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SURVEY OF SOLAR CYCLE PREDICTION MODELS

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ABSTRACT

As scientists make greater strides toward unfolding the secrets of the activities of the sun, we become more aware of its influence on the activities of the earth. In recent years, a more definite correlation has been found between the number of spots on the sun and the density of the upper atmosphere. Since the lifetimes of satellites in orbit depend upon this density, future plans for space research on these satellites depend upon an adequate forecasting of the sunspot cycle. Some prediction techniques now in existence are presented in this report.

Although much literature is available on the solar cycle, the literature on solar cycle prediction is limited. Since many of the techniques are essentially duplicates, an effort is made to present the basic types in terms of procedures and results.

As an evaluation of the techniques becomes necessary, the need for greater prediction reliability becomes obvious. The ultimate means to this end is the formulation of an adequate theory on the reason and formation of sunspots. As we try to achieve this, an immediate objective is the synthesis of current ideas and theories on solar activity into a comprehensive theory having a statistically acceptable degree of reliability when applied to prediction.

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SUMMARY

As scientists make greater strides toward unfolding the secrets of the activities of the sun, we become more aware of its influence on the activities of the earth. In recent years, a more definite correlation has been found between the number of spots on the sun and the density of the upper atmosphere. Since the lifetimes of satellites in orbit depend upon this density, future plans for space research on these satellites depend upon an adequate forecasting of the sunspot cycle. Some prediction techniques now in existence are presented in this report.

Although much literature is available on the solar cycle, the literature on solar cycle prediction is limited. Since many of the techniques are essentially duplicates, an effort is made to present the basic types in terms of procedures and results.

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I. INTRODUCTION

Mankind has been attracted by the phenomenal activity of the sun since ancient times. The earliest observations were made with the naked eye. Since the invention of the telescope, the sun has been under regular observation with accurate records being kept from the middle of the nineteenth century to the present.

Our daily life is influenced greatly by the effects of solar activity. Some of these effects are variations in radio-communication conditions, changes in climatic conditions, polar auroras, and geomagnetic storms. The effect of sunspots on upper atmosphere density is pronounced. Since the lifetime of orbiting satellites is directly related to the density of the upper atmosphere, experiments with these satellites make it necessary to predict long range solar activity. This is complicated by the limited amount of basic knowledge of solar activity. Within the framework of this restriction, many methods have been derived for predicting the sunspot cycle. This report is a survey of some typical solar cycle prediction techniques found in a search of the literature.

II. SUNSPOTS

2.1 General Characteristics

Sometimes in a particular region on the sun, granulation motions are replaced by more intense motions, setting up a magnetic field in this area on the solar surface. This is known as the birth of an active region. In the photosphere there comes into being a bright compact formation known as a solar facula, which gradually increases in area and brilliance. About 24 hours after the facula forms, a few dark dots called "pores" are observed in the facula. One or more pores then develop into dark regions with dimensions as great as or greater than the earth's diameter. These are called "sunspots," the characteristic attribute of an active region [1].

The beginning of a new solar cycle is usually marked by the appearance of new spots at high latitudes (30° approximately) on the sun's surface. As the cycle progresses, the spot zone descends toward the equator, reaching about 16° at the cycle maximum and about 8° at the next minimum.

Sunspots are encountered in groups in which there are usually two prominent spots - a leading (western) spot and a trailing (eastern) spot. The line joining this pair of spots is generally slightly inclined to the equator. The magnetic fields associated with these spots are often of different polarity. During a cycle, all the leader spots in one hemisphere have one polarity, while the leader spots in the other hemisphere have the opposite polarity.

2.2 Cyclic Characteristics

The variation of sunspots with time is well known. On the basis of observations between 1761-1769, Horrebow, a Danish astronomer, was first to discover this phenomenon. Schwabe, on the basis of twenty years of observations, established that solar activity varies with a period of about 10 years. This enabled the director of the Zurich Observatory, Rudolph Wolfe, to set up systematic observations of the variations in sunspot activity. These observations resulted in the discovery of the 11-year sunspot cycle. Wolfe showed that the number of sunspots, the Wolfe number, fluctuates with an average cycle duration of 11.1 years.

The length of a cycle can be determined by the interval of time between successive minima or by the interval of time between successive maxima. It has varied from 8 to 15 years in the first case and from 7 to 17 years in the second case. There is a variation also in the heights of the cycles at minimum and maximum. The minima have varied from 0.0 to 11.2 and the maxima have varied from 48.7 to 189.9. Here, the numbers quoted are smoothed mean annual sunspot numbers. This regularity is given much consideration in references 2 and 3.

2.3 Fundamental Activity Indices

The number of spots on the sun is not the same at all times. The need for a universal method of measuring the number of spots visible on the solar surface resulted in an introduction by Wolfe in 1849 of the "relative sunspot number,"

$$R = K(10g + f), \qquad (1)$$

where f is the number of individual spots and g is the number of groups. The factor K is a number assigned to each individual observer and/or his equipment to reduce the individual sunspot numbers to a consistent scale. The relative sunspot number, commonly called the Wolfe number, depends on the visibility conditions, the apparatus used, and the method of observation, as well as on such subjective factors as observer's fatigue and the way in which the sunspots are arranged into groups.

It is obvious then, that this index is not entirely objective. However, this is the longest existing series with acceptable results dating back to 1749. This perhaps is the reason, along with high correlation with other geophysical indexes, this index has been preserved.

Daily Wolfe numbers are not very significant because of irregular fluctuations. Monthly and yearly sunspot numbers are more suitable for forecasting and comparison with other geophysical indexes. The Wolfe numbers are smoothed according to formula 2,

$$R_i = \frac{1}{12} \sum_{i-5}^{i+5} \left[R_i + \frac{1}{2} (R_{i+6} + R_{i-6}) \right], \qquad (2)$$

which is used to eliminate terrestrial effects.

The second fundamental index of solar activity is the sunspot-group area. This index is determined only for the visible hemisphere of the sun. It is assumed that an analogous spot pattern exists on the unobservable hemisphere. This is also true of the Wolfe numbers. In their attempts to reduce this measuring defect, Becker and Kiepenheuer (see reference 3) used visibility functions which they derived for various types of sunspot groups in order to plot the curves for group development. They read from these curves the spot numbers during the 14-day period when the spots were invisible.

As cited by Vitinskii in reference 3, the sunspot-group area was first suggested by Carrington in Greenwich in 1784. In contrast to the Wolfe numbers, which are determined both photographically and visually, the sunspot areas are measured only photographically. They are usually given in millionth parts of the solar disk or in millionth parts of the visible solar hemisphere.

Although the spot area index is more objective than the Wolfe numbers, its forecasting value is much lower for two reasons. In the first place, its series is less than one-half as long as the Wolfe numbers. In the second place, it reflects the corpuscular component of solar radiation.

Again, monthly and yearly values are preferred to daily values of spot area. The correlation between Wolfe number, W, and sunspot area, S, is expressed as

$$S \approx 16.7\ W. \qquad (3)$$

Much detailed information about other indexes is accessible from many of the listed references. In view of the popular use of the Wolfe numbers in forecasting, one normally concedes that presently there is nothing better.

2.4 Theories of Formation

With existing tools our study of the sun is limited to the solar surface and solar atmosphere. Thus, we must guess about the internal structure of the sun. Although there are several theories, the mechanism of solar activity has not been satisfactorily explained.

One of these theories, formulated by H. H. Babcock, explains sunspot formation in terms of a magnetic field where the lines of force are drawn out longitudinally by differential rotation. His model accounts for sunspot polarity and provides a qualitative exploration of the preponderance of "preceding spots" of the forward tilt of the axes of older spots and of the recurrence of activity in preferred longitude.

According to H. Alfven, the preceding theory is unlikely. It appears to him that the nuclear energy released in the solar core is the most probable energy source of sunspots. He believes that the heat produced in the solar core is converted into mechanical and electromagnetic energy in the convection region, but he reaches no conclusion about the location of the convection region. If this theory holds, we have a way of exploring the sun's interior in the same way that we can explore the earth's interior - through a study of seismic waves. These theories, which are entirely speculative, are typical of the others which will not be considered here. Of great importance would be a general theory, which has not yet been formulated.

III. PREDICTION TECHNIQUES

Methods for predicting sunspots are usually classified as follows: They may be grouped according to the nature of the technique or according to the scope of the predictions. In the first case the groups could descriptively be called "Data Analysis" and "Causal." In the second case, they could be called "Short-Term" and "Long-Term."

Since the first method seems plausible for survey purposes, the discussion will proceed in that direction.

3.1 Data Analysis

3.1.1 King Hele's Method

King Hele's prediction method is based on a 7-cycle recurrence. His first prediction in 1963 came fairly close to the actual minimum. This encouraged him to apply the same method to get revised estimates of the next sunspot maxima. He then used another method to obtain the intensity of the maxima. His figures are used in the following discussion of his prediction methods.

Figure 1 shows t_R, the time for the rise from minimum to maximum sunspot numbers, for each of the past 15 cycles, from 1788 onward, plotted against the year in which the subsequent sunspot maximum occurred. It shows that the values of t_R for 1788-1870 (broken line) have an identical up and down sequence to those for 1870-1947 (unbroken line), thus indicating the 7-cycle recurrence tendency.

Based on the persistence of this regularity, he predicts the rise-time for the twentieth solar cycle as 3.4 years and for the subsequent cycle as 3.8 years. So, if 1964.7 is officially accepted as the date of the minimum of cycle No. 20, his prediction for the time of the maximum is 1968.1.

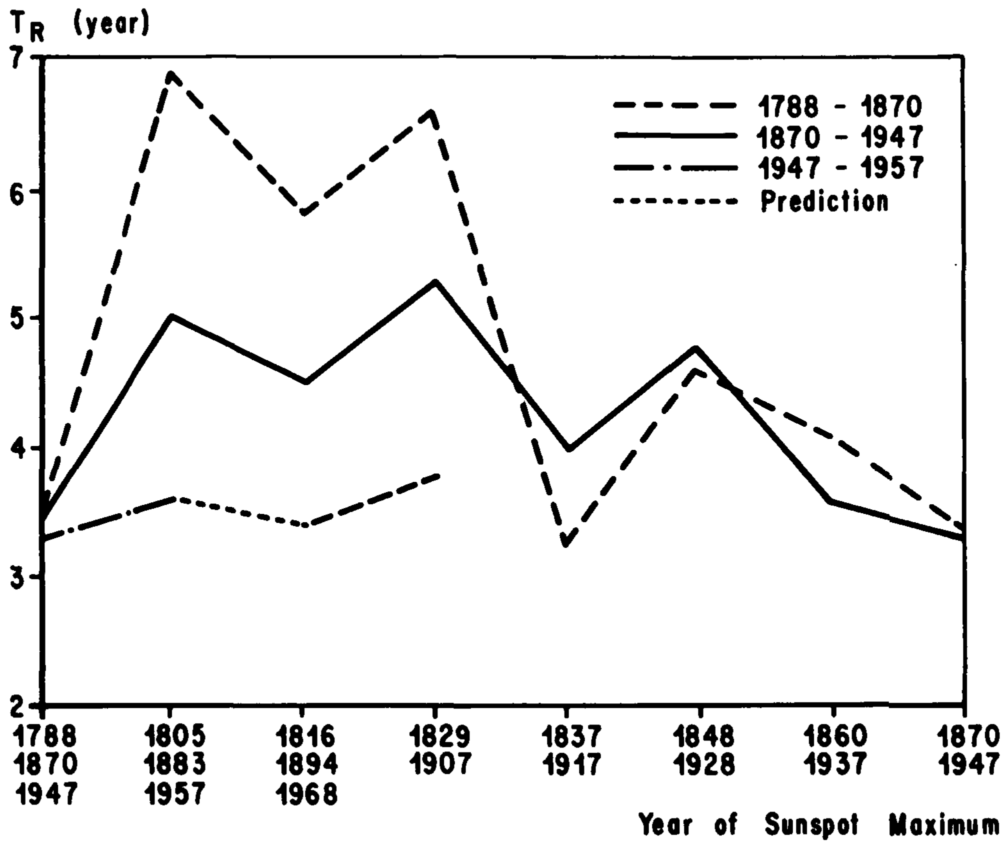

FIG. 1. TIME OF RISE

Figure 2 shows t_R expressed as a fraction of the total cycle length. The same regularity is observed for 1788-1870 and 1870-1947 except that t_R/T for 1937 is lower than 1928 instead of being equal like the preceding corresponding values. King Hele makes this accountable to the indefiniteness of the exact date of the minimum. If the regularity in figure 2 continues, he predicts that the twentieth cycle should last 10.0 years having a maximum at 1968.1 and a subsequent minimum at 1974.7.

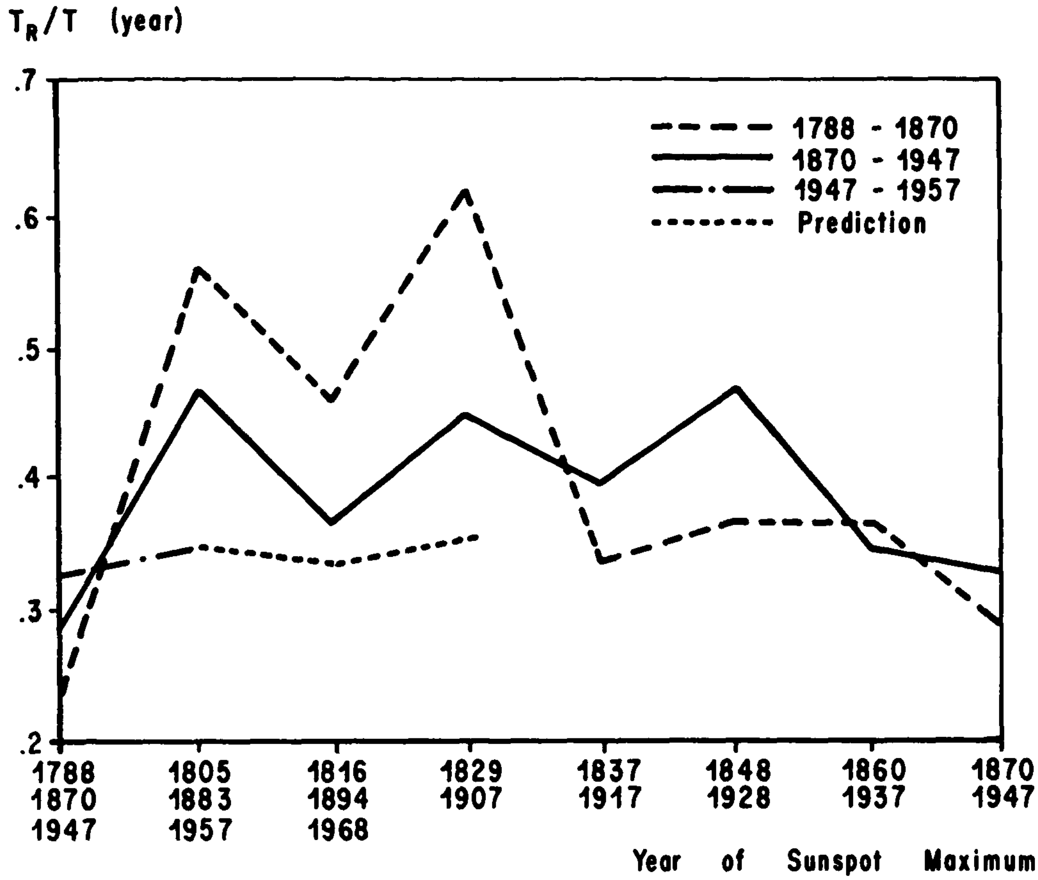

FIG. 2. TIME OF RISE EXPRESSED AS A FUNCTION OF TOTAL CYCLE LENGTH

King Hele found that relationship exists between R_m, maximum sunspot number, and $R_m t_R{}^2$ which usually has a value near 1750 (if t_R is in years). He modified this relationship and formulated equation (4):

$$(R_m - 16)\ t_R^2 - 10^{-5}(R_m - 100)^4 \simeq 1401. \tag{4}$$

With this equation and with his value of t_R already obtained, he predicts that the monthly smooth sunspot number at the maximum in 1968 will be about 140 (with R_m = 190 as a less likely alternative if the upper part of the curve applies).

3.1.2 Gleissberg's Method

Gleissberg originally developed a probability method of predicting certain features of the sunspot cycle. He later modified his method to give approximate average values instead of upper and lower limits. Later still, he again modified his method by introducing a new set of relationships.

His method presupposes the existence of an 80- to 90-year cycle of sunspots. He maintains that successive cycles are not entirely independent and that it is thus possible to forecast the next successive cycle. Because of the difficulty of determining the beginning and end of a cycle, Gleissberg used the times when the Wolfe number is equal to one-fourth of the maximum number. Gleissberg introduced the following characteristics of the 11-year cycle:

R_m = the maximum Zurich smoothed monthly relative spot number

T_r = the reduced length of the rising part of the cycle, defined as the time during which the smoothed monthly Wolfe number increases from 1/4 R_m to R_m, in months

t_f = the reduced length of the descending part of the cycle, defined as the time during which the smoothed monthly Wolfe number decreases from R_m to 1/4 R_m, in months

t_ℓ = the period of low activity, defined as the time interval between the end of the reduced descending part of one cycle and the beginning of the reduced rising part of the next cycle (in months).

With his original method, he used the following equations:

$$A = T_r^{(4)} + 0.2\ R_m^{(4)} \tag{5}$$

$$B = T_r^{(4)} + 0.4T_\ell^{(4)} \tag{6}$$

$$C = T_r^{(4)} + 0.5t_f^{(4)} \tag{7}$$

where $R_m^{(4)}$, $T_r^{(4)}$, $t_f^{(4)}$, and $t_\ell^{(4)}$ are obtained by taking an average of four successive 11-year cycles. The distribution of the differences between the actual values of A, B, and C and their respective averages agreed well with a Gaussian error distribution, and the mean error ϵ was nearly the same for A, B, and C (about ± 1.95). The constant of Gauss' law of errors was computed to be 0.36. Thus, the probability that the value of A, B, or C differs from its average by no more than δ is equal to erf (0.36δ), where erf denotes the error function.

Accordingly, Gleissberg derived the following probability laws:

I. The probability that $t_r^{(4)} + 0.2R_m^{(4)}$ for any two successive cycles lies between $55.5 - \delta$ and $55.5 + \delta$ may be expressed as $\mathrm{erf}(0.36\delta)$;

II. The probability that $t_r^{(4)} - 0.4t_\ell^{(4)}$ for these same cycles lies between $16.5 - \delta$ and $16.5 + \delta$ may be expressed as $\mathrm{erf}(0.36\delta)$;

III. The probability that $t_r^{(4)} + 0.8_f^{(4)}$ for these cycles lies between $77.5 - \delta$ and $77.5 + \delta$ may be expressed as $\mathrm{erf}(0.36\delta)$;

IV. The probability that $P(\delta) = \mathrm{erf}(0.16\delta + 0.08) - \mathrm{erf}(0.16\delta - 0.08)$.

With his first modification, he introduced the following equations:

$$t_r^{(4)} + 0.2R_m^{(4)} = 0.5 \tag{8}$$

$$t_r^{(4)} - 0.4t_\ell^{(4)} = 16.5 \tag{9}$$

$$t_r^{(4)} - 0.8t_f^{(4)} = 77.5 \tag{10}$$

$$a = 0.375t_\ell + .005t_\ell^2 \tag{11}$$

$$t_r^{(4)} + 1.42r_m^{(4)} = 41.85 \tag{12}$$

where r_m is the smallest Zurich smoothed monthly relative spot number and a is the interval expressed in months between the end of the reduced descending part and the month of the smallest Zurich smoothed monthly relative spot number.

His most recent modification resulted in the following relationship:

$$R_m \begin{matrix} > 1080 \\ < 1140 \end{matrix} - 20t_r^{(4)} - S_3(R_m), \tag{13}$$

where $S_3(R_m)$ is the sum of R_m for the three past cycles and where either the upper signs and values or the lower signs and values, according to whether $t_r^{(4)}$ is decreasing or increasing, are used.

Vitinskii applied Gleissberg's original method to the 18th cycle with the following results:

Table 1

	Predicted	Observed
R_m	145	152
Epoch of maximum	1948.3	1947.5
t_r	32	21
t_ℓ	40	37

The predicted maximum Wolfe number shows very good agreement with the observed Wolfe number. This prediction cannot be considered successful, however, because t_r, on which this method is based, is too much in error.

Vitinskii applied his first modification to the 19th cycle with the following results:

Table 2

	Predicted	Observed
r_m	16.5	3.6
Epoch of minimum	1955.2	1954.5
R_m	160	202
Epoch of maximum	1958.7	1958.1

The prediction for the epochs can be termed satisfactory, but the Wolfe values are not acceptable.

Black [3] used the most recent method of Gleissberg in predicting R_m for the 20th cycle. His prediction, on the basis of this approach, is that R_m will not exceed 82.7. This prediction is made with a probability of 95 percent.

3.1.3 Superposition Method

The superposition method is a rather simple one. Suggested by Wolfe in 1899, it is based on the hypothesis that the curve of growth represents the results of a superposition of many periodic processes. It, therefore, involves the discovery of all the possible periods which would give the best fit for the actual curve shape. Although this method has been used by many, it differs only in respect to the technique used (periodogram analysis, harmonic analysis, Fourier function, etc.) with none of them achieving any degree of success.

The most recent superposition method found in the literature was applied to the 20th cycle by Richards. The superpositions were effected by considering each cycle to consist of an eleven-year period beginning at the minimum points of the cycle. The method is based on the assumptions that the next cycle will be an average of the past cycles and that it will vary from this mean as the past cycle varied from it. The mean of the past cycles appears in figure 3 along with 1σ and 2σ confidence limits.

The significance of the superposition method in sunspot prediction certainly has not made itself obvious. According to Vitinskii, its real significance is just to draw attention to the study of long-period cycles and stress the importance of ultra-long-range forecast.

3.1.4 Waldmeier's Method

Waldmeier's forecasting method [2] is based on the following relations:

(1) The higher the maximum the shorter the ascending branch;

(2) The higher the maximum, the longer the descending branch;

(3) The higher the maximum, the stronger the spot forming activity five years after the maximum;

(4) The higher the maximum, the greater is the sum of the sunspot numbers during the time of decrease;

(5) The sum of the sunspot numbers during the time of increase is almost independent of the height of the maximum.

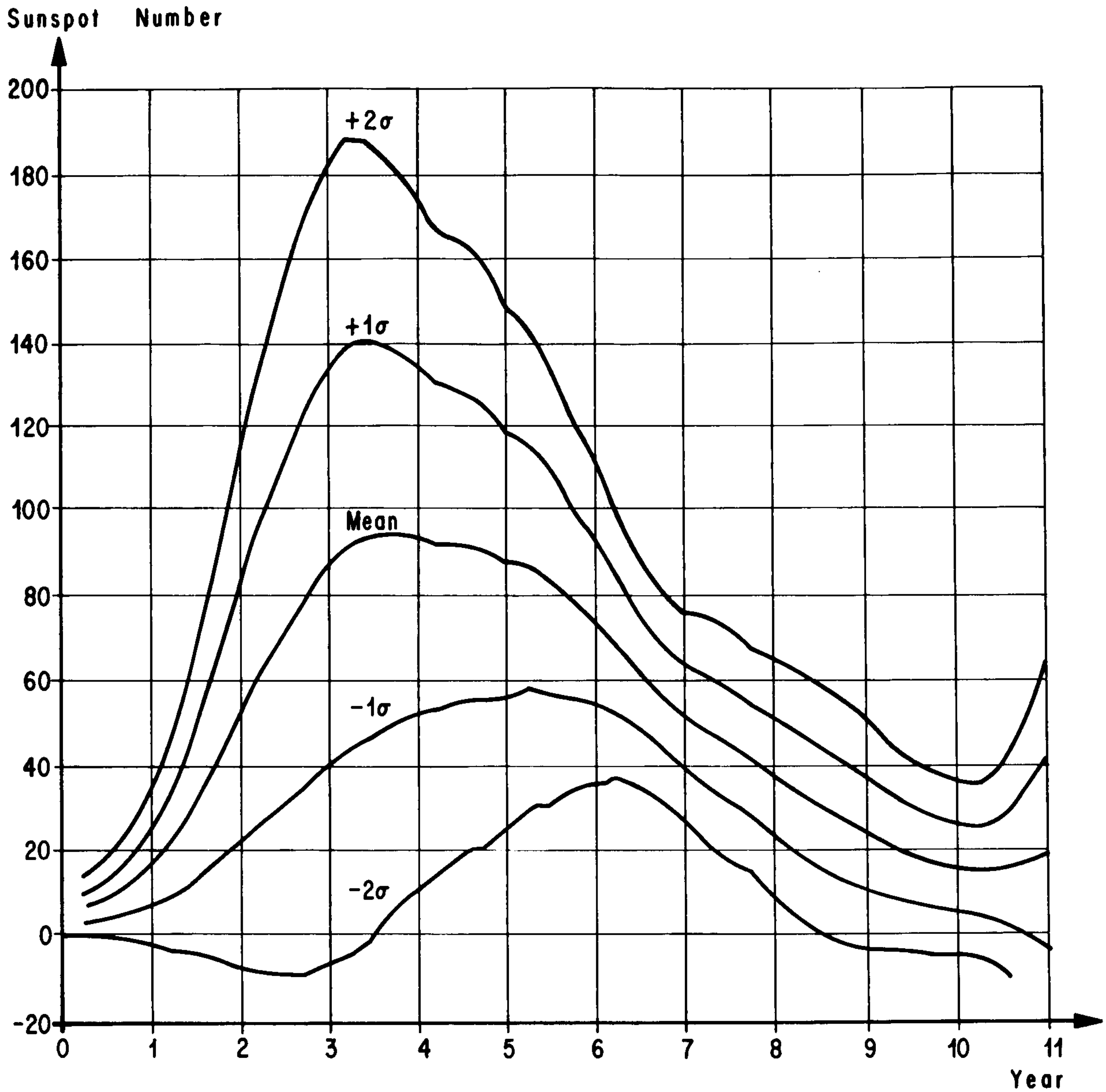

FIG. 3. MEAN CYCLE WITH "1σ" AND "2σ" CONFIDENCE LIMITS FOR CYCLE NUMBERS 1-19

From these relations, he established the following formulas:

$$\text{Log } R_m = 2.58 - 0.14T \quad (14)$$

$$\theta = 0.030\ R_m + 3.0 \quad (15)$$

$$R_5 = 0.29\ R_m - 11.4 \quad (16)$$

$$S_2 = 40.6\ R_m - 572 \quad (17)$$

$$S_1 = 2538 + 0.4\ R_m \quad (18)$$

where R_m is the largest smoothed mean monthly number, R_5 is the smoothed monthly Wolfe number five years after the maximum, T is the interval of time between the minimum and maximum, and θ is the interval of time between the maximum and the epoch when the smoothed mean monthly Wolfe numbers attain values near 7.5.

T. W. Bennington classified the sunspot cycles (18 of them completed at the time) into three categories: (1) a high maximum which exceeds 116; (2) a medium-high maximum which ranges from 80-116; and (3) a low maximum which falls below 80.

He found that there had been eight cycles with high maxima, four with medium-high maxima, and four with low maxima. Figure 4 gives three curves which represent the mean sunspot numbers of the cycles with high, medium-high, and low maxima, respectively. Table 3 gives the duration times for the different phases of the cycles in the three categories.

In consideration of the relationships stated by Bennington, a range of T's from three years to six years was used with Waldmeier's equations in a computer program to cover the categorical range. The results are given in Table 4. If we assume that T will fall between 3.0 and 6.0, we may assume that the maximum will fall between 145 and 55.

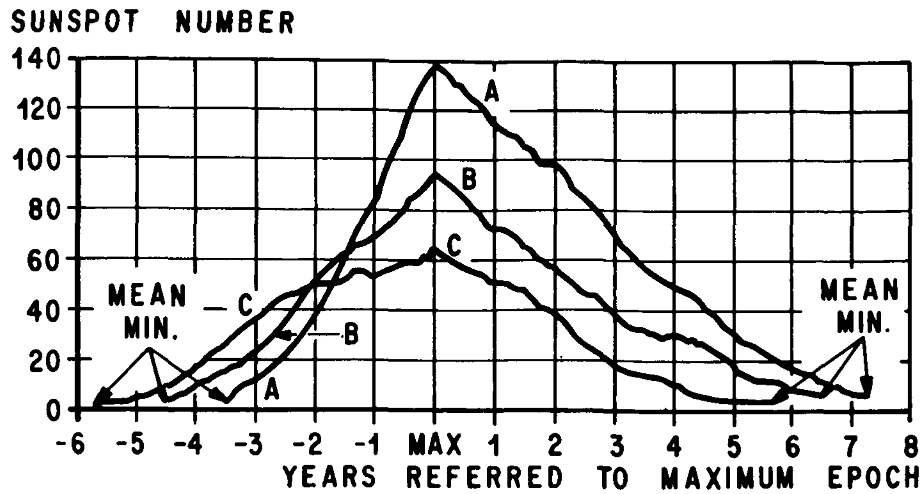

FIG. 4.

TABLE 3

TYPE	DURATION - YEARS		
	Minimum to Maximum	Maximum to Minimum	Whole Cycle
High Maxima	3.5	7.3	10.8
Medium-high Maxima	4.6	6.5	11.1
Low Maxima	5.8	5.7	11.5

Table 4

Sunspot Indicators Using Waldmeier's Equation

T	RM	THETA	R5	S1	S2
3.0	144.54	7.336319	30.52	2595.82	5296.49
3.1	139.96	7.198762	29.19	2593.98	5110.32
3.2	135.52	7.065568	27.90	2592.21	4930.07
3.3	131.22	6.936600	26.65	2590.49	4755.53
3.4	127.06	6.811722	25.45	2588.82	4586.53
3.5	123.03	6.690806	24.28	2587.21	4422.89
3.6	119.12	6.573726	23.15	2585.65	4264.44
3.7	115.35	6.460360	22.05	2584.14	4111.02
3.8	111.69	6.350590	20.99	2582.67	3962.46
3.9	108.14	6.244302	19.96	2581.26	3818.62
4.0	104.71	6.141386	18.97	2579.89	3679.34
4.1	101.39	6.041734	18.00	2578.56	3544.48
4.2	98.17	5.945244	17.07	2577.27	3413.90
4.3	95.06	5.851814	16.17	2576.02	3287.46
4.4	92.04	5.761349	15.29	2574.82	3165.03
4.5	89.13	5.673753	14.45	2573.65	3046.48
4.6	86.30	5.588936	13.63	2572.52	2931.69
4.7	83.56	5.506809	12.83	2571.42	2820.55
4.8	80.91	5.427288	12.06	2570.36	2712.93
4.9	78.34	5.350289	11.32	2569.34	2608.72
5.0	75.86	5.275733	10.60	2568.34	2507.82
5.1	73.45	5.203542	9.90	2567.38	2410.13
5.2	71.12	5.133641	9.23	2566.45	2315.53
5.3	68.87	5.065957	8.57	2565.55	2223.93
5.4	66.68	5.000420	7.94	2564.67	2135.24
5.5	64.57	4.936963	7.32	2563.83	2049.36
5.6	62.52	4.875518	6.73	2563.01	1966.20
5.7	60.53	4.816023	6.15	2562.21	1885.68
5.8	58.61	4.758414	5.60	2561.45	1807.72
5.9	56.75	4.702634	5.06	2560.70	1732.23
6.0	54.95	4.648623	4.54	2559.98	1659.14

3.1.5 Mayot's Method

Mayot's method [3] originally developed for monthly relative sunspot numbers, is based on the assumption that a multi-annual series of Wolfe numbers is representable in the form

$$W(t) = F(t) + E. \tag{19}$$

It involves the solution of a system of equations as follows:

$$W_i = a_1 W_{i-1} + a_2 W_{i-2} + \ldots + a_i W_o + \epsilon_i \tag{20}$$

$$W_{i+1} = a_1 W_i + a_2 W_{i-1} + \ldots + a_i W_1 + \epsilon_{i+1} \tag{21}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$W_n = a_1 W_{n-1} + a_2 W_{n+2} + \ldots \quad a_i W_{n-i} + \epsilon_n \, . \tag{22}$$

This system of equations can be solved only if the coefficients are sufficiently separable. The period up to the epoch of the maximum or near it must be used in order to satisfy this requirement.

This method has been discussed at length with Vitinskii along with his modification of it. Mayot's method has been found to involve smaller errors than the modified Mayot's method. It was also found that for Mayot's method the error increases appreciably upon transition from back calculation to forecasts. This indicates that the regression method is the most effective approach.

3.1.6 Lincoln-McNish Method

Lincoln and McNish developed a method for predicting the solar cycle. A regression technique, it is based on the following assumptions: (1) In a time series of cyclic tendencies, an estimate, to a first approximation of a future value in the series, is the mean of all past values for the same part of the cycle; and (2) this estimate can be improved by adding to the mean a correction proportional to the departures of earlier values of the same cycle from their respective means.

Lincoln and McNish used the following prediction formula:

$$R'_n = \bar{R}_n + \Delta R'_n = \bar{R}_n + K_{n-1}\,\Delta R_{n-1} + K_{n-2}\,\Delta R_{n-2} + K_{n-3}\,\Delta R_{n-3} + \cdots$$

$$+ K_{n-i}\,\Delta R_{n-i}, \tag{23}$$

where R'_n is the smoothed annual value to be predicted in a particular cycle for the nth year after the minimum, $\bar{R}_n$ is the mean of all the nth values of R in preceding cycles, ΔR_{n-i} is the departure of the particular R_{n-i} from $\bar{R}_{n-i}$, and K_{n-i} is a prediction coefficient calculated by the method of least squares.

Lincoln and McNish recommend that the prediction using smoothed yearly relative sunspot numbers be made only a year in advance. This method with the yearly numbers evaluated quarterly was used in reference 5 to predict the remainder of the twentieth sunspot cycle (fig. 5).

The regression method has been used extensively in forecasting, and its accuracy has been, for the most part, satisfactory. Vitinskii devotes a topic to the discussion of the validity of this method, and his opinion seems to be in harmony with the regression method. He points out, however, that this method is very sensitive to fluctuations in the data, and therefore, the results are usually too high.

3.1.7 Schove's Method

This method is based on sunspot data which go back to about 1610. These data were obtained from the available records of sunspots and polar auroras. Schove compiled these data into a table on the assumptions that (1) the time between successive maxima is not less than 8 and not more than 16 years; and (2) nine sunspot maxima occur every 100 years.

Schove discusses his table and gives a full explanation of the procedure of its compilation in reference 12. The table implies that the minimum generally precedes the maximum by four years if the maximum is strong or very strong, by five years if the maximum is moderate or moderately strong, and by six years if the maximum is weak. It also implies the existence of the 80-year to 90-year cycle, which is longer in aurorally weak periods.

Schove used the results of his careful analysis to forecast characteristics of cycles 19 through 25. Schove's method certainly warrants respect because of the amount of data upon which it is based.

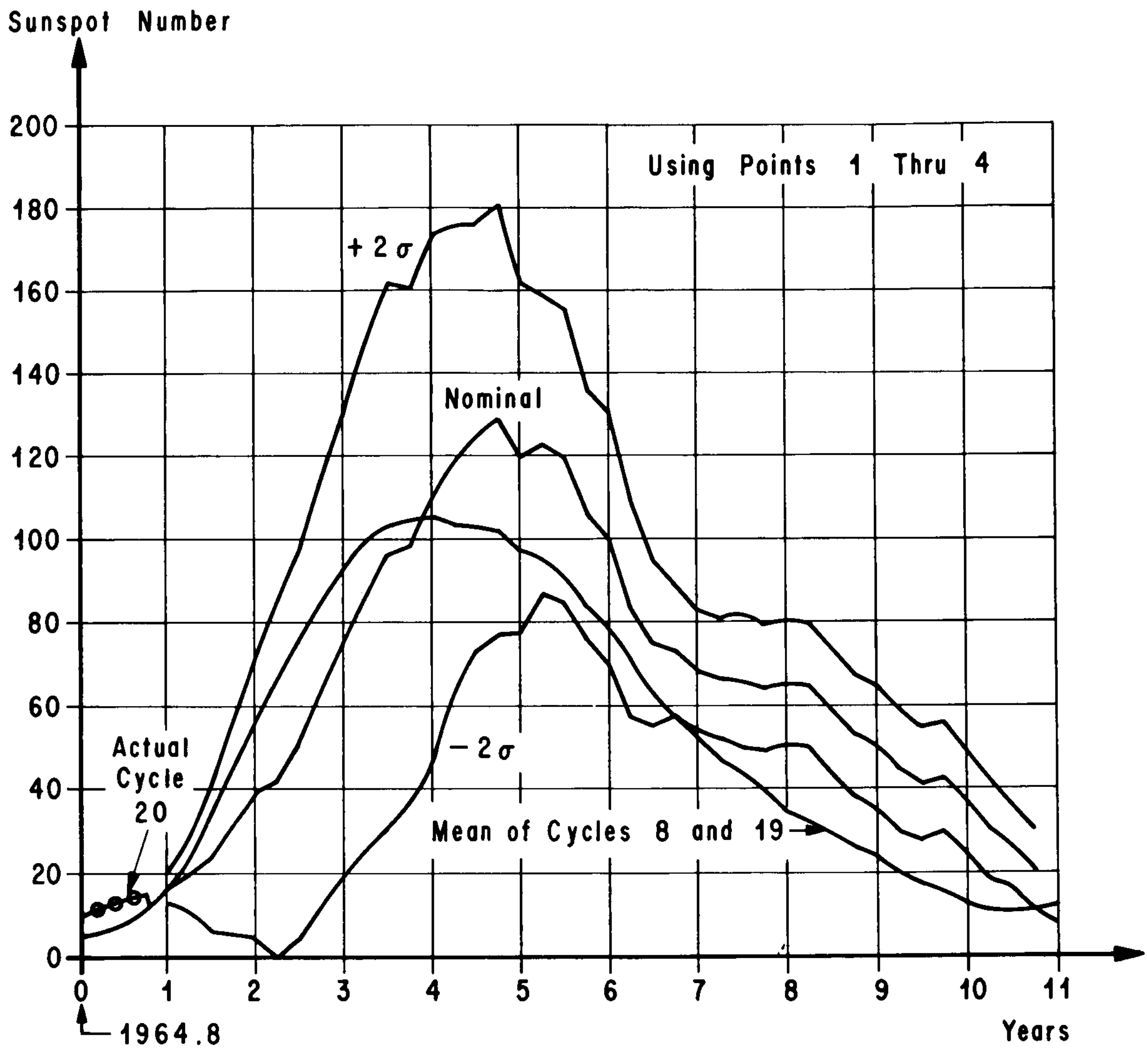

FIG. 5. PREDICTION OF CYCLE 20
(WITH 1965.8 AS THE LAST KNOWN POINT)

3.1.8 Minnis' Method

Minnis employed three statistical methods to forecast the height of the 20th sunspot cycle. He applied a directional sequence, a frequency distribution, and an autocorrelation function to estimate the probable limits within which the peak would lie.

His procedure was to examine the statistical evidence contained in the sequence of the peak values since 1750 and decide whether the next peak would be high or low. The individual results of his methods are given in Table 5; they are combined in Table 6.

Table 5

Individual Estimates of R

No.	Estimates Used	R(1968)	Probability
C1	3, 5	111-159	0.9
C2	7, 8, 9	104-194	0.68
C3	6, 7, 8, 9	104-218	0.68

Table 6

Combined Estimates of R

No.	Method Used	R(1968)	Probability
1	Direction Sequence	> 203	0.3
2	Direction Sequence	< 203	0.7
3	Distribution of R	< 159	0.95
4	Distribution of ΔR	> 128	0.95
5	Distribution of ΔR	> 111	> 0.95
6	Distribution of ΔR	238 ± 23	0.68
7	Distribution of ΔR	168 ± 23	0.68
8	Autocorrelation ($r = 1$)	154 ± 38	0.68
9	Autocorrelation ($r = 3$)	97 ± 36	0.68

From this study, Minnis adopted the range of 110-160 as the estimate of the peak sunspot number of the twentieth cycle. He made no attempt to justify the preciseness of the estimate.

3.1.9 Shapley's Method

Shapley made use of Brunner's formulas in forecasting the minimum and maximum of cycle 18. Brunner's formulas are based on the hypothesis that each spot-cycle represents a fresh recurrence of a phenomenon which, once started, follows a fairly standard pattern.

The formulas take the following forms:

$$\log R_m = (2.44 - 0.082T) \log R_m = (2.74 - 0.18T') \tag{24}$$

$$T = (2.44 - \log R_m)/0.082 \tag{25}$$

$$T' = (2.74 - \log R_m)/0.18 \tag{26}$$

$$E_m = (B + T) \tag{27}$$

$$E_m = (E_m - T'), \tag{28}$$

where R_m is maximum sunspot number, T is time lag in year between the beginning of the cycle and the epoch of the maximum, T' is time lag in years between the epoch of the maximum and epoch of minimum, B is the beginning of the cycle, E_m is the epoch of the maximum, and E_m is the epoch of the minimum.

Shapley determined the beginning of cycle 18 from the time of the appearance of the first high latitude spot. He advocates that there is an alternation of high and low values of sunspot numbers at maximum. A large "H" is followed by a large "L" and a small "H" by a small "L." The mean of the ratio (H/L) was demonstrated for successive cycles as 1.48. Based upon this, Shapley estimated the maximum of the 18th cycle as (119/1.48) or 80 since the maximum of the 17th cycle was 119.

His forecast using this estimated value of R_m and Brunner's formulas was as follows:

Table 7

	Predicted	Observed
R_m	1946.6	1947.4
r_m	1944.9	1944.3

Table 8

B	T	EPOCH OF MAX	TPRIME	EPOCH OF MIN
RM = 150.000				
1964.1	3.21839940E 00	1.96731840E 03	3.13282639E 00	1.96418557E 03
1964.2	3.21839940E 00	1.96741840E 03	3.13282639E 00	1.96428557E 03
1964.3	3.21839940E 00	1.96751840E 03	3.13282639E 00	1.96438557E 03
1964.4	3.21839940E 00	1.96761840E 03	3.13282639E 00	1.96448557E 03
1964.5	3.21839940E 00	1.96771840E 03	3.13282639E 00	1.96458557E 03
1964.6	3.21839940E 00	1.96781840E 03	3.13282639E 00	1.96468557E 03
1964.7	3.21839940E 00	1.96791840E 03	3.13282639E 00	1.96478557E 03
1964.8	3.21839940E 00	1.96801840E 03	3.13282639E 00	1.96488557E 03
1964.9	3.21839940E 00	1.96811840E 03	3.13282639E 00	1.96498557E 03
RM = 151.000				
1964.1	3.18320808E 00	1.96728321E 03	3.11679479E 00	1.96416641E 03
1964.2	3.18320808E 00	1.96738321E 03	3.11679479E 00	1.96426641E 03
1964.3	3.18320808E 00	1.96748321E 03	3.11679479E 00	1.96436641E 03
1964.4	3.18320808E 00	1.96758321E 03	3.11679479E 00	1.96446641E 03
1964.5	3.18320808E 00	1.96768321E 03	3.11679479E 00	1.96456641E 03
1964.6	3.18320808E 00	1.96778321E 03	3.11679479E 00	1.96466641E 03
1964.7	3.18320808E 00	1.96788321E 03	3.11679479E 00	1.96476641E 03
1964.8	3.18320808E 00	1.96798321E 03	3.11679479E 00	1.96486641E 03
1964.9	3.18320808E 00	1.96808321E 03	3.11679479E 00	1.96496641E 03
RM = 152.000				
1964.1	3.14824904E 00	1.96724825E 03	3.10086901E 00	1.96414738E 03
1964.2	3.14824904E 00	1.96734825E 03	3.10086901E 00	1.96424738E 03
1964.3	3.14824904E 00	1.96744825E 03	3.10086901E 00	1.96434738E 03
1964.4	3.14824904E 00	1.96754825E 03	3.10086901E 00	1.96444738E 03
1964.5	3.14824904E 00	1.96764825E 03	3.10086901E 00	1.96454738E 03
1964.6	3.14824904E 00	1.96774825E 03	3.10086901E 00	1.96464738E 03
1964.7	3.14824904E 00	1.96784825E 03	3.10086901E 00	1.96474738E 03
1964.8	3.14824904E 00	1.96794825E 03	3.10086901E 00	1.96484738E 03
1964.9	3.14824904E 00	1.96804825E 03	3.10086901E 00	1.96494738E 03
RM = 153.000				
1964.1	3.11351925E 00	1.96721352E 03	3.08504766E 00	1.96412847E 03
1964.2	3.11351925E 00	1.96731352E 03	3.08504766E 00	1.96422847E 03
1964.3	3.11351925E 00	1.96741352E 03	3.08504766E 00	1.96432847E 03
1964.4	3.11351925E 00	1.96751352E 03	3.08504766E 00	1.96442847E 03
1964.5	3.11351925E 00	1.96761352E 03	3.08504766E 00	1.96452847E 03
1964.6	3.11351925E 00	1.96771352E 03	3.08504766E 00	1.96462847E 03
1964.7	3.11351925E 00	1.96781352E 03	3.08504766E 00	1.96472847E 03
1964.8	3.11351925E 00	1.96791352E 03	3.08504766E 00	1.96482847E 03
1964.9	3.11351925E 00	1.96801352E 03	3.08504766E 00	1.96492847E 03
RM = 154.000				
1964.1	3.07901572E 00	1.96717902E 03	3.06932938E 00	1.96410969E 03
1964.2	3.07901572E 00	1.96727902E 03	3.06932938E 00	1.96420969E 03
1964.3	3.07901572E 00	1.96737902E 03	3.06932938E 00	1.96430969E 03
1964.4	3.07901572E 00	1.96747902E 03	3.06932938E 00	1.96440969E 03
1964.5	3.07901572E 00	1.96757902E 03	3.06932938E 00	1.96450969E 03
1964.6	3.07901572E 00	1.96767902E 03	3.06932938E 00	1.96460969E 03
1964.7	3.07901572E 00	1.96777902E 03	3.06932938E 00	1.96470969E 03
1964.8	3.07901572E 00	1.96787902E 03	3.06932938E 00	1.96480969E 03
1964.9	3.07901572E 00	1.96797902E 03	3.06932938E 00	1.96490969E 03
RM = 155.000				
1964.1	3.04473550E 00	1.96714474E 03	3.05371284E 00	1.96409102E 03
1964.2	3.04473550E 00	1.96724474E 03	3.05371284E 00	1.96419102E 03
1964.3	3.04473550E 00	1.96734474E 03	3.05371284E 00	1.96429102E 03
1964.4	3.04473550E 00	1.96744474E 03	3.05371284E 00	1.96439102E 03
1964.5	3.04473550E 00	1.96754474E 03	3.05371284E 00	1.96449102E 03
1964.6	3.04473550E 00	1.96764474E 03	3.05371284E 00	1.96459102E 03
1964.7	3.04473550E 00	1.96774474E 03	3.05371284E 00	1.96469102E 03
1964.8	3.04473550E 00	1.96784474E 03	3.05371284E 00	1.96479102E 03
1964.9	3.04473550E 00	1.96794474E 03	3.05371284E 00	1.96489102E 03

Brunner's formulas were applied to the present cycle 20. With the ratio (H/L) procedure, R_m was estimated as 181.3. A preliminary guess of the beginning of the cycle is required. Since the minimum of the 20th cycle definitely occurred in 1964, the input beginnings ranged from 1964.1 to 1964.9. It became apparent that this should serve as a numerical correction to the estimated R_m. Table 8 contains the computer output that shows the best correlation of the minimum between the beginning input, B, to the computer program and the epochs that are computed. These results indicate that R_m might be corrected to range between 150-155.

3.1.10 Linder's Method

From his study of sunspot numbers from 1843 to 1943, Linder observed a regularity in the alternation of the height of the maximum in successive years. He then compiled the yearly maximum numbers (see Table 9). On the basis of this tabulation, he concluded that the maximum for the 18th cycle would fall somewhere between 95.7, the highest yearly peak of the "low" alternate year, and 63.5, the lowest peak of the "low" alternate year.

Table 9

Cycle	Wolfe Yearly Peak		Wolf Monthly Peak	
	Year	Max. No.	Month	Max. No.
1843-1856	1848	124.3	Oct. 1847	180.4
1856-1867	1860	95.7	July 1860	116.7
1867-1879	1870	139.1	May 1870	176.0
1879-1889	1883	63.7	April 1882	95.8
1889-1901	1893	84.9	Aug. 1893	129.2
1901-1913	1905	63.5	Feb. 1907	108.2
1913-1923	1917	103.9	Aug. 1917	154.5
1923-1934	1928	77.8	Dec. 1929	108.0
1934-1943	1937	118.8	July 1938	165.3

The availability of recent data as shown in Table 10 discredits this forecast since the peak of the 18th cycle actually was 151.6, higher than the preceding "high" peak. However, if one makes this deviation accountable to the uniqueness of this period, a reasonable conclusion according to recent trends might be that the maximum of the twentieth cycle will fall between 189.9, the highest yearly peak of "high" alternate years, and 129.2, the lowest yearly peak of "high" alternate years.

Table 10

Peak Sunspot Numbers

Cycle	Month	Monthly Peak	Year	Yearly Peaks
1943-1954	May 1947	201.3	1947	151.6
1954-1964	Oct. 1957	253.8	1957	189.9

3.1.11 Yule's Method

Yule used three methods to investigate periodicities in the sunspot cycle. His first method, a harmonic curve equation, was derived by finding the best (least square) linear equation relating $U_x + U_{x-2}$ to U_{x-1} where the U's are annual smoothed sunspot numbers. Yule viewed the period resulting from this equation as being too low. He then sought to determine whether or not there existed a secondary period in addition to the fundamental. His results showed no evidence of the existence of such a period.

He resorted to a more general method of determining the regression equation for U_x on U_{x-1} and U_{x-2} and solving it as a finite difference equation:

$$U_x = b_1 U_{x-1} - b_2 U_{x-2} \; . \qquad (29)$$

His last method was a periodogram analysis which involves finding the standard error of the amplitude of a period found from any number n of observations given the standard deviation of the disturbances. Yule found that this type of analysis, when applied to sunspot numbers, tends to give much too low an intensity for the fundamental. From his investigations, Yule concluded the following generalities:

(1) Sunspot numbers should be regarded as analogous to the data that would be given by observations of a disturbed periodic movement.

(2) The better method for investigating this type of data is determining the regression equation for U_x on U_{x-1} and U_{x-2} and solving as a finite difference equation.

3.1.12 Herrinck's Method

Herrinck was impressed by the striking similarity between the period 1749-85 and 1918-54. When he incorporated this fact into his ionospheric predictions, he obtained good results. He was able to obtain better results after multiplying figures observed 169 years before by a factor of proportion.

Using running means over 13 months and taking July 1784 as a starting date, linked to April 1954, the following equation was obtained in November 1956:

$$\bar{x} = 1.488\ \bar{y} - 12.5, \tag{30}$$

where $\bar{y}$ is the value for a particular month of the old cycle and $\bar{x}$ is the corresponding value for the present cycle.

Using the data of April 1954 through October 1958, a new equation was derived for a still better fit:

$$\bar{x} = 1.527\ \bar{y} - 13.4. \tag{31}$$

Table 11 contains predicted sunspot numbers (p) and observed sunspot numbers (o) from 1957 to 1967. The first equation was used to predict from 1957 through 1958. The last equation was used for the remaining years. When negative values were obtained, they were replaced by zeroes.

Although Herrinck's tabulation shows good results (see table 11), he does not appear optimistic about the continuing trend.

Table 11

	1957		1958		1959	1960	1961	1962	1963	1964	1965	1966
	p	o	p	o								
January	172	170	196	198	164	137	93	81	62	46	29	15
February	175	172	189	199	163	136	91	80	61	49	28	14
March	182	177	187	203	168	129	89	79	59	50	23	11
April	185	183	180	198	165	122	87	78	58	47	19	11
May	184	187	179	191	167	117	86	77	57	46	19	9
June	188	189	178	189	163	111	86	76	55	43	16	6
July	191	191	176	187	160	108	87	75	51	40	19	5
August	189	190	177	182	157	106	85	73	52	41	19	2
September	191	194	178	183	153	103	83	71	50	36	16	1
October	196	194	174	181	147	100	81	70	49	33	17	1
November	197	197	169	-	143	97	80	68	48	31	16	0
December	197	197	167	-	140	95	82	64	46	28	15	0

3.2 Causal Methods

3.2.1 Jose's Method

As early as 1936, Jose investigated the problem of the effect of planetary motions on sunspot activity. It was found that the relative sunspot numbers have very nearly the same period as the sun's instantaneous angular momentum. Jose explains quite thoroughly how the parameters of motion are used in formulas which relate the effect of a system of bodies upon a given body in that system. They are as follows:

$$R = (x^2 + y^2 + z^2)^{1/2} \tag{32}$$

$$v = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} \tag{33}$$

$$\rho = \frac{v^3}{\triangle} \quad \triangle = \left[(\dot{y}\,\ddot{z} - \dot{z}\,\ddot{y})^2 + (\dot{z}\,\ddot{x} - \dot{x}\,\ddot{z})^2 + (\dot{x}\,\ddot{y} - \dot{y}\,\ddot{x})^2 \right]^{1/2} \tag{34}$$

$$L = \left[(y\dot{z} - z\dot{y})^2 + (z\dot{x} - x\dot{z})^2 + (x\dot{y} - y\dot{x})^2 \right]^{1/2} \tag{35}$$

$$P = \rho v, \tag{36}$$

where R is the distance from the center of mass of the system to the center of the sun, V is the velocity of the sun, L is the rate of change, and P is the angular momentum about the instantaneous center of curvature.

A cycle of approximately 178 years was found in the data that covered the time from 1653 to 2060. It was observed that this period is nine times the synodic period of Jupiter and Saturn (9 x 19.858 = 178.72).

The functions R, ρ, dL/dT, and dP/dT (with the unit of time as 40 days) are plotted separately in figures 6 and 7 for the two periods 1655 to 1833 and 1833 to 2012, respectively. The similarity between the two periods is striking.

The main conclusion reached by Jose from his study was that certain dynamic forces exerted on the sun by the motions of the planets are the cause of the sunspot activity. This is supported by the period of 178+ years in the sun's cycle. He took the maximum of 1615 as the starting point and compiled the information in Table 12 which gives his prediction based on this conclusion.

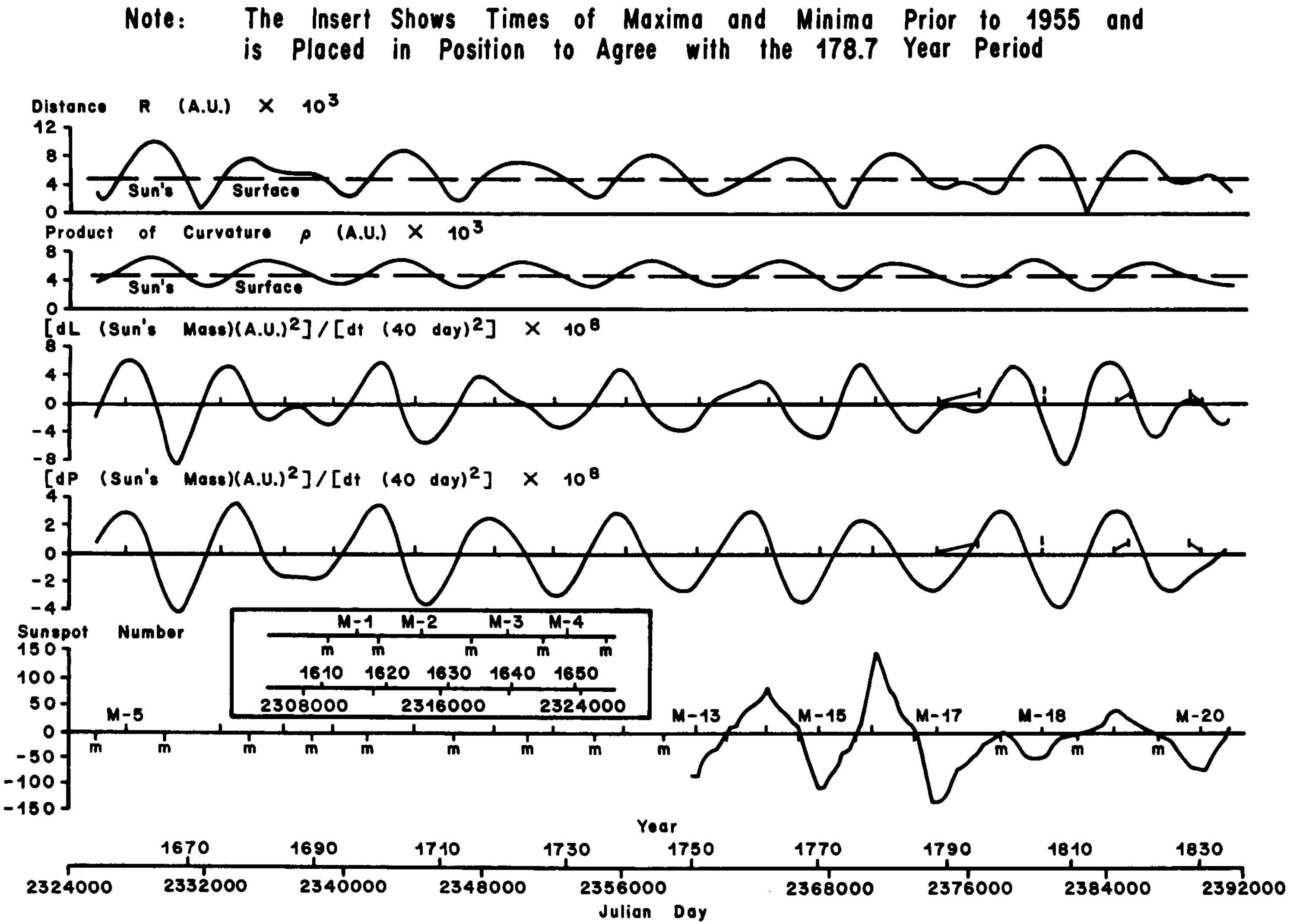

FIG. 6. THE QUANTITIES R, ρ, dL/dt, AND dP/dt FOR THE PERIOD 1655 TO 1833 RELATIVE SUNSPOT NUMBERS AND THEIR TIMES OF MAXIMA AND MINIMA ARE INDICATED

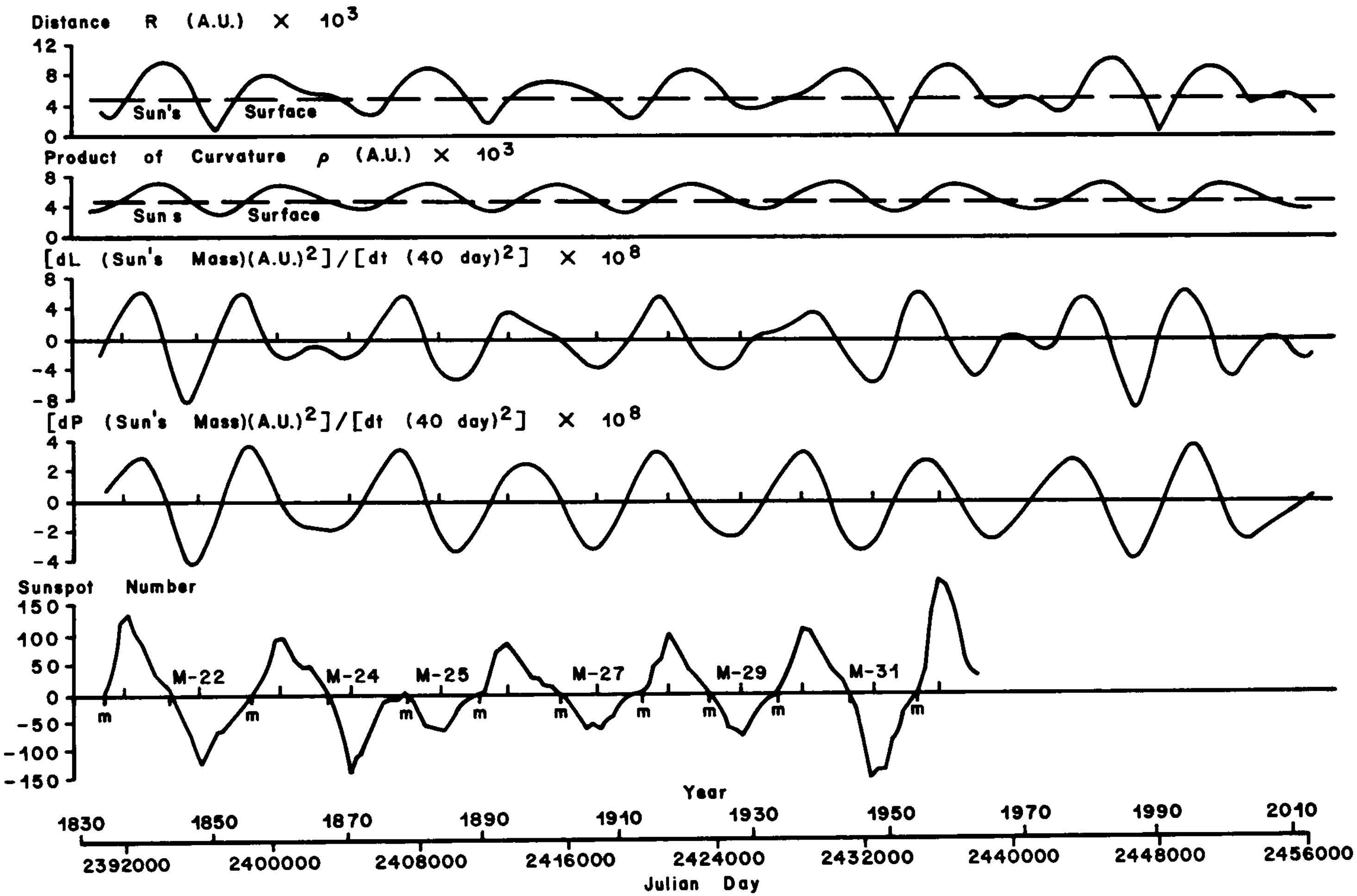

FIG. 7. THE QUANTITIES R, ρ, dL/dt, AND dP/dt FOR THE PERIOD 1833 TO 2012 RELATIVE SUNSPOT NUMBERS AND THEIR TIMES OF MAXIMA AND MINIMA ARE INDICATED

Table 12

Number of Cycle	Year of Minima	Year of Maxima	Differences between corre-sponding dates of minima	Differences between corre-sponding dates of maxima
1	1610.8	1615.5		
2	1519.0	1626.0		
3	1634.0	1639.5		
4	1645.0	1649.0		
5	1655.0	1660.0		
6	1666.0	1675.0		
7	1679.5	1685.0		
8	1689.5	1693.0		
9	1698.0	1705.5		
10	1712.0	1718.2		
11	1723.5	1727.5		
12	1734.0	1738.7		
13	1745.0	1750.3		
14	1755.2	1761.5		
15	1766.5	1769.7		
16	1775.5	1778.4		
17 (1)[a]	1784.7	1788.1	173.9[b]	172.6[b]
18 (2)	1798.3	1805.2	179.3	179.2
19 (3)	1810.6	1816.4	176.6[b]	176.9[b]
20 (4)	1823.3	1829.9	178.3	180.9
21 (5)	1833.9	1837.2	178.9	177.2
22 (6)	1843.5	1848.1	177.5	173.1[b]
23 (7)	1856.0	1860.1	176.5	175.1[b]
24 (8)	1867.2	1870.6	177.7	177.6
25 (9)	1878.9	1883.9	180.9	178.4
26 (10)	1889.6	1894.1	177.6	175.9[b]
27 (11)	1901.7	1907.0	178.2	179.5
28 (12)	1913.6	1917.6	179.6	178.9
29 (13)	1923.6	1928.4	178.6	178.1
30 (14)	1933.8	1937.4	178.6	175.9[b]
31 (15)	1944.3	1947.7	177.8	178.0
32 (16)	1954.3	1957.5	178.8	179.1
			Average of 24 intervals	
	Predicted		= 178.55	
			= 1.05	
33 (1)	1963	1967		
34 (2)	1977	1984		
35 (3)	1990	1995		
36 (4)	2002	2009		

[a]Corresponding cycle in first period.

[b]Dates omitted from average.

3.2.2 Suda's Method

Suda's theory on sunspot activity was along the same vein as Jose. He speculated that the tide-generating forces of the planets influenced solar activity. He investigated the individual effects of Earth and Jupiter and the combined effect of four planets (Earth, Jupiter, Mercury, and Venus) on solar activity.

Using a 178-year cycle, he classified the sunspots since 1750 according to latitude. From his study, he concluded that the tide-generating forces of Earth cause a 1/2- and 1-year change which does not show up in the yearly sunspot notation. Accordingly, Jupiter causes an irregular 11-year and 89-year (the semi-revolution of Jupiter) change. He concluded, in general, that the maximum amplitude occurs a short time after the planet's perihelion (with Jupiter exerting major influence).

3.2.3 Bell and Wolbach's Method

The following equation was derived some time ago by Danjon (1920) which predicted the dates of discontinuity in eclipse brightness with an average error of one year:

$$\text{Date of min} = t = 1584.8 + 10.87^{E}, \tag{37}$$

where E is the number of the minimum, beginning with E = 0 in 1583. He found that, by adding a correction term,

$$\triangle = 1.7 \sin 2\pi \, [(\tau - 1608)/136], \tag{38}$$

$$(\text{min} = 1584.8 + 10.87E + \triangle) \tag{39}$$

he could reduce the average error to .04 years.

Bell and Wolbach revised Danjon's equation to predict sunspot minima by adding 0.4 to the first term of Danjon's equation.

$$\text{Spot min} = 1585.2 + 10.87E + \triangle. \tag{40}$$

The dates of sunspot minima predicted by equation (4) and the resulting residuals O-C are given in Table 13, which shows remarkably good agreement.

Table 13

Danjon Epoch	Sunspot Minimum	Residual O-C
22	1823.4	-0.1
23	1833.7	+0.2
24	1844.4	-0.9
25	1855.5	+0.5
26	1866.8	+0.4
27	1878.5	+0.4
28	1890.3	-0.7
29	1901.8	-0.1
30	1913.0	+0.6
31	1923.8	-0.2
32	1934.1	-0.3
33	1944.3	0.0
34	1954.3	0.0
35	1964.4	+0.1 Est
36	1974.9	
37	1985.7	
38	1997.0	
39	2008.6	
40	2020.0	

IV. SUMMARY

This survey has covered only a few of the available sunspot prediction techniques in an effort to present some which would be most representative of the various proposals at the present. For comparative purposes, the results of some of these techniques appear in the following table as they apply to the oncoming maximum:

Table 14

Smoothed Maxima of Twentieth Cycle

Source	Prediction
Minnis	110-160
Shapley-Bruner	150-155
Waldmeier	55-145
Linder	129.2-189.2
King-Hele	140 (1968.1)
Gleissberg by Black	82.7
Jose	(1967.0)
Lincoln-McNish by Lockheed	128-180 (1967.8)
Standard Deviation by Lockheed	$+2\sigma$ 188 (1967)

These techniques cannot be examined without an awareness of their lack of reliability. In attacking this reliability problem, the ideal approach would be to formulate a theory on the reason for the formation of sunspots. The actuality of this, however, seems in the very distant future. An alternate approach would be a comprehensive theory on sunspots drawn from the many individual theory on sunspots drawn from the many individual theories already in existence. Meanwhile, efforts are underway to continue the study of current solar cycle literature in order to formulate a mathematical and/or physical concept that will permit prediction of future cycles and their expected statistical variation.

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SURVEY OF SOLAR CYCLE PREDICTION MODELS

by Jeanette A. Scissum

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