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A "HUMMINGBIRD" FOR THE L_2 LUNAR LIBRATION POINT

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GODDARD SPACE FLIGHT CENTER

GREENBELT, MARYLAND

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ABSTRACT

Consider as an example a spacecraft, contrary to all previously published investigations, NOT in an orbit but in a more or less permanent position in the vicinity of the far-sided lunar libration point L_2 , as commonly called. Such a spacecraft would be very useful for a communications relay between the back side of the moon and an earth tracking station. It could be placed "above" the lunar libration point in such a fashion that it is never occulted, thus making the stated communication link a continuous one, independent of time.

Obviously, the spacecraft will be in an accelerating force field and thus will need a permanent thrusting to "stay" in place, or move slowly around a predetermined point in space similar to a "Humming Bird."

The purpose of this report is to show, in simple analytical form, what the accelerations are in the vicinity of L_2 and what specific impulse values are needed to keep a spacecraft there economically, that is, with a reasonable fuel to useful spacecraft mass ratio (m_f/m_o = 0.05 to 0.15). This in turn will dictate the kind of engines to be used for missions like those where constant but extremely small accelerations are needed over the lifetime of such a spacecraft, say in the order of one to three years.

SUMMARY

All of the many suggestions made to use the far sided lunar libration point to "anchor" a communication satellite assume this spacecraft to be in an appropriate orbit "around" this point. This is by now text book knowledge.

The purpose of this brief paper is to make a slide rule type investigation of a "stationary" lunar libration satellite. The idea here is not to consider an orbiting spacecraft but a humming or hovering craft not in motion with respect to the earth-moon system.

Acceleration expressions are derived giving the reader a feeling of the situation considered. To quote an example, a spacecraft hovering 3500 km above L_2 experiences an acceleration of about 1.10^{-2} cm/s^2 or 10^{-5} g.'s (earth acceleration). Above here means along a perpendicular line from L_2 parallel to the earth-moon (bary center) rotational axis. This, of course, is not a necessity, the craft may also be in the earth moon plane located on either side of the moon by, say, 3500 km. As a matter of fact, the station keeping requirements may be a little less than the example quoted. The 3500 km (or more) distance quoted is needed in order to "see" the earth at all times, that is to prevent a lunar occulation of the spacecraft thus guarantying continuous communication between the backside of the moon and an earth bound tracking station.

Due to the fact that this spacecraft is not in motion and is an accelerating field, continuous thrusting is necessary. On the other hand, since the acceleration experienced is extremely small, the use of low thrust, high specific impulse electric space propulsion systems seems to be suitable. Using a 190 kg spacecraft, an ion engine with a specific impulse of 4300 sec, 2000 dyn thrust would suffice consuming only 23 kg of fuel operating over a year.

Additional thrusting will be needed for the necessary antenna pointing since the spacecraft has to rotate around its axis once every 27.3 days (lunar month). Due to the extremely small angular lunar motion, $\omega = 2.66 \times 10^{-6}$ sec^{-1} the thrusting requirements are extremely small compared to the station keeping requirements. Therefore not much rotational control fuel is required.

In summary, based upon this rather simple slide rate analysis it seems that such a spacecraft would be feasible to build and operate in the future. Launch and guidance operations for lunar orbiting spacecraft are within the state of the art and are proven at this time.

A spacecraft of this kind would be a necessary "extension of the ground tracking network" into space. Only with this (or similar systems) can a communications link be established between the earth and the backside of the moon needed for unmanned and manned operations on or at the "invisible" side of the moon.

A "HUMMINGBIRD" FOR THE L_2 LUNAR LIBRATION POINT

F. O. Vonbun

I. ACCELERATION EXPERIENCED BY THE "HUMMINGBIRD"

A. General

In order to get a "feeling" for station keeping of a satellite "above" the lunar L_2 libration point, a simple analytical model is developed of the acceleration as a function of r and φ as shown in Figure 1.

Using vector notation as indicated in Figure 1, we can write for the acceleration $\vec{\ddot{x}}$ of the satellite mass m*:

$$\vec{\ddot{x}} = \vec{d}^{\circ} \, p\,\omega^2 - \vec{\rho}^{\circ} \frac{\gamma m}{\rho^2} - \vec{R}^{\circ} \frac{\gamma M}{R^2} \tag{1}$$

where $\vec{d}^{\circ}$ is the unit vector along the earth-moon axis, p is the distance from the bary center to the spacecraft projection on the earth-moon line, $\omega = 2.66 \times 10^{-6}$ sec^{-1} is the moon's angular speed around the earth,[1] $\vec{\rho}^{\circ}$ is the unit vector from the moon to the spacecraft, ρ is the distance between moon and spacecraft, γ is the gravitational constant[1] ($\gamma = 6.668 \times 10^{-8}$ dyn cm^2 g^{-2} or cm^3 g^{-1} sec^{-2}), m is the mass of the moon (m = 7.350 10^{25} g),[1] $\vec{R}^{\circ}$ is the unit vector from the earth center to the satellite, R is the distance between earth and satellite, and M is the mass of the earth[1] (M = 5.977 10^{27}g).

The first term represents the centrifugal acceleration due to the moon's motion around the earth, the second and third terms represent the moon's attraction of the spacecraft to the moon and the earth respectively (thus, the negative signs of the unit vectors $\vec{\rho}^{\circ}$ and $\vec{R}^{\circ}$).

Of special interest here is the acceleration $\vec{\ddot{x}}$ as a function of $\vec{r}$, the satellite position vector from the lunar L_2 libration point, as shown in Figure 1.

*m cancels out

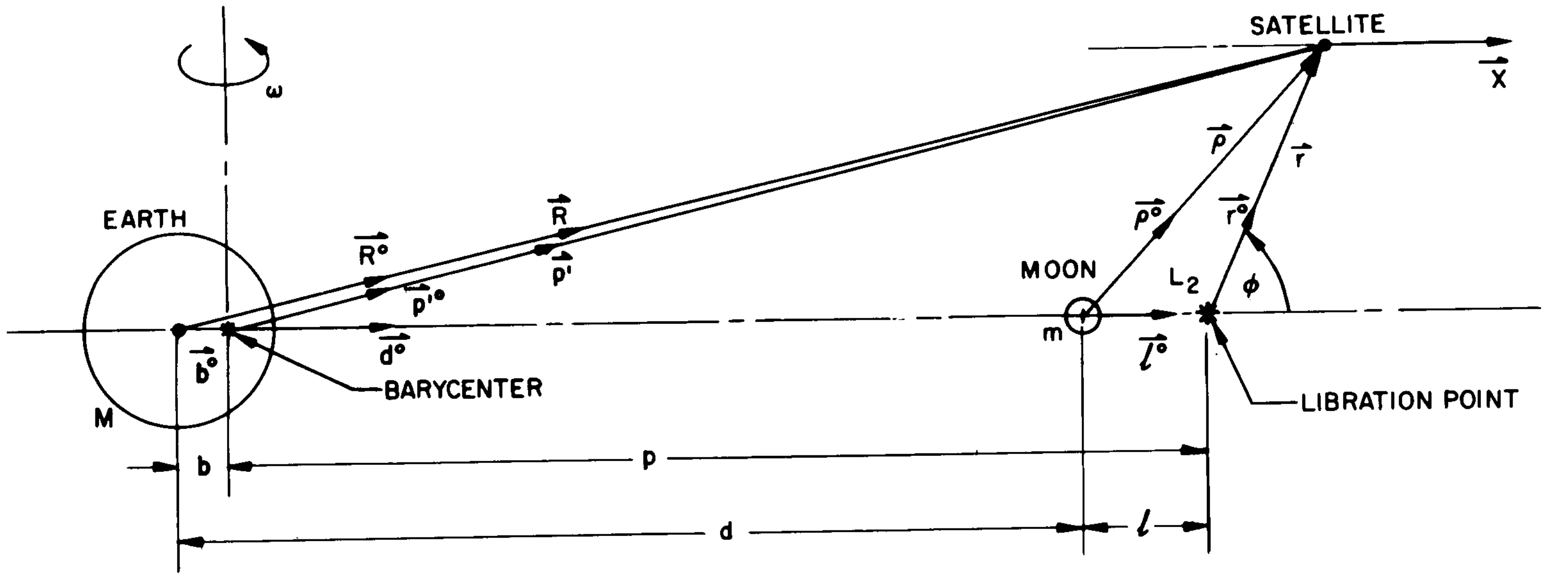

Figure 1—"Hummingbird" Geometry

From Figure 1 it is also evident that:

$$R^2 = (d + \ell + r \cos \varphi)^2 + (r \sin \varphi)^2$$

$$\rho^2 = (\ell + r \cos \varphi)^2 + (r \sin \varphi)^2$$

$$p = (d - b + \ell + r \cos \varphi)$$

where b is the bary center distance[2,3,4,5] ($b = d/81 = 4720$ km), d is the earth moon distance ($d = 3.84 \; 10^5$ km)[1] 1 is the lunar libration point distance[3] ($1 \doteq d \sqrt[3]{m/3M} \doteq 6.16 \cdot 10^4$ km), r is the distance (magnitude of $\vec{r}$) from the libration point L_2 to the spacecraft and φ is the angle between the earth moon axis and the vector $\vec{r}$, the satellite position vector as mentioned.

The influence of the sun being only ±10% is being considered later.

The magnitude of x, namely $x = \sqrt{(\vec{x} \cdot \vec{x})}$ can now easily be calculated from equation 1.

Setting for ease of writing:

$$p\omega^2 = A, \quad -\frac{\gamma m}{\rho^2} = B$$

$$-\frac{\gamma M}{R^2} = C, \tag{2}$$

one obtains after some manipulation

$$|\vec{x}| = x = \Big[A^2 + B^2 + C^2 + 2AB\,(\vec{d}^\circ \cdot \vec{\rho}^\circ) + 2AC\,(\vec{d}^\circ \cdot \vec{R}^\circ) + 2BC\,(\vec{\rho}^\circ \cdot \vec{R}^\circ)\Big]^{1/2} \tag{3}$$

Expressing the vector dot products in terms of known quantities yields:

$$(\vec{d}^\circ \cdot \vec{\rho}^\circ) = \frac{1}{\rho}(\ell + r\cos\varphi)$$

$$(\vec{d}^\circ \cdot \vec{R}^\circ) = \frac{1}{R}(d + \ell + r\cos\varphi) \tag{4}$$

$$(\vec{\rho}^\circ \cdot \vec{R}^\circ) = \frac{1}{\rho R}\left(r^2 + \ell^2 + \ell d + r\cos\varphi\,(d + 2\ell)\right)$$

Equation (3) can now be used to calculate the acceleration x of the "Hummingbird" as a function of r and φ. That is

$$x = f(\vec{r}) = g(r, \varphi) \tag{3a}$$

Equations (3), (3a) represents the acceleration of the spacecraft, which has to be compensated by rocket control (ion engines for instance) if one wants to "keep" the craft hovering "over" L_2 as shown in Figure 1.

In Figure 2, the acceleration x, equation (3), is shown as a function of r and φ in graph form in order to get an idea of the acceleration magnitudes involved.

B. Acceleration Along the Earth-Moon Line

Equation (1) can be simplified if one considers a spacecraft located at a distance r from the libration point along the earth moon line ($\varphi = 0$ that is). For this case (1) reads then:

$$x = p\omega^2 - \frac{\gamma m}{\rho^2} - \frac{\gamma M}{R^2} \tag{5}$$

If $p = (d-b+1)$ then the spacecraft would be directly located at L_2 and no acceleration x would be experienced by the spacecraft. (Sun's influence excluded.) This is actually the definition of L_2.

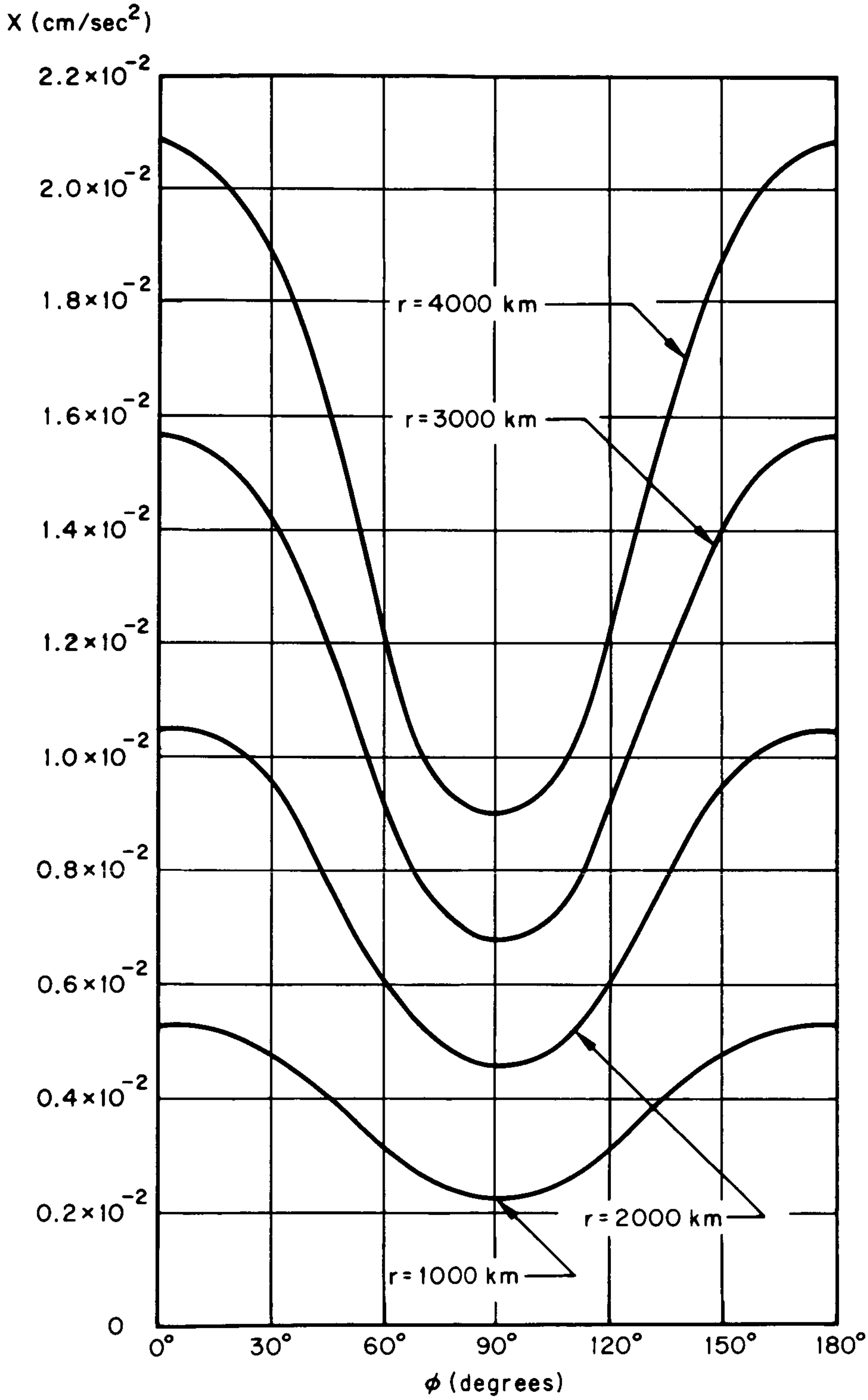

Figure 2–Acceleration of the "Hummingbird" Near L_2

What happens now if one takes the spacecraft out of L_2 along the earth moon line? This can best be studied by varying (5) which yields

$$\delta x = p\omega^2\left(\frac{\delta p}{p}\right) + 2\frac{\gamma m}{\rho^2}\left(\frac{\delta\rho}{\rho}\right) + 2\frac{\gamma M}{R^2}\left(\frac{\delta R}{R}\right) \tag{6}$$

Further: $\delta r = \delta p = \delta\rho = \delta R$ for $\varphi = 0$ as seen from Figure 1 and therefore approximately:

$$\frac{\delta\rho}{\rho} \doteq \frac{\delta R}{R} \tag{7}$$

and

$$\frac{\delta\rho}{\rho} \doteq \left(\frac{384}{61}\right)\frac{\delta R}{R}$$

since the value of $\delta\rho$ becomes δr and $\rho \doteq 61.000$ km and $R \doteq 584.000$ km. Introducing (7) into (6) and setting $\delta r = \delta R$ are obtained with $m/M \doteq 1/81$ an approximate equation for the variation δx as a function if δr:

$$\delta x \doteq \delta r\left(\omega^2 + 8\frac{\gamma M}{R^3}\right) \tag{8}$$

Example: Assume $\delta r = 3500 \text{ km} = 3.5 \cdot 10^8 \text{ cm}$

$$\omega = 2.66 \cdot 10^{-6}\ \text{sec}^{-1}$$

$$\gamma = 6.67 \cdot 10^{-8}\ \text{cm}^3\ \text{g}^{-1}\ \text{sec}^{-2}$$

$$M = 5.98 \cdot 10^{27}\ \text{g}$$

$$R = (d+\ell) = 4.45 \cdot 10^{10}\ \text{cm}.$$

as given for instance in Reference 1, one obtains

$$\delta x \doteq 1.5 \cdot 10^{-2}\ \text{cm/s}^2$$

or

$$\overline{\delta} x = 1.53 \cdot 10^{-5}\ g_o \tag{9}$$

in terms of the earth acceleration where $g_o = 981\ \text{cm/sec}^2$ = the acceleration at the earth surface.

Equation (8) gives a representative number for the acceleration a spacecraft experiences about 3500 km from L_2 along the earth moon line. This equation also indicates that relatively small accelerations are experienced at these distances which can easily be compensated for by ion engines.[4,5,6,7]

C. The Sun's Influence

As briefly mentioned, the sun's gravitational influence on the "Hummingbird" has been neglected and shall now be approximately determined. No secondary acceleration influences due to the sun are considered since they are small compared to those discussed.[8]

The equilibrium condition for L_2 really exists only for the case of a rotating earth-moon system as represented by equation (5) using $\vec{r} = 0$.

In case the sun is considered, a nearly sinusoidal perturbation acceleration is superimposed due to the fact that L_2 rotates around the bary center (~27.3 days, lunar period) and thus changes its distance from the sun as shown in Figure 3. Only if L_2 lies on the earth orbit is the sun's attractive force approximately[8] compensated by the angular velocity ω_e of the earth (or bary center) around the sun.

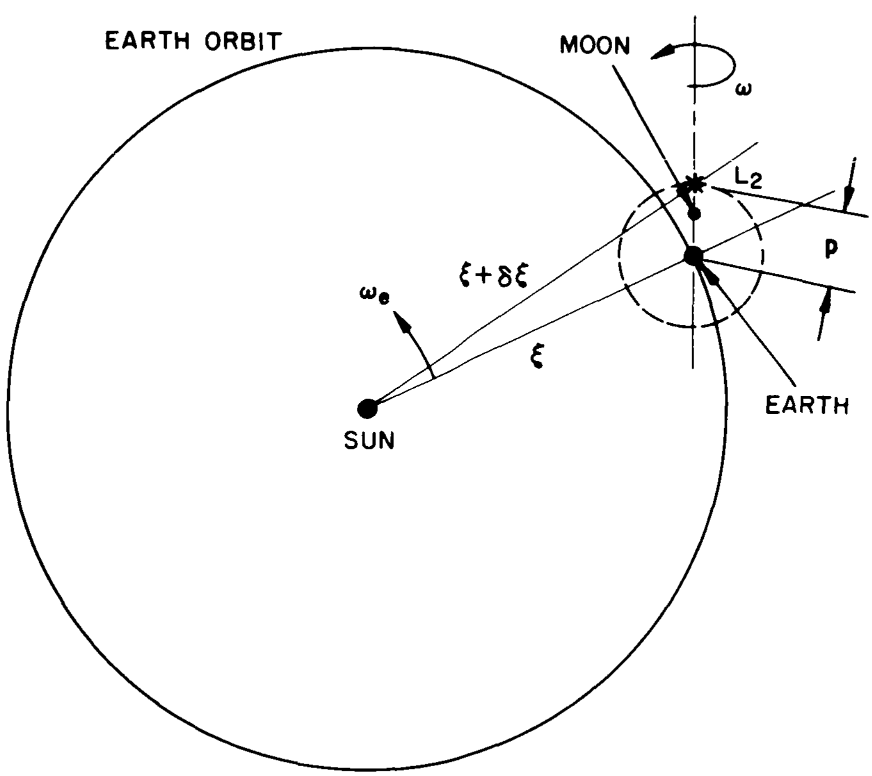

Figure 3–Sun's Influence on the Acceleration of L_2

The acceleration y of a mass point in a sun orbit is given again by:

$$y = \xi \omega_e^2 - \frac{\gamma M_s}{\xi^2} \tag{10}$$

where ξ is the distance from the sun to the mass point, $\omega_e = 1.99 \cdot 10^{-7}$ sec^{-1} is the earth angular velocity around the sun, $\gamma = 6.67 \cdot 10^{-8}$ cm^3 g^{-1} sec^{-2} the gravitational constant and $M_s = 1.99 \cdot 10^{33}$ g, the mass of the sun.

If $\xi = \xi_o$ = 1AU (astronomical unit) then, of course, y must be zero.

Varying (10) yields:

$$\delta y = \delta \xi \omega_e^2 + 2 \gamma M_s \frac{\delta \xi}{\xi^3} \tag{11}$$

Using $\xi = \xi_o$ = 1AU from (10), one obtains since y = 0 as mentioned in this case

$$\delta y \doteq 3 \omega_e^2 \; \delta \xi \tag{12}$$

as the variation for the sun's accelerative due to a change in ξ and ω_e which is assumed to be constant.

Example: $\omega_e^2 = 3.96 \cdot 10^{-14}$ sec^{-2}

$\delta \xi = \rho = (d - b - \ell) = 4.4 \cdot 10^5$ km $= 4.4 \cdot 10^{10}$ cm
(see also Figures 1 and 3).

moon's orbit assumed circular for simplicity.

then: $\delta y = 1.74 \cdot 10^{-3}$ cm/s^2

or $\overline{\delta y} = 1.7 \cdot 10^{-6}\ g_o$ (in terms of earth's acceleration)

As can be seen the sun's influence is only ~10% for the case considered as shown by equation (12).

Both equation (8) and (12) show that the accelerations experienced by "Hummingbird" type spacecraft are very small indeed (say from 0 to 2.10^{-2} cm/s^2).

II. NECESSARY SPECIFIC IMPULSE FOR ECONOMIC STATION KEEPING

In the foregoing pages, an estimate was obtained of the magnitude of the expected accelerations of a "Hummingbird" satellite.

The next question to be answered is the necessary specific impulse I_{sp} needed to "keep" the spacecraft on station say for one year using a reasonable (0.05 to 0.10 fuel to spacecraft mass ratio (m_f/m_o) .

A. Fuel to Spacecraft Mass Ratio

The acting force $\vec{F}$ on the spacecraft is given by

$$\vec{F} = m\,\vec{\ddot{x}} \tag{14}$$

where m is the total mass and $\vec{\ddot{x}}$ is the experienced accelerations shown in equation (5) or $\delta\ddot{x}$ as indicated in equation (8).

Using the common simple equation[2,3] for the force F (magnituded needed only for this consideration) of a rocket one obtains

$$F = \dot{m}\, g_o\, I_{sp} \tag{15}$$

where $\dot{m}$ is the mass flow (flow rate of exhaust material), g_o is the earth acceleration ($g_o = 981$ cm/s²) and I_{sp} is the specific impulse.

For station "keeping" the forces in (14) and (15) have to be equal, that means:

$$m\ddot{x} = \dot{m}\, g_o\, I_{sp} \tag{16}$$

Integrating equation (10) over a time T the "useful" station keeping time (say T = 1 year = $3.1 \cdot 10^7$ sec) one obtains

$$\frac{m_f}{m_o} = \exp\left\{\frac{\int_T \ddot{x}\,dt}{g_o I_{sp}}\right\} - 1 \tag{17}$$

where $m = (m_f + m_o)$ was used, that is, the total "flying mass" is always the sum of the fuel mass and the spacecraft mass (engine included). Figure 4 shows the ratio (m_f/m_o) as a function of the specific impulse I_{sp} for a "Hummingbird" with a station lifetime of one year. ($\delta\ddot{x} \doteq \ddot{x} \doteq 1.5 \cdot 10^{-2}$ cm/s²) $T = 3.1 \cdot 10^7$ sec).

B. Needed Station Keeping and Altitude Engines

As can be seen from Figure 4, using a rather reasonable (small) fuel to spacecraft mass ratio of say 0.1, specific impulses I_{sp} of the order of 4000 to 5000 sec are needed. This, coupled with the fact that these control accelerations, are really small ($\sim 1.5 \cdot 10^{-2}$ cm/s^2) points directly to the use of electric space propulsion engines for this kind of operation. The matter of fact they seem to be "just made" for this purpose. Thrust levels, lifetime, power consumption, etc. are all within very reasonable limits for these engines. [5,6,7] Even deflecting beams can be used for better control purposes.

For instance, in Ref. 6 an engine is cited that was built and tested over hundreds of hours with the following characteristics:

Thrust = 1900 dyn

Total power = 500 watt

Power to Thrust Ratio = 260 mW per dyn

Specific Impulse = 4330 sec

Using a total spacecraft mass of say 190 kg ($m_f + m_o$) we obtain an acceleration of

$$x = \frac{F}{M} = \frac{1.900}{1.9 \cdot 10^5} = 1.\,10^{-2} \text{ cm/s}^2 \tag{18}$$

This is well in the vicinity of the control thrust accelerations needed to "keep" the spacecraft humming. As can be seen from Figure 4, a favorable fuel to spacecraft mass ratio $m_f/m_o = 0.12$ for one year lifetime also results for the high specific impulse obtainable with ion engines. The same holds for electrical power requirements and engine weight. Solar cells may provide the required electrical power for years of space operation.

Another needed control system is one to "turn" the spacecraft, in such a fashion that it will always point its high gain antenna toward the earth during the rotation of the earth moon line as shown in Figure 3. This means the spacecraft has to rotate once every lunar month ($\sim$27.3 days) around its own axis (see Figures 3 and 5). For this motion, power for ion engines could again be made available. Depending on the coverage, antenna beams engulfing earth and moon respectively, as shown in Figure 6 the control systems have to work within certain angular accuracies.

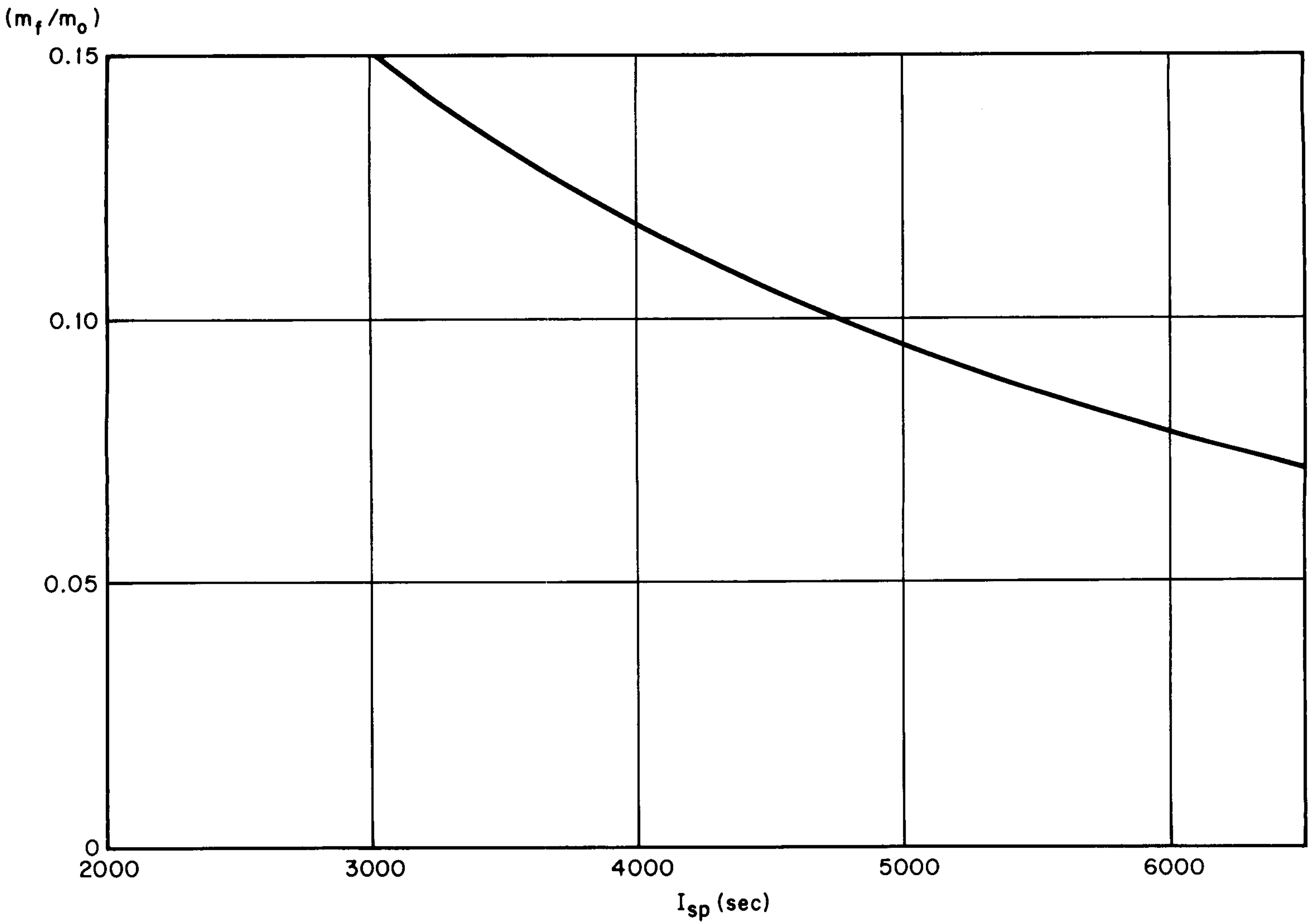

Figure 4–Fuel To Spacecraft Ratio Vs. Specific Impulse (1 Year Lifetime)

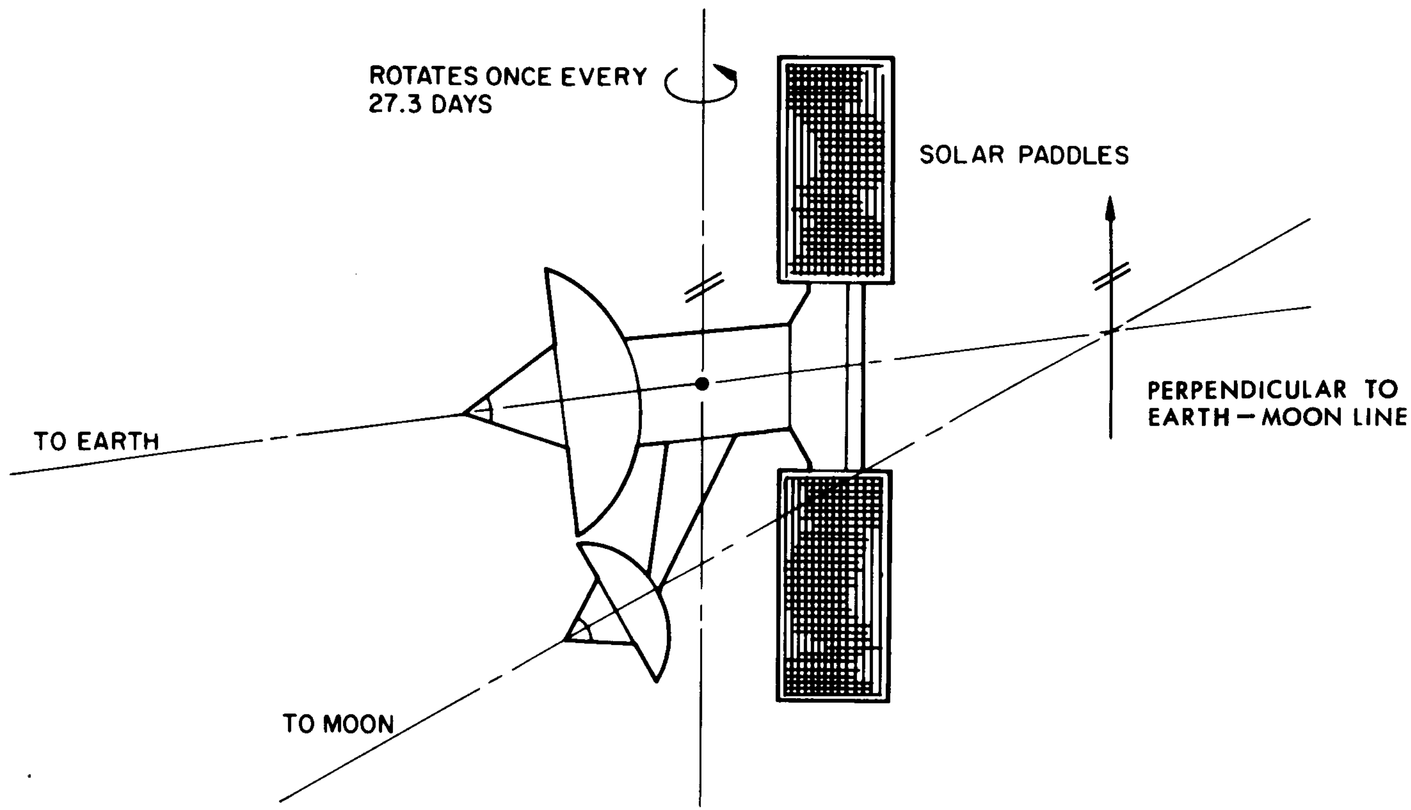

Figure 5–Schematic of the "Hummingbird"

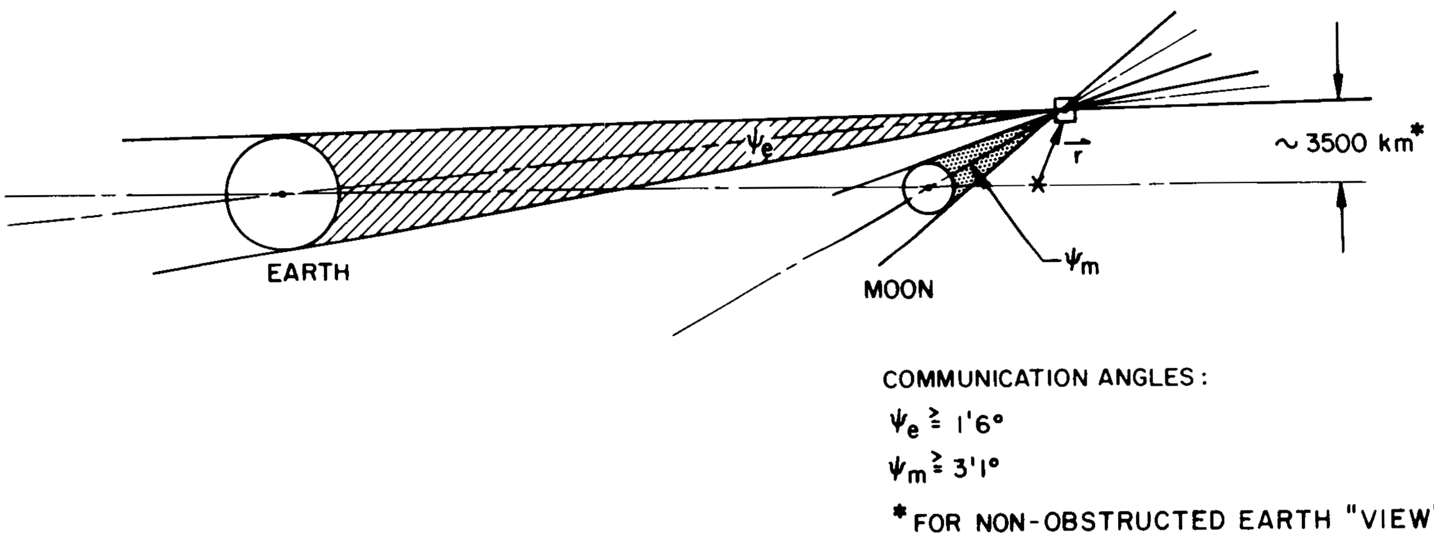

Figure 6–"Humming Bird" – Earth and Lunar Communications Coverage.

Assume one wants to cover the earth and a nearby earth satellite (an orbiting tracking station for instance), than one would make the earth antenna beam with $\psi_e = 3°$ and the moon antenna beam with $\psi_m = 5°$. Thus, a correction of say ±1° would be adequate for spacecraft stabilization. In brief, one has to control the rotational motion of the spacecraft during its lunar cycle to say this accuracy as stated.

The necessary thrust can be calculated from the basic equation of a system in angular rotation, that is

$$J \ddot{\varphi} = M = DT \tag{19}$$

where J is the moment of inertia about the spacecraft rotational axis (see Figure 5), $\ddot{\varphi}$ is the angular acceleration, M is the moment about the axis caused by a control engine of thrust T mounted at a distance D from the axis.

Integrating equation (19) one obtains, assuming a constant thrust:

$$\dot{\varphi} = \frac{T}{J} \Delta t \tag{20}$$

where Δt is the time the thrust is acting resulting in an angular motion $\dot{\varphi}$ which has to be equal the moon angular motion $\omega = 2.66 \cdot 10^{-6}$ sec^{-1}.

The needed thrust is then from (20):

$$T = \frac{J \cdot \omega}{D \cdot \Delta t} \tag{21}$$

Example:

$J = 1.9 \cdot 10^9$ g cm^2 (which is equivalent of a dumbell system using two 95 kg masses 2 meters apart)

$\omega = 2.66 \cdot 10^{-6}$ sec^{-1}

$D = 100$ cm

$\Delta t = 1000$ sec (arbitrary chosen)

then

$$T = \frac{1.9 \cdot 10^9 \; 2.66 \cdot 10^{-6}}{10^2 \cdot 10^3} = 0.5 \text{ dyn.}$$

This shows that for the rotational motions a few dyns of force are adequate. (Sun pressure = 1 dyn/m^2 for instance). This is a thrust level much smaller than that needed for station keeping as shown by the example stated in section II-B and should therefore not add anything too substantial to the total spacecraft fuel needed for station keeping.

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