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ANALYSIS AND DESIGN OF SPACE VEHICLE FLIGHT CONTROL SYSTEMS

VOLUME VIII - RENDEZVOUS AND DOCKING

by Daniel Chiarappa

Prepared by
GENERAL DYNAMICS CORPORATION
San Diego, Calif.
for George C. Marshall Space Flight Center

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FOREWORD

This report was prepared under NASA Contract NAS 8-11494 and is one of a series intended to illustrate methods used for the design and analysis of space vehicle flight control systems. Below is a complete list of the reports in the series:

Volume I	Short Period Dynamics
Volume II	Trajectory Equations
Volume III	Linear Systems
Volume IV	Nonlinear Systems
Volume V	Sensitivity Theory
Volume VI	Stochastic Effects
Volume VII	Attitude Control During Launch
Volume VIII	Rendezvous and Docking
Volume IX	Optimization Methods
Volume X	Man in the Loop
Volume XI	Component Dynamics
Volume XII	Attitude Control in Space
Volume XIII	Adaptive Control
Volume XIV	Load Relief
Volume XV	Elastic Body Equations
Volume XVI	Abort

The work was conducted under the direction of Clyde D. Baker, Billy G. Davis and Fred W. Swift, Aero-Astro Dynamics Laboratory, George C. Marshall Space Flight Center. The General Dynamics Convair program was conducted under the direction of Arthur L. Greensite.

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1. STATEMENT OF PROBLEM

Several space missions require spacecraft rendezvous and docking. The most immediate is the Apollo lunar landing, which requires rendezvous and docking of the LEM (Lunar Excursion Module) vehicle with the C&S (Command and Service) vehicle in lunar orbit. Large, earth-orbiting space vehicles (too large to be injected in orbit as a single launch) that are constructed in space and stay in orbit for long periods also require this capability, for the initial construction and the subsequent rotation of personnel and resupply of material and provisions.

Typically, a rendezvous and docking mission has a target vehicle in circular (or nearly circular) orbit passing above a manned rendezvous vehicle ready to be launched. The target vehicle orbit is known from ground tracking. The rendezvous vehicle is launched into a slightly lower coplanar orbit 100 miles or more behind the target. Because the lower-altitude orbit is faster, the rendezvous vehicle will close on the target. When the range is short enough for the rendezvous vehicle on-board sensors to acquire target vehicle relative position and velocity data, thrusting is applied to cause the rendezvous vehicle to assume the same orbit as the target.

Up to this point, the pilot may or may not have entered actively into the guidance and control loop, possibly at his option. He is now in visual contact with the target and can be considered to be in a short-term, station-keeping mode. His objective is to connect the two vehicles in some desired configuration. This maneuver is known as "docking." The target is maintained in a selected attitude, the rendezvous vehicle is piloted into the mating receptacle at some small closing velocity, and securing latches are actuated.

Rendezvous and docking may be broken up into several phases:

a. Launch to Rendezvous. Begins with the rendezvous vehicle on the launching pad. Ends when the rendezvous vehicle has acquired the target with on-board radar.

b. Orbital Rendezvous. Begins when the rendezvous vehicle on-board radar has acquired the target vehicle. Ends when the rendezvous vehicle has been thrust to the desired end condition — close-proximity station-keeping with the target vehicle.

c. Orbital Docking. Begins at the termination of orbital rendezvous. Ends when the rendezvous vehicle has been piloted into the target vehicle mating interface and latching has been completed.

Ideally, the objective of rendezvous and docking would be unrestricted by mass ratio, launch point, time control, etc. Unlimited rendezvous and docking would be possible at any time, with no restrictions on target orbit or rendezvous vehicle launch, and with manned or unmanned vehicles. However, several assumptions were made in the typical example to reflect current limitations in the state of the art. These are:

a. Extensive preplanning has resulted in the design of a compatible target orbit and rendezvous vehicle launch point and time to successfully accomplish the launch-to-rendezvous.

b. Ground tracking of target and rendezvous vehicle is not accurate enough to allow ground control of the rendezvous phase. Tracking equipment must be aboard the rendezvous vehicle.

c. A human pilot is needed to direct the rendezvous vehicle into the target vehicle mating interface.

d. The target vehicle is maintained at a fixed attitude during docking. It has been designed to mate with the rendezvous vehicle.

This monograph emphasizes spacecraft control during the orbital rendezvous and docking phases. Launch-to-rendezvous is treated only briefly to identify basic constraints affecting spacecraft design (mostly orbit parameters). Launch is primarily a ground computer and booster design problem. (For summary information on launch to rendezvous, see Reference 1.)

Control of the spacecraft is emphasized in this monograph. Guidance system discussion is restricted to a description of software in the control diagrams.

A primary source of information on rendezvous and docking is the symposium held at Edwards Air Force Base in September, 1963.*

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*See Advances in the Aeronautical Sciences, Volume 16, The Macmillan Company, New York, N.Y., 1963.

2. STATE OF THE ART

The state of the art of rendezvous and docking is reflected in the Gemini program. Four successful manned rendezvous flights have been accomplished. In the first, two Gemini vehicles were used, but docking was not possible because there was no suitable mechanical interface. In the second, the Gemini vehicle was joined to the target Agena but could not maintain the connection because of a failure of one of the Gemini rockets. This failure was accompanied by a tumbling motion, causing the pilot to disconnect from the Agena and transfer to a backup Gemini attitude control system. In the third, docking was again unsuccessful, because of the incomplete jettison of the shroud covering of the docking mechanism on the target vehicle. In the fourth, and most recent flight (18 July), with Gemini 10, two successful rendezvous and one docking were accomplished. Thus rendezvous and docking capability has been demonstrated by in-flight test.

Gemini is the testbed for the Apollo lunar landing mission (possibly in late 1968). This mission requires rendezvous and docking maneuvers between the LEM and C&S modules of the Apollo spacecraft. Just after translunar injection, the C&S module will detach from the S-IVB stage, turn around, and dock with the upper tunnel of the LEM. Much later, after the LEM has completed its lunar stay and has injected into lunar orbit, it will rendezvous and dock (again with its upper tunnel) with the C&S module.

The Gemini and LEM vehicles are illustrated in Figs. 1 and 2. These are edited versions of illustrations in Refs. 2 and 3. Note that the LEM includes lunar descent and ascent stages. Only the ascent stage is involved in the lunar rendezvous and docking.

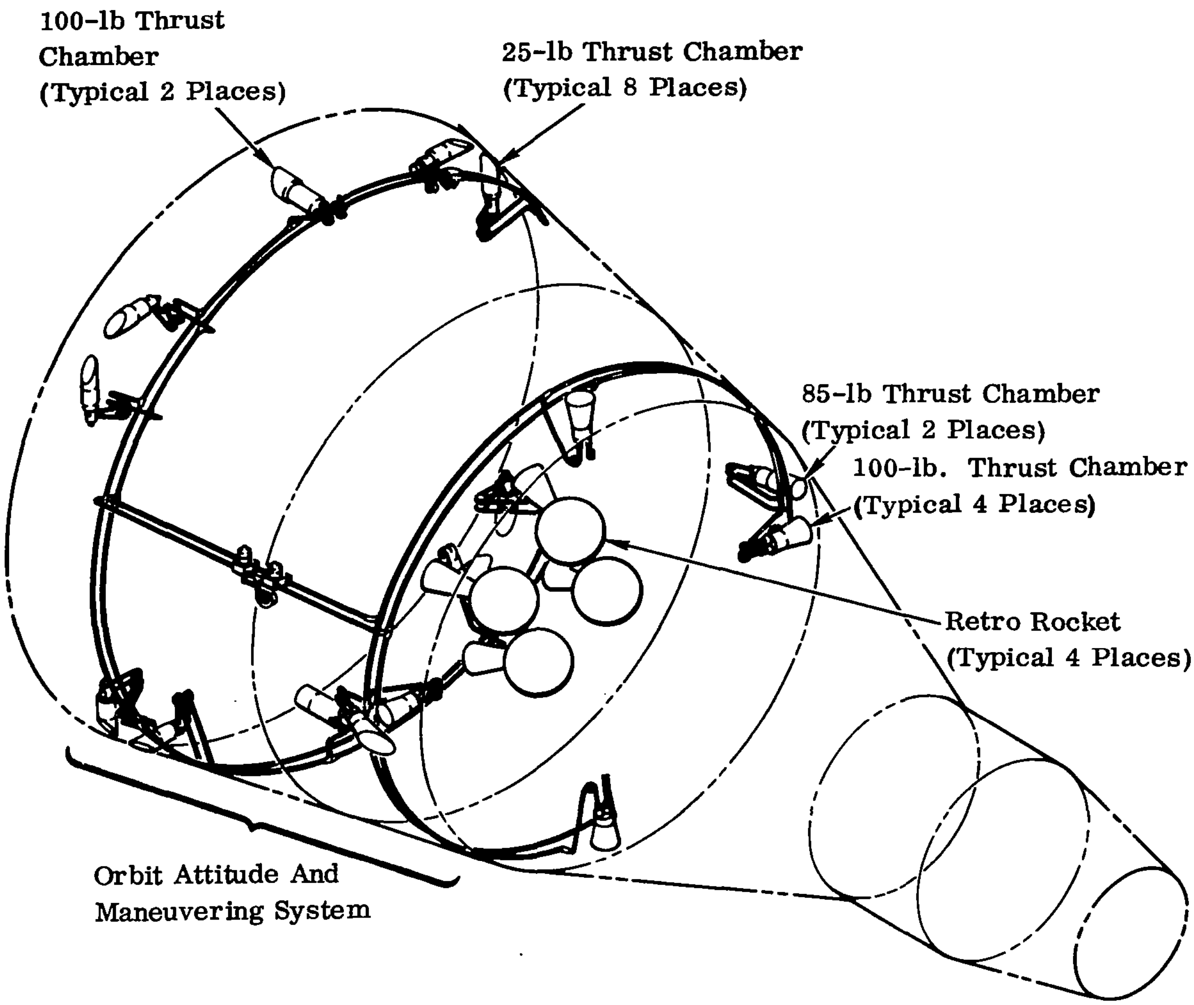


Figure 1. Gemini Vehicle

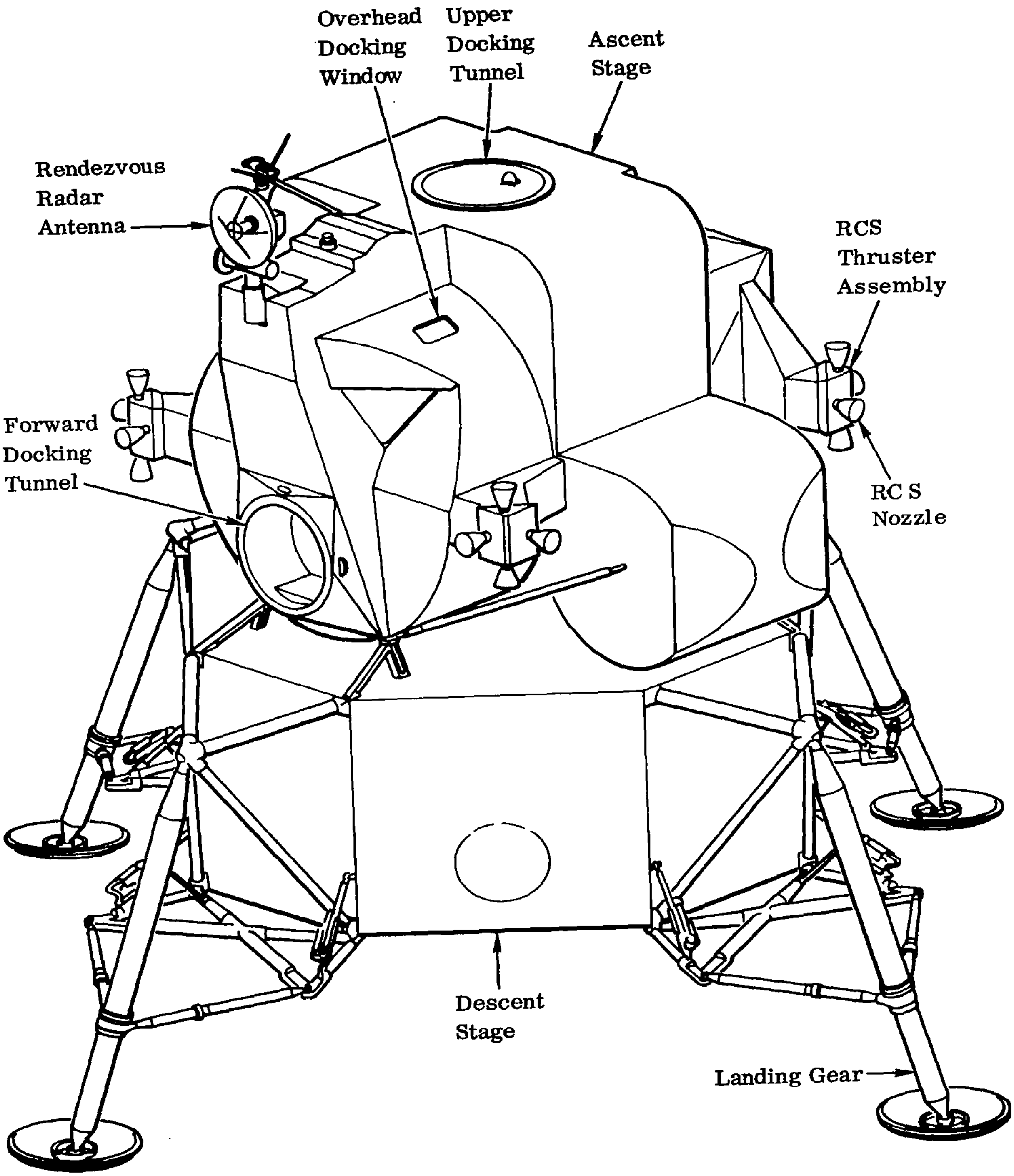


Figure 2. LEM Vehicle

3. RECOMMENDED PROCEDURES

In broad terms, the design criterion for control of rendezvous and docking is completion of the mission with a minimum of control equipment, propellant weight, and electrical power.

Launch-to-rendezvous, briefly considered in Section 3.1, reduces the gross requirements of subsequent phases by placing the rendezvous vehicle in the approximate vicinity of the target vehicle with a small difference in velocity vector.

Orbital rendezvous is divided into two subphases: midcourse and terminal. The rendezvous midcourse phase is concerned with driving the rendezvous vehicle within a few miles of the target. Terminal position is the prime factor. Terminal relative velocity is important but subordinate to position. The rendezvous terminal phase is concerned with closing the last few miles to the target, simultaneously eliminating relative velocity. These subphases are discussed as they occur in time, with the major theme of the discussion being derivation of control criteria through examination of guidance system maneuver instructions to be performed by the control system.

The docking phase differs from the rendezvous phase in that is is essentially a control problem. There is no guidance system in the usual sense. Control instructions are originated by the pilot looking out the cockpit at the target vehicle. In this monograph, the interface dynamics associated with mating the two vehicles is discussed first. Maneuvering the rendezvous vehicle into the target vehicle mating mechanism is then considered. The integrated control system requirements for rendezvous and docking, including control, thruster system configuration, and sizing, are then developed.

3.1 LAUNCH CONSIDERATIONS

The basic launch problem is illustrated in Fig. 3. As shown, the target vehicle is down-orbit from the launch site. The rendezvous vehicle launch site is moving at earth rate. As shown, the launch site moves from an out-of-plane position, point A, to an in-plane position, point B.

It is best to launch from point B to avoid requiring a later plane change. With the target down-orbit, the rendezvous vehicle would be launched into a coplanar lower altitude, "catch up" parking orbit. Should the target be overhead, the launch would be directly to target altitude. The launch site will be in the coplanar position twice a day, assuming the launch latitude is less than the target orbit inclination.

An out-of-plane launch, from point A, would require a later plane change. The fuel penalty for this change is significant for out-of-plane angles greater than a fraction

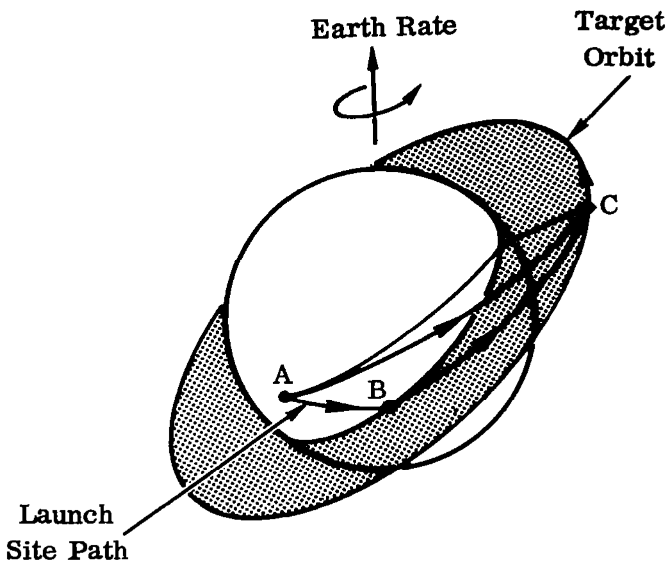


Figure 3. Launch-to-Rendezvous Phase

of a degree. For example, one degree of out-of-plane error in a 500-mile earth orbit requires a change in velocity (ΔV) of 426 ft/sec. Furthermore, a rendezvous orbit should be at such an altitude that its period is a subharmonic of the earth's rotational period. This will place the target vehicle in the same position relative to the launch site daily for even subharmonics, or twice daily for odd subharmonics, when the site is in the target orbit plane.

Assume now that the rendezvous vehicle is launched in the target orbit plane but that the target vehicle is ahead of or behind the rendezvous vehicle orbit injection point. This situation is illustrated in Fig. 4.

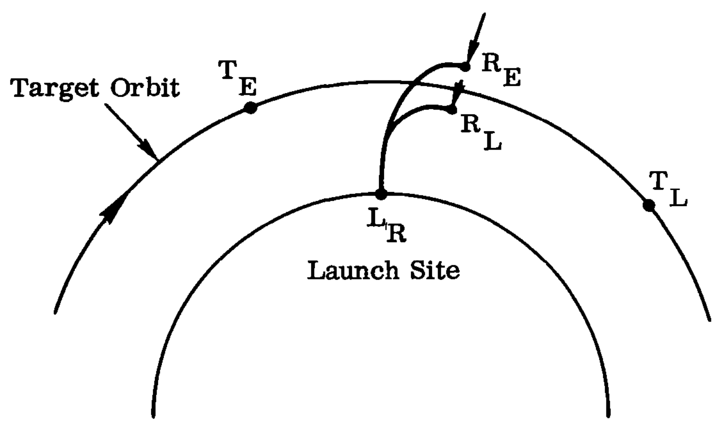


Figure 4. Early and Late Launches to Rendezvous

When the launch site is in or near the target vehicle orbit plane, the target may be upstream in its orbit, point T_E, or downstream, point T_L. Launches to interception are termed early when the target is at point T_E, or late when the target is at point T_L. In either case the launch will be made to observe the important coplanar requirement. Furthermore, the rendezvous injection into the chase orbit could theoretically be at a higher or lower altitude, because the target and rendezvous vehicles will eventually be in the proper positions for start of the midcourse phase. However, to reduce the time required to assume this phase, it is advantageous to launch the rendezvous vehicle to a higher altitude when the launch is early and to a lower altitude when the launch is late. Or if the target is in the right position (roughly overhead), a direct launch to target-vehicle altitude could be used. This procedure is planned for the LEM lunar rendezvous.[4] The early, on-time, and late launches are shown on Fig. 5.

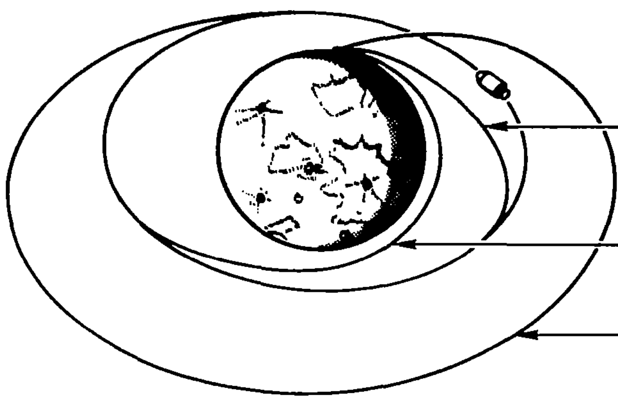


Figure 5. LEM Early, On-Time, and Late Trajectories

3.2 RENDEZVOUS GUIDANCE/CONTROL

3.2.1 Rendezvous Midcourse Guidance

Conditions for initiation of midcourse guidance correspond to the rendezvous and target vehicle being in a near-circular orbit. The rendezvous vehicle is at a lower or higher altitude in a nearly coplanar orbit and is closing on the target vehicle. When the distance between the two satellites is such that on-board radar can acquire relative position and velocity data, the midcourse phase begins.

The general technique involves the application of a number of short-period thrusts to change the velocity of the rendezvous vehicle such that it is brought into the same orbit as the target vehicle, with theoretically zero relative displacement.

A number of investigators have contributed information on this method [5, 6, 7, 8]. It involves setting up a convenient reference frame, deriving the differential equations of relative motion of the rendezvous and target vehicles, and solving these equations to determine a suitable guidance law. The method described below mainly follows Ref. 5.

The assumptions made are as follows.

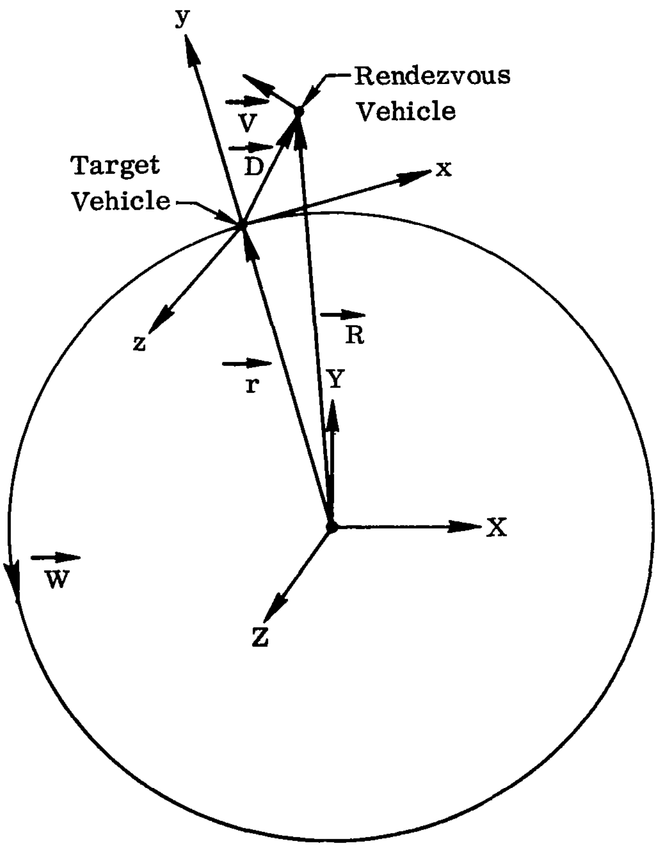
a. The relative distance between the two vehicles is small (100 to 200 n.mi.) compared with the distance to the earth center.

b. The target vehicle is in a circular orbit.

The reference frame is shown in Fig. 6. The y direction is along the target local vertical, the z direction is aligned with the orbit angular momentum vector (or orbit normal), and the x direction is in the orbit plane. The x, y, z axes form a right-handed orthogonal reference system. The directions of the inert reference system (X, Y, and Z) are arbitrary but inertially fixed.

The angular rate of the x, y, z rotating frame relative to the inert frame is

$$\vec{W} = \sqrt{\frac{g_E r_E^2}{r^3}} \; \vec{z} \equiv W\vec{k} \tag{1}$$



NOTES

1. x, y, z is Rotating Frame of Reference Fixed to Target
2. X, Y, Z is Inert Frame Fixed to Earth Center

Figure 6. Reference Frame for Midcourse Rendezvous Guidance

where

$\vec{W}$ is the orbit angular rate vector of the target vehicle; $\vec{W} = W\vec{k}$

g_E is the gravitational constant at earth surface

r_E is the earth radius

r is the distance to the target from the earth center; $\vec{r} = x\vec{i} + y\vec{i} + z\vec{k}$

The distance from the earth center to the rendezvous vehicle, in x, y, z coordinates, is

$$\vec{R} = \vec{r} + \vec{D} = \begin{bmatrix} x \\ r + y \\ z \end{bmatrix} \tag{2}$$

where

$\vec{R}$, $\vec{r}$ are respectively the distance vectors from the earth center to the rendezvous vehicle and target vehicle

$\vec{D}$ is the distance vector from the target to the rendezvous vehicle

x, y, z are the components of the vector $\vec{D}$ in the x, y, z reference system.

The velocity vector components of the rendezvous vehicle, in x, y, z coordinates, are

$$\vec{V} = \dot{\vec{R}}^{*} = \underset{\cdot}{\vec{R}}^{*} + \vec{W}X\vec{R} = \begin{bmatrix} \dot{x} - W(y + r) \\ \dot{y} + Wx \\ \dot{z} \end{bmatrix} \tag{3}$$

where $\vec{V}$ is the velocity vector of the target vehicle with respect to the earth center.

The acceleration vector components of the rendezvous vehicle, in x, y, z coordinates, are

$$\vec{A} = \dot{\vec{V}} = \underset{\cdot}{\vec{V}} + \vec{W}X\vec{V} = \begin{bmatrix} \ddot{x} - 2W\dot{y} - W^2x \\ \ddot{y} + 2W\dot{x} - W^2(y + r) \\ \ddot{z} \end{bmatrix} \tag{4}$$

where $\vec{A}$ is the acceleration vector of the target vehicle with respect to the earth center.

---

*The dot above the vector sign ($\dot{\vec{R}}$) indicates the vector derivative. The dot below the vector sign ($\underset{\cdot}{\vec{R}}$) indicates the time derivative of the components of R.

The significant forces acting on the rendezvous vehicle are the sum of applied thrusts and the gravitational force from the earth.

$$\vec{T} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}, \quad \vec{F}_G = -\frac{r_E^2 g_E m}{R^3} \vec{R} \tag{5}$$

where

$\vec{T}$, $\vec{F}_G$ are respectively the applied thrust and gravitational forces acting on the rendezvous vehicle

T_x, T_y, T_z are the components of $\vec{T}$ in the x, y, z frame

m is the rendezvous vehicle mass

The components of the gravitational force, in x, y, z coordinates, are

$$\vec{F}_G = -\frac{r_E^2 g_E m}{\left[x^2 + (y+r)^2 + z^2\right]^{3/2}} \begin{bmatrix} x \\ y + r \\ z \end{bmatrix} \tag{6}$$

However, using the fact that the relative distance is small compared with the distance to the earth center yields an approximation for the gravitational force. Retaining only first-order terms of a power series expansion of the gravity force components about the point, x, y, z = 0, yields

$$\vec{F}_G = -\frac{r_E^2 g_E m}{r^3} \begin{bmatrix} x \\ r - 2y \\ z \end{bmatrix} = mW^2 \begin{bmatrix} x \\ r - 2y \\ z \end{bmatrix} \tag{7}$$

Equating the force to acceleration components yields the differential equations of motion as follows.

$$\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = m \begin{bmatrix} \ddot{x} - 2W\dot{y} \\ \ddot{y} + 2W\dot{x} - 3W^2 y \\ \ddot{z} + W^2 z \end{bmatrix} \tag{8}$$

If the applied thrust is zero, the left side of Eq. (8) is zero, and the equations may be solved to yield expressions for the relative rate and distance, subject to the initial values of rate, $\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$, and distance, x_o, y_o, z_o. When this is done the rate and position at some later time is

$$\left.\begin{aligned}
\dot{x} &= (4\dot{x}_o - 6y_oW)\cos Wt + 2\dot{y}_o\sin Wt + 6Wy_o - 3\dot{x}_o \\
\dot{y} &= (3y_oW - 2\dot{x}_o)\sin Wt + \dot{y}_o\cos Wt \\
\dot{z} &= -z_oW\sin Wt + \dot{z}_o\cos Wt
\end{aligned}\right\} \quad (9)$$

$$\left.\begin{aligned}
x &= 2\left(\frac{2\dot{x}_o}{W} - 3y_o\right)\sin Wt - \frac{2\dot{y}_o}{W}\cos Wt + (6Wy_o - 3\dot{x}_o)t + x_o + \frac{2\dot{y}_o}{W} \\
y &= \left(\frac{2\dot{x}_o}{W} - 3y_o\right)\cos Wt + \frac{\dot{y}_o}{W}\sin Wt - \frac{2\dot{x}_o}{W} + 4y_o \\
z &= z_o\cos Wt + \frac{\dot{z}_o}{W}\sin Wt
\end{aligned}\right\} \quad (10)$$

Several guidance software schemes, which basically differ in the amount of time and number of "major" impulses used to cause rendezvous, have been developed from these equations. Specifically, rendezvous occurs when the initial velocities in Eq. (10) are set by velocity impulses to yield x, y, z = 0 at some preset time (t_R). This set of initial velocities would be the first "major" impulse correction. Several subsequent "minor" impulses would be needed to correct for errors in application of the major impulse and errors due to the approximations caused by assuming eccentricity to be zero and x, y and z to be small.

A reasonable rendezvous time for a two-major-impulse rendezvous would be between one-half and one orbit. One could dispense with "major" z corrections by waiting for z_o to be zero (which must occur within a half orbit of any start period) and applying an impulse correction causing $\dot{z}_o$ to be zero. Excluding errors, there would be no subsequent out-of-plane motion. The values of x and y at rendezvous time t_R should be zero. Inserting these values in Eq. (10) and solving for the needed $\dot{x}_o$ and $\dot{y}_o$ yields

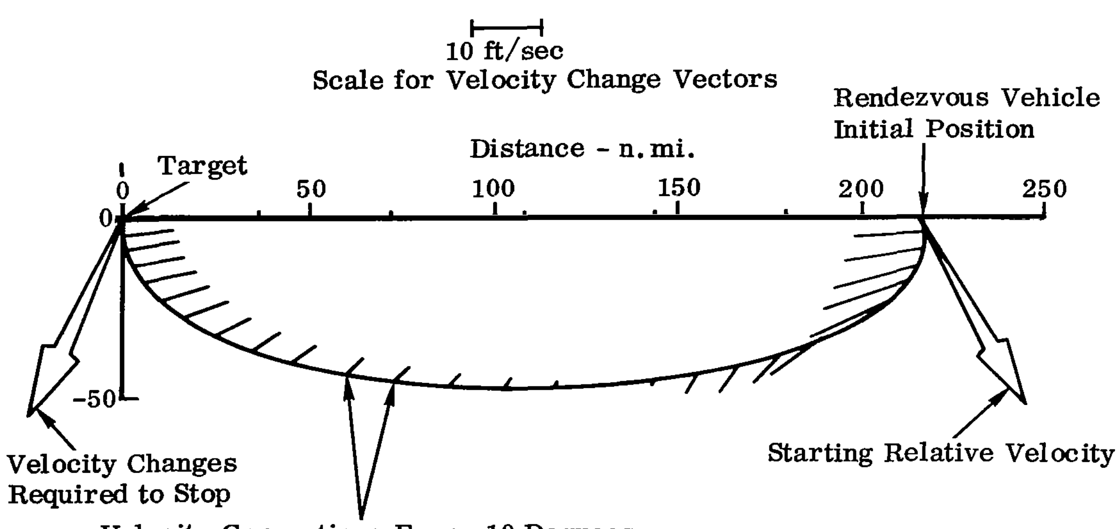
$$\left.\begin{aligned}
\dot{x}_o &= \frac{\left[x_o\sin Wt_R + y_o\{6Wt_R\sin Wt_R - 14(1 - \cos Wt_R)\}\right]}{3Wt_R\sin Wt_R - 8(1 - \cos Wt_R)}\,W \\
\dot{y}_o &= \frac{\left[2x_o(1 - \cos Wt_R) + y_o\{4\sin Wt_R - 3Wt_R\cos Wt_R\}\right]}{3Wt_R\sin Wt_R - 8(1 - \cos Wt_R)}\,W
\end{aligned}\right. \quad (11)$$

Note that the denominator of Eq. (11) is zero for Wt_R equal to multiples of 2π. This means that for this technique, the rendezvous can be at less than or greater than one orbit period but not at exactly one orbit.

For the simplest case, selecting a value of t_R yields the magnitude of the required $\dot{x}_0$ and $\dot{y}_0$ initial conditions. These values are compared with the existing values of velocity. The difference determines the required velocity correction. Assuming perfect execution of the velocity correction, the relative velocity of the rendezvous vehicle when it reaches the target vehicle is given by Eq. (9), using the initial values of position, the velocity after the impulse correction, and the rendezvous time. This relative velocity is reduced to zero by an impulse correction, thereby completing the rendezvous.

The value of t_R was assumed to be selected and subsequently fixed. In practice, it is expected that various values about a point would be run through a computation, using the radar-derived initial conditions to determine a t_R to achieve rendezvous using minimum propellant.

Reference 6 uses the technique described but includes the effect of added orbital eccentricity. Fig. 7 shows the effect of two major impulses at the beginning and end of rendezvous in three quarters of an orbit. Note the magnitude of the "secondary" corrective impulses needed as a result of the added orbital eccentricity. Presumably these impulses would not be necessary if the eccentricity were zero.



NOTES

1. Rendezvous accomplished in 270 degrees true anomaly travel of target.
2. Target orbit eccentricity = 0.01; Semi-major axis = 3592 n. mi.

Figure 7. Relative Motion of Rendezvous Vehicle with Respect to Target

Ref. 7 also uses the technique described above and suggests a general equipment block diagram given as Fig. 8. In this particular mechanization of a software scheme, the value of rendezvous time, once selected, is maintained. Corrections after the first one are based on the same equations, with existing initial conditions. A clock is used, and the remaining time to rendezvous is obtained by subtracting clock time from the original value of rendezvous time.

Although the emphasis here is on two-major-impulse correction techniques, not all guidance software schemes use this procedure. Ref. 8 suggests a slightly different scheme based upon four major corrections and a fixed time of one orbit to rendezvous.

The functional control requirements for the midcourse phase, therefore, are the execution of guidance system instructions to:

a. Rotate the vehicle attitude to align the vehicle axis accurately in the desired ΔV direction.

b. Change ΔV magnitude.

c. Provide for a coast attitude phase between corrections. This amounts to reducing the attitude accuracy requirement in order to conserve propellant.

Typical design figures are attitude accuracies of ±5 degrees during coast and ±0.5 degree during ΔV corrections. An execution accuracy of 1 fps for ΔV corrections seems sufficient. Completion of the midcourse phase within one orbit also seems reasonable.

3.2.2 Rendezvous Terminal Guidance

The rendezvous terminal phase begins just before the "last" major midcourse correction. As indicated in Section 3.2.1, the objective of this last correction (ideally) is to reduce the closing velocity to zero when the position has been closed to zero. This correction is actually applied as a series of smaller corrections. The gradual reduction in relative velocity as a function of range is the purpose of the rendezvous terminal phase.

A typical initial range is a few nautical miles. The velocity vector is roughly in line with the range vector in a direction closing the distance between the two vehicles. The closing velocity depends upon the midcourse initial conditions of target and rendezvous vehicles but typically is about 100 ft/sec. The end conditions are simultaneous reduction of relative velocity to a fraction of a foot per second and of range to a few feet. These conditions would be the start of the docking phase.

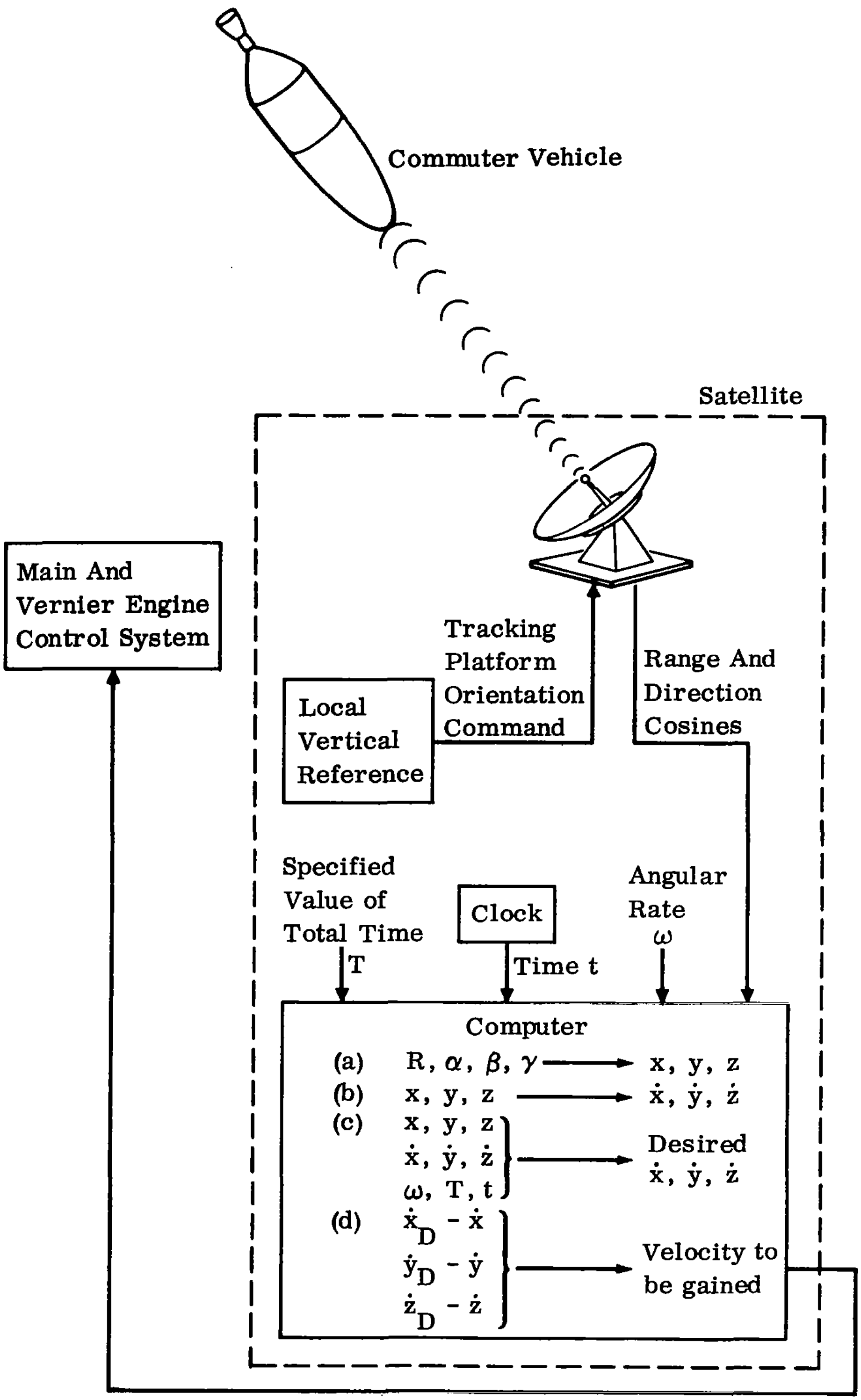


Figure 8. General Steering Control Configuration

Establishing an acceptable range vs. range rate criterion is the fundamental concept in terminal rendezvous. Many investigators[9, 10] call for a range-rate reduction proportional to the square root of range. The reason for doing so is explained below.

Assume perfect control of the range rate proportional to some power of the range,

$$\dot{R} = -KR^N \tag{12}$$

where

R is the range

K is a positive constant

N is a positive constant

Integrate Eq. (12) to find the time to rendezvous as follows.

$$\frac{dR}{dt} = -KR^N, \quad \int_{R_o}^{R} \frac{dR}{R^N} = -K\int_{o}^{t} dt$$

$$t = \frac{1}{(N-1)K}\left(R^{-(N-1)} - R_o^{-(N-1)}\right), \quad N \neq 1 \tag{13}$$

At time of rendezvous, t_R, R is to be zero. Inserting this end condition in (13) and using (12) yields

$$t_R = -\frac{1}{(N-1)K} R_o^{-(N-1)} = \frac{1}{(N-1)} \frac{R_o}{\dot{R}_o} \tag{14}$$

Eq. (14) shows that as N approaches 1, the time to rendezvous approaches infinity. With $N > 1$, the time to rendezvous is negative and has no physical significance. Therefore N must be less than 1. In addition, differentiate (12) to find

$$\ddot{R} = -KNR^{N-1}\dot{R} = K^2NR^{2N-1} \tag{15}$$

Eq. (15) shows that if $N < 1/2$, the exponent of R will be negative. When R is small, as it is when approaching rendezvous, the acceleration will approach infinity. Therefore we must have $1 > N \geq 1/2$ to avoid the problems mentioned. Further, if $N = 1/2$, the acceleration is constant for all values of range. This is desirable, because the acceleration is inherently supplied by constant thrust propulsion (neglecting mass change). Under constant acceleration, N = 1/2 and the time to rendezvous acceleration and range rate are

$$t_R = \frac{2}{K}\sqrt{R_o}\ , \quad \ddot{R} = \frac{K^2}{2}\ , \quad \dot{R} = -\sqrt{2\ddot{R}R_o} \tag{16}$$

Fig. 9 gives plots of range rate and of time versus range for various values of acceleration.

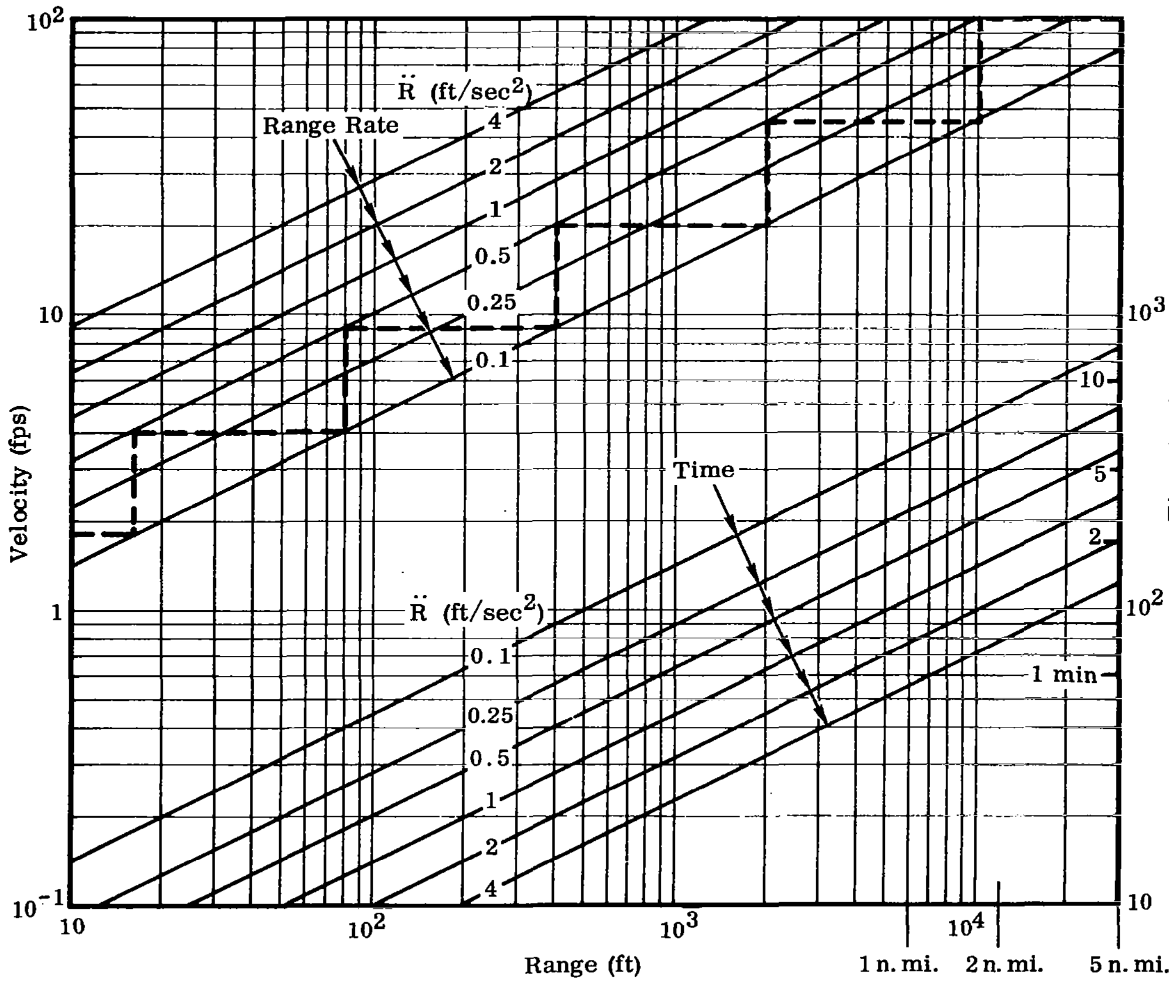


Figure 9. Range Rate; Time vs. Range

A log scale is used because the parabolic curve is reduced to a straight line and different accelerations yield parallel lines. Fig. 9 is useful for determining the distances and times required to stop a vehicle at various acceleration levels. It is directly applicable for continuous burn and is helpful for analyzing the more practical system of using an on-off thrust control in accordance with a preselected range vs. range rate criterion.

An example of off-on control is shown by the dotted line on Fig. 9. An initial approach velocity of 100 ft/sec is assumed; impulsive thrusts are turned on using the 0.5-ft/sec^2 line and turned off using the 0.1 ft/sec^2 line. The result is a bounded range vs. range rate profile with an average acceleration of about 0.25 ft/sec^2. The terminal velocity is about 2 ft/sec at a range of 10 feet. If this velocity is considered too high at so close a distance, a range bias of a few hundred feet can be used. When this range is nearly reached, a lower deceleration set of switch lines can be used to complete closing that few hundred feet with a resultant reduction in terminal velocity to a fraction of a foot per sec. Obviously, the impulsive change in velocity used on Fig. 9 is a simplification of the exact case, where the deceleration would be finite rather than infinite. For finite and decreasing deceleration, the vertical line tends to decrease in slope toward the left, showing a decrease in range during application of thrust.

The range vs. range-rate criterion example described above would result in a controlled drop in velocity as a function of range. However, it is also necessary to direct the rendezvous vehicle toward the target. Having selected an initial range and range-range rate criterion (this amounts to selecting the time to go to the end of terminal rendezvous), the time to go is established. The target vehicle equations of motion can be integrated out to that time, establishing the position of the target at rendezvous. The vehicle would be thrust toward that point. For example, Eq. (8), given for midcourse guidance, could be used. More complicated equations making fewer assumptions in regard to orbit eccentricity, spherical earth field, etc., could be used for both vehicles. The choice depends upon the computational capability within the rendezvous vehicle.

Alternatively, direct measurement could be used to generate directional thrust commands. A method (9) using line-of-sight rate in addition to the range-rate criteria is illustrated by Fig 10.

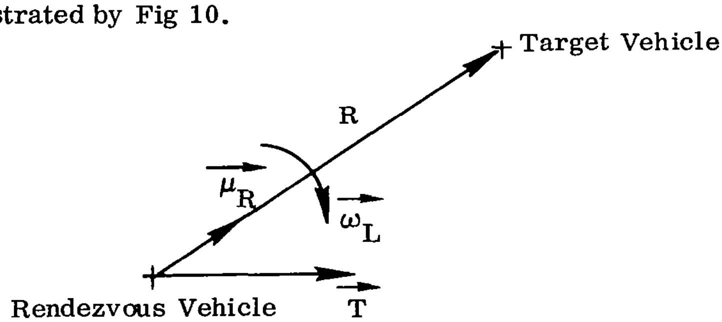


Figure 10. Terminal Guidance (Angular Rate of Line of Sight)

The thrust or acceleration vector is directed as indicated by the following expression.

$$\vec{A} = \frac{\vec{T}}{m} = S\left[(\dot{R} + K\sqrt{R})\vec{\mu}_R + R\vec{\mu}_R X \vec{\omega}_L\right] \tag{17}$$

where

R is the magnitude of the range vector

$\vec{\mu}_R$ is the line-of-sight unit vector

$\vec{\omega}_L$ is the inertial angular rate of the line-of-sight vector

$\vec{T}$ is the thrust vector

m is the vehicle mass

$\vec{A}$ is the thrust acceleration vector

S and K are positive control constants

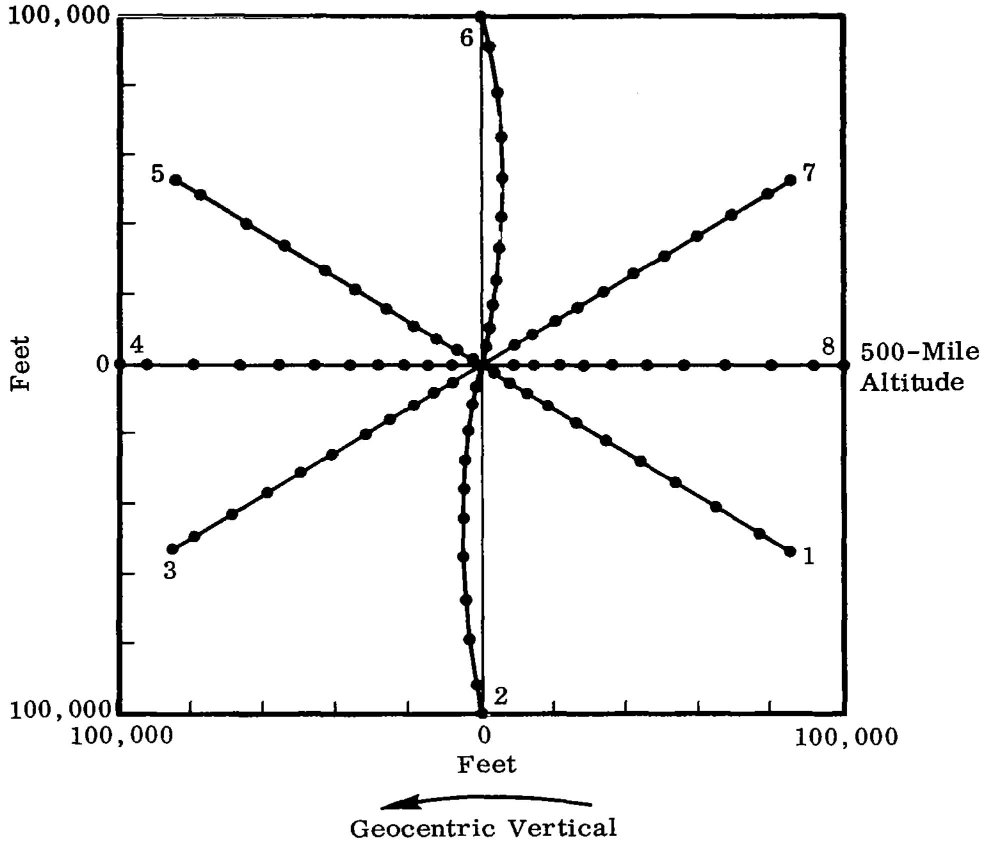
As indicated by Eq. (17), the rendezvous vehicle is thrusted both along and normal to the line of sight. Note the similarity of this technique to that used in missile homing. The first term of Eq. (17) provides the controlled closure in accordance with the previously explained range-range rate criterion. The second term directs the thrust such that the inertial rotation of the line of sight is minimized. Fig. 11 shows a typical case(9) using continuous thrusting from various initial conditions. Fig. 12 shows the result of not using the line-of-sight rotation term. As indicated, the rendezvous vehicle continues to rotate about the target rather than come to a complete stop. It should be noted that the initial conditions for Figs. 11 and 12 correspond to initial displacements of about 20 n. mi. Later work(8) has shown that the method is inefficient in terms of propellant consumption but is still useful for the terminal phase of rendezvous if its use is delayed until the range has been reduced to a couple of miles.

It should further be noted that in manned vehicles, the pilot may be able to control the terminal phase without aid of computation. He may be able to perform the entire maneuver himself with the aid of short-range optical distance measurements.

3.3 RENDEZVOUS AND DOCKING INTEGRATED CONTROL SYSTEM DESIGN

Docking control requirements greatly exceed those of rendezvous but also tend to include them. Accordingly, the docking control system is discussed first. Additional requirements for rendezvous control are then covered.

The docking control system requirements are derived in part from the critical nature of the mechanical mating of the two vehicles.



NOTES

1. Points are at 20-second intervals.
2. Velocity at each starting position equals circular velocity.
3. For all Trajectories:
   a. Time of transfer = 260 - 280 seconds.
   b. Characteristic velocity = 1450 ft/sec.
   c. Maximum applied acceleration = 70 ft/sec^2.

Figure 11. Terminal Rendezvous Trajectories (Angular Rate of Line of Sight)

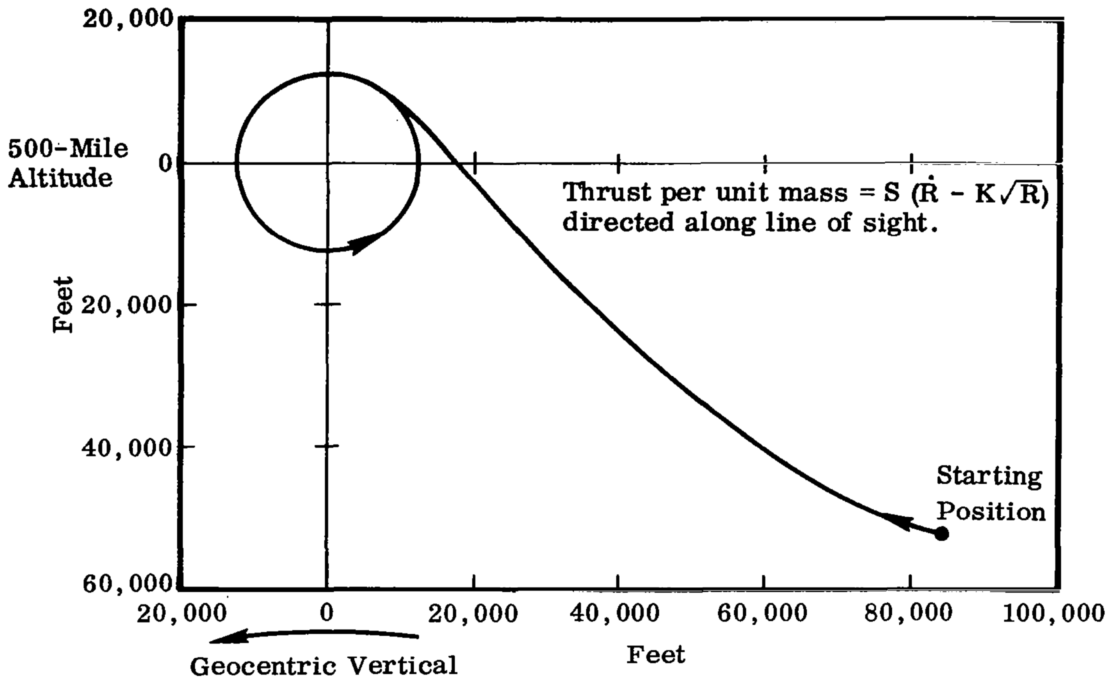


Figure 12. Terminal Rendezvous (No Line of Sight Information)

3.3.1 Docking Interface Dynamics

Obviously, if two vehicles are to be joined into a single body, they must be designed to mechanically mate. There are two basic categories of docking interfaces — extendible and impact. Typical configurations[2] are shown in Fig. 13.

Gemini and LEM use the impact type. Typical maximum initial impact conditions are 1 ft/sec longitudinally and ±0.5 ft/sec laterally, with a maximum angular misalignment of 10 degrees and a relative angular rate of 1 deg/sec.

Gemini uses the mating system illustrated by Fig. 14. The Agena cone absorbs energy when the spacecraft nose collides with it. This absorption of energy minimizes rebound. The pilot sets the attitude of the Gemini such that the index device enters the Y-shaped cut-out of the Agena receptacle, where spring-loaded latches are activated. The vehicles are then tightly drawn together.

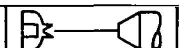
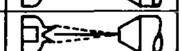
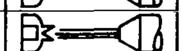
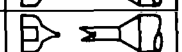
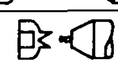
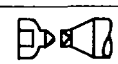
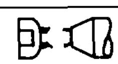
Extendible	Inflatable Probe	
	Stem	
	Stem & Cable	
	Inflatable Tunnel	
Impact	Center Probe & Drogue	
	Cone & Ring	
	Gemini	

Figure 13. Docking Concepts

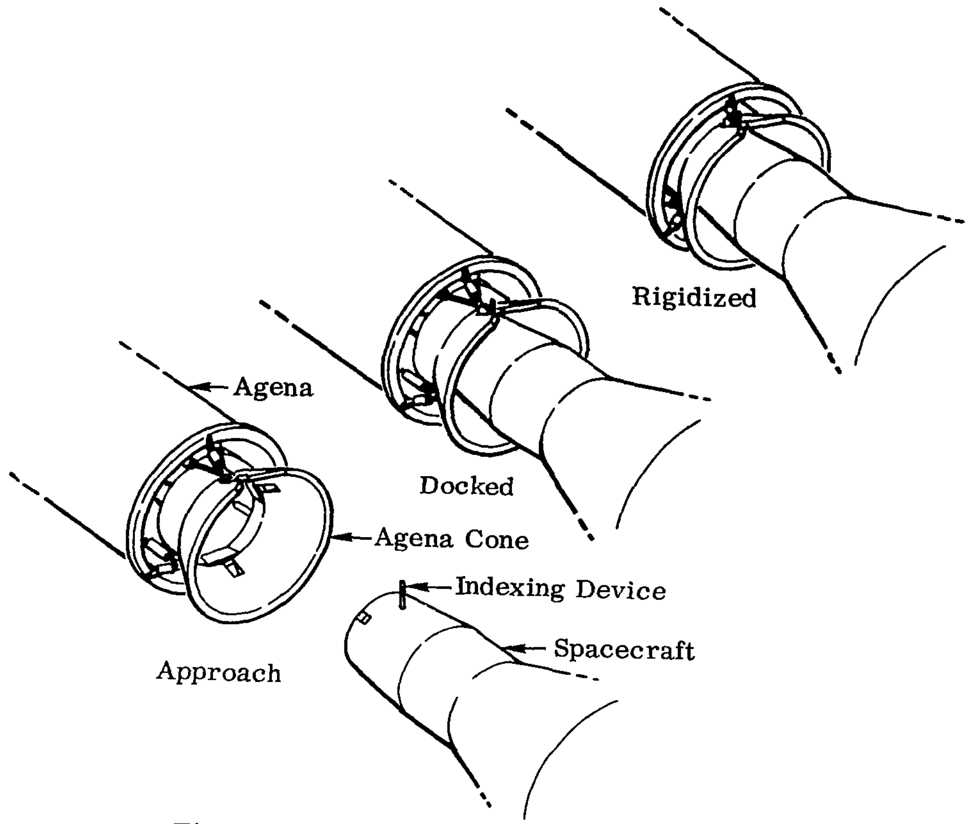


Figure 14. Gemini-Agena Docking System

The LEMC&S module mating interface, termed the probe-and-drogue concept,[2] is shown in Fig. 15. (See also Ref. 21.) On impact, the probe and drogue are latched.

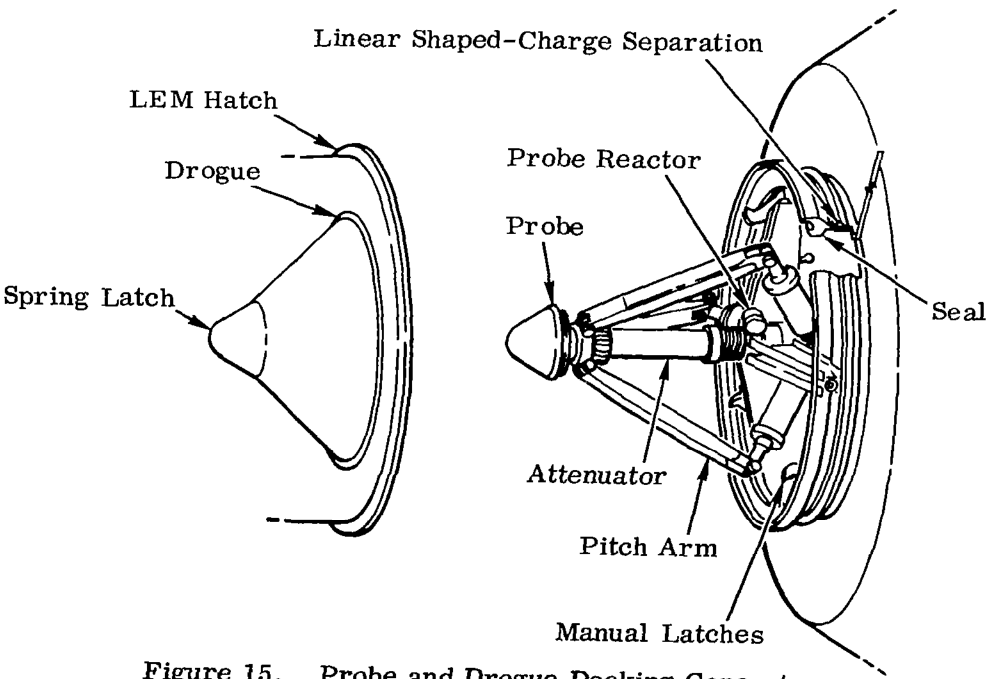


Figure 15. Probe and Drogue Docking Concept

The vehicles are then drawn together. It is assumed that the probe-and-drogue connection is later replaced by manual latches to allow the two-man crew of the LEM to pass through the docking interface to the C&S module. A charge separation is used to separate the C&S module from the LEM after the crew has transferred.

The analysis of such devices can be extremely lengthy because of the complexity of the mechanical interface. This interface, when reduced to a dynamic model, would be of the order of complexity illustrated by Fig. 16.[11] Following initial contact, simultaneous rotary and translational motion would take place as the rendezvous vehicle settled into the target vehicle receptacle. (See Fig. 17.)

It is impractical to demonstrate basic docking criteria with the complex device shown in Fig. 16. It is preferable to use much simpler problems. First consider translational motion only, as illustrated by Fig. 18. As indicated, only one degree of freedom is included. Prior to contact, the target vehicle is assumed to be moving at constant velocity and carrying a plate connected to its main body through a spring and viscous damper. The rendezvous vehicle is also traveling at constant velocity at a relative closing velocity, V (about one foot/second), and has just contacted the rendezvous vehicle damper plate at t = 0.

The system center of mass is used as X axis origin to derive the equations of motion. In particular, it is desired to calculate distance X as a function of time and initial relative velocity, V_0. The system center of mass can be used as an inert origin because it moves at constant velocity (equal to that existing before contact). This is true because no significant external forces are acting on the two vehicles considered

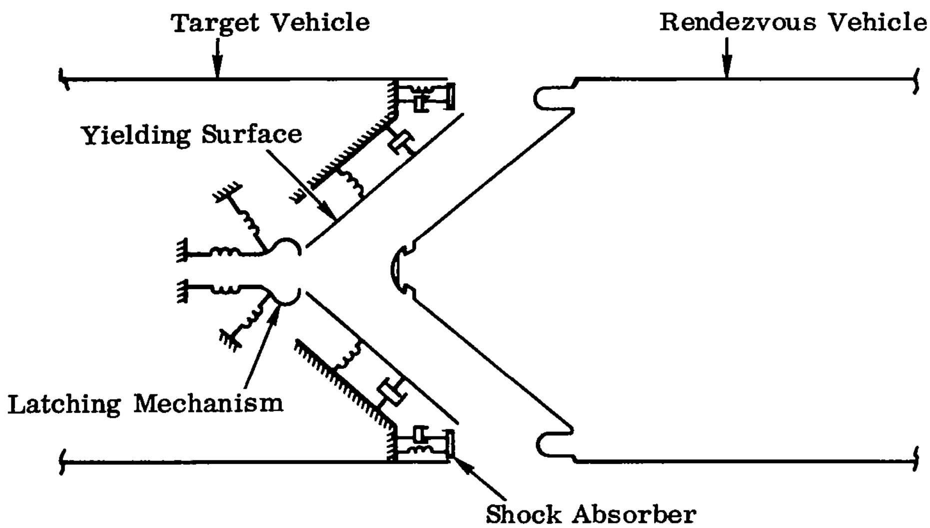


Figure 16. Schematic Cross Section of Docking Assembly

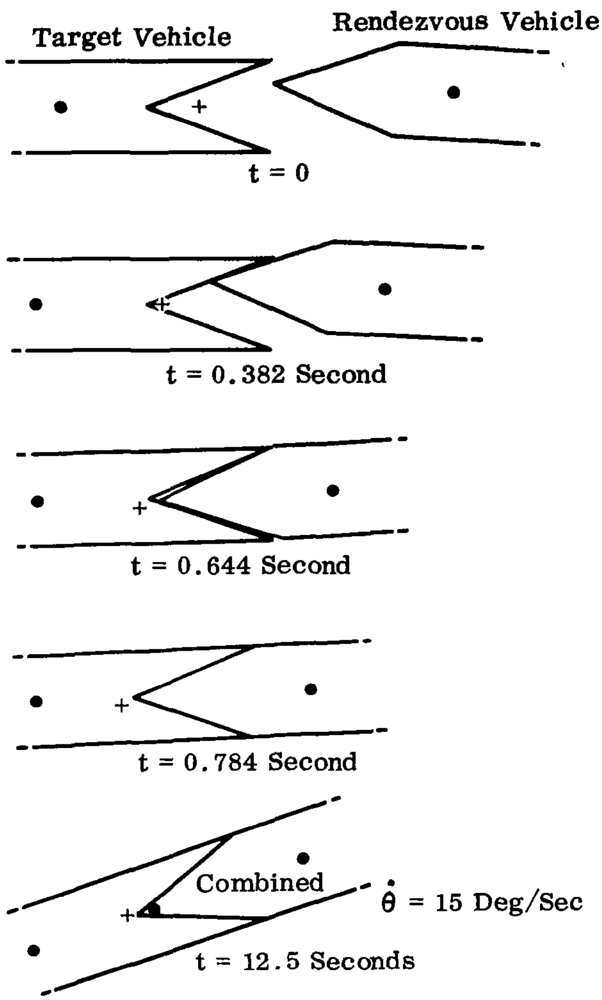


Figure 17. Relative Motion of Vehicles During Docking

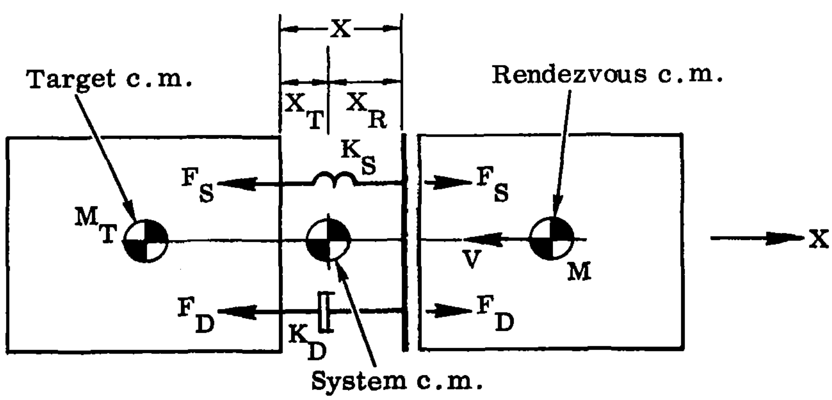


Figure 18. Final Docking Translation Motion Model

as a system. The sum of the internal forces (those of the damper and spring) is equal to zero because each force is balanced by an equal reaction. The spring and damper forces are

$$F_S = (X_o - X) K_S \tag{18}$$

$$F_D = -K_D \dot{X} \tag{19}$$

and

$$X = X_T + X_R \tag{20}$$

where

F_S is the force on both vehicles caused by spring compression

F_D is the force on both vehicles caused by the damper

K_S is the spring constant

K_D is the damper constant

X_T and X_R are the target and rendezvous vehicle displacements respectively from the system center of mass

The motion of the rendezvous and target vehicles centers of mass relative to the system center of mass is

$$F_S + F_D = M_T \ddot{X}_T = M_R \ddot{X}_R \tag{21}$$

where M_T and M_R are the masses of the target and rendezvous vehicles respectively.

It is desired to calculate the sum of X_T and X_R rather than the individual quantities, because knowledge of X gives the relative motion following contact. From Eqs. (20) and (21) the quantity X_T can be replaced by X. When this is done it is found that

$$M_T(\ddot{X} - \ddot{X}_R) = M_R \ddot{X}_R, \quad \ddot{X}_R = \frac{M_T}{M_R + M_T} \ddot{X} \tag{22}$$

Substituting (22), (18), and (19) into (21) yields one equation permitting calculation of X:

$$M\ddot{X} + K_D \dot{X} + K_S X = K_S X_o \tag{23}$$

where

$$M = \frac{M_R M_T}{M_T + M_R}$$

Solving Eq. (23) subject to the initial conditions

$$X_o = X_o, \dot{X}_o = -V$$

where

the subscript o indicates the value at $t = 0$

V is the magnitude of the initial closing velocity of the rendezvous vehicle relative to the target vehicle

gives the distance X as a function of time.

However, before solving the equation, it is helpful to define the penetration distance Y in terms of the distance X between the two vehicles as follows.

$$Y = X - X_o \tag{24}$$

Replacing X in Eq. (23) by Y gives

$$M\ddot{Y} + K_D\dot{Y} + K_S Y = 0 \tag{25}$$

The new initial conditions are easier to handle. They are

$$Y_o = 0, \dot{Y}_o = -V$$

The solutions for the penetration distance and velocity are

$$Y = -\frac{V}{\beta}\epsilon^{-\alpha t}\sin\beta t, \; \dot{Y} = -V\epsilon^{-\alpha t}\left(\cos\beta t - \frac{\alpha}{\beta}\sin\beta t\right) \tag{26}$$

where

$$\alpha = \frac{K_D}{2M}, \; \beta = \sqrt{\frac{K_S}{M}\left(1 - \frac{K_D^2}{4MK_S}\right)}$$

The angular rate, β, is, of course, the damped frequency. Additional control parameters are the familiar damping ratio and natural frequency as given by

$$W_n = \sqrt{\frac{K_S}{M}}, \quad \zeta = \frac{K_D}{2\sqrt{MK_S}} \tag{27}$$

If the masses are latched together sometime during the initial penetration, the maximum penetration occurs when $\sin \beta t = 1$ or at

$$t_M = \frac{\pi}{2\beta}, \quad Y_M = -\frac{V}{\beta}\epsilon^{-\frac{\alpha\pi}{2\beta}} \tag{28}$$

The value Y_M must not exceed the available penetration depth, X_o. One further criterion is that of the force loads on both vehicles:

$$F = F_D + F_S = K_D\dot{Y} + K_SY \tag{29}$$

where F is the total force on both vehicles.

A relatively simple planar analysis model can also be used for rotary docking motion. This model is illustrated in Fig. 19. As shown, the rendezvous and target vehicles have contacted and latched in such a way that a common hinge point has been established between the two vehicles.

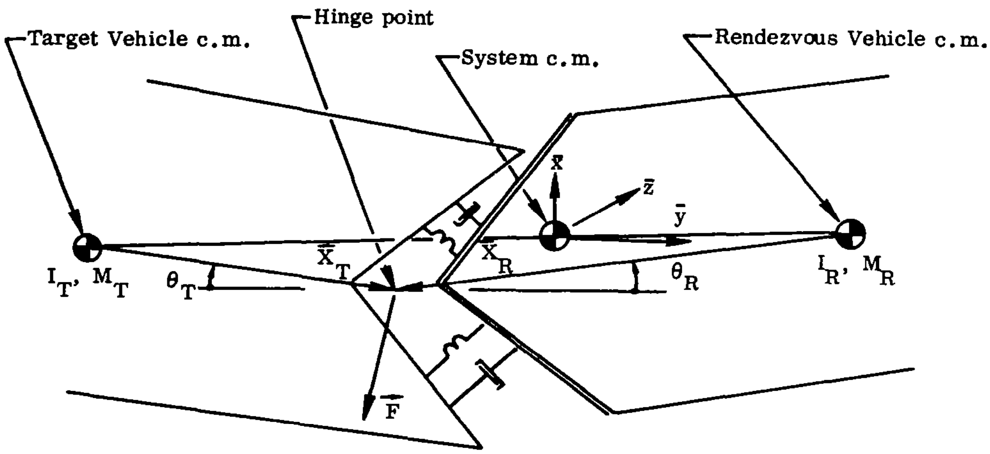


Figure 19. Final Docking Angular Motion Model

The equations of angular motion are not as simple to obtain as those given previously for linear motion because coupling between these two basic motions cannot be ignored. Appendix A was prepared to provide a general approach to the derivation of the coupled equations of motion for a two-body system. Derivation of the rotary equations of motion for the system shown in Fig. 19 starts from basic relations in this Appendix.

Conservation of angular momentum, calculated about the system center of mass (differentiate equation A14)* yields,

$$(I_T\ddot{\theta}_T - I_R\ddot{\theta}_R)\vec{z} + M(\vec{x}_T - \vec{x}_R) \times (\ddot{\vec{x}}_T - \ddot{\vec{x}}_R) = 0 \qquad (30a)$$

where $M = \dfrac{M_R M_T}{M_R + M_T}$

θ_R and θ_T are respectively the angular deviation of the rendezvous and target vehicles from an inert reference ($\vec{y}$ axis)

I_R and I_T are respectively the rendezvous and target vehicle moments of inertia about their center of mass

M_R and M_T are respectively the rendezvous and target masses

X_R and X_T are respectively the distances to the hinge point from the rendezvous and target centers of mass.

Conservation of linear momentum, calculated about the system center of mass, yields (see equation A7),

$$M(\ddot{\vec{x}}_T - \ddot{\vec{x}}_R) = -\vec{F} \qquad (30b)$$

where $\vec{F}$ is the hinge point reaction force acting on the target vehicle.

The torque on the target vehicle is due to the force $\vec{F}$ and that due to the spring damper assembly (assumed to produce a pure couple proportional to the relative angular displacement and rate of the two vehicles) is as follows.

$$\vec{T} = \vec{X}_T \times \vec{F} - \left[R_s(\theta_T + \theta_R) + R_D(\dot{\theta}_T + \dot{\theta}_R)\right]\vec{z} = I_T\ddot{\theta}_T\vec{z} \qquad (30c)$$

where $\vec{T}$ is the total torque acting on the target vehicle calculated about the target vehicle center of mass.

Also due to the restraint of the hinge and for small angular motions,

$$\vec{X}_T = X_T(\vec{y} - \theta_T\vec{x}), \qquad \vec{X}_R = X_R(-\vec{y} - \theta_R\vec{x}) \qquad (30d)$$

Substituting equation 30b into 30c eliminates the force $\vec{F}$. Performing the differentiations of 30d to expand the cross products indicated in 30a and 30c yields the following two relations,

*A14 is equation 14 of Appendix A.

$$\ddot{\theta}_T = \frac{I_R + MX_R(X_R + X_T)}{I_T + MX_T(X_R + X_T)}\ddot{\theta}_R \tag{30e}$$

$$R_s(\theta_T + \theta_R) + R_D(\dot{\theta}_T + \dot{\theta}_R) = -(I_T + MX_T^2)\ddot{\theta}_T + MX_T X_R \ddot{\theta}_R \tag{30f}$$

Define the relative angle

$$\theta = \theta_T + \theta_R \tag{31}$$

Using equations 30e and 31 to obtain θ_T and θ_R in terms of θ and substituting the result into 30f yields the desired result,

$$J\ddot{\theta} + R_D\dot{\theta} + R_s\theta = 0 \tag{32}$$

where $$J = \frac{I_T I_R + M(I_T X_R^2 + I_R X_T^2)}{I_T + I_R + M(X_T + X_R)^2} \tag{32}$$

This equation is identical in form to that for translation. See Eq. (23). The appropriate initial conditions are

$$\theta_o = \theta_{R_o} + \theta_{T_o}, \; \dot{\theta}_o = \dot{\theta}_{T_o} + \dot{\theta}_{R_o}$$

where the subscript o again means the value at $t = 0$ (contact time).

If the initial relative angular displacement is zero, the angular deviation and rate functions of time are identical in form to those given in Eq. (26):

$$\theta = \frac{\dot{\theta}_o}{\beta}\epsilon^{-\alpha t}\sin\beta t, \; \dot{\theta} = \dot{\theta}_o\epsilon^{-\alpha t}(\cos\beta t - \frac{\alpha}{\beta}\sin\beta t) \tag{33}$$

where

$$\alpha = \frac{R_D}{2J}, \; \beta = \sqrt{\frac{R_S}{J}\left(1 - \frac{R_D^2}{4JR_S}\right)}$$

The natural frequency and damping ratio are

$$W_n = \sqrt{\frac{R_S}{J}} \quad , \quad \zeta = \frac{R_D}{2\sqrt{JR_S}} \tag{34}$$

The same criteria identified for translational motion apply for rotary motion. However, the effect of an initial condition in angular displacement (not given herein) should be included.

It should be noted that if the models for translational and rotary motion are combined, the more complex configuration of Fig. 16 results. If the same configuration is repeated in a plane normal to that shown in Fig. 16, a complete system for damping out relative motion in all directions results.

After relative motion has been damped out, the rendezvous and target vehicles are drawn together. In effect, this drawing together rigidizes the damper spring assemblies and thereby rigidizes the two bodies into a single body. Having the cone-shaped receptacle on the target and plug on the rendezvous vehicle seems popular but apparently the location can be reversed with essentially the same results.

The cone angle is, of course, an important design figure. The procedure for determining a suitable angle is not considered in detail here. Its function is to guide sliding into a latching mechanism located at the cone apex. The sliding action of the plug into the cone is greatly enhanced by slippery contact surfaces and small cone angles. On the other hand, small angles reduce the aperture that the plug must enter for a specific cone length. The choice of angle is, then, a compromise between cone aperture and sliding action.

In the final design phase, a complete simulation of the relative motion equations is necessary. Refs. 11, 12, 13, and Appendix A will be found helpful in generating the equations and identifying desired results.

3.3.2 Docking Control

The initial docking phase conditions essentially correspond to station-keeping at a distance of from up to a couple of hundred feet to about 50 feet, depending upon the point at which the closeness of the target precludes the radar determination of relative range data. In the docking phase, this distance is closed while the rendezvous vehicle is nosed into the docking receptacle with minimum attitude and attitude rate relative to the target vehicle. Manned control of this phase has been the technique considered to date. There is little information on completely automatic docking.

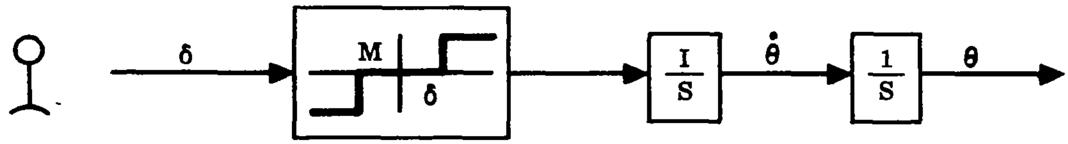
All control design parameters have been determined to date by simulation of the docking problem. A variety of simulators has been used. In all cases, it has been assumed that the target vehicle could be held stationary; only the motion of the rendezvous vehicle has been simulated. There are two basic types of simulators: fixed-base and moving-base. The fixed-base type(16, 17, 18) involves a fixed rendezvous vehicle, with relative motion between the two vehicles simulated by a TV camera and projector. The pilot "sees," through a cockpit window, the variation in attitude and distance to the target as he applies control inputs. The moving-base type(14, 15, 16) involves actual motion of the rendezvous vehicle in response to pilot commands. Both simulators are further identified by the degrees of freedom provided. The maximum, of course, is six — three for translation and three for rotation. Some simulators eliminate degrees of freedom in translation, since translating the entire vehicle requires great mechanical complexity. In these simulations, two three-axis hand controls are provided: one for rotation commands, the other for translation commands. Normally, the pilot uses both controls simultaneously.

Control moments and forces are generated using on-off reaction jets, which, in all reported cases, were considered adequate for docking. There is no information on the use of other control power sources. The configuration and sizing of these jets will be discussed later in this monograph. For present purposes, it is assumed that three-axis relative position and attitude control is available from these jets.

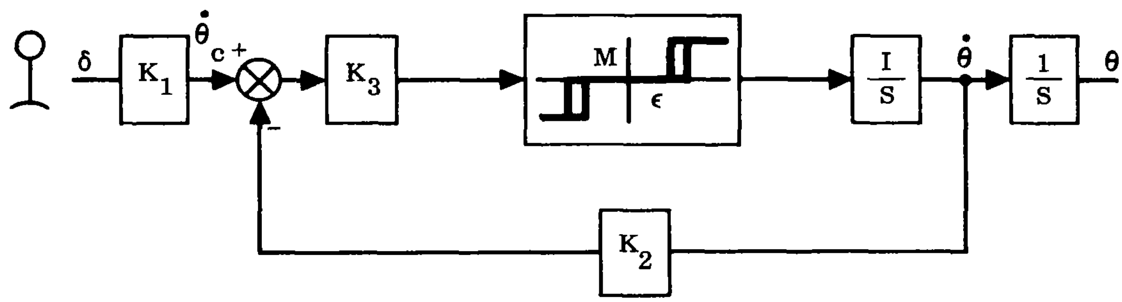
Simulation results indicate that translation control requires no stabilization feedback signals. The pilot controls translation in any or all of the three axes by commanding the appropriate jets to fire. Three configurations have been considered for attitude control: direct, rate command, and rate command with attitude hold. Control block diagrams for the three are illustrated in Fig. 20. The direct type has no stability augmentation; the pilot flies the vehicle without the aid of rate or position feedback (the same as used for translation control). The rate command type uses attitude rate feedback and incorporates the rate deadzone and hysteresis commonly used in reaction jet control systems. When the pilot reaches the attitude he wishes to maintain, the control system will allow only low rate changes in attitude if the attitude control is released or centered. This enables the pilot to concentrate more on translational control. The rate command with attitude hold type further aids the pilot in that when he reaches the desired attitude and centers the control handle, the attitude is subsequently maintained automatically.

Simulation results indicate that the direct type can be used for a backup in the event of control circuitry failure. Either the rate command type or rate command with attitude hold type will be the primary control.

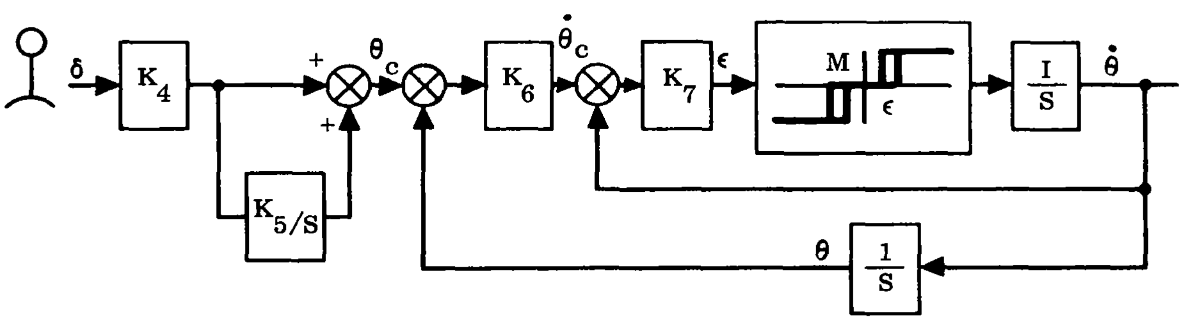
Three possible engine-on modes should be considered. In the first, the pilot turns the appropriate jet on by displacing the control handle away from center and off when he returns it to center. In the second, moving the control handle away from center causes a short time interval thrust (a particular value of impulse). The pilot must return the handle to center and displace it again in order to command another impulse. In the third, the on and off commands are the same as in the first mode, but



Direct Attitude Control



Rate Command Attitude Control



Rate Command with Attitude Hold Attitude Control

Figure 20. Autopilot Attitude Control Configuration

the command to the engine is a series of "on" pulses. The frequency of the fixed-width pulses may be increased with stick deflection, or the frequency may be held constant and the pulse width increased. Either pulse mode produces the same effect. That is, the integrated effect of the "on" command is the average of the pulses. During an "on" command an effective acceleration somewhat proportional to stick deflection is produced. This fundamental feature of pulsed systems may be helpful.

The procedures for analysis of the rate and rate command with attitude hold are covered in Vol. II, Part 2, of this series of monographs, Nonlinear Systems. Some typical numerical results arrived at in past Gemini-Agena and LEM-C&S docking simulations are listed below.

Rate Command Dead Band	0.5 deg/sec
Rate Command with Attitude Deadband	±2 deg
Maximum Rate Command	25 deg/sec
Rate Hysteresis	0.1 deg/sec
Control Stick Sensitivity Rate Command	~10 deg/sec/in.

3.3.3 Visual Docking Aids

The addition of short-range visual docking aid devices is considered of sufficient value that they are included in the LEM/CSM systems. The requirements for these aids in this specific application may be found in Ref. 22.

a. Visual acquisition and gross attitude determination at a minimum distance of 1000 feet.

b. Visual acquisition of relative attitude and alignment (docking) from a minimum distance of 200 feet.

c. Visual range and range rate indication from a distance of 200 feet.

Requirement a. has been implemented by the use of running lights similar to those used in marine applications. Several different colored lights are mounted on the target vehicle so that the human pilot's capability to visually detect the relative location and grossly determine attitude is enhanced. Fig. 21 illustrates the placement of most of these lights on the LEM vehicle. These lights aid the CSM pilot when the LEM is the target vehicle.

There is also a different, but similar, set of running lights on the CSM which aid the LEM pilots when the docking situation is reversed.

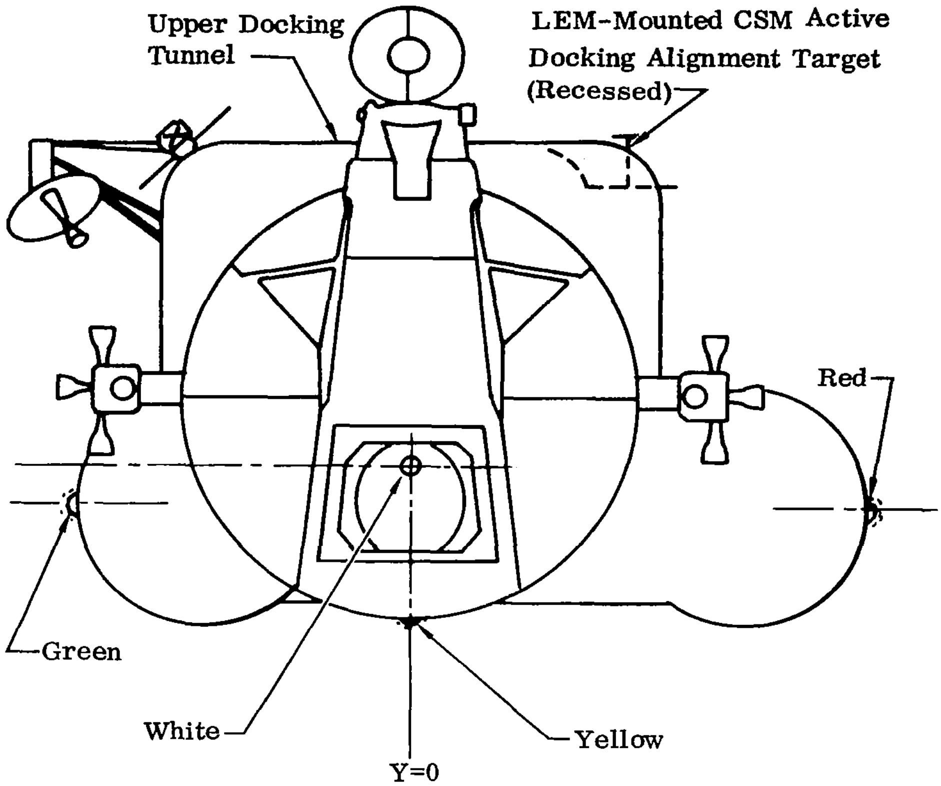


Figure 21. LEM Mounted Orientation Lights (Forward View)

Also note the location of the CSM active docking alignment target device on Fig. 21. The device is termed a standoff cross (SOC) and is illustrated in Fig. 22. Illumination of the designated light and dark areas would be provided if night-time operation is required. In operation the CSM pilot looks through an optical sight and drives the vehicle to center the cross in the target circle. The deviation from the true center position is visually indicated by the position of the legs of the cross relative to the corresponding radial markings on the background circle.

Practical considerations require offsetting the SOC from a natural position in the center of the docking mating interface. The mounting locations of the pilot optical sight on the rendezvous vehicle, the SOC on the target vehicle, and the mating interface all enter into configuring the complete system. In operation, the mounting constraints are illustrated in Fig. 23. Note the identical off-centering of the SOC and pilot optical aid and the alignment of the optical aid line of sight with the desired entry line into the docking receptacle.

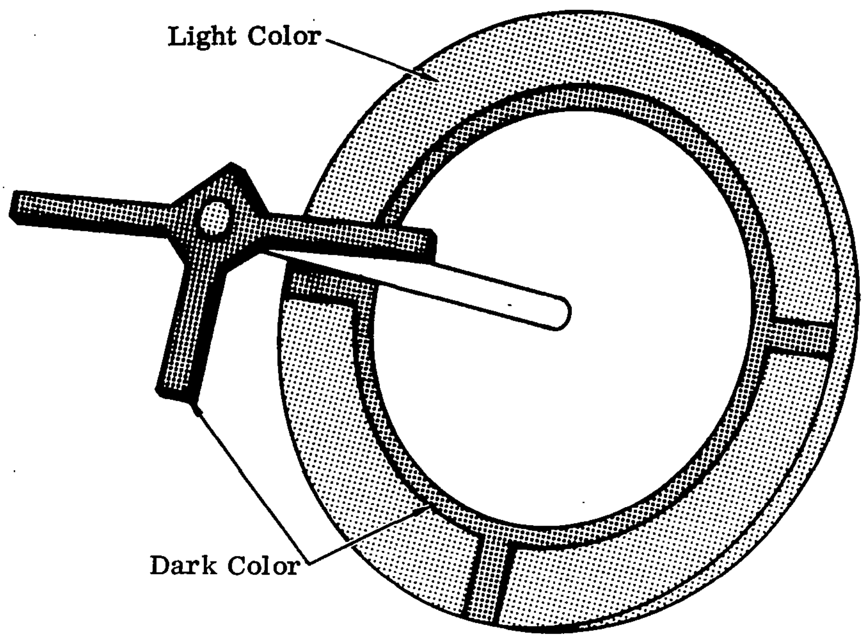


Figure 22. LEM and CSM Mounted Docking Alignment Targets

The SOC image as seen in the pilot optical sight gives relative attitude and alignment. In addition, the SOC image size in the pilot optics changes as a function of range. This image, intercepted by appropriate markings on the optical sight, will afford a gross indication of range. When timed, a gross indication of range rate results.

3.3.4 Docking Propulsion System Configuration and Sizing

Docking requires propulsion jet configuration capable of generating simultaneous rotary and translation accelerations. Ideally, an attitude command should generate pure rotary couples, and a translation command should generate forces that do not simultaneously place a torque on the vehicle. This ideal configuration would allow translation corrections to be effected without causing an attitude change and vice-versa. Consider the example illustrated by Fig. 24.

Assume that the pilot has moved the rendezvous vehicle up to the mechanical interface, with the attitudes aligned, and now desires to center the rendezvous vehicle nose into the target vehicle receptacle. The lateral offset acceleration of the rendezvous vehicle nose from the target vehicle receptacle center caused by activating the jet is

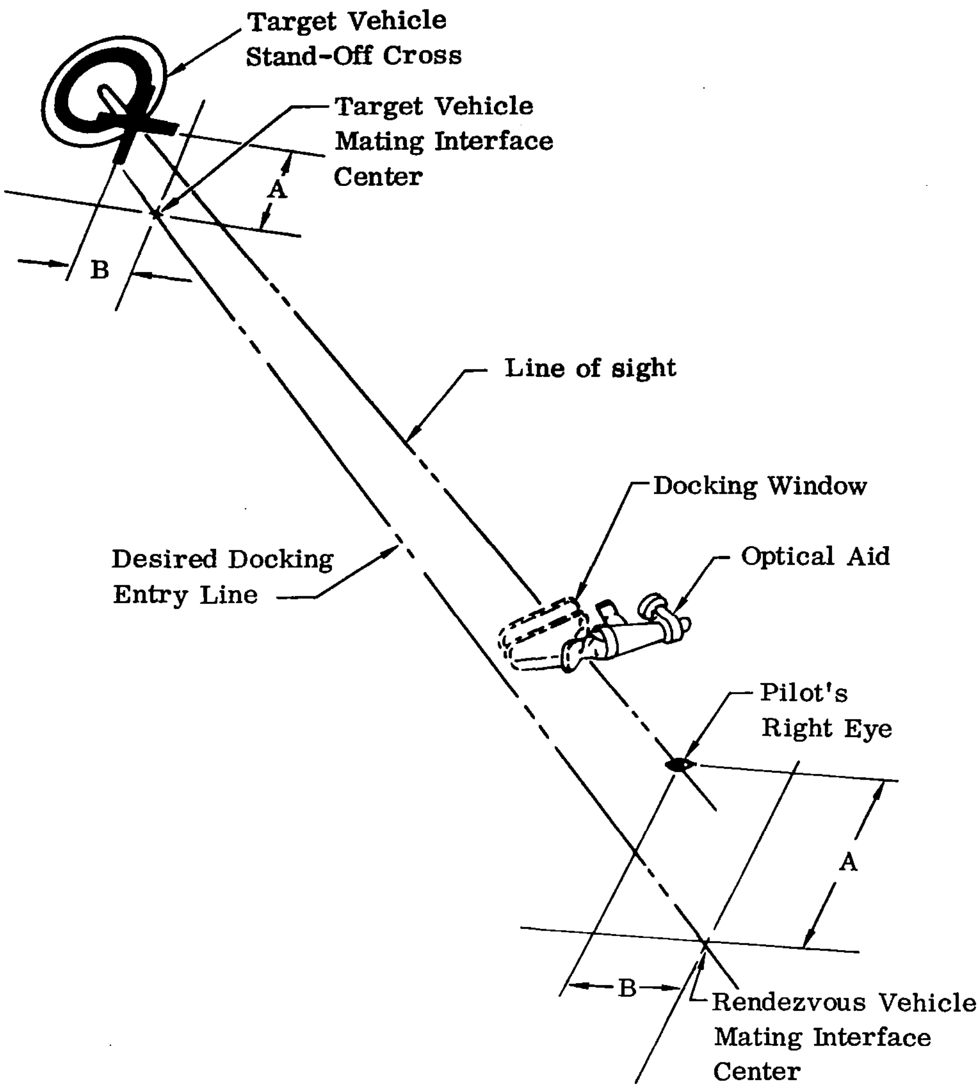


Figure 23. Docking Alignment Aid Geometry

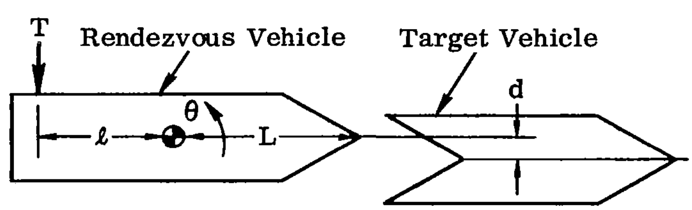


Figure 24. Effect of Offsetting Translation Jets from the C.M.

$$\ddot{\theta} = \frac{T\ell}{J}, \ \ddot{d} = T\left(\frac{1}{M} - \frac{\ell L}{J}\right) = \frac{T}{M}\left(1 - \frac{\ell L}{R^2}\right) \tag{35}$$

where

- θ is the relative angle between the two vehicles
- T is the reaction jet thrust level
- d is the lateral offset
- ℓ is the distance between the vehicle c.m. and the jet
- L is the distance between the vehicle c.m. and the rendezvous vehicle nose
- R is the radius of gyration of the rendezvous vehicle
- M is the vehicle mass
- J is the vehicle moment of inertia

For maximum rear locations of the jet, the quantity ℓL would be greater than the radius of gyration squared. The result would be an undesired angular offset accompanied by a change in lateral position due more to angular deviation than translation. The distance from the c.m. to the mating interface is not usually under control. It is a design input. However, locating the jet so that its firing line is directly through the c.m. (making $\ell = 0$) removes the undesirable simultaneous angular offset. The point of this example is that the control configuration should use translation jet locations whose effective firing line is as close as possible to passing through the vehicle c.m.

There is more design latitude in approaching the angular control ideal -- generating pure moments. This is because the pilot will tend to align the vehicle attitude when the separation distance is large. The attitude rate or attitude rate with attitude hold control configurations help the pilot to maintain this attitude adjustment. Subsequently, he will concentrate on lateral translation corrections and maintain a closing rate somewhat under 1 ft/sec. Thus a relatively small translation accompanying a attitude command is tolerable.

A docking design should:

a. Provide for simultaneous execution of attitude and translation commands.

b. Minimize coupling between translation and rotation commands, especially from translation to rotation.

c. Provide for jet fail safety.

d. Have primary attitude control through the attitude rate or attitude rate with attitude hold configurations and a backup mode of direct command coupling to the jets.

Control jet configurations satisfying the above criteria are presently used on the LEM and Gemini vehicles. Consider the LEM configuration shown in Figure 22, which shows the LEM c.m. in the plane of the jets. In the actual design, this may not exactly be the case. It is expected that one purpose of detail simulation study will be to determine a tolerable longitudinal and control offset.

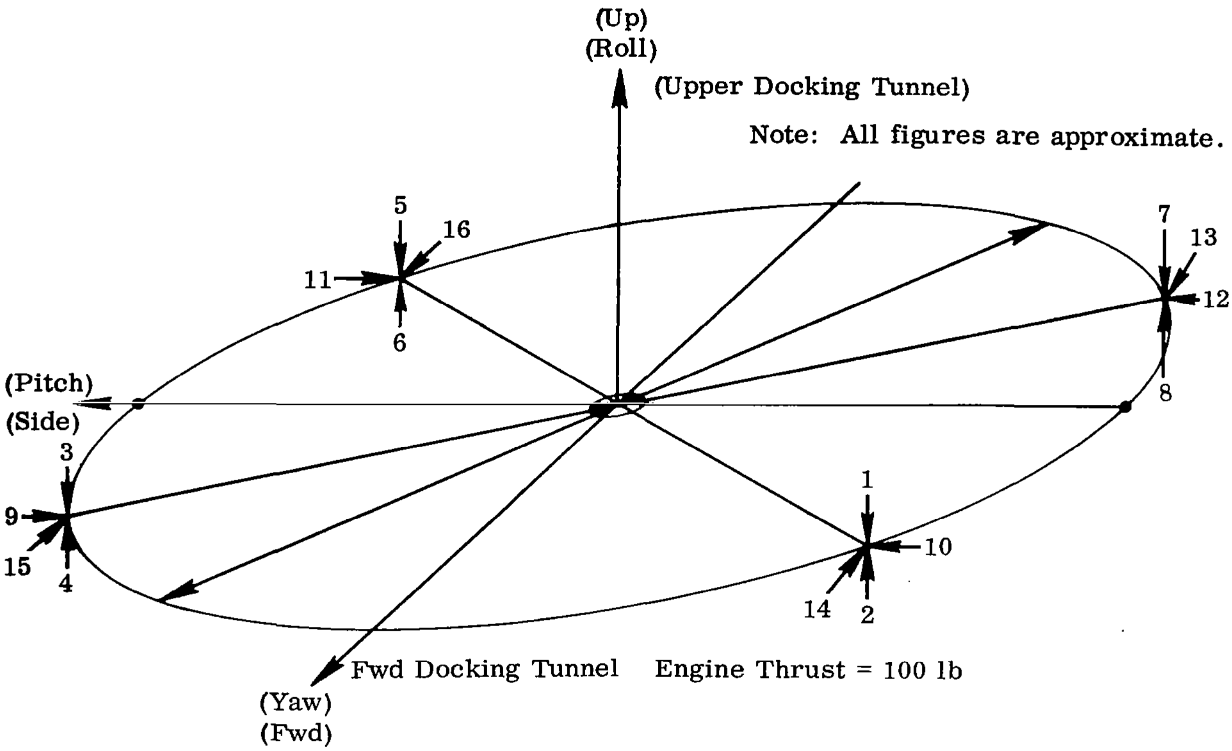


Figure 25. LEM Docking Propulsion Configuration

Figure 25 identifies pitch, yaw, and roll rotational control axes and up, side, and forward translational axes. Pure couples and forces (excluding combined commands) are generated by firing the engines as shown in Table 1. Engines 1 through 8 are used for plus and minus pitch, yaw, and up; engines 9 through 16 are used for plus and minus roll, side, and forward. A one in the table indicates that the engine is on.

It is desired to both analyze the configuration according to criteria a through d listed above and also to write the engine logic equations relating engine on-off to the commands. The basic individual commands are identified in Table 1 and it has been indicated that the entire problem can be broken up into two smaller problems involving engines 1 through 8 and engines 9 through 16. Only engines 1 through 8 will be considered here; the procedure for engines 9 through 16 is identical.

Table 1. Basic Engine Firing Logic

Desired Command	Engine No. 1	2	3	4	5	6	7	8
-Pitch (P_-)	1		1			1		1
+Pitch (P_+)		1		1	1		1	
-Yaw (Y_-)	1			1		1	1	
+Yaw (Y_+)		1	1		1			1
-Up (U_-)	1		1		1		1	
+Up (U_+)		1		1		1		1

Desired Command	Engine No. 9	10	11	12	13	14	15	16
-Roll (R_-)		1	1		1		1	
+Roll (R_+)	1			1		1		1
-Side (S_-)	1		1					
+Side (S_+)		1		1				
-Fwd (F_-)						1	1	
+Fwd (F_+)					1			1

All possible command combinations must be considered. In addition to the 6 basic uncombined modes for engines 1 through 8, there are 20 combined possibilities, as shown in Table 2. The table shows what engines are commanded, with each individual command making up the combined command taken into account. If two separate commands out of the combined command cause the same engine to fire, that engine is already on and only the additional engines need be noted. If two engines having the same line of fire but in opposite directions are commanded on, the result is the same as if they are both off; for purposes of fuel conservation, neither is commanded on. In Table 2, this situation is indicated by a zero over a one.

There are a number of useful bits of information which can be derived from inspection of Tables 1 and 2. Not commanding opposite engines to fire is worthwhile because of the large number of zeros superimposed over the ones when combined commands are applied. Further, two engines are commanded on for two-axis commands and one for three-axis commands. Obviously, for three-axis commands the system is not fail safe, while for one- and two-axis commands, it is. However, if a jet fails for one- or two-axis commands, coupling between translation and rotation motion will make the system harder to control.

The tables define the logical relations between the commands and the jet firing solenoids. Consider the engine-on logic for engine 1.

$$\left.\begin{aligned} E_1 = \; & P_-(\bar{P}_+\bar{Y}_-\bar{Y}_+\bar{U}_-\bar{U}_+) && \text{- Pitch Command} \\ & + Y_-(\bar{P}_-\bar{P}_+\bar{Y}_+\bar{U}_-\bar{U}_+) && \text{- Yaw Command} \\ & + U_-(\bar{P}_-\bar{P}_+\bar{Y}_-\bar{Y}_+\bar{U}_+) && \text{- Up Command} \\ & + (P_-Y_-)(\bar{P}_+\bar{Y}_+\bar{U}_-\bar{U}_+) && \text{- Pitch and - Yaw Command} \end{aligned}\right\} \quad (36)$$

$$+ \quad (P_- U_-)(\bar{P}_+ \bar{Y}_- \bar{Y}_+ \bar{U}_+) \qquad \text{- Pitch and - Up Command}$$

$$+ \quad (Y_- U_-)(\bar{P}_+ \bar{P}_- \bar{Y}_+ \bar{U}_+) \qquad \text{- Yaw and - Up Command}$$

$$+ \quad (P_- Y_- U_-)(\bar{P}_+ \bar{Y}_+ \bar{U}_+) \qquad \text{- Pitch, - Yaw, and - Up Command}$$

where the bar over the quantity means the logical complement; e.g. $\bar{P}_+$ equals 1 when + pitch command is not true.

Table 2. Combined Commands Firing Logic

Commands						Engine No.								
P_-	P_+	Y_-	Y_+	U_-	U_+	1	2	3	4	5	6	7	8	
1		1				1		0	0		1	0	0	Two-Axis Commands
1			1			0	0	1		0	0		1	
1				1		1		1		0	0	0	0	
1					1	0	0	0	0		1		1	
	1	1				0	0		1	0	0	1		
	1		1				1	0	0	1		0	0	
	1			1		0	0	0	0	1		1		
	1				1		1		1	0	0	0	0	
		1		1		1		0	0	0	0	1		
		1			1	0	0		1		1	0	0	
			1	1		0	0	1		1		0	0	
			1		1		1	0	0	0	0		1	
1		1		1		1		0	0	0	0	0	0	Three-Axis Commands
1		1			1	0	0	0	0		1	0	0	
1			1	1		0	0	1		0	0	0	0	
1			1		1	0	0	0	0	0	0		1	
	1	1		1		0	0	0	0	0	0	1		
	1	1			1	0	0		1	0	0	0	0	
	1		1	1		0	0	0	0	1		0	0	
	1		1		1		1	0	0	0	0	0	0	

Algebraic manipulation of Equation (36) reduces it to the simpler form:

$$E_1 = \left[P_-\bar{Y}_-\bar{U}_- + Y_-\bar{P}_-\bar{U}_- + U_-\bar{P}_-\bar{Y}_- + P_-Y_-\bar{U}_- + P_-U_-\bar{Y}_- + Y_-U_-\bar{P}_- + P_-Y_-U_-\right]\bar{P}_+\bar{Y}_+\bar{U}_+ \tag{37}$$

$$E_1 = \left[P_-(\bar{Y}_-\bar{U}_- + Y_-\bar{U}_- + U_-\bar{Y}_- + Y_-U_-) + \bar{P}_-(\bar{U}_-Y_- + U_-\bar{Y}_- + Y_-U_-)\right]\bar{P}_+\bar{Y}_+\bar{U}_+ \tag{38}$$

$$E_1 = \left\{P_-(\bar{Y}_- + Y_-)(\bar{U}_- + U_-) + \bar{P}_-\left[\bar{U}_-Y_- + U_-(\bar{Y}_- + Y_-)\right]\right\}\bar{P}_+\bar{Y}_+\bar{U}_+ \tag{39}$$

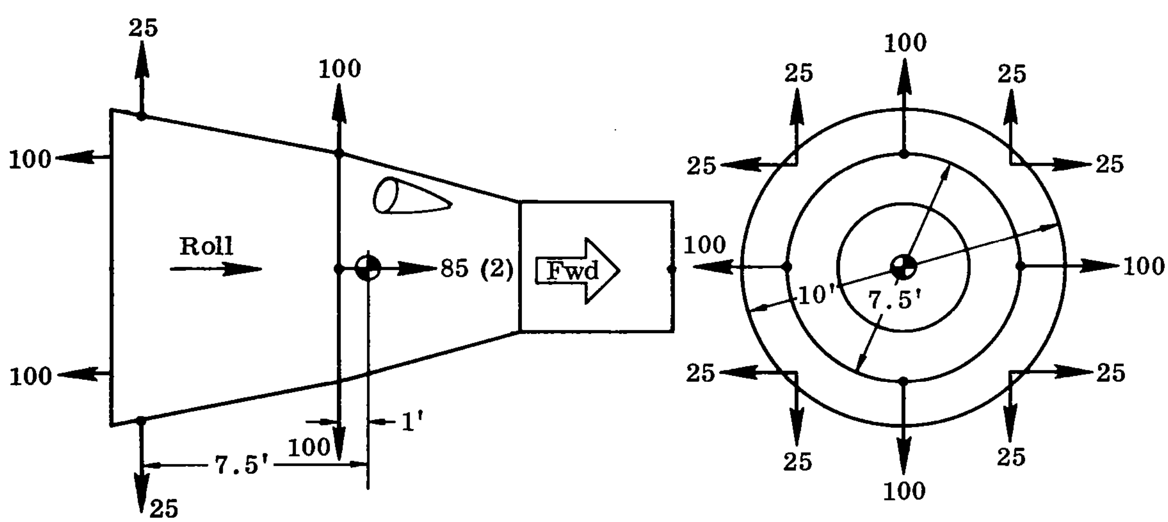
But $\bar{Y}_- + Y_- = 1$ and $\bar{U}_- + U_- = 0$.

Therefore

$$E_1 = \left[P_- + \bar{P}_-(\bar{U}_-Y_- + U_-)\right]\bar{P}_+\bar{Y}_+\bar{U}_+ \tag{40}$$

The same procedure is followed for all engines.

For reference, the Gemini configuration is shown in Fig. 26.



NOTES
1. All figures are approximate.
2. Engine thrusts are in pounds.

Figure 26. Gemini Docking Attitude and Translation Propulsion Control Configuration

The four aft 25-lb, three-jet clusters are used for pitch, yaw, and roll attitude control. The remaining engines are used for translation. As indicated, the effective line of fire of the translation jets is nearly through the c.m. Table 3 shows approximate angular and linear acceleration capability for the LEM and Gemini vehicles except that in accordance with Ref. 18, an effective thrust level of 25 lb rather than 100 lb is used for LEM. It is not clear from the literature whether LEM will use 100-lb engines or use the pulse mode to reduce their effectivity. The sizing shown applies when the attitude control rate command mode rather than the direct mode is used.

Table 3. Sizing of Gemini and LEM Docking Controls

Translation (ft/sec^2)	Gemini	LEM
Axial	0.8	0.4
Lateral	0.4	0.4
Rotation (deg/sec^2)		
Roll	12.6	25
Pitch and yaw	4.3	10

When the pilot uses the rate command (or rate command with attitude hold) configuration, it would be expected that the permissible control accelerations would be increased relative to those for the direct mode. This effect is illustrated by Fig. 27.

Data for this figure was obtained from Ref. 14. The "acceptable," "satisfactory," "unacceptable," and "undefined" regions all pertain to the direct mode of attitude control. The Gemini and LEM data of Table 3 is also graphed on Fig. 27 to show the increase in permissible control acceleration resulting from the use of the rate command attitude configuration.

3.3.5 Rendezvous Control System Design

The control and propulsion configurations presented earlier contain the essential elements needed to provide control during the midcourse and terminal phases. These elements are, however, used differently.

Consider the midcourse phase. As mentioned in paragraph 3.2.1, the major control requirement is accurate execution of several ΔV vector corrections and relatively coarse attitude control between these corrections. These requirements are not peculiar to rendezvous. They are, for example, functionally identical to those for translunar trajectory correction.

There are a number of ways to accomplish the ΔV correction[19]. Perhaps the most direct method applicable to manned rendezvous involves rotating the vehicle longitudinal axis into alignment with the required ΔV correction. The alignment is

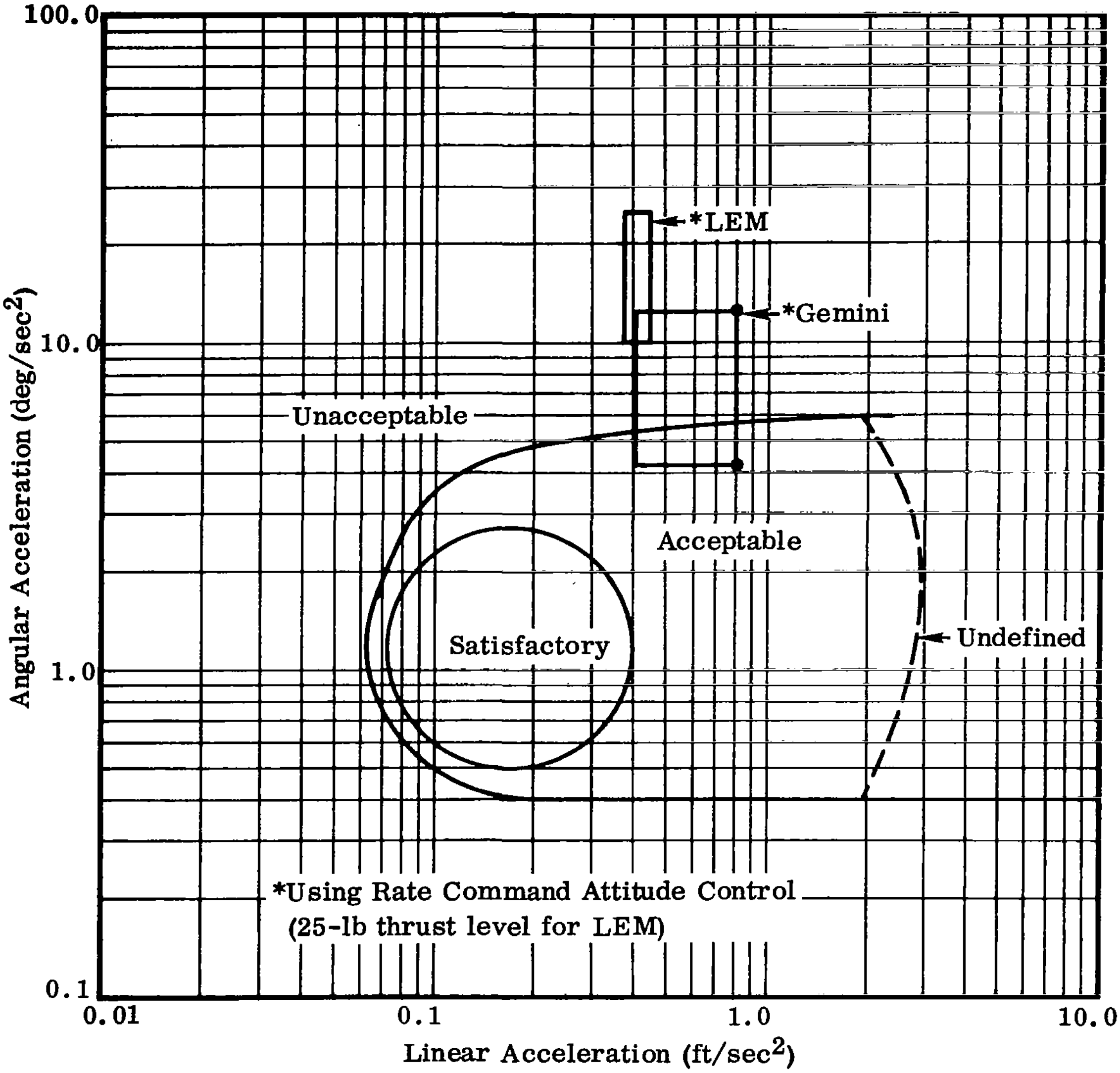


Figure 27. Propulsion System Sizing for Direct Attitude Control Mode

then maintained while axial jets are fired until the ΔV correction is completed. This attitude alignment could be accomplished by the pilot through his attitude control stick as directed by a display. He could also, again by reference to a ΔV display, command the axial jets on until the correction was completed. The maneuver could be automatic, by inserting the attitude and velocity correction commands into the autopilot.

The attitude error signal would depend upon the existing and desired attitudes of the vehicle thrust axis. The existing thrust axis can be brought into alignment by rotating the vehicle as illustrated by Fig. 28.

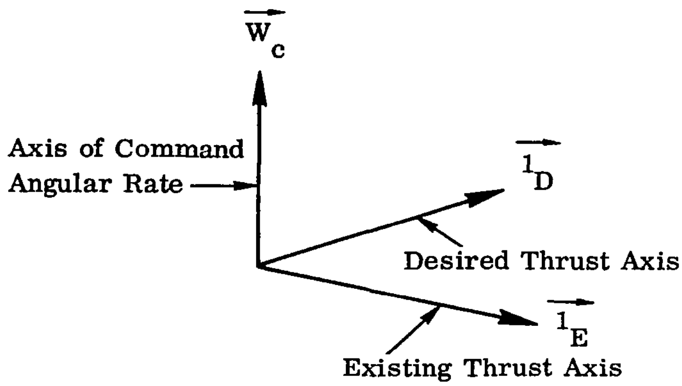


Figure 28. Attitude Steering

The control law is

$$\vec{W}_c = K(\vec{1}_E \times \vec{1}_D) \tag{41}$$

where

$\vec{W}_c$ is the angular rate command vector

$\vec{1}_E$ is a unit vector aligned with the existing thrust axis

$\vec{1}_D$ is a unit vector aligned with the desired ΔV vector

K is a fixed control constant

It is usually most convenient to evaluate the desired ΔV unit vector in a frame of reference fixed to the rendezvous vehicle axes because the value of the existing thrust vector is invariant here. Assume that this is done and that the thrust axis is aligned with the X axis of the vehicle. Then

$$\vec{W}_C = K \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} X_D \\ Y_D \\ Z_D \end{bmatrix} = K \begin{bmatrix} 0 \\ -Z_D \\ Y_D \end{bmatrix} \tag{42}$$

where X_D, Y_D, Z_D are respectively the x, y, z axis components of $\vec{1}_D$ in the vehicle fixed frame of reference.

Z_D and Y_D are directly related to the attitude angles of the vehicle with respect to an arbitrary inert reference frame and are termed pitch and yaw attitude errors (assuming that the thrust axis is also the vehicle roll axis). The pitch and yaw angular rate commands are proportional to these error signals. They become zero when the vehicle roll or thrust axis has assumed the proper attitude. The pitch and yaw errors are inserted in the autopilot (see Fig. 20) to command the appropriate proportional angular rates until the desired attitude is reached. Subsequently, during application of axial thrust, attitude is automatically maintained. As for the docking phase, it is again desirable that axial thrust coupling into rotation be minimized.

3.4 RENDEZVOUS AND DOCKING INTEGRATED GUIDANCE/CONTROL SYSTEM

An integrated configuration of the guidance and control system is briefly discussed here. Fig. 29 is a block diagram of the system used by the Gemini vehicle. The complete system is divided into pilot displays, the sensing and computing system, and the control system.

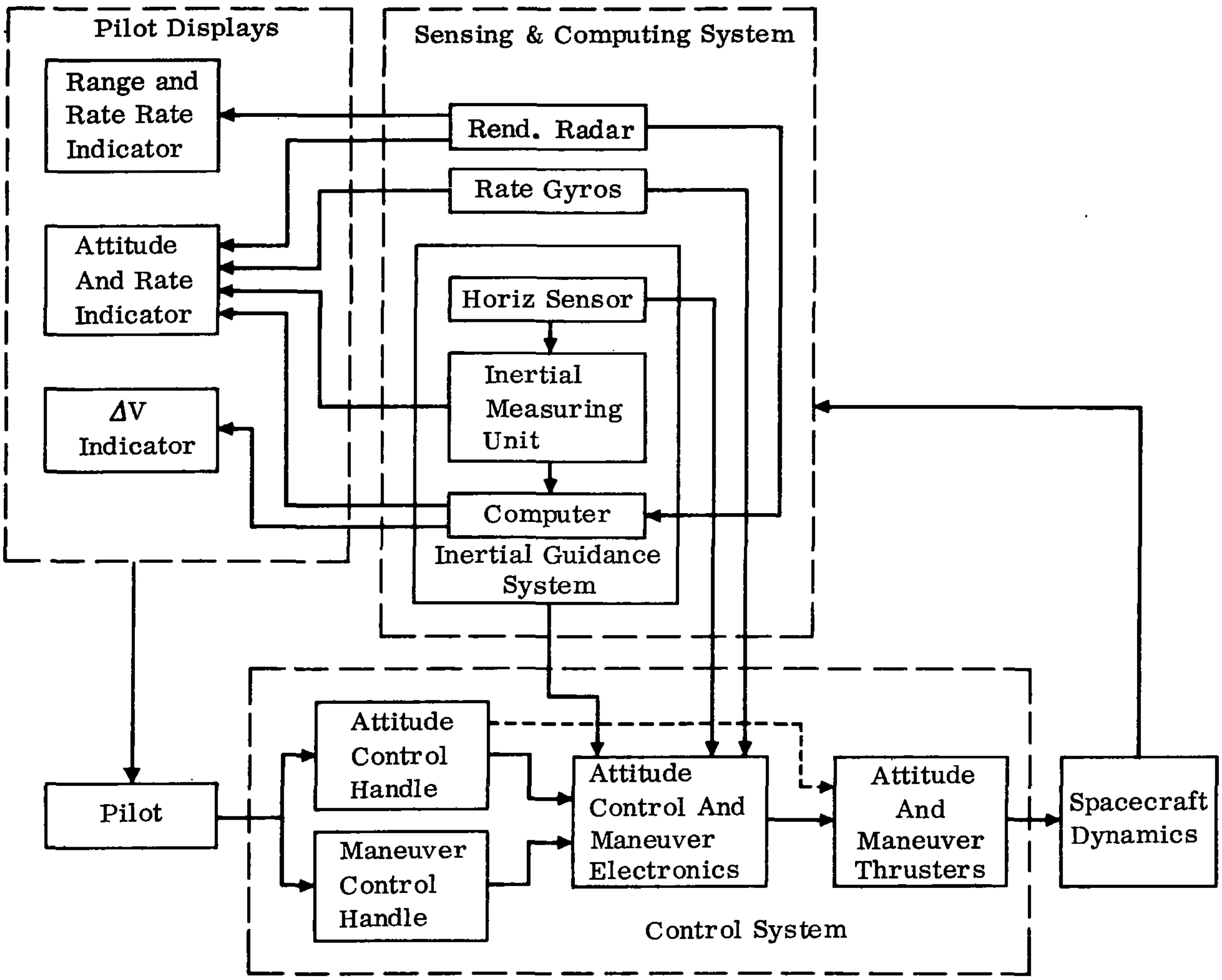


Figure 29. Gemini Guidance and Control System

Within the sensing and computing system, the on-board reference system is established by a horizon scanner and the inertial measuring unit. The attitude of the rendezvous vehicle with respect to this reference frame is also established by the inertial measuring unit. The vehicle attitude rate is acquired by rate gyros. The attitude rate and displacement information is fed to both the control system and to the pilot attitude indicator. The rendezvous radar[20] supplies digital relative target angle and range information to the computer from 250 n.m. to 500 feet. Analog line-of-sight range and range rate are also supplied to the range and range-rate indicator from 300,000 to 20 feet. The indicator is laid out so that it displays selected range/range-rate criteria to the pilot. Inputs to the attitude and attitude rate indicator from the rendezvous radar, rate gyros, and computer direct the pilot in turning the vehicle to, and maintaining, an attitude for application of the midcourse rendezvous velocity corrections. The required velocity correction is fed to the ΔV indicator from the computer.

The pilot accepts the various display inputs and operates the attitude and maneuver control handles to provide inputs to the control system. The attitude control and maneuver electronics system accepts these inputs and converts them to on-off commands of the attitude and maneuver thrusters.

4. REFERENCES

1. Dowlen, E.M., and Seddon, J. — "Orbital Rendezvous Techniques," J. British Interplanetary Soc., Vol. 19, 1963-64, p. 498-510.

2. Purser, P.E., Faget, M.A., and Smith, N.F., Editors — Manned Spacecraft: Engineering Design and Operation, Fairchild Publications, Inc., New York, N.Y.

3. Lunar Excursion Model Familiarization Manual, Grumman Report LMA 790-1, 15 March 1965.

4. Shepard, A.B., Jr. — "The Apollo Mission," Preprint 64-9, AAS Annual Meeting, New York, N.Y., May 1964.

5. Clohessy, W.H., and Wiltshire, R.S. — "Terminal Guidance System for Satellite Rendezvous," J. Aerospace Sci., Vol. 27, 1960, p. 653-658.

6. Soule, P.W. — "Rendezvous with Satellites in Elliptical Orbits with Low Eccentricity," Advances in the Aeronautical Sciences, Vol. 7, The Macmillan Company, New York, N.Y., 1961, p. 138-147.

7. Spradlin, L.W. — "The Long-Time Satellite Rendezvous Trajectory," Aerospace Engineering, June 1960.

8. Swanson, R.S., Peterson, N.W., and Hoover, L.R. — "An Astrovehicle Rendezvous-Guidance Concept," Advances in the Aeronautical Sciences, Vol. 6, The Macmillan Company, New York, N.Y., 1961, p. 147-160.

9. Sears, N.E., Jr., and Felleman, P.G. — "Terminal Guidance for Satellite Rendezvous," ARS paper 778-59, 30 April-1 May, 1959.

10. Harrison, E. — "Some Considerations of Guidance and Control Techniques for Coplanar Orbital Rendezvous," Proc. Nat. Spec. Mtg. on Guidance of Aerospace Vehicles, May 1960, p. 14.

11. Ward, J.W., and Williams, H.M. — "Orbital Docking Dynamics," AIAA Journ., Vol. 1, No. 6, June 1963.

12. Hopper, F.W. "Dynamic Behavior of Large, Flexible, Rotating Space Stations During Docking Maneuvers," Advances in the Astronautical Sciences, Vol. 16, Part 1, The Macmillan Company, New York, N.Y., 1963.

13. Grubin, C. "Docking Dynamics for Rigid Body Spacecraft," Preprint 62-43, AIS Thirtieth Annual Meeting, New York, N.Y., Jan. 22-24, 1962.

14. Montgomery, J.E. "Manned Control of Space Vehicle Docking," Advances in the Astronautical Sciences, Vol. 16, Part 1, The Macmillan Company, New York, N.Y., 10-12 Sept., 1963.

15. Little, A.C. "A Manned Docking Simulator With Five Degrees of Freedom," Advances in the Aeronautical Sciences, Vol. 16, Part 1, The Macmillan Company, New York, N.Y., 10-12 Sept. 1963, p. 497-506.

16. Hatch, H.G., Jr., Riley, D.R., and Cobb, J.B. "Full-Scale Gemini-Agena Docking Using Fixed and Moving Base Simulators, AIAA Paper 64-334, 29 June-2 July, 1964.

17. Riley, D.R., and Suit, W.T. A Fixed-Base Visual Simulator Study of Pilot Control of Orbital Docking of Attitude-Stabilized Vehicles, NASA TN D-2036, January 1964.

18. A Simulation Study of the Control Problems Encountered when Docking the LEM with the Command Module-Service Module Combination, North American Aviation Report NA 63H-82, Columbus, Ohio, 1963.

19. Leondis, C.T., Editor "Guidance and Control of Aerospace Vehicles," University of California Engineering and Sciences Extension Series, McGraw-Hill Book Co., Inc., New York, N.Y., 1963, p. 231-239.

20. Quigley, W.W. "Gemini Rendezvous Radar," AIAA Paper 63-650, August 12-14, 1963.

21. Brayton, W. D. "Dynamic Analysis of Probe and Drogue Docking Mechanism", J. of Spacecraft and Rockets, Vol. 3, No. 5, May 1966.

22. Anon. "CSM/LEM Docking Aids Interface Control Document," North American Aviation, Space and Inf. Div., Document No. MH01-05057-416.

23. Lass, H. Vector and Tensor Analysis, Chapter 6, McGraw-Hill Company, 1950.

APPENDIX A

TWO BODY EQUATIONS OF MOTION*

Figure A1 illustrates the definitions pertinent to the two-body problem.

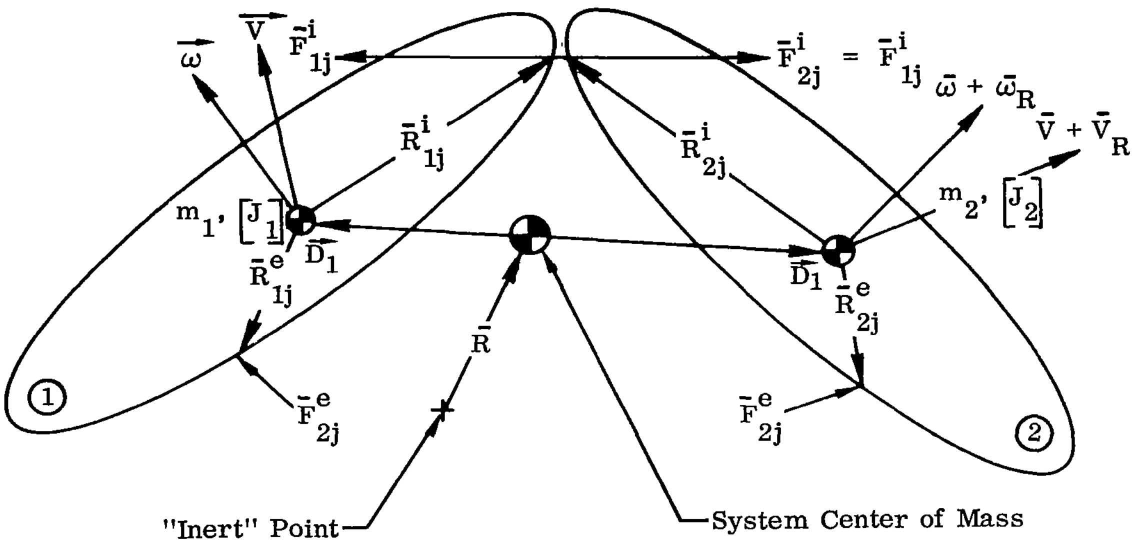


Figure A1. Two-Body Problem Definitions

As illustrated in Fig. A1, an "inert" point (unaccelerated) is defined and a vector $\vec{R}$ is drawn to the system center of mass (smc). The smc, of course, lies on a line between body 1 center of mass (cm 1) and body 2 center of mass (cm 2). $\vec{D}_1$ and $\vec{D}_2$ are colinear vectors from the smc to cm 1 and cm 2 respectively.

Both external and internal forces acting on the two bodies are considered. The internal forces, insofar as the docking problem is concerned, are those arising from contact of the two bodies at the mating interface. The internal forces are identified by the superscript i and external forces by the superscript e. Only one of each is shown, but the subscript j is used to indicate the jth force of any number. Each force has a line of action and a magnitude forming the force vector. The vector distance to this line of action is defined from the body center of mass. It could be drawn to any point on the line of action but is illustrated on Fig. A1 as the vector distance to the point of application of the force on the body.

*A suitable reference for the basic principles involved in the equations developed in this Appendix is Ref. 23.

It is planned to use Eulers equations to solve for the rotary motion of each body. As is standard procedure with the use of Eulers equations, a coordinate system is centered at the body cm with its three orthogonal axes fixed to the body in some orientation judged convenient. Doing this allows definition of a 3 x 3 matrix of constant moment-of-inertia values. The angular rate vector of body 1, $\vec{\omega}$, is then the angular rate of the body 1 fixed frame, relative to a non-rotating reference frame also centered at cm 1. The body 1 moment-of-inertia matrix is designated $[J_1]$. Another reference frame is identically fixed to body 2. The angular rate vector of this frame is designated $\vec{\omega} + \vec{\omega}_R$ to allow definition of the angular rate of body 2 relative to body 1 to be $\vec{\omega}_R$. The moment-of-inertia matrix for body 2 is designated $[J_2]$. Similarly, the velocity of the cm's of bodies 1 and 2, with reference to the "inert" point, are designated $\vec{V}$ and $\vec{V} + \vec{V}_R$ respectively to allow the definition of $\vec{V}_R$ to be the velocity of body 2 with respect to body 1. It is assumed desirable to define the relative motion vectors because internal forces will be functions of relative motion between the bodies.

The masses of bodies 1 and 2 are defined to be m_1 and m_2 respectively.

This completes the definitions of the quantities involved. Now consider the following two developments of the vector equations of motion. Method 1 focuses on determining the forces and moments on each body straightforwardly. These forces and moments enable the writing of the equations for mass center translational motion, and rotational motion about the mass center for each body. Method 2 focuses on developing useful relationships derived from considering the two bodies as a single system.

METHOD 1

The total force acting on each body is calculated as follows,

external $$\sum_j \vec{F}^{\,e}_{1j} \equiv \vec{F}_1^{\,e} \quad , \quad \sum_j \vec{F}^{\,e}_{2j} \equiv \vec{F}_2^{\,e} \qquad \text{(A1)}$$

internal $$\sum_j \vec{F}^{\,i}_{1j} \equiv \vec{F}_1^{\,i} \quad , \quad \sum_j \vec{F}^{\,i}_{2j} = \sum_j (-\vec{F}^{\,i}_{1j}) = -\vec{F}_1^{\,i} \qquad \text{(A2)}$$

It was arbitrarily decided to evaluate the internal forces acting on body 1 due to contact with body 2. Use of the law of equal and opposite reacting forces determines the force on body 2.

The motion of each body is obtained as follows:

$$\begin{aligned} \vec{F}_1^{\,e} + \vec{F}_1^{\,i} &= m_1\dot{\vec{V}} = m_1(\ddot{\vec{R}} + \ddot{\vec{D}}_1) \\ \vec{F}_2^{\,e} - \vec{F}_1^{\,i} &= m_2(\dot{\vec{V}} + \dot{\vec{V}}_R) = m_2(\ddot{\vec{R}} + \ddot{\vec{D}}_2) \end{aligned} \qquad \text{(A3)}$$

It may be more convenient to express $\vec{D}_1$ and $\vec{D}_2$ in terms of $\vec{D}$, the total distance between the mass centers. $\vec{D}_1$ and $\vec{D}_2$ are defined relative to the smc, so

$$m_1\vec{D}_1 + m_2\vec{D}_2 = 0, \quad \vec{D}_2 - \vec{D}_1 \equiv \vec{D} \tag{A4}$$

From equation A4,

$$\vec{D}_1 = -\frac{m_2}{m_1 + m_2}\vec{D}, \quad \vec{D}_2 = \frac{m_1}{m_1 + m_2}\vec{D} \tag{A5}$$

Substituting A5 into A3 yields

$$\begin{aligned} \vec{F}_1^{\,e} + \vec{F}_1^{\,i} &= m_1\left(\ddot{\vec{R}} - \frac{m_2}{m_1 + m_2}\ddot{\vec{D}}\right) \\ \vec{F}_2^{\,e} - \vec{F}_1^{\,i} &= m_2\left(\ddot{\vec{R}} + \frac{m_1}{m_1 + m_2}\ddot{\vec{D}}\right) \end{aligned} \tag{A6}$$

Or, a single differential equation resulting from elimination of $\vec{R}$ from A6 is,

$$\ddot{\vec{D}} = \frac{\vec{F}_2^{\,e}}{m_2} - \frac{\vec{F}_1^{\,e}}{m_1} - \frac{m_1 + m_2}{m_1 m_2} F_1^{\,i} = \ddot{\vec{D}} = \dot{\vec{V}}_R \tag{A7}$$

To develop rotary motion equations, first calculate the torques about the body cm as follows,

$$\text{external} \qquad \vec{T}_1^{\,e} \equiv \sum_j \vec{R}_{1j}^{\,e} \times \vec{F}_{1j}^{\,e}, \quad \vec{T}_2^{\,e} \equiv \sum_j \vec{R}_{2j}^{\,e} \times \vec{F}_{2j}^{\,e} \tag{A8}$$

$$\begin{aligned} \text{internal} \qquad \vec{T}_1^{\,i} \equiv \sum_j \vec{R}_{1j} \times \vec{F}_{1j}^{\,i}, \quad \vec{T}_2^{\,i} &\equiv \sum_j \vec{R}_{2j}^{\,i} \times \vec{F}_{2j}^{\,i} \\ &= \sum_j (-\vec{D} + \vec{R}_{1j}^{\,i}) \times (-F_{1j}^{\,i}) \\ &= \vec{D} \times \vec{F}_1^{\,i} - \vec{T}_1^{\,i} \end{aligned} \tag{A9}$$

The equations giving rotary motion of the two bodies are

$$\begin{aligned} \vec{T}_1^{\,e} + \vec{T}_1^{\,i} &= \dot{\vec{H}}_1 \\ \vec{T}_2^{\,e} + \vec{T}_2^{\,i} &= \vec{T}_2^{\,e} + \vec{D} \times \vec{F}_1^{\,i} - \vec{T}_1^{\,i} = \dot{\vec{H}}_2 \end{aligned} \tag{A10}$$

where $\vec{H}_1$ and $\vec{H}_2$ are the conventional angular momentums of body 1 and 2 calculated about the body cm.

$$\vec{H}_1 = \left[J_1\right] \vec{\omega}, \quad \vec{H}_2 = \left[J_2\right] (\vec{\omega} + \vec{\omega}_R) \tag{A11}$$

Equations A11 are differentiated and inserted into A10 to yield Eulers equations of motion (see Ref. 22). Equations A10 and A7 contain the vector unknowns $\vec{\omega}$, $\vec{\omega}_R$ and $\vec{V}_R$. The forces are functions of these quantities and angular and translational position obtained by integration of A11 and A7. The angular position is obtained by defining a set of Euler angles or direction cosines relating the body fixed frame to the inert frame.

METHOD 2

If, the two bodies are considered to be a single system, the smc is an important reference point. The sum of internal forces is zero. The sum of internal torques calculated with reference to the smc is also zero. Translational motion of the smc is given by

$$\vec{F}_1^{\,e} + \vec{F}_2^{\,e} = (m_1 + m_2)\ddot{\vec{R}} \tag{A12}$$

The time derivative of total angular momentum, calculated about the smc, equals the sum of the external torques, again calculated about the smc. The torque is

$$\vec{T}_{1c}^{\,e} = \vec{T}_1^{\,e} + \vec{D}_1 \times \vec{F}_1^{\,e}, \quad \vec{T}_{2c}^{\,e} = \vec{T}_2^{\,e} + \vec{D}_2 \times \vec{F}_2^{\,e}$$

$$\vec{T}_{1c}^{\,e} + \vec{T}_{2c}^{\,e} = \vec{T}_1^{\,e} + \vec{T}_2^{\,e} + \frac{m_1 m_2}{m_1 + m_2}\vec{D} \times \left(\frac{\vec{F}_2^{\,e}}{m_2} - \frac{\vec{F}_1^{\,e}}{m_1}\right) \tag{A13}$$

where the original notation for the torque about the single body cm is retained and the corresponding torque about the smc is designated by the additional subscript c. The angular momentums referred to the smc are denoted in the same way as:

$$\vec{H}_{1c} = \vec{H}_1 + m_1 D_1 \times \dot{\vec{D}}_1, \quad \vec{H}_{2c} = \vec{H}_2 + m_2 \vec{D}_2 \times \dot{\vec{D}}_2$$

$$\vec{H}_{1c} + \vec{H}_{2c} = \vec{H}_1 + \vec{H}_2 + \frac{m_1 m_2}{m_1 + m_2} D \times \dot{\vec{D}} \tag{A14}$$

The rotational equation of motion is then

$$\vec{T}_1^{\,e} + \vec{T}_2^{\,e} + \frac{m_1 m_2}{m_1 + m_2}\vec{D} \times \left(\frac{\vec{F}_2^{\,e}}{m_2} - \frac{\vec{F}_1^{\,e}}{m_1}\right) = \dot{\vec{H}}_1 + \dot{\vec{H}}_2 + \frac{m_1 m_2}{m_1 + m_2}\vec{D} \times \ddot{\vec{D}}$$

or $$T_1^e + T_2^e + \frac{m_1 m_2}{m_1 + m_2} \vec{D} \times \left(\frac{\vec{F}_2^e}{m_2} - \frac{\vec{F}_1^e}{m_1} - \ddot{\vec{D}} \right) = \dot{\vec{H}}_1 + \dot{\vec{H}}_2 \tag{A15}$$

The rotational (A15) and translational equation (A12) obtained from smc considerations do not give the motion of each body. However, it should be noted that A12 could replace one of the equations of A6; similarly A15 could replace one of the equations of A10 to give a set of equations. This again permits solving for body 1 and 2 angular and translational motion.

The major advantage in developing the equations stemming from smc considerations is that in the absence of external forces (an adequate predesign approach) system linear and angular momentum is conserved.

The smc velocity never changes ($\ddot{\vec{R}} = 0$). It could be used as an "inert" reference. Assume that the bodies are approaching each other but have not yet contacted. Conservation of linear momentum relative to the smc yields

$$m_1 \vec{V} + m_2 (\vec{V} + \vec{V}_R) = 0^* \tag{A16}$$

Given $\vec{V}_R$, the relative approach velocity, the velocity of body 1 from A17 is,

$$\vec{V} = - \frac{m_2}{m_1 + m_2} \vec{V}_R \tag{A17}$$

The translational kinetic energy during the approach must be dissipated during the docking by the docking interface mechanism because the energy (relative to the smc) of the joined bodies is zero. The kinetic energy to be dissipated is,

$$\frac{1}{2} \left[m_1 V^2 + m_2 (V + V_R)^2 \right] = \frac{m_1 m_2}{m_1 + m_2} V_R^2 \tag{A18}$$

Conservation of angular momentum calculated before and after joining yields,

$$\left[J_1\right] \vec{\omega} + \left[J_2\right] (\vec{\omega} + \vec{\omega}_R) + \frac{m_1 m_2}{m_1 + m_R} (\vec{D} \times \vec{V}_R) \equiv \vec{H}_o = \left[J_T\right] \vec{\omega}_T \tag{A19}$$

where $\left[J_T\right]$ is the moment-of-inertia matrix for the joined body about the smc.

*The total momentum relative to the smc is zero when this point is used as an "inert" reference.

$\vec{\omega}_T$ is the angular rate vector of the joined body relative to an inert frame

$\vec{H}_o$ is the constant total angular momentum vector

Equation A19 permits solving for the angular rate of the joined body. This angular rate gives the final rotational kinetic energy of the joined body, whereas the initial angular rates of the two bodies give the original amount. The difference is the amount that the docking interface is required to dissipate. Taking the simple case of a single rotational degree of freedom, the rotational kinetic energy to be dissipated is

$$\frac{1}{2}\left[J_1 \omega + J_2(\omega + \omega_R)^2 - J_3 \omega_T^2\right]$$

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