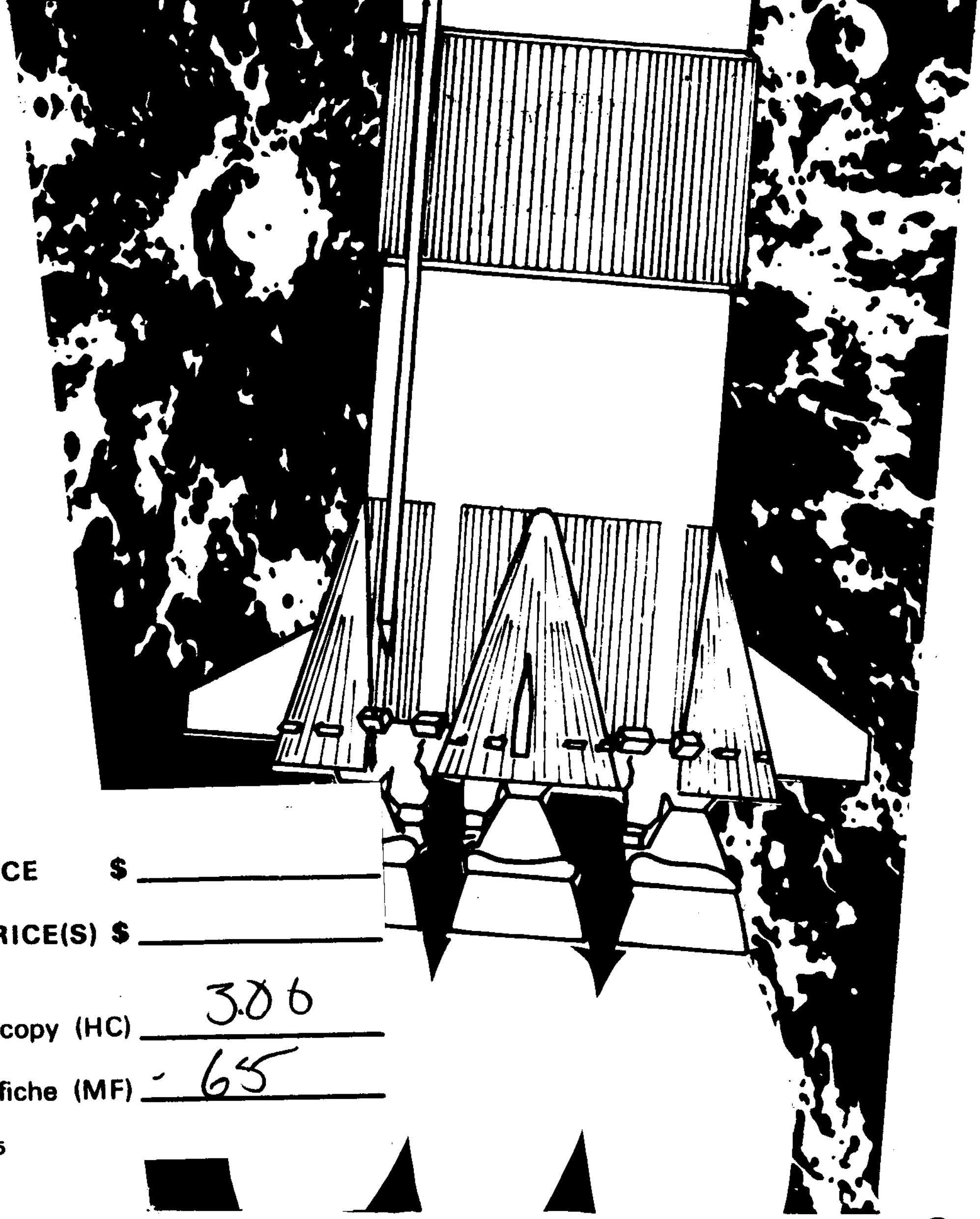

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NONLINEAR PROBLEMS IN WAVE PROPAGATION

By Klaus Oswatitsch

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NONLINEAR PROBLEMS IN WAVE PROPAGATION

By

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Vienna, Austria

RESEARCH PROJECTS OFFICE
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NONLINEAR PROBLEMS IN WAVE PROPAGATION

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ABSTRACT

This report presents a new method for the theoretical predictions of weak shocks and Prandtl-Meyer expansions in unsteady flow or in three-dimensional steady supersonic flow.

Present applications include: wings with sonic leading edges in supersonic flow, propagation of spherical and cylindrical waves including the projectile firing band, unsteady separation of bow shocks, wave propagation in magnetohydrodynamics.

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BIOGRAPHICAL NOTE

Dr. Klaus Oswatitsch is head of the Department of Fluid Mechanics at Vienna Institute of Technology, Austria. His fields of interest include flows with condensation, flows in the transonic to the hypersonic regime, similarity and equivalence rules, wave propagation and the solution of hypersonic systems.

A citizen of Austria, Dr. Oswatitsch was born in Marburg a. d. Drau. He was awarded his doctorate from the University of Graz in 1935 and spent the next eight years in Gottingen where he worked as an assistant to Dr. L. Prandtl. He later spent a year at the Royal Aircraft Establishment, Farnborough, and two years with Bureau D'Etudes in Emmendingen. Following that, Dr. Oswatitsch taught at the Royal Institute of Technology in Stockholm. In 1956 he founded and directed the Department of Theoretical Gasdynamics within the German Federal Research Organization DVL in Aachen. Dr. Oswatitsch returned to Austria in 1960 to take his present position. An authority in his field, Dr. Oswatitsch is the author of numerous technical papers and several books.

NONLINEAR PROBLEMS IN WAVE PROPAGATION

INTRODUCTION

The propagation of weak waves, which I will treat in this lecture, is a problem of unsteady flow or of steady supersonic flow. This is, perhaps, a little out of your present interests because it has very little to do with space flight. On the other hand, it is quite a general problem. In fact, you may have to contend with it or with a similar problem in some part of your research.

It is not possible to present in one hour the new theory we have developed during the past five years, a development which is not finished yet. Therefore, I prefer to illustrate the main steps of the theory with a simple case and to consider the physical significance in connection with the results. At the end of the manuscript of this lecture, you will find a list of pertinent references. I will refer to some of these references during my talk.

TWO-DIMENSIONAL SUPERSONIC FLOW

One of the simplest cases of wave propagation is the two-dimensional steady supersonic flow around a profile.

In the case of small disturbances, the flow can be described by the well-known theory of Ackeret, that is, by the linearized gas dynamic equation and the equation of irrotationality.

$$(1 - M_0^2)\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0; \quad \frac{\partial u_1}{\partial y} - \frac{\partial v_1}{\partial x} = 0. \tag{1}$$

The values, u and v, are the components of the velocity in the x and y direction; M is the Mach number. The index 0 is related to the undisturbed flow, and the index 1 designates disturbances of the first order.

The inclination of the Mach lines of the configuration drawn in Figure 1 is given by the coefficients of equation (1). Because the coefficients are constant, the inclination of the Mach lines is constant. The Mach lines are straight and lie parallel to the Mach lines of the undisturbed flow; for example, the two

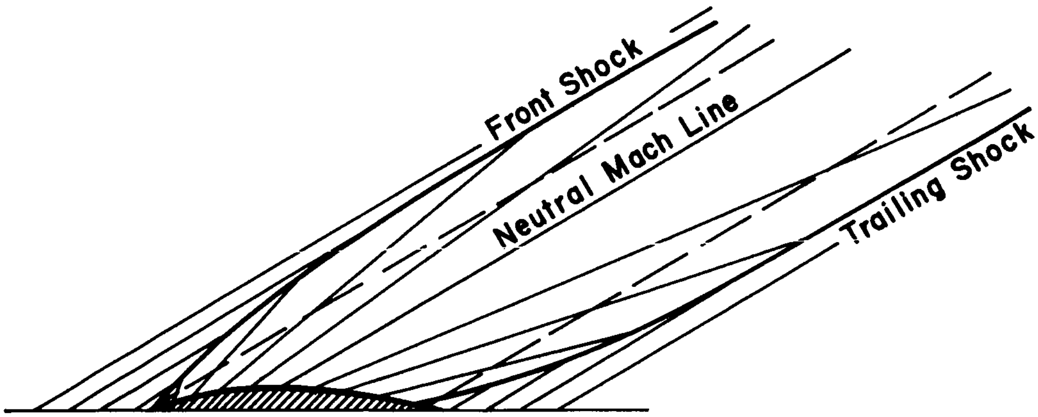


FIGURE 1. TWO-DIMENSIONAL STEADY SUPERSONIC FLOW AROUND SLENDER PROFILE

dashed Mach lines in Figure 1. Only between these two dashed Mach lines, which originate at the tip and at the end of the profile, does the theory of Ackeret give disturbances. In the case of two-dimensional flow, there is a jump in the two velocity components at these two Mach lines. This jump represents the leading shock wave and the trailing shock wave. It is well-known that this linear theory gives quite good results in the neighborhood of the profile. However, at great distances the results are absolutely wrong because the linear theory yields no attenuation of the disturbances in the two-dimensional case. The exact solution of this problem was first given by the well-known Prandtl-Busemann characteristic method. The disturbance caused by the profile propagates as you know along the Mach lines, which are straight lines, but does not propagate parallel to the undisturbed Mach lines. With respect to the divergence of these Mach lines, the shock strength decreases at the beginning as well as at the end of the disturbance. If the shocks are weak enough, the shock front can be considered as a bisectrix of the Mach line inclination immediately ahead and behind the shock front. This well-known formula of Pfriem plays quite an important role in the calculation of the shock wave fronts in our new theory.

The equivalent linearization of the subsonic flow, which leads to the well-known Prandtl-Glauert analogy, gives much better results. As is well-known, the linearized subsonic theory gives quite good results of the disturbances at the profile boundary, and the accuracy of the theory increases with increasing distance. There is no doubt that the error of the supersonic linearization in the physical plane is caused by the wrong inclination of the Mach lines, and it seems strange that none of the theoreticians have tried to overcome the weaknesses of the already classical linearized supersonic theory. The overcoming of these weaknesses is the purpose of our efforts.

The importance of the inclination of the Mach lines is also easy to see in the three-dimensional case. The z direction in Figure 2 can be considered as the direction of the undisturbed supersonic flow at Mach number $M_0 = \sqrt{2}$. We can also consider z as the time coordinate of the unsteady two-dimensional problem of wave propagation in quiet air. In this case, you may consider the horizontal coordinate as the x axis and the coordinate normal to the plane as the y axis. In each case, the straight lines trace out the surface of the Mach cone, which limits the zone of influence of a disturbance placed in the tip of the Mach cone. The hyperbolas are curves of constant influence of the classical linearized theory. As shown in the figure, the influence jumps from 0 on the outside of the surface of the Mach cone to infinity on the inside of the surface of the Mach cone. The influence is greatest immediately on the inside of the Mach cone. If you turn the Mach cone upside down, you can consider the cone as a surface which limits

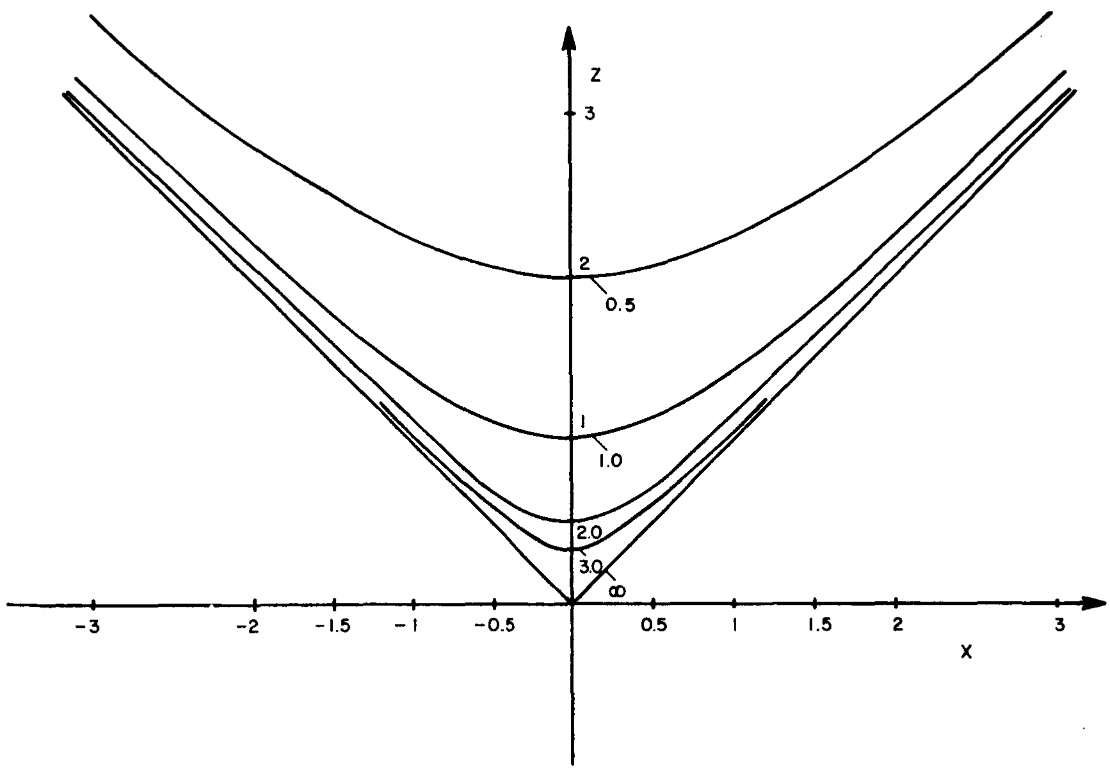


FIGURE 2. MACH CONE AND SURFACES OF CONSTANT INFLUENCE IN PLANE y = 0 OF DISTURBANCE PLACED IN ORIGO

the dependence. This means that the dependence of a certain point in the space because of other influences is greatest on the inside of the cone of dependence. In Figure 3, you see the cone of dependence with the vertex at point P_2, together with a cone of influence at another point, P_1. If the point of influence, P_1, is very close to the inside of the surface of the Mach cone of depencence at P_2, the influence and the dependence is very strong. On the other hand, if the point of influence moves outside the Mach cone of dependence, there is no immediate influence and no dependence at all. A little change in position causes a jump from low influence to maximum influence. Therefore, it is clear that a small change in Mach line inclination (or a small deformation of the Mach cone) can cause in many cases an important change in the results.

In the last 30 years, there has been a tremendous development of linearized theory starting from the linearized gas dynamic equation. Many examples of flow around bodies of revolution and wings in steady supersonic flow with and without inclination, supersonic flutter problems and flow around accelerated and decelerated bodies were calculated. The unsteady case of wave propagation in quiet air is investigated in papers of Lord Raleigh, and many formulas of the

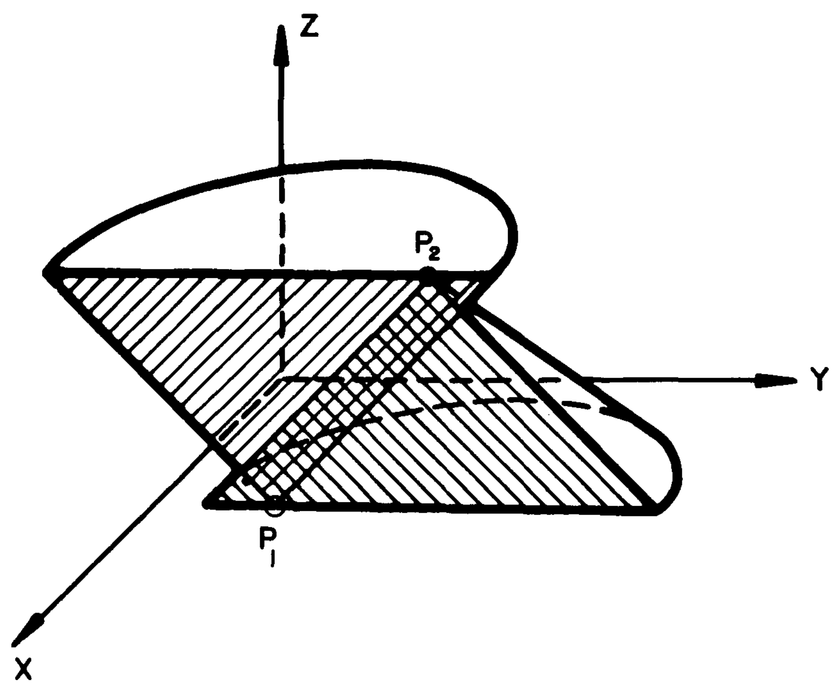


FIGURE 3. INFLUENCE OF DISTURBANCE IN POINT P_1 TO FLOW IN POINT P_2

supersonic flow theory are closely related to older results by this author. Therefore, we would call this already classical method of linearization in physical space, the "acoustical method." You will see that the new theory in its first approximation also includes a linearization. Therefore, the word linearization cannot be used to distinguish the older from the new method of treatment.

To avoid fixing the Mach line, the Mach surfaces and the Mach cones in the physical plane in our new theory, we introduce the Mach elements as independent variables. What we do is develop an analytical characteristic theory for small disturbances. As in all characteristic theories the flow properties, such as velocity components and pressure, are dependent variables depending on certain characteristic independent variables. The coordinates, and in unsteady flow the time, are also dependent variables. Therefore, we expand the Mach number, the components of the velocity and the coordinates in series,

$$M = M_0 + M_1 + M_2 + \ldots ; \; u = u_0 + u_1 + \ldots ; \; v = v_1 + \ldots ; \; w = w_1 + \ldots \tag{2}$$

$$x = x_0 + x_1 + \ldots ; \; y = y_0 + y_1 + \ldots ; \; z = z_0 + z_1 + \ldots \quad , \tag{3}$$

where the index 0 is related to the undisturbed flow and the indices 1 and 2 ... designate the terms of the first and second order, etc., of a parameter as, for example, the thickness ratio or the angle of inclination.

In equation (2), we get v_0 and w_0 equal to 0, and thus we assume undisturbed parallel flow in the direction of the x axis. To solve the problem, we now need equations for the velocity components and also for unsteady flow the components for pressure or velocity of sound. We need another group of equations for the coordinates and possibly for the time. In the case of two-dimensional or axisymmetrical flow, the equations for the components are the well-known compatibility equations. These are certain relations which the velocity components or the pressure have to satisfy along the Mach lines. In the more complicated case of three-dimensional flow or two-dimensional unsteady flow, the equations are certain relations between the so-called inner variables. We can derive the necessary equations for the coordinates, depending on the characteristic variables, from the equations for the inclination of the characteristics. I would like to explain this with the help of the two-dimensional steady supersonic flow.

If we assume a steady supersonic parallel flow in the x direction with the Mach angle α_0, we have two families of characteristics, ξ = constant and η = constant.

$$\xi = x_0 - y_0 \cot \alpha_0 \; ; \; \eta = x_0 + y_0 \cot \alpha_0 \, , \tag{4}$$

$$x_0 = \frac{1}{2} \, (\xi + \eta) \; ; \; y_0 = \frac{1}{2} \, \tan \alpha_0 \, (-\xi + \eta) \, . \tag{5}$$

These characteristics or Mach lines in the parallel flow are two families of straight lines (Fig. 4). Equation (4) gives us the relation between the coordinates of the undisturbed flow x_0, y_0 ... and the characteristic variables ξ and η. For the inclination of the distrubed Mach lines, we obtain the well-known equations (6), in the first one for the left-hand Mach line with ξ = constant and in the second one for the right-hand Mach line with η = constant.

$$y_\eta = \tan (\alpha + \vartheta) \, . \, x_\eta \; ; \; y_\xi = \tan (-\alpha + \vartheta) . \, x_\xi \, . \tag{6}$$

If we expand the equations with respect to small disturbances of the Mach angle α and the flow angle δ, we can express the Mach angle and the flow angle by the disturbances of the components. We can use equation (3) to the first order and equation (5) to derive the two equations for the disturbances of the coordinates x_1 and y_1.

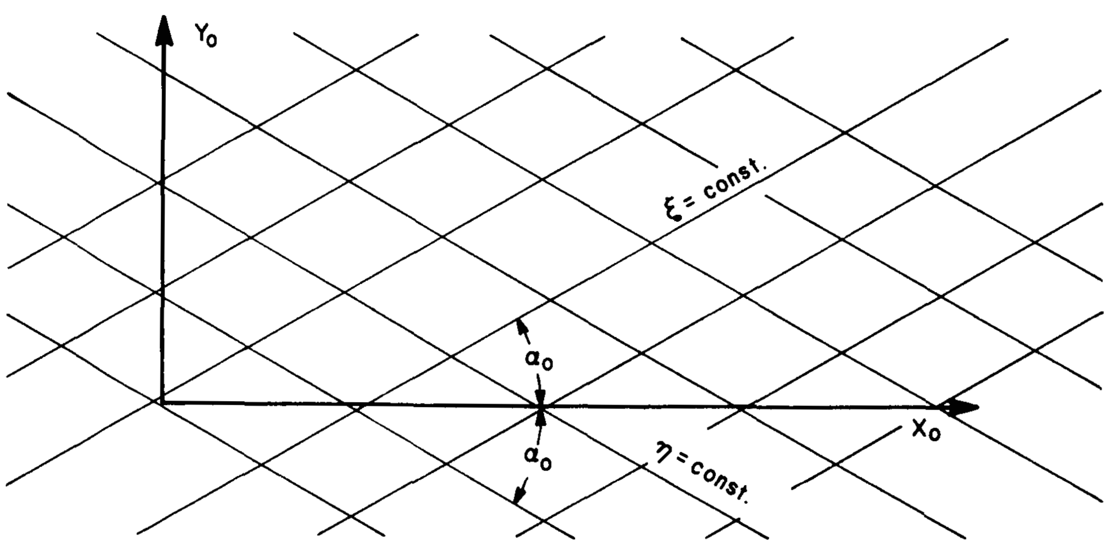


FIGURE 4. SYSTEM OF MACH LINES IN UNDISTRUBED PARALLEL FLOW

$$x_{1\eta} - y_{1\eta} \cot \alpha_0 = - M_0/2\, c_o\, [v_1 \tan \alpha_0 - u_1 k] \; ;$$
$$x_{1\xi} + y_{1\xi} \cot \alpha_0 = + M_0/2\, c_o\, [v_1 \tan \alpha_0 + u_1 k] \; ; \; k = \frac{1}{2}\, [(k+1) \tan^2 \alpha_0 + (k-1)] . \tag{7}$$

These two linear differential equations are easy to integrate, and we finally arrive at two linear equations for the disturbances of the coordinates x_1 and y_1.

$$x_1 - y_1 \cot \alpha_0 = -M_0/2\, c_o \int^{\eta} [v_1 \tan\alpha_0 - u_1 k]\, d\bar{\eta} \; ;$$
$$x_1 + y_1 \cot \alpha_0 = M_0/2\, c_o \int^{\xi} [v_1 \tan\alpha_0 + u_1 k]\, d\bar{\xi} . \tag{8}$$

The right-hand integrals have no kernel, and we can easily make the integration if we know the disturbances, u_1 and v_1, depending on the characteristic coordinates. The equations (8) are for the two-dimensional case already derived in a paper of C. C. Lin [1]. The derivations in our paper are much more general. We obtain similar equations in the three-dimensional case and in the three-dimensional unsteady flow, and we have shown that we can also apply equations (8) to axisymmetrical or similar cases. In this later generalization, u_1 and v_1 are naturally not solutions of the two-dimensional flow case but of more general flows.

The equations (4) between ξ and η on one side and x_0 and y_0 on the other side are not approximations for the Mach lines as in the acoustical theory. They are exact relations between ξ, η and the coordinates of no disturbance. Therefore, instead of ξ and η we can introduce with the help of equation (4) or (5) x_0 and y_0 as independent variables. If we do that in the compatibility equations, we get for the first order disturbances, u_1 and v_1, the linear differential equation

$$(1 - M_0^2)\, \frac{\partial u_1}{\partial x_0} + \frac{\partial v_1}{\partial y_0} = 0 \; ; \; \frac{\partial u_1}{\partial y_0} - \frac{\partial v_1}{\partial x_0} = 0 . \tag{9}$$

These equations have the same form as equation (1) of the acoustical theory with the important difference that the independent variables x_0 and y_0 in the first order of the analytical characteristic theory have been substituted for the independent variables x and y of the acoustical theory. In general, one can show that in the three-dimensional and in the unsteady case the equations are the same in the first order as in the acoustical theory except x, y, z and the time t have been replaced by x_0, y_0, z_0 and the time t_0. In the special case of axisymmetrical flow in the equation of continuity, the term v_1/y is to be replaced according to equation (10).

$$v_1/y \longrightarrow v_1/y_0. \tag{10}$$

In equation (3) you also see certain relationships to the method of Whitham and to the Poincaré-Lighthill-Kuo method. The difference is that the coordinate disturbances in the latter theory are introduced on certain singular places in the space. In our theory, the coordinate disturbances are functions of the characteristic variables and give, therefore, the location of characteristic surfaces or lines in the physical space.

In the new theory, we do not work in the physical space as in the acoustical theory but in a characteristic space with the coordinates x_0, y_0, z_0 and the time t_0. In this space, the Mach lines are exact straight lines. We can introduce plane Mach surfaces and exact circular Mach cones, and we have to introduce the initial and boundary conditions of the characteristic space. After we have solved the problem in the characteristic space, we have to find the location of the characteristic independent variables in the physical plane by integration, according to equations (8). In the case of three or more independent variables, we do not have to integrate along characteristics but along so-called "bicharacteristics." It would take too much time to explain the general three-dimensional method in this lecture. However, even in this case the mathematical handling is not complicated.

The translation of the boundary and initial conditions from the physical space to the characteristic space is easy in these cases where the acoustical theory gives good results near the body. In the cases where the acoustical theory gives wrong or no results near the body, the translation of the boundary conditions can cause difficulties and several problems are still not solved. However, one of the solved problems, which I will illustrate later, is the solution of the flow field about a delta wing with sonic leading edges.

In the cases where the acoustical theory gives good results near the body, the work for the new theory is less than twice the old theory because the integrations to obtain the coordinates according to equations (8) are much easier than the solution of the acoustical problem. In the many cases where we already had acoustical results, we very easily applied them to our theory by putting in the results for the components and the pressure instead of x, y ... and x_0, y_0 ...

In all cases we get not direct shock waves but an overlapping in the physical plane around the place we would get shock waves. We can then calculate the front of the shock wave in the region of overlapping by the formula of Pfriem and fit in the shock front in our results.

In Figure 5 you see the results at the cone cylinder in supersonic flow calculated by W. Schneider [2]. Naturally, one can also calculate smooth bodies much easier. According to Whitham, the front shock wave has a strength of the fourth power of the thickness ratio. At the shoulder of the body originates a generalized Prandtl-Meyer expansion as you see. In all cases where comparison with results of Whitham or with results of the second order theory of Van Dyke was possible, we obtain complete agreement [3 - 8].

In the next six figures I will show results of E. Y. C. Sun [9, 10] concerning the flow around the conical delta wings without inclination and with near sonic edges. It is a cone with a rhombic cross section, as shown in Figure 6. It is well-known that for a conical wing with sonic leading wedges the shock is pushed away from the leading edge to an upstream position because of the wedge like cross section. This means that these are Mach lines going from the sonic edge upstream to the front shock. We can obtain such Mach lines only by using a solution with the subsonic leading edge in the characteristic plane. Mr. Sun has used these solutions and obtains in the physical space wings with barely

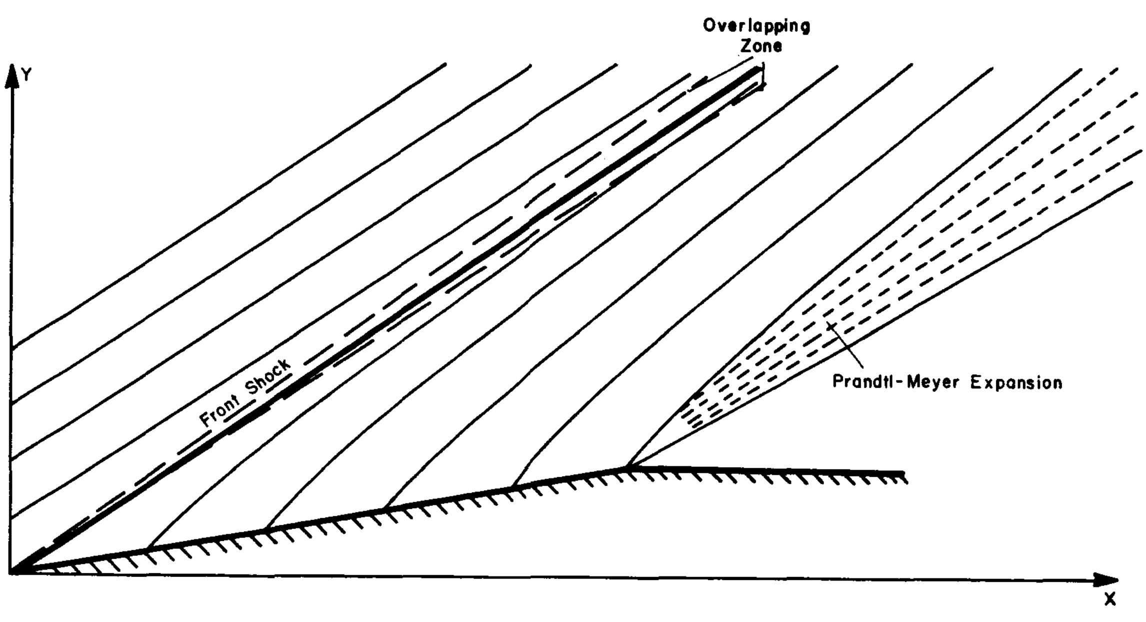


FIGURE 5. SUPERSONIC FLOW ALONG CIRCULAR CONE-CYLINDER

subsonic leading edges, with sonic leading edges and with barely supersonic leading edges. The shock waves are ahead of the leading edges as I will show in the following figures. Figure 7 is the case of the delta wing with 10 percent thickness ratio and a Mach number of $M_0 = \sqrt{2}$. The angle of the leading edge is 90 percent of the Mach angle. The shock wave is the dotted line, and a family of left-hand Mach lines is drawn. Also shown are the undisturbed 45-degree Mach lines ahead of the shock front and the disturbed Mach lines behind the shock front flowing into the shock.

Figure 8 shows the same delta wing with the addition of a sonic leading edge. The leading edge is parallel to the undisturbed Mach lines, and the shock front is pushed ahead of the leading edge. Figure 9 shows the wing with the same aspect ratio [sonic leading edge] but with a thickness ratio of 16 percent. Now with respect to the thicker wing, the shock wave is pushed farther ahead of the leading edge. The results of Figures 8 and 9 are related by the similarity law by S. B. Berndt [11] for sonic leading edges. The new theory satisfies all of these similarity laws.

In Figure 10 you see four different cases of the flow around the rhombic cone. In the cross-sectional plane of the wing, the full circle is the trace of the cross section of the undisturbed Mach cone, and the dotted line is a trace of the cross section of the shock wave in that plane. In the first case, we have just a sonic edge; the leading edge touches the undisturbed Mach cone. The shock wave in the plane of the wing is pushed farther away as in the plane perpendicular to the wing. In the second case, the shock wave is just touching the leading edge of the wing. No solution exists for this case. In the third case, we already have a small supersonic leading edge. The shock has already attached the leading edge, but there is still a curvature of the shock at the leading edge because immediately behind the shock on the wing we have here Mach lines parallel to the leading edge. The fourth case is a typical case of supersonic leading edge. The shock wave in the whole cross section is still outside of the undisturbed Mach cone. The shock wave forms a plane surface at the leading edge. These results [10] are still to be published. In Figure 11 the drag coefficient, c_o, of this wing is plotted versus the ratio $\tan\theta/\tan\alpha_0$.

For $\tan\theta/\tan\alpha_0 < 1$ we obtain subsonic leading edges and for $\tan\theta/\tan\alpha_0 > 1$ supersonic leading edges. As is well-known, the acoustical theory gives at $\tan\theta = \tan\alpha_0$ a certain corner in the curve of the drag coefficient. The full line gives the result of our theory. You see that the drag coefficient for the sonic leading edge is about 20 percent less than the result of the acoustical theory. This is in accordance with the experiments.

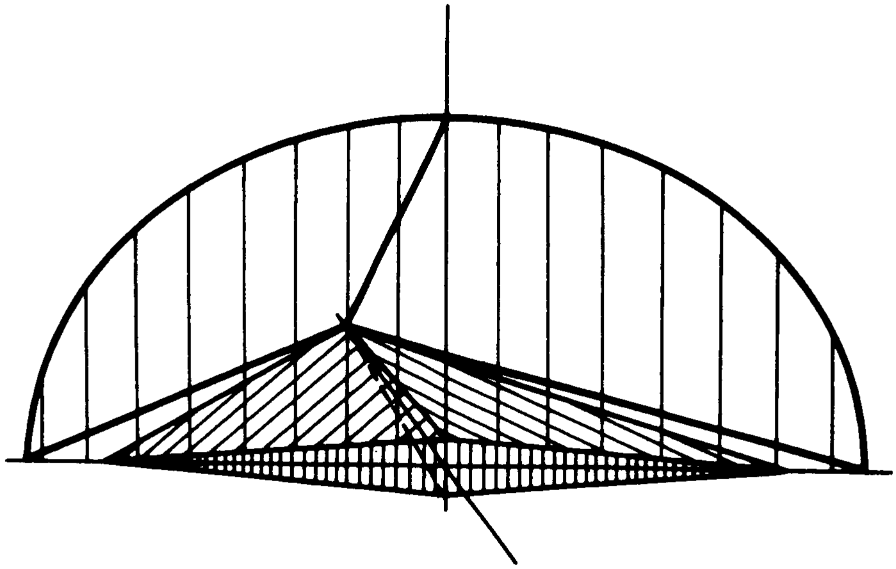


FIGURE 6. DELTA WING WITH RHOMBIC CROSS SECTION AND NEAR SONIC EDGES

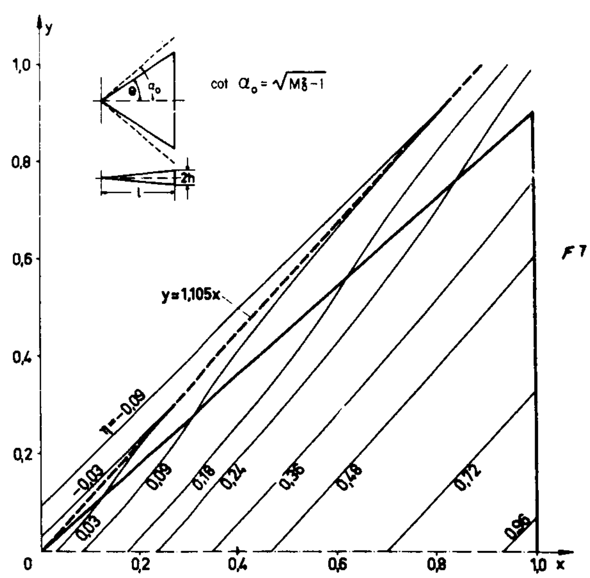


FIGURE 7. RHOMBIC DELTA WING WITH 10 PERCENT THICKNESS RATIO AND LEADING EDGE 90 PERCENT OF MACH ANGLE

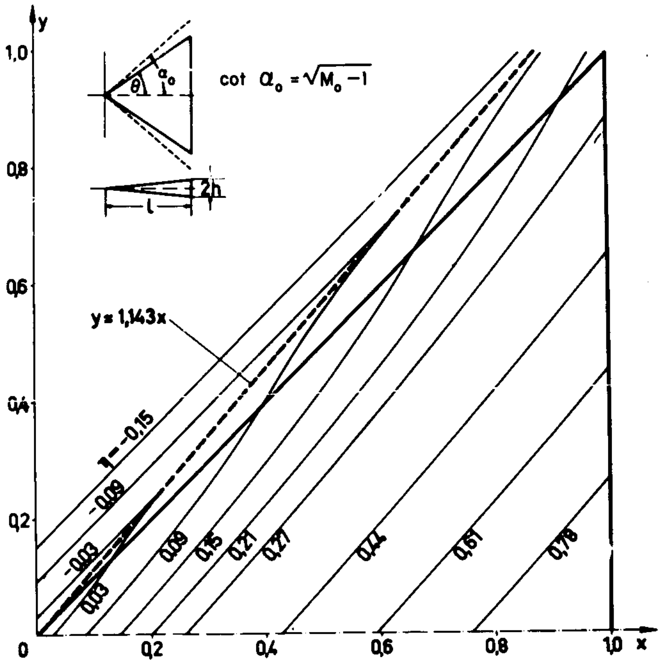


FIGURE 8. RHOMBIC DELTA WING WITH 10 PERCENT THICKNESS RATIO AND SONIC LEADING EDGE

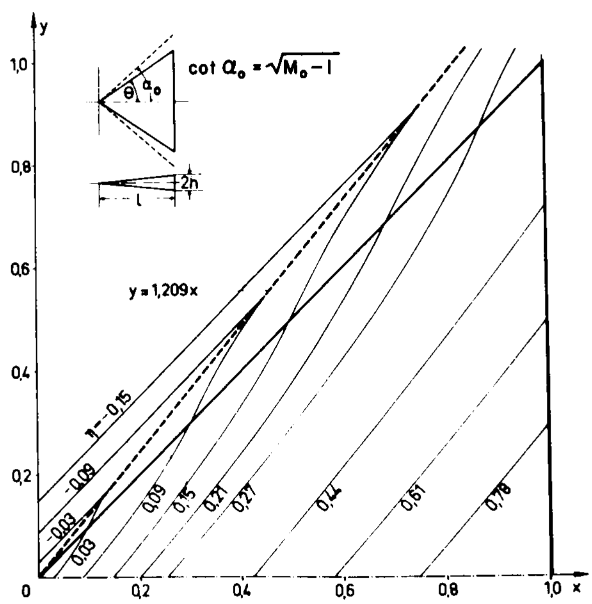
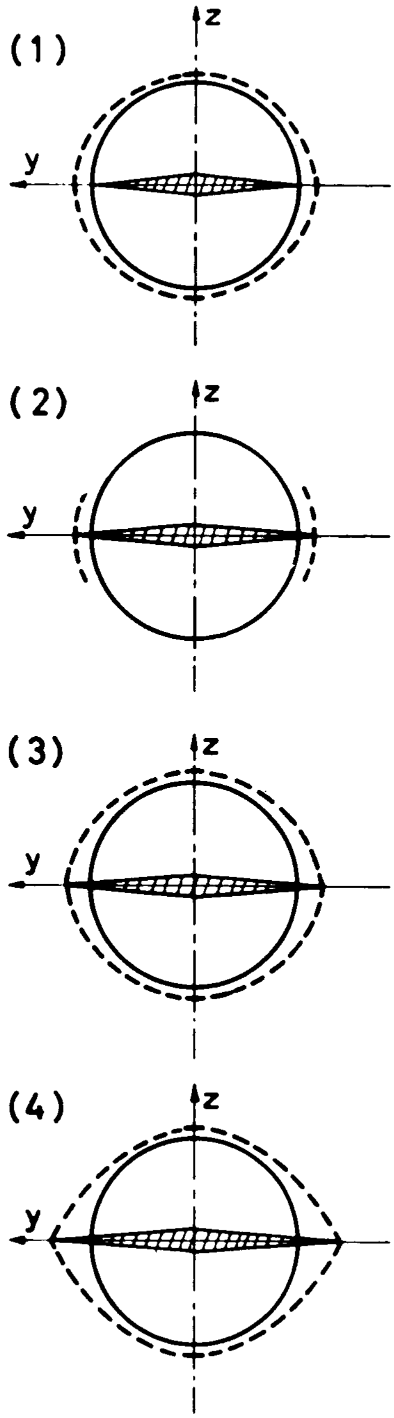


FIGURE 9. RHOMBIC DELTA WING WITH 16 PERCENT THICKNESS RATIO AND SONIC LEADING EDGE



Sonic leading edge front shock detached.
Solution: Subsonic leading edge.

Supersonic leading edge front shock immediately in front of leading edge.
Solution: Not known.

Supersonic leading edge front shock curved and oblique to plane at wing.
Solution: Supersonic leading edge.

Supersonic leading edge front shock oblique to plane of wing with straight piece.
Solution: Supersonic leading edge.

FIGURE 10. CROSS SECTION OF CONICAL SUPERSONIC FLOW AROUND RHOMBIC CONE WITH SONIC LEADING EDGE AND THREE TYPES OF SUPERSONIC LEADING EDGES

The propagation of a spherical wave from a center is shown in Figure 12. The x axis is the distance from the center; the y axis gives the disturbance of density. The dotted line between $1.5 \leq x \leq 2.5$ is the result of the acoustical distribution at a time $t = 2.5$. The dotted line between $2.5 \leq x \leq 3.5$ is the acoustical result for the later time $t = 3.5$. The two full curves give the result of the new theory. The spherical wave starts with a shock, has the well-known N form and terminates with a shock. The difference between the acoustical theory and the new theory is quite large with respect to the relative small density disturbance of less than five percent for this distance from the center.

Sonic boom results are shown in Figures 13 and 14. Figure 13 is in fact a picture drawn by L. Prandtl [12]. Prandtl explains in this picture the separation of the front shock of a projectile at decelerating speed. This picture is quite old, and it is, therefore, not surprising that it is not right in all details. Prandtl marks the front shock as still attached at Mach number 1. In reality, the shock has already separated from the tip of the projectile. Figure 14 shows the conditions in the characteristic space, that is, in the x_0, c_0t_0 plane with $y_0 = 0$. The full curve gives the trajectory of the tip of the projectile in the characteristic space. The origin is just at Mach number 1, the trajectory of the tip having a 45-degree inclination. In the subsonic region, the disturbances propagate faster than the projectile; therefore, the disturbances separate ahead of the body. To obtain the right physical results, it is necessary to provide the trajectory curve of the body with a small corner in the supersonic region at the separation point. The corner must have such qualities so that the curve has no corner in the physical plane at the separation point. The report of Stuff [13], my collaborator who is in charge of this work, will be published this year.

Figure 15 is related to a more academic question which also has some practical interest. The calculation was made by H. Sonn [14]. This result could already be calculated with the formulas given by C. C. Lin [1]. At the top of the figure a biplane is shown flying at Mach number $M_0 = 2$. It is not exactly the Busemann biplane but for our consideration this is not very important. The leading and trailing edges of the biplane are cusped to avoid all shock waves at Mach number $M_0 = 2$. If one now changes the Mach number of flight, if one decreases the Mach number as in the middle of the figure or increases the Mach number as at the bottom of the figure, very weak shock waves appear. They not only appear at the trailing edges but also at a certain distance behind the biplane. Such very weak shock waves are only found with analytical methods. It should not be possible to find them with a graphic characteristic method or in an experiment, for instance, in a Schlieren-picture. You would not detect any difference between such weak shock waves and a Mach line. This result shows that a steady supersonic flow has, in general, no shockless neighboring solutions.

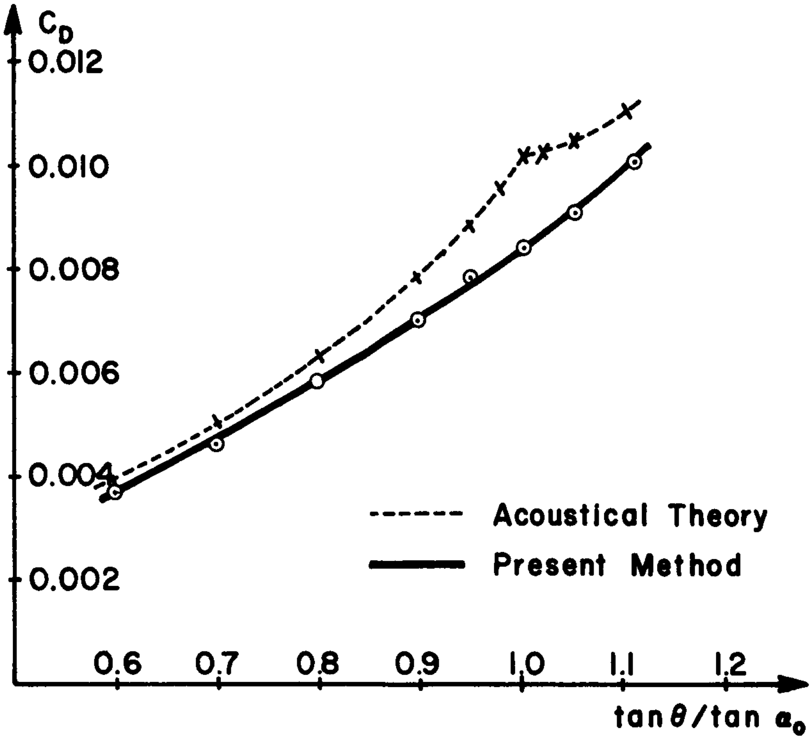


FIGURE 11. DRAG COEFFICIENT VERSUS RATIO TAN θ/TAN α_0

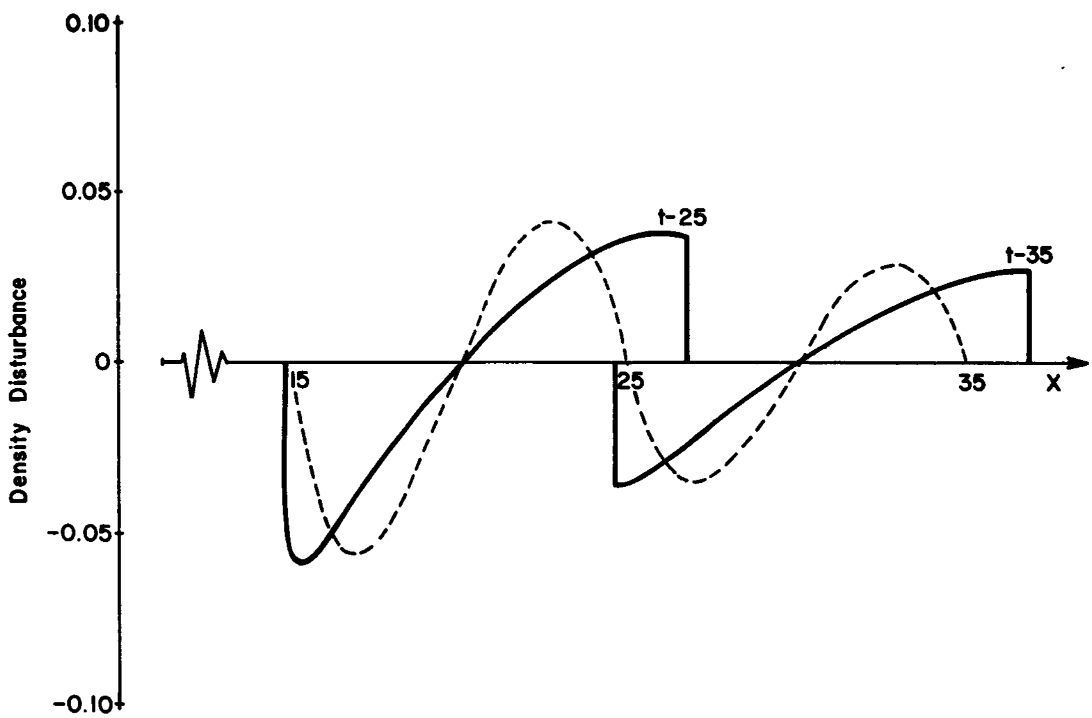


FIGURE 12. SPHERICAL WAVE PROPAGATING FROM CENTER

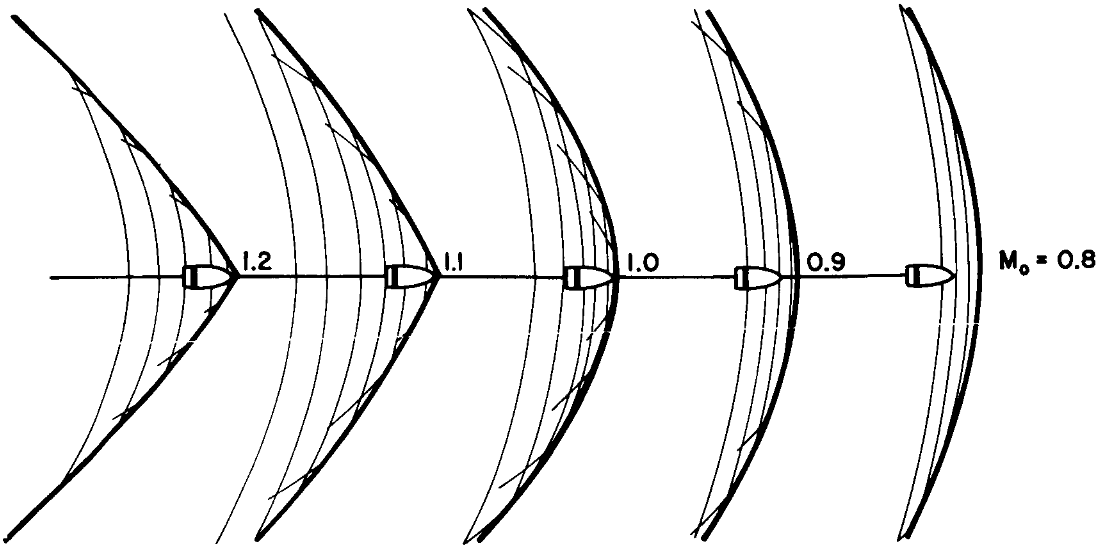


FIGURE 13. DECELERATING BODY WITH SEPARATING SHOCK WAVE NEAR SONIC SPEED

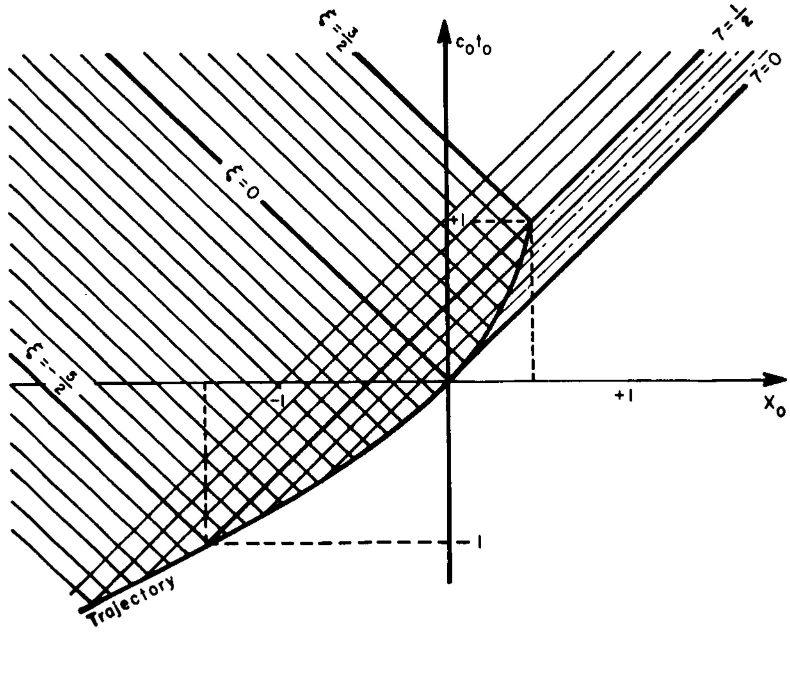


FIGURE 14. TRAJECTORY AND MACH LINES IN x_0, c_0, t_0 PLANE

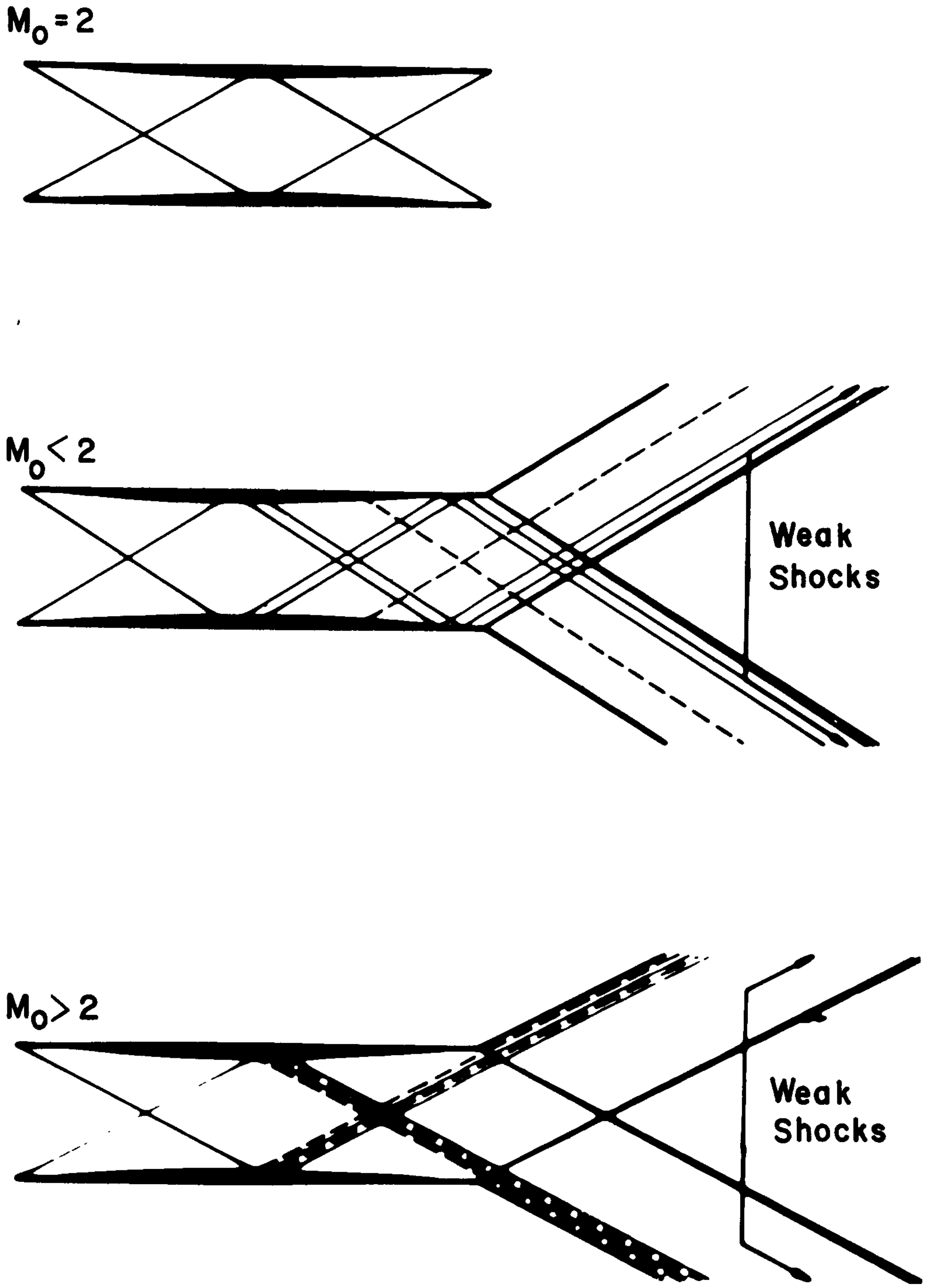


FIGURE 15. NEIGHBOURING SOLUTION OF SHOCKLESS BIPLANE IN STEADY SUPERSONIC FLOW

Therefore, this property -- that a flow has no shockless neighboring solutions -- is not a property of the transonic flow as many may assume, but a property of the supersonic flow. It is not at all necessary that a rectangular shock appear. The shock can be an extremely weak one which has no practical importance.

DISCUSSION

The following is the answer to a question of higher order results. In the papers of W. Schneider [2] you can find a second order result of the new theory. Schneider has calculated the second order to the supersonic flow around a circular cone and found full agreement with the results of Van Dyke. Concerning the higher orders, one has to remark that one calculates first the order 0 of the coordinates. One continues then to calculate the first order of the components and in the third step with the help of equation (8) the first order of the coordinates. Then one can continue to the second order of the components and if necessary to the second order of the coordinates. For example, it is justified to relate the first order results of the components to the second order results of the components because the first order results of the coordinates lies just between the two orders of the components. If we give the first number to the coordinates and the second number to the components, we can distinguish 0, 1; 1, 1; 1, 2... The 0, 0 order is again the undisturbed flow. The 0, 1 order -- that is, no disturbance of the characteristics but disturbances of the component -- is the acoustical theory; 1, 1 is first order disturbance of the coordinates and of the components. Almost all results given in this report were calculated in this order. In the 1, 2 theory we already have second order approximation in the components, as for example in the results of Van Dyke, but we also have disturbance of the Mach lines to the first order.

The following is the answer to a question concerning the trailing wave of a body of revolution. If we have a body of revolution which ends with a tip it is well-known that one has on the end of this body a very small local subsonic region. One should therefore assume that the new theory yields no results in the neighborhood of the end of the body. Nevertheless, Schneider [2] obtained a result because the logarithmic singularity is harmless. Also, in the case of an axisymmetrical free jet which is not parallel we hope to obtain results in the whole physical plane including the very small subsonic region at the axis.

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