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ANALYSIS OF INTEGRATION ERROR IN THE ODP-L PROGRAM AND THE EFFECT OF SELENOPOTENTIAL PARAMETERS ON THE SOLUTION VECTOR

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TRW Systems, Redondo

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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1. SUMMARY

This report describes an investigation into the ODP-L orbit determination program integration characteristics and the sensitivity of the solution vector to the coefficients of the lunar potential model.

A reference lunar orbit trajectory was integrated on the ODP-L program at various fixed integration step sizes. Simulated range and range rate observations were computed from these reference trajectories. The error due to integration in the computation of the observables was fit to an equation that related the error and the step size. Subsequent evaluation of the coefficients of the equation for various cases revealed that this method of predicting the error due to integration could not be generalized and would involve cumbersome computations in order to calculate the biases for all observations in a typical tracking orbit determination.

The ODP-L generated observations were input to the TRW AT-85 program to determine the bias in the solution vector introduced by the observation bias (due to the integration package in the ODP-L program). Because of possible program incompatibilities, it has not been possible to evaluate the bias in the solution vector that is introduced by the ODP-L integration package.

A study of the sensitivities produced by different models has led to the conclusion that no selenopotential constant of degree greater than four need be included in the model, provided all constants through C, S44 are included in the solution vector.

The most significant new technology developed under this contract is the computation of the sensitivity of unsolved parameters to solved parameters. This capability was developed at TRW and implemented into the AT-85 orbit determination program.

2. INTRODUCTION

Due to a limitation in the difference table used in the ODP-L trajectory integration package, the computed observations based on the trajectory will include an error with both random and secular components. This results in biased observational residuals and ultimately prevents the differential correction process from converging on the appropriate solution parameters. This anomaly in the ODP-L program has been known to NASA/LRC for some time. This error has been compensated for by adjusting the standard deviations on the observations in such a way that they no longer represent the performance of the participating radar stations, but rather the performance of the integration package. The objectives of this part of the study are to estimate the errors introduced into the statistics of the lunar orbiter solution vector by the behavior of the integration package and the error resulting from using adjusted standard deviations.

To answer the above question completely would be a very difficult task analytically and is beyond the scope of this study. It would be more difficult to assess the effects of the error sources and all of the statistical implications analytically than it would be to correct the source of error. As a compromise, TRW has attempted to satisfy the objectives of this study by a pragmatic approach, that is, a direct comparison of the ODP-L integration package with TRW's AT-85 program. The accuracy of the AT-85 integration package is discussed in section 3.3. The theory of numerical integration indicates that the size of the integration error can be related approximately to step size by means of a simple formula. The computed observations were successfully related to the step size formula; this is discussed in section 3.2. Hence, it is possible to predict the bias in the ODP-L observations when the coefficients of the step size formula have been evaluated.

The process of evaluating the coefficients of the step size formula was done off-line. Unfortunately, this formulation could not be generalized and therefore required a separate iterative curve fit procedure for the evaluation of each observable bias. Since it proved too cumbersome to compute the bias in each of the observations required in an orbit determination run (which is statistically determinate), the uncorrected observations were input to the AT-85 program directly.

This attempt also resulted in unsurmountable difficulties, as outlined in section 3.3. Essentially, it was not possible to match the computation of the observable at epoch, that is, before the integration started. Although the initial parameters of the orbit and the physical constants were checked for compatibility, there are other possible sources of error, which were not checked; for example, the observable computation depends upon the ephemeris of the moon, the coordinate rotations (mean of 1950.0 to true of date), and the actual computation of the observable given the vehicle-observing station geometry. These latter formulations were not checked for ODP-L and AT-85 compatibility since it would have required a level of effort well beyond the scope of this task.

For the second part of the study, a series of runs were made on the TRW AT-85 program in order to establish the dependence of the solution vector statistics upon the uncertainties in progressively more complicated models of the lunar potential field. Two kinds of results are obtained, depending on whether a particular potential coefficient is actually included in the solution vector or only considered for the sensitivity which the solution vector has to it.

When a lunar potential coefficient is included in the solution vector, the standard deviations of the observations result in some uncertainty in its determination. Further, the uncertainty in this determined value influences the determination of the state vector of the orbiter. The greater the number of elements included in the solution vector, the greater will be the uncertainty in the determination of the state of the orbiter. Because of the coincidences of tracking geometry and orbit geometry, the state vector will be more strongly influenced by some of the lunar potential coefficients' uncertainties than by others. Also, the state vector will be correlated more or less with the potential coefficients depending upon this geometry. The correlation coefficient gives a measure of sensitivity of the state vector to the solved gravitational parameter. The increase in the variance associated with the state vector, due to including an additional gravitational parameter in the solution vector, gives an estimate of the additional uncertainty in the state vector because of lack of information for the gravitational coefficient in this particular geometry. Because of correlations between the lunar potential coefficients, the increases in variance derived from adding any one coefficient depend on the order in which the coefficients are added.

When a parameter is included in the consider option, it is assumed that nothing is known about it, that is, its value or its statistics. The partial derivative of all solved parameters with respect to the considered parameter are calculated. These partial derivatives are interpreted as sensitivity coefficients to multiply any estimated change and/or uncertainty in the considered parameter to determine the effect which the considered parameter has on the solution vector. This effect is called the sensitivity of the solution variable to the considered parameter. These sensitivities were determined for four different solution vectors. By studying sensitivities for different cases, it is possible to determine which parameters are most influential and are to be included in the solution vector. The results of this study are reported in section 4.

3. INTEGRATION ERROR IN THE ODP-L PROGRAM

3.1 Integration of Reference Trajectory

The reference trajectory (see initial conditions in table 3-1) was integrated on the ODP-L version of the Jet Propulsion Laboratory orbit determination program. The integration was carried out at a fixed step size for the following step size increments: 30, 50, 75, 100, 125, 150, and 200 seconds; for comparison purposes, the orbit was also integrated at its natural step size.

TABLE 3-1

INITIAL CONDITIONS AND THE LUNAR CONSTANTS USED FOR THE REFERENCE TRAJECTORY

Epoch		State vector (selenocentric, mean of 1950.0)			Lunar constants				
Year	1966	X	2324.15	km	GMM	4902.58	km^3/sec^2	C42	-0.14 E-4
Month	08	Y	90.61		J20	0.2048 E-3		C44	0.017E-4
Day	17	Z	616.73		J30	0.98 E-4		S31	0.21 E-4
Hr	17	$\dot{X}$	-0.5796	km/sec	J40	-0.48 E-4		S41	0.54 E-4
Min	0.0	$\dot{Y}$	1.3494		C22	0.23 E-4		S33	0.018E-4
Sec	0.6	$\dot{Z}$	0.3792		C32	-0.15 E-4		S43	0.032 E-4

Table 3-2 is a summary of the position and velocity at the change of phase; this event occurs at the first pericynthion passage and the trajectory conditions are computed and printed at this time to satisfy some internal logic requirements in the ODP-L program. The state vector is presented in both geocentric (table 3-2a) and selenocentric (table 3-2b) coordinates. Although the initial conditions of the trajectory at epoch were identical for all step sizes, it can be seen that the numerical integration already has a noticeable effect on the components of the state vector, especially for the larger integration steps. Depending on the step size, between 9 and 59 integration steps were taken to propagate the trajectory from epoch to the phase change. The secular trend of the numerical integration error in the components of the state vector is not shown in this report since the intermediate trajectory prints were not available on the computer runs; only the initial and final points of the trajectory (plus the phase change) were available. However, it can be inferred that the trend would approximate the behavior of the computed observables, since the observable computations are based on the trajectory parameters. Tabulations of range and range rate observables at selected times from epoch are presented in tables 3-4, 3-5, and 3-6.

TABLE 3-2a

COMPUTED GEOCENTRIC EQUATORIAL VEHICLE POSITION AND VELOCITY AT PHASE CHANGE

Step size	X (km)	Y (km)	Z (km)	$\dot{X}$ (km/sec)	$\dot{Y}$ (km/sec)	$\dot{Z}$ (km/sec)
30	-339660.58	91474.465	71292.952	-2.1156187	-0.75414611	-0.81504039
50	-339660.57	91474.465	71292.952	-2.1156187	-0.75414509	-0.81503993
75	-339660.57	91474.464	71292.952	-2.1156187	-0.75414529	-0.81504001
100	-339660.57	91474.464	71292.952	-2.1156187	-0.75414547	-0.81504007
125	-339660.57	91474.464	71292.952	-2.1156185	-0.75414606	-0.81504033
150	-339660.57	91474.465	71292.954	-2.1156178	-0.75414084	-0.81503808
200	-339660.51	9.474.487	71292.977	-2.1156119	-0.75410028	-0.81502087
NATURAL	-339660.57	91474.464	71292.952	-2.1156187	-0.75414522	-0.81503997

TABLE 3-2b

COMPUTED SELENOCENTRIC EQUATORIAL VEHICLE POSITION AND VELOCITY AT PHASE CHANGE

Step size	X (km)	Y (km)	Z (km)	$\dot{X}$ (km/sec)	$\dot{Y}$ (km/sec)	$\dot{Z}$ (km/sec)
30	40.783167	1801.9034	712.07836	-1.7645769	0.18602625	-0.36967321
50	40.784336	1801.9032	712.07862	-1.7645769	0.18602728	-0.36967275
75	40.784116	1801.9032	712.07856	-1.7645769	0.18602708	-0.36967283
100	40.783912	1801.9031	712.07846	-1.7645769	0.18602691	-0.36967289
125	40.783226	1801.9024	712.07800	-1.7645767	0.18602631	-0.36967315
150	40.789132	1801.9008	712.07888	-1.7645760	0.18603153	-0.36967091
200	40.834959	1801.8920	712.08702	-1.7645700	0.18607206	-0.36965372
NATURAL	40.784201	1801.9032	712.07860	-1.7645768	0.18602715	-0.36967279

Table 3-3 is a summary of the position and velocity of the vehicle in geocentric and selenocentric equatorial coordinates seven days after epoch. The individual components of selenocentric position and velocity have been plotted as a function of the step size (see figures 3-1 to 3-6.)

Tables 3-4, 3-5, and 3-6 are tabulations of range and range rate of stations number 11, 42, and 41 respectively. Since these observations are based on the integrated trajectory, it is expected that the observations demonstrate a buildup in error as the step size gets larger, and also as the time of the observation gets farther away from epoch (the time at which the trajectory integration is initiated).

Figures 3-7 to 3-12 are plots of range and range rate as a function of step size; the associated time corresponds to the last observation in each case. Figures 3-13 to 3-15 illustrate how the error in the computed observation tends to increase with time. Some of the intermediate observations have been omitted for the sake of clarity.

3.2 Integration Error Formula

The tabulations and graphs of the computed range and range rate observables (see section 3.1) indicate that the error grows with increasing integration step size. The theory of numerical integration indicates that the size of the integration error can be related to the step size by a single formula.

In terms of the computed observable, equation (3.1) specifies the value as a function of step size.

$$R(h) = R_o + \gamma h^k \qquad (3.1)$$

TABLE 3-3a

COMPUTED GEOCENTRIC EQUATORIAL VEHICLE POSITION AND VELOCITY AFTER SEVEN DAYS

Step size	X (km)	Y (km)	Z (km)	$\dot{X}$ (km/sec)	$\dot{Y}$ (km/sec)	$\dot{Z}$ (km/sec)
30						
50						
75	-68619.806	-346117.36	-169382.22	1.4755882	-1.1927498	-0.41283726
100	-68619.350	-346118.26	-169382.44	1.4759084	-1.1925048	-0.41266216
125	-68616.606	-346123.71	-169383.76	1.4778573	-1.1909996	-0.41158796
150	-68604.827	-346146.92	-169389.37	1.4861081	-1.1845248	-0.40700115
200	-68457.096	-346402.91	-169446.54	1.5741306	-1.1070880	-0.35492835

TABLE 3-3b

COMPUTED SELENOCENTRIC VEHICLE POSITION AND VELOCITY AFTER SEVEN DAYS

Step size	X (km)	Y (km)	Z (km)	$\dot{X}$ (km/sec)	$\dot{Y}$ (km/sec)	$\dot{Z}$ (km/sec)
30						
50						
75	-2257.6644	-1780.8383	-1258.4152	0.49592572	-1.0197591	-0.25199471
100	-2257.2106	-1781.7336	-1258.6329	0.49624593	-1.0195141	-0.25181961
125	-2254.4637	-1787.1884	-1259.9580	0.49819483	-1.0180089	-0.25074541
150	-2242.6865	-1810.3988	-1265.5660	0.50644566	-1.0115341	-0.24615860
200	-2094.9556	-2066.3822	-1322.7312	0.59446819	-0.93409728	-0.19408580

TABLE 3-4a

COMPUTED RANGE OBSERVATIONS FOR STATION 11 AT SELECTED TIMES FROM EPOCH AS A FUNCTION OF STEP SIZE

Days from epoch / step size	0.0	1.0	2.0	3.5	4.5	5.5	6.5
30	353162.11	358280.06	365171.06	370424.15	374709.21		
50	353162.13	358280.06	365171.03	370424.26	374708.99	382853.55	382348.65
75	353162.14	358280.02	365171.22	370424.48	374708.87	382853.62	382348.66
100	353162.13	358279.95	365171.36	370425.02	374708.35	382853.80	382348.69
125	353162.13	358279.61	365172.17	370427.90	374705.30	382854.85	382348.85
150	353162.14	358278.77	365174.49	370438.51	374693.86	382859.17	382349.90
200	353162.13	358271.88	365197.10	370550.23	374567.82	382901.28	382400.05
NATURAL	353162.14	358280.02	365171.20	370424.44	374708.92	382853.60	382348.66

TABLE 3-4b

COMPUTED RANGE RATE OBSERVATIONS FOR STATIONS 11 AT SELECTED TIMES FROM EPOCH AS A FUNCTION OF STEP SIZE

Days from epoch / step size	0.0	1.0	2.0	3.5	4.5	5.5	6.5
30	0.69768284	-1.2433861	0.65021198	2.0631918	-0.71851856		
50	0.69768287	-1.2433868	0.65023942	2.0631695	-0.71852563	0.64863159	0.30403268
75	0.69768286	-1.2433883	0.65009531	2.0631280	-0.71852732	0.64852336	0.30452439
100	0.69768288	-1.2433889	0.64997799	2.0630261	-0.71853580	0.64825870	0.30553745
125	0.69768307	-1.2433922	0.64934866	2.0624758	-0.71859242	0.64677183	0.31154414
150	0.69768373	-1.2434057	0.64752140	2.0604104	-0.71881294	0.64042657	0.33775785
200	0.69768960	-1.2435369	0.62965422	2.0366486	-0.72029529	0.57059050	0.64065832
NATURAL	0.69768290	-1.2433883	0.65010633	2.0631379	-0.71852642	0.64854793	0.30443246

TABLE 3-5a

COMPUTED RANGE OBSERVATIONS FOR STATION 42 FROM EPOCH AS A FUNCTION OF STEP SIZE

Days from epoch step size	0.0	1.0	2.0	3.0	4.0	5.0	6.0
30	352620.10	355848.47	361218.81	362179.07	372395.64	374119.55	
50	352620.12	355848.47	361218.78	362178.99	372395.64	374120.01	383317.30
75	352620.13	355848.43	361218.97	362178.94	372395.58	374120.12	383316.87
100	352620.11	355848.36	361219.11	362178.70	372395.46	374120.82	383316.16
125	352620.11	355848.02	361219.91	362177.57	372394.76	374124.98	383312.22
150	352620.12	355847.18	361222.17	362173.48	372392.19	374141.42	383294.62
200	352620.12	355840.28	361244.31	362132.36	372363.07	374332.89	383092.94
NATURAL	352620.12	355848.43	361218.95	362178.96	372395.60	374120.05	383316.93

TABLE 3-5b

COMPUTED RANGE RATE OBSERVATIONS FOR STATION 42 AT SELECTED TIMES FROM EPOCH AS A FUNCTION OF STEP SIZE

Days from epoch step size	0.0	1.0	2.0	3.0	4.0	5.0	6.0
30	1.3031935	-0.68042717	1.1537521	-0.64053836	-0.20305882	1.3529692	
50	1.3031936	-0.68042774	1.1537799	-0.64051006	-0.20307510	1.3532484	-1.0326062
75	1.3031936	-0.68042896	1.1536341	-0.64048722	-0.20313733	1.3533096	-1.0327142
100	1.3031936	-0.68042900	1.1535153	-0.64040186	-0.20328012	1.3537343	-1.0328868
125	1.3031938	-0.68042957	1.1528783	-0.63997199	-0.20407857	1.3561947	-1.0338659
150	1.3031945	-0.68043631	1.1510291	-0.63844696	-0.20701723	1.3657849	-1.0382233
200	1.3032004	-0.68051098	1.1329530	-0.62250332	-0.23847061	1.4619267	-1.0855519
NATURAL	1.3031936	-0.68042899	1.1536452	-0.64049497	-0.20312341	1.3532693	-1.0326985

TABLE 3-6a

COMPUTED RANGE OBSERVATIONS FOR STATION 41 AT SELECTED TIMES FROM EPOCH AS A FUNCTION OF STEP SIZE

Days from epoch / step size	0.5	1.5	2.5	3.5	4.5	5.5	6.5
30	356488.27	353754.87	361828.16	365618.66	371101.36		
50	356488.26	353754.86	361828.13	365618.78	371101.15	380429.06	381294.04
75	356488.27	353754.86	361828.05	365618.99	371101.03	380429.13	381294.03
100	356488.26	353754.86	361827.88	365619.53	371100.50	380429.32	381294.03
125	356488.24	353754.87	361826.98	365622.41	371097.44	380430.45	381294.00
150	356488.21	353754.88	361824.08	365633.00	371085.98	380435.11	381294.18
200	356487.93	353755.20	361795.33	365744.52	370959.55	380480.98	381334.45
NATURAL	356488.27	353754.86	361828.06	365618.95	371101.08	380429.11	381294.04

TABLE 3-6b

COMPUTED RANGE RATE OBSERVATIONS FOR STATION 41 AT SELECTED TIMES FROM EPOCH AS A FUNCTION OF STEP SIZE

Days from epoch / step size	0.5	1.5	2.5	3.5	4.5	5.5	6.5
30	-0.021366125	-0.00050178111	-0.82932787	1.5116263	-1.2938704		
50	-0.021369384	-0.00051132709	-0.82933297	1.5116053	-1.2938794	0.10488288	-0.27419984
75	-0.021373425	-0.00042624787	-0.82935724	1.5115664	-1.2938822	0.10477507	-0.27370917
100	-0.021394057	-0.00035455358	-0.82940315	1.5114710	-1.2938954	0.10451129	-0.27269805
125	-0.021479992	0.000030875803	-0.82963510	1.5109556	-1.2939796	0.10302904	-0.26670266
150	-0.021682906	0.0010445029	-0.83039720	1.5090188	-1.2943039	0.096703588	-0.24053102
200	-0.023061445	0.010521095	-0.83792450	1.4865965	-1.2969487	0.027038119	0.062741105
NATURAL	-0.021371802	-0.00043277698	-0.82935316	1.5115757	-1.2938808	0.10479957	-0.27380091

where R_o is the value of the computed observable using a "0" step size that is, the true value; h is the step size, and γ and k are constants. A similar equation exists for the range rate observable.

Assuming a linear log-log function of integration error $[R(h) - R_o]$ versus step size, equation (3. 1) was solved for four representative values of range and range rate for each of the observing stations. An iterative algorithm was devised for solving equation (3. 1) and programed on a small computer. The results of a typical case are itemized in table 3-7. The case shown below is for the range rate observation of station 11, 4.5 days after epoch (see table 3-4b).

TABLE 3-7

A TYPICAL RESULT OF CURVE-FITTING THE INTEGRATION ERROR VERSUS STEP SIZE

Step size h	ODP-L observations (km/sec)	Curve fit observations (km/sec)
30	-0.71851856	-0.71852623
50	-0.71852563	-0.71852628
75	-0.71852732	-0.71852732
100	-0.71853580	-0.71853680
125	-0.71859242	-0.71857862
150	-0.71881294	-0.71872903
200	-0.72029529	-0.72029544

$$\dot{R}(h) = \dot{R}_o + \gamma h^k$$

where
$$\dot{R}_o = -.71852622$$
$$\gamma = -8.37 \times 10^{-21}$$
$$k = 7.529$$

As can be seen from table 3-7, the ODP-L observations can be fit to the empirical formula (equation (3. 1)). Therefore, the error in integration due to step size can be computed directly by comparing R_o and R(h). As was mentioned previously, this was done for 12 values of range and 12 of range rate. However, it soon became apparent that this formulation only holds true for the particular observable time for which it was computed. That is, the constants γ and k of equation (3. 1) differ for each time the equation is evaluated.

In order to evaluate the bias in the solution vector introduced by the integration error (and hence the bias in the computed observables), it is necessary to remove the observation bias which is introduced by the ODP-L integration package. This would require an evaluation of the empirical equation for each observation in the orbit determination run designed to evaluate the bias in the solution vector. It would also require the evaluation of the ODP-L observations from trajectories that are integrated over a span of step sizes.

This method of evaluating the bias is much too cumbersome and tedious to carry out. Hence, it was decided, in consultation with LRC, to evaluate the bias in the solution vector by using ODP-L generated observations (uncorrected) in the TRW AT-85 program. The results of this task are discussed in the next section.

3.3 AT-85 Program Bias Evaluation

The results of experimentation concerning the effect of integration error on the computed observables and the solution vector are presented in this section.

The ODP-L program was operated at Langley Research Center and 3447 range rate observations were generated for three DSIF stations. The reference orbit was a typical Lunar Orbiter orbit with a period of 220 minutes. The force model used included a two body moon, the earth (with J_2, J_3, and J_4 zonal harmonics), and the sun. The range rate observations were input into the TRW AT-85 program and an 11 component solution vector run was made to determine the effects of the integration error in the ODP-L integration package.

In order to justify that the TRW AT-85 integration routine (TRAJ) yields more accurate results than the ODP-L program, the TRAJ integration subroutine was compared with a two body analytic program. The reference orbit was propagated for 12 revolutions on both the AT-85 integrating routine and the analytic program. The step size is tested at each integration step to determine if its current value is adequate in order that a desired accuracy will be maintained. Thus, after an integration step has been completed, an internal integration quantity is computed and compared with a specified error tolerance, ϵ. If the step size is too large, it is halved; if too small, doubled. See section 5.7.3 of reference 1 for a more complete discussion of the step size control and error tolerances in subroutine TRAJ of the AT-85 program. The reference orbit was integrated with an integration error tolerance of 10^{-7}, 10^{-8}, and 10^{-9}. The resulting trajectories were compared with the analytic program results and the actual error growth behavior in position for 12 orbits is plotted in figure 3-16. For an error tolerance of 10^{-9}, the difference in selenocentric range after 36 hours of trajectory propagation is 38 meters.

The bias evaluation method can be expressed as follows:

$$\delta x_o = \left(\sum_i a_i^T w_i a_i \right)^{-1} \sum_i a_i^T w_i \delta O_i$$

where δx_o denotes the solution vector (11 x 1), and where δO_i denotes the difference between LRC's and TRW's computed range rate observations, which is due to integration error only.

The solution vector consisted of the following 11 components:

$$\left.\begin{array}{l}\Delta x\\ \Delta y\\ \Delta z\end{array}\right\}\quad \text{change in position}$$

$$\left.\begin{array}{l}\Delta \dot{x}\\ \Delta \dot{y}\\ \Delta \dot{z}\end{array}\right\}\quad \text{change in velocity}$$

$$\left.\begin{array}{l}\Delta J_{20}\\ \Delta J_{30}\\ \Delta J_{40}\end{array}\right\}\quad \text{change in moon's zonal harmonics}$$

$$\left.\begin{array}{l}\Delta C_{22}\\ \Delta S_{22}\end{array}\right\}\quad \text{change in moon's sectorial harmonics}$$

The range rate observation is a function of the state vector (11 x 1), the station's coordinates, time, lunar ephemeris, coordinate rotations, and the force model.

The method explained above will show the effect of integration error on the state vector and selenopotential constant corrections if the only difference between the two programs is in the integration packages. However, the results of the experiment run under this contract cannot be related to the integration error because of one or more of the following possible program differences:

a. The AT-85 program force model cannot simulate the earth's zonal harmonics (J_2, J_3, J_4) when integrating in the moon phase.

b. It has not been verified that LRC and TRW have the same ephemeris for the moon.

c. It has not been verified that the ODP-L and AT-85 programs have compatible coordinate rotations from mean of 1950.0 (integration coordinate system) to true of date (output coordinate system).

d. It has not been verified that ODP-L and AT-85 compute range rate similarly from the trajectory and station coordinate information.

e. An adequate description of the ODP-L integration package has not been received.

To reconcile the above was beyond the scope of the contract. However, some compatibility was achieved:

a. The initial conditions of the trajectory and the physical constants were identical.

b. The identical station locations were used.

The following errors were uncovered:

a. For this run, geodetic latitudes were input to the ODP-L program at LRC instead of the geocentric latitudes of the participating tracking stations.

b. At the time of the first observation ($t = 0$), the computation of the range rate observations should be identical since no integration has been carried out yet. However, this did not occur, indicating that a difference exists in the computation of the range rate observable independent of the actual trajectory (trajectory parameters were verified for compatibility). This difference is probably a function of one or more of the possible program differences listed above.

Several runs were made at TRW using geodetic station coordinates before it was realized that geocentric coordinates should have been used. However, it should be noted that the erroneous station latitudes (by as much as 0.2°) did not produce significant differences in the run results.

Because of the known errors and possible unaccounted for program incompatibilities, it has not been possible to assess the error in the solution vector that is introduced by the ODP-L integration package.

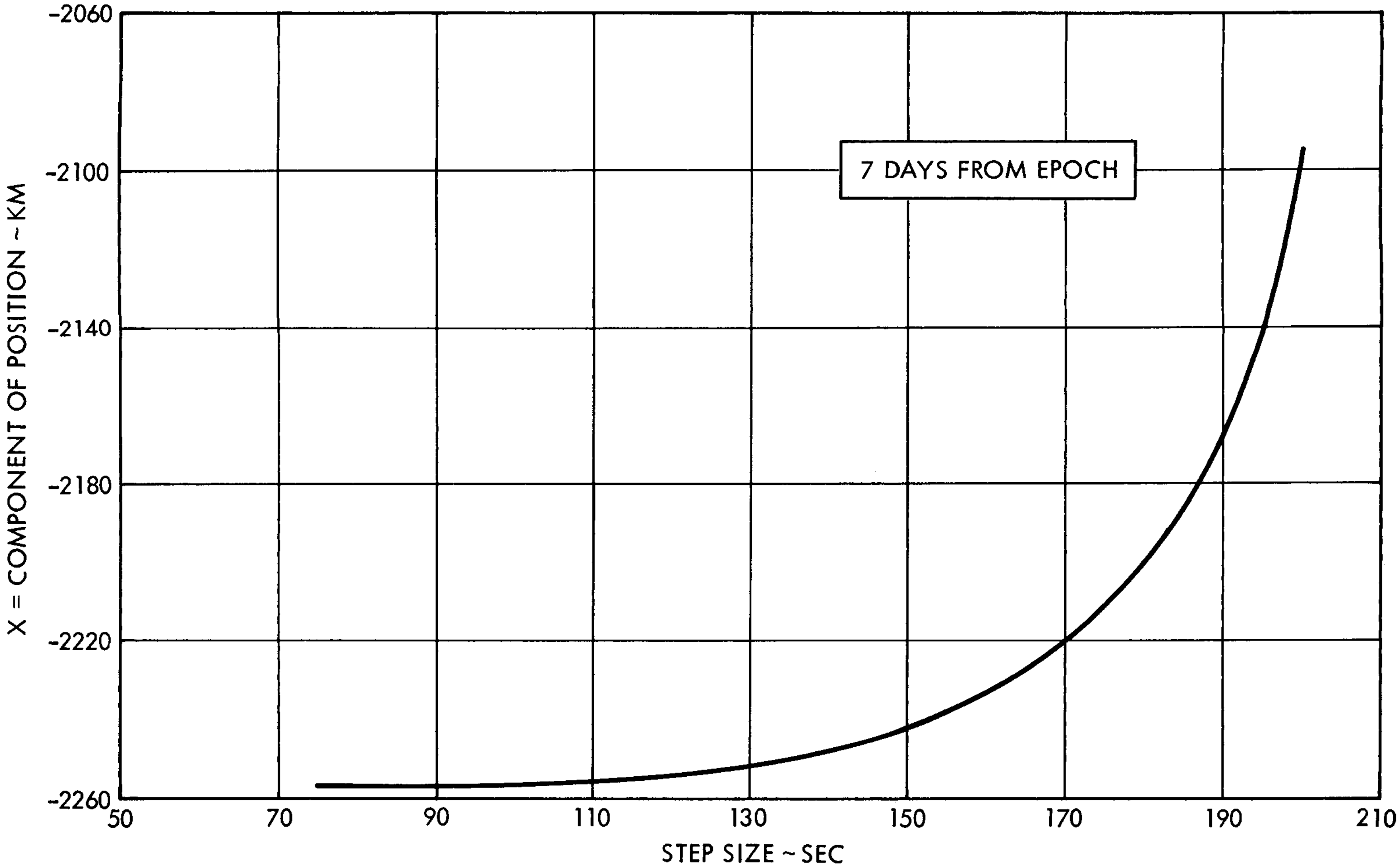

Figure 3-1.—X-Component (Selenocentric) of Position Versus Step Size

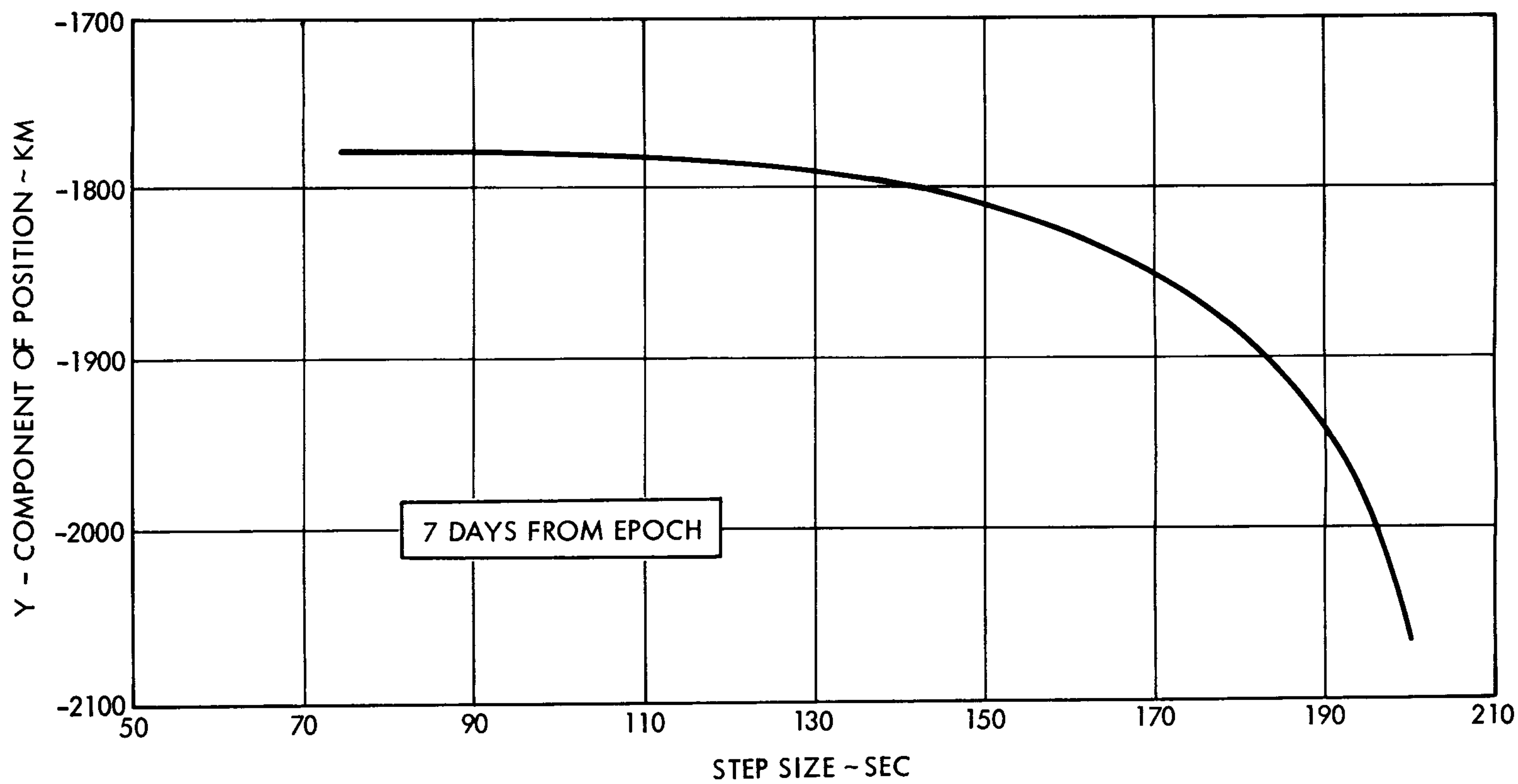

Figure 3-2.—Y-Component (Selenocentric) of Position Versus Step Size

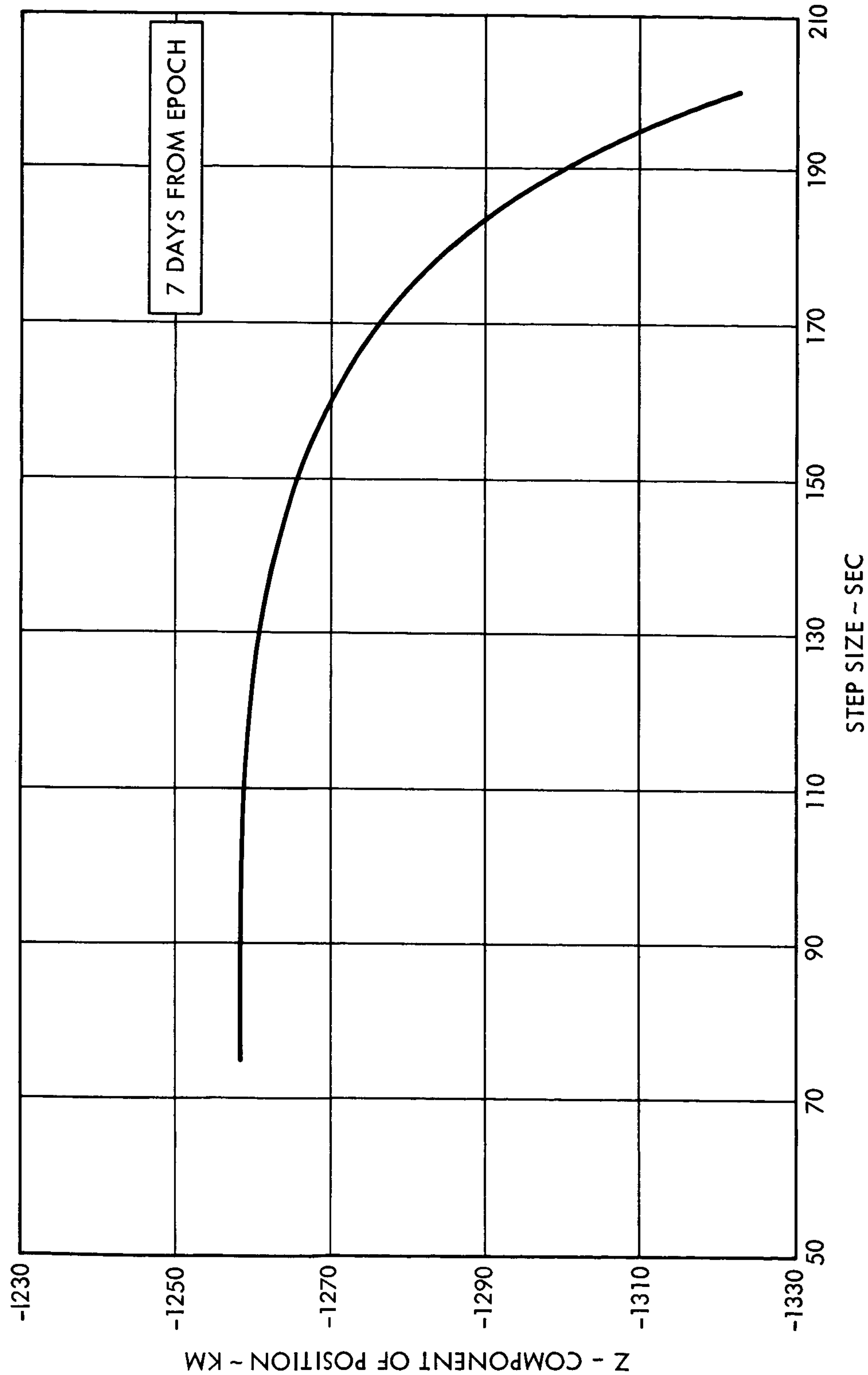

Figure 3-3.—Z-Component (Selenocentric) of Position Versus Step Size

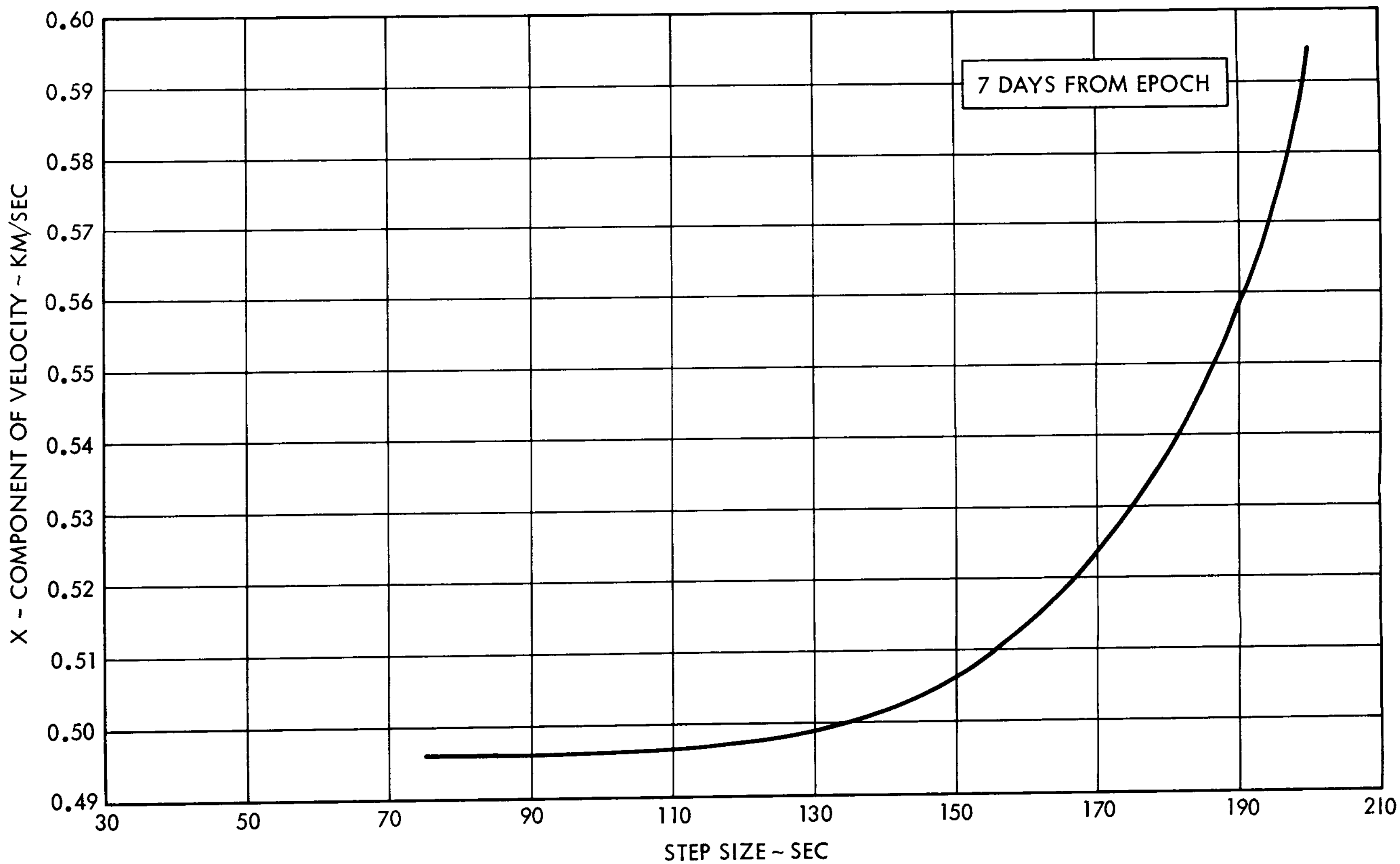

Figure 3-4.—X-Component (Selenocentric) of Velocity Versus Step Size

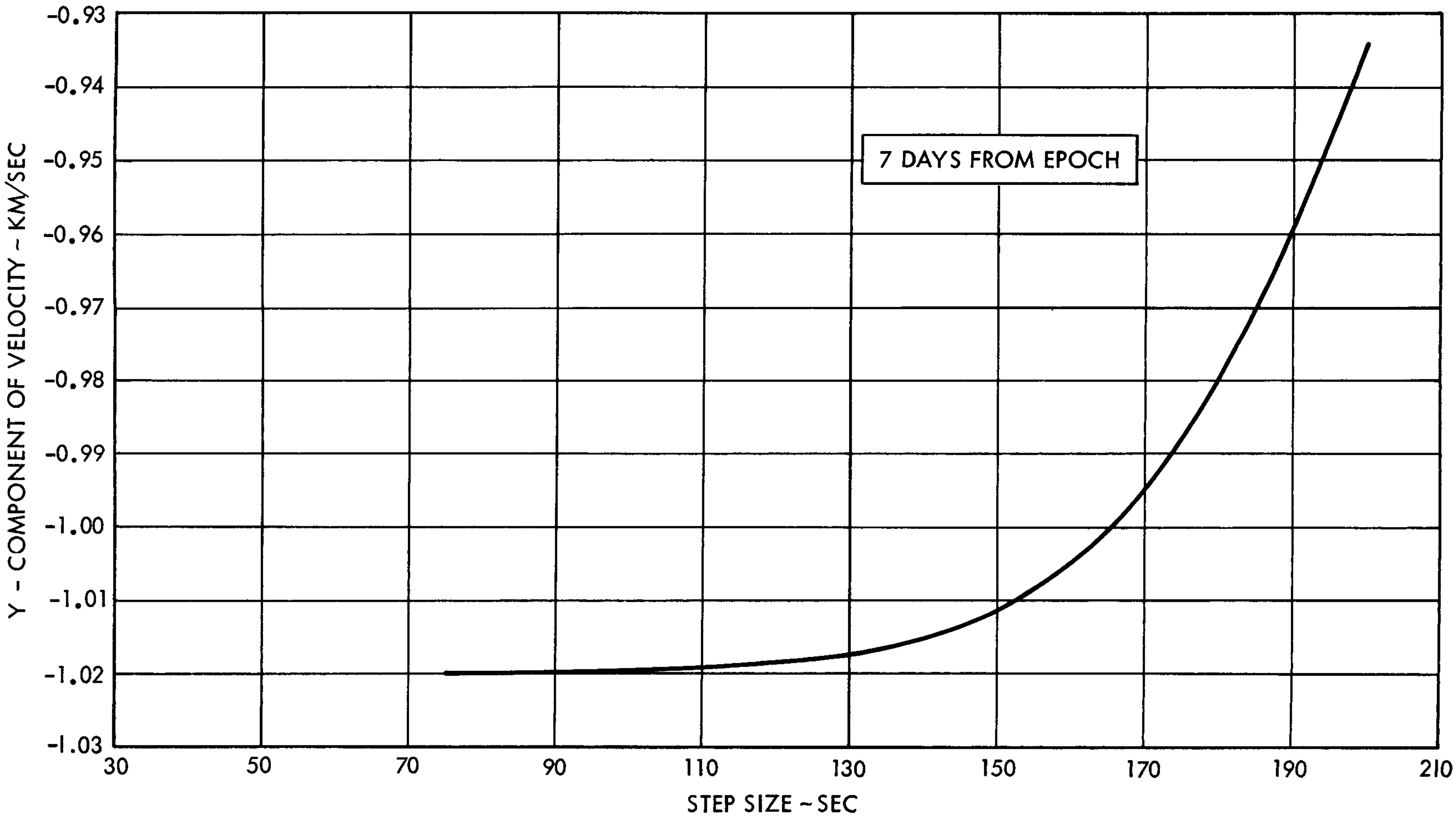

Figure 3-5.—Y-Component (Selenocentric) of Velocity Versus Step Size

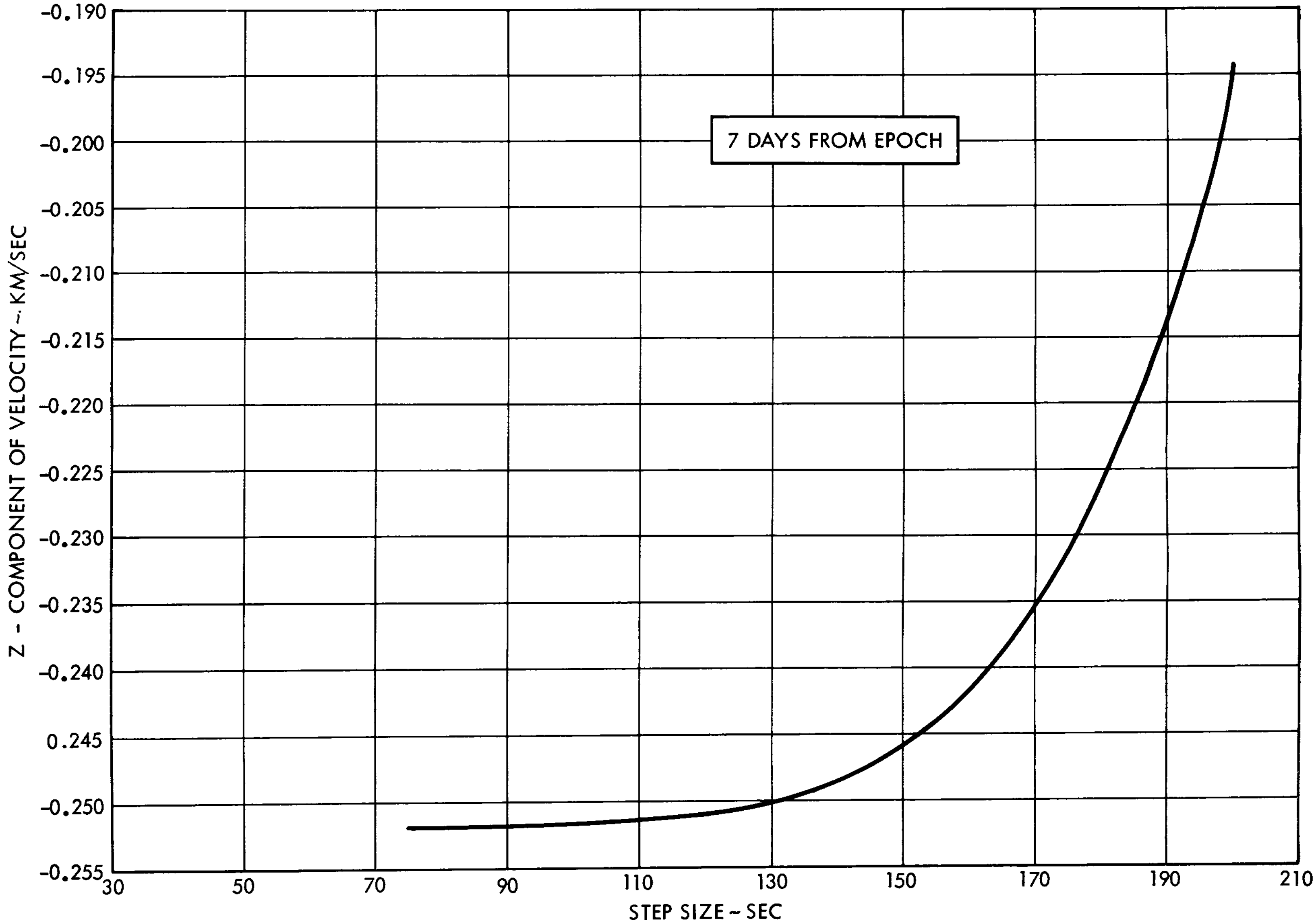

Figure 3-6.—Z-Component (Selenocentric) of Velocity Versus Step Size

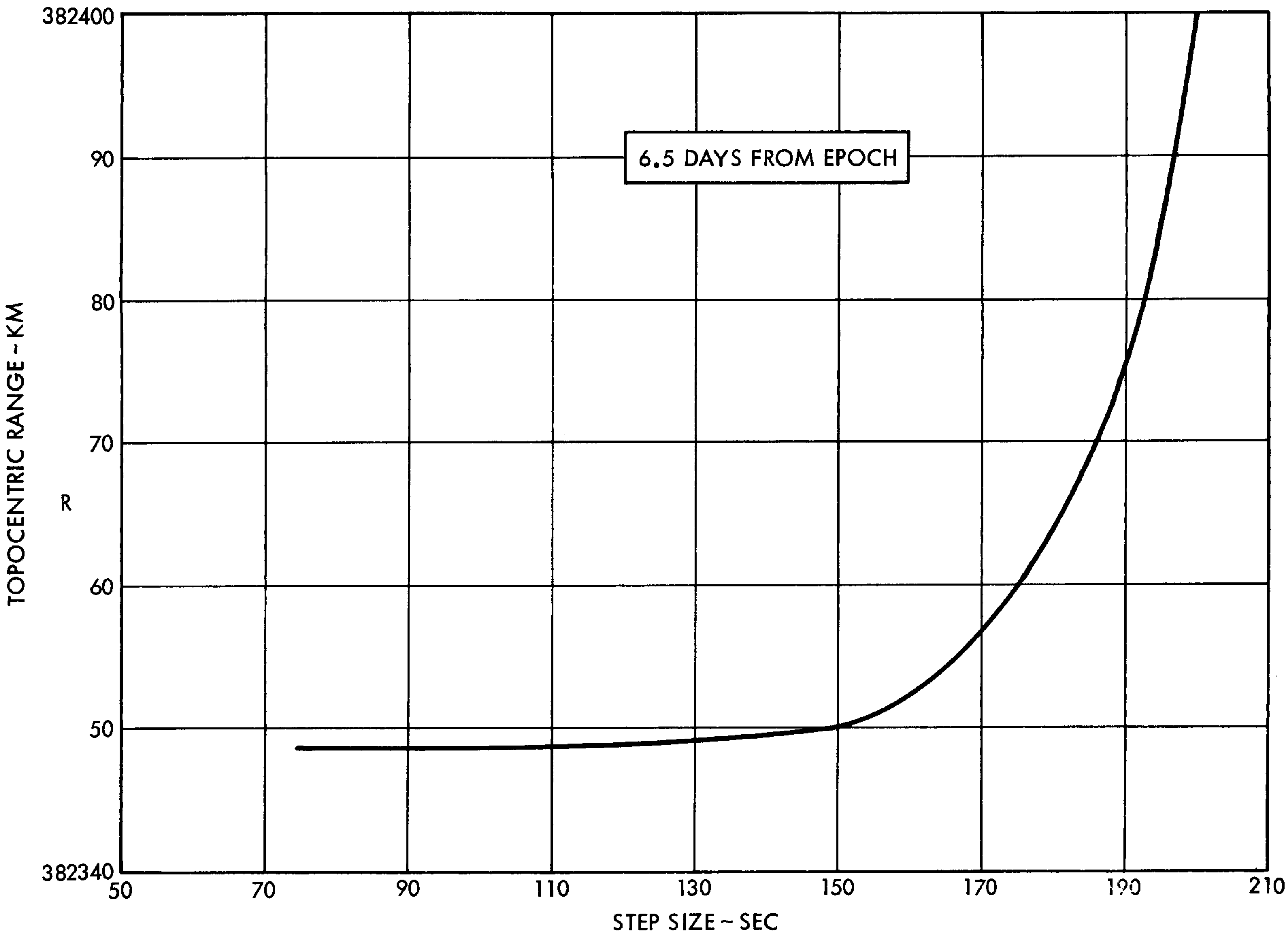

Figure 3-7.—Computed Topocentric Range for Station 11 Versus Step Size

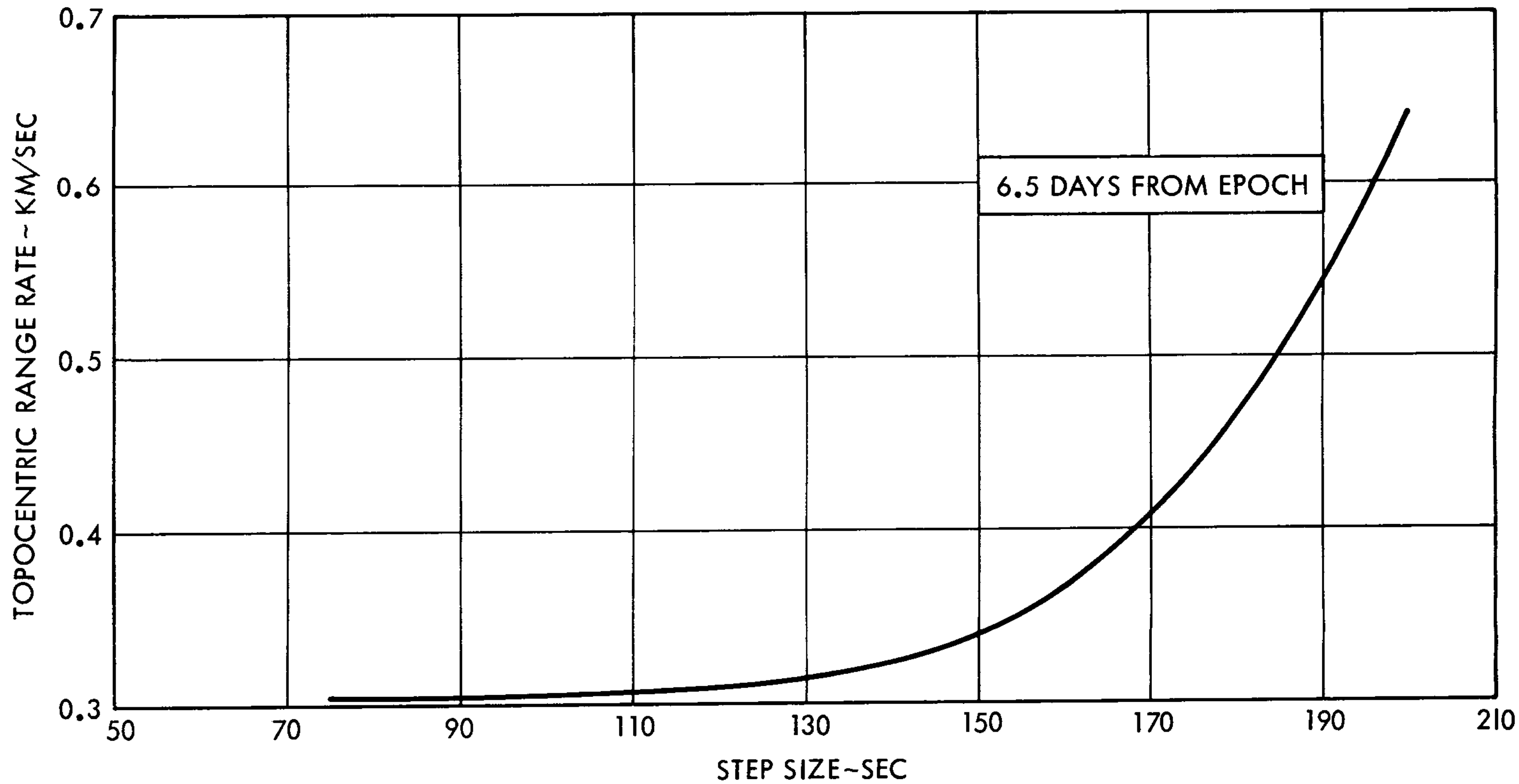

Figure 3-8.—Computed Topocentric Range Rate for Station 11 Versus Step Size

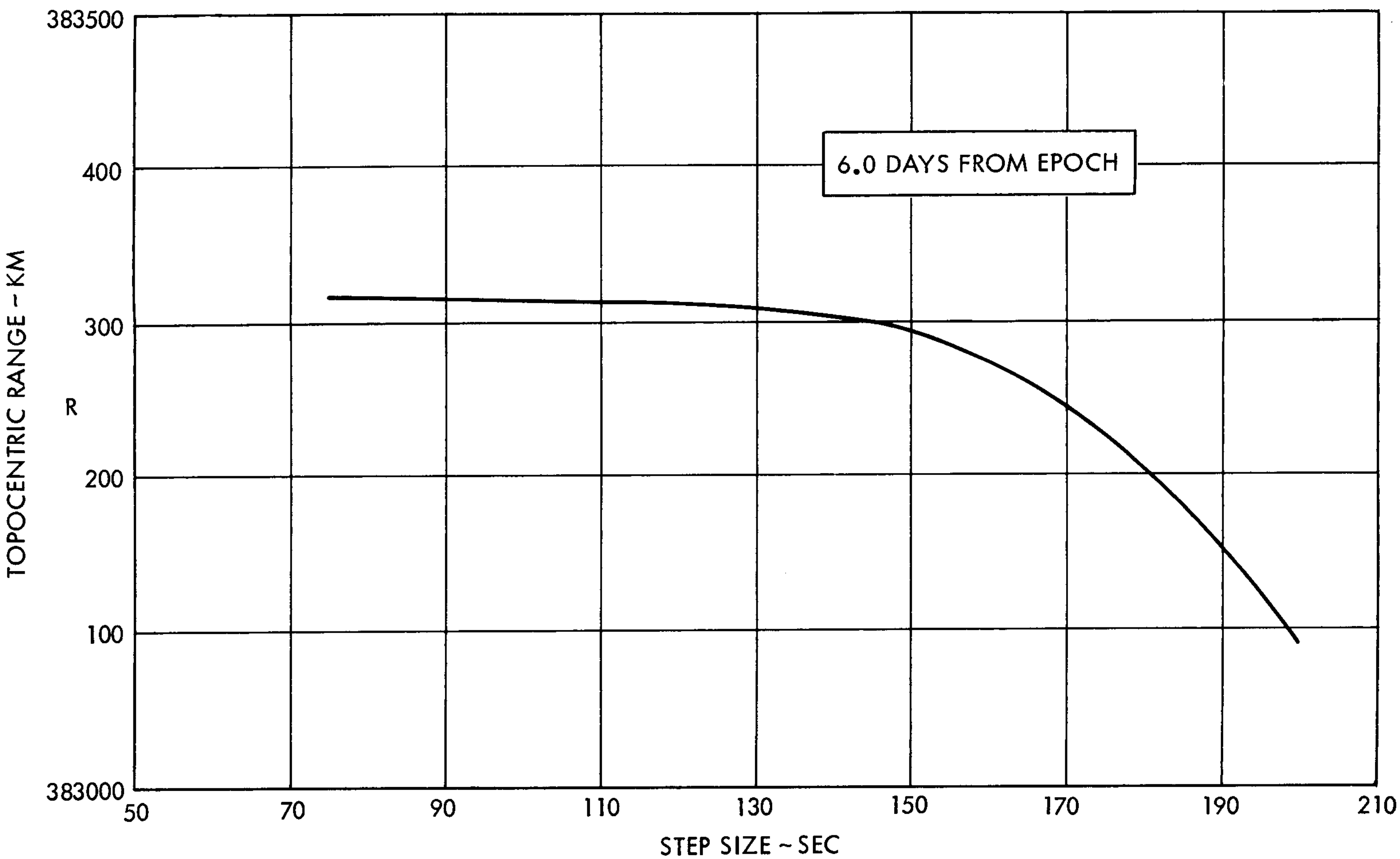

Figure 3-9.—Computed Topocentric Range for Station 42 Versus Step Size

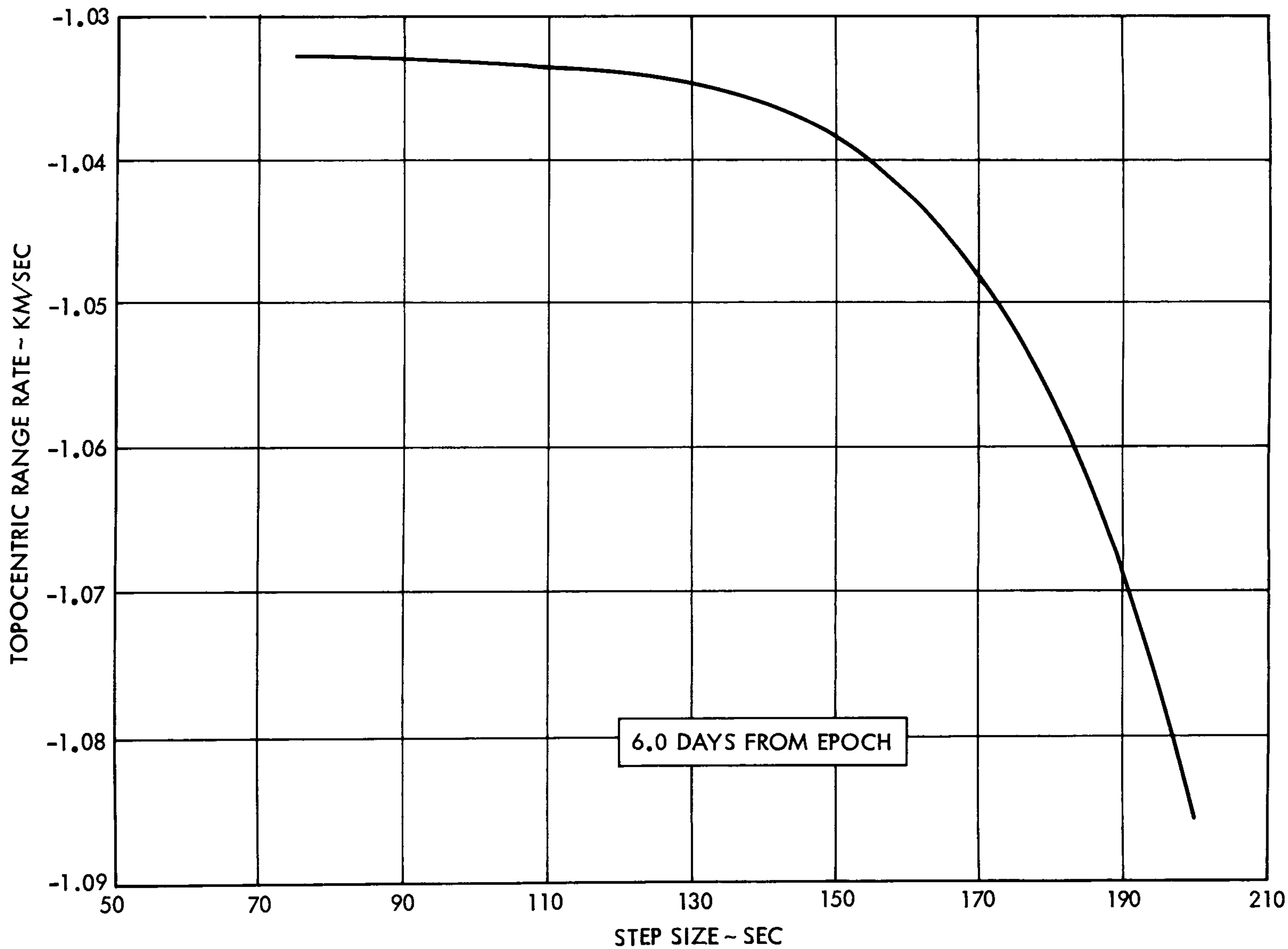

Figure 3-10.—Computed Topocentric Range Rate for Station 42 Versus Step Size

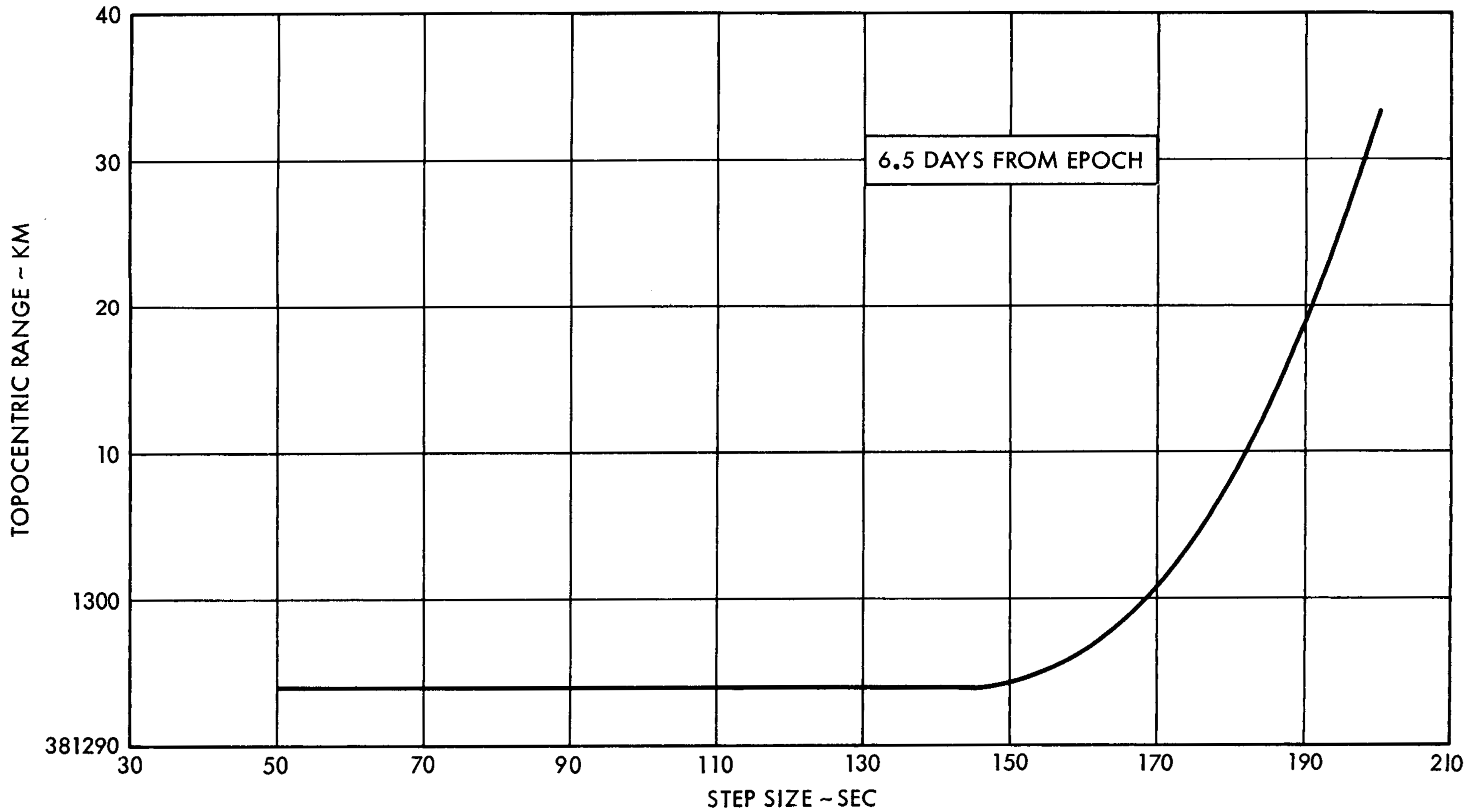

Figure 3-11.—Computed Topocentric Range for Station 41
Versus Step Size

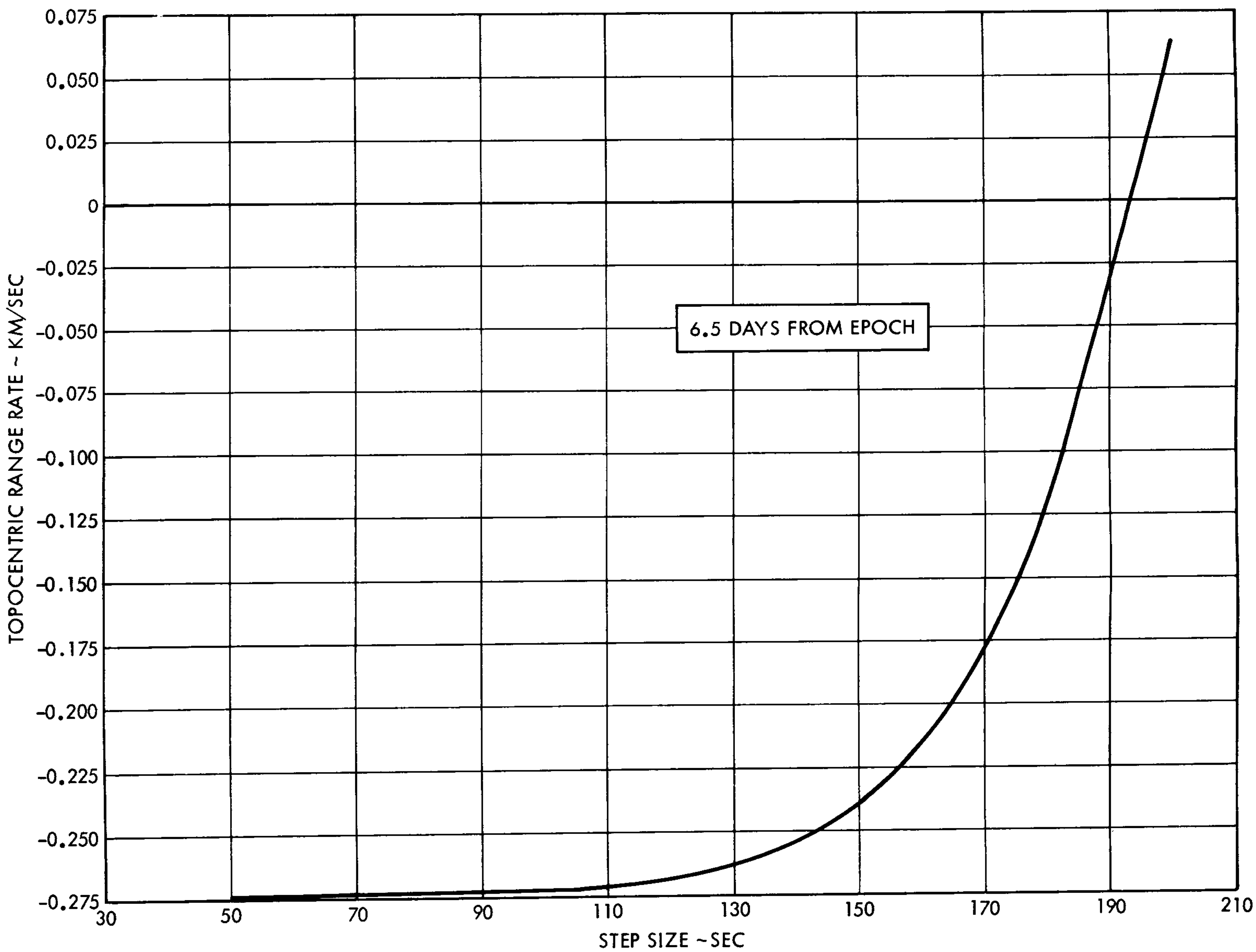

Figure 3-12.—Computed Topocentric Range Rate for Station 41 Versus Step Size

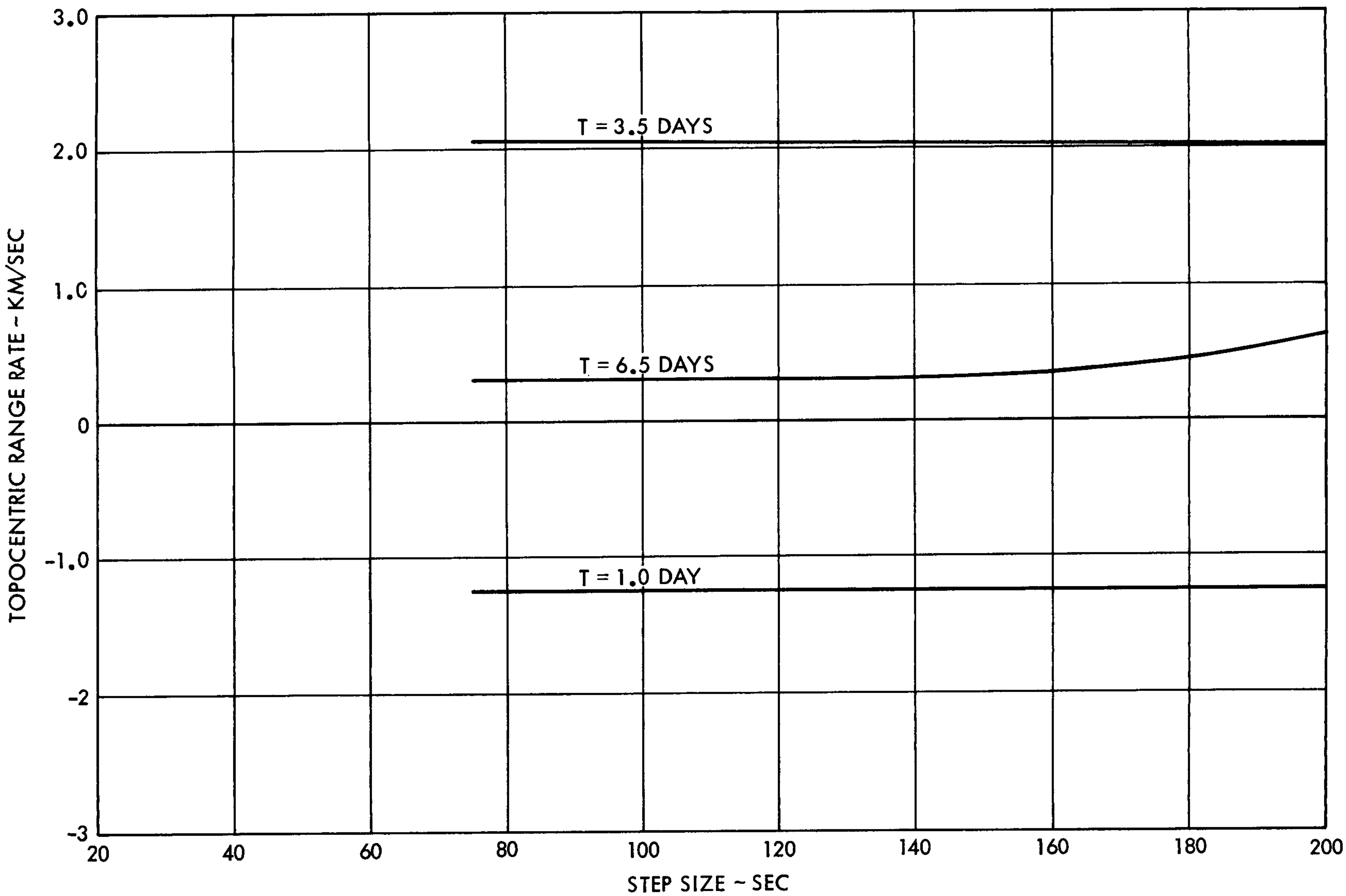

Figure 3-13.—Computed Topocentric Range Rate for Station 11 at Selected Times from Epoch Versus Step Size

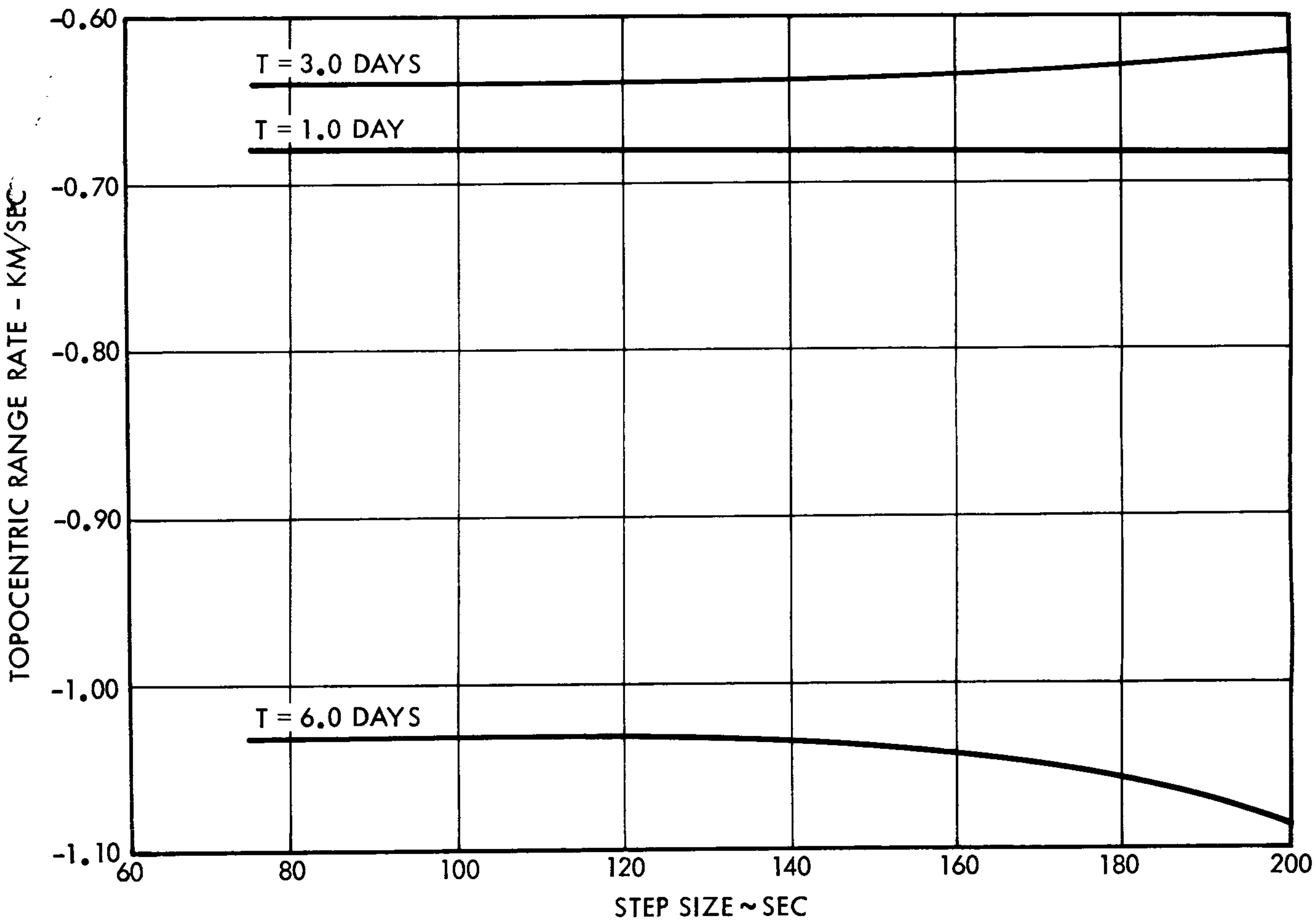

Figure 3-14.—Computed Topocentric Range Rate for Station 42 at Selected Times from Epoch Versus Step Size

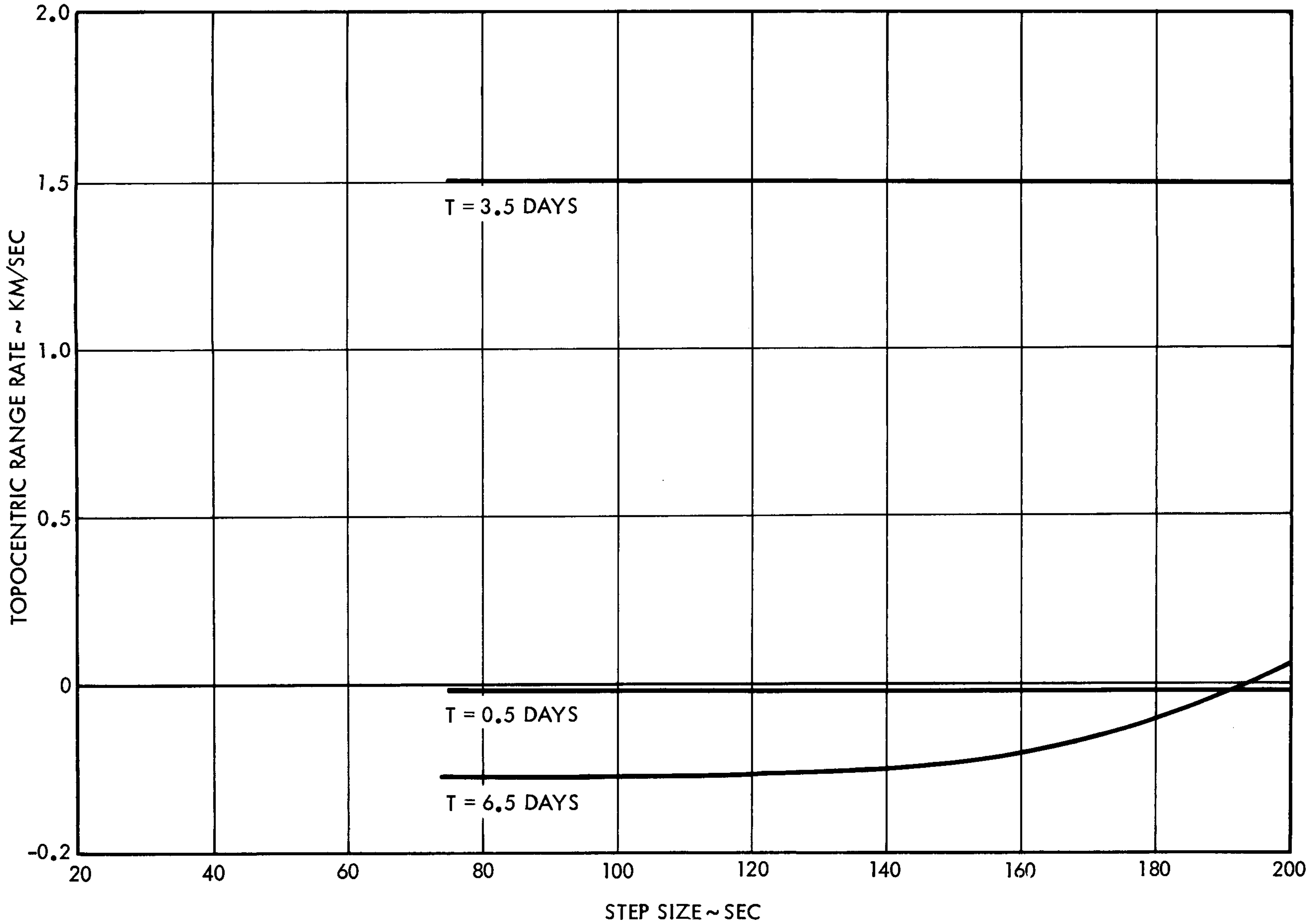

Figure 3-15.—Computed Topocentric Range Rate for Station 41 at Selected Times from Epoch Versus Step Size

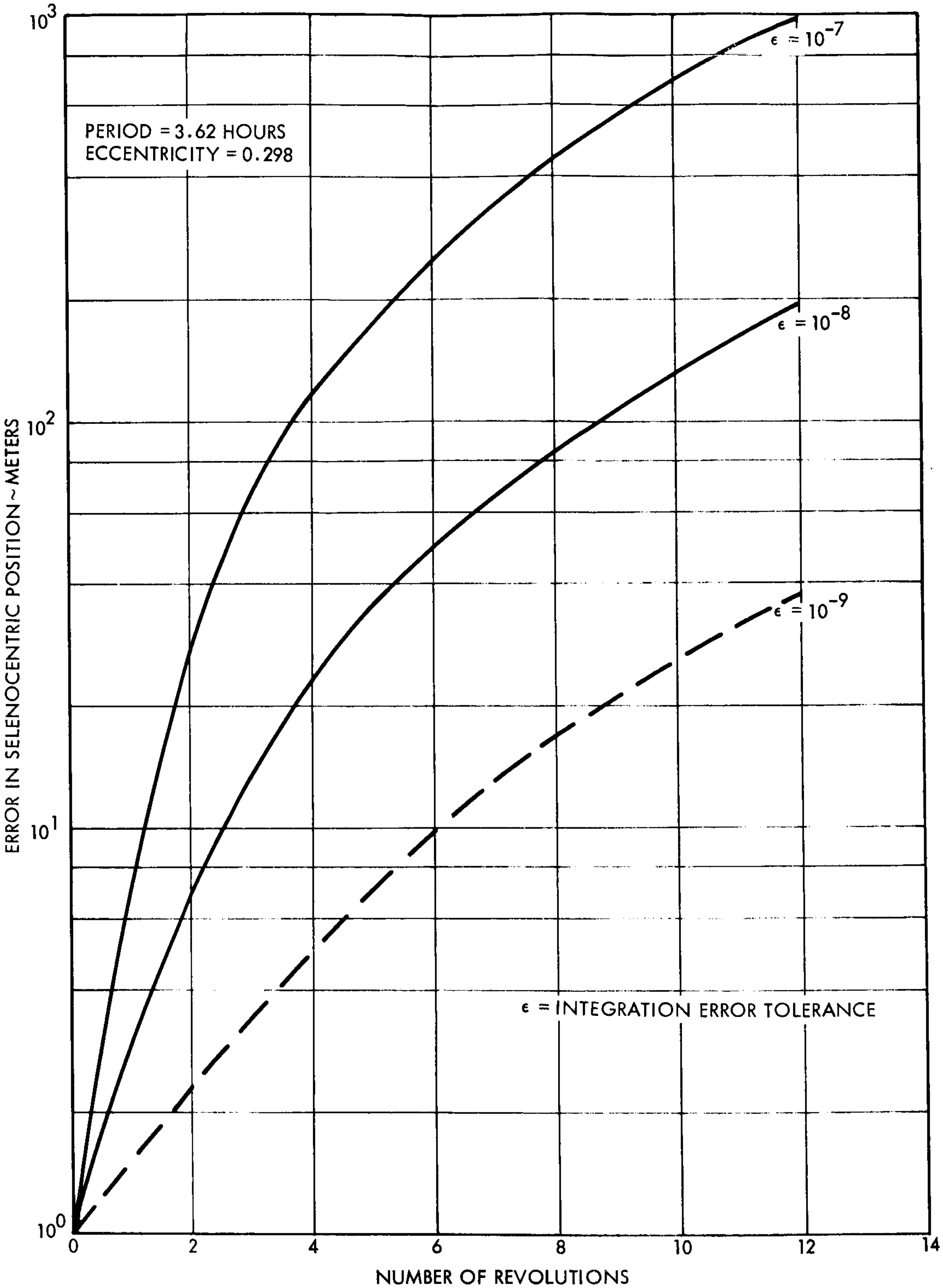

Figure 3-16.—Error Growth in Position in AT-85 Integration Routine (Compared with Analytic Formulation)

4. SENSITIVITY OF SOLUTION VECTOR TO SELENOPOTENTIAL CONSTANTS

To study the sensitivity of the solution vector to considered parameters, the following cases were analyzed:

Case number	Dimension (solve and consider)	Parameters solved	Parameters considered
1	6S + 25C	State	μ, Jn (n=2, . . . , 7), C, Sn, m(n=2, 3, 4; m $\leq$ n)
2	11S + 20C	State, J2, J3, J4, C22, μ	J5, J6, J7, C, Sn, m (n=2, 3, 4; m $\leq$ n), excluding C22
3	17S + 50C	Above plus six most influential considered parameters in Case 2	All remaining parameters through n, m= 7, 7
4	28S + 39C	State, μ, J2, J3, J4, C, Sn, m (n= 2, 3, 4; m $\leq$ n)	All remaining parameters through n, m= 7, 7

All cases were run on the TRW AT-85 program for a tracking interval of 5040 minutes with observations simulated every minute from those stations (of the set Goldstone, Madrid, and Woomera) that can see the satellite. Another set of runs was made for a tracking interval of 10,080 minutes to examine the longer term effects of the potential parameters. These are analyzed under Case 5. For Cases 3 and 4, the variances of the pericynthion and line of nodes are also investigated. The estimated uncertainties in parameters that were considered in the different runs are presented in table 4-1.

TABLE 4-1

ESTIMATED UNCERTAINTIES IN CONSIDERED PARAMETERS ($\times 10^4$)

J2	0.2
J3	0.1
J4	0.5
J5	0.1
J6	0.1
J7	0.01
C21	0.07
S21	0.07
C31	0.2
S31	0.2
C41	0.3
S41	0.3
C22	0.1
S22	0.1
C32	0.06
S32	0.06
C42	0.042
S42	0.042
C33	0.01
S33	0.01
C43	0.05
S43	0.05
C44	0.01
S44	0.01
for $N \geq 5$	
CNM, SNM	$10^{-(N+M+4)/2}$ for N+M even; $10^{-(N+M+5)/2}$ for N+M odd
μ	0.00001

The sensitivity coefficients produced by the runs give the partial derivatives of the solved parameters with respect to the considered parameter. The product of the uncertainty and the sensitivity coefficient gives the sensitivity. The sensitivities are in the same units as the solved variable.

The initial conditions used for the runs are the following:

X	=	2324.1447 km	DX	=	-0.57962222 km/sec
Y	=	90.669095 km	DY	=	1.3494068 km/sec
Z	=	616.74111 km	DZ	=	0.37914168 km/sec

The results of this analysis are valid for these initial conditions only, since the sensitivity coefficients are dependent on the trajectory being used.

The Goudas II coefficients used for the selenopotential are the following:

J20	=	2.048 E-4	C44	=	0.017 E-4
J30	=	-0.98 E-4	S31	=	0.21 E-4
J40	=	-0.48 E-4	S41	=	0.54 E-4
C22	=	0.23 E-4	S33	=	0.018 E-4
C32	=	-0.15 E-4	S43	=	0.032 E-4
C42	=	-0.14 E-4			

In addition, the force model contains perturbations due to the Sun, Earth, Venus, Mars, Saturn, and Jupiter.

4.1 Case 1

The selenopotential constants which are most influential on the state vector are determined by solving for the state and considering the selenopotential parameters through C, S44. The sensitivities are shown in table 4-2.

To find the most influential of the considered parameters, the sensitivities were ranked, forming table 4-3. For example, in the X column of table 4-3, 1 and 2 were placed opposite C41 and S31 because the sensitivities due to them were the two largest in X column of table 4-2.

TABLE 4-2

CASE 1: SENSITIVITIES

	X (m)	Y (m)	Z (m)	$\dot{X}$ (m/sec) $\times 10^{-4}$	$\dot{Y}$ (m/sec) $\times 10^{-4}$	$\dot{Z}$ (m/sec) $\times 10^{-4}$
J_2	830	61.8	-224	1730	-2,440	-5,820
J_3	26.2	-193	-361	498	374	1,040
J_4	-2,330	-152	612	-4,870	6,610	16,500
J_5	-19.8	224	406	-546	-464	-1,250
J_6	378	22.2	-108	799	-1,040	-2,720
J_7	0.610	-18.7	-33.5	42.7	42.1	110
C_{21}	201	-254	157	411	-2,140	3,110
S_{21}	175	380	-732	-934	-754	1,350
C_{31}	119	2,690	921	-7,190	-4,390	-4,960
S_{31}	-2,620	462	333	-5,930	9,800	10,800
C_{41}	-2,690	1,230	-531	-5,590	16,200	-5,200
S_{41}	-723	-1,710	4,350	4,740	1,640	-6,380
C_{22}	-87.1	1,310	-77.4	-9.45	1,003	-1,840
S_{22}	-117	-264	430	533	233	-412
C_{32}	-148	-1,190	209	3,370	2,170	183
S_{32}	813	-379	146	1,890	-4,410	185
C_{42}	1,090	-536	126	2,550	-5,670	370
S_{42}	244	1,430	-618	-3,770	-2,090	-920
C_{33}	17.7	78.6	-44.8	-145	-137	38.7
S_{33}	-47.5	32.4	-15.8	-118	216	-143
C_{43}	1,440	846	-275	-3,320	7,970	-1,620
S_{43}	-350	-2,250	616	6,170	3,500	1,350
C_{44}	111	-101	39.3	661	-374	262
S_{44}	46.6	139	-109	-177	-755	588
μ	-5.18	29.0	-37.5	-27.1	-32.3	-181

TABLE 4-3

CASE 1: RANKING OF SENSITIVITIES

	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$	Cumulative rank	Final rank
J2	6	21	13	12	9	4	65	10
J3	21	15	10	18	20	14	98	18
J4	3	16	6	5	4	1	35	5
J5	22	14	9	16	18	13	92	17
J6	9	24	19	14	14	8	88	15
J7	25	25	24	23	24	24	145	24
C21	12	13	15	19	11	7	77	14
S21	13	10	3	13	17	12	68	11
C31	15	1	2	1	7	6	32	3
S31	2	9	11	3	2	2	29	2
C41	1	5	7	4	1	5	23	1
S41	8	3	1	6	13	3	34	4
C22	18	18	20	25	15	9	105	19
S22	16	12	8	17	21	17	91	16
C32	14	6	14	8	10	22	74	13
S32	7	11	16	11	6	20	71	12
C42	5	8	17	10	5	18	63	9
S42	11	4	4	7	12	15	53	8
C33	23	20	21	21	23	25	133	22
S33	19	22	25	22	22	23	133	22
C43	4	7	12	9	3	10	45	7
S43	10	2	5	2	8	11	38	6
C44	17	19	22	15	19	19	111	21
S44	20	17	18	20	16	16	107	20
μ	24	23	23	24	25	21	140	23

All the selenopotential parameters were ranked in this manner for each column in the sensitivity table. Each row of table 4-3 was summed to form the cumulative rank column and this column itself was ranked to give the final rank column.

The five selenopotential constants contributing the greatest change to the state vector are C41, S31, C31, S41, and J4.

4.2 Case 2

In Case 2, the parameters J2, J3, J4, C22 and μ are included in the solution vector along with the state. The remaining parameters through degree 4 and J5, J6, J7 are considered for their effect on the solution vector. The resulting sensitivities are listed in table 4-4 and ranked in table 4-5. The ranking is done for the effects on the state only (columns 1 and 2), for the effects on J2, J3, J4, C22 and μ (columns 3 and 4), and for the entire solution vector (columns 1 + 3). The most influential of the considered parameters are S41, C31, S43, S42, C32, and S21, respectively.

4.3 Case 3

The six most influential of the considered parameters in Case 2 (S41, C31, S43, S42, C32, S21) are added to the solution vector. All remaining parameters through C, S77 are considered. The resulting sensitivities are listed in table 4-6. Table 4-7 contains the covariance matrices of the state and the ascending node and pericynthion, respectively, in σ - ρ form. The σ - ρ form of the covariance matrix is a lower triangular array (since the covariance matrix is symmetric) of numbers which represent the standard deviations (the square root of the variances) of the solution variables on the diagonal and the correlation coefficients (ρ) off the diagonal.

TABLE 4-4

CASE 2: SENSITIVITIES

	X (m)	Y (m)	Z (m)	$\dot{X}$ (m/sec)	$\dot{Y}$ (m/sec)	$\dot{Z}$ (m/sec)	J2	J3	J4	C22	μ (km^3/sec^2)
J5	1.10	2.59	3.17	0.104-3	0.046-3	0.116-2	0.22-5	0.114-4	0.17-5	0.21-7	0.27-2
J6	0.107	0.137	0.671	0.079-3	0.204-3	0.073-3	1.1-5	0.145-6	1.8-5	0.19-7	0.012
J7	0.216	0.277	0.134	0.247-4	0.198-4	0.219-3	0.35-6	0.94-6	0.26-6	0.44-8	0.36-3
C21	24	55	30	0.183-1	0.049	0.183-1	0.28-3	0.422-5	0.25-3	0.67-5	0.63
S21	114	1710	2900	0.198	4.9	0.23	0.084-2	0.112-4	0.075-2	0.66-6	3.0
C31	116	4420	7900	0.457	1.34	0.427	0.29-2	1.22-4	0.26-2	1.17-5	7.0
S31	34.1	39.9	244	0.271-1	0.171-1	0.223	0.78-3	0.54-5	0.65-3	0.70-5	1.7
C41	110	119.	402	0.701-1	0.271	0.774-1	1.38-3	1.97-5	1.18-3	0.38-4	2.9
S41	344	8230	14000	9.1	2.26	1.28	0.45-2	0.40-4	0.40-2	1.52-5	13
S22	43	1010	1710	0.107	0.266	0.128	0.55-3	0.77-5	0.49-3	0.79-6	1.7
C32	67	.2700	4900	2.7	0.90	0.110	0.196-2	0.55-4	0.175-2	0.089-4	4.1
S32	54	78	570	0.177-1	0.119	0.079-1	0.23-3	0.44-5	0.187-3	0.080-4	1.4
C42	61	27	450	0.023	0.125	0.036	0.213-3	0.152-5	0.170-3	0.148-4	1.4
S42	70	3400	6200	3.5	1.10	0.30	0.242-2	0.060-3	0.216-2	0.105-4	5.1
C33	0.052	207	400	0.165-1	0.079	0.271-2	0.147-3	0.35-5	0.131-3	0.591-6	0.37
S33	0.155	16.2	55	0.091-1	0.034-1	0.241-1	0.33-4	0.13-5	0.30-4	0.175-5	0.020
C43	137	5.8	1700	0.094	0.33	0.32	0.372-3	0.234-4	0.36-3	0.28-4	2.2
S43	11.6	4900	9400	4.9	0.017-2	0.059	0.42-2	0.106-3	0.38-2	0.23-4	6.8
C44	16.5	158	43	0.079	0.271-1	0.098	0.12-3	0.420-5	0.11-3	0.65-5	0.26
S44	12.8	4600	1010	0.265-1	0.265	0.076	0.36-3	0.86-5	0.32-3	0.36-6	1.0

TABLE 4-5

CASE 2: RANKING OF SENSITIVITIES

	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$	Cum. rank 1	Rank 2	J2	J3	J4	C22	μ	Cum. rank 3	Rank 4	1 + 3	Final rank 1 + 3
J5	16	18	18	18	19	18	107	18	19	8	19	18	19	83	17	190	18
J6	19	20	19	19	18	20	115	19	18	20	18	19	18	93	19	208	19
J7	17	19	20	20	20	19	115	20	20	19	20	20	20	99	20	214	20
C21	12	13	17	14	14	15	85	15	12	14	12	11	14	63	14	148	15
S21	4	6	6	6	6	5	33	4	7	9	7	16	6	45	8	78	6
C31	3	3	3	3	3	2	17	2	3	1	3	6	2	15	3	32	2
S31	11	14	14	11	16	6	72	13	8	12	8	10	9	47	9	119	10
C41	5	11	12	10	8	10	56	9	6	7	6	1	7	27	6	83	8
S41	1	1	1	1	1	1	6	1	1	5	1	4	1	12	1	18	1
S22	10	7	7	7	9	7	47	8	9	11	9	15	10	54	10	101	9
C32	7	5	5	5	5	8	35	6	5	4	5	8	5	27	5	62	5
S32	9	12	10	15	12	16	74	14	13	13	13	9	11	59	12	133	12
C42	8	15	11	13	11	13	71	11	14	17	14	5	12	62	13	133	13
S42	6	4	4	4	4	4	26	3	4	3	4	7	4	22	4	48	4
C33	20	9	13	16	13	17	88	16	15	16	15	17	15	78	16	166	16
S33	18	16	15	17	17	14	97	17	17	18	17	14	17	83	18	180	17
C43	2	17	8	8	7	3	45	7	10	6	10	2	8	36	7	81	7
S43	15	2	2	2	2	12	35	5	2	2	2	3	3	12	2	47	3
C44	13	10	16	9	15	9	72	12	16	15	16	12	16	75	15	147	14
S44	14	8	9	12	10	11	64	10	11	10	11	13	13	58	11	122	11

	X (m)	Y (m)	Z (m)	$\dot{X}$ (m/sec)	$\dot{Y}$ (m/sec)	$\dot{Z}$ (m/sec)	J2	J3	J4
J5	0.525	5.344	6.418	0.4537-3	0.9925-4	0.13032-2	0.905236-6	0.113613-4	0.603271-6
J6	0.386	1.250	0.8021	0.1697-3	0.2349-3	0.9081-4	0.102305-4	0.239677-6	0.172622-4
J7	0.102	0.8064	0.6652	0.9005-3	0.5975-4	0.2916-3	0.166251-6	0.933435-6	0.109471-6
C21	77.8	21.15	19.56	0.2823-3	0.07587	0.1126	0.395104-3	0.479589-4	0.348248-3
S31	86.05	983.2	348.1	0.1698	0.2210	0.9128	0.653424-3	0.830535-4	0.523005-3
C41	426.6	48.04	186.7	0.0414	0.3453	0.4254	0.220574-2	0.195207-3	0.190977-2
C51	7.402	3.424	14.85	0.9392-3	0.2082-2	0.6209-4	0.940032-5	0.638368-5	0.741790-5
S51	65.78	454.5	197.3	0.07519	0.1169	0.4488	0.366036-3	0.328484-4	0.280992-3
C61	14.07	11.45	7.544	0.3486-2	0.7706-2	0.3470-2	0.655678-4	0.395396-5	0.555262-4
S61	7.093	1.854	0.2061	0.3190-3	0.1331-2	0.2886-2	0.131497-5	0.975073-6	0.111845-5
C71	0.9161	2.199	3.538	0.2461-3	0.1190-2	0.2741-2	0.122904-5	0.164360-5	0.957483-6
S71	7.845	25.22	15.98	0.3619-2	0.8391-2	0.02854	0.269531-4	0.115653-5	0.190522-4
S22	21.39	59.30	142.4	0.6483-2	0.01193	0.2932-2	0.795175-4	0.271816-5	0.689797-4
S32	131.4	1016.0	586.1	0.1755	0.1409	0.8626	0.410672-3	0.654159-4	0.344908-3
C42	161.2	960.5	517.2	0.1559	0.1855	0.9084	0.369709-3	0.628362-4	0.303122-3
C52	6.532	5.804	0.0646	0.1846-3	0.1180-3	0.4421-2	0.465159-5	0.432247-5	0.389271-5
S52	72.15	522.7	323.16	0.0840	0.07225	0.45279	0.217006-3	0.207547-4	0.178610-3
C62	95.43	547.6	346.9	0.0857	0.09172	0.5064	0.214605-3	0.131934-4	0.171268-3
S62	8.470	8.797	18.96	0.1828-2	0.1429-2	0.4703-2	0.211179-4	0.405813-5	0.181498-4
C72	1.528	0.6611	1.181	0.2033-3	0.1391-2	0.1369-2	0.523474-6	0.128670-5	0.389826-6
S72	12.81	77.80	54.94	0.0120	0.0100	0.06794	0.335648-4	0.488710-6	0.268894-4
C33	7.320	0.2418	23.42	0.3555-2	0.6736-2	0.02808	0.500791-4	0.103368-4	0.455604-4
S33	4.783	75.35	33.62	0.01459	0.02661	0.10982	0.775215-4	0.262930-5	0.701084-4
C43	284.4	2806.	1688.	0.4440	0.2523	2.282	0.176178-3	0.502623-4	0.115557-3
C53	5.376	180.3	2.547	0.03544	0.05890	0.2720	0.233553-3	0.158332-4	0.210690-3
S53	188.7	1641.	956.2	0.2654	0.1742	1.406	0.283924-4	0.916115-5	0.134803-5
C63	40.55	377.7	245.7	0.06012	0.02998	0.3044	0.340589-4	0.274579-5	0.235966-4
S63	4.292	20.53	3.595	0.2622-2	0.5154-2	0.02832	0.199871-4	0.181019-5	0.177297-4
C73	9.060	65.09	16.76	0.9666-2	0.01432	0.08311	0.688102-4	0.668108-5	0.615163-4
S73	75.73	627.0	433.4	0.09811	0.04769	0.5078	0.535517-4	0.187009-4	0.353581-4
C44	41.77	70.46	72.59	0.01938	0.02142	0.01920	0.238932-3	0.118210-3	0.211130-3
S44	46.36	432.6	342.3	0.07182	0.04752	0.5253	0.353957-3	0.389359-4	0.322507-3
C54	0.1544	11.008	18.49	0.11102-2	0.9575-2	0.03512	0.546997-4	0.124742-5	0.491020-4
S54	22.23	256.2	157.1	0.03791	0.01728	0.21168	0.383677-4	0.751335-5	0.369240-4
C64	83.53	857.6	587.2	0.1322	0.02559	0.6333	0.557851-4	0.617564-4	0.600443-4
S64	2.629	246.6	120.8	0.0399	0.0494	0.3116	0.22444-3	0.260031-5	0.202477-3
C74	2.250	38.26	10.25	0.5149-2	0.9428-2	0.0522	0.414736-4	0.216721-5	0.371999-4
S74	24.72	253.0	180.3	0.0388	0.8889-2	0.1916	0.114644-4	0.125214-4	0.135655-4
C55	32.06	271.6	229.2	0.05504	0.02314	0.3112	0.157145-3	0.182806-4	0.144171-3
S55	22.75	267.7	338.0	0.03002	0.5523-2	0.2191	0.130738-3	0.653671-4	0.113507-3
C65	5.032	105.8	84.42	0.01559	0.6123-2	0.0649	0.135102-4	0.160903-4	0.127202-4
S65	1.537	41.57	18.05	0.3635-2	0.9650-2	0.0602	0.541206-4	0.797766-6	0.486803-4
C75	6.476	204.3	141.1	0.03013	0.02908	0.2368	0.155847-3	0.688706-5	0.140751-3
S75	43.66	378.9	285.5	0.0540	0.01485	0.2361	0.395032-4	0.559061-4	0.389920-4
C66	8.939	286.0	325.1	0.0436	0.6313-2	0.2133	0.688190-4	0.163532-4	0.58952-4
S66	20.57	161.3	202.5	0.03971	0.2326-2	0.1308	0.399305-4	0.433895-5	0.378664-4
C76	0.6139	38.65	28.72	0.5073-2	0.2404-2	0.0441	0.363405-4	0.588277-5	0.327717-4
S76	0.4670	8.974	9.475	0.3841-2	0.01086	0.01520	0.288644-5	0.122263-4	0.246018-5
C77	40.27	59.42	102.0	0.01692	0.9977-2	0.0434	0.585623-4	0.455528-4	0.513057-4
S77	23.46	187.8	237.0	0.0450	0.02589	0.0813	0.749821-4	0.929979-5	0.658013-4

TABLE 4-6

CASE 3: SENSITIVITIES

S21	C31	S41	C22	C32	S42	S43	μ (km^3/sec^2)
0.166034-6	0.607312-7	0.190405-6	0.222297-7	0.721408-8	0.200931-7	0.279504-8	0.291092-2
0.101853-8	0.126775-7	0.207425-7	0.851008-8	0.467583-8	0.206770-8	0.561781-8	0.912347-2
0.324850-7	0.139828-7	0.384065-7	0.364367-8	0.284551-8	0.453250-8	0.232686-9	0.355129-3
0.108484-4	0.122906-4	0.164835-4	0.286271-5	0.221632-6	0.580228-6	0.560224-6	0.975583
0.428668-4	0.819566-5	0.610224-4	0.152593-6	0.165676-4	0.842369-6	0.477328-6	1.34955
0.533078-4	0.572658-4	0.715477-4	0.158032-4	0.323682-5	0.225181-5	0.291954-5	5.192546
0.102783-4	0.123871-4	0.859442-5	0.212434-6	0.703613-6	0.250819-6	0.482315-7	0.0174667
0.221518-4	0.468251-5	0.294076-4	0.824516-6	0.766178-5	0.212811-6	0.548308-7	0.804366
0.136904-5	0.163920-5	0.146382-5	0.557340-6	0.360262-6	0.416043-7	0.919665-7	0.151177
0.122627-5	0.175066-6	0.189098-5	0.123321-7	0.437641-7	0.205931-7	0.306415-8	0.186894-2
0.1919537-5	0.122580-5	0.159284-5	0.665713-7	0.143268-6	0.430721-7	0.146249-7	0.380301-2
0.159103-5	0.331847-6	0.182958-5	0.186751-6	0.421943-6	0.543829-9	0.203640-7	0.0664830
0.114413-4	0.103852-5	0.807373-5	0.190576-5	0.698069-5	0.339058-5	0.947259-6	0.415898-2
0.324718-4	0.386316-5	0.44932-4	0.155894-5	0.108384-4	0.193056-5	0.934755-6	0.556185
0.334438-4	0.606093-5	0.467052-4	0.773617-5	0.129881-4	0.108649-5	0.116560-7	0.754523
0.582005-5	0.136034-6	0.540564-5	0.834267-7	0.245444-5	0.269516-6	0.538912-7	0.020125
0.173944-4	0.554010-5	0.219458-4	0.966264-6	0.561621-5	0.505011-6	0.411865-6	0.285284
0.188299-4	0.854041-5	0.227203-4	0.25035-5	0.70768-5	0.332015-6	0.151741-6	0.356261
0.102178-4	0.445018-8	0.8591197-5	0.364784-6	0.122761-5	0.18265-5	0.1828106-6	0.0375451
0.207656-5	0.114358-7	0.19529-5	0.1193488-7	0.272949-6	0.602195-7	0.157331-7	0.0854416
0.270568-5	0.140859-5	0.29442-5	0.208276-6	0.766198-6	0.344119-8	0.545810-7	0.4053801
0.429215-5	0.163104-6	0.728025-5	0.532788-6	0.188227-5	0.254424-5	0.51239-7	0.028834
0.473914-6	0.494332-5	0.160471-5	0.203921-5	0.368356-5	0.145342-6	0.402753-6	0.047047
0.687524-4	0.427659-4	0.878480-4	0.148962-4	0.270499-4	0.797018-6	0.263964-5	0.694184
0.912344-6	0.145396-4	0.162309-4	0.129599-5	0.722793-5	0.913709-5	0.511932-6	0.315265
0.360065-4	0.380396-4	0.417674-4	0.750626-5	0.236350-4	0.220495-5	0.856322-6	0.382032
0.943562-5	0.833482-5	0.105997-4	0.840265-6	0.396078-5	0.135642-7	0.338854-6	0.0939253
0.311341-5	0.162380-5	0.164272-5	0.357503-7	0.137341-5	0.452818-6	0.25997-6	0.04080307
0.739175-5	0.621560-5	0.32463-5	0.169131-6	0.363422-5	0.210351-5	0.974715-7	0.139295
0.146862-4	0.169688-4	0.140375-4	0.138559-5	0.739913-5	0.454040-6	0.575521-6	0.174392
0.203529-4	0.308831-4	0.395724-4	0.131865-4	0.156820-4	0.832737-5	0.895648-6	0.523625
0.237965-4	0.292190-4	0.540088-4	0.173048-5	0.637457-5	0.212357-4	0.476761-5	0.820158
0.957877-6	0.200615-5	0.242002-5	0.963630-6	0.608029-7	0.996324-6	0.367791-6	0.64999
0.360296-5	0.793766-5	0.454659-5	0.134828-5	0.418644-5	0.295236-6	0.221261-6	0.187975
0.106298-4	0.297213-4	0.308544-5	0.457145-5	0.127667-4	0.344863-5	0.127222-5	0.606161
0.252042-5	0.188891-4	0.15592-4	0.109854-5	0.816247-5	0.714766-5	0.850266-6	0.456508
0.115540-5	0.310347-5	0.472025-6	0.215171-6	0.126756-5	0.730927-6	0.167360-6	0.078207
0.452099-5	0.809618-5	0.39067-5	0.585850-6	0.301900-5	0.894489-7	0.134151-6	0.1721512
0.123260-4	0.159624-4	0.27921-4	0.108870-5	0.407824-5	0.108816-4	0.26935-5	0.416631
0.141786-4	0.521214-5	0.329330-4	0.75935-5	0.226990-5	0.943283-5	0.266974-5	0.0634383
0.800172-6	0.544195-5	0.287618-5	0.89079-6	0.238590-5	0.708370-6	0.503910-7	0.128865
0.882234-6	0.385072-5	0.296113-5	0.705170-6	0.123994-5	0.137062-5	0.174284-6	0.102734
0.2297308-5	0.157041-4	0.986430-5	0.755837-6	0.604292-5	0.455976-5	0.593411-6	0.396076
0.123561-5	0.165869-4	0.896684-5	0.306676-5	0.555765-5	0.388363-5	0.135214-5	0.372345
0.644420-5	0.622047-5	0.170835-4	0.514269-5	0.177404-5	0.694254-5	0.224341-5	0.260204
0.714228-5	0.104083-4	0.150293-4	0.631459-6	0.276251-5	0.595050-5	0.154233-5	0.275471
0.794562-7	0.390502-5	0.747663-6	0.167948-6	0.101926-5	0.914154-6	0.140272-6	0.094135
0.147209-5	0.183223-5	0.444808-5	0.89838-6	0.614633-6	0.131612-5	0.366362-6	0.0390896
0.900845-5	0.532909-5	0.188745-4	0.201772-6	0.986730-6	0.276343-5	0.112115-5	0.0599722
0.467010-5	0.526092-5	0.112419-4	0.187085-5	0.297129-5	0.397632-5	0.160095-5	0.237826

TABLE 4-7a

CASE 3: COVARIANCE MATRIX OF STATE IN σ - ρ FORM (σ in m, m/sec)

	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$
X	2.48					
Y	-0.308	18.8				
Z	0.442	-0.914	33.28			
$\dot{X}$	0.239	-0.850	0.625	0.250-2		
$\dot{Y}$	0.531	-0.643	0.876	0.339	0.671-2	
$\dot{Z}$	-0.337	-0.417	0.0438	0.570	-0.414	0.0118

TABLE 4-7b

CASE 3: COVARIANCE MATRIX OF ASCENDING NODE (Ω) AND PERICYNTHION (r_p) IN σ - ρ FORM (σ in deg and m)

	Ω	r_p
Ω	0.00283	
r_p	-0.853	17.38

4.4 Case 4

In Case 4, all selenopotential constants through degree 4 are included in the solution vector in addition to the state and μ. All remaining parameters through C, S77 are considered for their effects on the solution vector. Table 4-8 contains the sensitivities of the solution parameters due to the considered parameters. Table 4-9 contains the covariance matrices for the state and the ascending node and pericynthion, respectively, in σ - ρ form. The standard deviations, σ, are larger for Case 4 than for Case 3 because more variables are included in the solution vector in Case 4.

	X (m)	Y (m)	Z (m)	$\dot{X}$ (m/sec)	$\dot{Y}$ (m/sec)	$\dot{Z}$ (m/sec)	J2	J3	J4	C21
J5	0.49	2.7	-7.0	0.34-4	-0.12-3	0.40-3	0.68-6	-0.11-4	0.25-6	0.64-7
J6	-0.11	-0.43	0.23	0.12-3	0.30-3	0.37-3	-0.12-4	0.48-6	-0.19-4	0.47-7
J7	-0.10	-0.27	0.85	-0.27-5	0.24-4	-0.58-3	-0.45-6	0.90-6	-0.30-6	-0.27-7
C51	3.7	11	-12	-0.13-2	0.43-2	-0.013	0.14-4	-0.78-5	0.12-4	0.82-6
S51	1.7	4.9	-1.7	-0.11-3	0.24-2	-0.24-2	-0.76-4	0.36-5	-0.71-4	-0.86-5
C61	0.24	0.61	-1.4	-0.58-5	0.12-3	-0.26-3	-0.10-4	-0.16-6	-0.95-5	-0.16-5
S61	-0.55	-2.7	2.0	0.043-2	-0.92-3	0.34-2	-0.91-5	0.11-5	-0.72-5	-0.43-6
C52	-5.2	3.4	-0.14	-0.30-3	0.14-2	-0.34-2	0.35-4	-0.13-5	0.26-4	0.41-5
S52	-0.46	-0.61	-3.1	-0.12-2	0.19-3	-0.52-2	0.18-5	0.92-6	-0.42-5	0.75-5
C62	-0.98	-0.67	-2.9	-0.25-3	-0.13-2	-0.82-3	0.13-5	-0.29-5	-0.63-5	0.11-4
S62	5.8	14	-12	-0.18-2	0.52-2	-0.016	-0.53-4	-0.10-5	-0.37-4	-0.60-5
C53	18.	-4.9	-52	0.49-2	-0.021	0.049	-0.32-3	-0.13-4	-0.24-3	-0.31-4
S53	8.8	7.6	-4.9	-0.016	0.022	-0.095	0.31-3	0.10-4	0.26-3	-0.53-5
C63	0.52	17	-5.5	-0.20-2	0.46-2	-0.016	0.26-4	-0.41-6	0.29-4	-0.63-5
S63	-5.5	4.3	9.5	-0.18-2	0.43-2	-0.014	0.10-3	0.37-5	0.83-4	0.98-5
C54	-4.0	3.0	8.5	0.15-4	0.34-2	-0.61-2	0.72-4	0.73-6	0.53-4	0.69-5
S54	0.022	-17	-7.3	0.46-2	-0.15-2	0.023	-0.34-4	-0.70-6	-0.29-4	0.32-5
C55	18	-52	64	0.67-2	0.010	-0.027	0.22-3	0.31-4	0.15-3	0.31-4
S55	-19	-61	24	0.016	-0.021	0.10	0.15-3	-0.24-4	0.87-4	0.54-4
C71	-0.58	-4.6	4.0	0.58-3	-0.15-2	0.49-2	-0.70-5	0.20-5	-0.57-5	-0.36-6
S71	-0.27	-0.79	1.1	0.30-4	-0.64-3	0.55-3	0.19-4	-0.94-6	0.18-4	0.17-5
C72	1.6	2.2	-1.9	-0.40-3	0.80-3	-0.30-2	-0.66-5	-0.11-6	-0.45-5	-0.11-5
S72	0.067	-0.18	0.064	0.40-3	-0.52-4	0.18-2	-0.87-5	-0.14-6	-0.53-5	-0.31-5
C73	-13	-6.4	29	-0.98-3	0.30-2	-0.98-2	0.24-3	0.49-5	0.17-3	0.24-4
S73	-1.0	-30	-0.49	0.55-2	-0.85-2	0.034	-0.16-3	-0.47-5	-0.14-3	0.88-5
C64	-3.0	-88	0.70	0.020	-0.023	0.11	-0.41-3	-0.89-5	-0.34-3	-0.90-5
S64	21	6.1	-49	0.16-2	-0.013	0.019	-0.33-3	-0.70-5	-0.25-3	-0.36-4
C74	6.7	-1.7	-16	0.16-2	-0.37-2	0.013	-0.15-3	-0.46-5	-0.12-3	-0.14-4
S74	0.20	29	-3.0	-0.52-2	0.61-2	-0.030	0.10-3	0.15-5	0.89-4	-0.20-5
C65	-1.5	6.1	11	-0.43-3	0.34-2	-0.010	0.11-4	-0.14-5	0.93-5	-0.57-6
S65	-6.4	-2.8	11	0.17-2	0.64-3	0.43-2	0.12-3	-0.93-6	0.97-4	0.10-4
C75	-27	0.64	23	0.55-3	0.40-2	0.014	0.23-3	-0.53-5	0.18-3	0.23-4
S75	6.7	-49	-22	0.014	-0.013	0.055	-0.34-3	0.47-5	-0.27-3	-0.24-4
C66	13	55	-12	-0.37-2	0.023	-0.11	-0.26-5	0.39-4	0.36-4	-0.43-4
S66	-0.49	-110	12.0	0.020	-0.82-2	0.024	0.61-3	0.29-4	0.44-3	0.78-4
C76	4.3	-11	8.8	0.11-2	0.67-3	-0.10-2	-0.45-4	0.36-5	-0.37-4	-0.21-5
S76	-6.1	-0.91	18	0.26-2	0.17-2	0.82-3	0.11-3	-0.51-5	0.84-4	0.12-4
C77	-17	73	-30	-0.015	-0.43-2	0.79-2	-0.61-4	-0.20-4	0.10-4	-0.60-4
S77	14	110	15	-0.61-2	0.030	-0.21	0.74-3	0.51-4	0.61-3	0.19-4

S21	C31	S31	C41	S41	C22	S22	C32	S32	C42
0.34-6	0.44-7	0.92-7	0.99-9	0.34-6	0.43-7	0.12-6	0.20-6	-0.48-7	0.26-
0.69-7	0.65-7	-0.66-7	0.12-7	0.14-6	0.28-7	-0.80-8	0.11-7	-0.75-7	0.31-
-0.58-7	-0.61-8	-0.27-7	-0.12-7	-0.60-7	-0.11-7	-0.23-7	-0.39-7	0.91-8	-0.55-
0.80-5	-0.12-4	0.27-6	0.10-5	0.66-5	0.15-6	-0.12-5	-0.23-5	-0.30-6	0.37-
0.13-5	0.48-6	-0.10-4	-0.80-5	0.17-5	0.64-6	-0.42-7	0.12-6	-0.96-6	0.62-
0.66-7	0.29-8	0.12-6	-0.23-5	-0.91-9	-0.10-6	0.14-7	0.19-7	0.16-6	-0.85-
-0.11-5	0.18-6	-0.21-6	-0.37-6	-0.17-5	-0.13-6	-0.41-7	-0.43-8	0.11-6	-0.90-
-0.55-5	0.62-6	0.66-6	0.35-5	-0.46-5	0.31-6	0.81-6	-0.19-5	-0.84-6	0.60-
0.71-6	-0.19-6	0.94-6	0.57-5	0.10-5	-0.99-6	-0.46-8	0.21-6	-0.17-5	-0.75-
-0.20-7	-0.64-6	0.57-6	0.83-5	-0.58-6	-0.26-5	0.85-7	-0.23-6	0.92-6	-0.30-
0.88-5	-0.16-5	-0.13-5	-0.46-5	0.70-5	-0.30-6	-0.21-5	-0.67-6	0.76-7	0.19-
0.87-5	0.18-5	-0.71-5	-0.23-4	0.13-5	-0.18-5	-0.68-5	0.19-5	0.20-5	-0.57-
-0.10-5	-0.43-6	0.97-5	0.37-5	0.34-5	0.10-4	0.42-6	-0.57-6	-0.32-5	0.98
-0.40-6	-0.25-6	0.26-6	-0.34-5	-0.48-6	0.17-5	0.14-6	-0.40-6	-0.51-6	0.14
-0.39-5	0.27-6	0.19-5	0.83-5	-0.16-5	0.88-6	0.12-5	-0.52-6	-0.15-5	0.11
-0.31-5	0.32-6	0.22-5	0.61-5	-0.27-5	0.13-5	-0.27-6	-0.10-5	-0.15-7	0.89
0.66-6	0.11-6	-0.17-5	0.23-5	0.62-6	-0.55-6	0.42-6	0.32-6	-0.11-5	0.47
-0.11-4	0.30-5	0.11-4	0.22-4	0.15-5	0.11-4	-0.14-4	-0.84-5	0.20-5	0.16
-0.88-5	0.65-6	-0.89-6	0.35-4	-0.15-4	0.12-4	0.62-5	0.40-6	-0.14-4	0.97
-0.14-5	0.13-5	-0.14-6	-0.39-6	-0.11-5	-0.10-6	0.19-6	0.42-6	0.17-6	-0.16
-0.30-6	-0.13-6	0.71-6	0.16-5	-0.42-6	-0.93-7	0.11-7	-0.58-7	0.13-6	-0.10
0.19-5	-0.35-6	-0.12-6	-0.97-6	0.16-5	-0.54-8	-0.20-6	0.12-6	0.13-6	-0.10
-0.91-7	0.98-7	-0.24-6	-0.24-5	-0.21-6	0.21-6	0.13-7	-0.21-7	0.17-6	0.17
-0.10-4	0.54-6	0.50-5	0.19-4	-0.64-5	0.17-5	0.33-5	-0.44-6	0.23-5	0.12
0.18-5	-0.40-6	-0.36-5	0.31-5	0.17-6	-0.57-5	-0.95-7	0.42-6	0.25-5	-0.47
0.38-5	0.24-6	-0.10-4	-0.11-4	0.21-6	-0.11-4	-0.64-6	0.10-5	0.61-5	-0.66
0.82-5	-0.41-6	-0.65-5	-0.28-4	0.54-5	-0.16-5	-0.70-5	0.11-6	0.30-5	-0.10
0.41-5	-0.63-6	-0.33-5	-0.12-4	0.21-5	-0.13-5	-0.11-5	0.34-6	0.14-5	-0.13
-0.10-5	-0.11-6	0.22-5	0.29-6	-0.52-6	0.24-5	0.15-6	-0.42-6	-0.13-5	0.18
-0.10-5	-0.11-6	0.55-6	-0.10-5	-0.15-5	0.81-6	-0.11-6	-0.44-6	0.36-6	-0.77
-0.35-5	0.69-6	0.24-5	0.95-5	-0.37-5	0.17-5	0.32-6	-0.47-6	-0.18-5	0.18
-0.43-5	0.71-6	0.28-5	0.20-4	-0.75-5	-0.14-5	0.84-5	0.19-5	-0.34-5	0.18
0.67-5	-0.64-7	-0.72-5	-0.18-4	0.77-5	-0.11-4	-0.24-5	0.94-6	0.80-5	-0.31
0.34-5	-0.77-7	0.11-5	-0.26-4	0.16-4	-0.11-4	-0.92-5	-0.46-5	0.11-4	0.19
-0.22-4	0.74-5	0.15-4	0.57-4	-0.91-5	0.19-4	-0.13-4	-0.69-5	-0.97-5	0.47
-0.40-7	-0.20-6	-0.48-6	-0.31-5	0.16-5	0.56-6	-0.18-5	-0.11-5	0.11-5	-0.12
-0.40-5	0.33-6	0.16-5	0.93-5	-0.53-5	0.28-5	0.85-6	-0.53-6	-0.29-5	0.50
0.11-4	-0.56-5	-0.68-5	-0.37-4	0.22-5	-0.13-4	0.11-4	0.41-5	0.44-5	0.10
-0.15-4	0.44-5	0.18-4	0.26-4	0.33-5	-0.15-5	-0.11-4	-0.76-5	0.20-5	0.73

TABLE 4-8

CASE 4: SENSITIVITIES

	S42	C33	S33	C43	S43	C44	S44	μ (km^3/sec^2)
7	0.11-6	-0.28-7	0.11-7	-0.74-8	-0.10-7	0.90-9	0.52-9	-0.14-2
7	0.23-7	-0.13-8	0.13-7	-0.19-8	0.47-8	0.24-9	-0.87-9	0.016
8	-0.24-7	0.58-8	-0.24-8	0.19-8	0.17-8	-0.25-9	-0.68-10	0.79-3
6	-0.11-5	0.25-6	0.22-7	0.66-8	0.92-7	-0.55-8	-0.66-8	-0.022
6	0.15-6	-0.29-7	0.88-7	-0.11-7	0.15-7	0.22-8	-0.44-8	0.12
7	-0.13-7	-0.42-8	-0.25-7	0.82-8	-0.61-8	-0.84-9	0.96-9	0.016
7	-0.36-7	0.43-8	-0.20-7	0.16-7	0.95-9	-0.16-8	-0.13-9	0.013
6	0.52-6	0.43-7	0.12-6	-0.35-7	-0.71-7	-0.18-8	-0.18-8	-0.063
6	0.36-6	-0.47-7	-0.14-8	0.87-7	0.29-7	0.20-8	-0.76-8	-0.94-2
5	-0.18-6	0.42-7	-0.19-6	0.56-7	-0.27-7	-0.10-7	0.52-8	-0.019
8	-0.25-5	0.12-6	-0.35-7	0.74-7	0.32-7	-0.92-8	-0.80-8	0.10
6	-0.45-5	-0.11-5	-0.27-6	0.53-6	0.74-6	-0.56-7	-0.16-6	0.51
5	0.13-5	-0.12-6	-0.19-6	-0.81-6	0.11-6	0.16-6	-0.25-7	-0.42
5	-0.27-6	0.81-7	0.10-6	-0.33-6	-0.46-7	0.10-8	0.41-8	-0.031
5	0.66-6	0.18-7	0.27-6	-0.79-6	-0.33-6	0.88-8	0.49-8	-0.17
6	-0.11-5	0.49-6	0.43-6	0.17-5	-0.52-6	0.52-7	0.23-8	-0.12
6	0.38-6	-0.28-6	0.75-6	0.39-5	0.84-7	0.12-7	0.32-7	0.055
5	-0.77-6	-0.14-5	0.30-5	0.33-5	0.14-5	0.64-6	-0.87-6	-0.26
6	-0.18-5	-0.11-5	0.14-5	-0.32-4	-0.15-6	0.10-5	0.45-6	-0.34
6	0.18-6	-0.47-7	-0.22-7	0.61-8	-0.10-7	0.65-9	0.92-9	0.94-2
6	-0.58-7	0.11-7	-0.14-7	-0.30-8	-0.60-8	-0.29-9	0.14-8	-0.031
6	-0.14-6	0.17-8	-0.22-7	-0.16-8	0.20-7	0.10-8	0.12-9	0.013
6	-0.97-7	0.76-8	-0.53-8	-0.16-7	-0.13-7	-0.91-9	0.24-8	0.015
5	0.17-5	0.25-7	0.37-6	-0.36-6	-0.32-6	0.30-7	0.53-7	-0.41
5	-0.14-6	-0.18-7	-0.51-6	0.44-6	-0.36-7	-0.58-7	0.40-8	0.22
5	-0.45-6	0.85-7	-0.10-5	0.16-5	-0.10-7	-0.41-6	-0.16-7	0.62
5	-0.48-6	-0.22-6	-0.25-6	0.61-6	0.18-5	-0.17-7	-0.38-6	0.58
5	-0.10-6	-0.87-7	-0.39-6	0.66-7	0.34-6	-0.28-7	-0.21-7	0.26
5	-0.22-6	0.13-6	0.93-7	-0.35-6	-0.80-7	0.15-7	-0.61-8	-0.15
6	-0.64-6	0.33-6	-0.18-6	-0.41-6	-0.83-7	0.93-7	-0.13-7	-0.021
5	-0.11-5	0.82-7	0.66-6	-0.49-8	-0.42-6	0.37-7	0.96-7	-0.21
5	-0.13-5	0.50-6	-0.30-6	-0.22-6	-0.17-5	-0.16-6	0.43-6	-0.45
5	0.14-5	0.13-6	-0.10-5	0.25-5	0.35-6	-0.49-6	-0.15-6	0.56
6	0.22-5	0.23-5	-0.98-6	0.29-5	0.14-5	-0.78-6	-0.62-6	0.086
5	-0.18-5	-0.92-6	0.56-5	-0.20-5	0.12-5	0.87-6	-0.67-6	-0.86
5	0.46-6	-0.44-7	-0.69-7	-0.10-6	0.51-6	0.74-7	-0.12-6	0.079
6	-0.11-5	0.20-6	0.30-6	-0.81-6	-0.11-6	0.14-6	0.65-7	-0.19
5	0.88-6	0.10-5	-0.44-5	0.21-5	-0.90-6	-0.74-6	0.49-6	0.031
5	0.79-6	0.25-5	0.50-6	0.18-5	0.20-5	-0.53-6	-0.54-6	-1.1

TABLE 4-9a

CASE 4: COVARIANCE MATRIX OF STATE IN σ - ρ FORM (σ in m, m/sec)

	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$
X	7.89					
Y	-0.0131	27.0				
Z	-0.156	-0.564	37.6			
$\dot{X}$	0.234	-0.339	0.317	0.439-2		
$\dot{Y}$	0.503	-0.314	0.546	0.136	0.954-2	
$\dot{Z}$	-0.203	-0.169	0.0227	0.191	0.170	0.0215

TABLE 4-9b

CASE 4: COVARIANCE MATRIX OF ASCENDING NODE (Ω) AND PERICYNTHION (r_p) in σ - ρ FORM (σ in deg and m)

	Ω	r_p
Ω	0.00449	
r_p	-0.899	35.5

4.5 Case 5

The runs described in the previous cases were made for a tracking interval of 5040 minutes. Cases 1, 2, and 4 were also run for a 10,080-minute interval to study the long term effects of the selenopotential constants. Table 4-10 compares the standard deviations of the solution variables in each of the cases for the two intervals. It is seen that the standard deviations are significantly lower for the longer tracking interval.

TABLE 4-10

COMPARISON OF SOLUTION VECTOR STANDARD DEVIATIONS FOR 5040 AND 10,080-MINUTE INTERVALS

Solution vector	Standard Deviation (σ) 5040 min interval	10,080 min interval	
Case 1			
X	0.80	0.60	m
Y	1.7	1.03	m
Z	1.3	0.85	m
$\dot{X}$	0.82E-3	0.58E-3	m/sec
$\dot{Y}$	0.33E-3	0.24E-3	m/sec
$\dot{Z}$	0.91E-3	0.53E-3	m/sec
Case 2			
X	2.5	1.7	m
Y	19	3.1	m
Z	33	4.1	m
$\dot{X}$	0.25-2	0.94-3	m/sec
$\dot{Y}$	0.67-2	0.88-3	m/sec
$\dot{Z}$	0.012	0.27-2	m/sec
J2	0.14E-4	0.027E-4	
J3	0.32E-5	0.043E-5	
J4	0.12E-4	0.024E-4	
S21	0.46E-6	0.12E-6	
C31	0.93E-6	0.13E-6	
S41	0.87E-6	0.18E-6	
C22	0.18E-6	0.067E-6	
C32	0.26E-6	0.10E-6	
S42	0.20E-6	0.057E-6	
S43	0.60E-7	0.15E-7	
μ	0.35E-1	0.40E-2	km^3/sec^2

TABLE 4-10. — (Concluded)

COMPARISON OF SOLUTION VECTOR STANDARD DEVIATIONS FOR 5040 AND 10,080-MINUTE INTERVALS

Solution vector	Standard Deviation (σ) 5040 min interval	10,080 min interval	
	Case 4		
X	7.9	3.14	m
Y	27	8.24	m
Z	37	0.14	m
$\dot{X}$	0.43E-2	0.16E-2	m/sec
$\dot{Y}$	0.94E-2	0.18E-2	m/sec
$\dot{Z}$	0.20E-1	0.58E-2	m/sec
J2	0.15E-03	0.024E-03	
J3	0.52E-05	0.081E-05	
J4	0.12E-03	0.020E-03	
C21	0.90E-05	0.055E-05	
S21	0.36E-05	0.076E-05	
C31	0.13E-05	0.018E-05	
S31	0.25E-05	0.046E-05	
C41	0.80E-05	0.055E-05	
S41	0.32E-05	0.080E-05	
C22	0.17E-05	0.021E-05	
S22	0.75E-06	0.027E-05	
C32	0.12E-05	0.035E-05	
S32	0.22E-05	0.028E-05	
C42	0.15E-05	0.016E-05	
S42	0.58E-06	0.021E-05	
C33	0.26E-06	0.086E-06	
S33	0.36E-06	0.070E-06	
C43	0.26E-06	0.036E-06	
S43	0.20E-06	0.044E-06	
C44	0.26E-07	0.078E-07	
S44	0.24E-07	0.076E-07	
μ	0.26	0.028	km^3/sec^2

4.6 Conclusions

Comparisons of Cases 3 and 4 leads to the following conclusions. In Case 3, where only 11 selenopotential parameters (degree not greater than 4) are included in the solution vector, the sensitivities due to parameters greater than degree 4 are significant and comparable in magnitude to the sensitivities produced by the remaining parameters of degree less than 4. However, in Case 4, where all parameters through degree 4 are included in the solution vector, the sensitivities due to the remaining parameters through degree 7 can be considered negligible. Thus, if all parameters through degree 4 are included in the solution vector it is not necessary to include any of the remaining parameters through degree 7. Because of the large sensitivities produced by the parameters of degree greater than 4 in Case 3, it seems necessary to solve for all parameters through degree 4.

5. NEW TECHNOLOGY

This section is included to comply with the requirements of the "New Technology" clause of the Master Agreement under which this report was prepared. The most significant new technology developed under this contract is the computation of the sensitivity of unsolved parameters to the parameters being solved for. This capability was developed at TRW Systems and implemented into the AT-85 orbit determination program.

6. REFERENCES

1. ESPOD Mathematical and Subroutine Description; TRW No. 8497-6065-RU-000, 24 June 1964.