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RSIC-637

COMMENTS ON "WAVE DAMPING COMPUTATION FOR A VISCOUS LIQUID OF FINITE DEPTH," BY F. BIESEL, LA HOUILLE BLANCHE, NO. 5, 630-634 (1949)

by

C. Carry

La Houille Blanche, No. 1, 75-79 (1956)

Translated from the French

January 1967

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Translation Branch
Redstone Scientific Information Center
Research and Development Directorate
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INTRODUCTION

Shortly after publication of the article referred to, we had some doubts about the validity of the energy method used to determine the rate of damping in space from the rate of time damping.

A more complete analysis of the problem seemed to show, in fact, that only first order terms (in $\nu^{1/2}$) were appropriately calculated, whereas terms of second order (in ν) contained possible errors.

To finally settle this question and, at the same time, give the exact expression for the spacial damping coefficient, it was necessary to repeat entirely the calculation made for spacial damping. This quite long and delicate calculation, carried out by Carry, shows definitely that the second order terms given by the energy method were incorrect. Nevertheless, the error introduced in this way was negligible for almost all practical cases. For this reason La Houille Blanche, busy with other things, asked us to suspend a publication which is nonessential and repeats a previous one. However, the importance of the question was renewed with the publication by Miche of a work on the "Properties of Ocean and Laboratory Wave Trains" which, furthermore, was recently analyzed by La Houille Blanche. In a note in his work (page 93), Miche claims without formal proof that the principle term in spacial damping is in ν and not in $\nu^{1/2}$, and suggests that the fact of having obtained a term in $\nu^{1/2}$ results from the use of the energy balance method.

The calculations of Carry which are independent of the energy balance method show that that is not the case and that our conclusions with respect to the existence and importance of the term in $\nu^{1/2}$ remain unchanged; when the speed of the wave on the bottom becomes appreciable, this term predominates. The term becomes negligible only when the depth is greater than one-half the wave length and when, as a result, the speed on the bottom is less than a few percent of the speed on the surface.

We believe that Miche's error results from his omitting to write that the speeds go to zero on the bottom. This condition, as is known, corresponds to the usual hypotheses of theories of viscous fluid flow.

While Biesel has calculated the time damping of a wave, it is sometimes useful to have the expression for spacial damping; the calculational scheme is the same, but we shall repeat it for the sake of clarity.

We know that there exists a current function of the form $\psi = \psi_1 + \psi_2$, where ψ_1 and ψ_2 satisfy, respectively,

$$\Delta\psi_1 = 0$$

and

$$\Delta\psi_2 - \frac{1}{\nu}\frac{\delta\psi_2}{\delta t} = 0 \ .$$

The excess of pressure over the hydrostatic pressure is given by the total differential,

$$dp = \rho\frac{\delta}{\delta t}\left[\frac{\delta\psi_1}{\delta x}\,dz - \frac{\delta\psi_1}{\delta z}\,dx\right] ,$$

or with Φ_1 being the conjugate function of ψ_1,

$$p = -\rho\frac{\delta\Phi_1}{\delta t} + C' \ .$$

For the case of waves propagating in a finite depth H, ψ can be written in the form,

$$\psi = (A\cosh\alpha z + B\sinh\alpha z + C\cosh\beta z + D\sinh\beta z)\,e^{-iax + ibt} , \qquad (1)$$

where the x axis coincides with the free surface, and the z axis is perpendicular to it. β is determined from the relation,

$$\beta^2 = \alpha^2 + \frac{ib}{\nu} \ . \qquad (2)$$

The aim of our study is to find the complex numbers α and β, thus defining the function ψ.

The conditions which must be satisfied on the bottom are

$$\frac{\delta\psi}{\delta z} = 0$$

and

$$\frac{\delta\psi}{\delta x} = 0$$

for

$$z = - H.$$

If, for simplification, we let

$$L = \cosh \alpha H, \qquad M = \sinh \alpha H,$$

$$P = \cosh \beta H, \text{ and } Q = \sinh \beta H,$$

these conditions can be written as

(a) $AL - BM + CP - DQ = 0$

(b) $(AM - BL)\alpha + (CQ - DP)\beta = 0.$

If $\eta(x)$ designates the ordinate of the free surface, one also has

$$\frac{\delta\eta}{\delta t} = -\left(\frac{\delta\psi}{\delta x}\right)_{z = 0} = - i\alpha \,(A + C)\, e^{iax + ibt}$$

from which one finds

$$\eta + \frac{\alpha}{b} (A + C)\, e^{iax + ibt} = 0 \ .$$

Furthermore, the expression for the pressure is

$$p = - \rho g z - \rho b \,(A \sinh \alpha z + B \cosh \alpha z)\, e^{iax + ibt} \ .$$

At the free surface ($z = \eta$), the normal pressure should be zero so that

$$p + 2\rho\nu \frac{\delta^2\psi}{\delta x \delta z} = 0$$

or

$$\text{(c)}\qquad Ag\,\frac{a}{ib} + B\,(2a^2\,\nu + ib) + Cg\,\frac{a}{ib} - 2Da\beta\nu = 0.$$

The condition of zero tangential force is written as

$$\frac{\delta^2\psi}{\delta x^2} - \frac{\delta^2\psi}{\delta z^2} = 0$$

or, using Equation (2),

$$\text{(d)}\qquad 2Aa^2\nu + (2a^2\,\nu + ib)\,C = 0.$$

Eliminating A, B, C, and D from the preceding equations gives

$$\begin{vmatrix} 2a^2\nu & L & Ma & \dfrac{ga}{ib} \\ 0 & -M & -La & 2a^2\nu + ib \\ 2a^2\nu + ib & P & Q\beta & \dfrac{ga}{ib} \\ 0 & -Q & -P\beta & 2a\beta\nu \end{vmatrix} = 0\,.$$

Solving this determinant, one gets

$$-4a^3\nu^2\beta^2QM + \beta\,\left|PMga + PL\,(2a^2\,\nu + ib)^2 + 4a^2\nu^2PL\right.$$

$$\left.-4a^2\nu\,(2a^2\nu + ib)\right| - ga^2\,QL - QMa\,(2a^2\,\nu + ib)^2 = 0.$$

Dividing by PL and replacing L, M, P, and Q by their values, one finds that

$$-\;4a^3\nu^2\beta^2\tanh aH\tanh\beta H + \beta\,\left|\left\{ga\tanh aH + (2a^2\nu + ib)^2\right\}\right.$$

$$\left.+\;4a^4\nu^2 - \frac{4a^2\nu}{\cosh aH\cosh\beta H}(2a^2\,\nu + ib)\right| - ga^2\tanh\beta H \qquad (3)$$

$$-\;a\tanh\beta H\tanh aH\,(2a^2\,\nu + ib)^2 = 0.$$

Equations (2) and (3) can be used to find α and β; β^2, on the order of $1/\nu$, will be very large so that we can set $\tanh \beta H = 1$ and $\cosh \beta H = \infty$. Thus Equation (3) becomes

$$- 4\alpha^3\nu^2\beta^2 \tanh \alpha H + \beta (g\alpha \tanh \alpha H - b^2 + 4ib\, \alpha^2 \nu$$

$$+ 8\alpha^4 \nu^2) - g\alpha^2 - \alpha \tanh \alpha H (2\alpha^2 \nu + ib)^2 = 0.$$

Solving Equation (2) for β by a series expansion, we have

$$\beta = \frac{\sqrt{b}\,(1 + i)}{\sqrt{2}\ \nu^{1/2}} + \frac{\alpha^2 (1 - i)\sqrt{2}\ \nu^{1/2}}{4\sqrt{b}} \quad .$$

Replacing β in Equation (4) by this expression, we get

$$- 4\alpha^5\nu^2 \tanh \alpha H - i\, 4\alpha^3\nu\, b \tanh \alpha H - g\alpha^2 - \alpha \tanh \alpha H (2\alpha^2\nu + ib)^2$$

$$+\left(\frac{\sqrt{b}\,(1 + i)}{\sqrt{2}\ \nu^{1/2}} + \frac{\alpha^2 (1 - i) \sqrt{2}\ \nu^{1/2}}{4 \sqrt{b}}\right)(\tanh \alpha H\, g\alpha - b^2 + 4ib\alpha^2\nu$$

$$+ 8\alpha^4 \nu^2) = 0.$$

Neglecting terms of order greater than $\nu^{1/2}$, this becomes

$$\frac{\sqrt{b}\,(1 + i)}{\nu^{1/2} \sqrt{2}} (\tanh \alpha H\, g\alpha - b^2) - g\alpha^2 + \alpha \tanh \alpha H\, b^2 \tag{6}$$

$$+ \nu^{1/2} \left[4ib\alpha^2\sqrt{b}\ \frac{(1 + i)}{\sqrt{2}} + (\tanh \alpha H\, g\alpha - b^2)\ \frac{\alpha^2 (1 - i) \sqrt{2}}{4 \sqrt{b}}\right] = 0.$$

A solution for α is looked for of the form

$$\alpha = C_1 + C_2\nu^{1/2} + C_3\nu + \ldots \quad .$$

Noting that

$$\tanh \alpha H = \tanh C_1H + \frac{C_2H}{\cosh^2 C_1H}\ \nu^{1/2} + \left[\frac{C_3H}{\cosh^2 C_1H} - \frac{C_2^2H^2 \tanh C_1H}{\cosh^2 C_1H}\right]\nu$$

and equating the different powers of ν in Equation (6), we have the three equations:

$$gC_1 \tanh C_1 - b^2 = 0,$$

$$-gC_1^2 + b^2 C_1 \tanh C_1H + \frac{\sqrt{b}\,(1+i)}{\sqrt{2}} \left[gC_2 \tanh C_1H + \frac{gC_1C_2 H}{\cosh^2 C_1H} \right] = 0,$$

and

$$\frac{4\, b^{3/2}\, C_1^2}{\sqrt{2}} (i - 1) - 2\, gC_1C_2 + b^2 \left(C_2 \tanh C_1H + \frac{C_1C_2H}{\cosh^2 C_1H} \right)$$

$$+ \frac{\sqrt{b}\,(1+i)}{\sqrt{2}} \left[gC_1 \frac{(C_3H - C_2^2H^2 \tanh C_1H)}{\cosh^2 C_1H} + g \frac{C_2^2 H}{\cosh^2 C_1H} \right.$$

$$\left. + \; gC_3 \tanh C_1H \right] = 0.$$

As is known, the wave number $a = 2\pi/L$ of waves of angular frequency b in a perfect fluid is given by $ga \tanh ah = b^2$. By analogy we shall use here the notation $C_1 = a$, but it must be emphasized that a does not represent the same concept for waves in viscous fluids.

Thus, we have

$$C_2 = \frac{\sqrt{2}\;(1 - i)\, a^2}{\sqrt{b}\,(\sinh 2aH + 2aH)}$$

and

$$C_3 = - \frac{4ia^3}{b\,(2aH + 2 \sinh 2aH)} \left[\sinh 2aH + \frac{1}{\sinh 2aH + 2aH} \right.$$

$$\left. + \frac{2 \sinh aH \cosh^3 aH}{(\sinh 2aH + 2aH)^2} \right]$$

from which

$$\alpha = a \left[1 + \sqrt{\frac{2a^2 \nu}{b}} \frac{(1 - i)}{\sinh 2aH + 2aH} - i4 \frac{a^2 \nu}{b\,(2aH + 2 \sinh 2aH)} \right.$$

$$\left. \times \left(\sinh 2aH + \frac{1}{\sinh 2aH + 2aH} + \frac{2 \sinh aH \cosh^3 aH}{(\sinh 2aH + 2aH)^2} \right) \right] \qquad (7)$$

and

$$\beta = \sqrt{\frac{b}{2\nu}}\,(1 + i) + \frac{a^2\,(1 - i)}{4} \sqrt{\frac{2\nu}{b}} \quad . \qquad (8)$$

From Equation (7), the formula giving the damping law as a function of distance traveled can be written as

$$2h = 2h_o \exp \left[\frac{-2}{\sinh 2aH + 2aH} \left\{ \sqrt[3]{\frac{a^2 \nu}{2b}} + \left(\frac{2a^2 \nu}{b} \right) \left(\sinh 2aH + \frac{1}{\sinh 2aH + 2aH} + \frac{2 \sinh aH \cosh^3 aH}{(\sinh 2aH + 2aH)^2} \right) \right\} \right] .$$

Before comparing this formula to that obtained by Biesel using the energy method, a preliminary remark must be made on the definition of a and b.

In the case of damping in time and not in space, treated "in extenso" by Biesel, the wave length, L, was defined unambiguously, allowing the determination of a from the formula,

$$a = \frac{2\pi}{L} .$$

b was then defined from a:

$$b = \sqrt{ag \tanh aH} .$$

For the case of spacial damping, the concept of wave length is no longer obvious since the profile of the wave at a given time is no longer periodic in space, the amplitude decreasing from one crest to another. a could be defined as the real part of α, but we have preferred to proceed more rationally starting from the periodicity in time. The latter is free from ambiguity. Thus, we have taken

$$b = \frac{2\pi}{T}$$

and have defined a as the solution of

$$b = \sqrt{ag \tanh aH} .$$

In this way, we avoid talking of a wave length that does not correspond to an obvious physical reality, as Biesel avoided speaking of a period in his article.

Thus, the definitions of a and b cannot automatically be transposed from the time damping case to the spacial damping case; strictly, Biesel would have had to have given the definitions of these quantities at the same time as the last formula of his article. Furthermore, note that the various definitions that can be logically considered differ by quantities of the order of $\nu^{1/2}$. As a result, to the order of approximation used, only terms in ν in the damping formula should be affected by the choice of definitions of a and b.

Thus, the differences that can be noted between the terms in ν in our formula and that of Biesel can be partially explained. However, there exists another possible cause for the difference in the estimation of the rate of energy propagation; the latter may be effected by the viscosity, the most important term in the expression for this effect being probably of the order of $\nu^{1/2}$. Strictly, it would be necessary to understand this modification to perfect Biesel's energy method and to obtain correctly the terms of the order of ν.

Ultimately, if one wanted to obtain the terms in ν using the energy method, it would be necessary to complicate the method and, in doing so, take away the simplicity that is its primary advantage.

The comparison between our formula and that of Biesel is clarified by the preceding remarks. Such a comparison shows that the term in $\nu^{1/2}$ is correctly given by the energy method whereas the term in ν is in error.

The practical importance of the differences between the two formulas is, nevertheless, not great, since:

1) For large relative depths the difference disappears.
2) For moderate depths (less than a half wave length) the difference in the term in ν is small and, furthermore, the term $\nu^{1/2}$ predominates.
3) For very small relative depths the term in ν becomes the most important, and the asymptotic values are different, the damping being

$$2h = 2h_o \exp\left[\frac{3}{8\,(aH)^2} \cdot \left(\frac{a^2\nu}{b}\right) ax\right] = 2h_o \exp\left[\frac{3\nu}{8bH^2} \quad ax\right]$$

instead of

$$2h = 2h_o \exp\left[\frac{1}{4\,(aH)^2} \cdot \left(\frac{a^2\nu}{b}\right) ax\right] = 2h_o \exp\left[\frac{\nu}{4bH^2} \; ax\right] .$$

In any case, it is important to note that the method used cannot give sure results for these orders of magnitude, since the assumptions made during the calculation imply that ν is very small and that h is not too small (hypothesis $P/Q = 1$). But the numerical examples given by Biesel show that that difference is significant only for liquids with viscosities much greater than that of water or for very small relative depths (on the order of 1/60 of a wave length).

Thus, the general conclusions of Biesel's article remain tenable, at least qualitatively. In particular, it is true that in depths that are not too large, the damping is linked particularly to the term in $\nu^{1/2}$. This is in contradiction to the statement of Miche on page 93 of his book, Proprietes des Trains d'Ondes Oceaniques et de Laboratoire (Properties of Ocean and Laboratory Wave Trains), edited by COEC.

To finish the calculation we shall look for the expression for the current function, ψ. Equation (d) gives

$$A = -\frac{C}{2}\left(1 + \frac{\beta^2}{\alpha^2}\right) .$$

Equations (a) and (b) give

$$D = \frac{A\alpha + C\left[\alpha \cosh \alpha H \cosh \beta H - \beta \sinh \beta H \sinh \alpha H\right]}{\alpha \cosh \alpha H \sinh \beta H - \beta \sinh \beta H \sinh \alpha H}$$

and

$$B = \frac{C\beta + \alpha \left[\beta \cosh \beta H \cosh \alpha H - \alpha \sinh \alpha H \sinh \beta H\right]}{\beta \cosh \beta H \sinh \alpha H - \alpha \cosh \alpha H \sinh \beta H} .$$

Inserting A, B, and D in Equation (1), we get

$$\psi = C_1 e^{iax + ibt} \left[\sinh \alpha \; (H + Z)\right] - \frac{\alpha}{\beta} \left[\tanh \beta H \cosh \alpha \, (H + Z)\right.$$

$$\left. + \frac{\sinh \beta Z}{\cosh \beta H}\right] - 2 \left(\frac{\alpha^2}{\beta^2}\right) \left[\frac{\sinh \alpha H \cosh \beta \, (H + Z)}{\cosh \beta H} + \frac{\sinh \alpha Z}{\cosh \beta H}\right.$$

$$\left. - \frac{1}{2} \sinh \alpha \, (H + Z)\right] - 2 \left(\frac{\alpha}{\beta}\right)^3 \left[- \frac{\cosh \alpha H \sinh \beta (H + Z)}{\cosh \beta H}\right.$$

$$\left. + \frac{1}{2} \tanh \beta H \cosh \alpha \, (H + Z) + \frac{\sinh \beta Z}{2 \cosh \beta H}\right]$$

with

$$C_1 = - \frac{1}{2} \left(\frac{\beta}{\alpha}\right)^3 \frac{C \, \alpha \cosh \beta H}{\beta \sinh \alpha H \cosh \beta H - \alpha \cosh \alpha H \sinh \beta H} ,$$

α and β being defined by Equations (7) and (8).

It should be noted that the form of the current function is the same for time damping if one sets $\alpha = a$ and $ib = K$, a and K being defined by Biesel, and where

$$K = ib \left(1 - \frac{1}{\sinh 2aH} \sqrt{\frac{a^2 \nu}{b}}\right) + b \left[\frac{1}{\sinh 2aH} \sqrt{\frac{a^2 \nu}{b}}\right.$$

$$\left. + 2 \; \frac{a^2 \nu}{b} \; \frac{\cosh 4aH + \cosh 2aH - 1}{\cosh 4aH - 1}\right]$$

and

$$\beta = \sqrt{\frac{b}{2\nu}} \left[(1 + i) - \frac{a}{2 \sinh 2aH} \sqrt{\frac{2a^2 \nu}{b}} + \frac{a^2 \nu}{8 \; b \sinh^2 2aH}\right.$$

$$\left. \times \; (1 - 4 \cosh 2aH - 2 \cosh 4aH) \; (1 - i)\right] .$$

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13. ABSTRACT

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To finally settle this question and to give the exact expression for the spacial damping coefficient it was necessary to repeat entirely the calculation that was made for spacial damping. This delicate calculation was carried out by Carry who showed definitely that the second order terms given by the energy method were incorrect. Nevertheless, the error introduced in this way was negligible for almost all practical cases. The question became important again with the publication by Miche of a work on the "Properties of Ocean and Laboratory Wave Trains", which was recently analyzed by La Houille Blanche. In a note in his work (page 93) Miche claimed that the principle term in spacial damping is in ν and not in $\nu^{1/2}$. Miche suggests that the fact of having obtained a term in $\nu^{1/2}$ results from the use of the energy balance method.

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of the term in $\nu^{1/2}$ remain unchanged. When the speed of the wave on the bottom becomes appreciable, this term predominates. The term becomes negligible only when the depth is greater than 1/2 the wave length, and when, as a result, the speed on the bottom is less than a few percent of the speed on the surface.