U. Houston

SEMI-ANNUAL STATUS REPORT ON NASA GRANT NGR-44-005-037

This is to submit a semi-annual status report covering the period from December 1, 1966 through May 31, 1967 on NASA Grant NGR-44-005-037. Individual reports are enclosed for each of the projects contained in the grant.

ON THE ESTIMATION OF THE TRANSITION MATRIX OF A FINITE MARKOV PROCESS

The research effort has been directed toward the problem of determining which nonnegative matrices are diagonally equivalent to stochastic or doubly stochastic matrices and that of determining the minimum permanent of the n x n doubly stochastic matrices.

The following results have been obtained. Corresponding to a given nonnegative square matrix A there is a unique doubly stochastic matrix of the form D_1AD_2 where D_1 and D_2 are diagonal matrices with positive main diagonals if and only if every positive element of A lies on at least one positive diagonal. Furthermore if A is an n x n nonnegative fully indecomposable matrix whose positive diagonal products are equal, then there exists a unique positive n x n matrix B of rank one such that $b_{ij} = a_{ij}$ when $a_{ij} > 0$. As a consequence of this latter result it can be shown that for such an A the nth term of the sequence of matrices obtained by alternately dividing the elements in each row by the maximal element in the row and then the elements in each column by the minimal positive element in the column is a (0,1) matrix. It then follows that distinct n x n doubly stochastic matrices A and B do not have proportional corresponding diagonal products, i.e. there is no $k > 0$ such that for each permutation σ, $\prod_{i=1}^{n} a_{i\sigma(i)} = k \prod_{i=1}^{n} b_{i\sigma(i)}$. Since the permanent of a matrix is the sum of the diagonal products, a partial solution regarding uniqueness has been obtained for the minimum permanent problem.

Related to the D_1AD_2 problem is the following problem which is still not completely solved. Given two nonnegative n x n matrices A and B, under what conditions do there exist diagonal matrices D_1 and D_2 with positive main diagonals such that D_1AD_2 and D_2BD_1 are each stochastic? The special case $B = A^T$ is the aforementioned D_1AD_2 problem. Some partial results are known. If for every zero submatrix $B[E|F]$ in B there is a zero submatrix $A[G|E]$ in A where F and G are disjoint, then there is a unique positive number μ and diagonal matrices D_1 and D_2 with positive main diagonals such that D_1AD_2 and μD_2BD_1 are each stochastic. Of course the same result holds if the roles of A and B are interchanged.

The problem is connected with that of finding positive eigenvalues and eigenvectors for nonlinear but homogeneous monotone operators in n space. The number μ above is a eigenvalue for such an operator. An important question in this study is that of determining for which A,B pairs μ is equal to one. This is the case if B is diagonally similar to either A or A^T, but such a drastic condition is hardly necessary.

Research will be continued toward the end of determining necessary and sufficient conditions on nonnegative matrices A and B so that D_1AD_2 and D_2BD_1 are simultaneously stochastic for appropriate diagonal D_1 and D_2 and the related minimum permanent problem. The research already completed seems promising and gives difinite indication of a satisfactory completion of the problem.

RESEARCH IN ABELIAN GROUPS

The research stimulated by this grant is a continuation of earlier work done by the principal investigator in the theory of semigroups. The present results are concerned, for the most part, with conditions under which continuous characters of closed subsemigroups of compact abelian semigroups admit extensions.

More specifically, we work both with characters and unit-characters of semigroups. A unit-character of a semigroup S is a homomorphism from S into the multiplicative group D of complex numbers z such that $|z| = 1$. A character of a semigroup S is a homomorphism from S into the multiplicative semigroup D^0 of complex numbers z such that either $|z| = 1$ or $z = 0$. Assume S is a compact abelian semigroup and that H is a closed subsemigroup of S. We have the following results:

(A) if χ is a continuous unit-character of H, then χ has an extension which is a continuous unit-character of S if and only if

$$(a,b,x) \in H \times H \times S \quad \text{and} \quad ax = bx \Longrightarrow \chi(x) = \chi(y),$$

and (B) each continuous unit-character of H has an extension to a continuous unit-character of S if and only if

$$(a,b,x) \in H \times H \times S \quad \text{and} \quad ax = bx \Longrightarrow ae = be$$

where e is the least idempotent of H.

We also obtain (much more difficult) extension theorems analogous to those above regarding continuous characters of compact abelian semigroups.

The corresponding condition for (B) is in terms of "approximating" certain groups by means of direct limits of other groups. We will not state the results due to their technical nature.

Various other results are obtained regarding uniqueness of extensions, and concerning the special case of idempotent characters.

This research has been submitted in the form of a paper to a leading mathematical research journal and copies have been sent to the Technical Reports office of NASA in Washington.

THE STRUCTURE OF NEAR-RINGS AND NEAR-RINGS HOMOMORPHISMS

Under the grant an investigation was made, jointly with James R. Clay of the University of Arizona, of the properties of some finite near-rings with identities. The results of this investigation will appear in MATHEMATICA SCANDINAVICA under the title "The Near-Rings with Identities on Certain Finite Groups".

The principal investigator has also considered questions concerning automorphisms of abstract affine near-rings. A paper presenting some of these results was given before the 638th meeting of the American Mathematical Society (see:"Automorphisms of Abstract Affine Near-Rings", NOTICES AMER. MATH. SOC. 13 (1966), 823. Abstract 638-29.). This material is currently being prepared for submission for publication.

METRIZATION OF TOPOLOGICAL SPACES

In the investigation of the relationship between real-valued continuous functions on a topological space and various properties such as normality and countable paracompactness the following was completed and presented at the Arizona State University Conference on Topology March 20-25, 1967.

(1) There exists a connected, locally connected, separable complete Moore space on which every real-valued continuous function is constant.

(2) A necessary and sufficient condition that the characteristic functions of closed sets in a space S be expressible as the greatest lower bound of a family of continuous functions is that the space S be completely regular.

Investigation is continuing on the relationships between insertion theorems which allow continuous functions to be "inserted" between upper and lower semi-continuous functions and various paracompactness conditions.

The following theorem was proved:

Theorem. Suppose A and B are chainable continua which intersect. (1) $A + B$ is a chainable continuum if and only if $A + B$ is atriodic and $A \cdot B$ is connected. (2) $A + B$ is a non-chainable circle-like continuum if and only if $A + B$ is atriodic and $A \cdot B$ is not connected.

Note that (1) is a slightly strenghtened version of a theorem of Fugate (Decomposable chainable continua, Trans. Amer. Math. Soc., 123 (1966), 460-468.

This theorem together with several other results related to it are found in a paper (Decomposable circle-like continua by W. T. Ingram) which has been submitted for publication. Copies of the paper have already been submitted to NASA under the provisions of the grant.