \frac{1}{2}\, p_{11}(\tau)x^2 + p_{12}(\tau)xy + \frac{1}{2}p_{22}(\tau)y^2 + \ldots, \tag{22}$$

where the first coefficients will be determined by formulas

$$\left.\begin{aligned}p_{11}(\tau) &= p - 6\alpha x_0 + 12\beta x_0^2 - 6\beta_0^2 + \ldots,\\ p_{12}(\tau) &= 3\alpha y_0 - 12\beta x_0 y_0 + \ldots,\\ p_{22}(\tau) &= -q + 3\alpha x_0 - 6\beta x_0^2 + \frac{9}{2}\,\beta y_0^2 + \ldots\end{aligned}\right\} \tag{23}$$

These coefficients are periodic functions of τ, with common period 2π and, in addition, are dependent on parameter c. Since these coefficients are values of partial derivatives of holomorphic function $\Omega\,(x + x_0,\ y + y_0)$ with $x=y=0$, series (23) will be absolutely convergent in the area where this function is holomorphic, and therefore coefficients $p_{11}(\tau), p_{12}(\tau), p_{22}(\tau), \ldots$ can be represented in the form of series placed according to increasing degrees of parameter c and absolutely convergent at all values of τ and with $|c| \leq \bar{c}$. These series will have the form

$$\left.\begin{aligned} p_{11}(\tau) &= p + cp_{11}^{(1)}(\tau) + c^2p^{(2)}(\tau) + \ldots, \\ p_{12}(\tau) &= cp_{12}^{(1)}(\tau) + c^2p_{12}^{(2)}(\tau) + \ldots, \\ p_{22}(\tau) &= -q + cp_{22}^{(1)}(\tau) + c^2p_{22}^{(2)}(\tau) + \ldots, \end{aligned}\right\} \tag{24}$$

where all coefficients $p_{11}^{(1)}(\tau)$, $p_{12}^{(1)}(\tau)$, $p_{22}^{(1)}(\tau)$. . . are periodic functions of τ with common period 2π. Thus, our problem of periodic motion stability is reduced to a problem of stability of trivial equation

$$x = 0, \; y = 0 \tag{ST}$$

of a system with periodic coefficients (20).

According to the general Lyapunov theory, we shall first examine the equations in variations corresponding to system (20). These equations in the variations will be written as follows:

$$\left.\begin{aligned} \frac{d^2x}{d\tau^2} - 2n\frac{T}{2\pi}\frac{dy}{d\tau} &= \frac{T^2}{4\pi^2}\left[p_{11}(\tau)x + p_{12}(\tau)y\right], \\ \frac{d^2y}{d\tau^2} + 2n\frac{T}{2\pi}\frac{dx}{d\tau} &= \frac{T^2}{4\pi^2}\left[p_{12}(\tau)x + p_{22}(\tau)y\right] \end{aligned}\right\} \tag{25}$$

and will represent a system of linear equations with periodic coefficients. The problem of stability (ST) of system (25) depends, as is known, on the nature of the roots of the characteristic equation of system (25). Since system (25) can be reduced to canonical form, its characteristic equation will have the form[3]

$$\rho^4 + A\rho^3 + B\rho^2 + A\rho + 1 = 0, \tag{26}$$

and coefficients A and B will be holomorphic functions of parameter c and can be represented in the form of series located by increasing degrees of c and absolutely convergent at $|c| \leqslant \tau^4$.

To determine the nature of the roots of Equation (24), we shall see what we shall have with c = 0. Assuming in Equation (25) c = 0, we shall obtain a system with constant coefficients

$$\left.\begin{aligned}\frac{d^2x}{d\tau^2}-\frac{2n}{\lambda}\frac{dy}{d\tau}-\frac{p}{\lambda^2}x&=0,\\ \frac{d^2y}{d\tau^2}+\frac{2n}{\lambda}\frac{dx}{d\tau}+\frac{q}{\lambda^2}y&=0,\end{aligned}\right\} \tag{27}$$

the determining equation of which will have the form

$$x^4-\frac{1}{\lambda^2}(p-q-4n^2)x^2-\frac{pq}{\lambda^4}=0. \tag{28}$$

Therefore, the roots of Equation (28) will be

$$i, \quad -i, \quad \frac{\lambda_2}{\lambda}, \quad -\frac{\lambda_1}{\lambda} \qquad (i=\sqrt{-1}), \tag{29}$$

but system (27) can also be viewed as a system with periodic coefficients; therefore, for system (28), we can also build a characteristic equation of the same form as Equation (26). There is no need to set up the latter equation. The roots of the determining and characteristic equations of a system with constant coefficients are connected by formula[5]

$$\rho = e^{2\pi x}, \tag{30}$$

and since values of x are known, we immediately obtain corresponding values of ρ:

$$1, \quad 1, \quad e^{\frac{2\pi\lambda_1}{\lambda}}, \quad e^{-\frac{2\pi\lambda_1}{\lambda}}. \tag{31}$$

Hence, it is evident that with $c = 0$ Equation (26) possesses one root, the modulus of which is greater than one. Since A and B are holomorphic functions of c and since the above-mentioned root is simple, according to the theorem of the existence of implicit functions[6], Equation (26) with $c \neq 0$ will have a single root, tending to $e^{\frac{2\pi\lambda_1}{\lambda}}$, when c tends to zero. Therefore, with sufficiently small values of $|c|$ Equation (26) will obviously have a root the modulus of which is greater than one. According to Lyapunov's[7] results, we conclude that (ST) of system (25) and (ST) of system (20) will be unstable.

This means that all periodic motions close to collinear points of libration in the restricted problem of three bodies also possess orbital instability, and the problem of stability of these periodic motions has thus been elucidated by us.

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Examined is the problem of periodic motions near collinear points of libration in the plane restricted problem of three bodies. With the aid of the Lyapunov methods, it is demonstrated that around each collinear point of libration a continuous family of periodic motions exists, dependent on one arbitrary constant. Examined is the stability of the indicated periodic motions; it is shown that they are unstable. Also established is the orbital instability of the periodic motions under study.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Periodic motions						
Collinear points of libration						
Taylor's series						
Holomorphic integral						
Lyapunov theory						

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