

NASA-179

SPACE RESEARCH COORDINATION CENTER



GPO PRICE \$ _____

CFSTI PRICE(S) \$ _____

Hard copy (HC) 3.00

Microfiche (MF) .65

653 July 65

THE PROBLEM OF USING MARINER IV
IONOSPHERIC DENSITIES TO DEDUCE A
MODEL OF THE MARTIAN
ATMOSPHERIC STRUCTURE

BY

T. M. DONAHUE

DEPARTMENT OF PHYSICS

SRCC REPORT NO. 41

UNIVERSITY OF PITTSBURGH
PITTSBURGH, PENNSYLVANIA

7 APRIL 1967

(THRU)
1
(CODE)
30
(CATEGORY)

N67-28785
(ACCESSION NUMBER)
24
(PAGES)
CR-84825
(NASA CR OR TMX OR AD NUMBER)

THE PROBLEM OF USING MARINER IV IONOSPHERIC DENSITIES TO
DEDUCE A MODEL OF THE MARTIAN ATMOSPHERIC STRUCTURE

T. M. Donahue

University of Pittsburgh
Pittsburgh, Pennsylvania

April 1967

Reproduction in whole or in part is permissible for any purpose of the
United States Government.

The Problem of Using Mariner IV Ionospheric Densities
to Deduce a Model of the Martian Atmospheric Structure

(Presented at the Conference on The Atmospheres of Mars and Venus
Kitt Peak National Laboratory, February, 1967)

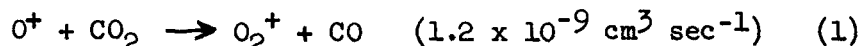
T. M. Donahue
University of Pittsburgh

ABSTRACT

An F_1 layer maximum would apparently occur fifteen kilometers too high in the Martian ionosphere even though the atmosphere were pure CO_2 in a model with high temperature mesopause and thermopause. It is pointed out, however, that a similar calculation of electron densities in the earth's ionosphere starting with a reasonable model for neutral structure and temperature would also disagree with the observed profile of electron density in the F_1 region. It is argued that where no ionospheric model based on unforced assumptions manifestly supports any atmospheric model it is unreasonable to rely on ionospheric properties to discriminate between models.

Recently I suggested⁽¹⁾ that the Martian ionosphere observed by Mariner IV⁽²⁾ might be an F₁ layer if the temperature profile was close to that calculated by Chamberlain and McElroy⁽³⁾ and the lower atmosphere was almost pure CO₂ with a surface pressure of 6 mb. In the F₁ model an F₂ maximum was to be suppressed by the rapid conversion of O⁺ to O₂⁺ by CO₂ up to 230 km where diffusion would begin to control the ion distribution. To achieve this the ratio of O to CO₂ would have to be less than 100 to one below 230 km.

I have now calculated the photo-ionization rate in this model of the Martian atmosphere (Fig. 1) for a solar elevation angle of 20°. The results are shown in Fig. 2. Maximum ionization occurs at 140 km and in the absence of ad hoc assumptions about recombination rates the maximum electron density will also be found near that altitude. If O⁺ is converted to O₂⁺ through the reaction⁽⁴⁾



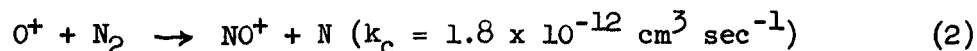
and if CO⁺ is converted to CO₂⁺ by charge transfer⁽⁴⁾ with a rate coefficient of $1.1 \times 10^{-9} \text{ cm}^3 \text{ sec}^{-1}$ the ion densities plotted in Fig. 3 result. It is assumed that both CO₂⁺ and O₂⁺ recombine dissociatively at a rate of $2 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$ independent of altitude. Clearly a thermal model like that of Chamberlain and McElroy, even if the atmosphere is pure CO₂ and has a low surface pressure of 6 mb, will not produce the observed ionosphere if the simplest and most natural assumptions concerning ion production and loss processes are correct. There are, of course, numerous ad hoc alterations which can be made in the model that will force it to yield the desired ionosphere. A lower mesopause temperature might reduce the atmo-

spheric densities sufficiently that optical depth of unity in the ionizing ultra violet can occur at 125 km instead of 140 km. Alternatively, the dissociative recombination coefficient of CO_2^+ can be manipulated, since it has not yet been measured. For example taking $\alpha(\text{CO}_2^+)$ to be $2.5 \times 10^{-8} \text{ cm}^3 \text{ sec}^{-1}$ at 125 km and letting it increase to $4 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$ by 140 km would result in a fair reproduction of the Mariner IV electron densities. Such exercises, however, seem somewhat frivolous, and it is distasteful to accept the results as serious arguments for one model of atmospheric composition and thermal behavior as against another. If a simple and unforced explanation of the Martian ionosphere had followed from some atmospheric model that fact could perhaps have been used as supporting the adoption of that model. Even such a satisfying state of affairs, had it occurred, should have been accepted with reservation and suspicion in view of the experience with the ionosphere of the earth. For here, even with a wealth of information at hand on composition, temperatures and reaction rates it has not been possible to put together a consistent model of that ionosphere.

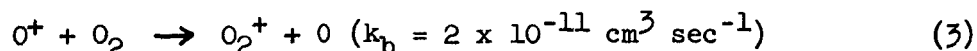
To illustrate this difficulty suppose that we had only as much information about Earth's atmosphere and ionosphere as we do about Mars. Suppose that for the morning of 15 February, 1963 with the sun 30° above the horizon at White Sands, New Mexico we had a measurement of atmospheric density as a function of altitude and based on this a calculation of temperatures and the densities of O, N_2 and O_2 shown in Fig. 4. These are in fact the densities which are given by the observations of Hinteregger, et al.⁽⁵⁾ for that epoch. Suppose also that the measured electron density as a function of altitude were that shown in Fig. 6. This profile agrees with that measured by Holmes, Johnson and Young⁽⁶⁾ on the day selected. Suppose also that there were at hand all of the information which is in

fact available concerning ionization cross sections, solar flux, ion-molecule rates and dissociative recombination rates and armed with these we attempted to predict the electron density as a function of altitude for the proposed atmospheric model. As a by product the steady state densities of the positive ion species would also be produced in the calculation but would not be susceptible to experimental verification under the hypothetical circumstances. It is conceivable that the proposed calculation might reproduce the desired total ion density but predict a breakdown into individual species completely at variance with the ion composition data. Thus the agreement between the model electron densities and those observed would be fortuitous but would probably be used as a strong argument for the validity of the model. Conversely - and this in fact is the case - the calculated electron density profile might differ sufficiently from the observed one to cast doubt on the neutral model even though it is in reality close to the correct one.

Let us, therefore, calculate the rate of production of O^+ , N_2^+ and O_2^+ by ionizing solar flux and fast photo-electrons for this model atmosphere in the standard fashion. The rates obtained are plotted in Fig. 5. The O^+ created is converted to NO^+ and O_2^+ via the reactions



and



The rates shown are the latest quoted by the ESSA group at 300°K⁽⁷⁾. Although no data on temperature dependence of these rate coefficients are yet available from flowing afterglow studies, the results obtained from beam studies at low energy⁽⁸⁾ as well as theoretical arguments strongly

support the supposition that they increase slowly with increasing temperature.

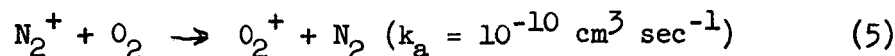
The steady state O^+ density is given by

$$n_1^+ = \frac{q_1}{k_b n_3 + k_c n_2} \quad (4)$$

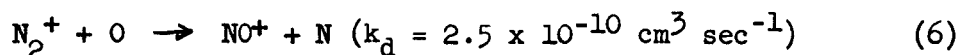
where the subscript 1 refers to O , 2 refers to N_2 , 3 to O_2 and 4 to NO .

The q_i are the solar production rates. The charge transfer reaction of N_2^+ with O is neglected as a minor contributor to the O^+ production.

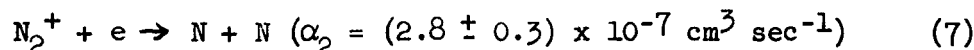
Similarly N_2^+ is converted to O_2^+ and NO^+ by the reactions



and



Again the rate constants are the ESSA room temperature values. N_2^+ is also neutralized by dissociative recombination



The latest information on the temperature dependence of α_2 is curious⁽⁹⁾.

Under conditions where $T_e = T_{ion} = T_{gas}$ almost no change in α_2 occurs between 200° and 480° . However, when T_e alone varies and the ions and neutrals are held at $300^\circ K$ α_2 varies as $T_e^{-1/3}$. At least in the case of N_2^+ these results suggest that α increases with the vibrational excitation of the ion but decreases with the electron temperature. The safest assumption to apply to the atmospheric ion is that α_2 is constant.

The steady state N_2^+ density is determined by

$$n_2^+ = \frac{q_2}{k_a n_3 + k_d n_1 + \alpha_2 n_e} \quad (8)$$

There is no information available on the variation of k_a and k_d with temperature. It would be a great surprise, however, if they were to decrease rapidly with T .

As for O_2^+ it is created by solar uv and photo electrons as well as by processes a and b from N_2^+ and O^+ . A non negligible portion of O_2^+ may be lost by charge exchange with NO at low altitudes. The rest recombines dissociatively at a rate

$$\alpha_3 = (2.2 \pm 0.5) \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1} \quad (9)$$

at 300°K. Between 200° and 1000°K under conditions in which $T_e = T_{\text{gas}} = T_{\text{ion}}$ α_3 appears to follow a $T^{-1/2}$ law⁽⁹⁾.

The steady state gives

$$n_3^+ = \frac{q_3 + k_a n_2 + k_b n_1}{\alpha_3 n_e} \quad (10)$$

Finally, NO^+ created from N_2^+ and O^+ (and O_2^+ via charge transfer with NO) recombines with a rate coefficient⁽⁹⁾

$$\alpha_4 = (5 \pm 1) \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1} \quad (11)$$

at 300°K. The data on temperature dependence⁽⁹⁾, again for $T_{\text{ion}} = T_{\text{gas}} = T_e$ support a variation between T^{-1} and $T^{-3/2}$ between 200° and 5000°K. The density of NO^+ is given by

$$n_4^+ = \frac{k_c n_2 + k_d n_1}{\alpha_4 n_e} \quad (12)$$

Finally a neutrality condition given by

$$n_e = n_1^+ + n_2^+ + n_3^+ + n_4^+ \quad (13)$$

must prevail.

At the F_2 peak, measured to be at 260 km with $n_e = 6 \times 10^5 \text{ cm}^{-3}$ the rate of diffusion loss of O^+ and the chemical loss to NO^+ by process c are in balance as expressed by the relationship

$$k_c n_2 = D/H^2, \quad (14)$$

where the diffusion coefficient D is approximately given by

$$D = 10^{17} \sqrt{T} / n_1 \quad (15)$$

and H is the scale height. Taking k_c to be $2 \times 10^{-12} \text{ cm}^3 \text{ sec}^{-1}$, H to be 50 km and T to be 1000°K this condition demands that at the F_2 peak

$$n_2 n_1 = 6 \times 10^{16} \text{ cm}^{-6}. \quad (16)$$

At 260 km in the model the densities of oxygen and nitrogen are so high that their product is an order of magnitude larger than required by (16). The F_2 peak should occur at 300 km where

$$n_1 = 7 \times 10^8 \text{ cm}^{-3} \quad (17)$$

and

$$n_2 = 9 \times 10^7 \text{ cm}^{-3} \quad (18)$$

Furthermore, n_1^+ should be given at the peak by

$$n_1^+ = q_1 / (k_c n_2) \quad (19)$$

or

$$n_1^+ = 1.2 \times 10^6 \text{ cm}^{-3} \approx n_e \quad (20)$$

with $q_1 = 250 \text{ cm}^{-3} \text{ sec}^{-1}$. The observed F_2 maximum density is a factor of two smaller.

In the F_1 region, however, there are problems. At 130 km where the temperature is close to 300°K the O^+ condition (4) becomes

$$n_1^+ = \frac{10^3}{.35 + .13} = 2.1 \times 10^3 \text{ cm}^{-3} \quad (21)$$

The N_2^+ condition (8) leads to

$$n_2^+ = \frac{1.26 \times 10^3}{1.75 + 8.75 + 3 \times 10^{-7} n_e} \approx 1.2 \times 10^2 \text{ cm}^{-3} \quad (22)$$

with the recombination loss negligible compared to that in channels a and d. The NO^+ condition (12) gives

$$5 \times 10^{-7} n_4^+ n_e = 2.7 \times 10^2 + 1.05 \times 10^3 \quad (23)$$

and the O_2^+ condition (10) gives

$$2 \times 10^{-7} n_3^+ n_e = 1.5 \times 10^3 + 2.1 \times 10^2 + 7.3 \times 10^2 \quad (24)$$

Since it is obvious that n_1^+ and n_2^+ are small compared to n_3^+ and n_4^+ the relationships (23) and (24) give

$$n_e^2 \approx 1.49 \times 10^{10} \text{ cm}^{-6} \quad (25)$$

$$n_e \approx 1.22 \times 10^5 \text{ cm}^{-3} \quad (26)$$

and

$$n_3^+ = 1 \times 10^5 \text{ cm}^{-3} \quad (27)$$

$$n_4^+ = 2.2 \times 10^4 \text{ cm}^{-3} \quad (28)$$

If only the value of n_e is to be compared with observation the agreement with $1.1 \times 10^5 \text{ cm}^{-3}$ is excellent. Where ion spectrometer results are also at hand the situation is not quite so cheerful. The O^+ density computed is a factor of 2.5 too large, the NO^+ density a factor of 2 too low and the O_2^+ density almost a factor of 2 too high. Changing the

neutral model considerably has small effect on the discrepancies. For example, one based on the mass spectrometric data of Nier et al.^(10, 11) with

$$\begin{aligned}n_1 &= 2.2 \times 10^{10} \text{ cm}^{-3} \\n_2 &= 1.1 \times 10^{11} \text{ cm}^{-3} \\n_3 &= 1.1 \times 10^{10} \text{ cm}^{-3}\end{aligned}\tag{29}$$

(less O and O₂ but more N₂ than the first model) leads to

$$\begin{aligned}n_1^+ &= 1.65 \times 10^3 \text{ cm}^{-3} \\n_2^+ &= 3.2 \times 10^2 \text{ cm}^{-3} \\n_3^+ &= 7.1 \times 10^4 \text{ cm}^{-3} \\n_4^+ &= 3.9 \times 10^4 \text{ cm}^{-3}\end{aligned}$$

and
$$n_e = 1.1 \times 10^5 \text{ cm}^{-3}\tag{30}$$

This is rather better all around agreement, but it will develop that this model is in more serious difficulty than the first one at higher altitudes.

Either model can be brought much closer to agreement with observed densities by invoking a sizeable flow of O₂⁺ to NO⁺ by charge transfer of O₂⁺ with NO⁽¹¹⁾. Since the rate coefficient for this reaction is measured⁽⁷⁾ to be $8 \times 10^{-10} \text{ cm}^3 \text{ sec}^{-1}$ at 300°K the rate of transfer would be $8 \times 10^{-3} n_3^+$ if the NO density were as high as 10^7 cm^{-3} at 130 km. The result would be to reduce n_3^+ to $8.2 \times 10^4 \text{ cm}^{-3}$ and raise n_4^+ to $2.9 \times 10^4 \text{ cm}^{-3}$ with $n_e = 1.1 \times 10^5 \text{ cm}^{-3}$ in the first model. In the second low atomic oxygen model, the NO⁺ density would increase to $5 \times 10^4 \text{ cm}^{-3}$ and the O₂⁺ density go down to $5.5 \times 10^4 \text{ cm}^{-3}$ which are very close to the observed values.

In the upper reaches of the thermosphere a problem is what to do about the temperature variation of the rate coefficients. In the spirit of the present exercise there is no reason not to accept the indications of the laboratory measurements. Accordingly all ion molecule rates will be held constant - with the understanding that this may underestimate the rate of O^+ loss (the corresponding production rates of O_2^+ and NO^+ are controlled entirely by q_1). The dissociative recombination coefficients will be assumed to follow the laboratory trends discussed already. Thus α_2 will be held at $2 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$; α_3 will decrease to $1.6 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$ at 160 km and to $1 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$ at 220 km; α_4 will be $1.5 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$ at 160 km and $3 \times 10^{-8} \text{ cm}^3 \text{ sec}^{-1}$ by 220 km. This provokes a rapidly decreasing loss rate with altitude particularly for NO^+ and tends to hold NO^+ densities up to high levels.

The ion densities calculated as a function of altitude under these assumptions for the two models of atmospheric composition are plotted in Figs. 6 and 7. The O^+ density calculated is too low by a factor of about two from 160 km to nearly 220 km. In model 2 the discrepancy is much worse at high altitude. The NO^+ densities are too low by almost a factor of 2 also from 130 km to 160 km and then are much too high at high altitude. N_2^+ is much too low in model 1 - by a factor of from 2 to 3 at all altitudes. Only O_2^+ in model 1 follows the measured profile well. The result is a large deficiency in electron density in the F_1 region from 140 km to about 220 km even in model 1. Near 210 km a recovery in calculated density occurs caused mainly by the four fold excess of NO^+ above its measured value. This anomaly can be traced to the strong assumed decrease in α_4 with the electron temperature since T_e increases to 2200°K at 220 km. If α_4 at 220 km were allowed to be $2 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$ (appropriate if T_i or T_{gas} controls α_4 rather than T_e) the theoretical value of n_4^+ would only be $2 \times 10^4 \text{ cm}^3$ at

220 km. However, this would be at the expense of n_e which in model 1 would be only $3 \times 10^5 \text{ cm}^{-3}$ instead of $4.5 \times 10^5 \text{ cm}^{-3}$ required. The disagreement between the calculated and observed densities are at least as serious as those turning up in the case of Mars.

Flow diagrams showing the rate of production and loss of the various ionic species in various channels are shown in Fig. 8. Such diagrams are quite helpful in showing at a glance what effect a modification in the model or in the rate constants will have on the ionic densities.

The problem of the low values of O^+ and N_2^+ over most of the F_1 region is a knotty one. An ad hoc solution is to let k_b , k_c and k_d decrease with temperature but there is no way to justify such a measure. Some new source of ionization above 140 km while helping with O^+ and N_2^+ would create a new problem by creating too much O_2^+ . Some improvement is possible by taking account of O^+ production in excited states⁽¹²⁾. If the 4D state is produced at a rate about 20% of the 4S state and is rapidly converted to N_2^+ as McElroy and Dalgarno propose it contributes little to the O^+ density but will increase the N_2^+ and NO^+ densities. The effect is only about 30% in N_2^+ and 20% in NO^+ , however.

In brief there is a serious excess in the observed degree of ionization from 140 to about 240 km above the expected level. To avoid doing violence to reaction rates or invoking esoteric sources of ionization the observer who does not have available direct measurements of atmospheric densities or ion composition would probably be tempted to solve this problem by greatly increasing the proportion of atomic oxygen or some other atomic constituent in the upper atmosphere. O^+ tends to be removed more slowly than either N_2^+ or O_2^+ partly because the loss rate for O^+ itself is independent of electron density and partly because the recombination is a two step process involving diatomic ion formation and subsequent

recombination. However, in fact, building up the relative O concentration will push up the N_2^+ loss rate through channel d and make the observed excess of that ion even worse. The calculated NO^+ density correspondingly tends to increase even higher above observed values at high altitude. There also arises a risk of reducing the ionization rate at 130 km because of absorption above that altitude thus losing the agreement between computed and observed electron density at that altitude. This could be avoided by reducing the N_2 density so as to keep the optical depth down at the wavelengths which ionize both N_2 and O. This measure also reduces the loss rate per O^+ ion in channel c. Thus our hypothetical observer in trying to force a model to fit electron density alone would probably be tempted by measures that we, with more information, would be quite reluctant to take. It is for this reason that I believe the Martian ionosphere may present problems that would persist even if the composition of the neutral atmosphere were well known. It seems to me, therefore, that even serious difficulties in predicting ionospheric electron densities in the F_1 region should not be taken as weighty arguments at this time against any model of the neutral atmosphere that otherwise makes sense.

Acknowledgements

This work was supported, in part, by the National Aeronautics and Space Administration under contract NASr-179 and by the National Science Foundation, Atmospheric Sciences Division (Aeronomy), under Grant GP-4435.

Figure Captions

- Figure 1 A model of the Martian upper atmosphere based on 6 mb of pure CO_2 at the surface and the temperatures of Chamberlain and McElroy.
- Figure 2 Ion production rates in the Martian ionosphere for solar zenith angle of 70° .
- Figure 3 Calculated and observed Martian electron densities.
- Figure 4 A model of the upper atmosphere of Earth.
- Figure 5 Ion production rates in the Earth's Ionosphere for a solar zenith angle of 60° .
- Figure 6 Observed electron and O^+ densities (solid lines) for Earth and calculated densities for model 1 (dashed lines) and model 2 (dotted lines).
- Figure 7 Observed densities of O_2^+ , NO^+ and N_2^+ (solid lines) and calculated densities (dashed lines for model 1 and dotted lines for model 2).
- Figure 8 Flow diagrams showing densities of N_2^+ , O_2^+ , O^+ and NO^+ , solar production rates, transfer rates from N_2^+ to O_2^+ (a) and NO^+ (d) and from O^+ to O_2^+ (b) and NO^+ (d), and recombination rates of N_2^+ to O^+ and NO^+ . Channels e from N_2^+ to O^+ and f from O_2^+ to NO^+ via NO are not shown.

References

1. T. M. Donahue, Science 152, 763 (1966).
2. A. J. Kliore, et al., Science 149, 1243 (1965).
3. J. W. Chamberlain and M. B. McElroy, Science 152, 21 (1966).
4. F. C. Fehsenfeld, E. E. Ferguson, and A. L. Schmeltekoff, J. Chem. Phys. 44, 3022 (1966); 45, 23 (1966).
5. H. E. Hinteregger, L. A. Hall and G. Schmidtke, in Space Research V, eds. D. G. King-Hele, P. Muller and G. Righini (Amsterdam: North Holland), 1175 (1965).
6. J. C. Holmes, C. Y. Johnson and J. M. Young, in Space Research V, eds. D. G. King-Hele, P. Muller and G. Righini (Amsterdam: Holland), 756 (1965).
7. F. C. Fehsenfeld, A. L. Schmeltekopf, P. D. Goldan, H. I. Schiff and E. E. Ferguson, J. Chem. Phys. 44, 4087 (1966); 44, 4095 (1966).
8. R. F. Stebbings, B. R. Turner, J. A. Rutherford, J. Geophys. Res. 71, 771 (1966).
9. M. A. Biondi, Recombination Processes (Charged Particles), to appear as Chapter VII in the DASA Reaction Rate Handbook, 1967.
10. A. O. Nier, J. H. Hoffman, C. Y. Johnson and J. C. Holmes, J. Geophys. Res. 69, 979 (1964).
11. T. M. Donahue, Planet. Space Sci. 14, 33 (1966).
12. A. Dalgarno and M. B. McElroy, Planet. Space Sci. 13, 947 (1965); 14, 1321 (1966).

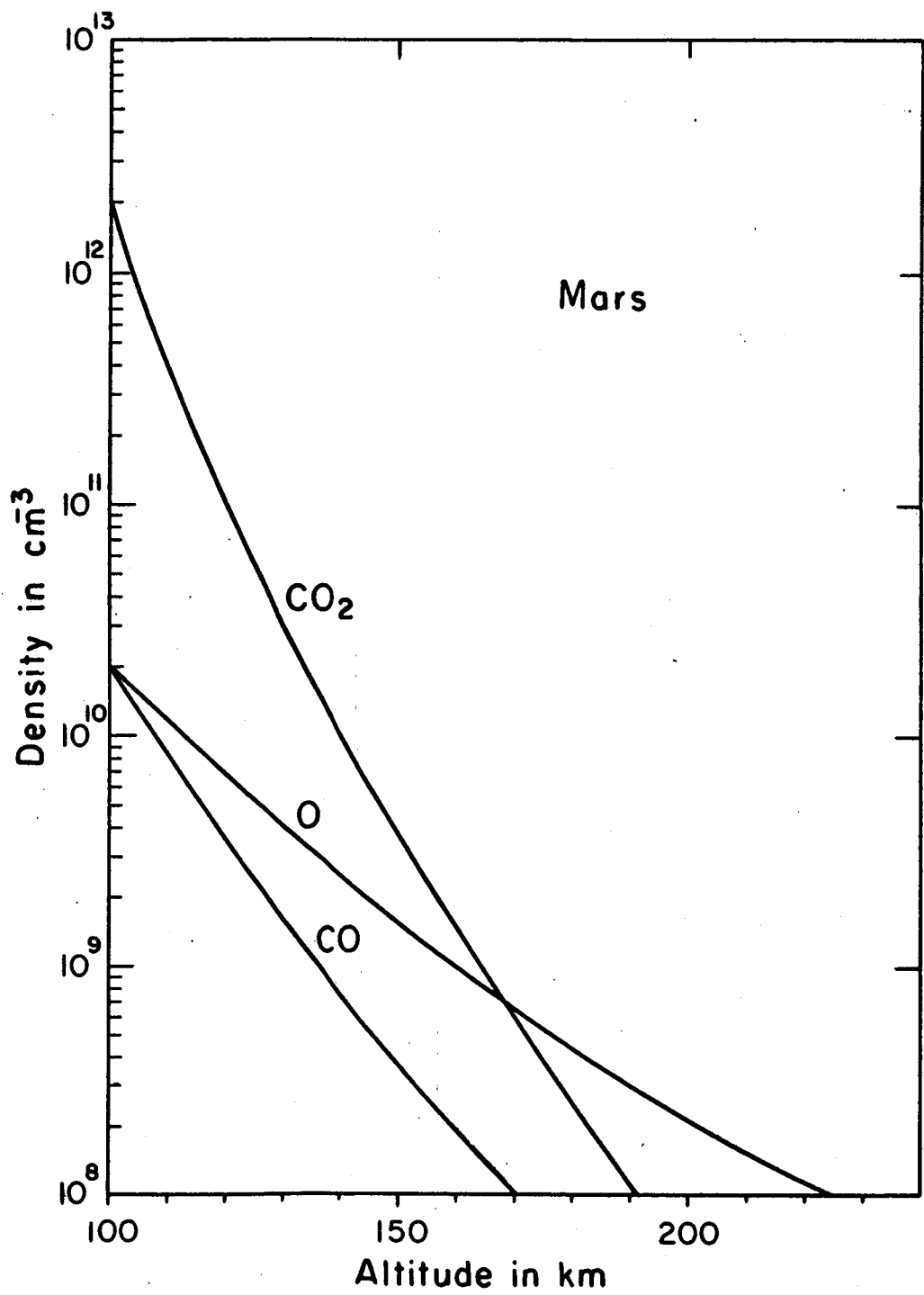


Figure 1

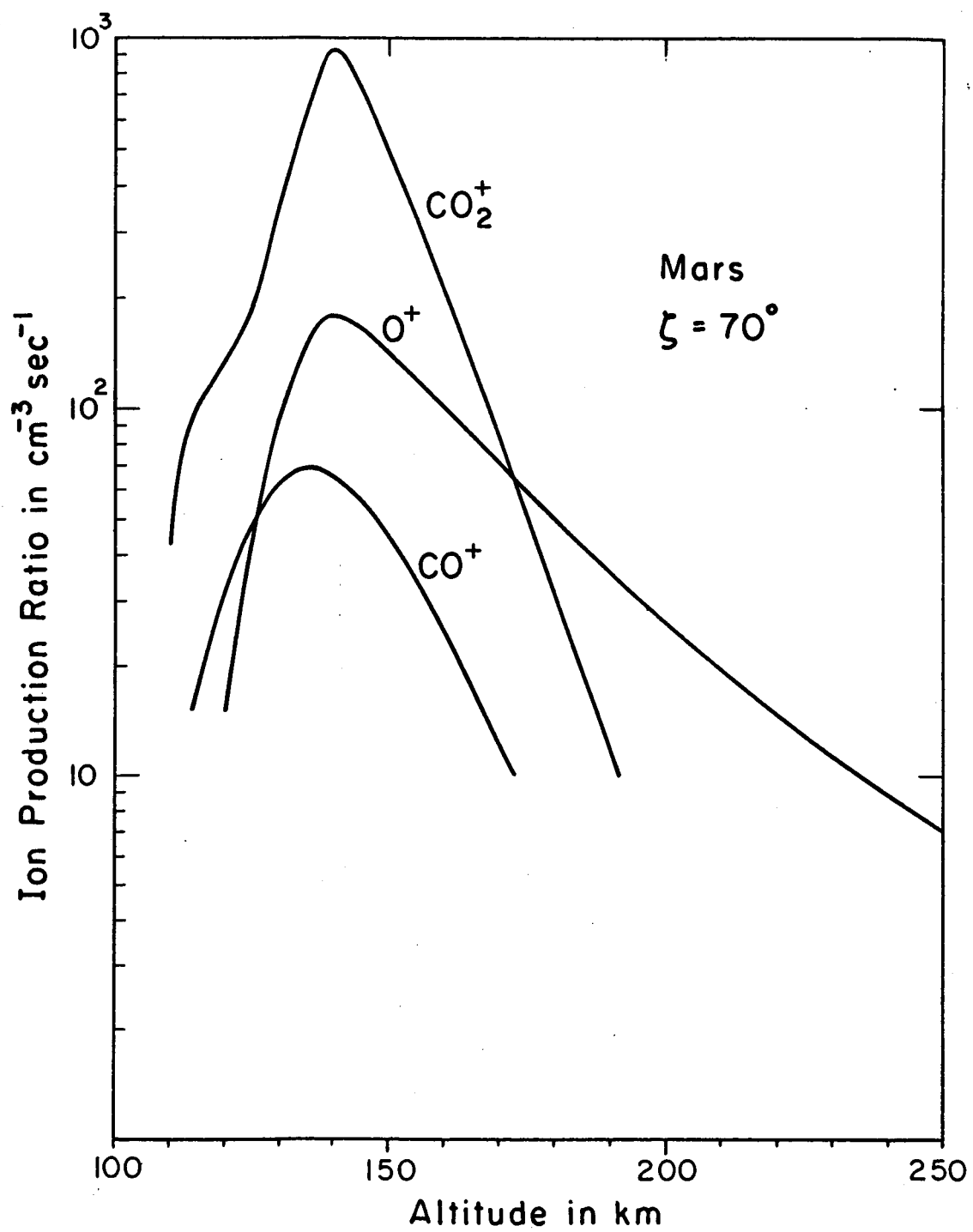


Figure 2

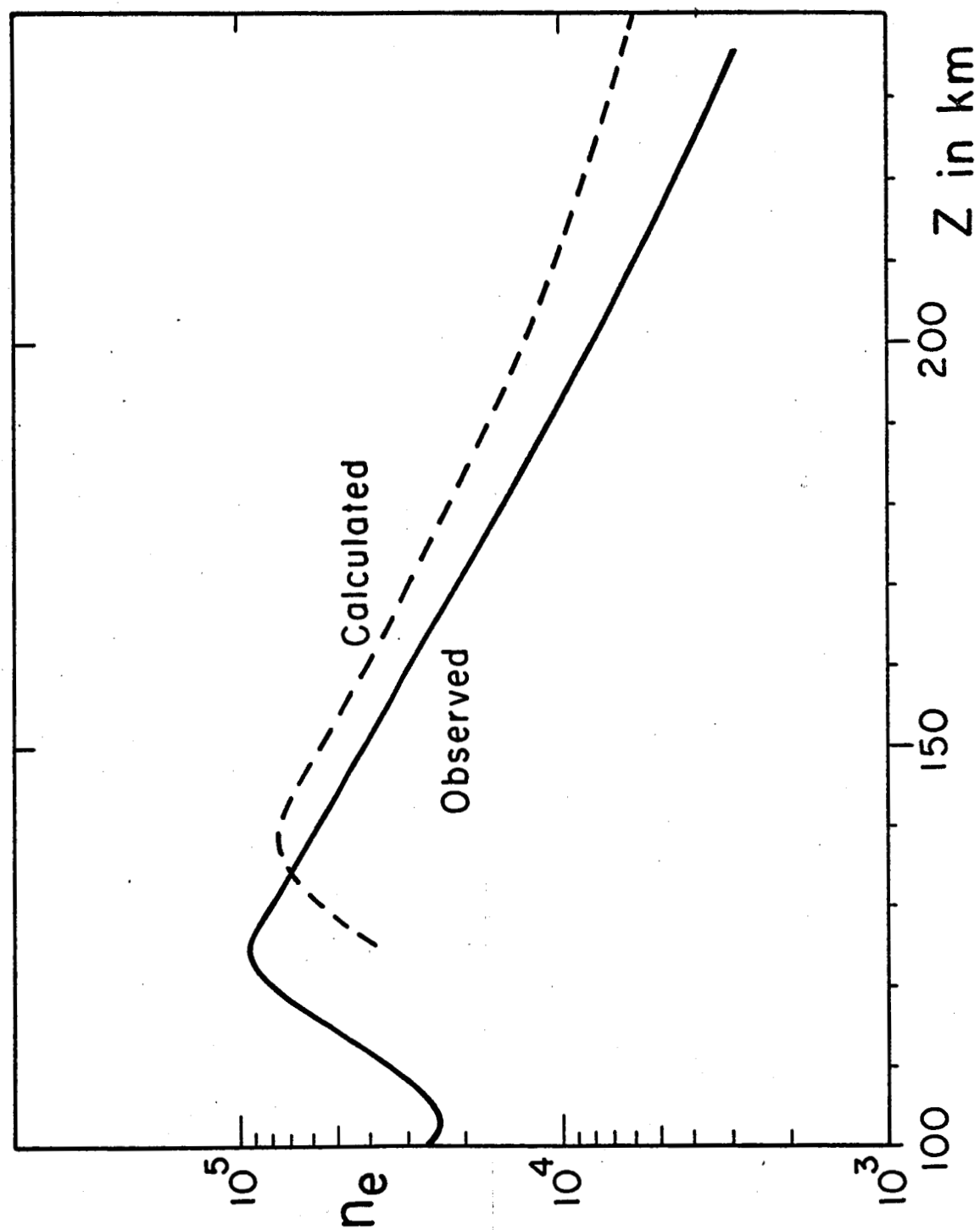


Figure 3

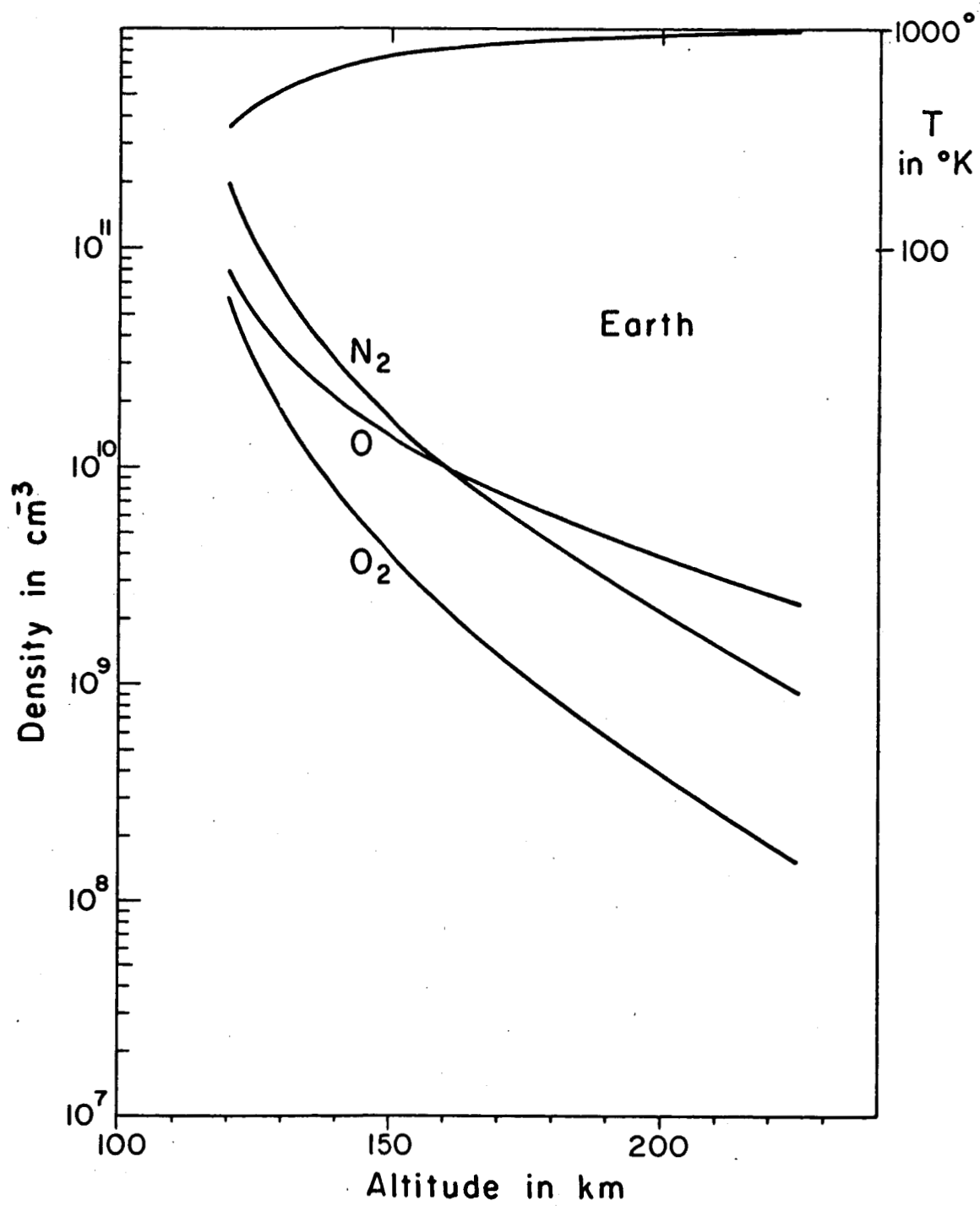


Figure 4

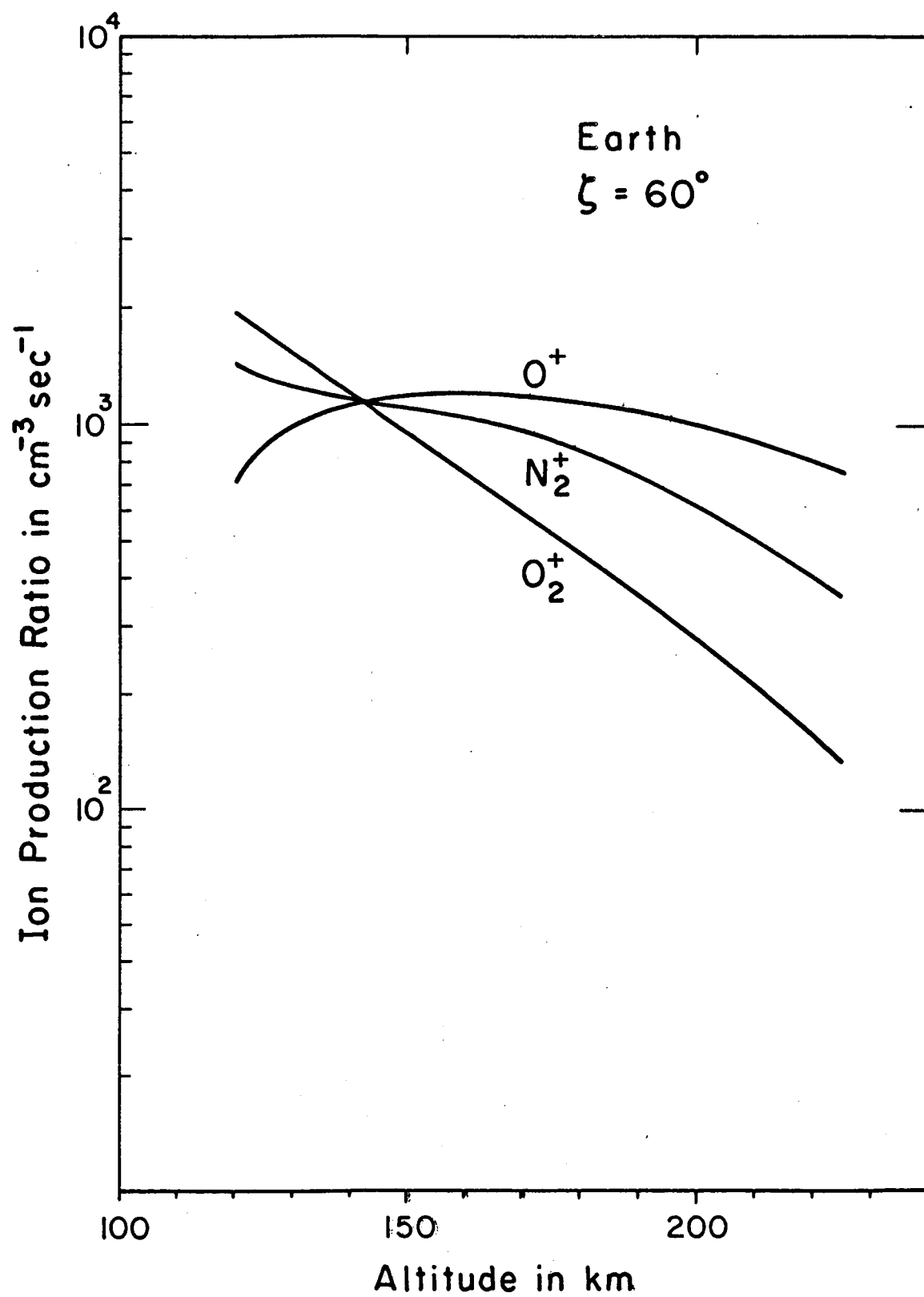


Figure 5

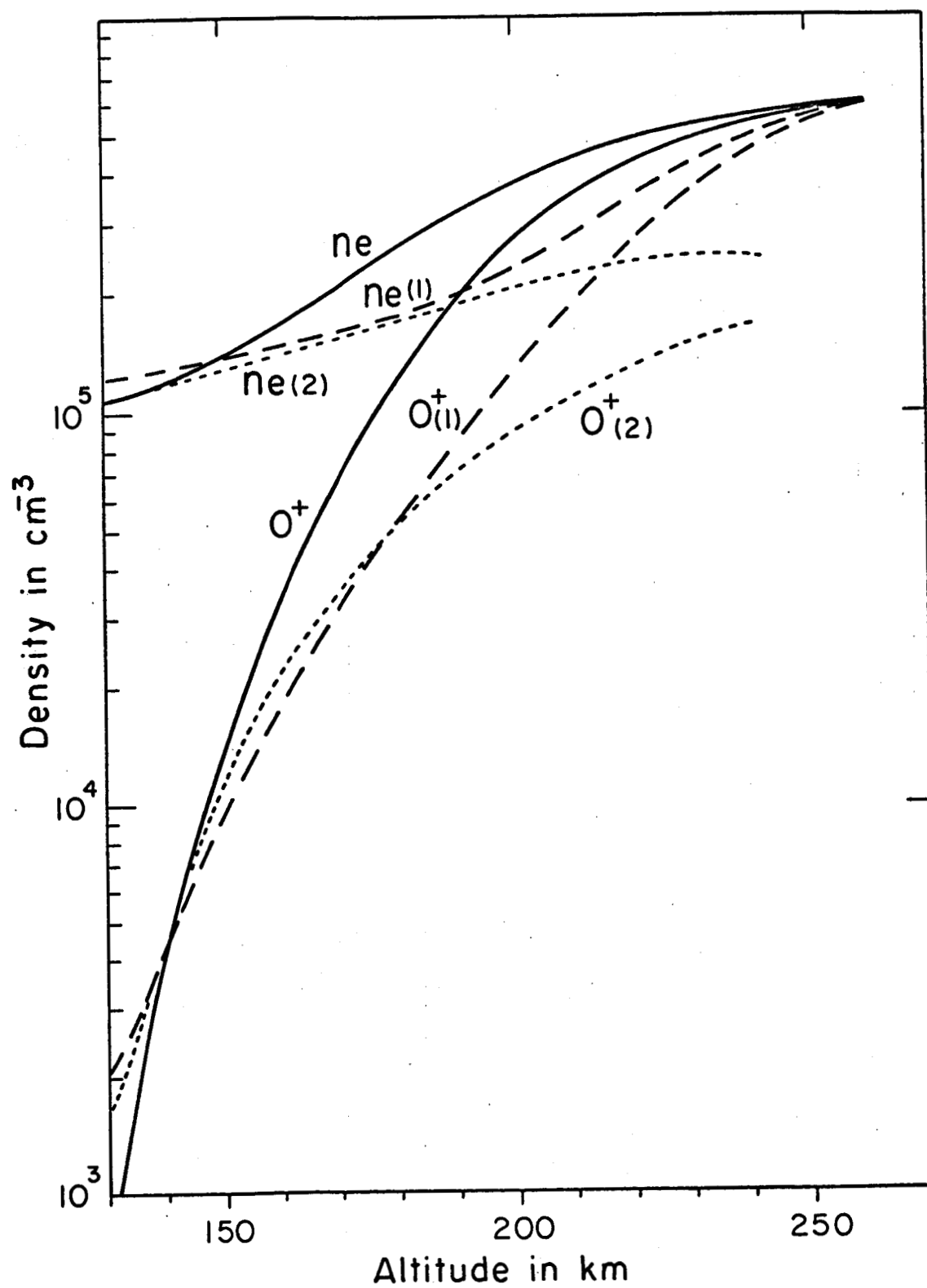


Figure 6

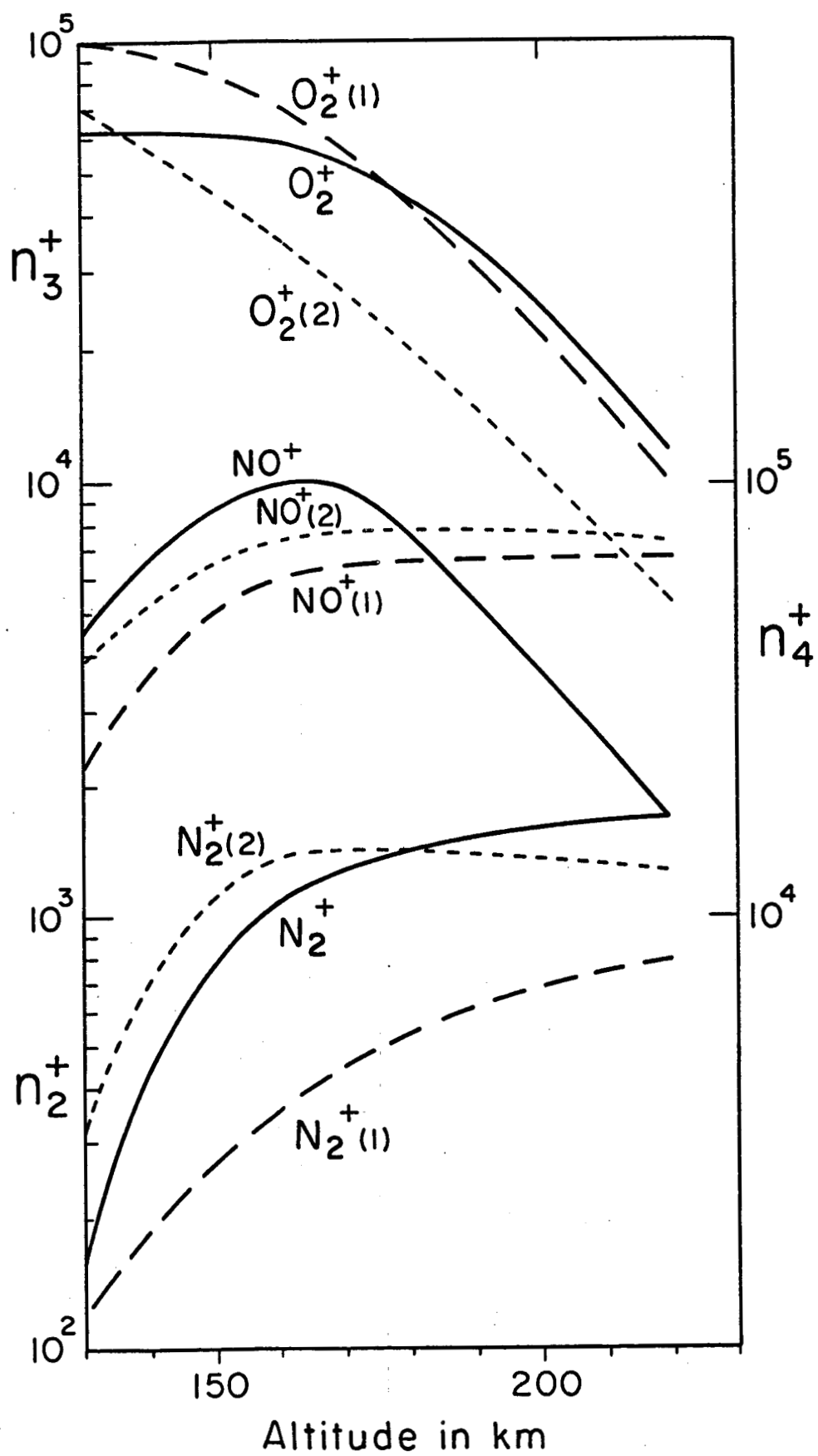


Figure 7

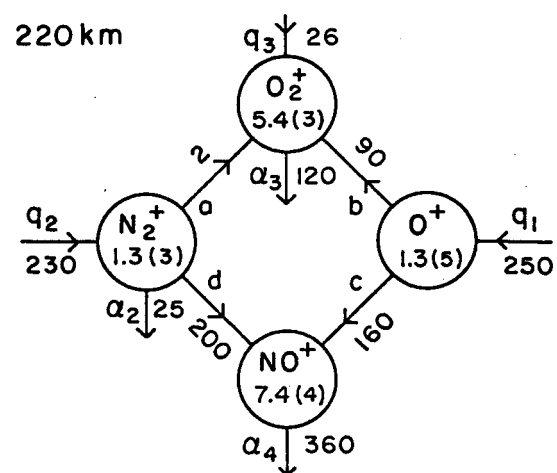
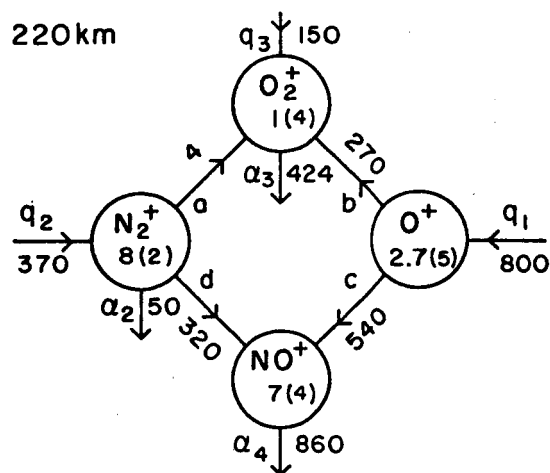
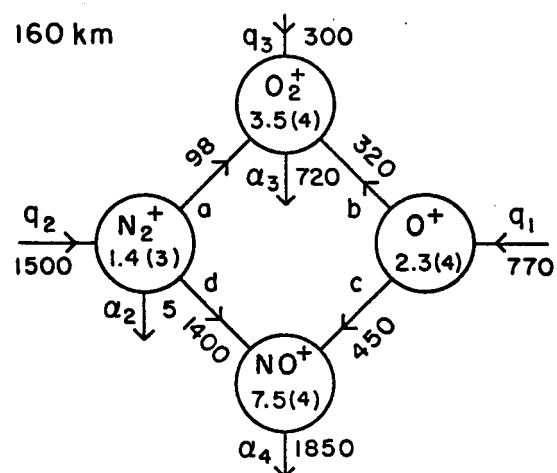
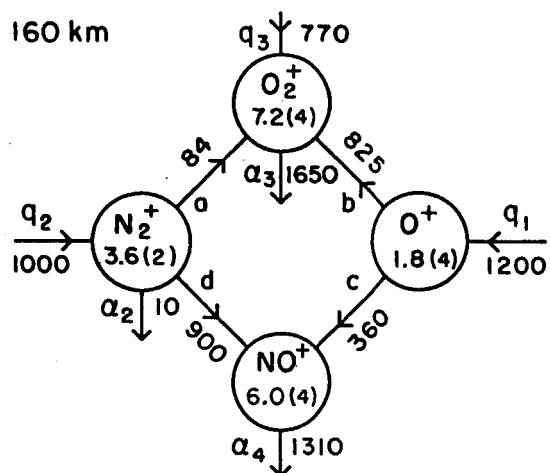
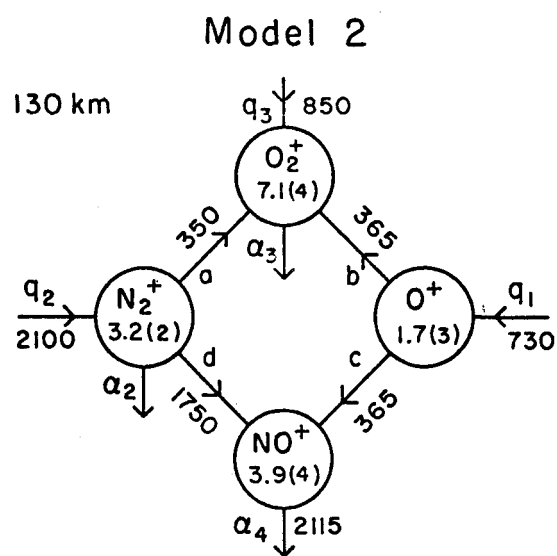
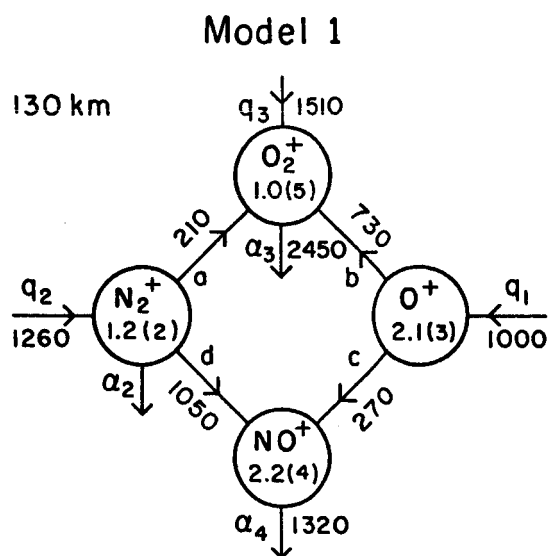


Figure 8

$$\begin{aligned}
q_1 &= \sqrt[6]{a_6} \times \\
q_2 &= (q_1 + A)^2 \\
q_3 &= (q_2 + B)(q_1 + C) \\
C(X) &= (q_2 + q_3 + D)(+q_3 + E) + F \quad \text{on } [-h, h]
\end{aligned}$$

$$h = 1$$

$$\begin{aligned}
A &= +0.4513408582627891 \times 10^0 \\
B &= +0.3744865190483202 \times 10^1 \\
C &= -0.2769272800754423 \times 10^1 \\
D &= +0.9433565393074166 \times 10^1 \\
E &= +0.1055413968372178 \times 10^2 \\
F &= +0.6288190624578802 \times 10^{-2} \\
M &= 0.5828670879282069 \times 10^{-14}
\end{aligned}$$

$$h = 2$$

$$\begin{aligned}
A &= +0.4512008438957284 \times 10^0 \\
B &= +0.3743936586512442 \times 10^1 \\
C &= -0.2769076432349482 \times 10^1 \\
D &= +0.9429681327692907 \times 10^1 \\
E &= +0.1055053194443676 \times 10^2 \\
F &= +0.6284207351324604 \times 10^{-2} \\
M &= 0.9935108291614367 \times 10^{-13}
\end{aligned}$$

$$h = 4$$

$$\begin{aligned}
A &= +0.4507019590625572 \times 10^0 \\
B &= +0.3740448131455307 \times 10^1 \\
C &= -0.2768317205414987 \times 10^1 \\
D &= +0.9414899439055362 \times 10^1 \\
E &= +0.1053701311149719 \times 10^2 \\
F &= +0.6272339359526764 \times 10^{-2} \\
M &= 0.1224323420423443 \times 10^{-10}
\end{aligned}$$

$$h = 16$$

$$\begin{aligned}
A &= +0.4405766736959988 \times 10^0 \\
B &= +0.3669989101432331 \times 10^1 \\
C &= -0.2752824996097087 \times 10^1 \\
D &= +0.9117484138879304 \times 10^1 \\
E &= +0.1026464040151929 \times 10^2 \\
F &= +0.6161613003116790 \times 10^{-2} \\
M &= 0.2015977389469012 \times 10^{-6}
\end{aligned}$$