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On RANKING PROBLEMS IN MULTIVARIATE

NORMAL (STATISTICAL) POPULATIONS

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RANKING PROBLEMS IN MULTIVARIATE NORMAL (STATISTICAL) POPULATIONS

The major problem posed in the proposal, namely the selection of t out of k non-central chi-squared and non-central F populations with the largest non-centrality parameters, has been solved by the Principal Investigator (Supervisor) and a co-author. These investigations are reported in the paper by K. Alam and M. H. Rizvi entitled "Selection from Multivariate Normal Populations," Annals Inst. Stat. Math. 18 (1966) 307-318. Five copies of this paper are enclosed. The paper also considers the problem of selecting a subset containing the t populations with largest non-centrality parameters. Suitable procedures have been proposed for these problems and their operating characteristics studied in this paper. It should be noted that the ranking of k p-variate normal populations with mean column vectors μ_i and covariance matrices Σ_i ($i = 1,\ldots,k$) in terms of the Mahalanobis distance function $\theta_i = \mu_i' \Sigma_i^{-1} \mu_i$ reduces to ranking (with respect to the non-centrality parameters) the non-central chi-squared or non-central F populations. This parametric distance function has wide applications in multivariate analysis. An application that might be of some interest to NASA is indicated here. When $p = 1$, the measure of distance becomes $\theta_i = \mu_i^2/\sigma_i^2$ and $\theta_i^{1/2} = |\mu_i|/\sigma_i$. The ratio $|\mu_i|/\sigma_i$ is called the measurement signal-to-noise ratio in communications theory and plays a basic role in the evaluation of modern electronic apparatus (see, for instance, J. J. Freeman: Principles of Noise, John Wiley, New York, 1958, Chapters 7 and 9 and E. Parzen: Modern Probability Theory and its Applications, John Wiley, New York, 1960, Chapter 8, Section 6). An apparatus is considered superior if it has a larger signal-to-noise ratio. Thus the ranking procedures of the above paper should be of value to those interested in selecting better electronic equipment.

Some interesting differential difference equations involving non-central chi-squared (non-central F) distribution and density functions are obtained for solving the minimization problem in the above-mentioned paper. These equations have possibility of great use in problems of obtaining extrema of functions involving non-central chi-squared (non-central F) distribution and density functions. A short note entitled "On Non-Central Chi-Squared and Non-Central F Distributions," by Dr. Kursheed Alan and myself, giving these equations has been accepted for publication in The American Statistician. This note is expected to appear in the October, 1967, issue. Five preprints of this note have already been sent to NASA along with the first semiannual progress report of this grant.

The work on numerical computations of the ranking integrals and the integral equations occurring in the above-mentioned paper is approaching completion. Suitable quadrature techniques for the evaluation of the ranking integrals involving non-central chi-squared (non-central F)

distribution and density functions have been employed in the preparation of the tables to be used in conjunction with the above-mentioned paper. The Principal Investigator and his two co-authors hope to finalize soon a write-up on this aspect of the problem and submit the same for possible publication in some appropriate journal.

Certain preliminary results were obtained for the nonparametric selection procedures for the problem of comparison of several univariate distributions (with unknown forms of distributions) with a control; these results were presented by the Principal Investigator at the International Congress of Mathematicians held at Moscow, U.S.S.R. during August 16-26, 1966. The Principal Investigator hopes to develop these nonparametric results further. Attempt will be made to formulate a coherent theory for nonparametric selection procedures for the univariate distributions. Some extensions to the multivariate case will also be studied. This work will be done in the next few months on the extension of the present grant expected from the National Aeronautics and Space Administration.