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AN EXPERIMENTAL INVESTIGATION OF TWO VISUAL METHODS OF ALTITUDE DETERMINATION

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SUMMARY

An investigation was made to measure the ability of an astronaut to determine the altitude of his spacecraft above the lunar surface by purely visual means. Two techniques were used: one consisted of matching calibrated curved arcs to the projected horizon curvature, and the other consisted of measuring the visual arc subtended by a known surface feature. The slides used for projection were photographs of a relief map of the lunar surface. A second set of slides with a smooth arc for the horizon was used in order to evaluate the effect of horizon irregularity.

For the limited field of view used (about 40°) in the horizon-arc matching technique, the average errors for these measurements were as large as 36 miles (58 km) and the standard deviation was about 28 miles (45 km). Repeating the slides a second time or using the smooth-arc slides decreased the error but did not seriously affect the standard deviation. The results indicate that a learning process is involved and that features on the horizon do influence the altitude estimations.

The surface-feature technique, when the surface feature was viewed from directly above, seemed to be considerably more accurate for determining altitude than the horizon-matching technique.

INTRODUCTION

As part of a continuing research effort directed toward defining the role of man in space operations, considerable emphasis has been placed on evaluating how well a man can perform certain tasks, necessary for the successful accomplishment of the mission, with a minimum of equipment. These investigations generally are aimed at aiding in the decision of whether to continue certain portions of the mission after failure of some equipment components and/or simplifying the onboard equipment.

The altitude above the surface of a planetary body is probably the most important parameter to determine accurately because it enters into the evaluation of almost all the other orbit parameters. Presented in this paper are the results of an experimental attempt to measure the ability of an astronaut to determine the altitude of his vehicle by two different visual methods. One of the methods used was the horizon-curvature matching technique, in which the view of the curvature of the horizon is compared with a previously calibrated template, and the other method consisted of measuring the visual angle of a known surface feature and utilizing conversion tables to determine the altitude.

SYMBOLS

c'	chord of the small circle on the actual usable portion of the slide, that is, 1.32 inches (3.35 cm)
c_M	chord of the small circle on the moon containing the horizon
c_t	chord of the template
D	diameter of the object
d_p	distance from the optical center of the projection lens to the screen
$d_{v,1}$	distance from viewer's eye to the screen
$d_{v,2}$	distance from viewer's eye to the template
f_c	focal length of the camera
f_c'	theoretical focal length of the camera
f_p	focal length of the projector
h	altitude
h_a	actual altitude
h_e	estimated altitude
i	integer
K	constant

N	number of readings
R_M	radius of moon
r	correlation coefficient
r_d	radius of the projected small circle on the screen
r_M	radius of the small circle on the moon containing the horizon
r_s	radius of the small circle on the slide
r_t	radius of the arc on the template
S_1,S_2,S_3	sagitta of different arcs on the template
S_M	sagitta of the small circle on the moon containing the horizon
S_t	sagitta of the small circle on the template
X	distance from the camera lens to the horizon on the moon
X'	distance from the camera lens to the chord of the small circle on the moon
Y	distance from the camera lens to the center of the small circle on the moon
α	central angle subtending the chord of the horizon viewed
β	angle of view
γ	central angle of the small circle subtending the chord of the horizon viewed
θ	half the optical angle of the object
σ	standard deviation
ϕ	angle between the local vertical and the line of sight to the horizon

METHODS AND APPARATUS

As mentioned previously, two methods of estimating the altitude were used in this investigation. These consisted of (1) matching the visual curvature of the horizon with a previously calibrated template, which will be called the horizon-curvature technique; and (2) comparing the measured visual arc that a crater or other object of known size on the surface subtends with a previously calibrated table, which will be called the surface-feature technique. In the latter technique the angle between the line of sight and the local vertical must be known in order to determine the altitude. However, if the surface feature is vertically below the vehicle, as was assumed in these tests, the altitude is obtained directly.

The apparatus used for both techniques was basically the same; it differed only in the slides that were presented to the viewer and the templates that were compared with the slide presentation. A sketch of the apparatus is shown in figure 1. The observers

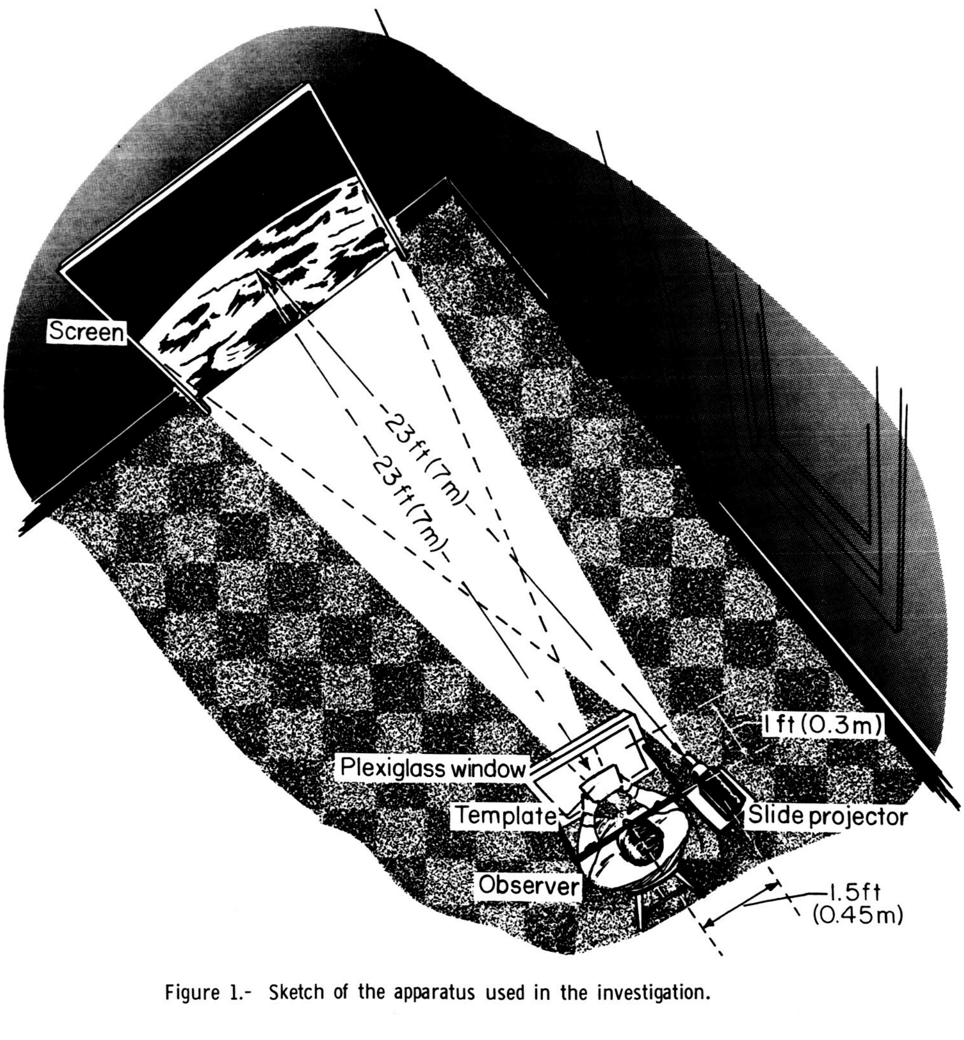
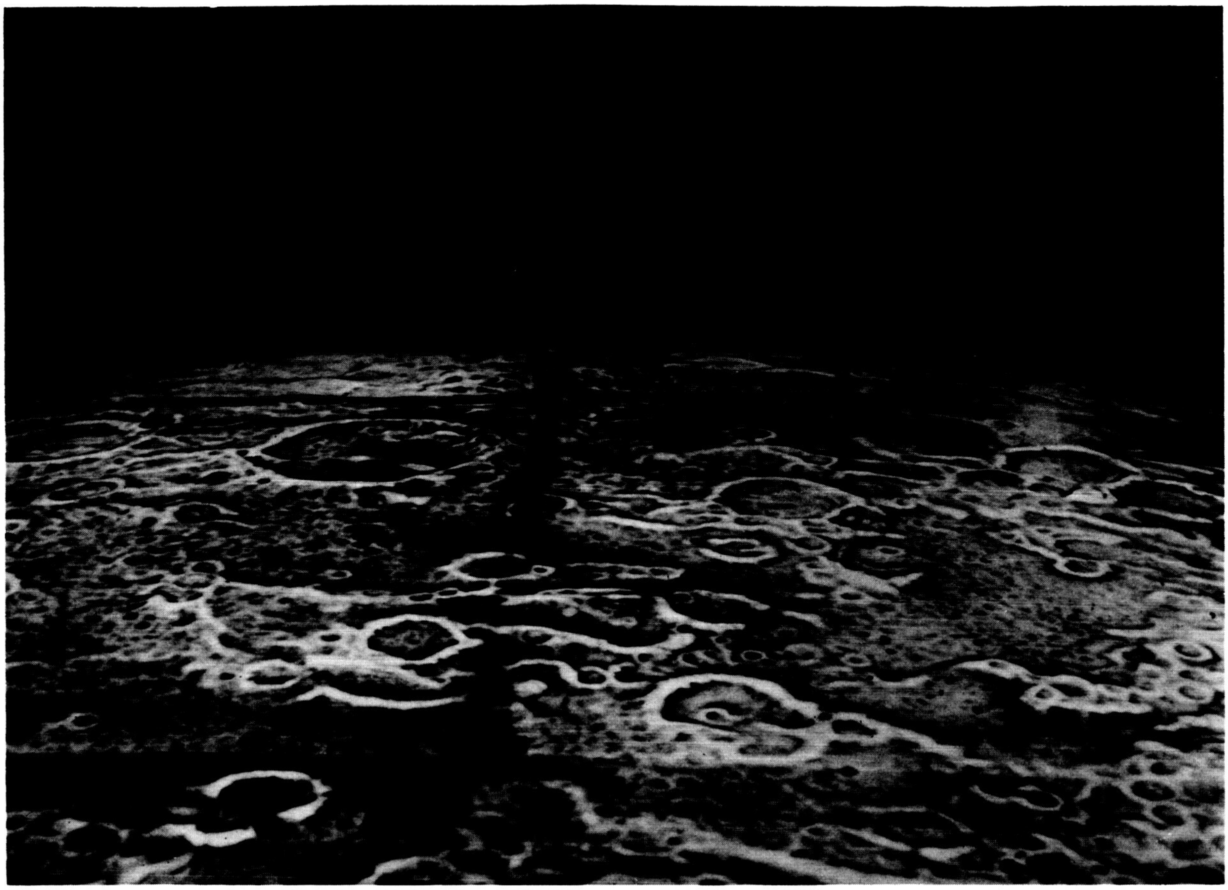


Figure 1.- Sketch of the apparatus used in the investigation.

were instructed to rest their heads against a padded bar so that their eyes were positioned 1 foot (0.3 m) from the plexiglass window that supported the templates. Slides of either the curvature of the moon's horizon for various altitudes or direct overhead views of a crater for various altitudes were projected on the screen. The observer tried to match the projected horizon curvature with a suitable template or tried to measure the diameter of the crater with a visual-angle template, depending upon which of the techniques was being investigated.

The slides used for the horizon-curvature technique were photographs of a model of a portion of the lunar surface. This was a curved relief model constructed to a scale of 126 720 to 1. The photographs were taken with the camera aimed at the horizon and at various scaled distances from the surface. These scaled distances between pictures represented 12-mile (19.3-km) increments in altitude up to an altitude of 216 miles (292.8 km). Since the model was in relief some of the photographs show the irregularities of mountains and crater lips. A 35-mm camera with a 2-inch (5.1-cm) focal length lens was used; this resulted in a horizontal field of view of about 40°. At some of the higher altitudes this field of view was wider than the width of the model and so the useful information was reduced somewhat, to about 32°. In an attempt to measure the influence of the mountains and craters on the observers' readings, some slides were made with a smooth-arc horizon curvature for those altitudes at which the irregularities seemed most prominent. Examples of both types of horizon-curvature slides are presented in figure 2 and a typical template is shown in figure 3. The curvature of the various templates was computed to match the projected curvature of the slides. Since both the slides and templates were made in 12-mile (19.3-km) increments in altitude, any error by the observer in matching the two would automatically be in units of 12 miles (19.3 km). The tests were made with 22 observers for this portion of the program and the slides were presented in a random order. There are, however, more than 22 readings for each slide because many readings were repeated when the error, in the opinion of the tester, was too large or when a particular error in reading varied considerably from the pattern established by each observer. For the comparison between the photographs of the lunar model and the smooth-arc horizon slides, only 10 of the observers were tested. Since both types of slides were used, the photographs of the lunar model for the second time, this test not only afforded comparison between the two types of slides but also gave some insight into the learning effect.

For the surface-feature technique, pictures were taken looking directly down on a typical crater (Abulfeda) at various simulated altitudes up to about 536 miles (863 km). A typical view is shown in figure 4(a). For the different altitudes the diameter of the crater would subtend different visual angles at the eye of the observer, who was furnished with a template for measuring the angle (shown in fig. 4(b)). Once the visual angle was ascertained (usually an average of several readings taken along different diameters), he



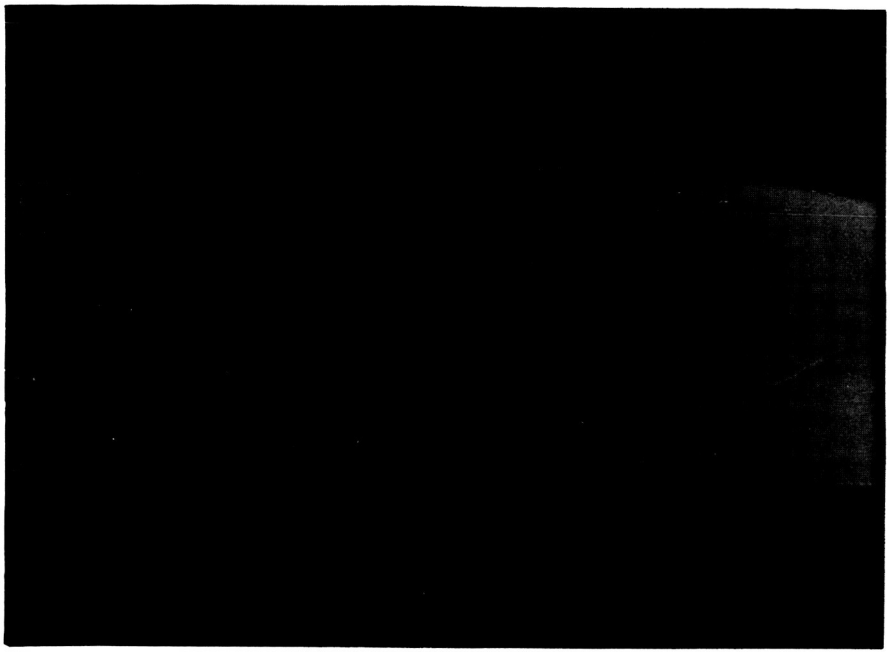
(a) Photograph of moon-surface model.

L-67-968

Figure 2.- Reproductions of typical slides used in the investigation. These slides were for the horizon-curvature technique and are for an altitude of 156 miles (251 km).

could determine from a table his altitude for any known crater diameter. A typical table is shown as table I. This method could be used for any size crater at any altitude. In the present investigation only one crater was used because of the time involved in presenting the series of slides.

The transverse displacement between the slide projector and the observer introduced some distortion in the viewed image, and some computations were made to evaluate this error. For the horizon-curvature matching technique, the computations showed that an error of 1/10 mile (0.16 km) high was introduced for low altitudes (12 miles, or 19.3 km), and the error increased with altitude to a value of $1\frac{1}{4}$ miles (2.01 km) at an altitude of 200 miles (322 km). Errors of this magnitude would not noticeably affect the altitude estimations because the adjacent horizon arcs on the template were 12 miles (19.3 km) apart. For the surface-feature technique, the angular error was about $2/100^{\circ}$



(b) Smooth-arc slide. L-67-969

Figure 2.- Concluded.

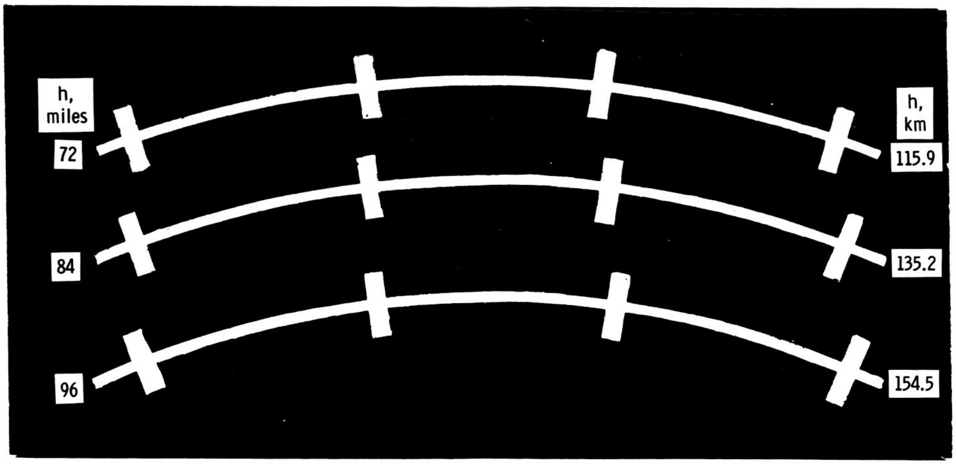
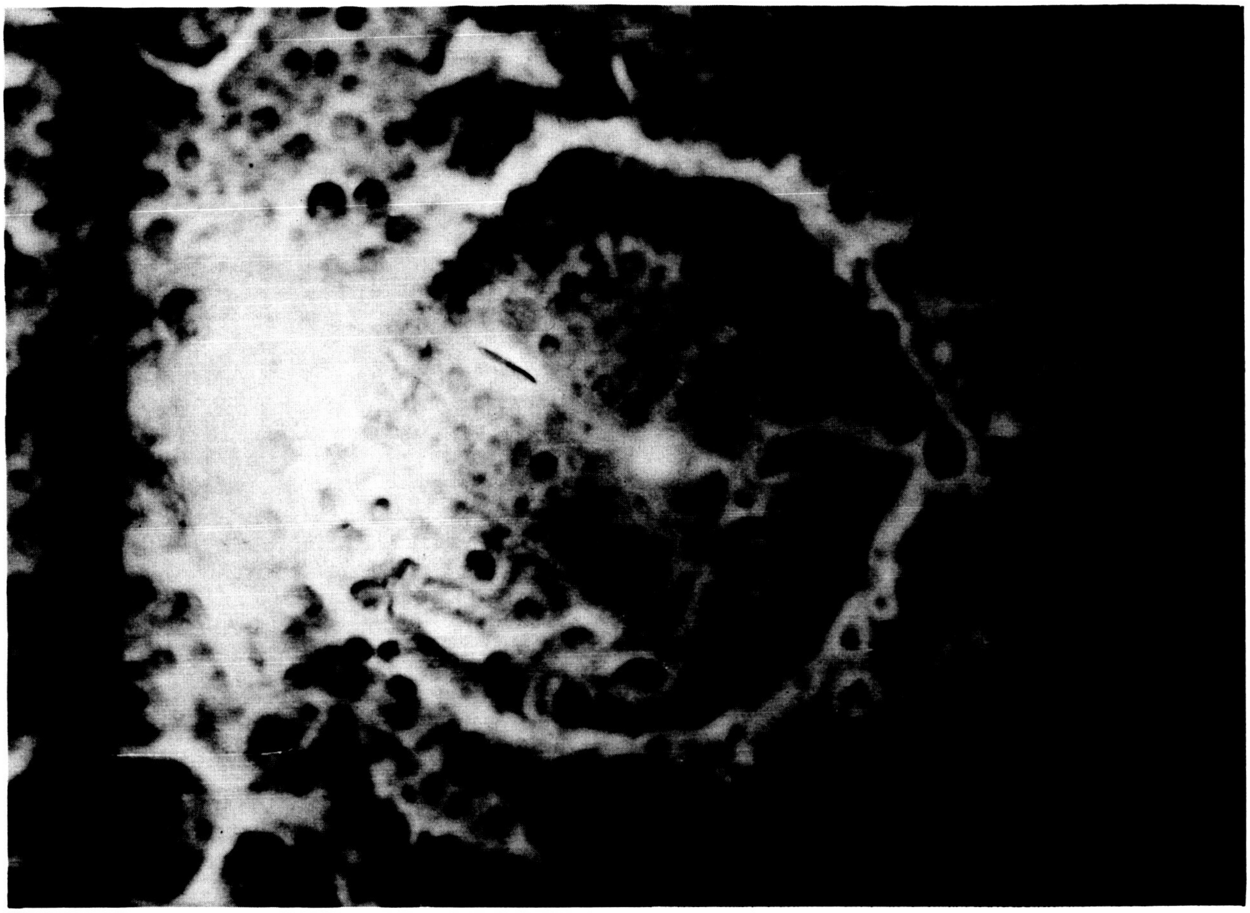
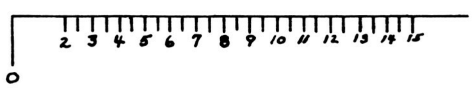


Figure 3.- Typical template used for determining altitude by comparison with horizon curvature. L-67-970



(a) Typical slide used for the surface-feature technique. Altitude is 156 miles (251 km). Crater is Abulfeda. L-67-971



(b) Visual-angle scale used in the surface-feature technique.

Figure 4.- Reproduction of a typical slide for the surface-feature technique and visual-angle measuring device.

or less for a measurement made to a $1/4^{\circ}$ accuracy. This also would have a negligible effect on the altitude estimations.

Although the test subjects were not instructed as to whether one or two eyes should be used, they almost immediately discovered that it was virtually impossible to make the observations with two eyes and they all resorted to a one-eye technique.

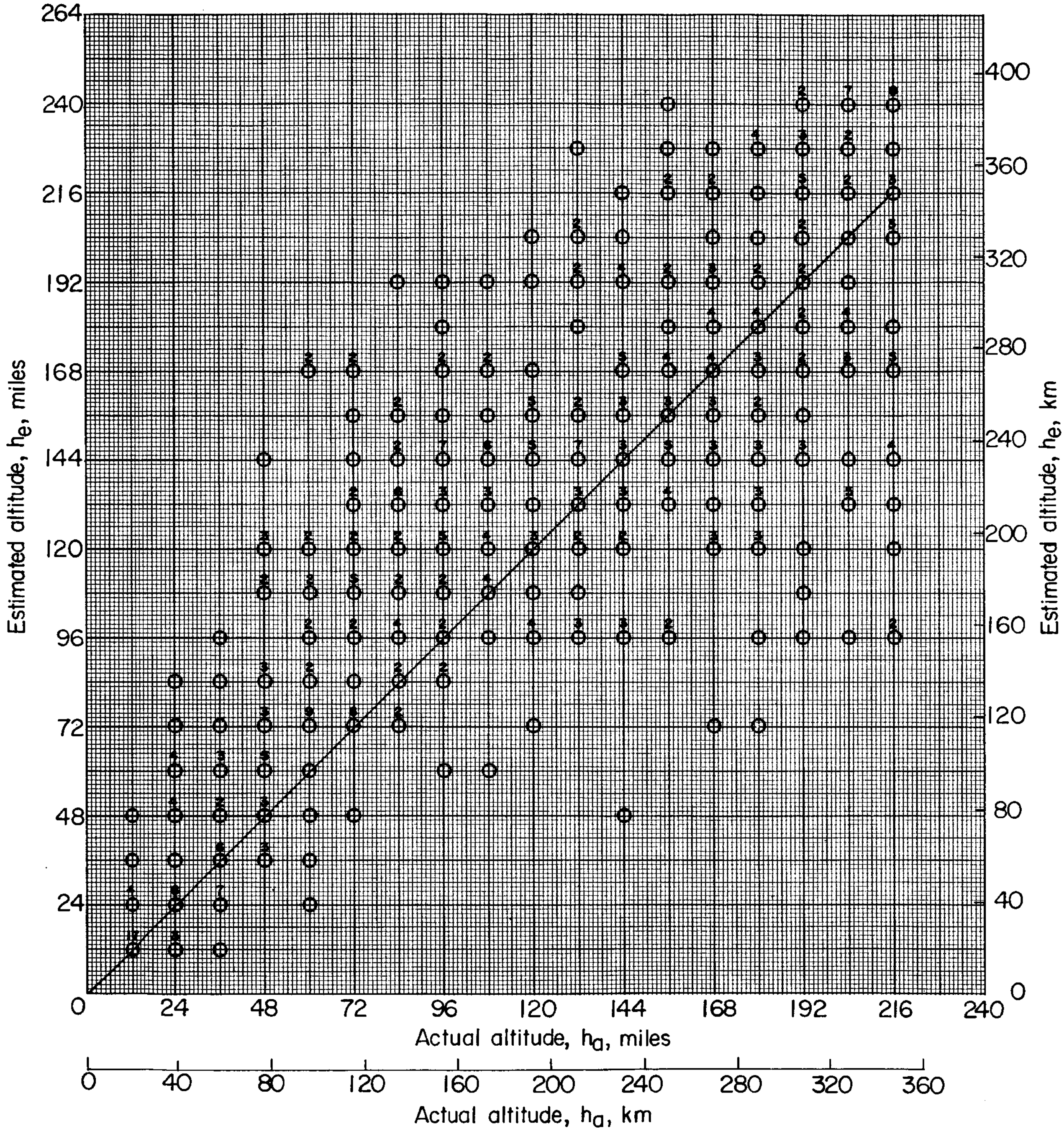
TABLE I.- TYPICAL TABLE USED TO CONVERT VISUAL ANGLE READINGS TO ALTITUDE FOR THE LINE OF SIGHT ALONG THE LOCAL VERTICAL

α, deg	Altitude (miles or km) for crater diameter (miles or km) of –								
	10	15	20	25	30	35	40	45	50
2.00	286	430	573	716	859	1003	1146	1289	1432
2.25	255	382	509	637	764	891	1019	1146	1273
2.50	229	344	458	573	687	802	917	1031	1146
2.75	208	312	417	521	625	729	833	937	1042
3.00	191	286	382	477	573	668	764	859	955
3.25	176	264	353	441	529	617	705	793	881
3.50	164	245	327	409	491	573	655	736	818
3.75	153	229	306	382	458	535	611	687	764
4.00	143	215	286	358	430	501	573	644	716
4.25	135	202	270	337	404	472	539	606	674
4.50	127	191	255	318	382	445	509	573	636
4.75	121	181	241	301	362	422	482	542	603
5.00	115	172	229	286	344	401	458	515	573
5.25	109	164	218	273	327	382	436	491	545
5.50	104	156	208	260	312	364	416	468	521
5.75	100	149	199	249	299	348	398	448	498
6.00	95	143	191	239	286	334	382	429	477
6.25	92	137	183	229	275	321	366	412	458

RESULTS AND DISCUSSION

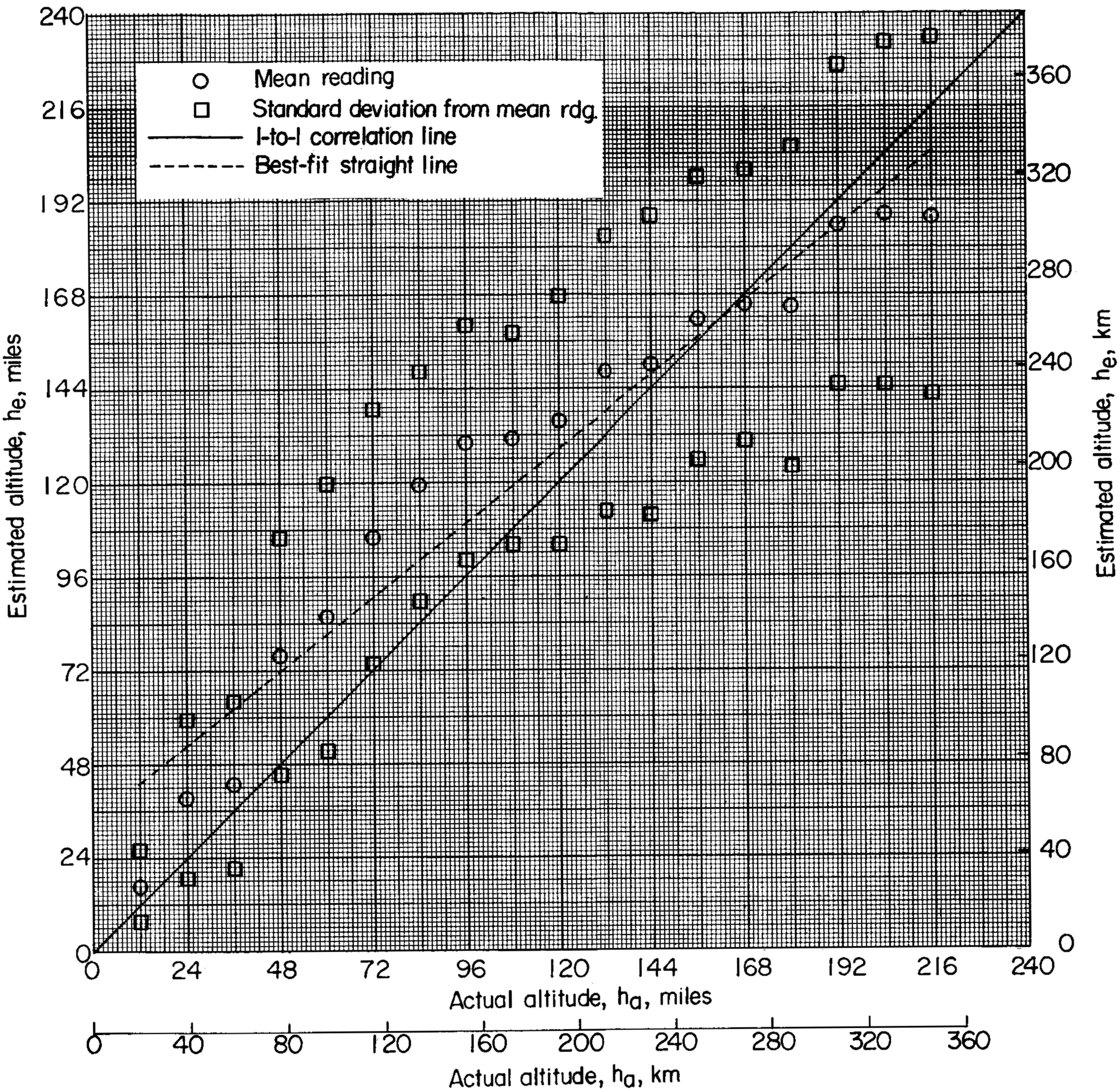
Horizon-Curvature Technique

The data shown in figure 5(a) are a compilation of the readings of all the observers for the initial presentations. Figure 5(b) shows the curves for 1-to-1 correlation and the best straight-line fit through the data, the mean estimated altitude, and the band of standard deviation σ from this mean altitude. The magnitude of the average error shown in this figure varies from about 6 to 36 miles (9.7 to 58 km) and the standard deviation from



(a) Compilation of all the readings for all the observers.

Figure 5.- Compilation and summary of the data for the initial presentation of the horizon-curvature technique. Numerals above points indicate the number of times the estimate was made.



(b) Summary of the data presented in figure 5(a).

Figure 5.- Concluded.

about 15 to 40 miles (24.1 to 64.4 km). The skewness of the best-fit straight line with respect to the 1-to-1 correlation is due to the generally high readings at lower altitudes (below 144 miles, or 232 km) and the generally low readings at high altitudes (above 192 miles, or 309 km). At the lowest altitudes, below 36 miles (58 km), the data were biased toward the high side because no provision was made for estimating altitudes of zero or below and only positive errors were possible. At the high altitudes, above

192 miles (309 km), the data were biased toward the low side because the greatest altitude on the templates was 240 miles (386.4 km) and this was not high enough to permit readings with an unrestricted high-side error. This effect can be seen in figure 5(a) in the high number of readings at the highest available template curvature (240 miles, or 386.4 km). In the center altitude region (48 to 144 miles, or 77 to 232 km), where there was no template bias, the readings in general were too high.

An attempt was made to determine whether the high estimations of altitude in the middle altitude range were a natural tendency of the observers or due to the display. Examination of the slides for altitudes between 48 and 144 miles (77 and 232 km) showed that in this altitude range a group of mountains in the center of the slide was quite prominent along the horizon. At lower altitudes, these mountains were behind the horizon, and some seas in the foreground were the most noticeable feature. At higher altitudes, the mountains moved into the foreground and became less noticeable. Because it was believed that this group of mountains could influence the altitude estimations, another set

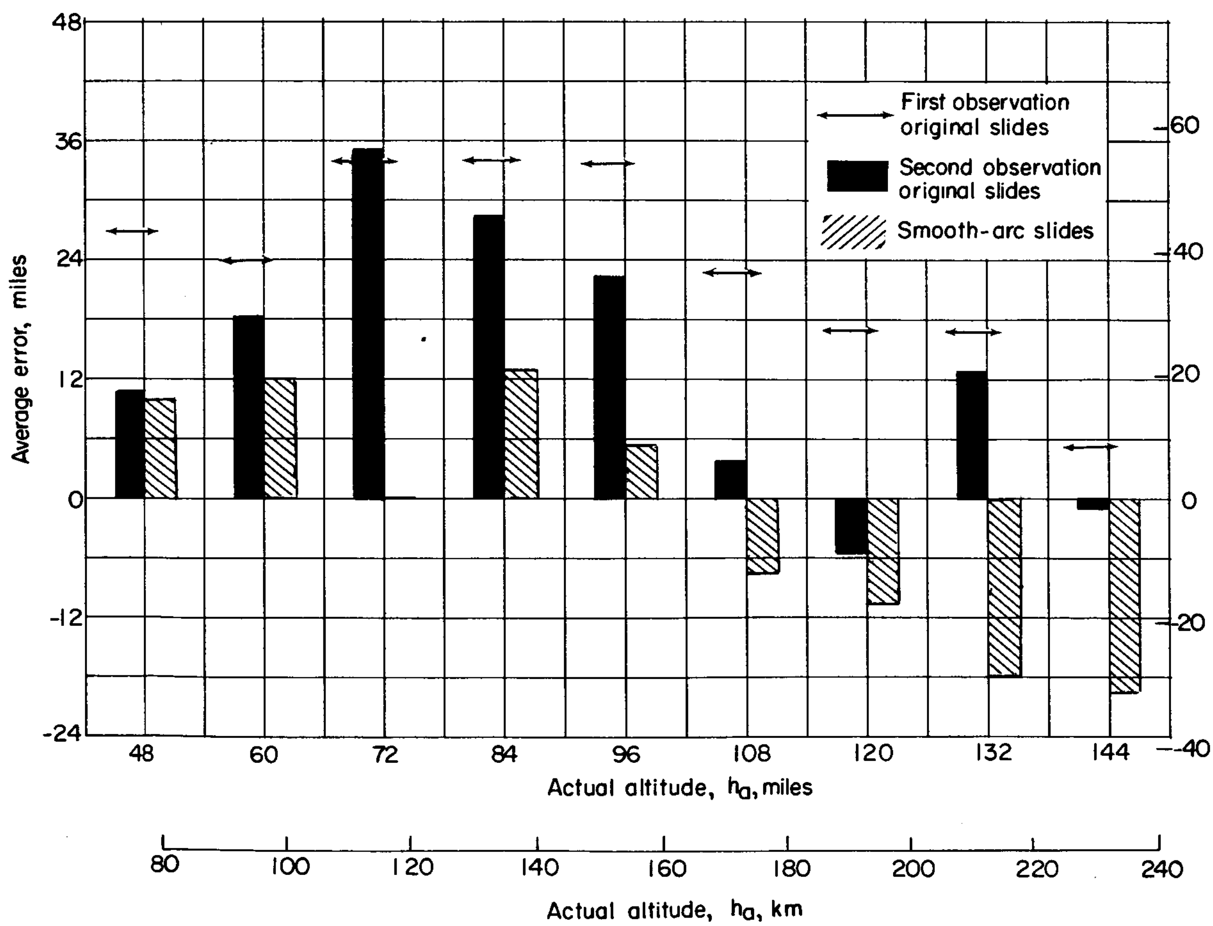


Figure 6.- Effect of repeated observations and horizon irregularities on the average error.

of slides, with a smooth arc for the horizon, was made for the middle altitudes. A comparison of the average errors and standard deviations obtained by using the original slides and the smooth-arc slides is shown in figures 6 and 7, respectively.

The data in figure 6 show two important results. One is that for the second presentation of the slides of the lunar model the average error was considerably smaller than for the initial presentation. The other is that when the smooth-arc slides were used the average altitude readings were lower (that is, smaller positive values or larger negative values) than for either presentation of the slides of the lunar model. The fact that repeated presentations decreased the magnitude of the error indicates that there is a definite learning trend, but the mountains situated in the center of the slides influenced the estimation of altitude toward the high side. The results are shown in another manner in figure 8, which presents the average estimated altitudes for the various actual altitudes of the three presentations. Generally, the smooth-arc slides resulted in estimates closer to the correct values (the 1-to-1 correlation line).

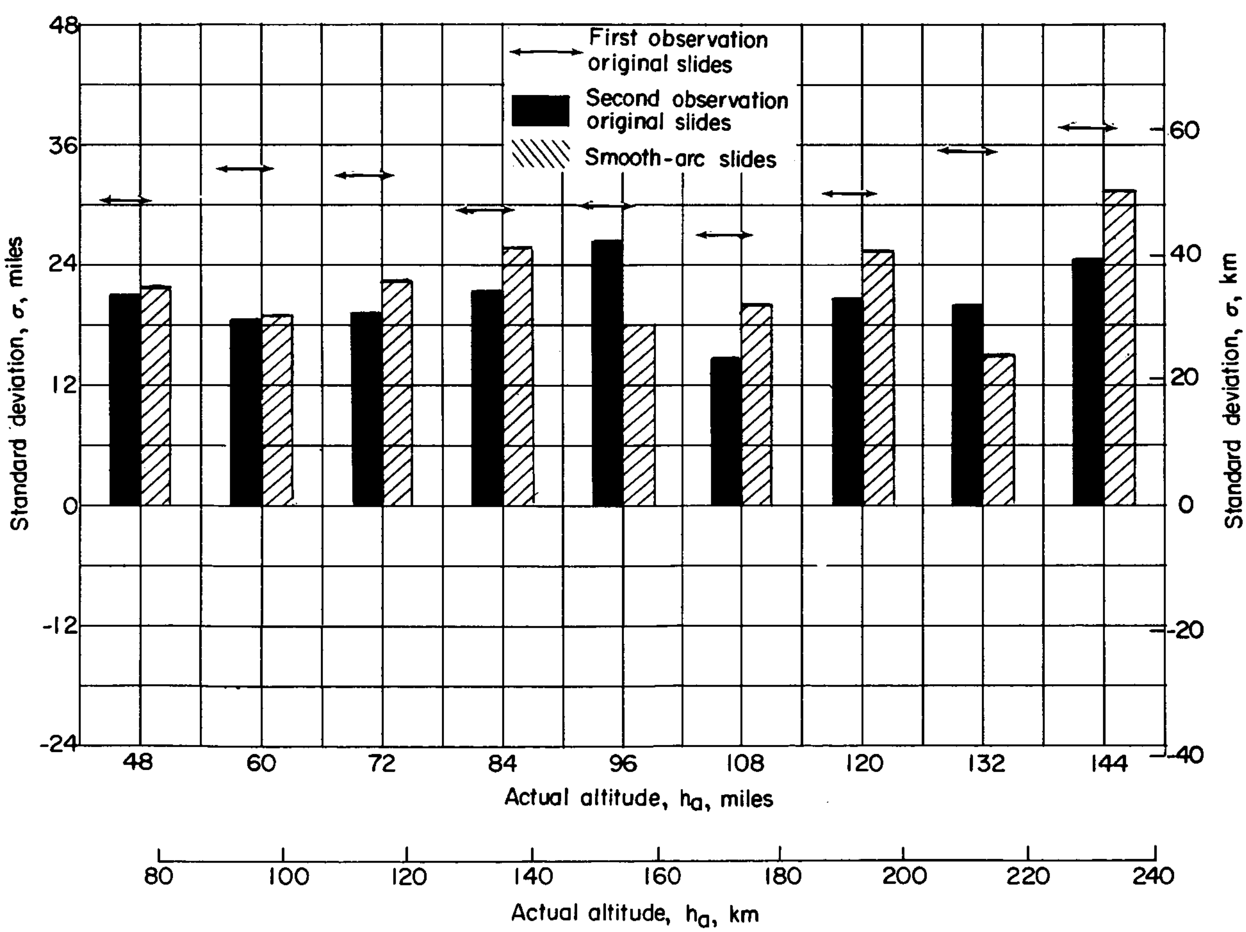


Figure 7.- Effect of repeated observations and horizon irregularities on the standard deviation.

The standard-deviation data of figure 7 present a picture generally similar to that for the average error in that the second presentation improved the results; however, the smooth-arc slides did not result in any improvement over the second presentation. The standard deviation σ of the estimated altitude was about 28 miles (45 km) for the first observation of the original slides and about 20 miles (32 km) for the second observation and for the smooth-arc slide observations. The correlation coefficient r, as discussed in reference 1, is a measure of the degree of relationship of the estimated altitude to the

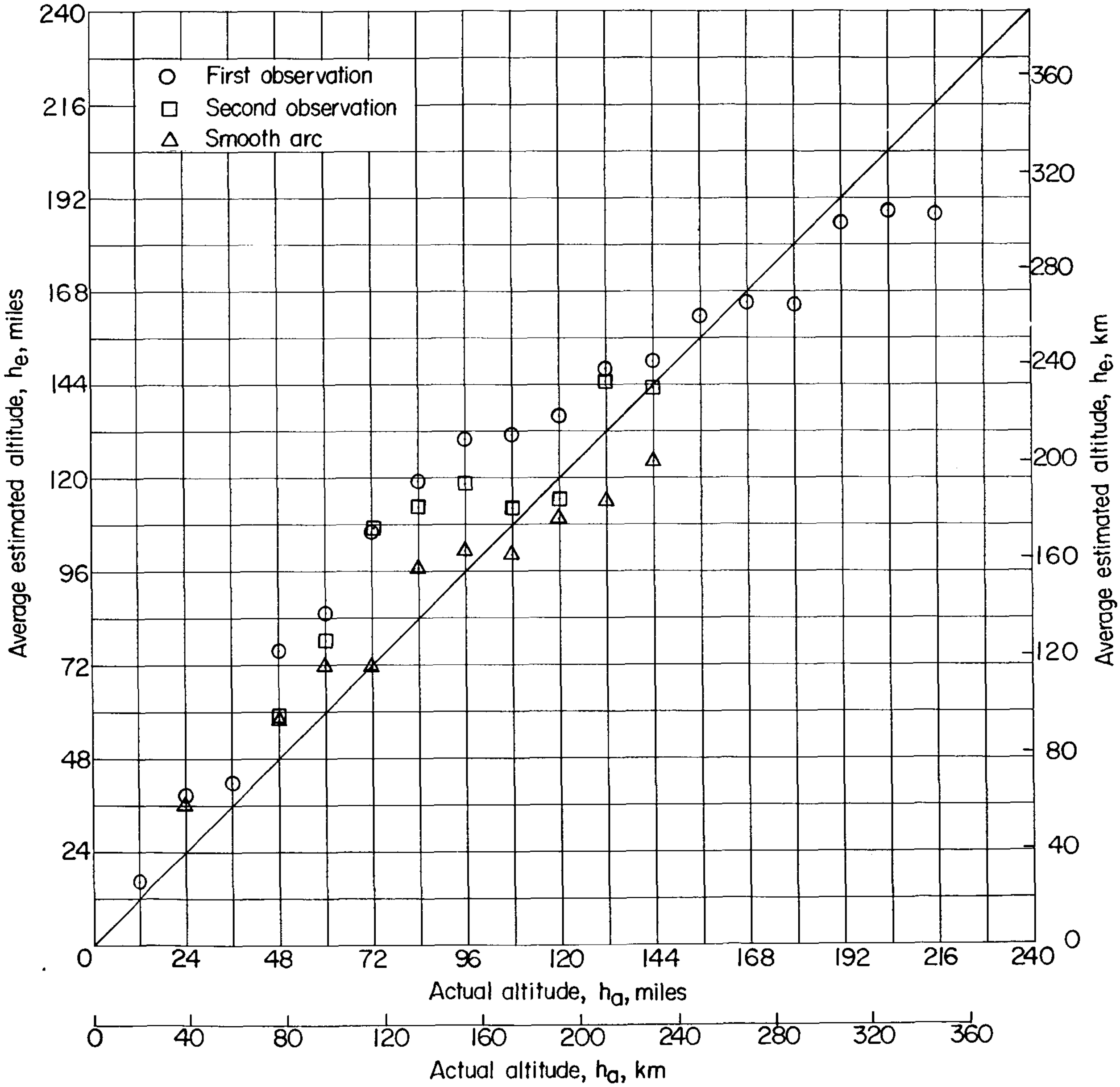


Figure 8.- Average altitude estimated for first observation with original slides, second observation with original slides, and observation with smooth-arc slides.

actual altitude and was obtained from the expression

$$r = \frac{N\sum(h_{e,i}h_{a,i}) - \sum(h_{a,i})\sum(h_{e,i})}{\sqrt{N\sum(h_{a,i}^2) - \left(\sum h_{a,i}\right)^2}\sqrt{N\sum(h_{e,i}^2) - \left(\sum h_{e,i}\right)^2}}$$

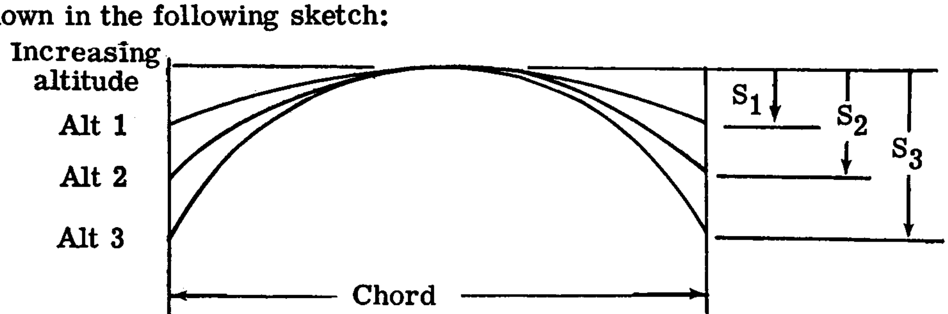
where h_a and h_e are the values of the actual and estimated altitudes, respectively, N is the number of readings, and i varies from 1 to N. For the data discussed in this paper the correlation coefficient had a value of 0.805. A value of this magnitude indicates reasonable correlation between the estimated and actual altitudes.

The best-fit straight line through the data is given by the regression equation for the estimated altitude, $h_r = A + Bh_a$ (see ref. 1), where

$$A = \frac{\sum(h_{a,i}^2)\sum(h_{e,i}) - \sum(h_{a,i})\sum(h_{a,i}h_{e,i})}{N\sum(h_{a,i}^2) - \left(\sum h_{a,i}\right)^2} \qquad B = \frac{N\sum(h_{a,i}h_{e,i}) - \sum(h_{a,i})\sum(h_{e,i})}{N\sum(h_{a,i}^2) - \left(\sum h_{a,i}\right)^2}$$

The symbols h_a and h_e are the same as defined previously.

In attempting to find some explanation for the apparently large errors in altitude estimation (standard deviation of about 28 miles, or 45 km), the decision was made to relate an error in choice of the arc on the template to the error in altitude. An error in choice of the proper arc on the template results from misjudgment of the horizon curvature. The horizon curvature is determined by the radius of the small circle described by the tangent points of the line of sight from the vehicle to the surface of the sphere (see appendix, fig. A1). This radius, decreased in magnitude as dictated by the geometry of the display system, was used to determine the radii of the template arcs for the different altitudes. However, since only a portion of the circle is presented, and the chord of all the presented arcs was the same, the different curvatures are manifest in different sagittas shown in the following sketch:



The relationships for computing the sagitta for the template from the altitude and field of view are given in the appendix, and these data were used to determine the error curves, presented in figure 9. These curves show the error in estimated altitude that results from a misjudgment of the horizon curvature such that the sagitta of the chosen arc differs from that of the correct arc by only 0.01 inch (0.25 mm). The error curves for three angles of view for the astronaut, 40°, 60°, and 80°, are shown. The 40° field-of-view curve is the one that is applicable to the data in this paper (the actual field of view is 36.4°); the other two curves will be discussed subsequently.

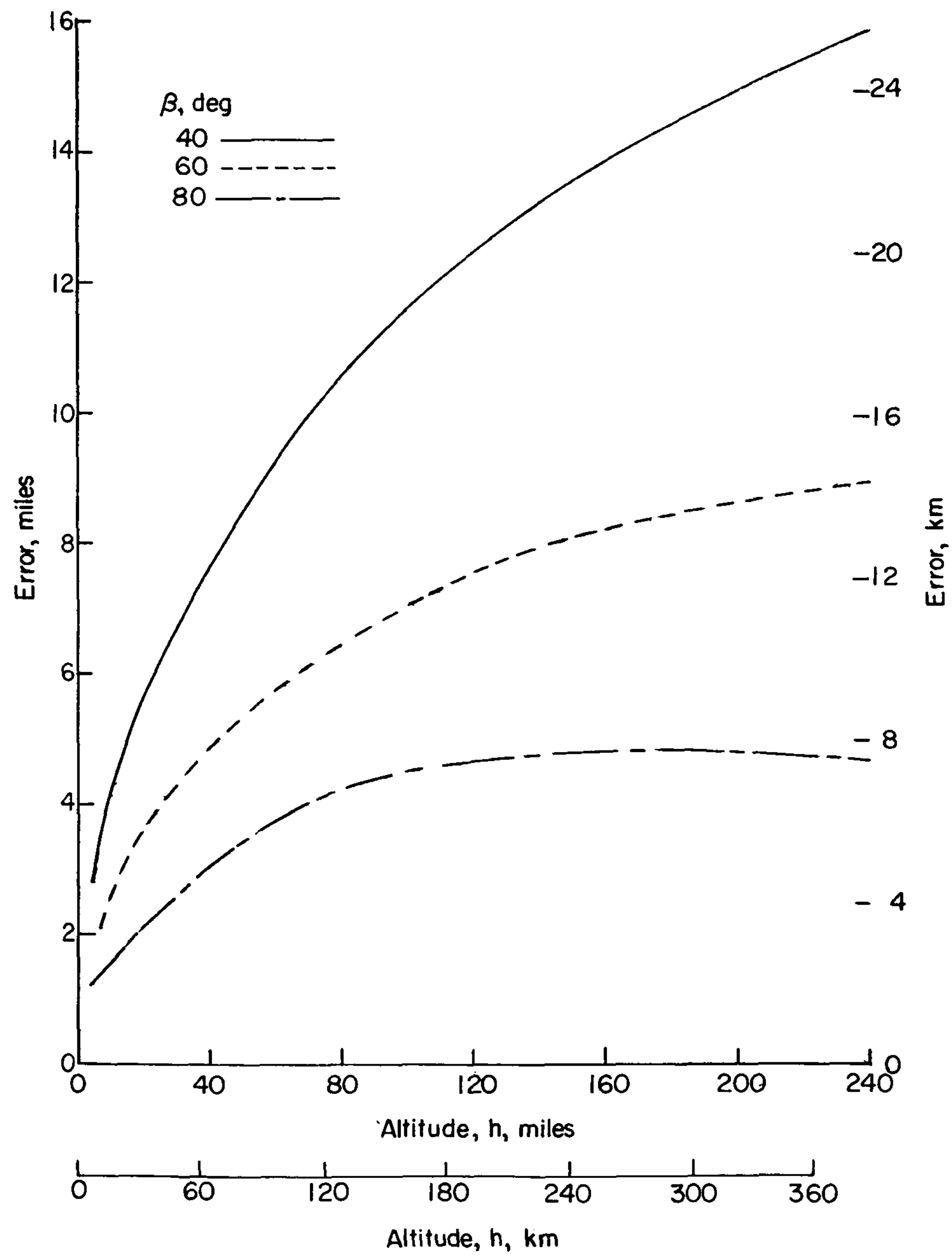


Figure 9.- Error in altitude for each 0.01-inch (0.25-mm) error in sagitta on template.

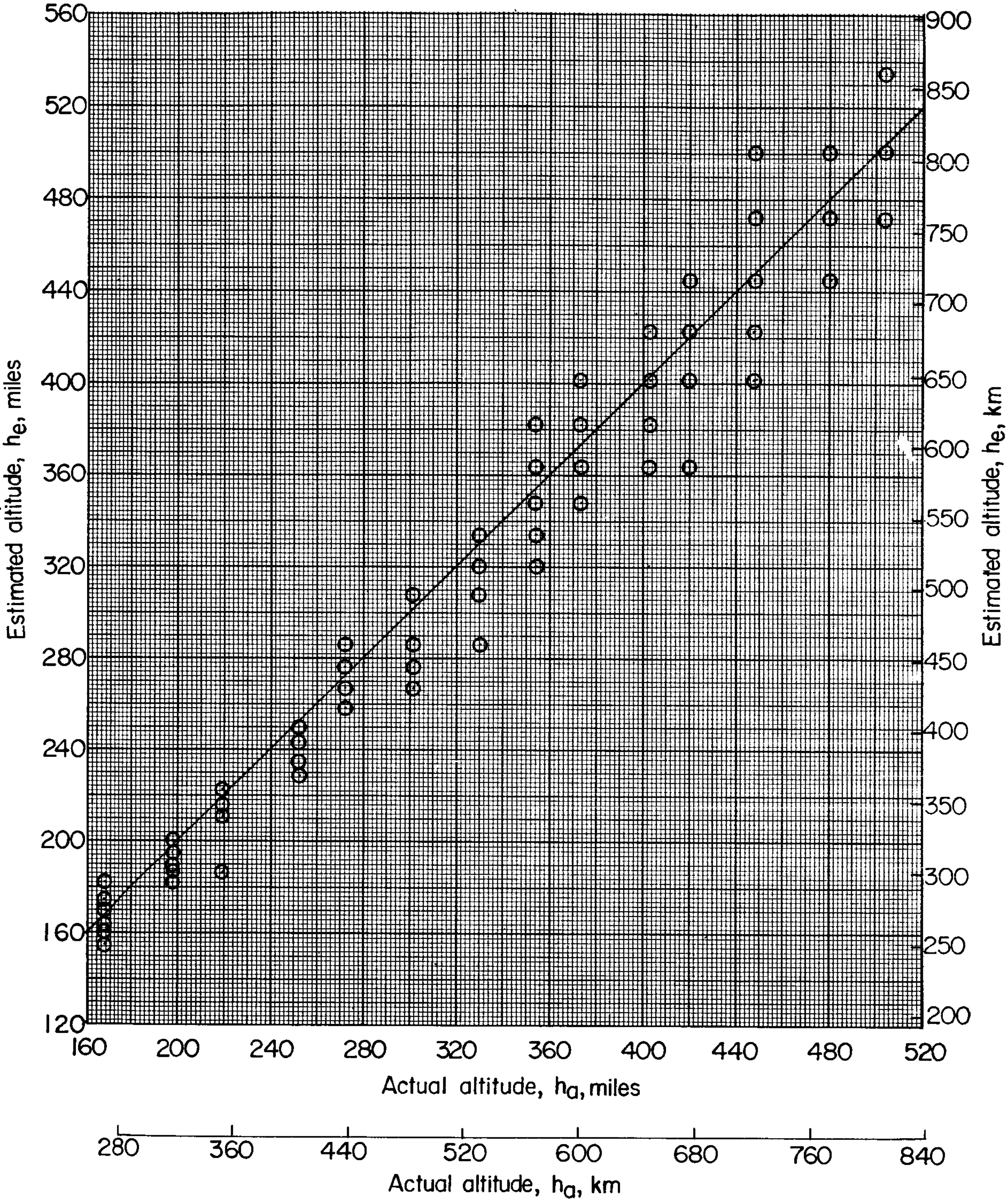
The 40° curve in figure 9 shows that if the observer were to choose an arc on the template that differed in sagitta from the correct curve by only 0.01 inch (0.25 mm), the estimate of the altitude would be in error by about 3 miles (4.8 km) at the low altitudes (about 12 miles, or 19.3 km) and about 15 miles (24 km) at the high altitudes (about 240 miles, or 386 km). For altitudes from 20 to 100 miles (32 to 161 km), the theoretical error in altitude for each 0.01-inch (0.25-mm) error in reading the template varies from about 4 to 11 miles (6.4 to 17.7 km). The test results showed that the standard deviation of the estimated altitude from the correct altitude was about 20 to 30 miles (32 to 48 km). Interpreting the standard deviation of the measured results as an error in selecting the template arc shows that, generally, the error in selection of the proper template arc was such that the sagitta was off by 0.05 to 0.03 inch (1.27 to 0.76 mm). The setup used for the tests was such that the horizon was projected on a screen about 24 feet (7.3 m) in front of the observer, whereas the template was only 1 foot (0.3 m) in front of the observer. Because the template was located so close to the observer, the possibility that errors due to the inability of the eye to focus sharply on both the template and the screen had to be considered. However, the information contained in reference 2 points out that the accommodation (that is, changing focus from far to near) process is very rapid, occurring in less than half a second, and is to a large extent instinctive. The normal eye therefore would be shifting focus back and forth from template to horizon trying to maintain both in optimum focus.

The three error curves in figure 9 show that, for a 0.01-inch (0.25-mm) error in sagitta, there is a considerable decrease in the error of the estimated altitude with an increase in the field of view. If the field of view is increased from 40° to 80° the altitude estimation error theoretically can be reduced by more than 50 percent. This beneficial effect results from the fact that there is a greater separation between the ends of successive arcs as the viewing angle is increased. In addition to this purely geometric benefit, the wider field of view, because it offers a longer arc with greater separation at the ends, allows the astronaut to do a much better job of matching the template arc to the horizon curve.

Surface-Feature Technique

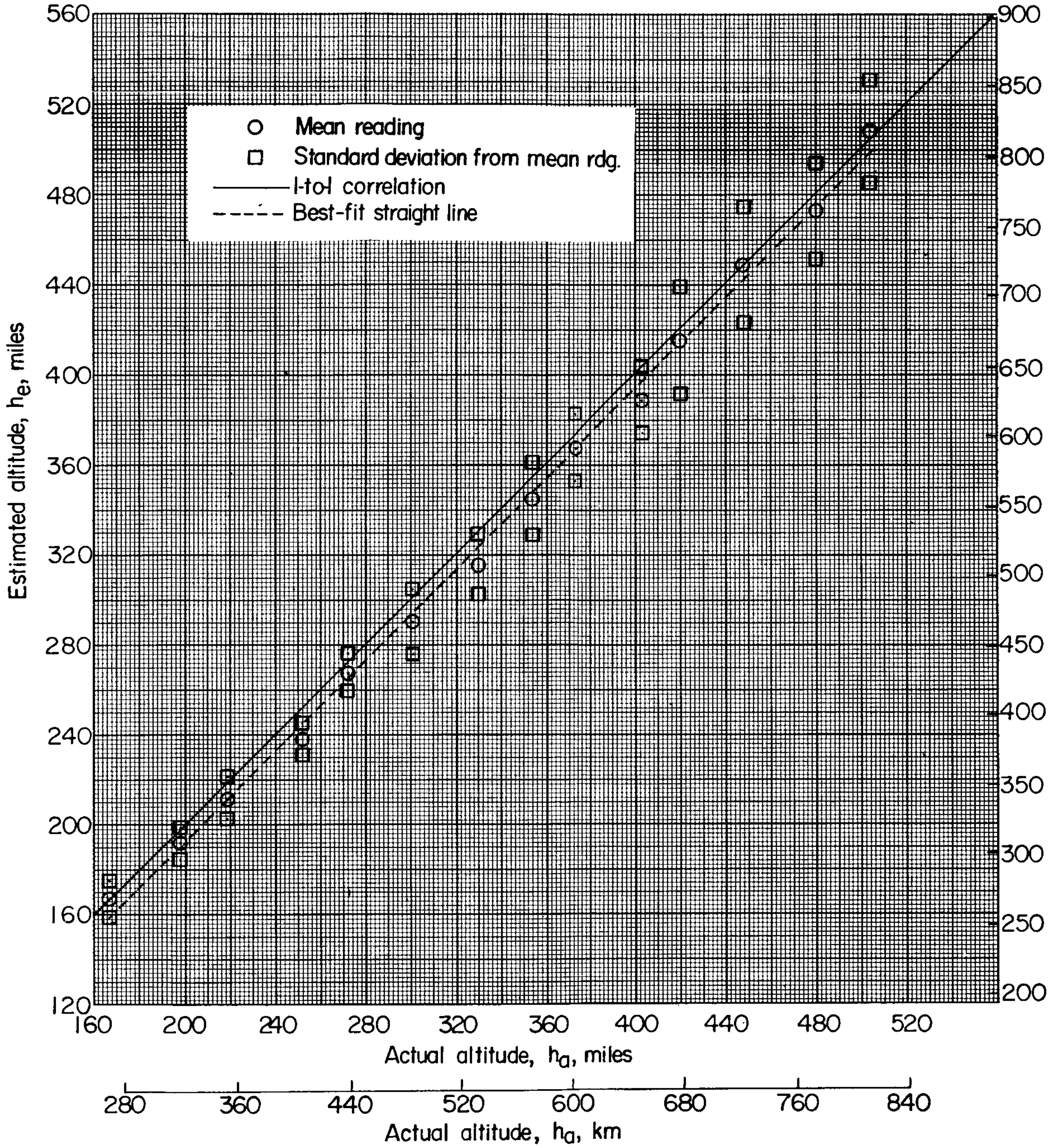
The data shown in figure 10(a) are a compilation of the surface-feature readings for all the observers tested. In figure 10(b) these data are reduced to mean readings and standard deviations from these readings.

The range of altitudes presented resulted from the fact that the crater chosen (Abulfeda) is about 35 miles (56.3 km) in diameter and the camera was limited in how close it could approach the model and still capture the entire crater on the film. If a smaller crater had been chosen the data would be shifted toward zero altitude in direct



(a) Compilation of all the readings for all the observers.

Figure 10.- Compilation and summary of the data for the surface-feature technique.

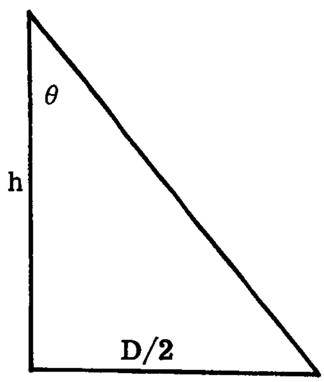


(b) Summary of the data presented in figure 10(a).

Figure 10.- Concluded.

ratio to the size of the craters; for instance, if a crater 10 miles (16.1 km) in diameter were used the data would be shifted to the left down to an altitude of approximately 50 miles (80.5 km).

From the data in figure 10 it can be seen that the correlation between the estimated altitude and the actual altitude was quite good. This is shown by the close agreement between the line of perfect correlation and the line of best fit and by the high value of the correlation coefficient, 0.9867. A comparison of these data for the surface-feature technique with the data for the horizon-curvature technique (fig. 6) shows the much better correlation obtained with the surface-feature technique. The average-error data (fig. 11) show no definite trend with changes in altitude within the accuracy of the data. The data for the standard deviation (fig. 12) show a definite trend of increasing magnitude with increasing altitude. This trend is a result of the geometry of the situation and the manner in which the altitude error is dependent upon the actual altitude, as shown in the following discussion. In the adjacent sketch, if θ is half the optical angle of the object, h the altitude above the object, and D the diameter of the object, then the expression for the altitude is



$$h = \frac{D/2}{\tan\theta} \tag{1}$$

The derivative of this expression is

$$\partial h = -\frac{D/2}{\sin^2\theta}\,\partial\theta \tag{2}$$

If θ is assumed to be small so that $\sin\theta \approx \tan\theta$, the expression for h becomes

$$\partial h = -\frac{h^2}{D/2}\,\partial\theta \tag{3}$$

If the observer can measure the optical angle to the same accuracy at all altitudes, then it is obvious that the altitude error will be larger at the high altitudes.

The comparison of the average error and the standard deviation for this method with those for the horizon-curvature technique (figs. 11 and 12) shows that generally both are much smaller for this method (σ of about 12 miles, or 19.3 km) than for the horizon-curvature technique (σ of about 28 miles, or 45 km).

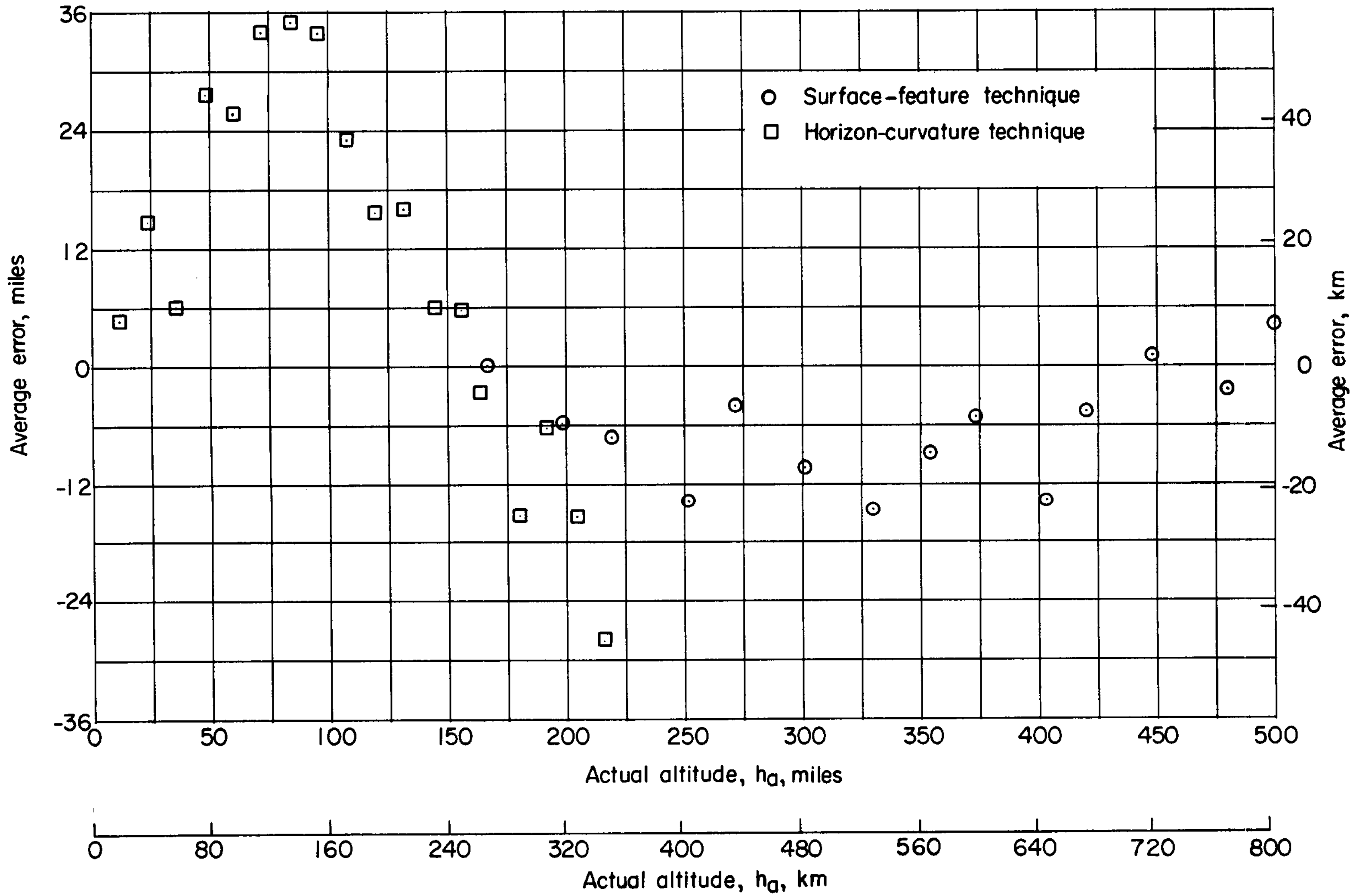


Figure 11.- Comparison of the average errors obtained by the surface-feature technique and the horizon-curvature technique.

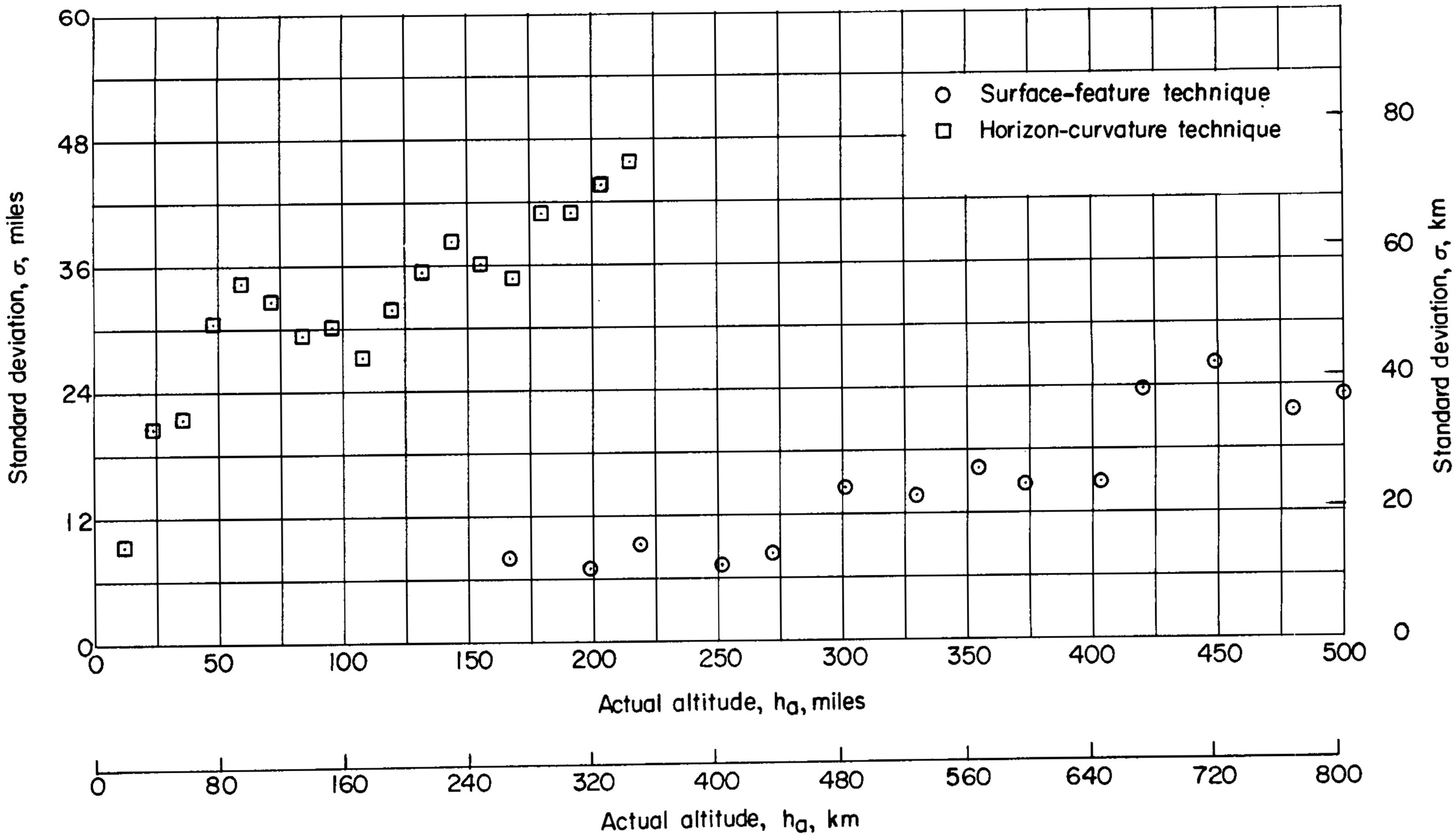


Figure 12.- Comparison of the standard deviations obtained by the surface-feature technique and the horizon-curvature technique.

CONCLUDING REMARKS

An investigation was made to measure the ability of an astronaut to determine, by visual means, the altitude of his spacecraft above the lunar surface. Two techniques were used: one consisted of matching calibrated curved arcs to the projected horizon curvature, and the other consisted of measuring the visual arc subtended by a known surface feature (that is, a crater of known diameter). Although no mention was made as to whether one or two eyes should be used, the observers almost immediately found that a one-eye technique was necessary.

The results of the horizon-matching tests, at least for the limited field of view of this investigation (40^{o}), showed that the average error of the estimated altitude from the actual altitude was quite large, varying from about 6 to 36 miles (9.7 to 58 km), and that the standard deviation also was large, about 28 miles (45 km). An increase in the field of view theoretically would improve the accuracy. The results also showed that mountains or crater lips on the horizon during the attempt to match the curvature could seriously influence the reading toward the high side if located near the center of the field of view, and toward the low side if located near the edges.

A noticeable improvement in the observer's ability to estimate altitude was obtained when he was put through the tests a second time, indicating that there is a learning process.

The surface-feature technique, for those cases in which the surface feature is viewed from directly above, seemed to be considerably more accurate for determining altitude than the horizon-curvature technique. The standard deviation from the correct value, for instance, was only about one-third as large.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., February 15, 1967,
127-51-01-02-23.

APPENDIX

DEVELOPMENT OF EQUATIONS FOR VIEWER'S TEMPLATE AND THEORETICAL SIGHTING-ERROR CURVE

In this appendix, the relations used to compute the radii of the viewer's template and to generate the data for the theoretical sighting-error curve (fig. 9) are developed.

The following relations are apparent from figure A1:

$$\frac{r_M}{Y} = \frac{r_s}{f_c} \tag{A1}$$

$$\tan \phi = \frac{r_M}{Y} = \frac{R_M}{X} \tag{A2}$$

and also

$$\sin \phi = \frac{r_M}{X} = \frac{R_M}{R_M + h} \tag{A3}$$

The expressions

$$\frac{r_s}{f_p} = \frac{r_d}{d_p} \tag{A4}$$

and

$$\frac{r_d}{d_{v,1}} = \frac{r_t}{d_{v,2}} \tag{A5}$$

are obtained from figures A2 and A3, respectively. Equations (A1) to (A5) are combined to obtain the following expression for r_t, the radius of curvature of the template:

$$r_t = f_c \frac{d_p}{f_p} \frac{d_{v,2}}{d_{v,1}} \frac{1}{\sqrt{\frac{h}{R_M}}\left(2 + \frac{h}{R_M}\right)^{1/2}} \tag{A6}$$

Once the projection and viewing geometry has been fixed, the radius of curvature of the template becomes a function of altitude only.

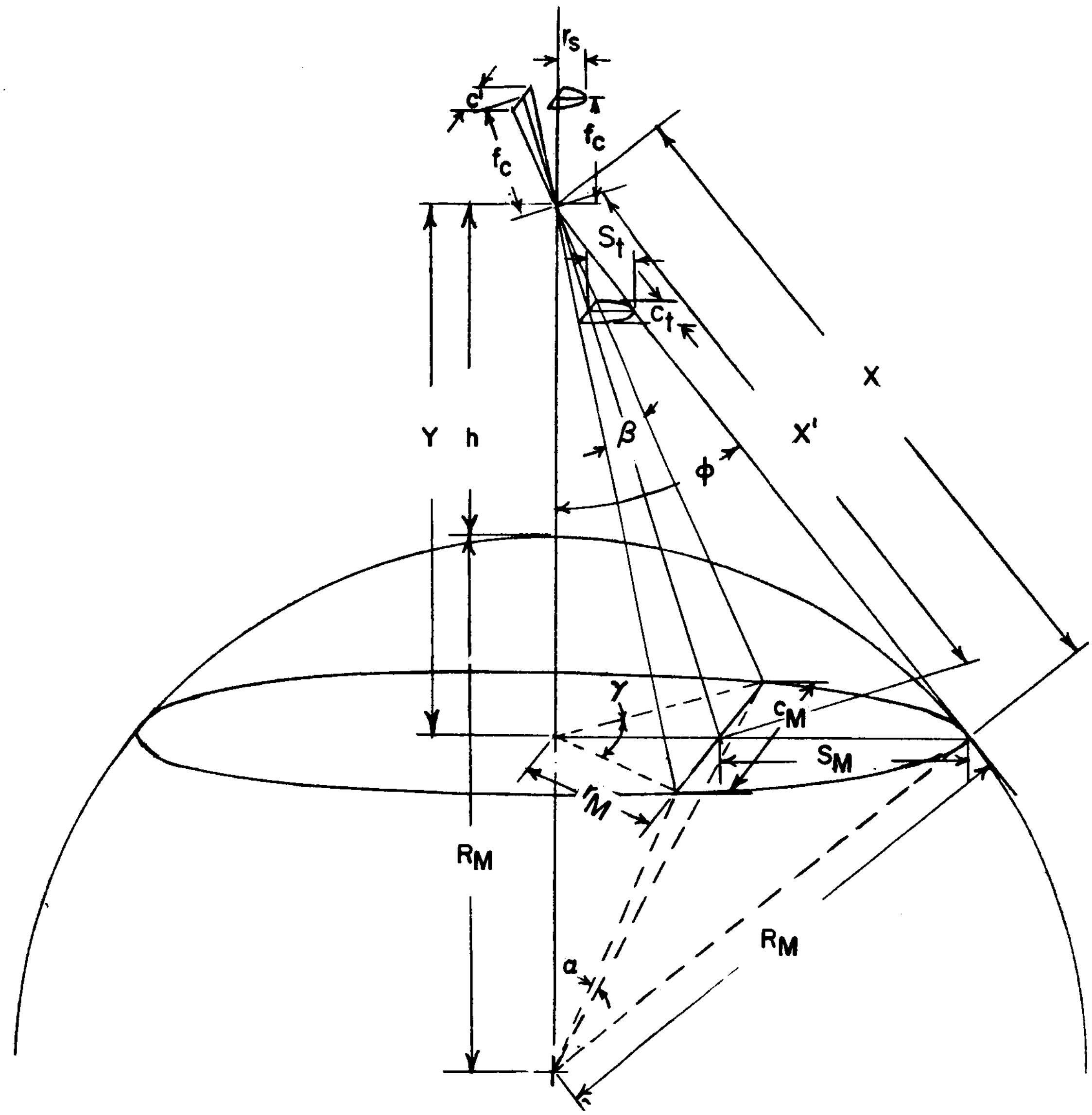


Figure A1.- Geometry of the horizon-viewing problem.

The expressions used to generate the data for the theoretical sighting-error curve (fig. 9) are derived next. The expressions

$$S_t = r_t - \sqrt{r_t^2 - \left(\frac{c_t}{2}\right)^2} \tag{A7}$$

and

$$\frac{c_t}{2} = d_{v,2} \tan\frac{\beta}{2} \tag{A8}$$

can be obtained from figure A3.

Equations (A7) and (A8) are combined to yield the following expression for the sagitta of the template:

$$s_t = r_t - \sqrt{r_t^2 - \left(d_{v,2} \tan \frac{\beta}{2}\right)^2} \tag{A9}$$

When equation (A6) is substituted into equation (A9) the resulting equation for the sagitta is dependent only on the altitude h and viewing angle β:

$$S_t = K\sqrt{\frac{h}{R_M}\left(2 + \frac{h}{R_M}\right)^{1/2}} - \sqrt{K^2 \frac{h}{R_M}\left(2 + \frac{h}{R_M}\right) - \left(d_{v,2} \tan \frac{\beta}{2}\right)^2} \tag{A10}$$

where

$$K = \frac{f_{c'} d_p d_{v,2}}{f_p d_{v,1}}$$

All of the terms in K are constants of the test setup. For those cases in which the angular field of view β is prescribed, it is necessary to compute a theoretical camera focal length $f_{c'}$ that would put the proper horizon curvature on the usable slide width c'. This is obtained from the following relation:

$$\frac{c'/2}{f_{c'}} = \tan \frac{\beta}{2}$$

which results in the expression

$$f_{c'} = \frac{c'}{2} \cot \frac{\beta}{2}$$

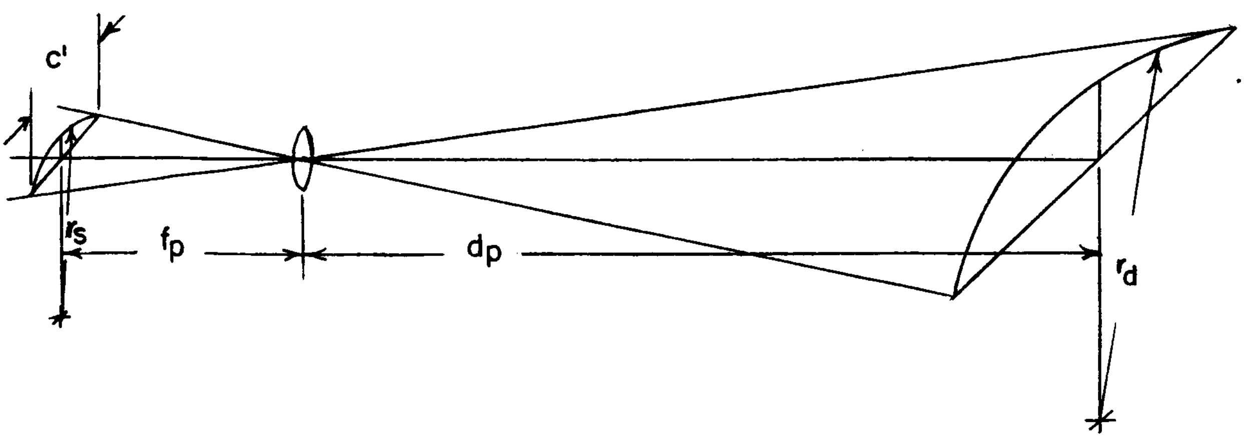


Figure A2.- Geometry of the projection setup.

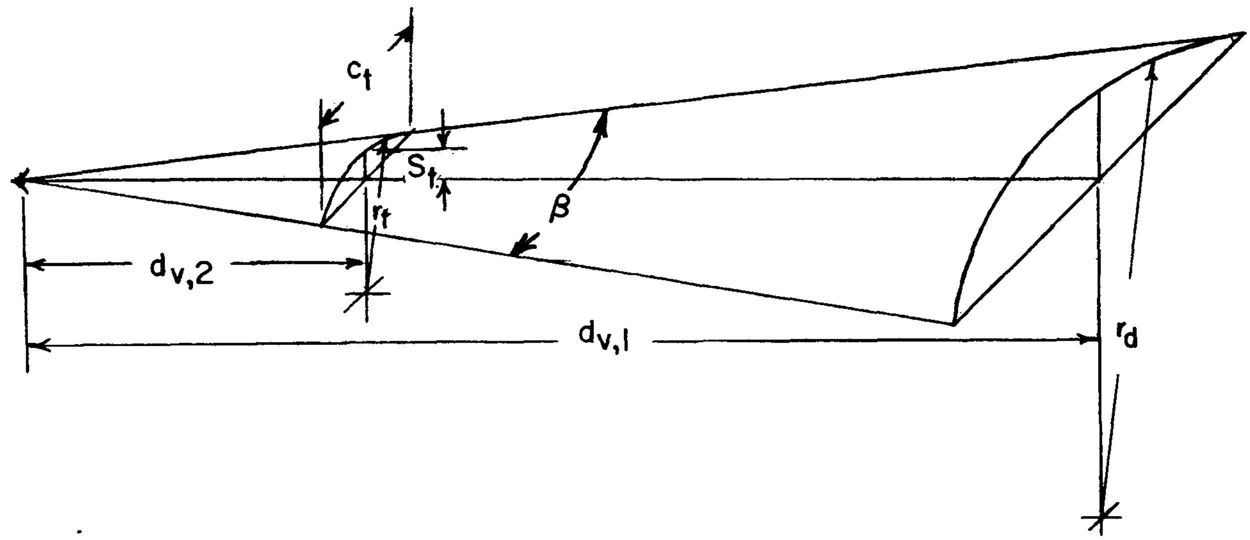


Figure A3.- Geometry of the viewing setup.

When this is substituted into the expression for the constant K, it becomes

$$K = \frac{c' \cot \frac{\beta}{2} d_p d_{v,2}}{2 f_p d_{v,1}}$$

In these tests the viewing angle β was about 40^0, so there was actually only one variable. Computations of the sagitta were made for altitude increments of 2 miles (3.2 km) up to an altitude of 240 miles (386.4 km). The differences between the sagitta for a given altitude and those for the preceding and succeeding altitudes were averaged to obtain the increment in sagitta for a 2-mile (3.2-km) altitude difference at the particular altitude of interest. Thus, it is possible to correlate the error in estimated altitude with an error in matching the proper template curvature with the projected scene. By using the error in sagitta as an indication of the error in matching the horizon curvatures, the data for figure 9 were developed.

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