SPACE TRAJECTORIES AND ERRORS IN TIME, FREQUENCY, AND TRACKING STATION LOCATION

F. O. VONBUN

APRIL 1967

NASA
GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

X-507-67-163

SPACE TRAJECTORIES AND ERRORS IN TIME, FREQUENCY, AND TRACKING STATION LOCATION

By

F. O. Vonbun

25 November 1966

Goddard Space Flight Center
Greenbelt, Maryland

i

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

SPACE TRAJECTORIES AND ERRORS IN TIME, FREQUENCY, AND TRACKING STATION LOCATION

By

F. O. Vonbun

ABSTRACT

The purpose of this paper is to show, in analytical fashion, how the errors in tracking station time and frequency synchronization, as well as the errors in station location influence accurate trajectory determination.

Two systems, which data are presently used for most of the more accurate orbit determination schemes, are described, namely, (a) a range and range rate system and (b) a radar system. All other systems are really a combination of these two basic systems and are therefore not mentioned specifically in this report.

Rather simple analytical expressions are derived relating measing errors with those of time and frequency synchronization, as well as ground tracking station location.

For the range and range rate system, the error in range rate ($\delta \dot{r} = 0.01$ cm/s) is used as a "yardstick" and all other quantities are derived from it. For the radar system (not measuring range rate directly), the total local position vector, as measured by such a system, is used as the "yardstick" to determine the necessary time synchronization and station location accuracy.

Numerical examples are presented which hopefully will show what accuracies of the mentioned quantities are needed for a good ground tracking system to make its data most useful for the determination of accurate orbits and space trajectories.

SPACE TRAJECTORIES AND ERRORS IN TIME, FREQUENCY, AND TRACKING STATION LOCATION

By

F. O. Vonbun

SUMMARY

A. General

Frequency (Doppler) or even more proper, a phase determination over a certain time interval, is one of the few basic physical measurements which can be performed with extreme accuracy. If this quantity can be used for the determination of orbits one would expect that these also will be very accurate. This indeed can be done and is used on an almost daily basis, particularly for deep space trajectory determination.

In brief, the Doppler frequency experienced by a station receiving an electromagnetic signal from a spacecraft is often used as a "measuring stick." One could also use, even more appropriately, as a measure of a system, an error quantity at a particular target. For instance, the closest approach to the moon or a planet should be within 100 m or 10,000 m, respectively. The problem with this approach is that system requirements may result which could not be met due to practical physical limitations such as systems noise, bias errors, tropospheric and ionospheric propagation disturbing the transmitted and received electromagnetic signals. This is the reason the range rate limited approach has been chosen here. In any event, the final results should almost be the same. Since, as mentioned, this "yardstick" is an extremely accurate one ($\Delta\nu/\nu$ ± 10^{-10} to 10^{-12} per day), all other quantities involved have to have appropriate accuracies. Thus, in order to really exploit the Doppler measurement to its fullest as an example, the sender frequency has to be very accurate, the time synchronization between tracking stations as well as their location on the surface on the earth have to be determined to a "certain" degree. How much and how all these quantities and their errors are approximately related to each other is outlined in this paper. Coherent two or three way phase locked Doppler systems as well as radar systems are discussed.

It should be pointed out here that a treatment of this nature, namely to establish "reasonable" design limits, cannot be a mathematical rigorous one.

Experience and judgement have to enter into results of this nature. Nevertheless, it is felt that the values summarized will give the reader a good picture of the problems and their solutions.

B. Relative Frequency Stability

As can be noted from equation (9) one needs a relative frequency stability of:

$$\left(\frac{\delta\nu_{D_2}}{\nu_{D_2}}\right)_{t_m} \doteq \left(\frac{\delta\nu_o}{\nu_o}\right)_{2\tau} < \frac{\delta\dot{r}}{\dot{r}} \doteq 2 . 10^{-9},$$

where the first term refers to the measured Doppler shift during the measuring time t_m, the second term refers to the relative stability of the local standard oscillator over the time the signal travels from the station to the spacecraft and back, $\delta\dot{r}$ = 0.01 cm/s, the assumed "yardstick," and $\dot{r}$ = 50 km/s is assumed for an average maximum interplanetary range rate.

In addition, it should be noted that the value ($\delta c/c$) has to be considered as an unknown in the orbit determination process in order to be able to "use" the above value.

C. Station Time Synchronization

As can be seen from equations (12) and (13), one obtains for δt = 0.1 to 0.2 msec for the earth moon space using $\delta\dot{r}$ = 0.1 cm/s and the full earth acceleration $\ddot{\rho}$ = 1000 cm/s^2 (a pessimistic value proper for near earth space).

Similar $\delta t \doteq$ 0.2 msec as the needed time synchronization error for a radar depending on the assumed angular bias error of the radar considered. Equations (16) and (17) and (17a) are applicable for this case.

For interplanetary missions, where $\ddot{\rho}$ is small (in this case $\ddot{\rho} \doteq$ 0.6 cm/s^2 is the acceleration due to the sun for 1 AU) even with $\delta\dot{r}$ = 0.01 cm/s (compared to 0.1 cm/s for near earth and lunar space) a value of $\delta t \doteq$ 2 msec is adequate as given by equation (13). Only in the special case of a planetary approach or a planetary fly by, timing errors of $\delta t \doteq$ 10 to 20 μsec seems to be needed. For this case however the method suggested by JPL, namely, using the orbit for time synchronization, can fulfill this requirement without the use of additional equipment or methods.

D. Station Location Errors

To fully utilize the Doppler error of only 0.01 cm/s a station location accuracy of:

$$|\delta \vec{R}| \doteq 1.5 \text{ to } 5.5 \text{ meters}$$

will be needed depending on the use of equation (25) or (25b). These numbers give a range to work toward in the future.

In case of a radar system, a station error range of

$$|\delta \vec{R}| \doteq 3.5 \text{ to } 11.0 \text{ meters}$$

will be necessary for future systems according to equation (28) and its associated assumptions.

E. Frequency Synchronization

For a two-way Doppler system, similar values hold as stated under B above. In this case one cannot speak of frequency synchronization as such but more of frequency stability during the travel time of the electromagnetic signal.

This is of course different for the three-way Doppler system as applied for lunar and planetary missions.

As can be seen from equation (18), in order to make full use of the Doppler ($\delta \dot{r} = 0.01$ cm/s) a, frequency synchronization between station 1 and 2 of

$$\frac{\delta \nu_{12}}{\delta \nu_o} \leq \frac{2}{3} 10^{-11}$$

is needed. Or in brief, field worthy hydrogen masers will have to be developed and used.

It should be pointed out that the errors considered, namely, the errors in frequency synchronization, tracking station time synchronization, and the errors in location of the tracking stations on earth are to be considered as bias errors. Obviously so, since all these quantities stay approximately constant during the time when tracking measurements are taken and the orbits determined.

I. FREQUENCY STABILITY

One of the first questions to be answered, when a coherent Doppler tracking system is to be designed, is: What frequency stability of the transmitter is required?

As can be seen by the schematic of Figure 1, the following principle [1,2,3,4,5] is used to "detect" the Doppler frequency shift ν_{D_2} for a two-way Doppler system.

The multiplied frequency of a stable source ν_o is transmitted to the spacecraft, received there, transplanted to insure no interference, retransmitted to the same ground receiving system together with the translation factor k, and finally fed into a mixer[5]. Here, the difference frequency (Doppler) between the signal sent to the spacecraft and received back on the ground is extracted as the Doppler ν_{D_2}. This Doppler is of course only as good as the oscillator is during the time the signal takes to transverse twice (up and down) the spacecraft distance r from the transmitting ground station.

Thus, the frequency of the source has to be "constant" during this travel time 2τ. What "constant" means here will be shown in the following.

A. The Doppler Shift

The special relativistic equation for the Doppler shift reads: [1,2,3,4]

$$\nu = \nu_0 \frac{1 - \frac{1}{c}(\vec{v} \cdot \vec{r}^{\circ})}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \tag{1}$$

where: $\vec{v}$ is the relative velocity between spacecraft and receiving station $v = |\vec{v}|$; c is the speed (scalar) of light; r the frequency at the spacecraft; ν_0 the transmitted frequency, $\vec{r}^{\circ}$ the unit position vector from the station to the spacecraft in the moving frame of reference.

Further

$$\dot{r} = (\vec{v} \cdot \vec{r}^{\circ}) \tag{2}$$

is the range rate of the spacecraft with respect to the tracking station. A similar equation holds for the return path as shown in references 1, 2, 3, and 4. After some manipulation, one obtains for the ratio of the received frequency ν^1 and the transmitted frequency ν_o the following:

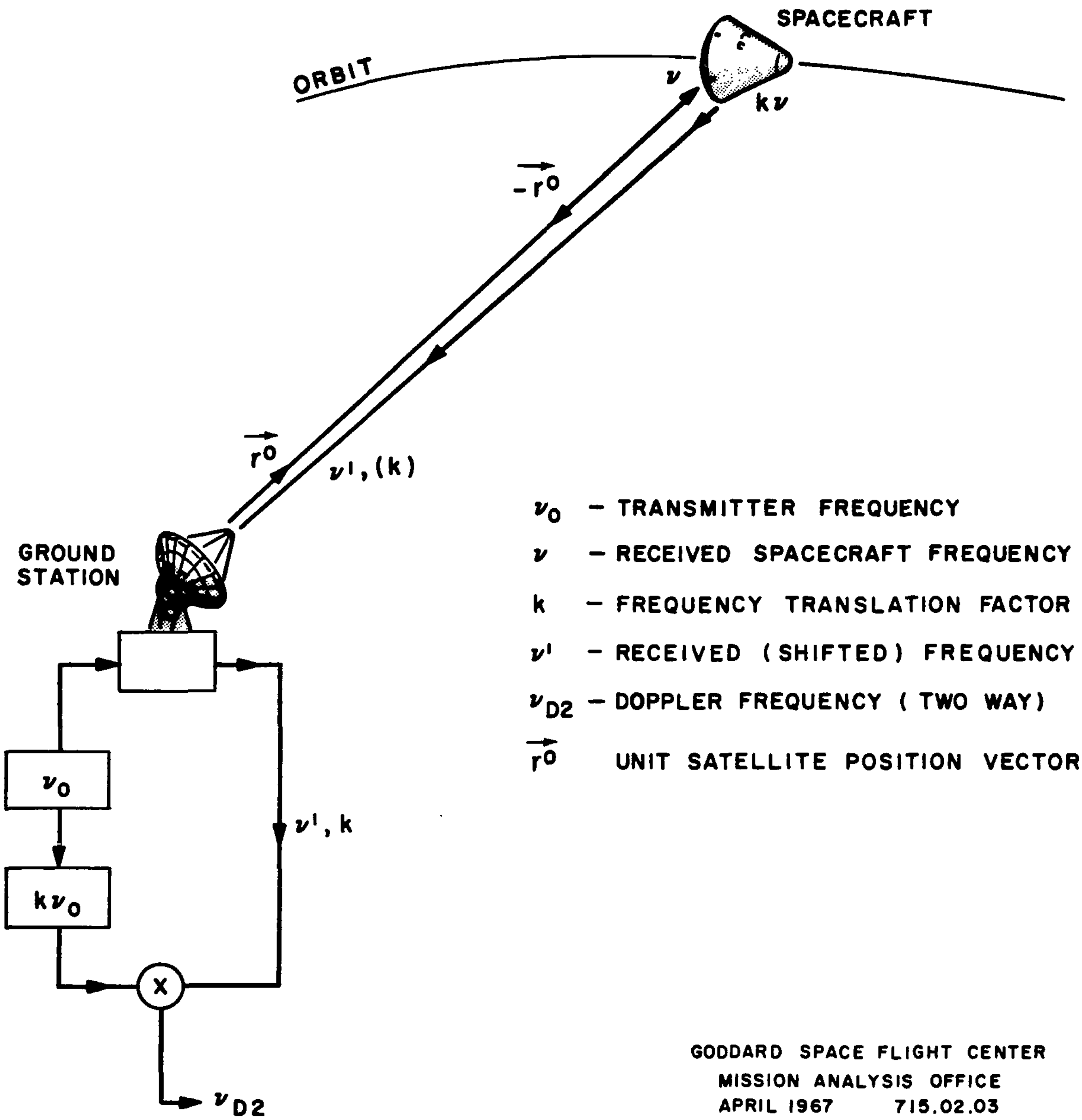


Figure 1–Two Way Doppler System

$$\frac{\nu^1}{\nu_o} = \frac{1-\frac{1}{c}\dot{r}}{1+\frac{1}{c}\dot{r}} \tag{3}$$

The Doppler frequency for the so-called two-way mode ν_{D_2} is known simply as:

$$\nu_{D_2} = (\nu^1 - \nu_o) \tag{4}$$

The factor k can be assumed as 1 since this value is compensated for on the ground, as shown in reference 5, and therefore is of no interest for the principle involved. As mentioned, the frequency translation (k) was performed only to rid interference at the spacecraft.

From equations (3) and (4) one obtains for the range rate:

$$\frac{dr}{dt} = \dot{r} = \frac{-c}{2}\frac{\nu_D}{\nu_0} \cdot \frac{1}{1+\left(\frac{\nu_D}{2\nu_o}\right)} \tag{5}$$

Equation (5) relates Doppler frequency ν_D, transmitter frequency ν_o, speed of light c and the range rate $\dot{r}$ between spacecraft and transmitting stations in exact form as stated by the special theory of relativity. (No gravitational terms are considered since these are small effects compared to these used here.)

B. The Relative Errors

Equation (5) will now be used to answer the question of frequency stability using $(d\dot{r}/\dot{r})$* as a "yardstick." Since only small variations are to be considered and $\nu_D/\nu_o \lll 1$ equation (5) can be simplified for the following variational analysis that is:

$$\dot{r} \doteq \frac{-c}{2}\frac{\nu_D}{\nu_o}^{**} \tag{6}$$

Varying equation (6) and adding the results in the Gaussian sense leads to

*During the course of this paper normalized errors such as δ_c/c, $\delta_{\dot{r}}/\dot{r}$, ... etc., are often used for simplicity of the mathematical expressions.

**Sign of Doppler frequency ν_D is neglected from now on since it has no real meaning in the sense of a "negative" frequency.

$$\left(\frac{\delta \dot{r}}{\dot{r}}\right)^2 = \left(\frac{\delta c}{c}\right)^2 + \left(\frac{\delta \nu_D}{\nu_D}\right)^2 + \left(\frac{\delta \nu_0}{\nu_0}\right)^2 . \tag{7}$$

This equation shows that the relative error $\delta \dot{r}/\dot{r}$ one is making in the determination of $\dot{r}$ is dependent on the relative error in the speed of light $\delta c/c$, the relative error in the Doppler frequency $\delta \nu_D/\nu_D$ and the transmitter frequency $\delta \nu_0/\nu_0$. Decreasing $\delta \dot{r}$ or $\delta \dot{r}/\dot{r}$ means one has to reduce also all three other quantities accordingly.

First, one has to improve the relative error ($\delta c/c$) of the speed of light (at present $(\delta c/c) = 3.10^{-7}$)[6,7,8,9] then one has to be able to measure the relative Doppler frequency ($\delta \nu_D/\nu_D$) to at least the same accuracy. This means that: a certain measuring time t_m, and measuring time error δt_m have to be secured to accomplish a frequency measurement to a given accuracy[10], that is:

$$\frac{\delta \nu_D}{\nu_D} \doteq \frac{\nu_D}{\nu_0} \cdot \frac{\delta t_m}{t_m} \doteq \frac{2\dot{r}}{c} \cdot \frac{\delta t_m}{t_m} \tag{8}$$

assuming that the quantities contributing to noise (noise modulation index, signal to noise ratio) are small enough compared to ($\delta t_m/t_m$) which is the case for the systems and methods considered here.

Assuming as an example a carrier frequency $\nu_0 = 2000$ Mc a measuring time $t_m = 1$ sec and an error in the measuring time $\delta t_m = 10^{-6}$ sec results in a value of

$$\frac{\delta \nu_D}{\nu_D} = \frac{2}{3} \cdot 10^{-4} \cdot 10^{-6} \cdot = \frac{2}{3}\, 10^{-10} \cdot (\text{for } \dot{r} = 10 \text{ km/s}).$$

In addition, of course, the transmitter standard oscillator has to be "constant" during the travel time 2τ of the electromagnetic wave from the transmitter to the spacecraft. Any shift during this time cannot be detected by the mixing process as shown in Figure 1 and described in reference 5 for instance.

In brief, one needs:

$$\frac{\delta c}{c} \doteq \left(\frac{\delta \nu_D}{\nu_D}\right)_{tm} \doteq \left(\frac{\delta \nu_0}{\nu_0}\right)_{2\tau} \tag{7a}$$

Assume, as an example, all three quantities are equal and are 10^{-6} (a conservative value) than $\delta \dot{r} = \sqrt{3}.10^{-6}\, r$. For flights between the earth and the moon $\dot{r}_{ar} = 3$ km/s $= 3.10^5$ cm/s and thus $\delta \dot{r} = 1.0$ cm/s which is at present adequate for purposes of trajectory determination for the Apollo project for instance[7,8,11 through 16]. (See also Figures 2a, 2b).

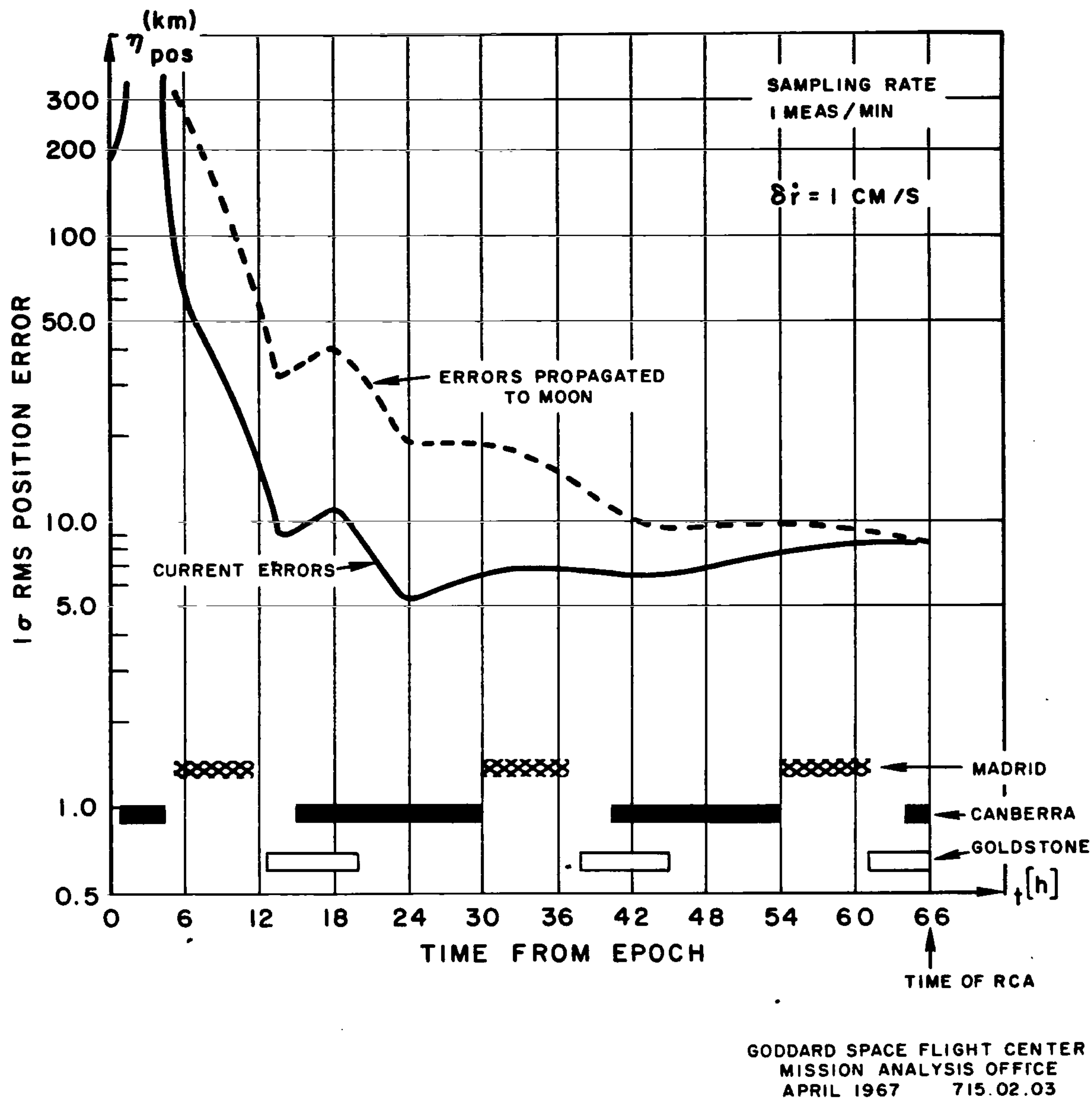


Figure 2a—Position Errors of a Lunar Transfer Trajectory

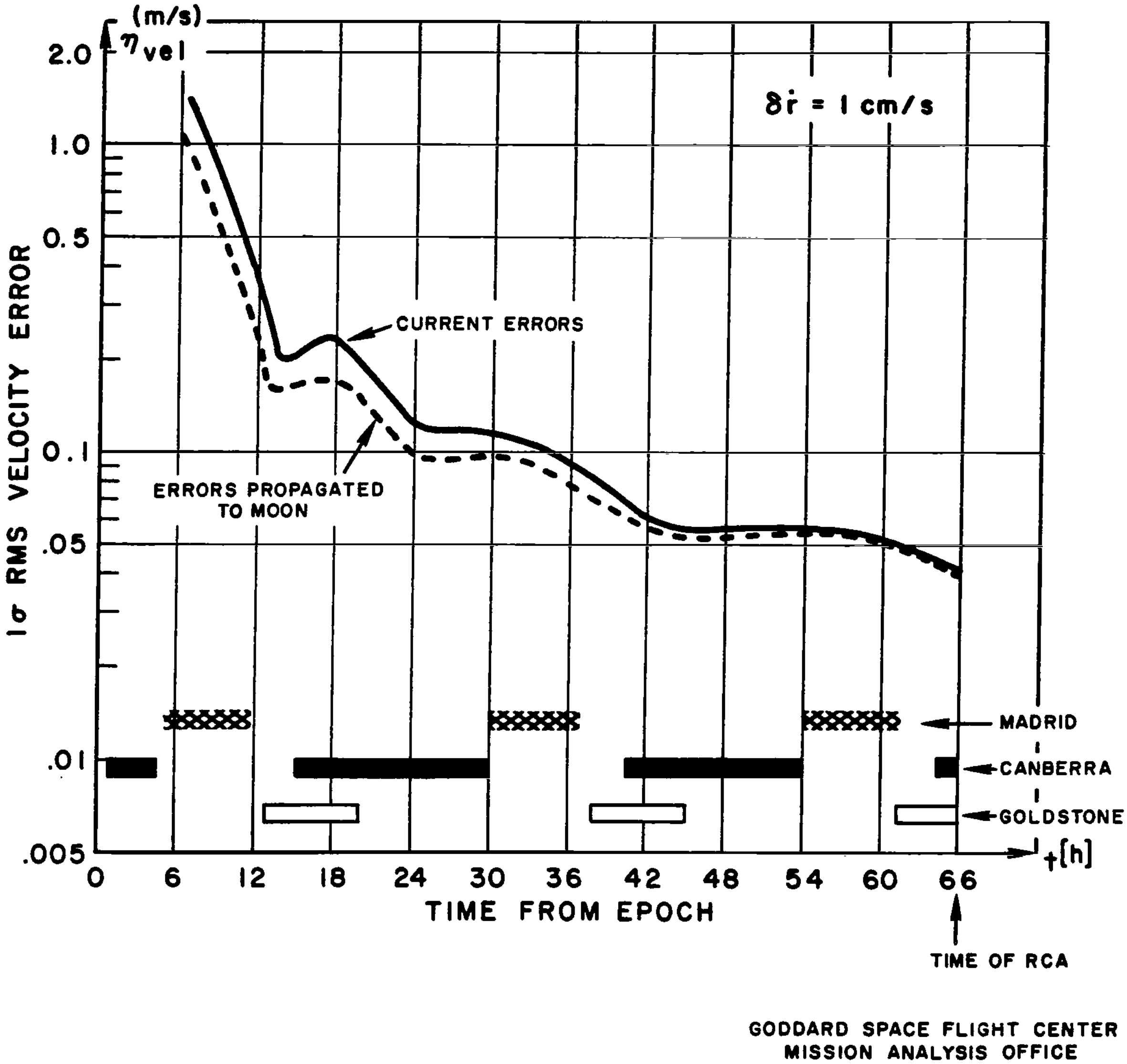


Figure 2b–Velocity Error of a Lunar Transfer Trajectory

Figure 2a depicts the instantaneous and propagated (to the moon) position error of a lunar transfer using a bias range rate error of 1 cm/s plus a 1 cm/sec noise component superimposed. Station errors as given in references 7 and 8 are also included to show realistic values as used for these Apollo Navigation studies. Figure 2b shows the same for the spacecraft velocity.

For interplanetary flights one has to decrease the errors in range rate in order to get a more reasonable trajectory. Figures 3a and 3b show the position and velocity errors of a Jupiter probe as an example when tracked by a two-way Doppler system using three ground stations approximately 120° separated in longitude. The errors of twenty days of tracking are shown, then these errors are propagated to 260, 458, and 500 days respectively (planets intercept) along the trajectory. This error propagation is done in order to be able to determine if it is reasonable to make a midcourse maneuver say after 20 days of tracking based upon our range rate accuracy requirements discussed here.

Since at present both quantities namely $(\delta v_D/\nu_D)$ and $(\delta v_0/\nu_0)$ can be determined far better than $(\delta c/c)$ one has to improve on the latter to improve $(\delta \dot{r}/\dot{r})$ as shown by equations (7) and (7a). This should possibly be done by an orbit independent method in a laboratory since the value of c and thus $(\delta c/c)$ can be "calculated" (considered as an additional unknown in the orbit computation) together with the orbit parameters station locations, etc. The problem involved here is that all our constants are based upon the velocity of light within 3.10^{-7} as quoted before[17]. It should be noted that the velocity of light is one of the primary physical constants and really occupies a "key position" in astronomy[17,18]. The velocity of light is involved via the "light time" (time the light needs to transverse 1 AU, the heliocentric distance of the center of mass of the earth + moon system) with the solar parallax, the mean earth radius, the eccentricity of the earth orbit, the Gaussian gravitational constant, etc. This means in essence all these quantities have to be changed accordingly to proper adjust our physical constants in the Universe in terms of distance, distance variation, time, etc[17].

For interplanetary flights[19,20,21] values of $\delta\dot{r} = 0.02$ cm/s have been obtained over a 1 minute measuring time. Thus, using 0.01 cm/s as a "standard" one would need for $\dot{r} = 5.10^6$ cm/s the following accuracies:

$$\frac{\delta \dot{r}}{r} \pm \left(\frac{\delta c}{c}\right) \pm \left(\frac{\delta v_D}{\nu_D}\right)_{tm} \pm \left(\frac{\delta v_0}{r_0}\right)_{2\tau} \leqq 2.10^{-9} \tag{9}$$

Assuming a good S/N ratio of say 10 db or more and Doppler measuring times in the order of minutes[10], atmospheric disturbances seem to be the only limiting factors[19,20,21,22]

As shown in reference 22, this limit is in the order to 0.01 cm/s rms when elevation angles $\epsilon \geqq 10°$ are used and daily tropospheric connection terms are

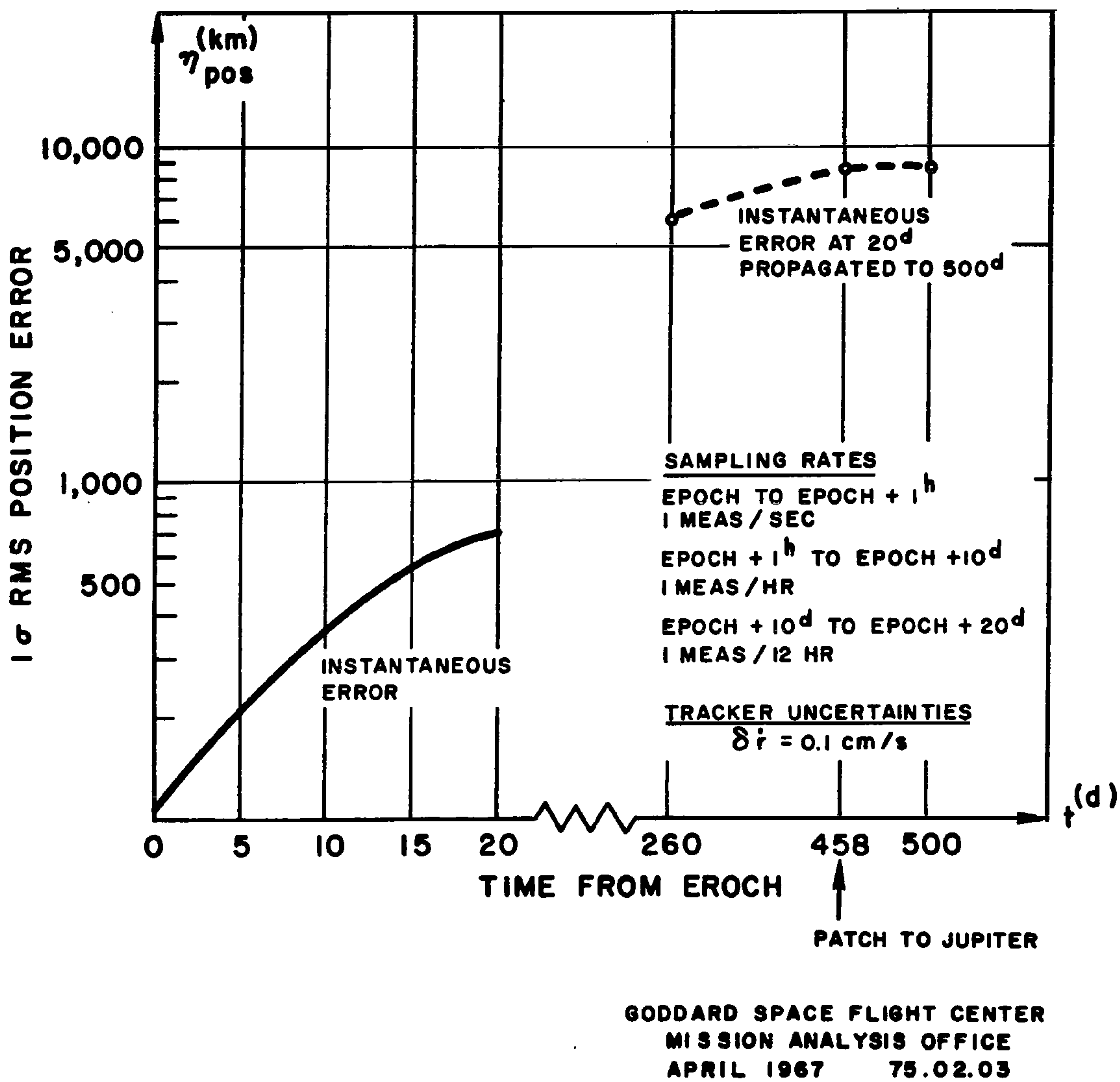
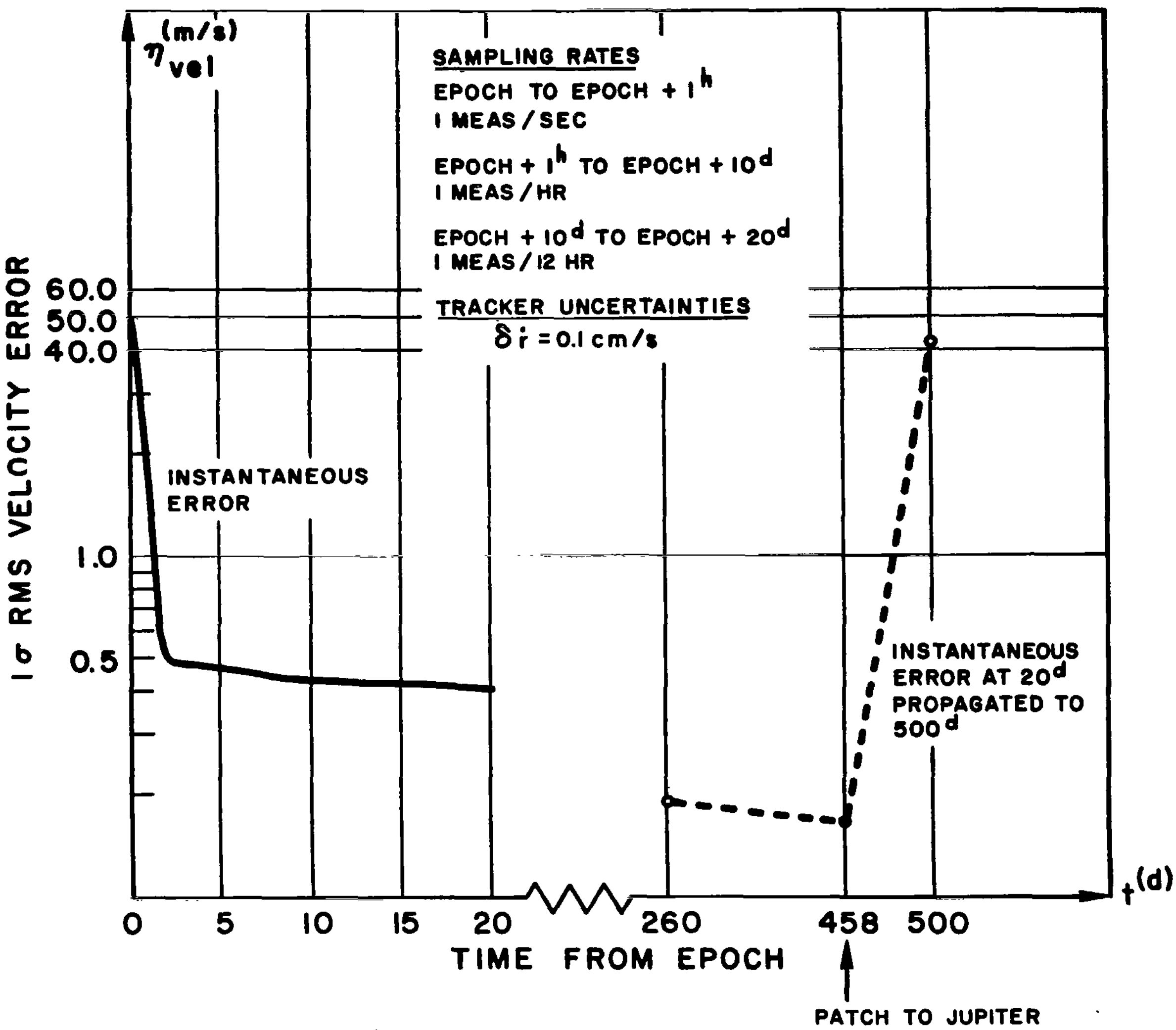


Figure 3a–Position Errors of a Jupiter Trajectory



GODDARD SPACE FLIGHT CENTER
MISSION ANALYSIS OFFICE
APRIL 1967 715.02.03

Figure 3b—Velocity Errors of a Jupiter Trajectory applied in the order of $\delta N = \pm 5$ where $N = (n-1) \cdot 10^{-6}$, and n is the index of refraction. The only unknown quantity is ($\delta c/c$) to the accuracy stated, thus a better determination of $\delta c/c = 3.10^{-7}$ as shown in reference 6 is needed. Laboratory measurements of the velocity of light will probably not reach an accuracy of 1p 10^{10} or better in the near future. One way to circumvent the errors of the "known" value of c is to introduce this quantity as an unknown into the orbit determination process.

II. STATION TIME SYNCHRONIZATION

One of the next logical questions to answer is: To what accuracy should the time be synchronized between tracking station used for orbit determination?

A relationship between the time error δt and the tracking quantities to be measured will be derived in the following. A CW range rate system and a radar system is considered since their time synchronization requirements are different since different "yardsticks" have to be used. (A radar does not "measure" $\dot{r}$ as such for instance.)

A. Synchronization Between Station Time and Orbit Time

For orbit determination, the local or station measurements r, $\dot{r}$, etc. (or any other measurement) are taken at a certain time t. These values are then transmitted from the station to the computing center and there used for orbit determination. Thus, the station "clock" has to be synchronized to the computing center "clock" to within a certain limit, say δt. What this value should be in order to make full use of our "standard" $\delta \dot{r}$ is shown in the following.

1. For a CW Range Rate System

Considering small deviations only one can write to a first order accuracy for the variation $\dot{r}$ the following:

$$\delta \dot{r} \doteq \ddot{r} \, \delta t \tag{10}$$

since

$$\ddot{r} = \frac{\delta}{\delta t} \dot{r},$$

where $\ddot{r}$ is the magnitude of the time derivative of the range rate $\ddot{r}$ and δt is the time error to be determined here.

The spacecraft inertial position vector (see Figure 4) $\vec{\rho}$ is:

$$\vec{\rho} = \vec{R} + \vec{r} \tag{11}$$

and there accelerations are:

$$\ddot{\vec{\rho}} = \ddot{\vec{R}} + \ddot{\vec{r}} \ . \tag{12}$$

Borrowing terms from vector analysis, that is, $\dot{\vec{R}} = (\vec{\omega} \times \vec{R})$ and further $\ddot{\vec{R}} = (\vec{\omega} \times (\vec{\omega} \times \vec{R}))$ one obtains:

$$\ddot{\vec{\rho}} = (\vec{\omega} \times (\vec{\omega} \times \vec{R})) + \ddot{\vec{r}} \tag{12a}$$

Using (12a) and (10) one obtains approximately:

$$\delta t = \frac{\delta \dot{r}}{|\ddot{\vec{\rho}} - \ddot{\vec{R}}|} \tag{13}$$

Here one has to remember two points, namely:

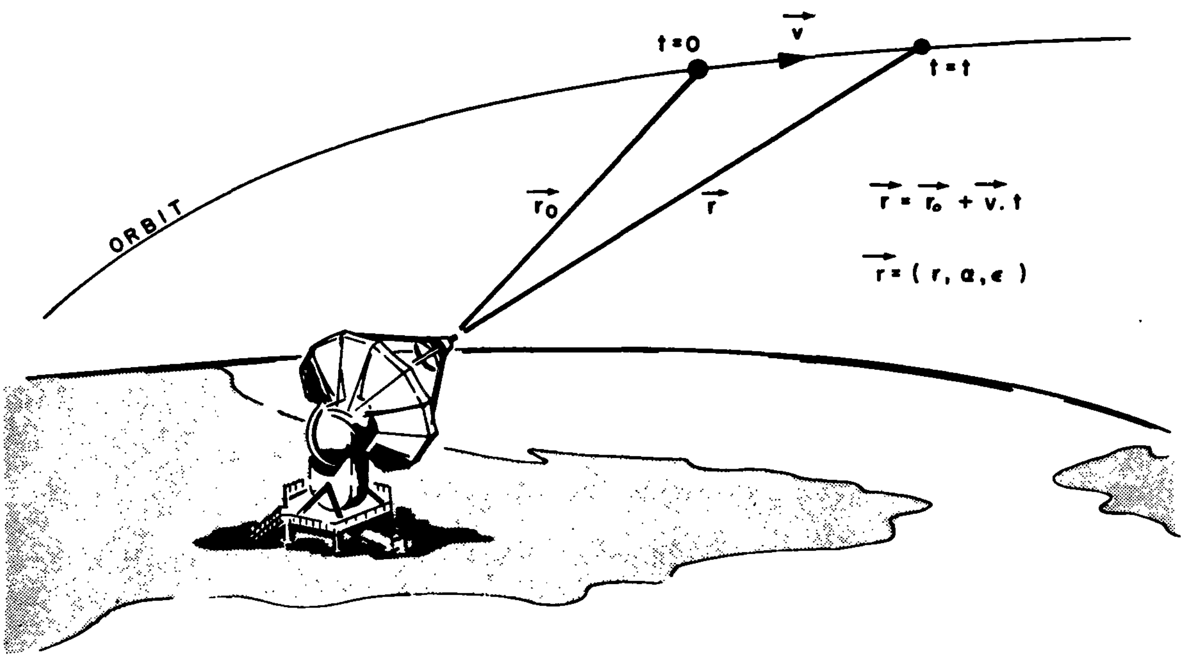
(a) the range rate error $\delta \dot{r}$ was previously assumed to be 0.01 cm/s based upon deep space mission results and

(b) range rate is NOT the only measurable used for orbit determination for near earth satellites except for the TRANET spacecraft. From experimental data such as our Goddard spacecraft as well as lunar orbiter evaluation a value of $\delta \dot{r} \leqq 1$ cm/s seems to emerge. Improvements are on the other hand under way so that an assumption of $d\dot{r} = 0.1$ cm/s for earth and lunar space satellites seems reasonable for the near future. Using this value namely $d\dot{r} \approx 0.1$ cm/s and $\ddot{\rho} = 10^3$ cm/s^2 as the near earth and lunar space "yardstick" one obtains using (13) for

$$\delta t = \frac{0.1}{10^3} = 10^{-4} \text{ sec} = 100 \ \mu \text{ sec} \tag{13a}$$

The value $|\ddot{\vec{R}}| = 3$ cm/s^2 has been neglected as compared to $\ddot{\rho} \doteq 1$ g $\doteq$ 1000 cm/s^2.

For interplanetary travel, where the acceleration terms are small (in the order of a few cm/s^2), timing errors can be appropriately large as dictated by equation (13).



$\vec{r}$ - SPACECRAFT POSITION VECTOR

$\vec{v}$ - SPACECRAFT RELATIVE VELOCITY

t - TIME OF MEASUREMENT

$\vec{r_0}$ - PROPER POSITION VECTOR FOR SAY t = 0 AND NO SYNCHRONIZATION ERROR.

GODDARD SPACE FLIGHT CENTER
MISSION ANALYSIS OFFICE
APRIL 1967 715.02.03

Figure 4—Radar Measurement of Slant Range Vector

The value given in (13a) should then really suffice for most of our space tracking problems.

The only time where more stringent requirements are to be considered is during a planetary approach as well as flyby missions. For such cases, the value given in (13a) may have to be eventually reduced to say 10 to 20 μsec. ($\ddot{\rho} \geqq$ 1000 cm/s^2, $\delta \dot{r} = 0.01$ cm/s). Here, on the other hand, single tracking stations are mostly involved for approximately 8 hours each day if necessary. The station time synchronization method as suggested by JPL (reference 27) can be applied for this case reducing the timing errors to the above mentioned values without elaborate additional station equipment and or other (VLF etc.) methods.

2. For a Radar System

For near earth orbit calculations more than just $\dot{r}$ information (or not at all) is used in many cases. Angular data (azimuth, elevation, or equivalent) and range data (radar) together are utilized to determine orbits for manned flight programs for instance[7,8,11,12]. In this case one can obviously not use $\dot{r}$ or $\delta \dot{r}$ as a yardstick, yet time errors do enter into the orbit analysis. They are but here somewhat more relaxed as shall be shown.

Assume a near earth spacecraft moves with say 10 km/s (during a lunar transfer orbit for instance) and the radar system would "look" in such a fashion that it would experience almost the full 10 km/s (pessimistic case). What would the time accuracy have to be to make the position measurement ($\vec{r} \sim$ a vector quantity) accurate to a certain predescribed number of meters (say 5 to 10 m)?

In this case one can write (see Figure 4)

$$\vec{r} = \vec{r}_0 + \vec{v}\, t \tag{14}$$

Where $\vec{r}$ is the measured slant range vector $\vec{r}$ at t = t (range r, azimuth α, elevation ϵ) $\vec{r}_0$ is the initial position vector at t = 0 and $\vec{v}$ is the velocity of the spacecraft relative to the radar system. If the stations would be synchronized completely, the value $\vec{r} = \vec{r}$ at t = 0. This of course, can never be achieved. Therefore, varying equation (14) yields

$$\delta \vec{r} = \delta \vec{r}_0 + \delta \vec{v}\, t + \vec{v}\, \delta t \tag{15}$$

Considering all these errors one obtains in the Gaussian sense rearranging the terms and introducing r, α, and ϵ:

$$v^2 \delta t^2 = \delta r^2 + r^2 (\delta\epsilon^2 + \delta\alpha^2) + \delta r_0^2 + r_0^2 (\delta\epsilon_0^2 + \delta\alpha_0^2) + \delta v^2 t^2$$

or

$$\delta t = \frac{1}{v} \sqrt{2 \delta r^2 + 2 r^2 (\delta\epsilon^2 + \delta\alpha^2) + \delta v^2 t^2} \tag{16}$$

since δr^2 and $\delta \nu^2$ t^2 (t is to be expected in the order of say msec or less) are always relatively small as experience has shown as compared to the middle term, one can write simply for

$$\delta t \doteq 2 \frac{r}{v} \delta \measuredangle_B \tag{17}$$

where $\delta \measuredangle_B$ is the bias error in elevation and azimuth assumed to be equal for reasons of simplicity. Equation (17) thus gives an estimate of the time synchronization needed for a good radar system.

To quote an example, assume a near earth orbit with $\nu \doteq 8000$ m/s, a radar slant range $r = 200$ km, and a elevation and azimuth bias $\delta \measuredangle_B = 0.02$ mrad, than $\delta t = 0.5$ msec.

For near earth orbit determination, the angular measurements of a radar system play a major role particularly when only short measuring (tracking) times say in the order of 60 to 100 sec are available. In this case, one obtains

$$\delta t \doteq \frac{\delta \epsilon_B}{\dot{\epsilon}_{max}} \doteq \delta \epsilon_B \frac{h}{v} \tag{17a}$$

where $\delta \epsilon_B$ is the bias error in the random elevation angle, v is the orbital speed of the spacecraft, h is its height, and $\dot{\epsilon}_{max}$ is the maximum angular acceleration. Equation (17a) represents a worst case overhead pass. Using again as an example $v = 8000$ ms, $h = 200$ km, $\delta \epsilon_B = 0.02$ mrad, one obtains for $\delta t = 0.25$ msec.

Thus even for sophisticated, and properly calibrated radars, a true synchronization of 0.2 msec occurs to be adequate since range, azimuth and elevation measurements play a role in the needed time synchronization as equation (17) indicates. The matter of fact we can see, as mentioned, from (17) that the angular errors push the needed time synchronization to a larger value.

In reference 26 it is shown how an error in station time synchronization influences the errors in the final Apollo orbits obtained. In general, under the present assumed Apollo systems errors [7,8] timing errors of approximately 10 msec do not appreciably influence the Apollo earth parking and lunar transfer orbits. This is different for lunar orbits where 10 msec do make a difference by a factor of 10 as compared to 1 msec. See Figure 5 (taken from reference 26).

It should be mentioned here that a rather simple and therefore elegant method for time synchronization was suggested by JPL, reference 27. In this method, the range is measured from 2 stations using say a CW ranging system. From one of the ranges the "time" of one of the stations can be determined with respect to the other. The error in time synchronization $\delta t = 1/c\ \delta$ pos. where δ pos is the position error of the spacecraft and c is the speed of light. One can see that even a relative large position error, say 3 km (see Figure 3), results in $\delta t = 1/3.10^{5} \cdot 3 = 10^{-5} = 10 \mu$sec, a rather small time synchronization error.

III. FREQUENCY SYNCHRONIZATION

As outlined under I. the frequency of a transmitting system has to be "constant" only to a specified value during the travel time of the signal.

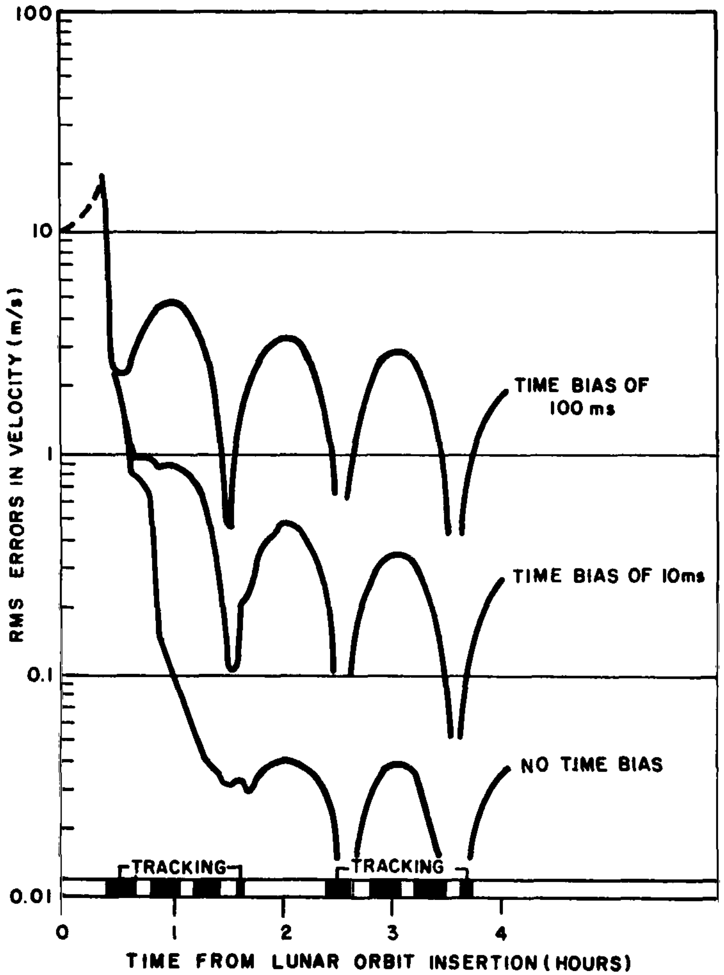
This is, of course, not the case when the transmitted signal is not only received at the transmitting site but also by a so-called "secondary" station. This technique, the 3-way Doppler, a "pseudo Doppler" is applied for orbit determination of the lunar orbiter and will be used for the lunar orbit determination during the Apollo mission [7,8,11] (See also Figure 6).

Using exactly the same approach as outlined under chapter I. one can write a similar equation as (9) but

$$\frac{\delta \nu_{D3}}{\nu_{D3}} \leq \frac{\delta \nu_{12}}{\nu_{01}} \doteq \frac{\delta \nu_{12}}{\nu_{02}} < \frac{\delta \dot{r}_{ps}}{\dot{r}_{ps}} \doteq 2.10^{-9} \tag{18}$$

where the pseudo Doppler frequency [7,8] at the second station is ν_{D3} (3-way Doppler) and $\delta \nu_{12} = (\nu_{01} - \nu_{02})$ is the difference frequency (error in frequency synchronization) between station 1 and 2 (see Figure 5) and $\dot{r}_{ps}$ is the pseudo range rate between the two stations. One has to modify (9) since the value $\delta \nu_{12}$, which is not known, will be included in the 3-way Doppler (pseudo Doppler) as can be seen by inspecting Figure 5. From equation (18) one obtains than for $\delta \nu_{12}$ a similar value than $\delta \nu_{D3}$. Obviously when ν_{D2} is measured, the error $\delta \nu_{12}$ is contained in the error of ν_{D2}, namely $\delta \nu_{D3}$. Equation (18) can be written as

$$\delta \nu_{12} \doteq 2 \nu_0 \frac{\delta \dot{r}_{ps}}{c} \tag{19}$$



ORBITAL PARAMETERS

EQUATOR OF DATE COORDINATES – MOON CENTERED

T = SEPT 12, 1969 77.170049

X_1 = 165.63936 nm $\dot{X}_1$ = -5208.365 ft/s

X_2 = -919.37695 nm $\dot{X}_2$ = -817.04316 ft/s

X_3 = -415.86151 nm X_3 = -268.21399 ft/s

INSERTION ERRORS
(1σ)

POSITION = 5196 m MEASUREMENT BIAS UPDATED

VELOCITY = 10.4 m/s STATION LOCATION UNCERTAINTY NOT UPDATED

TIME BIAS NOT UPDATED

HORIZON

ELEVATION ANGLE ≥ 5°

STATION	SYSTEM	STATION LOCATION UNCERTAINTIES			MEASUREMENTS	SAMPLING RATE	MEASUREMENT UNCERTAINTIES	
		NORTH (1σ) (m)	EAST (1σ) (m)	UP (1σ) (m)			NOISE (1σ) (m/s)	A-PRIOR BIAS(1σ) (m/s)
CNB	USBS	63.0	60.0	61.0	R	1 PER MIN	0.00061	0.0091
CRO	USBS	60.0	64.0	60.0	R	1 PER MIN	0.00131	0.0610
HAW	USBS	53.0	74.0	73.0	R	1 PER MIN	0.00131	0.0610

GODDARD SPACE FLIGHT CENTER
MISSION ANALYSIS OFFICE
APRIL 1967 715.02.03

Figure 5–RMS Velocity Errors During Lunar Orbital Phase (Measurement Biases Updated)

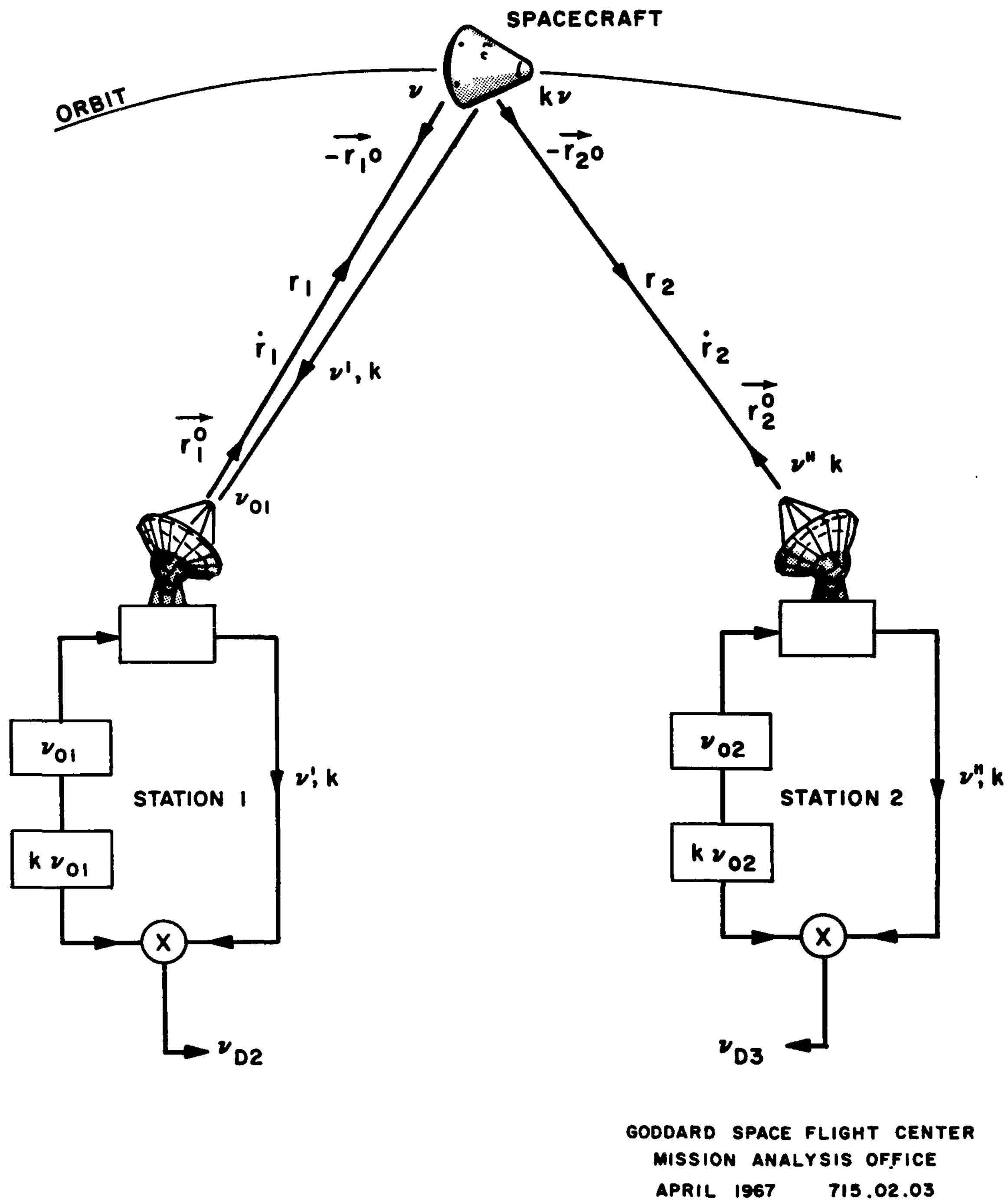


Figure 6–Three Way Doppler System

where $\dot{r}_{ps}$ represent the 3-way (pseudo) Doppler.

Example:

$$\delta \dot{r}_{ps} = 0.1 \text{ cm/s}$$

$$\nu_0 = 2.10^9 \text{ cps}.$$

than

$$\frac{\delta \nu_{12}}{\nu_0} = \frac{2}{3} \cdot 10^{-11}$$

is an extreme small value.

This number however is obtainable with hydrogen masers[23,24,25]. Thus a 3-way Doppler system[7,8] seems to be one of the first practical systems in the field requiring a frequency standard of the quality of a hydrogen maser. It should be noted that a frequency difference $\delta \nu_{12}$ between these two standards still result in bias error in $\dot{r}_{ps}$ which is more damaging for the orbit analysis (becomes less accurate) than a random error. A bias error is the same no matter how many measurements are made, a random error decreases with $N^{-1/2}$, the number of measurements[13,14,15].

IV. STATION LOCATION ERRORS

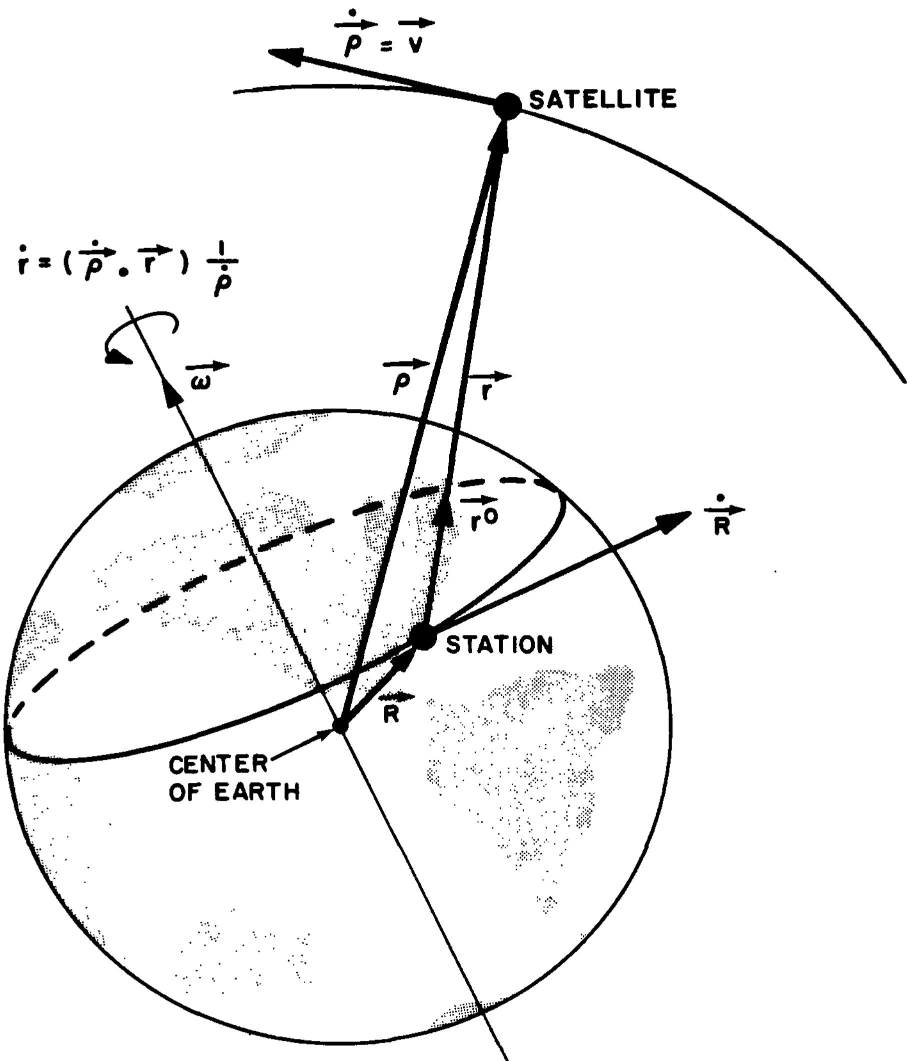
Possible location errors, compatible with a range rate system and the "yardstick $\delta \dot{r}$", as well as for a radar system, will be discussed in the following. These errors are considered as a limit for these systems based upon their accuracy assumptions.

A. For a CW Range Rate System

The aim here is to relate a station deviation $\delta \vec{R}$ (the variation of the station position vector R) to the range rate deviation $\delta \dot{r}$.

From Figure 7 one can see that the range rate error to an error in station location can be expressed as:

$$\delta \dot{r} = (\vec{r}^{\,0} \cdot \delta \dot{\vec{R}}) \tag{20}$$



$\vec{R}$ – STATION VECTOR

$\vec{\omega}$ – EARTH ROTATION VECTOR

$\vec{r}$ – SLANT RANGE VECTOR

$\vec{\rho}$ – SATELLITE POSITION VECTOR

GODDARD SPACE FLIGHT CENTER
MISSION ANALYSIS OFFICE
APRIL 1967 715.02.03

Figure 7–Tracking Station - Satellite Geometry

Where $\vec{r}^{\,0}$ is the unit position vector from the station to the spacecraft $\delta\dot{\vec{R}}$ is the variation of the station velocity vector $\dot{\vec{R}}$ (caused by a station error). Equation (20) is simply the projection of the ground station velocity error $\delta\dot{\vec{R}}$, caused by its location error, onto the position vector $\vec{r}$ as shown in Figure 7.

On the other hand, from basic vector analysis, the following holds:

$$\frac{\delta}{\delta t}\vec{R} = \dot{\vec{R}} = (\vec{\omega} \times \vec{R}) \tag{21}$$

Where $\vec{\omega}$ is the rotation vector of the earth and $\vec{R}$ is the station position vector as before (vector from center of earth to station).

Varying (11) yields

$$\delta\dot{\vec{R}} = (\vec{\omega} \times \delta\vec{R}) \tag{22}$$

assuming $\delta\vec{\omega} = 0$, or a constant rotation of the earth during the time interval considered.

Introducing (22) into (20) finally results to a first order approximation in:

$$\delta\dot{r} = (\vec{r}^{\,0} \cdot (\vec{\omega} \times \delta\vec{R})) \tag{23}$$

Equation (23) can also be written as

$$\delta\dot{r} = \omega\, \delta R \sin\psi \cos\varphi \tag{24}$$

Where ψ is the angle between $\vec{\omega}$ and $\delta\vec{R}$ and φ is the angle between $(\vec{\omega} \times \delta\vec{R})$ and $\vec{r}^{\,0}$. Since these two quantities are not known, it could be assumed that their product be equal to one, thus making the value of δR somewhat small. The minimum variation, or error in location of a tracking station δR is then from (24)

$$\delta R \doteq \frac{\delta\dot{r}}{\omega} \tag{25}$$

in terms of the "yardstick" $\delta\dot{r}$ and the known quantity of the earth's rotation W.

Since the product $\sin\psi \cos\varphi$ will never be equal to unity in practice, as assumed in equation (25), a quadratic (energy equivalent) average will give a more realistic value. That is

$$\frac{1}{4\pi^2} \int_{2\pi} \int_{2\pi} \sin^2\psi \cos^2\varphi \;\delta\psi\, \delta\varphi = \frac{1}{4},$$

than (25) reads

$$\overline{\delta R} \doteq 4 \frac{\delta \dot{r}}{\omega} \tag{25a}$$

Example: Assume again

$\delta \dot{r} = 0.01$ cm/s (deep space mission)

$\omega \doteq 7.3 \cdot 10^{-5}\ \mathrm{sec}^{-1}$

than

$$\overline{\delta R} = 4 \frac{10^{-2}}{7.3 \cdot 10^{-5}} = 5.5\ 10^{2}\ \mathrm{cm}$$

In brief,

$$\overline{\delta R} \doteq 5\ 1/2\ \mathrm{meters}$$

If this can be done, then one can make full use of the $\dot{r}$ measurement up to an accuracy of $\delta \dot{r} = 0.01$ cm/s providing the requirements discussed in Chapters I. and II. can be met.

B. For a Radar System

Again for a radar system, not measuring $\dot{r}$ as such, an other criterion, say the position error has to be introduced. In general, if the station position error $\delta \vec{R}$ is smaller than the radar error $\delta \vec{r}$, no improvements can be obtained anymore by the radar system. That is

$$\delta \vec{R} \doteq \delta \vec{r} \tag{26}$$

or

$$\delta \vec{R} = \sqrt{\delta r^2 + r^2 (\delta \epsilon^2 + \delta \alpha^2)} \tag{27}$$

Using the same reason (neglecting δr^2) as for equation (17) one obtains for the station error (square root of the number of the components):

$$\delta R \doteq \sqrt{2}\ r \left(\frac{\delta \measuredangle}{\sqrt{N}} + \delta \measuredangle_B \right) \tag{28}$$

Where $\delta \measuredangle = \delta \alpha = \delta \epsilon$, N the number of measurements used to "construct" one radar point (a vector, $\vec{r}(r, \alpha, \epsilon)$) and $\delta \measuredangle_B$ is the angular bias error.

Example

Assume as before, N = 10, r = 300 km, $\delta \measuredangle$ = 0.02 mrad, $\delta \measuredangle_B$ = 0 and 0.02 mrad than we obtain:

$$\delta R = 11 \text{ meters}$$

and

$$\delta R = 3\ 1/2 \text{ meters}$$

depending on the bias error $\delta \measuredangle_B$. A reasonable range of station accuracies is believed to be given by these figures.

ACKNOWLEDGEMENT

The author wishes to take the opportunity to thank Mr. C. Schroeder and Mr. C. Looney for their valuable criticism of the paper, and Dr. B. Kruger for his corrections and modifications of some of the equations used.

REFERENCES

1. Moller, C., "The Theory of Relativity," Oxford at the Clarendon Press, 1957.

2. Kruger, B., "The Doppler Equation in Range and Range Rate Measurement," GSFC Report X-507-65-385, October 8, 1965.

3. Joos, G., "Theoretical Physics," G. E. Stechart & Co., New York.

4. Weitzel, W., "Lehrbuch der Theoretischen Physik," Band I, Springer-Verlag, Berlin, 1955.

5. Habib, E., et al., "Development of a Range and Range Rate Spacecraft Tracking System," NASA TN D-2093, June 1964.

6. Froome, K. D., "A New Determination of the Free Space Velocity of Electromagnetic Waves," Prec. Roy. Soc., A247, 1958.

7. MSC-GSFC, "Apollo Missions and Navigation Systems Characteristics," NASA, ANWG Report AN-1.1, April 4, 1966.

8. MSC-GSFC, "Apollo Navigation, Ground and Onboard Capabilities," NASA, ANWG Report, AN-2.1, September 1, 1966.

9. The American Ephemeris and Nautical Almanac for the Year 1968, U.S. Government Printing Office, Washington, 1966.

10. Winkler, G. M., "Short Time Stability Measurements on Frequency Standards and the Notion of Resolution," J. R. E. Vol. 47, Jan. 1, 1959 (Note).

11. Vonbun, F. O., "Tracking the Apollo," Journal of Aeronautics and Astronautics, May 1966.

12. Vonbun, F. O., W. D. Kahn, "Tracking of a Lunar Transfer Orbit," GSFC Report X-513-64-353, November 1964.

13. Vonbun, F. O., W. D. Kahn, "Tracking Systems, Their Mathematical Models and Their Errors, Part I - Theory," NASA TN D-1471.

14. Kahn, W. D., F. O. Vonbun, "Tracking Systems, Their Mathematical Models and Their Errors, Part II - Least Square Treatment, NASA TN D-3776, December 1966.

15. Cooley, J. L., "Tracking Systems, Their Mathematical Models and Their Errors, Program Description," GSFC Report X-513-64-145, May 20, 1964.

16. Schmidt, S., "Mark II Error Propagation Program and Powered Flight Optimization Error Analysis Programs," User's and Programmer's Manuals prepared for Contract NAS5-9700, Philco Corp.

17. Kulikov, K. A., "Fundamental Constants of Astronomy," NASA publication Washington, D.C. (Moskow 1956, Russian translation).

18. Allen, C. W., "Astrophysical Quantities," Univ. of London, the Atlantic Press, 1955.

19. Wollenhaupt, W. R., "Tracking System Data Anlysis Report, Ranger 4," Final Report, JPL TR No. 32-523, March 1, 1964.

20. Sjogren, W. L., D. W. Trask, "Radio Tracking of the Mariner and Ranger Missions and its Implications," JPL TM No. 312-513 (order no. 65-418) March 15, 1965.

21. Mulholl, J. D., W. L. Sjogren, "Lunar Orbiter Ranging Data: Initial Results," Science, Vol. 155, January 6, 1967.

22. Schmid, P. E., "Atmospheric Tracking Errors at S- and C-Band Frequencies," NASA TN D-3470, August 1966.

23. Allen, D. W., "Statistics of Atomic Frequency Standards," Proc. IEEE, Vol. 54, No. 2, February 1966.

24. Johnson, E. H., T. F. McGunigal, "Hydrogen Maser Frequency Comparison with a Cesium Beam Standard," NASA TN D-3292, April 1966.

25. McCoubrey, A. U., "A Summary of Atomic Frequency Standards," Proc. IEEE, Vol. 54, No. 2, February 1966, p. 125.

26. Cooley, J. L., D. W. Koch, "The Effect of Station Time Synchronization Errors on Apollo Tracking," GSFC Report X-507-66-130, April 1966.

27. Hamilton, T. W., et al., "Tracking and Navigation Accuracy Analysis," JPL Report 37 ϵ 43, Vol. III, January 31, 1967.