X-612-67-275

NASA TM X-55801

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GPO PRICE $ ____________

CFSTI PRICE(S) $ ____________

Hard copy (HC) 3.00

Microfiche (MF) .65

ff 653 July 65

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JUNE 1967

NASA

GODDARD SPACE FLIGHT CENTER

GREENBELT, MARYLAND

PRESENTED AT THE CONJUGATE POINT SYMPOSIUM,
BOULDER, COLORADO, JUNE 15, 1967

THE ROLE OF THE MAIN GEOMAGNETIC FIELD IN LOCATING CONJUGATE POINTS

by

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Presented at the Conjugate Point Symposium, Boulder, Colorado, June 15, 1967.

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Abstract: A sample distribution of conjugate points is calculated using various models of the main geomagnetic field evaluated at different epochs. A summary of the conclusions is that (1) For $L < 4$ the errors are of the order of a few tens of kilometers; (2) Older models of the field such as Finch-Leaton give results within a few degrees of the latest more accurate models; (3) Use of the eccentric dipole approximation gives errors that range from a few tens of kilometers at high latitude to several hundred kilometers near the equator; and (4) The secular change of conjugate point locations is very small, averaging 1 to 10 Km/yr.

Inclusion of Mead's symmetrical boundary field for a trace from Macquarie Island to Alaska ($L = 5$) shifts the conjugate only about 100 Km. It is concluded that for $L \geq 5$ the field line passes through regions where the field intensity is sufficiently weak that conjugate point determinations should include realistic estimates (not now available) of the distortions due to plasma interactions.

Over the past few years our knowledge of the exact structure of the geomagnetic field has increased considerably. Not only is the main field known to a high accuracy but also some of the gross features of its distortions by external pressures are becoming clear. The purpose of this review is to discuss the significance of this increased knowledge in the context of conjugate point phenomena.

Description of the Main Field

In describing the improvements in our knowledge of the main field over the past few years it is useful to comment on the relative accuracies of the past field models used for conjugate point calculations. We have made such comparisons in various publications using as a criterion the degree to which each field model matches the available set of magnetic survey data. This match is made by comparing the root-mean-square deviation between the measured field component and that predicted by the model. Since the measured components are often the angles D (declination) and I (inclination), their deviations are converted to force units by multiplying by the values of the horizontal and total force respectively. The deviations whose squares are summed thus include ΔF, ΔH, ΔZ, $H\Delta D$, and $F\Delta I$. Also, since the distribution of residuals of observations about model predictions is non-gaussian due to surface crustal anomalies, the data are frequently "clipped" to remove those with high deviations (e.g. $< 2000\gamma$) before a comparison is made. Sometimes data are given relative weights in making comparisons but this sophistication rarely adds to the confidence in a given model. Usually, if the relative standing of two field models changes due to a change in the weights, they are normally regarded as equivalent.

One difficulty in making these comparisons is that many field models do not contain coefficients describing secular change. Inferences as to their accuracy can only be made for data taken very near their epochs, the data must be changed to the epoch of the coefficients, or estimates of secular change must be used to adjust the coefficients. Either of these last two alternates makes the comparison increasingly dependent on the accuracy of the secular change estimates as the time span of the data is broadened sufficiently to include a good global distribution. The sparseness of magnetic survey data is sufficiently great that it has previously been necessary to use data taken before 1925 to obtain a coverage of some areas.

We have published (Cain et al., 1965) a comparison for the interval 1940-1963 of field models including those by Vestine (1960), Jones and Melotte (1953), Nagata and Oguti (1962), Adam et al., (1962, 1963), Fanselau and Kautzleben (1956, 1964), Fougere (1965), Finch and Leaton (1957), Leaton, Malin, and Evans (1965), Jensen and Cain (1962), and a then new model which we have subsequently labelled GSFC(4/64). The results of this comparison were that of the older models which included secular change terms, those of Vestine and Nagata and Oguti gave reasonable ($\sim$ 300γ rms) matches to the data near or before their epochs, but the errors were clearly in the 500-600γ range by 1960. Other older models without secular change coefficients such as those by Fanselau and Kautzleben, and Jones and Melotte were out of date by 1950 and gave increasingly large residuals to more recent data. The Jensen and Cain model was seen to be of no improvement over Finch and Leaton until after 1962.

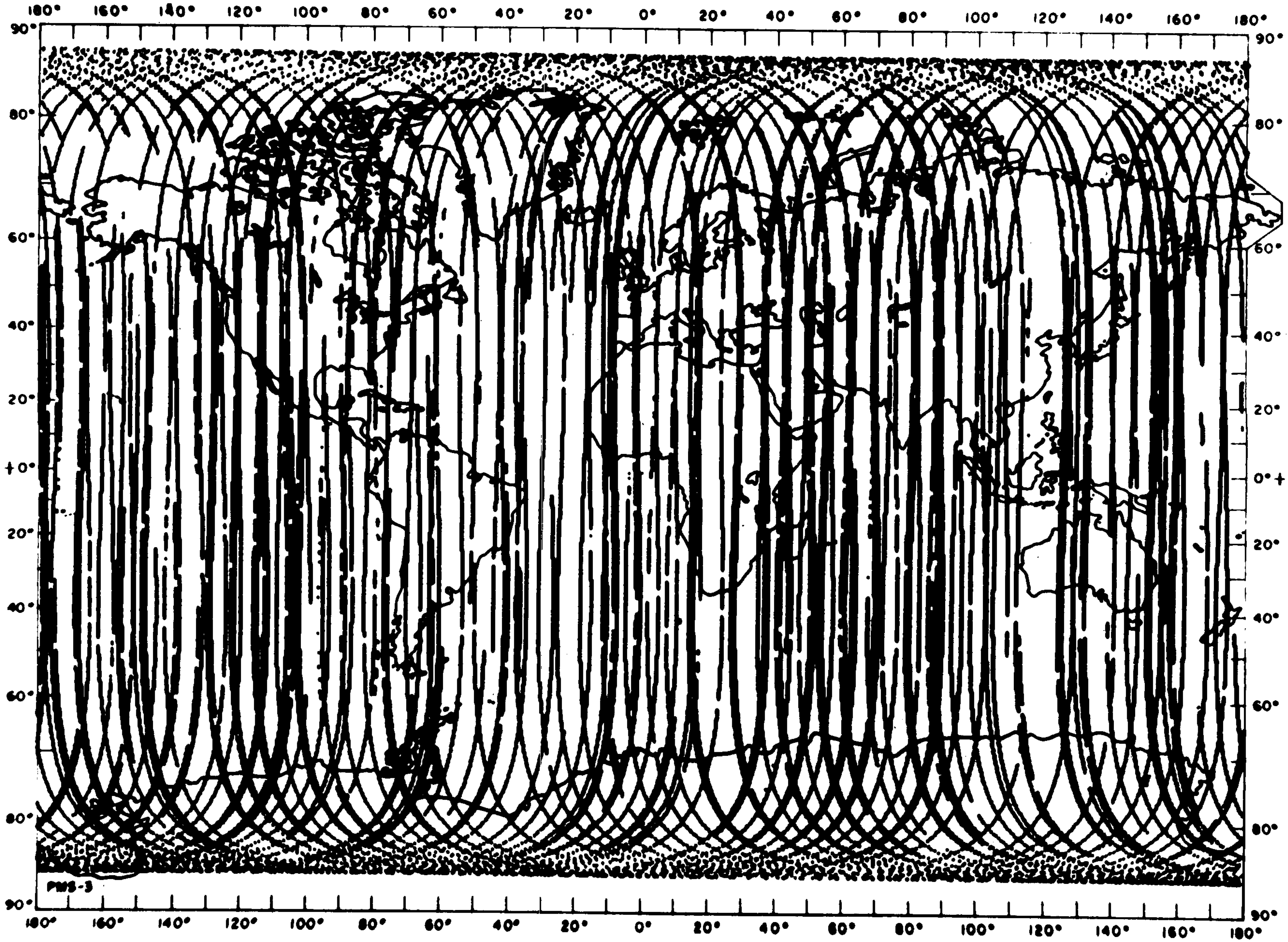
The Fougere model was noted to give erratic residuals over the interval 1955-1963 ranging from 280 to 740γ rms depending primarily on the area over which the survey data were taken. The two best ($\sim$ 250γ rms) matches to recent data at the time of this publication were the Leaton, Malin, and Evans and the GSFC(4/64) field. Their relative merits changed according to how the data were weighted. Although we have not comprehensively compared the Jensen and Whitaker (1960) model with survey data, our experience in attempting to use it as a reference for the Vanguard 3 data (Cain et al., 1962) indicated that it was not noticeably more accurate than Finch and Leaton's. This higher residual was also reported by Fougere (1965) in comparing survey data by using different field models adjusted to the epoch of the survey data by use of the Nagata and Oguti secular change terms. Although the application of these secular change coefficients to the field models makes his detailed conclusions debatable, the fact that the rms errors to the Jensen and Whitaker model were a factor of about two above that of several of the other models makes it another one to dismiss from further consideration.

In more recent publications (Cain, 1966; Hendricks and Cain, 1966) we have presented new models which agree with surface survey data even better than past models. We are currently testing (Cain et al., 1967) a model labelled GSFC(12/66) which gives a weighted rms residual of 99γ to a selection of all magnetic survey data since 1900, including a fit of 13γ rms to a sample of total field data taken during a magnetically quiet period in November, 1965 by the OGO-2 satellite. As shown in Figure 1 these last data cover the earth completely. We conclude that the present errors of the main field model are now near the level of the average time variations.

The previous lack of comprehensive data coverage had the result that even if a given field model fit the available data to a high degree, one would still not know its accuracy over the very large unsurveyed areas of the South Pacific and Southern Polar regions. The only remaining uncertainties in the GSFC(12/66) field description are to evaluate systematic errors in the orbital positions (now estimated as being equivalent to a field distortion $< 20\gamma$), to take into account the quiet daily contributions of sources external to the earth, and to improve the secular change estimates for the high order terms.

Calculating Conjugate Locations

Presentations of conjugate point locations have been given by Vestine and Sibley (1960), Dudziak et al. (1963), Roederer et al. (1965) and at this symposium by Campbell and Matsushita. Vestine



OGO-II Data October 29-November 15, 1965

FIGURE 1

and Sibley used the Finch and Leaton field and a Runge-Kutta-Gill method to integrate the field lines with estimated errors of less than 83 Km. Dudziak et al. used both the Jensen-Cain and the Jensen-Whitaker coefficients to produce a graphical representation. Roederer et al. also used the Jensen-Cain coefficients to list conjugates at various altitudes over selected stations. The map presented at this symposium by Campbell and Matsushita (1967) is based on a 48 term truncation of the GSFC(9/65) coefficients (Hendricks and Cain, 1966).

Both of the last two calculations use a simple line tracing program originally written by W. E. Daniels and employing the use of an Adams four point formula (see Ralston and Wilf, 1960, p. 97). Using integration steps of the order of 50 Km. at low latitudes one can make a two-way field line trace and return to the original starting point within 0.5 Km.

The Effect of Improvements in Field Models

There are not many conjugate phenomena for which an accuracy of more than a few degrees is necessary. Wescott and Mather (1965) have reported that the Macquarie Island magnetic fluctuations for the IGY sometimes appear to correlate better with data taken in central Alaska than with data taken at Kotzebue near the conjugate location. We have calculated in Table 1 the Macquarie Island Conjugate location for a few of the available field models. All of these positions lie within an area of a few tens of kilometers located about 100 Km. to the

TABLE 1

100 Km intersections and minimum field points of geomagnetic field line originating 100 Km over Macquarie Island Magnetic Observatory (54.50°S, 158.95°E).

		Conjugate		Minimum Field Point			
Date	Field Model	Lat	Long	Lat	Long	r/r_e	B
1960.0	J+C	67.3°	-163.9°	5.3°	176.3°	5.42	201γ
1958.0	LME	67.2	-165.0	5.3	176.0	5.32	214
1958.0	GSFC(4/64)	67.3	-165.0	5.4	175.9	5.33	212
1958.0	GSFC(12/66)	67.3	-164.8	5.4	176.0	5.32	212

northwest of Kotzebue. It is thus apparent that such effects as are observed by Wescott and Mather could not be due to uncertainties in the main field. Such a close agreement in conjugate point locations is typical for different points over the earth using different field models.

Perhaps a more sensitive test of conjugacy is that provided by Leonard (1963) in reporting observations of an artificial aurora in the South Pacific. In our paper presenting the GSFC(4/64) model (Cain et al., 1965) we noted an agreement of 27 Km. between the observed and predicted position by either the GSFC(4/64) model or that of Leaton, Malin, and Evans. We repeat these results here in Table 2 to show that the GSFC(12/66) model improves this agreement to within 15 Km.

Evaluating Conjugate Point Locations

Several devices can be helpful in conjugate point evaluations to shorten the computation time if a numerical integration is used. Roederer et al.(1965) used an integration step size of 50L - 30 kilometers, where L is McIlwain's (1966) parameter, and found it satisfactory. Since the computation time is directly proportional to the number of steps, the use of the largest possible interval which will maintain the necessary accuracy is desirable. The feature of dropping high order terms from the field expansion is another useful short-cut already built into the Fortran code distributed by McIlwain (1966)

TABLE 2

Distance R between position of observed artificial aurora [Leonard, 1963] and traces to 122 Km. altitude using field models indicated.

Model	R, km
Finch and Leaton	56
Jensen and Cain	46
Leaton, Malin, and Evans	27
GSFC(4/64)	27
GSFC(12/66)	15

for the computation of B and L (see also Cain et al., 1964). This expedient was designed mainly to reduce computation time as the terms in the potential expansion which are multiplied by $(a/r)^{n+2}$ are reduced to insignificance with increasing r for large n. However, it is easy to see that the result of this truncation is for the lower order terms to be more significant at the higher L values in controlling the conjugate point locations than for the conjugate points at lower L values. The use of only the lowest order terms in a numerical integration can thus be considered, with a degradation of the result mainly in a few low latitude areas. A set of sample conjugate points computed using the first two orders and degrees of the main field potential expansion as compared with the same points computed using all 120 terms of the GSFC(12/66) field model* is given in Table 3. Here we compute for epoch 1965.0 and an altitude of 100 Km. for both the beginning and ending point. For a comparison, values are listed alongside as read from the map by Campbell and Matsushita distributed at this symposium.

The nmax = 2 column corresponds to results that would be obtained using an eccentric dipole approximation to the field (see Chapman and Bartels, 1940, p. 651. Note correction of misprint g_1^0 instead of g_0^1 in their equation for L_1).

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* As noted earlier the higher order terms are discarded with increasing altitude as they become insignificant. The very conservative truncation algorithm used here was that the maximum degree and order of the expansion was $\text{nmax} = -1 + \frac{7.5}{\ln(1.001 + h/a)} \geq 2$ where h = altitude above geoid and a = 6371.2 Km.

TABLE 3

Conjugate point locations at 100 Km. altitude above given locations.

GSFC(12/66) field model evaluated at 1965.0.

ORIGIN			CONJUGATE LOCATIONS						
			nmax = 2		nmax = 10		From Map By Campbell & Matsushita		Minimum Field
Area	Lat	Long	Lat	Long	Lat	Long	Lat	Long	(γ)
Alaska	65°	-150°	-56.7°	169.3°	-56.5°	167.6°	-56°	168°	200
near Norway	70	10	-61.2	56.3	-62.9	64.1	-61	64	94
USSR	70	110	-49.4	111.3	-49.7	114.4	-50	114	229
USA	45	-90	-67.3	114.0	-66.3	119.8	-66	120	778
Kerguelen	-50	70	65.6	48.5	63.5	44.0	64	43	579
S. America	10	-70	41.0	-68.8	38.2	-71.5	-38	-72	15866
Egypt	30	30	-16.1	35.8	-11.6	32.4	-11	31	18948

As predicted, the greatest error in using only the lowest order field arises in low latitudes where the field line never rises high enough to be free of the influence of higher order terms.

The (α,β) coordinate system as introduced by Ray (1963) and Stern (1967) may also be another technique useful for conjugate point evaluations. However, at the present time the most accurate representations are obtained by a direct field line integration using at least the first 48 terms of a main field expansion as is presented at this symposium by Campbell and Matsushita.

Secular Change

The two main features of the secular change of the geomagnetic field continue to be its slow weakening and apparent westward drift. However, the effect of secular change on conjugate points is an even smaller effect in slowly distorting the field. That is, although the field as a whole appears to drift westward at rates estimated to be of the order of 0.2° per year (Bullard et al., 1950), the shift of the relative locations of conjugate points is another order of magnitude less.

To illustrate this small change we have listed in Table 4 the difference in position for the conjugate points given in Table 3 between 1960 and 1970 as computed using the GSFC(12/66) field model.

TABLE 4

Secular Change in Conjugate Points(1960-1970)

Area	Lat	Long	ΔLatitude (degrees)	ΔLongitude (degrees)
Alaska	65	-150	-.04	-.36
Norway	70	10	.23	-.05
USSR	70	110	.22	-.22
USA	45	-90	.00	.97
Kerguelen	-50	70	.26	-.57
S. America	10	-70	1.32	1.23
Egypt	30	30	-.05	.05

As can be seen in this table the usual change is only of the order of a few kilometers/year with the largest values of the order of 10 Km/yr.

Influence of External Fields

The previous calculations have all included only the main field contribution to the total ambient field at any point along a field line. In Tables 1 and 3 we give the minimum value of fields for the field lines traced. The minimum of 200γ from Macquarie Island to Alaska is sufficiently small that even the quiet field distortions, estimated to be of the order of 40γ from either the external boundary pressure (Mead, 1964) or from internal trapped particles, would cause a significant deflection if there were a large angle between the two vectors. However, most of the known pressures result in perturbation fields that are oriented at a relatively small angle to that of the earth's out to distances of the order of ten earth radii and hence have no serious effect. For example, the addition of the boundary model of Mead (1964) to the main field for Macquarie Island merely moves the conjugate location another 70 Km. west of the values in Table 1 for a sunward boundary distance of 10 earth radii and about 100 Km. for a boundary at eight earth radii. In both cases the conjugate location traces out an ellipse about 50 Km. across during the course of a day. While this model external field is probably only a rough approximation to the true external sources, which must also include trapped plasma and be drastically modified from Mead's representation on the evening side of the earth, it does help to place in some perspective the previous results. External effects will be less significant to field lines extending a smaller distance from the earth. This can be simply illustrated by noting the approximate field values at various distances from the earth using the dipole inverse cube approximation as follows:

r/r_e	Field
1	30000γ
1.5	9000
2	3800
3	1100
4	480
5	240
6	140

Since most of the distortion must occur in the outer regions it is likely that under moderately quiet magnetic conditions the conjugate point predictions using only the main field will be accurate for $L < 4$ to a few tenths of a degree and that the errors will increase to a degree by $L = 5$. Beyond $L = 5$ such predictions must become increasingly less certain and are likely to be only of marginal value above $L = 8$.

During magnetic disturbance where the field changes are known to be of the order of several hundred gammas at a few earth radii the degree of uncertainty must also increase and quantitative predictions are not now likely to be meaningful.

Conclusions

A firm conclusion from this review is that the errors of the main field representations contribute in no way to significant uncertainties in determining conjugate point locations. Unless a need arose for predicting positions to accuracies better than 100 Km., the addition of external field sources is unnecessary up to the auroral zones. The effect of the secular change of the main field on conjugate point maps is so slight that no significant errors will occur by updating such representations at intervals as long as a decade.

The most useful contributions that could be made to this subject in the future would thus be to define the distortions of the field at more than a few earth radii both during magnetically quiet and disturbed conditions.

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