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ORBIT TRIM PROPULSION REQUIREMENTS FOR SUN SYNCHRONOUS SATELLITES

BY

DANIEL L. ENDRES

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ABSTRACT

Two problems are considered in this report. The first is to determine what method of thrusting would be the best for correcting the precession rate of a near sun synchronous satellite to the precession rate of a sun synchronous satellite. The second is to compare five propulsion systems that could fulfill the requirements of such a mission. Special emphasis is placed on the Tiros M mission.

It is found that a tangential thrust is the best means for correcting to a sun synchronous orbit from a propulsion standpoint. The results of the propulsion system comparisons are that: 1) the 2 lb. hydrazine system and the 200 sec., I_{sp} resistojet weigh about the same, 2) the 2 lb. hydrazine system requires the least time and, 3) the cold gas resistojet and the 2 lb. hydrazine system require very little power. Thurst misalignment is not considered.

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LIST OF SYMBOLS AND CONSTANTS

I = Inclination, degrees

$\dot{\psi}$ = Average precession rate, radians/second unless otherwise specified

J_{20} = Second zonal harmonic in the series expansion of the gravity potential for the Earth, 1.082×10^{-3}

$\dot{\eta}$ = Orbital rate, radians/second

r_e = Mean equatorial radius of the Earth, 3440 nautical miles

r = Satellite radial distance measured from the center of the Earth, ft. unless otherwise specified

μ = Gravitational constant of the Earth, 62,746.8 n.mi.^{3}/sec.^{2} or 1.4076449×10^{16} ft.^{3}/sec.^{2}

$\vec{M}$ = Moment vector, ft.-lbs.

$\vec{H}$ = Angular momentum, slug.-ft.^{2}/sec.

η = Angle, in the orbit plane, between the line of nodes and radius vector, radians

$\hat{i}, \hat{j}, \hat{k}$ = Unit vectors along X, Y, and Z respectively

$\vec{M}_P$ = Precession moment due to oblateness of earth, ft.-lbs.

$\vec{M}_T$ = Moment due to thrust, ft.-lbs.

$\vec{F}_N$ = Normal thrust, lbs.

$\vec{F}_T$ = Tangential thrust, lbs.

m = Mass of satellite, slugs

T = Period of orbit, sec.

I_T = Total impulse, lb.-sec.

W_0 = Initial satellite weight, lbs.

I_{sp} = Specific impulse, sec.

ΔV_T = Total effective velocity increment for tangential thrust method, ft./sec.

ΔV_N = Total velocity increment for normal thrust method, ft./sec.

C_{eff} = $\Delta V_N / \Delta V_T$ = penalty factor

$\overline{\Delta r}$ = Final sun synchronous altitude minus reference altitude, n.mi.

R_1 = Radius of initial circular orbit, ft.

R_2 = Radius of final circular orbit, ft.

g = Acceleration due to gravity, ft./sec.^{2}

τ_T = Thrust on time, sec.

r, θ, ϕ = Polar coordinates

$\delta r, \delta\theta, \delta\phi$ = Perturbations of polar coordinates

F_r, F_θ, F_ϕ = Forces in r, θ, and ϕ directions respectively, lbs.

n = Constant angular velocity, rad./sec.

τ = Dimensionless time

I_{mean} = Mean change in inclination, radians unless otherwise specified

I_{orbit} = Change in inclination over an orbit, radians

λ = Longitude of satellite with respect to ascending node

L = Latitude of satellite

ω = Angle between ascending node and perigee

ξ = True anomaly of satellite, radians

$\dot{\xi}$ = Satellite orbital rate, rad./sec.

$\vec{f}$ = Oblateness force acting on satellite, lbs.

Δh = Change in altitude, n.mi.

W_{ps} = Weight of propulsion system, lbs.

W_p = Propellant weight, lbs.

W_t = Tankage weight, lbs.

W_{th} = Weight of thruster, lbs.

W_{fs} = Weight of feed system, lbs.

W_{pc} = Weight of power conditioner, lbs.

W_{tel} = Weight of telemetry. lbs.

K_i $(i=0, 3)$ = Structural constants for the propulsion systems

σ_1, σ_2 = Conversion factors

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ORBIT TRIM PROPULSION REQUIREMENTS FOR SUN SYNCHRONOUS SATELLITES

INTRODUCTION

There are two main purposes to this report. The first is to determine the thrust requirements necessary to adjust an initial near sun synchronous orbit to a final orbit which is sun synchronous. It will be assumed that the satellite is in a near-polar and near-circular orbit about a polar oblate Earth. The second is to compare the weights, the orbit correction times, and the power requirements of various propulsion systems that could fulfill the thrust requirements.

An obvious advantage of a near polar sun synchronous orbit is that the power output can be optimized. This is due to the larger amounts of radiant energy that can be encountered by the solar panels.

In various reports and texts it has been shown that a near circular satellite orbit subjected to the polar oblateness of the earth's gravity field will precess at a rate that depends on the orbit's semi-major axes and inclination (c.f. References 1 and 2). This then leads to the possibility of defining a reference orbit in which the satellite will precess at a sun synchronous rate without the use of any thrust when not subjected to other external orbit perturbations. Such a reference orbit can in fact be defined by an equation which is derived in Appendix A.

If the satellite is not put into a reference orbit, thrust can be applied to obtain the proper precession rate. There are two general ways this can be done: 1) use a continuous thrust to increase or decrease the precession that is due to the Earth's polar oblateness, or 2) use thrust to correct the injection errors. In this report only errors in semi-major axis and inclination will be considered. Also, all orbits will be assumed to be circular.

ANALYSIS

This section will deal with the analysis of the methods which could be used to obtain the sun synchronous precession rate. To reiterate, the two general methods are: 1) use a force normal to the plane of the orbit in such a manner that the desired value for the precession rate will be achieved, and 2) use thrust to correct either of the two injection errors that are being considered in this report (altitude and inclination). Also in this section, a relationship between altitude and inclination for a sun synchronous satellite will be obtained.

At the conclusion of this section a specific method for correcting the Tiros M precession rate will be given. The conclusion will be based on propulsion requirements only.

Relationship Between Inclination and Altitude for a Sun Synchronous Satellite

In this paragraph a formula which relates inclination and altitude for a sun synchronous satellite will be derived.

First it is imperative to have a clear idea of what sun synchronous means. When the angle between the satellite's line of nodes and the projection into the equatorial plane of the line between the center of the Earth and the Sun has an average constant value, an orbit is said to be sun synchronous. This would mean that the angle C in Figure 1 would have an average constant value.

In Appendix A it is shown that the average precession rate due to the Earth's oblateness is

$$\dot{\psi} = -\frac{3}{2} J_{20} \dot{\eta} \left(\frac{r_e}{r}\right)^2 \cos I \quad . \tag{1}$$

If it is assumed, as it will be throughout this report, that we have a near circular orbit, we have

$$r \dot{\eta}^2 \simeq \frac{\mu}{r^2} ,$$

and hence

$$\dot{\psi}(\text{deg/day}) = -\sigma_1 J_{20} \sqrt{\mu}\, r_e^2\, r^{-7/2} \cos I \quad ,$$

where σ_1 is a conversion factor. Putting in the constants gives

$$\dot{\psi}(\text{deg/day}) = -\left(2.3817 \times 10^{13}\right) r^{-7/2} \cos I$$

$$= +0.985 \text{ (the sun synchronous precession rate)}$$

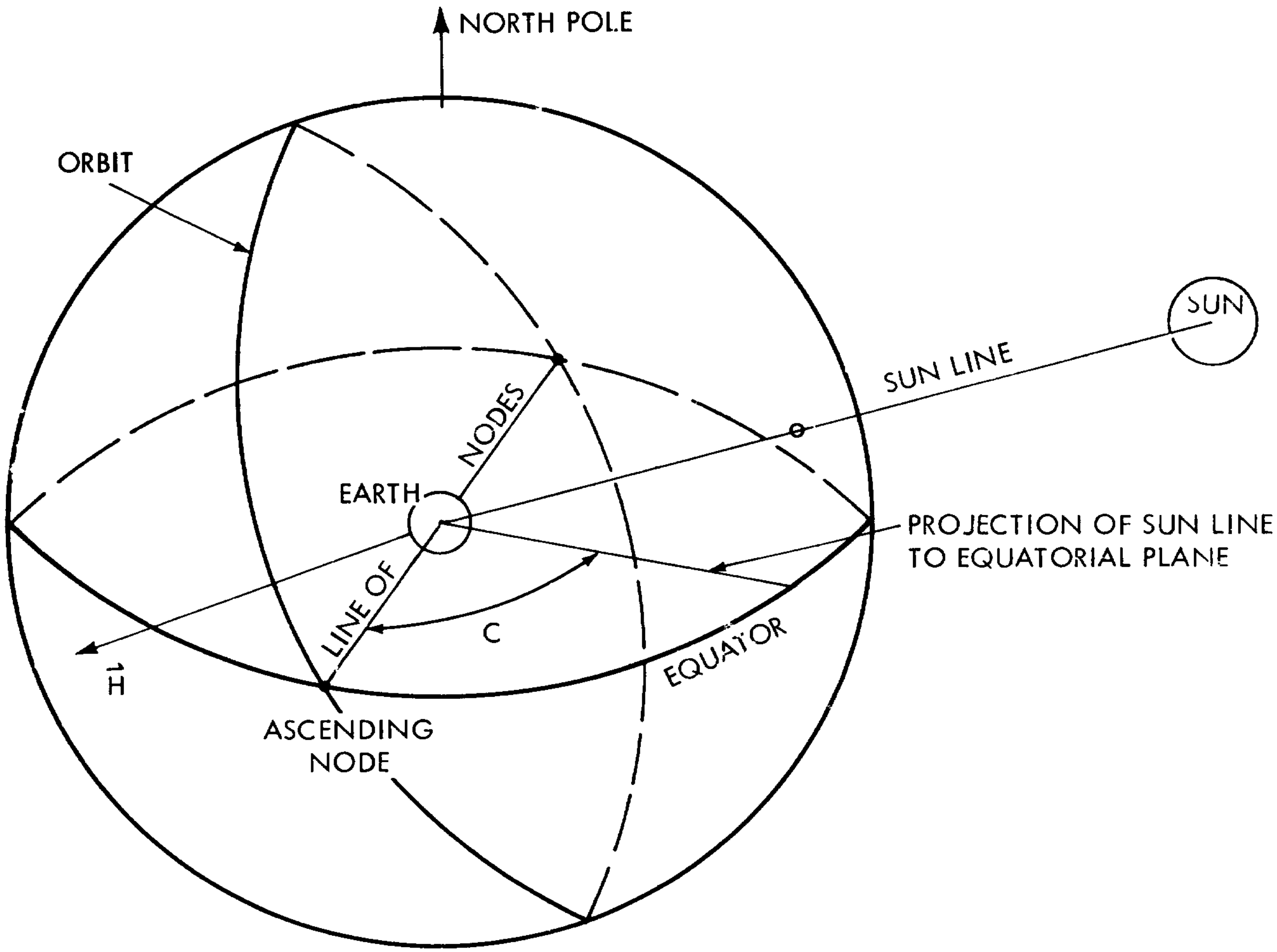

Figure 1. A Satellite is Sun Synchronous when the Average Value of the Angle C Remains Constant

or

$$I = \cos^{-1}\left[-4.136 \times 10^{-14}\, r^{7/2}\right] \tag{2}$$

where r is given in nautical miles. Equation (2) now gives us a useful relationship between altitude and inclination for a sun synchronous satellite. Figures 2 and 3 illustrate this relation in inclination versus altitude plots. Any point along these curves could be used for a reference orbit depending on the mission objectives. Note that atmospheric drag, and solar and lunar perturbations have been ignored here and will be throughout the report.

Normal Thrust (Aid Oblateness)

If a reference orbit is not obtained, two general ways to keep the precession at the correct value were given in the introduction. The first way mentioned was to use a thrust normal to the plane of the orbit in such a manner that it could change the precession rate.

Our basic equation is

$$\Sigma \vec{M} = \dot{\vec{H}} \tag{3}$$

where,

$\vec{M}$ = moment vector

$\dot{\vec{H}}$ = rate of change of angular momentum.

In the case under consideration

$$\Sigma \vec{M} = \vec{M}_P + \vec{M}_T$$

where,

$\vec{M}_P$ = precession moment due to Earth's oblateness

$\vec{M}_T$ = moment due to thrust.

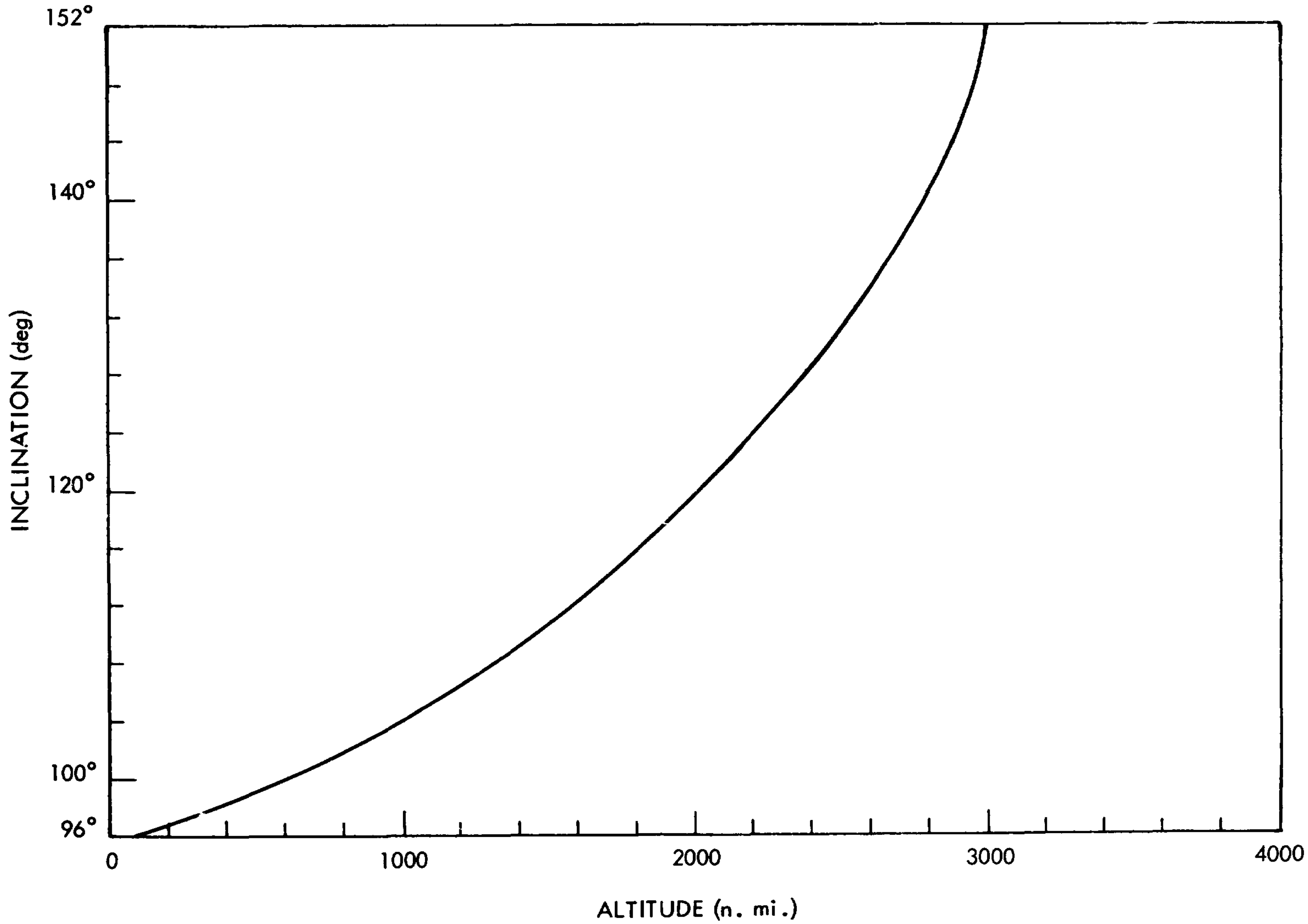

Figure 2. Inclination vs. Altitude for Sun Synchronous Rate

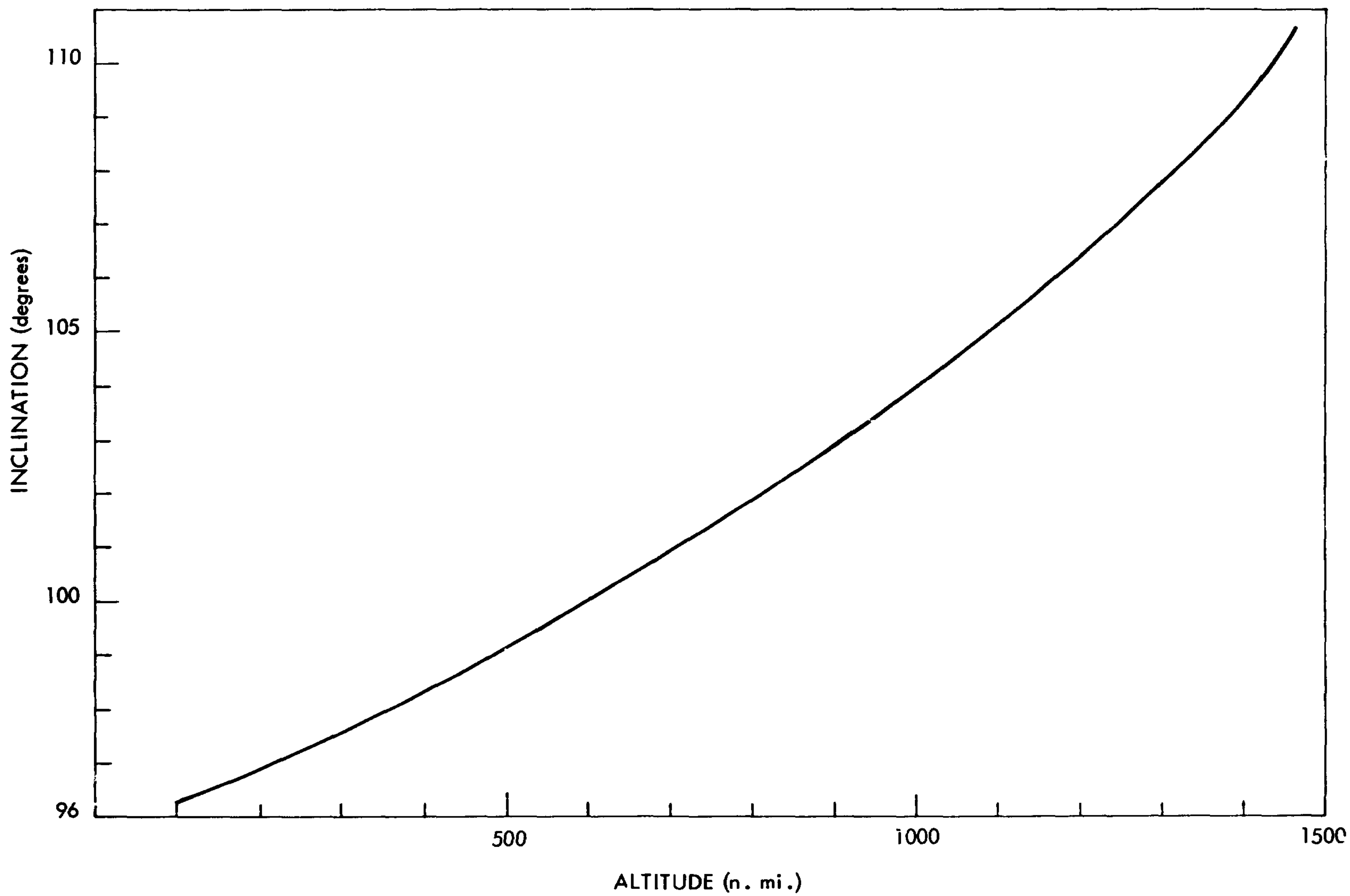

Figure 3. Inclination vs. Altitude for Sun Synchronous Rate

It is assumed that the thrust comes solely from a force normal to the plane of the orbit and thus with the aid of Figure 4 can be written

$$\vec{F}_N = |\vec{F}_N| \left[-\sin I\,\hat{j} + \cos I\,\hat{k}\right]$$

$$= F_N \left[-\sin I\,\hat{j} + \cos I\,\hat{k}\right] \tag{4}$$

where,

$$|\vec{F}_N| = F_N \ .$$

Also, according to the nomenclature of Figure 4

$$\vec{r} = |\vec{r}| \left[\cos\eta\,\hat{i} + \sin\eta\cos I\,\hat{j} + \sin\eta\sin I\,\hat{k}\right]$$

$$= r \left[\cos\eta\,\hat{i} + \sin\eta\cos I\,\hat{j} + \sin\eta\sin I\,\hat{k}\right] \tag{5}$$

where $|\vec{r}| = r$. The moment due to thrust can now be determined from Equations (4) and (5).

$$\vec{M}_T = \vec{r} \times \vec{F}_N$$

$$= r F_N \left\{\sin\eta\,\hat{i} - \cos\eta\cos I\,\hat{j} - \cos\eta\sin I\,\hat{k}\right\} \tag{6}$$

The components of the precession moment that are determined in Appendix A are

$$M_{PX} = -rJ\,[1 - \cos 2\eta]\,\sin I\cos I \tag{7}$$

$$M_{PY} = rJ\sin 2\eta\sin I \tag{8}$$

$$M_{PZ} = 0 \tag{9}$$

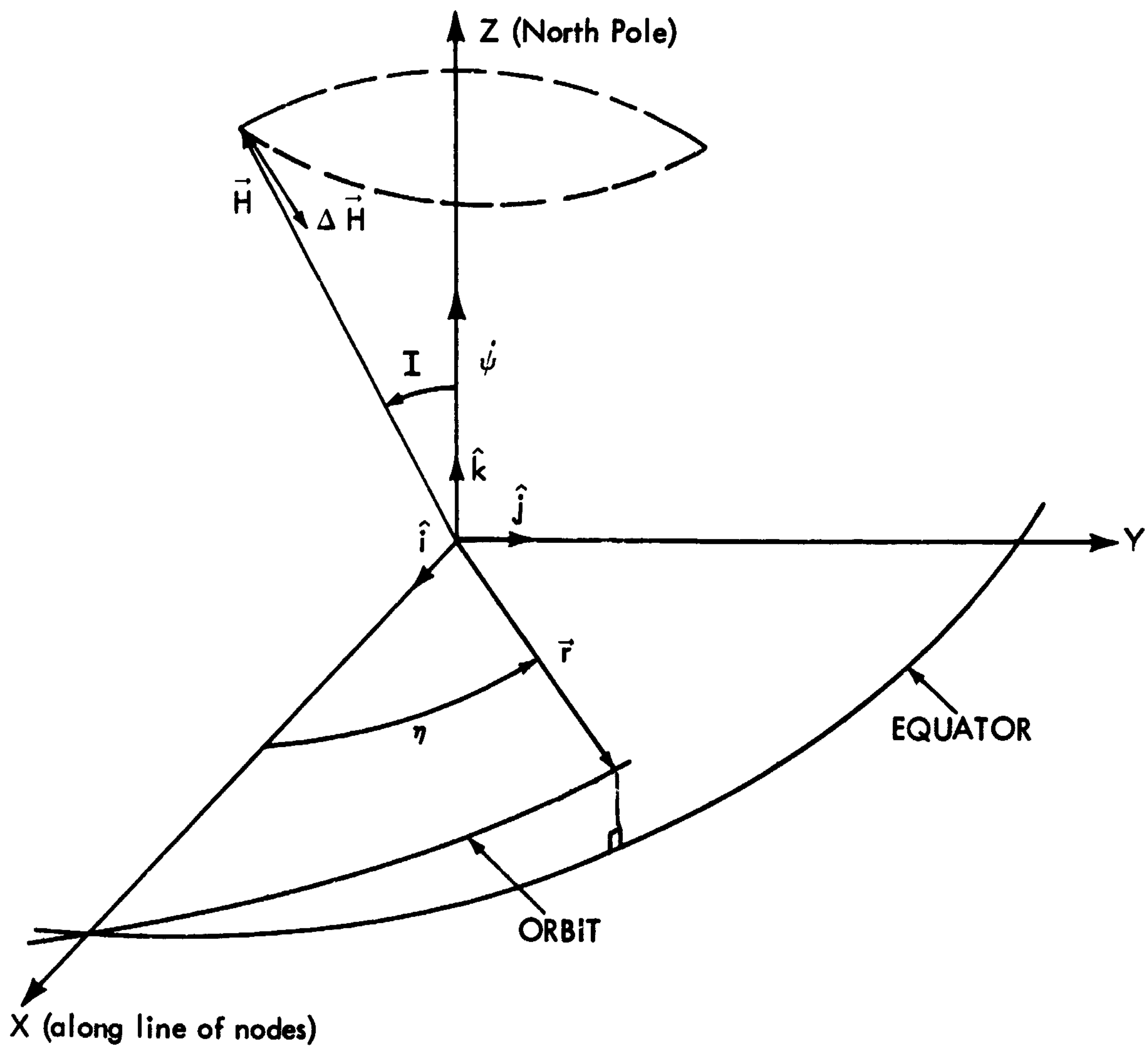

Figure 4. Reference Coordinate System used in the Normal Thrust (Aid Oblateness) Paragraph

where,

$$J = \frac{3}{2} m J_{20} \frac{\mu}{r^2} \left(\frac{r_e}{r}\right)^2 \tag{10}$$

The components of the moments in the Y and Z directions will be examined first.

$$\begin{aligned} \Sigma M_Y &= M_{PY} + M_{TY} \\ &= r J \sin 2\eta \sin I - r F_N \cos \eta \cos I \\ &= \dot{H}_Y \end{aligned} \tag{11}$$

where for reasons which will become apparent later, F_N is so chosen that

$$F_N = \begin{cases} +F_N & 0 \leq \eta < \pi \\ -F_N & \pi \leq \eta < 2\pi \end{cases} \tag{12}$$

If Equation (11) is now integrated over one orbit, the ΔH_Y resulting will be zero. A similar situation arises for the Z component. Thus, only the possibility of a change in $\vec{H}$ in the X direction over an orbit is left. Since this is instantaneously perpendicular to $\vec{H}$, the direction of $\vec{H}$ will be changed.

Now the X component can be examined. Using Equation (3) again gives

$$\begin{aligned} \Sigma M_X &= M_{PX} + M_{TX} \\ &= \dot{\vec{H}} \end{aligned} \tag{13}$$

where

$$\begin{aligned} \dot{\vec{H}} &= \hat{k}\dot{\psi} \times \vec{H} \\ &= m r^2 \dot{\eta} \dot{\psi} \sin I \quad . \end{aligned} \tag{14}$$

Rewriting Equation (13) using Equations (14), (6) and (7) gives

$$mr^2 \dot{\eta} \dot{\psi} \sin I = -rJ\,[1 - \cos 2\eta] \sin I \cos I + r F_N \sin \eta$$

Solving for $\dot{\psi}$,

$$\dot{\psi} = -\frac{J\,[1 - \cos 2\eta] \cos I}{mr\,\dot{\eta}} + \frac{F_N \sin \eta}{mr\,\dot{\eta} \sin I} \quad . \tag{15}$$

Assuming a circular orbit, i.e.

$$\begin{cases} \dot{\eta} = 2\pi/T \\ \eta = \dot{\eta} t \end{cases} \quad , \tag{16}$$

Equation (15) can be integrated using Equation (12). The result is

$$\psi(\text{rad/orbit}) = -3\pi J_{20} \left(\frac{r_e}{r}\right)^2 \cos I + \frac{4r^2 \; F_N/m}{\mu \sin I} \quad , \tag{17}$$

or in degrees/day this becomes

$$\dot{\psi}(\text{deg/day}) = \frac{1.9736 \times 10^8}{r^{3/2}} \psi(\text{rad/orbit})$$

$$= +0.985^\circ \text{ (for a sun synchronous satellite) .} \tag{18}$$

Solving for the thrust to mass ratio

$$F_N/m = \frac{\mu \sin I}{4} \left\{ \frac{0.985}{1.9736 \times 10^8 \sqrt{r}} + \frac{3\pi J_{20}\, r_e^2 \cos I}{r^4} \right\}$$

or

$$F_N/m = \sin I \left\{ \frac{4.757 \times 10^{-1}}{\sqrt{r}} + \frac{1.1502 \times 10^{13} \cos I}{r^4} \right\} \quad (19)$$

where r is in nautical miles. This equation indicates the amount of thrust that will be required at a given altitude and inclination to give a satellite of mass m a sun synchronous precession rate.

F_N/m versus I as determined from Equation (19) has been plotted for various altitudes in Figure 5.

It should be noted that this is a continuous thrust that will last as long as the satellite is desired to remain sun synchronous. This implies that for missions of one to two years the total impulse required would be extremely large.

Orbit Correction

The second method mentioned to correct the precession, if the nominal orbit is not obtained, is to correct altitude or inclination to a sun synchronous value as given in Figure 2.

Altitude Correction—To correct altitude, a low tangential thrust can be used to spiral in or out without significantly changing the eccentricity which is assumed to be initially near zero. In Reference 4, an equation is derived for total transfer time as a function of thrust and altitude change. It is

$$\tau_T = \frac{W_0 I_{sp}}{F_T} \left[1 - \exp\left(-\Delta V_T / g I_{sp}\right)\right] \quad (20)$$

where ΔV_T can be calculated from

$$\Delta V_T = \sqrt{\mu / R_1} - \sqrt{\mu / R_2} \, . \quad (21)$$

Equations (20) and (21) were derived under the assumptions: 1) a nearly circular orbit, 2) a small thrust to mass ratio, and 3) a spherical Earth.

It should be carefully noted that even though a spherical Earth was assumed, Equations (20) and (21) are still very good approximations for a nonspherical Earth. This can be shown by taking the gravitational forces for an

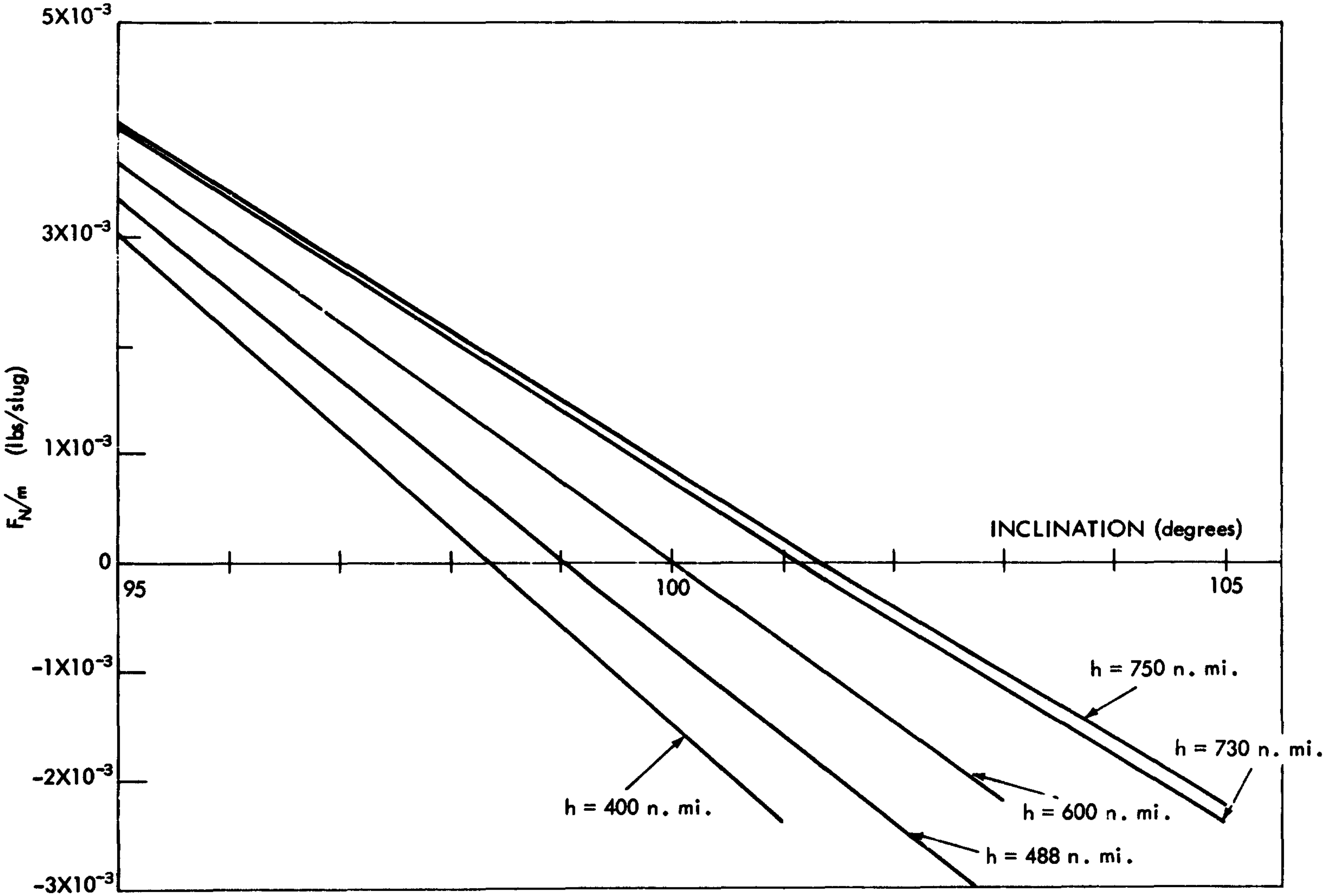

Figure 5. Thrust/Mass Requirements needed to Maintain a Sun Synchronous Precession Rate at an Altitude h vs. Inclination

oblate earth given in Reference 5 and rotating them through the azimuth angle (azimuth angle here is defined from the equator) so that they will be in the same coordinates Reference 4 used. Two cases which can then be analyzed are: 1) when the ratio of $\Delta r/r$ over an orbit is much less than one, and 2) when the ratio is not much less than one. When $\Delta r/r << 1$, the tangent gravitational force can be integrated over an orbit. This term integrates to zero, indicating that the Δr caused by this force is zero over an orbit since Δr is proportional to a tangential force. If $\dot{r}$ is large enough so that $\Delta r/r$ is not much less than one over an orbit, then the oblateness force is quite small in comparison to the central force term in the equations of motion and can thus be ignored.

It should also be noted that by a method exactly identical to that performed in the "Normal Force" discussion, it can be shown that the only effect a tangential force has on $\vec{H}$ is that it changes its magnitude but does not add a precession torque.

Using Equation (21) a plot of change in velocity versus change in altitude for various final altitudes has been made (see Figure 6). By using Figure 2, for a given injected inclination and altitude, a certain change in altitude can be found which will make a satellite sun synchronous with the injected inclination. This then can be used with Figure 6 to obtain the tangential change in velocity requirements.

Inclination Correction—To analyze the correction of inclination errors, a method involving the linearization of the equations of motion will be used. Even though this will be done using the potential for a spherical Earth, the derivation for the inclination is still quite valid for a non-spherical Earth when we are considering only a near circular orbit. This follows by considering the manner in which a plane change can be induced by a force. The required force is one that has a component normal to the orbit plane and (referring back to Figure 4) which points along $\vec{H}$ for $-\pi/2 \leq \eta < \pi/2$ and along $-\vec{H}$ for $\pi/2 \leq \eta < \pi/2$ (or the reverse directions for a plane change in the opposite direction). If the force points along $\vec{H}$ in the interval $0 \leq \eta < \pi$ and along $-\vec{H}$ in the other interval, no net change in inclination occurs over an orbit due to cancellation. This means that for an oblate Earth which is symmetrical to the equatorial plane, no net change in inclination will occur due to the gravitational force since the force is of the type just mentioned.

The equations of motion used in this derivation are:

$$\ddot{r} = \left(r\dot{\theta}^2 \cos^2\phi + r\dot{\phi}^2\right) + F_r/m - \mu/r^2 \qquad (22)$$

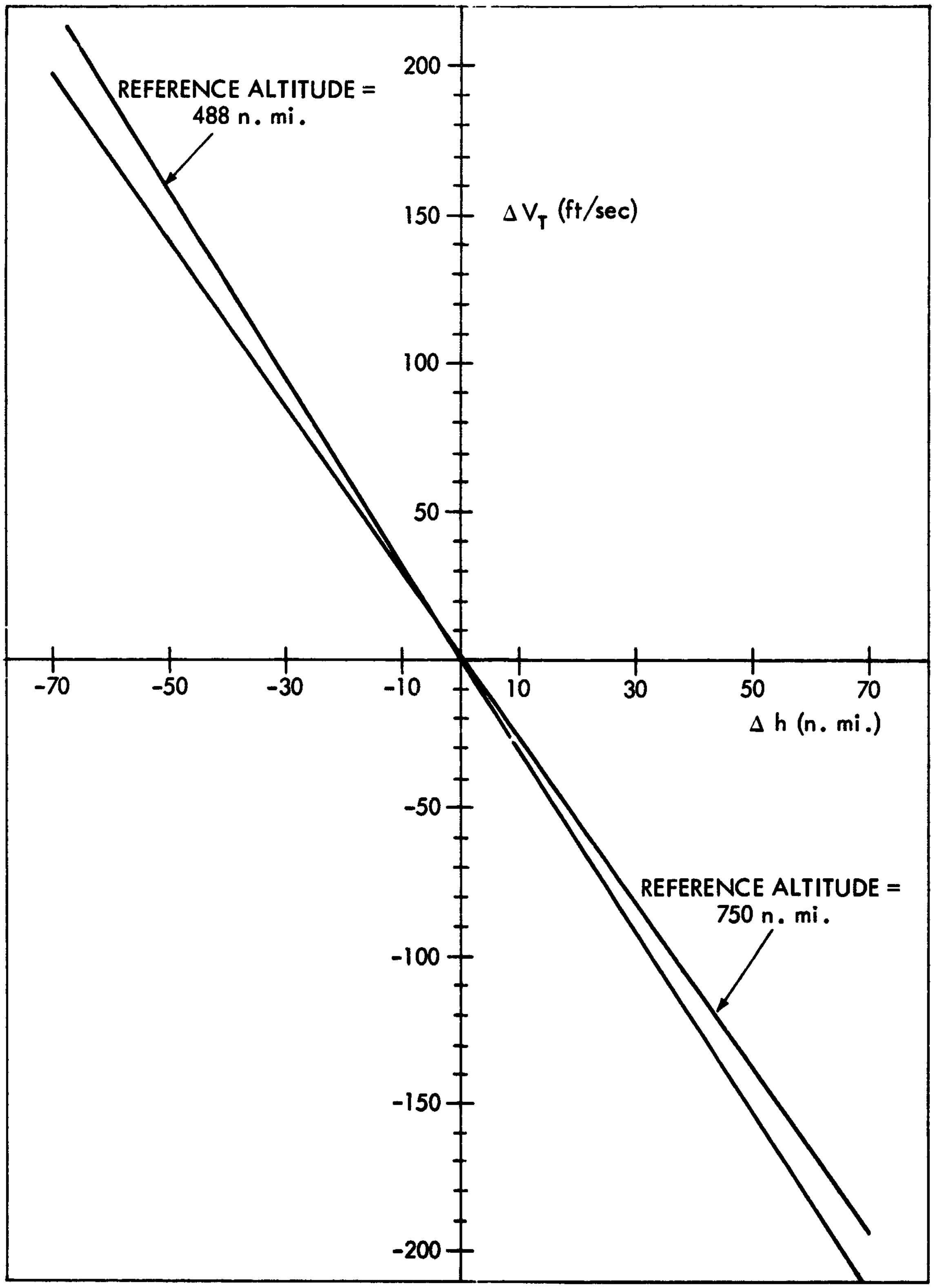

Figure 6. Change in Velocity vs. Change in Altitude

$$\frac{d}{dt}\left(r^2 \dot{\theta} \cos^2 \phi\right) = \left(F_\theta/m\right) r \cos \phi \tag{23}$$

$$\frac{d}{dt}\left(r^2 \dot{\phi}\right) = r\left(F_\phi/m - r\dot{\theta}^2 \sin \phi \cos \phi\right) \tag{24}$$

where r, θ and ϕ are defined as in Figure 7. The reference orbit for the linearization will be circular. The perturbed variables can be written

$$r = r_0 + \delta r \qquad \dot{r} = \delta \dot{r} \qquad \ddot{r} = \delta \ddot{r} \tag{25}$$

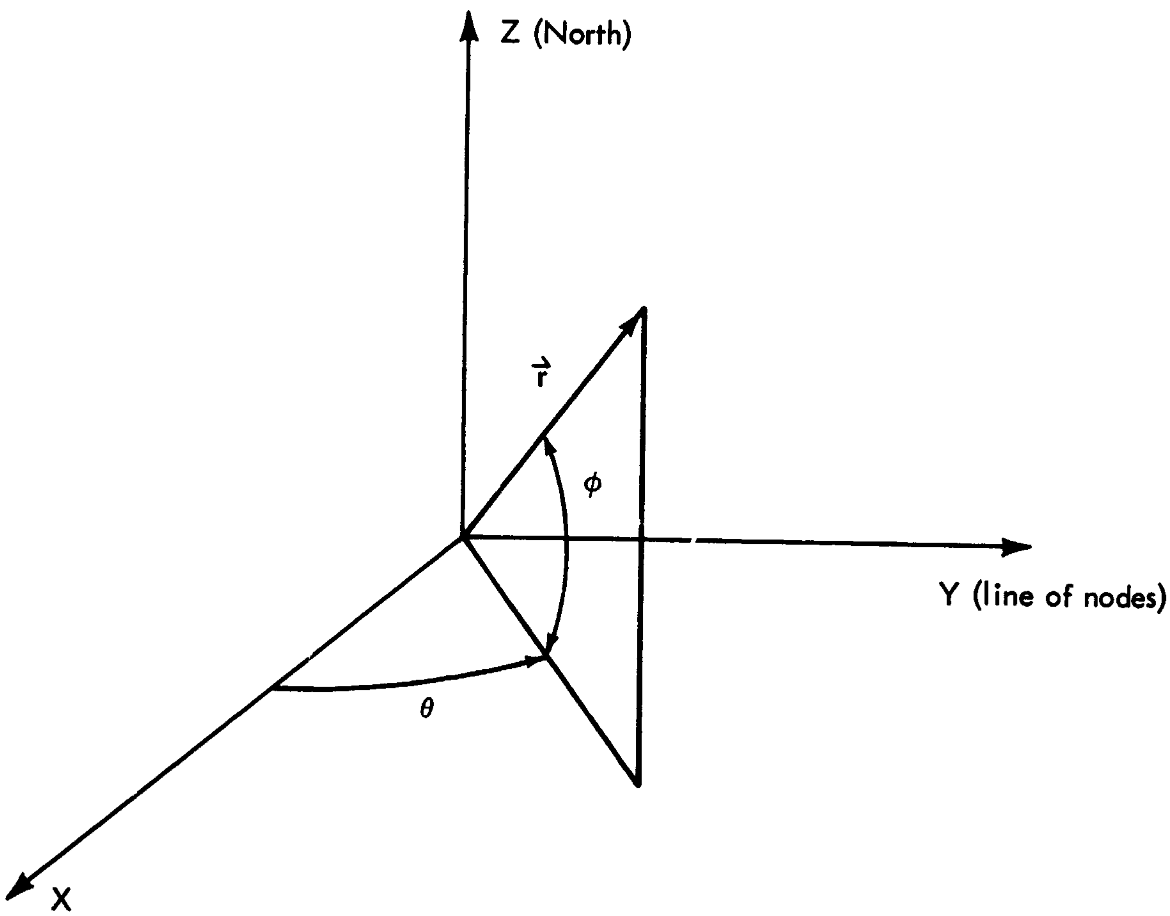

Figure 7. Reference Coordinate System Used in the Inclination Correction Paragraph

$$\phi = \phi_0 + \delta\phi \qquad \dot{\phi} = \delta\dot{\phi} \qquad \ddot{\phi} = \delta\ddot{\phi} \tag{26}$$

$$\dot{\theta} = n + \delta\dot{\theta} \qquad \ddot{\theta} = \delta\ddot{\theta} \tag{27}$$

where r_0, ϕ_0 and n refer to the circular reference orbit, and δr, $\delta\phi$ and $\delta\dot{\theta}$ are the perturbations with respect to this reference. Without any loss of generality let $\phi_0 = 0$. Equation (26) then becomes

$$\begin{aligned} \phi &= \delta\phi \\ \dot{\phi} &= \delta\dot{\phi} \\ \ddot{\phi} &= \delta\ddot{\phi} \; . \end{aligned} \tag{28}$$

Substituting Equation (25), (27) and (28) into Equation (22) and neglecting terms second order or more gives

$$\delta\ddot{r} = r_0 n^2 + n^2 \delta r + \left(r_0 + \delta r\right)\left\{2n\,\delta\dot{\theta} + (\delta\dot{\phi})^2\right\} + F_r/m - \mu/r_0^2 + 2\left(\mu/r_0^2\right)\delta r/r_0 \; . \tag{29}$$

Since the reference orbit is circular,

$$r_0 n^2 = \mu/r_0^2 \; . \tag{30}$$

If Equation (30) is substituted into Equation (29), and this result is divided by $r_0 n^2$, we have

$$\frac{1}{n^2}\frac{\delta\ddot{r}}{r_0} - \left\{3 + \left(\frac{1}{n}\,\delta\dot{\phi}\right)^2\right\}\frac{\delta r}{r_0} - \left[2\,\frac{1}{n}\,\delta\dot{\phi} + \left(\frac{1}{n}\,\delta\dot{\phi}\right)^2\right] = F_r/m r_0 n^2 \tag{31}$$

In the exact same manner Equations (23) and (24) become

$$\frac{1}{n^2}\,\delta\ddot{\theta} + 2\left(\frac{1}{n}\,\frac{\delta\dot{r}}{r_0}\right)\left(\frac{1}{n}\,\delta\dot{\theta}\right) + 2\left[\frac{1}{n}\,\frac{\delta\dot{r}}{r_0} - \frac{\delta r}{r_0}\,\frac{1}{n}\,\frac{\delta\dot{r}}{r_0}\right.$$

$$\left. + \frac{1}{2}\left(F_\theta/mr_0\,n^2\right)\frac{\delta r}{r_0} - \delta\phi\left(\frac{1}{n}\,\delta\dot{\phi}\right)\right] = F_\theta/mr_0\,n^2 \qquad (32)$$

$$\frac{1}{n^2}\,\delta\ddot{\phi} + \left[2\,\frac{1}{n}\,\frac{\delta\dot{r}}{r_0}\left(1 - \frac{\delta r}{r_0}\right)\right]\frac{1}{n}\,\delta\dot{\phi} + \delta\phi + \frac{F_\phi}{mr_0\,n^2}\,\frac{\delta r}{r_0} = \frac{F_\phi}{mr_0\,n^2}\;. \qquad (33)$$

By letting $\tau = nt$, Equations (31), (32) and (33) can be nondimensionalized. Also, all the non-linear terms will be dropped so that the system of equations can be solved. (The validity of this will be established later.) The system of equations then becomes

$$\frac{d^2}{d\tau^2}\left(\frac{\delta r}{r_0}\right) - 3\,\frac{\delta r}{r_0} - 2\,\frac{d}{d\tau}\,(\delta\theta) = A \qquad (34)$$

$$\frac{d^2}{d\tau^2}\,(\delta\theta) + 2\,\frac{d}{d\tau}\left(\frac{\delta r}{r_0}\right) + B\,\frac{\delta r}{r_0} = B \qquad (35)$$

$$\frac{d^2}{d\tau^2}\,(\delta\phi) + \delta\phi + C\,\frac{\delta r}{r_0} = C \qquad (36)$$

where,

$$A = F_r/mr_0\,n^2 \qquad (37)$$

$$B = F_\theta/mr_0\,n^2 \qquad (38)$$

$$C = F_\phi/mr_0\,n^2 \qquad (39)$$

The initial conditions and constraints on the system are:

1) $\delta r/r_0 = \delta\phi = \delta\theta = 0$ at $\tau = 0$, and

2) if $A = B = 0$, then $\delta r/r_0 = \delta\theta = 0$.

Also, since only low thrust to mass ratios, and radii less than 6,666 n.mi. (the maximum sun synchronous radius) are being considered, A, B and C << 1, and hence, terms second order or more in A, B and C will be ignored.

Using these approximations and conditions, the solutions of Equations (34), (35) and (36) are

$$\delta r/r_0 = 2B\tau \tag{40}$$

$$\delta\theta = -\frac{1}{2}\tau(A + 3B\tau) \tag{41}$$

$$\delta\phi = C\left\{1 - \cos\tau + 2B(\sin\tau - \tau)\right\} . \tag{42}$$

(It should be noted that if these solutions are substituted back into the original Equations, (31), (32) and (33), these equations will be satisfied if second order and above terms of A, B and C are neglected.)

Since no perturbations in r and θ are desired, let $A = B = 0$. This choice for A and B leaves only Equation (36) with a non-zero solution. Equation (36) becomes

$$\frac{d^2}{d\tau^2}(\delta\phi) + \delta\phi = C . \tag{43}$$

If C is defined as

$$C = \begin{cases} +|C| & 0 \le \theta < \pi \\ -|C| & \pi \le \theta < 2\pi \end{cases} , \tag{44}$$

Equation (43) will have a solution in $0 \leq \theta < \pi$ like Equation (42), but in $\pi \leq \theta < 2\pi$ another solution arises since there will be new initial conditions (continuity at boundaries) and the sign of C has changed. If C is continued as a periodic function of period 2π, the solution of Equation (43) will be

$$\delta\phi = \begin{cases} |C| \, (1 - \cos\tau) & 0 \leq \theta < \pi \\ -|C| \, (1 + 3\cos\tau) & \pi \leq \theta < 2\pi \\ |C| \, (1 - 5\cos\tau) & 2\pi \leq \theta < 3\pi \quad \text{etc.} \end{cases}$$

The solution is represented by a plot in Figure 8.

The relationship between inclination change and $\delta\phi$ is

$$\max |\delta\phi| = \Delta I \; .$$

From Figure 8 and the relation just given

$$\Delta I_{mean} = \frac{2|C|}{\pi} \tau \tag{45}$$

or

$$\Delta I_{orbit} = 4|C|$$

$$= 4F_{\phi}/mr_0 n^2$$

$$\Delta I_{orbit} \simeq 4F_N/mr_0 n^2 \tag{46}$$

The substitution of $F_N \simeq F_{\phi}$ is justified since $\delta\phi$ has been assumed to be small.

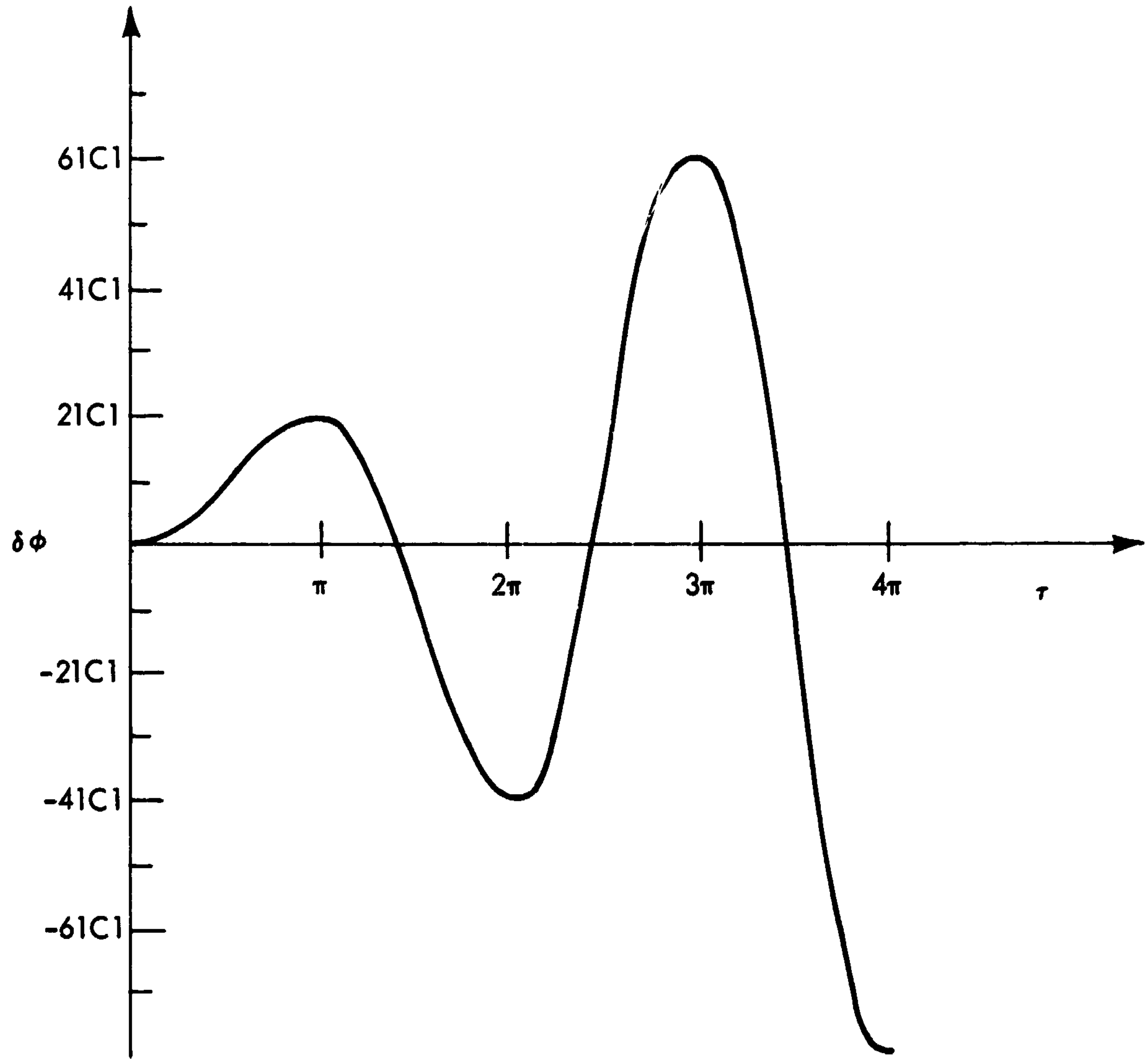

Figure 8. Solution of Equation (43)

The equation of prime concern here is Equation (45) rewritten as

$$\Delta I_{mean} \simeq \frac{2}{\pi} \frac{F_N}{m} \frac{1}{rn} \tau_T$$

$$\simeq \frac{2}{\pi} \sqrt{r/\mu} \frac{F_N}{m} \tau_T \quad (47)$$

where it was assumed $rn \simeq \sqrt{\mu/r}$. This can also be rewritten as

$$\Delta V_N = \sqrt{\mu/r}\left(\frac{\pi}{2}\right) \Delta I_{mean} \quad , \tag{48}$$

where the substitution $\Delta V_N = F_N \tau_T/m$ was used. To validate this substitution it will be assumed that the propellant mass is much less than the total initial mass of the satellite. Using Equation (48), Figure 9, which plots change in velocity versus mean change in inclination, was made.

General Comparisons of Precession Correcting Methods

In this paragraph the three methods just considered for the precession correction will be compared.

The method which uses a normal thrust to aid the Earth's oblateness requires a large total impulse for missions of a half year or longer. This is due to the fact that a thrust is needed as long as a sun synchronous precession rate is desired. This will be illustrated for a Tiros M mission in the next paragraph.

The two methods for orbit correction will now be compared. These two methods are: 1) use a tangential thrust to correct the altitude by spiraling in or out, and 2) use a normal thrust to cause an inclination change. The comparison will be based on the total impulse required for each method.

The two basic relationships that will be used are Equations (21) and (48) rewritten as

$$\Delta V_T = \sigma_2 \left(\sqrt{\frac{\mu}{r_0 - \Delta h}} - \sqrt{\frac{\mu}{r_0}}\right) \quad , \tag{49}$$

and

$$\Delta V_N = \sigma_2 \frac{\pi \Delta I_{mean}}{2} \sqrt{\frac{\mu}{r_0 - \Delta h}} \tag{50}$$

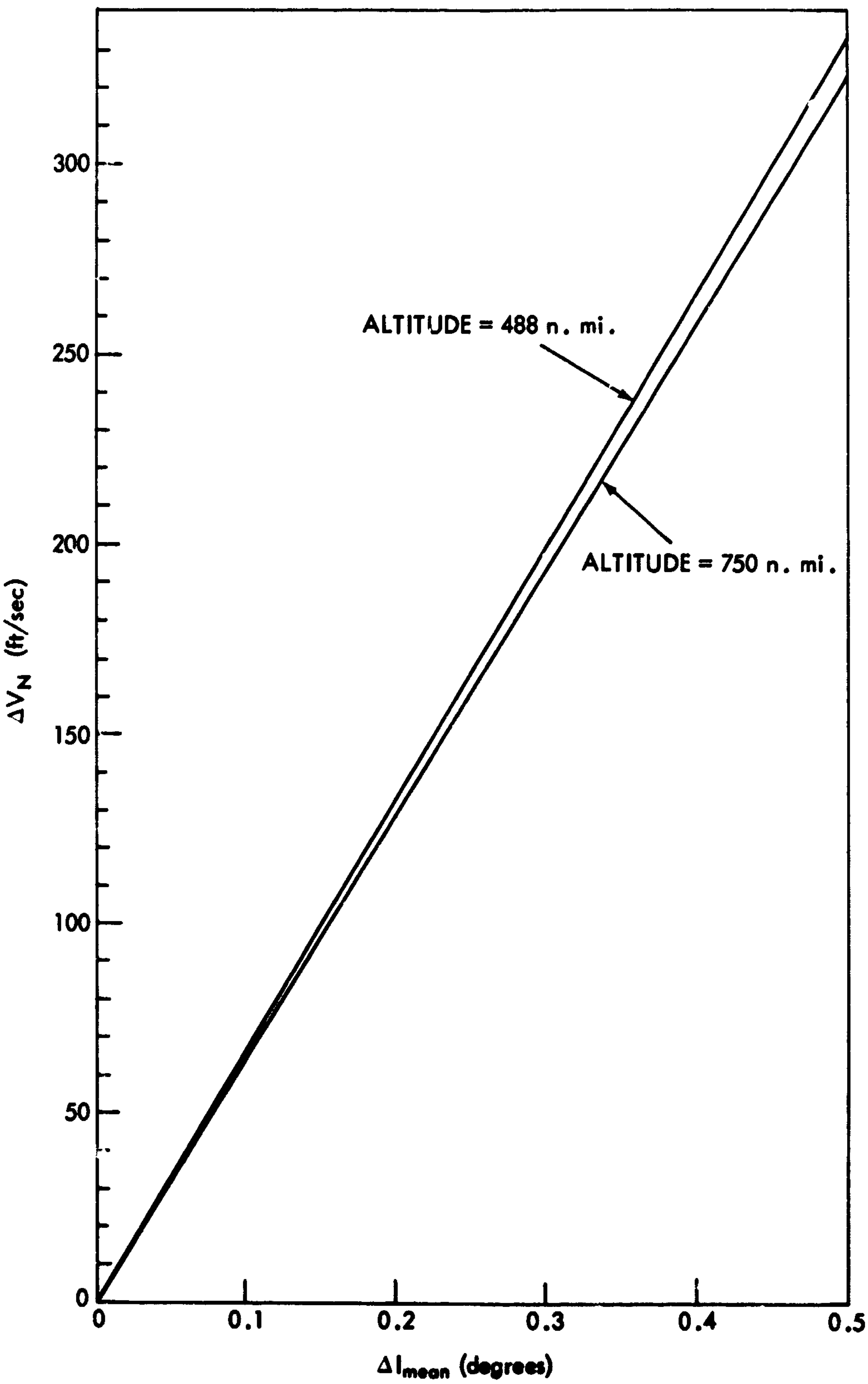

Figure 9. Change in Velocity vs. Mean Change in Inclination

where σ_2 is the needed conversion factor from n.mi./sec. to ft./sec. since r_0 is given in nautical miles. Let C_{eff} be defined as the ratio of the total impulse necessary to make a satellite sun synchronous using only an inclination change, to the total impulse necessary to make a satellite sun synchronous using only an altitude change, i.e.

$$C_{eff} = \frac{m\Delta V_N}{m\Delta V_T} \qquad \left(\text{assuming } W_p/W_0 << 1\right)$$

or

$$C_{eff} = \frac{\Delta V_N}{\Delta V_T} \; . \tag{51}$$

If it is also assumed that $\Delta h/r_0 << 1$, C_{eff} reduces to

$$C_{eff} = \frac{\pi r_0 \, \Delta I_{mean}\,(\deg)}{57.3\,\Delta h} \tag{52}$$

where $\Delta I_{mean}\,(\deg)/\Delta h$ is the slope of the sun synchronous curve, Figure 3, if Δh and ΔI_{mean} are sufficiently small.

To obtain a better understanding of C_{eff} and the variables which make it up, consider Figure 10. The solid curve represents the sun synchronous curve of Figure 3, and the dashed curve represents a constant precession rate error from the sun synchronous value. The point (r_1, I_1) represents the injection point; (r_0, I_1) represents the point on the sun synchronous curve which is reached by changing the altitude. From Figure 10 it can be seen that a certain Δh and ΔI_{mean} are associated with each injection point.

Figure 11 shows C_{eff} (which is in fact a penalty factor) as a function of altitude. This figure shows the interesting result that C_{eff} is always greater than one. This means that the total impulse required to correct a given injection by correcting only inclination is always greater than that required for an altitude correction.

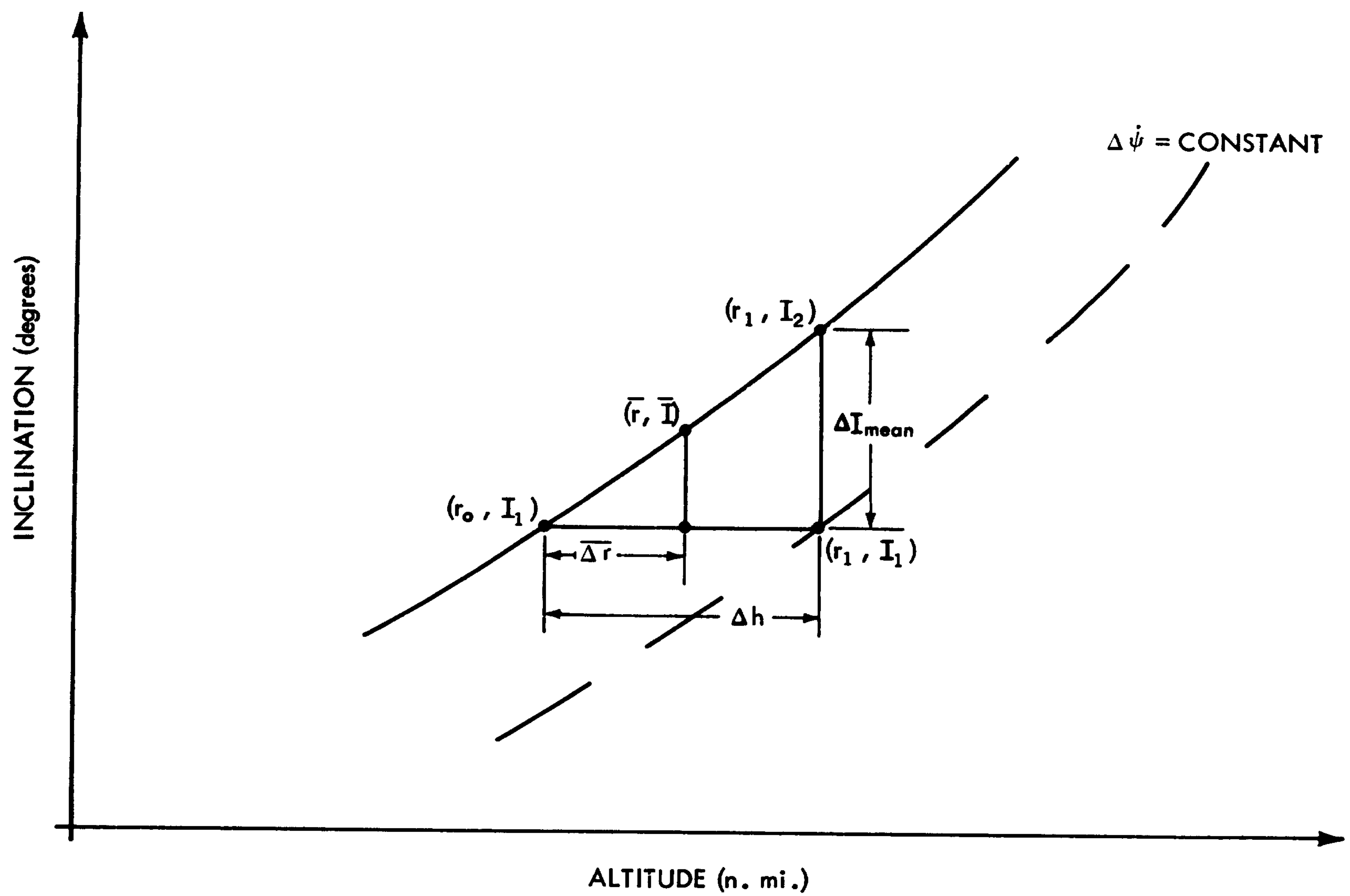

Figure 10. Illustration of Variables Used in C_{eff} Derivation

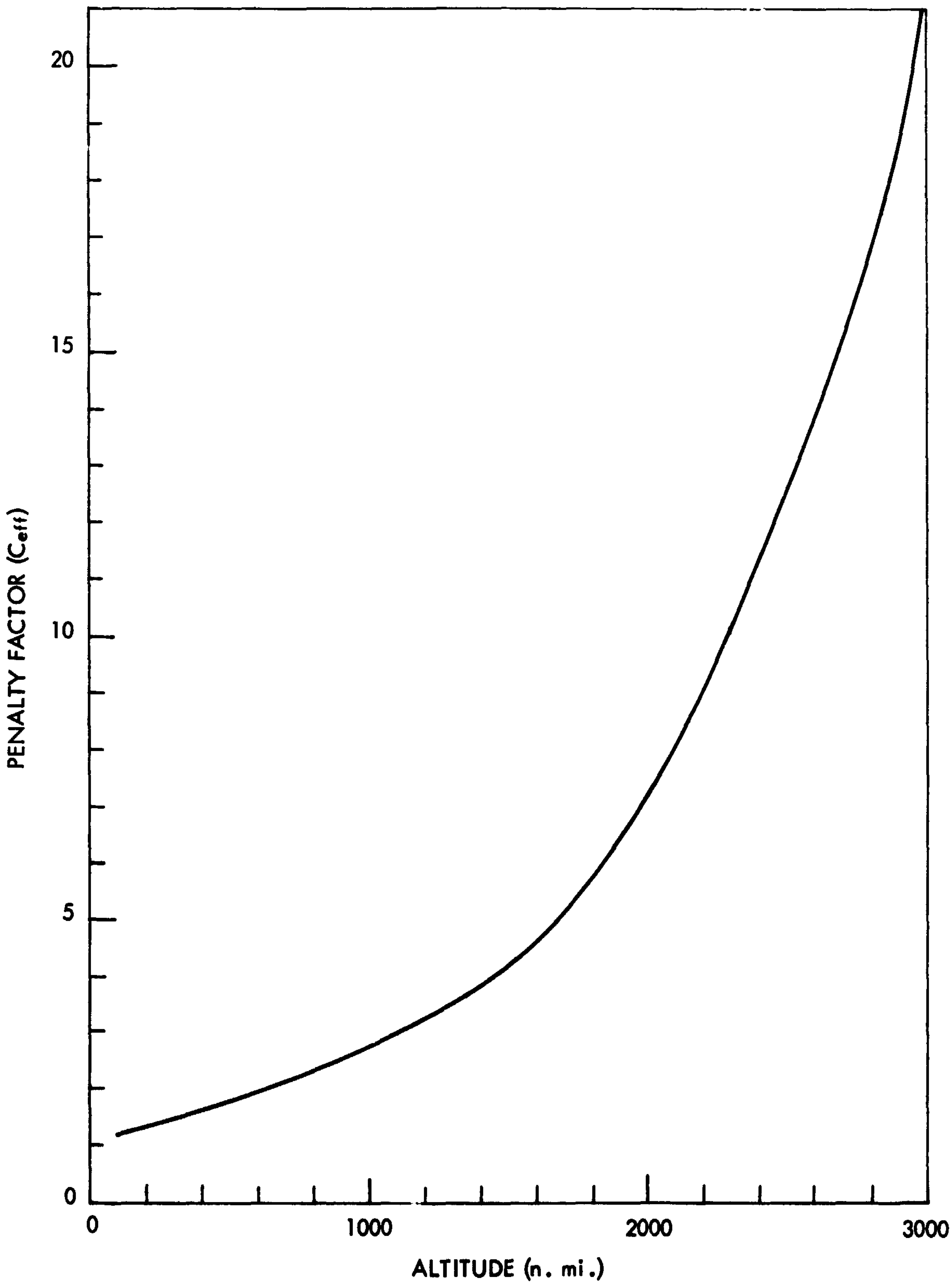

Figure 11. Penalty Factor vs. Altitude

The conclusion that can be drawn from this is that it is always better, from a propulsion standpoint, to correct to a sun synchronous orbit with a tangential thrust (altitude correction).

It should be pointed out, however, that the mission constraints may make this impossible. Referring again to Figure 10, where $(\bar{r}, \bar{I})$ represents the reference orbit, one can see that an altitude error can arise if one only spirals in. In the next paragraph this will be illustrated further using parameters for Tiros M.

Comparison of Precession Correcting Methods for a Tiros M Mission

To illustrate what has been said concerning the various methods for correcting the precession rate error, the Tiros M mission will be considered. Tiros M is a near-polar, near-circular sun synchronous meteorological satellite. The nominal orbit for Tiros M is given in Table 1 (Reference 3). The 1σ injection error, given in terms of a precession rate error, is 0.029 degrees/day (Reference 7). The total impulse required to correct the 1σ injection error will be used to compare the three methods.

So that the change in altitude and the change in inclination that are needed in Equations (28) and (41) can be found for a certain precession rate error, Figure 12 and 13 have been made. Figure 12 plots change in precession rate in degrees/day versus change in inclination in degrees. Figure 13 plots change in precession rate in degrees/day versus change in altitude in nautical miles.

The total impulse required to correct a precession rate error of 0.029 degrees/day can be determined quite readily now. First it is noted that the 1σ precession rate error could be caused by either an inclination error of 0.34 degrees or an altitude error of 35 n.mi. (these are found from Figures 12 and 13). Now using Figures 5 and 6 and Equation (48) the total impulse for each of the three methods can be obtained. The results are listed in Table 2.

Table 2 illustrates quite clearly what was said in general earlier. The total impulse for correcting inclination is approximately 2.2 times as large as that needed to correct altitude; this is in agreement with the value given on the

Table 1

Tiros M (nominal orbit)

inclination = 101.4°
altitude = 750 n.mi. (approx.)
weight of satellite = 670 lbs.

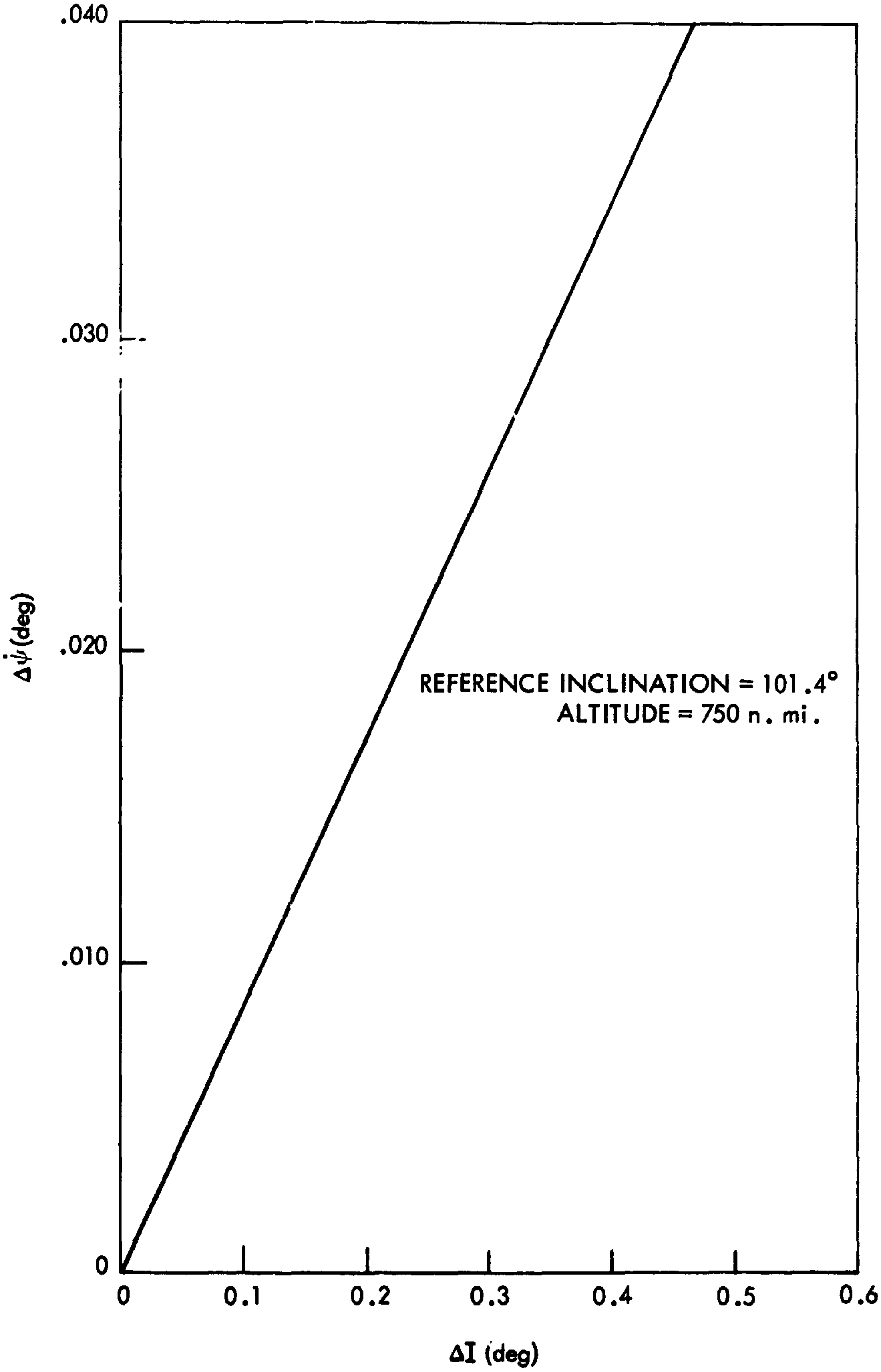

Figure 12. Change in Precession Rate vs. Change in Inclination

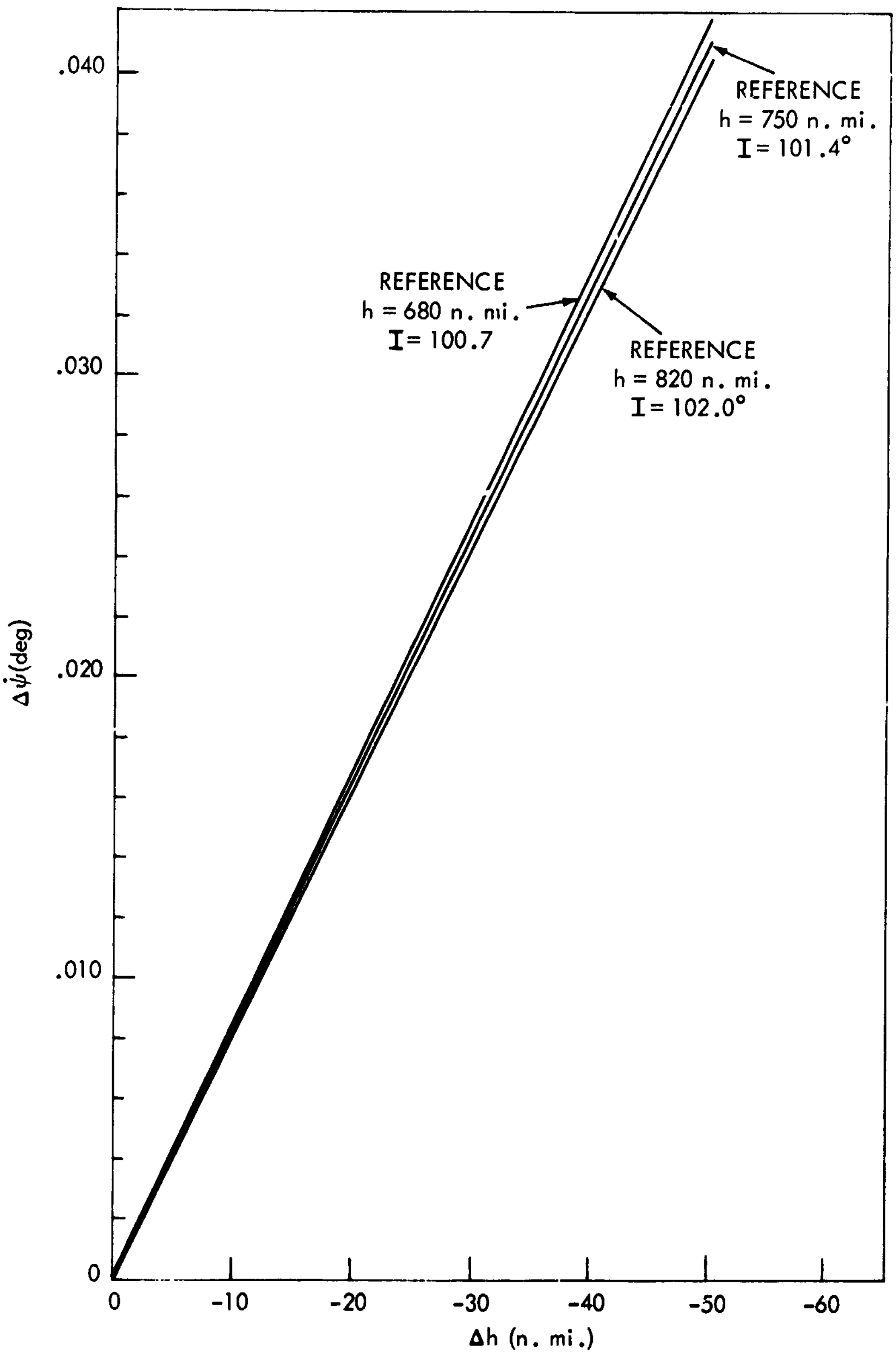

Figure 13. Change in Precession Rate vs. Change in Altitude

Table 2

Total Impulse Required to Correct
a Precession Rate Error of 0.029 deg./day

Normal Force (aid oblateness)	138,700 lb.-sec./year
Tangential Force (correct altitude)	2,058 lb.-sec.
Normal Force (correct inclination)	4,571 lb.-sec.

C_{eff} graph. The total impulse needed to aid the precession due to oblateness for one year is approximately 67.4 times that needed to correct altitude.

The above facts illustrate the basic conclusion: from a propulsion standpoint, it is best to use a tangential thrust, which corrects altitude, to get back to a point on the sun synchronous curve.

As was mentioned earlier, one objection to using a tangential thrust is the error in altitude that might arise. Figure 14, which plots error in altitude (final sun synchronous altitude minus the reference altitude) versus the injected inclination, illustrates this. For the 1σ injection error which corresponds to an inclination error of 0.34 degrees, the altitude error would be approximately 35 n.mi. An error of this magnitude could be tolerated by Tiros but for a satellite such as EROS, which has a very small allowable error in altitude, this would be far too much (Reference 6).

EVALUATION OF PROPULSION SYSTEMS

The purpose of this section is to compare propulsion systems that can be used to correct the Tiros M injection errors. Five propulsion systems are considered for the mission. They are: 1) a 20 μlb. ion engine system, 2) a 300 μlb. ion engine system, 3) two thermal storage resistojet systems, and 4) a 2 lb. hydrazine system. Each system is assumed to be mounted so that the thrusters are tangent to the orbit (the configuration that was just shown in the preceding section to be the best from propulsion standpoint, see Figure 15). It should be carefully noted that each propulsion system contains two thrusters so that the satellite can be either spiraled in or out. Three aspects of each system will be compared. These are: 1) total weight of the system, 2) total time needed to correct the orbit, and 3) total power needed to operate the system.

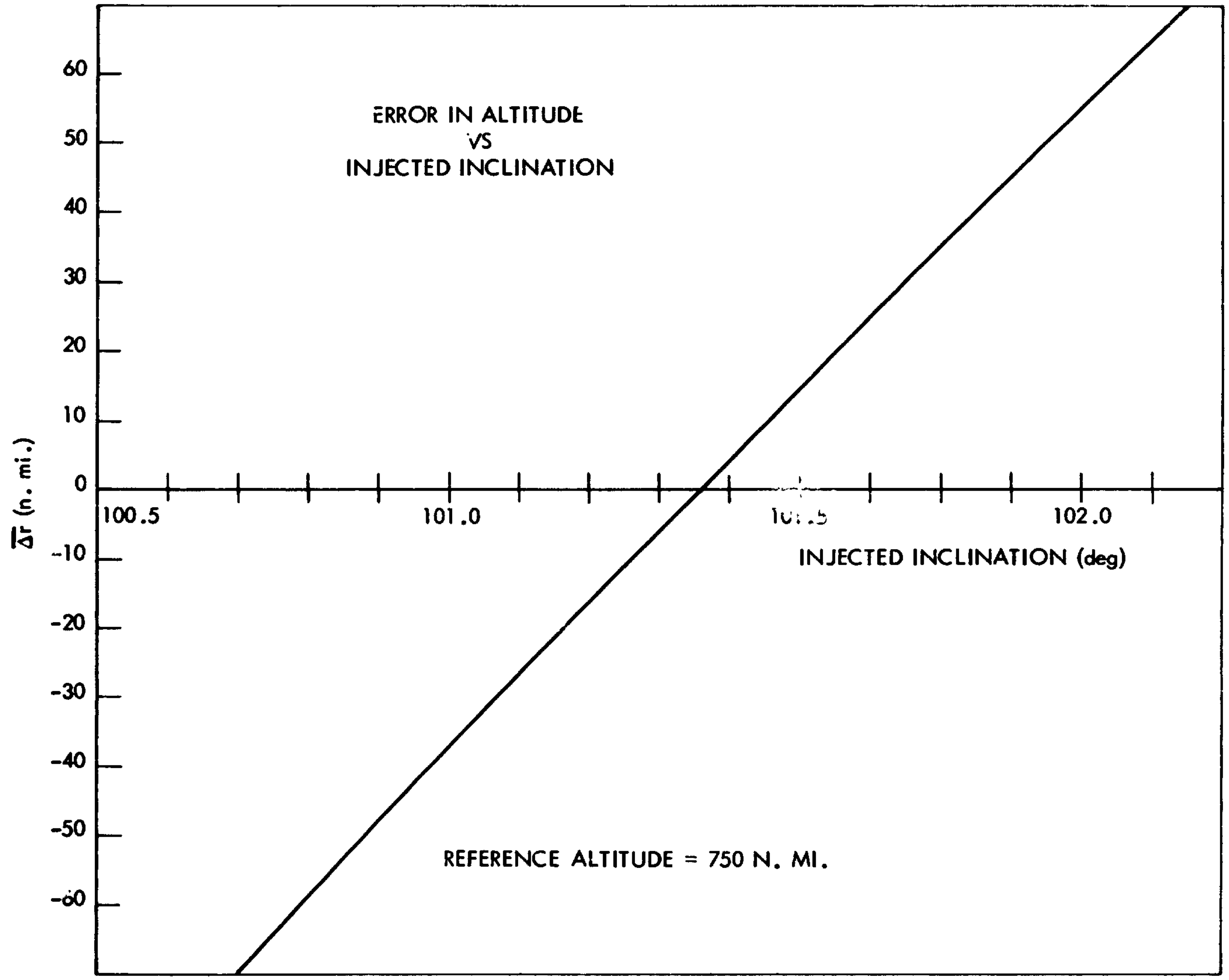

Figure 14. Error in Altitude vs. Injected Inclination

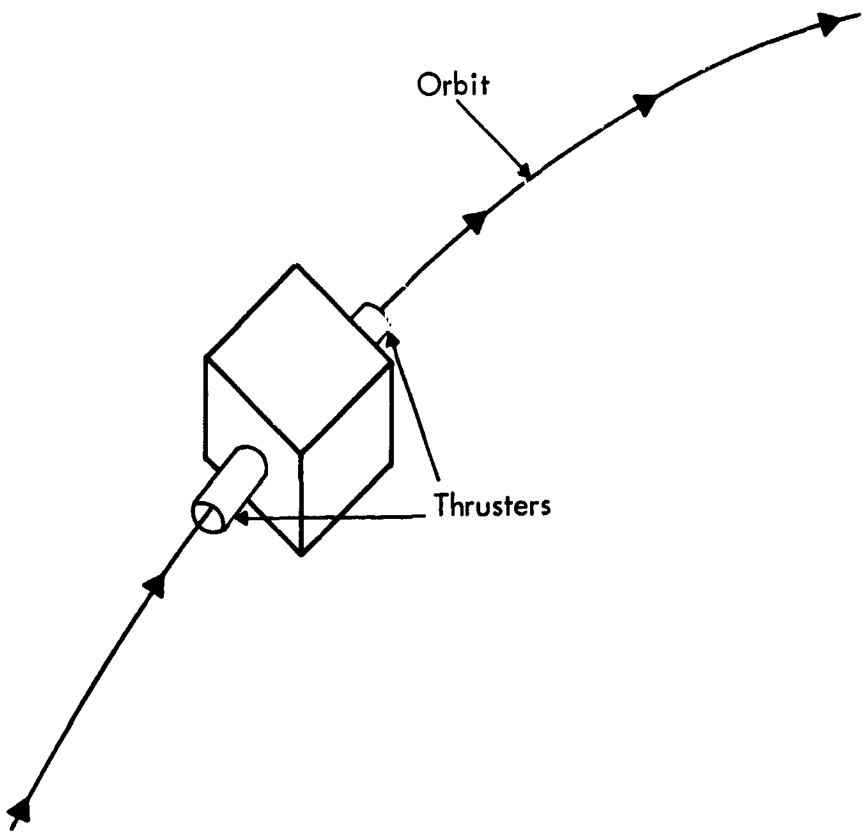

Figure 15. Thruster Configuration: Thrusters are Tangent to Orbit

The propulsion system weight is made up of six parts: 1) the propellant weight, W_p; 2) the weight of thrusters, W_{th}; 3) the weight of feed systems, W_{fs}; 4) the tankage weight, W_t; 5) the weight of the power conditioner, W_{pc} and 6) the telemetry weight, W_{tel}. Written as an equation this becomes

$$W_{ps} = W_p + W_{th} + W_{fs} + W_t + W_{pc} + W_{tel} \quad . \tag{49}$$

However,

$$W_{th} + W_{fs} + W_{pc} + W_{tel} \simeq K_0 = \text{constant} \quad , \tag{50}$$

and

$$W_t \simeq K_1 W_p + K_2 \qquad (51)$$

where K_1 and K_2 are system constants (Reference 6). Putting Equations (50) and (51) into Equation (49) gives

$$W_{ps} \simeq (1 + K_1) W_p + K_3 , \qquad (52)$$

where $K_3 = K_0 + K_2$. The propellant weight can be determined by

$$W_p = \left(\frac{I_t}{I_{sp}}\right) . \qquad (53)$$

Since for all the propulsion systems being considered $W_p << W_0$, the loss of satellite mass due to the expulsion of propellant is negligible. Thus total impulse can be written

$$I_t = \left(\frac{W_0}{g}\right) \Delta V_T . \qquad (54)$$

Combining Equations (52), (53), and (54) gives

$$W_{ps} = (1 + K_1) \frac{W_0 \Delta V_T}{g I_{sp}} + K_3 . \qquad (55)$$

Using Equation (55) and the constants for the various propulsion systems given in Table 3, propulsion system weight versus increment in velocity plots can be obtained (see Figure 16). Figure 16 shows that the 20 μlb. ion engine system is the lightest after 28 fps change in velocity; while from 0 to 28 fps, the resistojet (I_{sp} = 200 sec.) system is the lightest. The ranking of the systems

Table 3

Propulsion Systems (two thrusters)

System #1	20 μlb. ion engine
	I_{sp} = 5000 sec.
	K_1 = 0
	K_3 = 16 lb.
	Power = 25 watts
System #2	300 μlb. ion engine
	I_{sp} = 5000 sec.
	K_1 = 0
	K_3 = 30 lb.
	Power = 73 watts
System #3	thermal storage resistojet
	I_{sp} = 200 sec.
	F = 0.005 lb.
	K_1 = 0.4167
	K_3 = 12 lb.
	Power = 66 watts
System #4	thermal storage resistojet (cold gas)
	I_{sp} = 100 sec.
	F = 0.050 lb.
	K_1 = 0.4167
	K_3 = 12 lb.
	Power = negligible
System #5	2 lb. hydrazine engine
	I_{sp} = 225 sec.
	K_1 = 0.26
	K_3 = 16 lb.
	Power = negligible

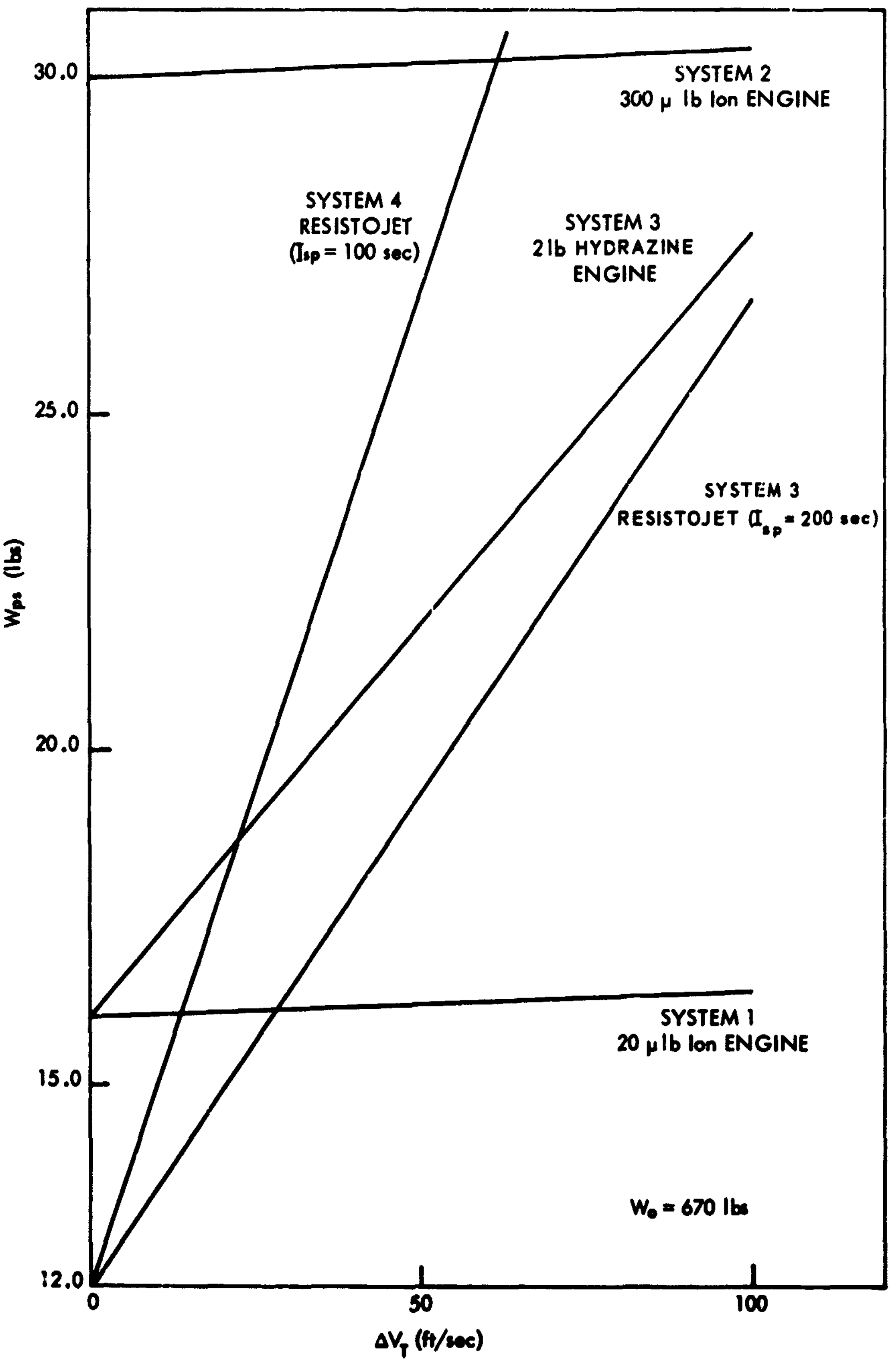

Figure 16. Propulsion System Weight vs. Change in Velocity Required to Correct Orbit

with respect to lightest average weight in the 0-100 fps ΔV range is: 1) 20 μlb. ion engine system, 2) 200 sec., I_{sp} resistojet system, 3) 2 lb. hydrazine system, 4) 100 sec., I_{sp} resistojet system, and 5) 300 μlb. ion engine system.

The next comparison that will be made concerns the time required for a given correction. Since the satellite mass remains nearly constant, the total impulse can then be written

$$I_t = \left(\frac{W_0}{g}\right) \Delta V_T = F\tau_T$$

or,

$$\tau_T = \frac{W_0 \, \Delta V_T}{gF} \, . \tag{56}$$

Using the values given in Table 3 and Equation (56), Figure 17, which plots thrust time versus change in velocity, was constructed. Figure 17 clearly shows that the 2 lb. hydrazine system takes the least amount of time for any ΔV_T required. The ion engine systems took considerably more time than any other system, e.g., the 20 μlb. ion engine system required approximately 486 days for a 40 fps change in velocity and the 300 μlb. ion engine system required approximately 32.2 days for the same change in velocity. Rating the systems according to least time required for a given change in velocity gives: 1) 2 lb. hydrazine system, 2) 100 sec., I_{sp} resistojet system, 3) 200 sec. I_{sp} resistojet system, 4) 300 μlb. ion engine system, and 5) 20 μlb. ion engine system.

The last comparison to be made is that of total power required. Using the powers listed in Table 3 (these were obtained from Reference 6) it can be seen that in terms of least power required the systems rate: 1) 2 lb. hydrazine system and 100 sec. I_{sp} resistojet system (cold gas), which required almost no power, 3) 20 μlb. ion engine system, 25 watts, 4) 200 sec., I_{sp} resistojet system, 66 watts, and 5) 300 μlb. ion engine system, 73 watts.

So that a comparison of all the propulsion systems can be more easily made, the Tiros M mission is again considered. Table 4 lists for each propulsion system the weight of the propulsion system, the thrust on time, and the power

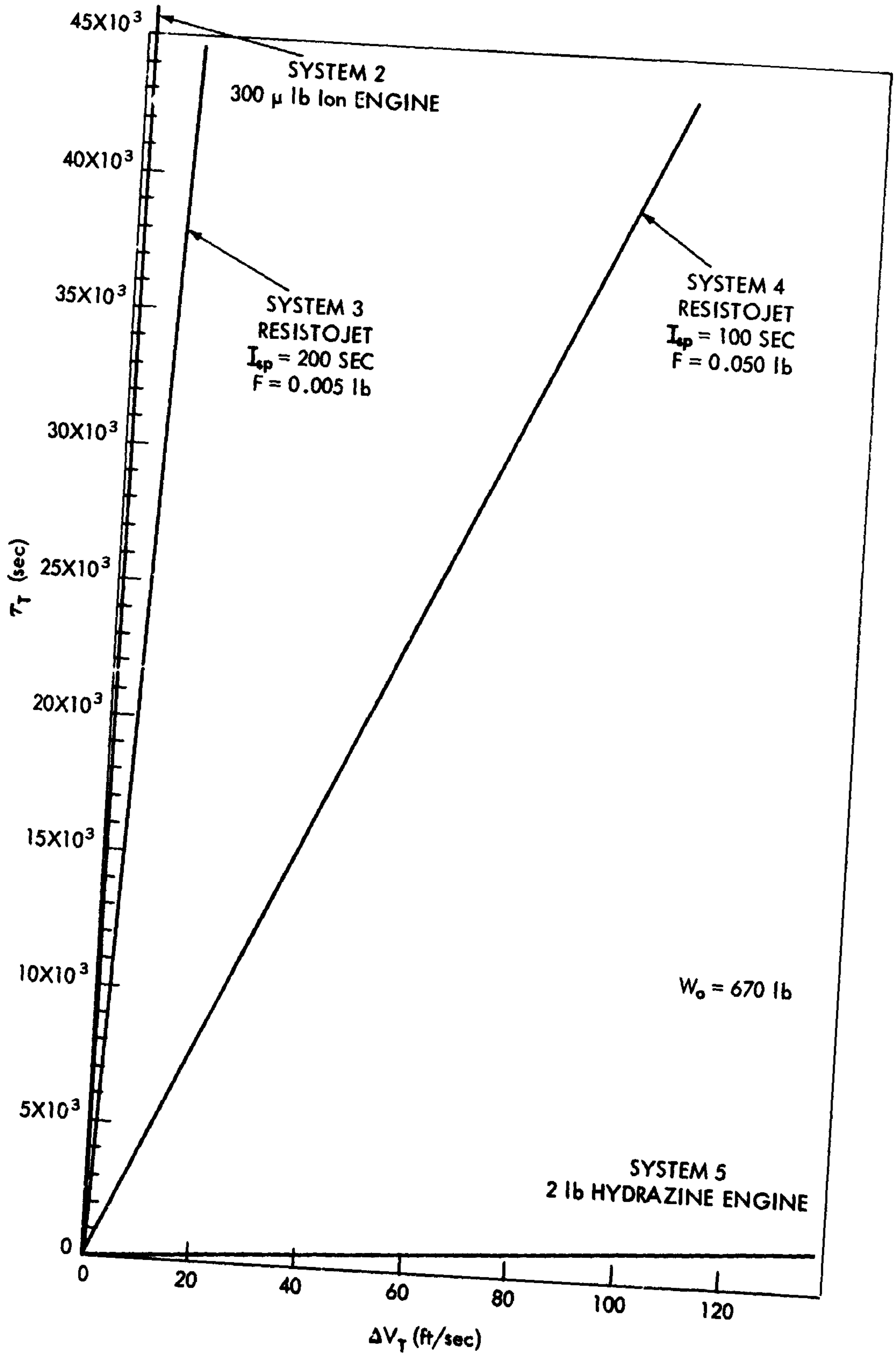

Figure 17. Thrust Time vs. Change in Velocity Required to Correct Orbit

Table 4

Tiros M Propulsion System Requirements

(Note: These are for an approximate precession rate error of 0.029 degrees/day)

Propulsion Systems	W_{ps} (lb.)	τ_T (days)	Power (watts)
20 μlb. ion engine	16	1208.	25
300 μlb. ion engine	30	80.5	73
200 sec., I_{sp} resistojet	27	4.8	66
100 sec., I_{sp} resistojet (cold gas)	42	0.5	negligible
2 lb. hydrazine	28	0.012 (1,043 sec.)	negligible

requirements needed to correct a precession rate error of 0.029 degrees/day (the 1σ error for Tiros M). A correction time of more than 30 days was considered unacceptable for Tiros; thus, the ion engines are ruled out as possible systems for Tiros M. This leaves only the resistojets and the hydrazine system for further consideration. The final decision, on which system would be the best for Tiros M, rests on the attitude control system. It is possible that a thrust misalignment on the hydrazine system could cause disturbances in the attitude that could not be corrected by the control system. This problem, however, is not being treated in this paper.

CONCLUSION

Three methods for correcting a precession rate error, that can result from injection errors, have been considered in this report. They are: 1) use a force normal to the plane of the orbit to aid the oblateness force, 2) use a tangential force to spiral in or out to an altitude which gives a sun synchronous precession rate at the injected inclination and 3) use a force normal to the plane of the orbit to obtain an inclination which causes a sun synchronous precession rate at the injected altitude. The analysis showed that of the three methods considered tangential thrusting would be the best method from a propulsion standpoint. It should be kept in mind, however, that the final altitude may be different than the reference altitude if the injected inclination is different than the reference one.

Comparisons were made of the total system weights, the orbit correction times, and total power requirements of five propulsion systems (the systems include two thrusters to allow for either spiraling in or out). The five systems are: 1) a 20 μlb. ion engine system, 2) a 300 μlb. ion engine system, 3) two resistojet systems, one with I_{sp} = 200 sec. and thrust equal to 0.005 lb., and the other with I_{sp} = 100 sec. and thrust equal to 0.050 lb., and 5) a 2 lb. hydrazine system. The comparisons of the five systems were based on the requirements needed to correct the 1σ error for Tiros M (0.029 deg./day). The ion engines required more time than was acceptable for the Tiros mission and were eliminated from further consideration. The weight of the 200 sec., I_{sp} resistojet and the 2 lb. hydrazine system were about the same. The hydrazine system (if fired continuously) required the least time for the correction. The cold gas resistojet and the hydrazine system required negligible amounts of power. An important factor that was not included in this paper was the effects of thrust misalignment. The attitude distrubances caused by a thrust misalignment must be compared with the control capability of Tiros before a final decision can be made.

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6. Private communication with Mr. W. C. Isley of the Systems Analysis and Ion Propulsion Section, April 6, 1967.

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APPENDIX

PRECESSION DUE TO THE EARTH'S OBLATENESS*

The purpose of this appendix is to derive expressions for the moments and the orbit precession rate due to the equatorial bulge.

From Reference 5 the forces due to the equatorial bulge are

$$f_r = J\left(3 \sin^2 L - 1\right) ,$$

$$f_L = -2J \sin L \cos L ,$$

and

$$f_\lambda = 0 . \tag{A1}$$

Where

$$J = \frac{3}{2} mJ_{20} \frac{\mu}{r^2} \left(\frac{r_e}{r}\right)^2 .$$

Equation (A1) transforms tc components in the X, Y, Z coordinates of Figure A-1 as

$$f_X = J\left(5 \sin^2 L - 1\right) \cos\lambda \cos L ,$$

$$f_Y = J\left(5 \sin^2 L - 1\right) \sin\lambda \cos L ,$$

and

$$f_Z = J\left(5 \sin^2 L - 3\right) \sin L . \tag{A2}$$

*Derivation was performed by C. C. Barrett of the Auxiliary Propulsion Branch, Systems Analysis and Ion Propulsion Section.

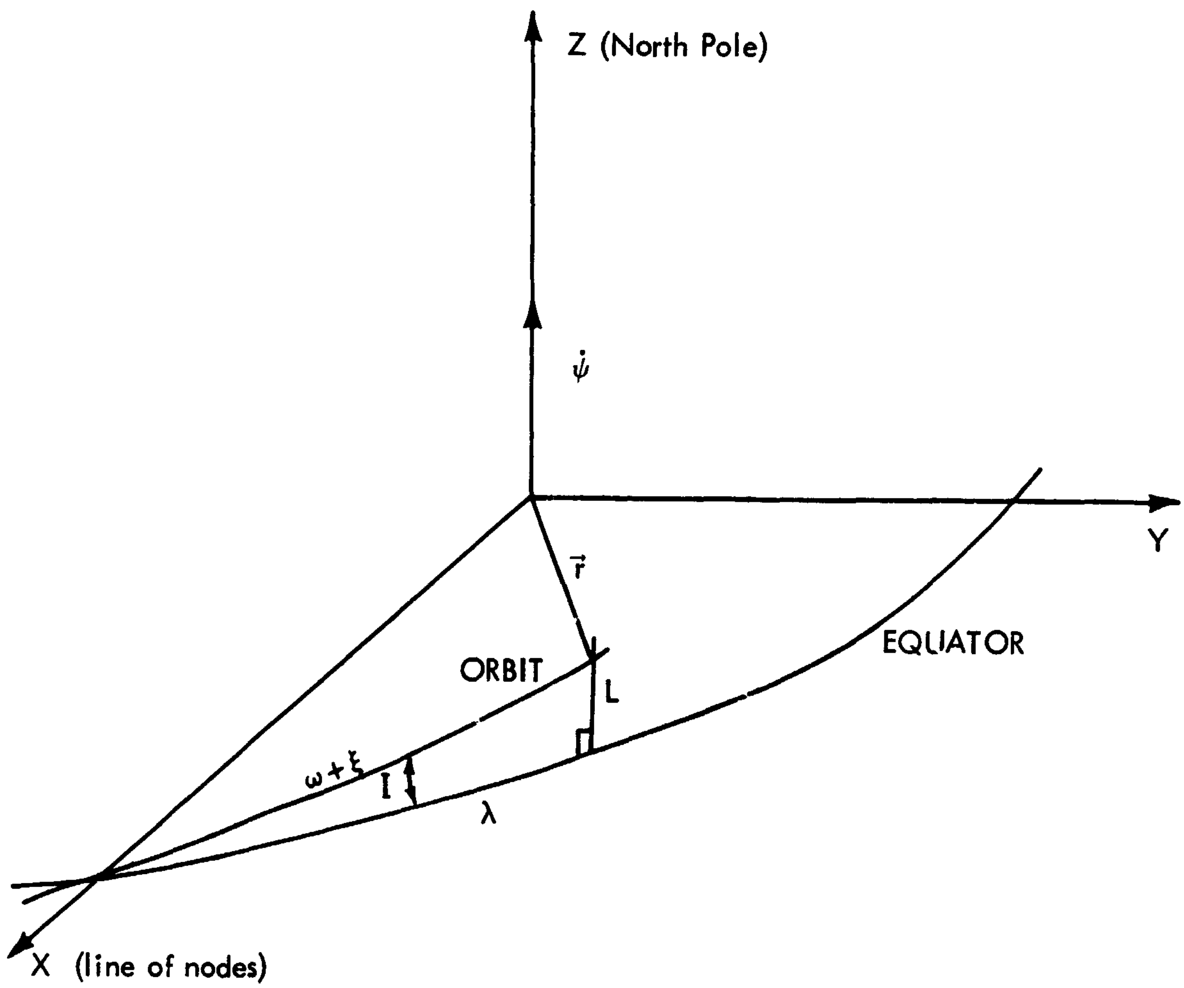

Figure A-1. Reference Coordinate System for Appendix

Noting that

$$\frac{X}{r} = \cos\lambda\cos L \ ,$$

$$\frac{Y}{r} = \sin\lambda\cos L \ ,$$

and

$$\frac{Z}{r} = \sin L \ , \qquad \text{(A3)}$$

and evaluating

$$\vec{M}_P = \vec{r} \times \vec{f}$$

gives

$$M_{PX} = -2rJ \left(\frac{Y}{r}\right) \left(\frac{Z}{r}\right)$$

$$M_{PY} = 2rJ \left(\frac{X}{r}\right) \left(\frac{Z}{r}\right)$$

and

$$M_{PZ} = 0 \quad . \tag{A4}$$

Noting that in terms of orbital variables $\omega + \xi$ and I

$$\frac{X}{r} = \cos(\omega + \xi) \quad ,$$

$$\frac{Y}{r} = \sin(\omega + \xi) \cos I \quad ,$$

and

$$\frac{Z}{r} = \sin(\omega + \xi) \sin I \quad , \tag{A5}$$

Equations (A4) become:

$$M_{PX} = rJ \left[1 - \cos 2(\omega + \xi)\right] \sin I \cos I \quad ,$$

$$M_{PY} = rJ \sin 2(\omega + \xi) \sin I \quad ,$$

and

$$M_{PZ} = 0 \quad . \tag{A6}$$

All of M_{PY} and that part of M_{PX} containing $\cos 2(\omega + \xi)$ have a net value of zero over one orbit, or as $(\omega \pm \xi)$ changes from 0 to 2π. Therefore the net moment over an orbit acts about the X-axis and has a value of

$$\tilde{M}_{PZ} = -rJ \sin I \cos I$$

$$= -\frac{3}{2} mJ_{20} \left(\frac{\mu}{r}\right) \left(\frac{r_e}{r}\right)^2 \sin I \cos I \ . \tag{A7}$$

This moment produces a change in momentum given by

$$\dot{\vec{H}} = \hat{k} \dot{\psi} \times \vec{H}$$

$$= \hat{i}\, mr^2 \dot{\xi} \dot{\psi} \sin I \ . \tag{A8}$$

Equating Equations (A7) and (A8) yields

$$mr^2 \dot{\xi} \dot{\psi} \sin I = -\frac{3}{2} mJ_{20} \left(\frac{\mu}{r}\right) \left(\frac{r_e}{r}\right)^2 \sin I \cos I \ .$$

Since this equation is valid for all inclinations, there follows

$$\dot{\psi} = -\frac{3}{2} J_{20} \left(\frac{1}{\dot{\xi}}\right) \left(\frac{\mu}{r^3}\right) \left(\frac{r_e}{r}\right)^2 \cos I \ . \tag{A9}$$

For a nearly circular orbit $\dot{\xi} \simeq \mu/r^3$ and Equation (A9) can be rewritten as

$$\dot{\psi} = -\frac{3}{2} J_{20} \dot{\xi} \left(\frac{r_e}{r}\right)^2 \cos I \ . \tag{A10}$$