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Technical Report 32-1139

Functional Design Of The Mariner Midcourse Maneuver Operations Program

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Abstract

This document specifies the functions of the *Mariner* Midcourse Maneuver Operations Program. The program has two functions: (1) establish the existence of maneuvers satisfying spacecraft constraints, and (2) compute these maneuvers and code them into commands acceptable to the spacecraft. Besides the decision programs that must be used if the flight goes according to plan, the program is capable of exploring alternate trajectories if some out-of-tolerance condition forces a change in plans. A discussion of the engineering and operational considerations governing the design of the program is given, as well as the derivation of most of the less obvious equations.

Functional Design of the *Mariner* Midcourse Maneuver Operations Program

I. Introduction

A. Purpose and Nature of the Program

The primary mission of the *Mariner* spacecraft is to perform near-planet measurements. To ensure the success of this mission, it may be necessary to correct the initial trajectory of the spacecraft with a midcourse maneuver, nominally between two and ten days after launch. If necessary, the trajectory can be re-adjusted with a second maneuver at a later time.

The *Mariner* Midcourse Operations Program serves two primary purposes:

(1) Perform calculations which enable the operations personnel to select the most advantageous maneuver after the orbit achieved by the spacecraft is known. If the mission proceeds according to plan, the selection of the maneuver will be straightforward; otherwise, an appreciable quantity of trajectory and performance information must be rapidly analyzed under operational pressure to ensure that the capabilities of the spacecraft are intelligently utilized. The spacecraft can still gather significant scientific information even if it follows a trajectory different from nominal. Operations personnel must make appropriate guidance decisions as long as there is any chance of obtaining useful scientific or engineering information.

(2) Code the commands that will make the spacecraft perform the desired maneuver. The midcourse maneuver requires three 26-bit stored quantitative commands (QC), which are transmitted to the spacecraft and stored until the real-time direct command (DC) that initiates the sequence is sent.

In computing the maneuver, various practical constraints imposed by the limitations of the spacecraft may have to be considered. Accordingly, the program described consists of two subprograms: (1) a set of drivers for the trajectory program, which can be used to explore the different classes of trajectories available to the spacecraft after a maneuver, with specialized output relevant to the operational situation; and (2) decision routines, which also use the trajectory program, and which are used to compute and code the desired maneuvers. These subprograms are discussed in the following sections. The subprograms, written as separate loops compatible with the SFOF monitor system, may be linked together as required.

B. Sequence of Guidance Operations

The sequence of events for computing and executing the midcourse maneuver is as follows:

(1) The spacecraft is tracked from launch. After at least two days of tracking, a definitive orbit determination is made.

(2) The midcourse velocity impulse required to modify the trajectory of the spacecraft so that it flies by the target planet in an acceptable way and at a favorable time is computed. Certain constraints must be satisfied since the velocity impulse delivered cannot exceed the maximum value set by the amount of propellant available, and the terminal phase must occur in view of a DSIF station or its backup station with the special equipment that may be needed. This restricts the time at which the terminal phase can take place.

If a maneuver that modifies the best-fit orbit so that it passes through the desired aiming point with an acceptable time of flight cannot be found, a failure situation exists. If the spacecraft is operating properly and is following a trajectory that takes it sufficiently close to the target planet, an attempt will be made to determine a midcourse maneuver that will place the spacecraft on the most advantageous trajectory. The trajectory evaluation features of this Maneuver Operations Program enable the operations personnel to choose a revised aiming point. Once a suitable aiming point has been chosen, the midcourse maneuver is computed.

(3) The vector impulse is converted to the appropriate coordinates: pitch turn angle, roll turn angle, and magnitude of impulse. These values are then converted to the binary-coded form acceptable to the spacecraft.

(4) The three stored commands are transmitted, properly coded, to the appropriate DSIF station (Goldstone) where the command operator checks and sends them to the spacecraft through the Read-Write-Verify system. The spacecraft stores the commands in registers in the Central Computer and Sequencer (CC&S).

(5) The direct command, EXECUTE MIDCOURSE MANEUVER, is transmitted.

(6) The pitch and roll turns are executed by the spacecraft.

(7) The midcourse motor is ignited and burns for the required time as counted down by the CC&S.

(8) After completing the maneuver, the spacecraft returns to the cruise mode, orienting itself through Sun and Canopus sensors.

C. Considerations Determining the Midcourse Maneuvers

1. Experimental requirements. Many constraints and experimental requirements affect the choice of a best trajectory for a particular operational situation. The nominal aiming point is chosen to satisfy all the constraints in an optimum way. If it is not possible to reach the nominal aiming point, a revised aiming point must be chosen during the operation.

2. Engineering subsystem requirements. Several constraints are imposed by the various subsystems:

a. Attitude control. The following restrictions are imposed by the attitude control system:

(1) The angular separation between the near limb of the target planet and Canopus should be greater than some value, nominally 36°, until completion of the mission.

(2) The angular separation between the near limb of the target planet and the Sun should be greater than some value, nominally 1°, until the completion of the encounter sequence.

b. Communications. The following restrictions are imposed by communications requirements:

(1) The orientation of the spacecraft during the midcourse maneuver should permit the transmission of telemetry over the low-gain antenna.

(2) The high-gain antenna must be pointed toward the Earth during the encounter sequence while data taking is in progress.

(3) The midcourse maneuver must be performed while the spacecraft is over a DSIF station with command capability.

(4) The encounter sequence must take place during a pre-determined time interval.

c. Propulsion. The following restrictions are imposed by propulsion requirements:

(1) The midcourse impulse cannot exceed the maximum provided.

(2) Because of the orientation of the midcourse motor and the pitch-roll turn sequence, an inaccessible cone of thrust directions exists around the initial pitch axis (Appendix A).

II. General Program Description

A. Inputs

The input to the operations program can be conveniently divided into the following three classifications.

1. Input from the tracking program. The basic inputs from the tracking program are the best estimates of the six injection coordinates and the injection time. These inputs are presented in Table 1.

2. Prelaunch input. The prelaunch input generally consists of items which can be considered constants of the spacecraft system and thus not subject to variation during changes or unexpected operating conditions. The input items within this category are presented in Table 2.

3. Post-launch input. Items in the post-launch input fall within two classifications: (1) items which are subject to change during changes or unexpected operating conditions and (2) control commands for the program. The post-launch input items for all operational options are presented in Table 3.

The tracking program input and the post-launch input will be entered automatically during the operation through the SFOF monitor and the prelaunch input will be the nominal values unless changes have been made. The capability to change any or all prelaunch input items will be provided in the same manner as the post-launch inputs. The capability to enter all inputs will be provided for nonoperational use of the program, which is expected to be extensive before each flight.

B. Functional Blocks

Figure 1 presents a functional block diagram of the Maneuver Operations Program. The six independent blocks presented in Fig. 1 were programmed separately. These blocks, with the exception of Nos. 3, 4, and 5, will require the trajectory program as a subroutine.

Table 1. Input from the orbit determination program

Item	Description
1a	$\left.\begin{matrix} X \\ Y \\ Z \\ \dot{X} \\ \dot{Y} \\ \dot{Z} \end{matrix}\right\} =$ best estimate of Earth-centered space-fixed equatorial inertial coordinates at injection
1b	Best estimate of injection GMT
2	$N = U\Lambda U^T$ Where Λ is the 6×6 noise moment matrix of the tracking uncertainty of injection coordinates, and $U_{ij} = \dfrac{\partial M_i}{\partial q_j} \quad i = (1, 2, 3) \quad j = (1, 2, \cdots, 6)$ $M_1 = \mathbf{B} \cdot \mathbf{R}$ $M_2 = \mathbf{B} \cdot \mathbf{T}$ $M_3 =$ linearized time of flight $q_j =$ quantities listed under Item 1a Here, the following are defined as $\mathbf{S} =$ a unit vector in the direction of the approach asymptote $\mathbf{T} =$ a unit vector which is normal to $\mathbf{S}$ and parallel to the ecliptic plane $\mathbf{R} = \mathbf{S} \times \mathbf{T}$ $\mathbf{B} =$ vector to point in $\mathbf{R}, \mathbf{T}$ plane pierced by the incoming asymptote
3[a]	$\sigma_{B(OD)} = \sqrt{N_{11} + N_{22}}$
4	$\sigma_{S(OD)} = V_\infty \sqrt{N_{33}}$ Here $V_\infty =$ hyperbolic velocity excess $= \sqrt{\dfrac{\mu}{A}}$
5	$\sigma_{TL(OD)} = \sqrt{N_{33}}$
6a[b]	λ_1, the semi-major axis of the 40% dispersion ellipse on the $\mathbf{R}, \mathbf{T}$ plane $\lambda_1^2 = \dfrac{1}{2}(N_{11} + N_{22}) + \sqrt{\left(\dfrac{N_{11} - N_{22}}{2}\right)^2 + N_{12}^2}$
6b[b]	λ_2, the semi-minor axis of the 40% dispersion ellipse on the $\mathbf{R}, \mathbf{T}$ plane $\lambda_2^2 = \dfrac{1}{2}(N_{11} + N_{22}) - \sqrt{\left(\dfrac{N_{11} - N_{22}}{2}\right)^2 + N_{12}^2}$
6c	θ, the angle from the $\mathbf{T}$ axis to the major axis of the ellipse $\theta = -\dfrac{1}{2}\tan^{-1}\dfrac{2N_{22}}{N_{11} - N_{22}}$

[a] Items 3 through 6c are calculated in the Orbit Determination Program but are not required as inputs to the Midcourse Maneuver Operations Program. These items will be recomputed in the Midcourse Program from Item 2.

[b] The λ_1 and λ_2 used here should not be confused with the vectors in the critical plane which are denoted by the same symbol. The notation here is consistent with that used by the Orbit Determination Group.

Table 2. Prelaunch input

Item	Symbol	Description	Typical value	Units
1	ACA	Angle of low-gain antenna permissible cone	90	deg
2	BTOL	Tolerance in $\mathbf{B} \cdot \mathbf{R}$, $\mathbf{B} \cdot \mathbf{T}$ location (used in Midcourse Decision Program)	75.0	km
3	C	Speed of light	3×10^5	km/sec
4	$DT1i$	CC&S computation lag (to be determined) $i = P, R, V$	0	sec
5	$DT2i$	CC&S turn command resolution in seconds; $i = P, R$	1	sec
6	ITER	Number of iterations in search before K matrix is to be recomputed	1	none
7	IA	The initial angle between $\mathbf{T}$ and $\mathbf{V}$ for the capability ellipse generator	0	deg
8	PADC	A tolerance angle used in evaluating the Canopus-lock constraint	36	deg
9	PADS	A tolerance angle used in evaluating the sun-lock constraint	1	deg
10	RA	Radius of target planet	6100	km
11	σ_k^2	Variance of k^{th} type execution error, $k = 1, 2, 3, 4$ (shutoff, pointing, resolution, autopilot)	vary	vary
12	$\dot{\theta}_{ci}$	The spacecraft turn rate $(i = P, R)$	0.18	$\frac{\text{deg}}{\text{sec}}$
13	TP_{max}	Time allotted from start of pitch turn to start of roll turn	1320	sec
14	TR_{max}	Time allotted from start of roll turn to motor ignition	1320	sec
15	TCA_{max}	Latest allowed GMT time to closest approach	—	days, min, sec
16	TCA_{min}	Earliest allowed GMT time to closest approach	—	days, min, sec
17	TM	Time from receipt by the Central Computer and Sequencer of the EXECUTE MIDCOURSE MANEUVER command to ignition of the midcourse motor	6210 ± 30	sec
18	TT	Time delay from start of transmission of EXECUTE COMMAND to initiation by the CC&S of the midcourse maneuver sequence, assuming no light-time delay	38	sec
19	VCAP	Nominal velocity capability of the midcourse propulsion system	0.080	km/sec
20	NPR	Number of steps in search, Midcourse Decision Program, divergent exit if not converged	10	none
22	TTOL	Convergence criterion for time of flight in Decision Program	0.005	days
23	β	Angle from projection of Canopus sensor optic axis on pitch-yaw plane to pitch axis	−45	deg
24	TLAM	Time increment for computing $\dot{\mathbf{B}}$	900	sec
25	TSI	Angle between roll axis and motor nozzle axis	88.5	deg
26	γ	Angle between pitch axis and projection of motor nozzle axis on pitch-yaw plane	45	deg

Table 3. Post-launch input[a]

Item	Description
	I. Operational Option A—Midcourse Maneuver Program
1	$T1$ Time at which midcourse motor is to start
2	$\mathbf{B\cdot R}$, $\mathbf{B\cdot T}$, TCA The desired values of the components of the $\mathbf{B}$ vector in the $\mathbf{R}$, $\mathbf{T}$ plane and the time of closest approach
3	$PT_{ARB\text{-}max}$, $PT_{ARB\text{-}min}$ maximum and minimum arbitrarily desired pitch turns
	II. Operational Option B—Mission Capability Study
1	$T1$ Time at which midcourse motor must start
	III. Operational Option C—Spacecraft and Trajectory Constraint Evaluation
	No additional post-launch input is required

[a]Input from the Orbit Determination Program, see Table 1, is required for all options.

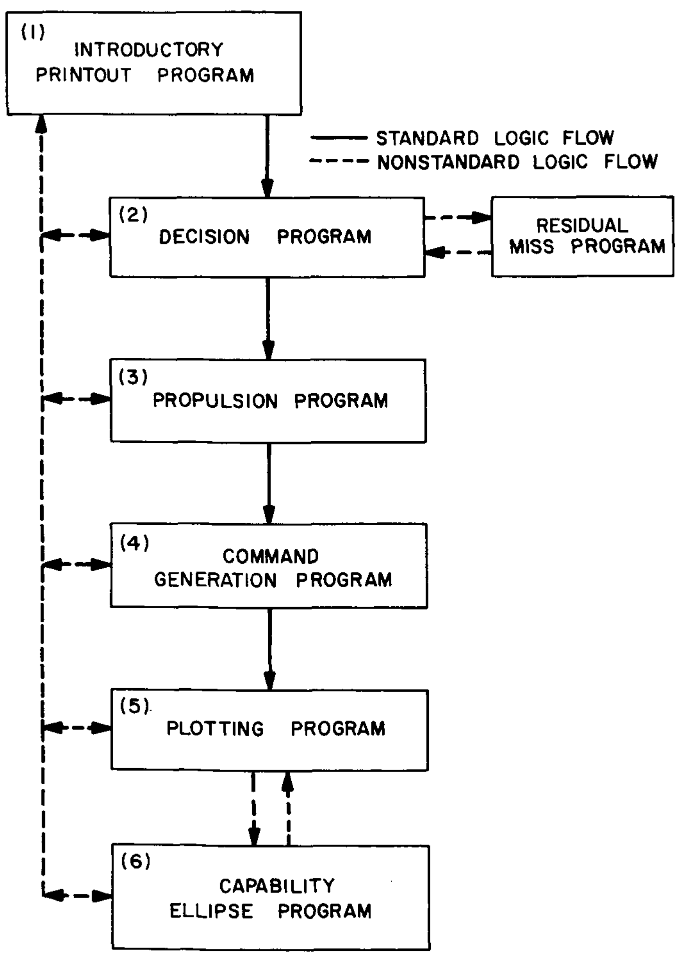

Fig. 1. Functional block diagram—maneuver operations program

Functional Block Nos. 1, 2, 3, and 4 are required for both standard operation (i.e., it is possible to place the spacecraft on a trajectory which passes near the nominal aiming point at a satisfactory time) and nonstandard operation (i.e., it is *not* possible to place the spacecraft on a trajectory which passes near the nominal aiming point at a satisfactory time.) Functional Block Nos. 5 and 6 were designed to assist in making operational decisions during nonstandard operating conditions. The following sections present a brief description of each operational block. Section III provides a detailed discussion of the program.

1. Midcourse Maneuver Program introductory printout. The information in the introductory printout describes the quality of the orbit determination and gives an initial estimate of the quality of the orbit obtained. Input quantities from the orbit determination program will be utilized to obtain an initial estimate of the miss distance and time-of-flight error between the best-fit trajectory and the nominal aiming point.

Table 4 provides information for the introductory printout.

Table 4. Output quantities—midcourse maneuver program introductory printout

Item	Description
1	All input quantities from the Orbit Determination Program will be printed out in the Midcourse Maneuver Program Introductory Printout (Table 1, Items 1 through 6)
2	The $\lvert \mathbf{B} \rvert$ and the $\mathbf{B\cdot R}$, $\mathbf{B\cdot T}$ components on the best-fit trajectory in km
3	The time of flight from injection to closest approach on the best-fit trajectory
4	The date, GMT, and PST of closest approach
5	All prelaunch input quantities (Table 2)

2. Midcourse Decision Program. The purpose of the Midcourse Decision Program is to calculate the required velocity increment subject to various constraints. The constraints which are evaluated are (1) propulsion, (2) time-of-flight, and (3) low-gain antenna pattern. The Propulsion and the time-of-flight constraints are absolute. If the location requested cannot be reached (Ref. 1) without violating an absolute constraint, then the Midcourse Decision Program will print a simple statement to this effect and proceed with the Mission Capability Study. If it is possible to reach the location requested without violating an absolute constraint, the output of the Midcourse Decision Program will be fed directly to the Midcourse Command Generation Program. If the midcourse maneuver violates the low-gain antenna constraint, some of the telemetry data may be lost.

The required output from the Midcourse Decision Program is presented in Table 5. The output falls within three groups. The first group specifies the trajectory conditions (i.e., position, velocity, and time) at which the midcourse rocket motor ignites, and the desired aiming point. The second group consists of a set of brief statements which will be printed out if called for by the decision sequence which formulates the maneuver finally chosen, and possible alternate maneuvers. The third group contains detailed numerical information related to the test for the antenna constraint, and to the Earth-Sun-spacecraft geometry.

Table 5. Output quantities—midcourse decision program

Item	Description	Units
	Group I	
1	Aiming point: (Post-Launch Input) desired $\mathbf{B} \cdot \mathbf{R}$, $\mathbf{B} \cdot \mathbf{T}$, TCA	km, date and time
2	The time past injection of the ignition time of the midcourse rocket motor ($T1$)	days
3	The X, Y, Z position coordinates and $\dot{X}$, $\dot{Y}$, $\dot{Z}$ velocity components in an Earth-centered, space-fixed, inertial coordinate system at time $T1$ (from space program, based on orbit determination input)	km, km/sec
4	The components and magnitude of the unmodified velocity vector $\mathbf{V}$ (also called the "ideal maneuver")	km/sec
5	The components and magnitude of the critical plane component vector $\mathbf{VCP}$	km/sec
6	The components and magnitude of the non-critical direction component vector $(\mathbf{V} \cdot \mathbf{N})\mathbf{N}$	km/sec
7	The components and magnitude of the no-pitch maneuver $\mathbf{VNP}$	km/sec
8	The required pitch and roll turns to make each of the maneuvers	deg
9	The resulting flight-time change, $DTL = \mathbf{VM} \cdot \boldsymbol{\lambda}_3$	days
10	$VMV = \cos^{-1}\left(\frac{VM \cdot \mathbf{V}_{sc}}{VM\ V_{sc}}\right) =$ the angle between initial velocity vector and the maneuver velocity	

Table 5 (contd)

Item	Description	Units
	Group II[a]	
	The appropriate messages specified in the flow sequence. There are twelve such messages:	
1	Available propellant insufficient to null miss components; enter mission capability study sequence	
2	Time-of-flight constraint cannot be satisfied with available propellant; enter mission capability study sequence	
3	Maneuver has been modified to allow for constraints	
4	Modified maneuver fails to satisfy absolute constra'nts. Unmodified maneuver used ignoring antenna constraint	
5	Maneuver satisfies all constraints	
6	Maneuver has been modified from inaccessible cone	
7	Alternate turns used	
8	Maneuver has been modified to adjust flight time for propulsion constraint	
9	Maneuver violates antenna constraint	
10	Modified maneuver still fails antenna constraint	
11	No VNP available	
12	Modified maneuver fails to satisfy absolute constraints. Thermal constraint cannot be satisfied	
	Group III	
1	$VNMX$ Maximum noncritical direction velocity magnitude	km/sec
2	$VNTL$ Minimum noncritical velocity component for flight-time constraint	km/sec
3	$VNTU$ Maximum noncritical velocity component for flight-time constraint	km/sec
4	$L1$, $L2$ Intervals which satisfy flight-time and propulsion constraints	km/sec
5	$P1$, $P2$ Pitch turn limits at which antenna constraint is violated	deg
6	The angle $LGE = \cos^{-1} [\mathbf{E} \cdot \mathbf{M1}(PT)]$ between the low-gain antenna and the Earth-probe line for the unmodified velocity vector $\mathbf{V}$	deg
7	The angle $LGS = \cos^{-1} [\mathbf{H} \cdot \mathbf{M1}(PT)]$ between the low-gain antenna and the Sun-probe line for the unmodified velocity vector $\mathbf{V}$	deg

[a] Each message shall be followed by a printout of the components of the maneuver involved as well as the corresponding pitch and roll turns.

Table 5 (contd)

Item	Description	Units
8	The Earth-probe-Sun angle *EPS*	deg
9	The antenna constraint angle ACA	deg
10	The angle OCP $= 90° - \cos^{-1} (\mathbf{V} \cdot \mathbf{N}/V)$ between the unmodified maneuver and the critical plane	deg
11	The probe's geocentric latitude (*PLAT*), longitude (*PLON*), and the distance from the center of the Earth in miles (*PDIST*) at the time *T*1	
12	The direction cosines of $\mathbf{E}$ and $\mathbf{H}$	
13	The angle between the spacecraft velocity vector at time *T*1 and the Earth-probe line $VEP = \cos^{-1}\left(\frac{\mathbf{V}_{sc} \cdot (-\mathbf{E})}{\lvert \mathbf{V}_{sc} \rvert}\right)$ where $\mathbf{V}^T_{sc} = (\dot{X}, \dot{Y}, \dot{Z})$ defined in Group 1, Item 2	deg
14	The angle between the spacecraft velocity vector at time *T*1 and the Sun-probe line $VSP = \cos^{-1}\left(\frac{\mathbf{V}_{sc} \cdot \mathbf{H}}{\lvert \mathbf{V}_{sc} \rvert}\right)$	deg
15	The angle between the spacecraft velocity vector at time *T*1 and the noncritical direction $VNC = \cos^{-1}\left(\frac{\mathbf{V}_{sc} \cdot \mathbf{N}}{\lvert \mathbf{V}_{sc} \rvert}\right)$	deg
16	The values of $\mathbf{B} \cdot \mathbf{R}$, $\mathbf{B} \cdot \mathbf{T}$, and *TCA* from last converged trajectory	

3. Propulsion Program. This program accepts required velocity magnitude and pertinent spacecraft parameters as input to compute motor burn time (Ref. 3).

4. Midcourse Command Generation Program. The Command Generation Program converts the velocity vector correction determined in the Midcourse Decision Program into three commands expressed in binary form and adjusted to a form the CC&S can use. The pitch and roll turn times are computed by dividing the required turns by the turn rates. If the spacecraft had an integrating accelerometer loop, only the velocity magnitude would be required to compute burn time instead of the separate program listed above. The commands will be transmitted via teletype tape machine. The midcourse Command Generation Program will also calculate a noise moment matrix of midcourse execution errors and accomplish a complete trajectory error analysis. The items required on the program printout are presented in Table 6. The Midcourse Command Generation Program may be followed by the Plotting Program.

Table 6. Output quantities—midcourse command generation program

Item	Description	Units
	Group I: Midcourse Guidance Information	
1	The *X*, *Y*, *Z* position coordinates and $\dot{X}$, $\dot{Y}$, $\dot{Z}$ velocity components in Earth-centered, space-fixed, inertial coordinates at the end of the maneuver (*T*1 + *TVM*)	km/sec
2	The time past injection of the ignition time of the midcourse rocket motor (*T*1)	days
3	The three components of the vector maneuver *VM* to nearest hundredth,[a] in true geocentric equatorial coordinates	km/sec
4	The pitch turn angle, to nearest hundredth[a]	deg
5	The roll turn angle, to nearest hundredth[a]	deg
6	The magnitude of the maneuver, *VM*, to nearest hundredth[a]	km/sec
7	The quantitative commands: QC1-1, QC1-2, QC1-3 (1) Decimal fraction: sign and magnitude (2) 26-bit binary commands (1 start bit, 7-bit decoder address, 5-bit CC&S address, 12 data bits, 1-bit polarity) (3) The binary commands in octal; octal representation should check with that on Read-Write-Verify console at Goldstone (4) *TP*, *TR*, and *TVM* (5) English language message to be transmitted	 sec
8	ADVMT to three decimals (1) The direction cosines of $\mathbf{VM}$ in spacecraft body-fixed coordinates ($\mathbf{I}, \mathbf{J}, \mathbf{K}$) (2) The direction cosines of $\mathbf{VMT}$ in spacecraft body-fixed coordinates ($\mathbf{I}, \mathbf{J}, \mathbf{K}$)	 deg
9	The GMT of the following events to 0.01 sec (1) Transmit the EXECUTE COMMAND, *TX* (2) Begin pitch turn (3) End pitch turn (4) Begin roll turn (5) End roll turn (6) Ignite motor (7) Cut off motor	hours, min, sec
10	Rate of change of the miss vector $\mathbf{B}$ if maneuver impulse is held fixed, but execution is delayed	km/sec
11	The 3 × 3 moment matrix *N* of miss caused by orbit-determination errors, already printed in the introductory printout	km, days
12	The 3 × 3 moment matrix N_g of miss caused by midcourse guidance component errors $N_g = K(LX)K^T$ where the *LX* matrix is defined in Appendix B	km, days

Table 6 (contd)

Item	Description	Units
13	The sum of the moment matrices N (miss from orbit determination) and N_g (miss from guidance error)	km, days
14	The same auxiliary quantities depending on the elements of the moment matrices N_g and $N + N_g$ that were given in the Introductory Printout (Table 4)	
	(1) $(N_{11}+N_{22})^{1/2}$, $(N_{g11}+N_{g22})^{1/2}$, $[(N+N_g)_{11} + (N+N_g)_{22}]^{1/2}$	km
	(2) $(N_{33})^{1/2}$, $(N_{g33})^{1/2}$, $[(N+N_g)_{33}]^{1/2}$	days
	(3) $V_\infty (N_{33})^{1/2}$, $V_\infty (N_{g33})^{1/2}$, $V_\infty [(N+N_g)_{33}]^{1/2}$	km
	(4)[b] $\lambda_1(N)$, $\lambda_1(N_g)$, $\lambda_1(N+N_g)$	km
	(5)[b] $\lambda_2(N)$, $\lambda_2(N_g)$, $\lambda_2(N+N_g)$	km
	(6)[b] $\theta(N)$, $\theta(N_g)$, $\theta(N+N_g)$	deg
	These quantities are printed in three parallel columns. The first column, containing the quantities depending on the elements of the moment matrix N of miss due to orbit determination errors, was printed in the Introductory Printout also	
15	Probability of spacecraft impacting the target planet	
16	Change in range rate during maneuver, $\Delta \dot{r}$	
17	Tabulated output quantities of cone and clock angles of the Sun and the Earth every DT sec during turns	
	Group II: Midcourse Injection Information for the Orbit Determination Group	
1	*TMI*: GMT of Midcourse Injection	hours, min, sec
2	VM	km/sec
3	*LX*	km/sec
	The information in Group II will be saved for use in the orbit determination programs on request	

[a] These quantities are also printed as output from the Midcourse Decision Program.

[b] Parenthesis imply functional relationship, e.g., λ_1 is a function of elements of matrix N.

5. *Plotting Program.* A set of dispersion ellipses will be computed from the covariance matrix of errors in the orbit, and the execution of the maneuver will be mapped to the target. These ellipses will be plotted to the same scale that is used for the plots of the general constraints. This program is used primarily as an aid in selecting new aiming points under nonstandard conditions.

6. *Capability Ellipse Program.* The Capability Ellipse Program will generate the maximum capability ellipse in the R, T plane, assuming that maximum maneuvers are applied in the critical plane. This ellipse may also be plotted by the Plotting Program.

C. Operation of the Program

The functional blocks described in Section II-B are combined to form the following three operational options:

(1) Operational Option A — Midcourse Maneuver Program.

(2) Operational Option B — Midcourse Capability Study.

(3) Operational Option C — Spacecraft and Trajectory Constraint Evaluation.

Generally, all operational options will be used before selecting the midcourse.

All three operational options require access to the prelaunch input, presented in Table 2, and the inputs from the tracking program (Table 1). Each option requires the trajectory program. The post-launch input is specified by options in Table 3.

In Fig. 1, the dashed lines represent nonstandard flow, and the solid lines indicate programmed connection of the functional blocks that are used in the various operational options. The starting point for each of the various operational options is always the Introductory Printout Program.

Operational Option A, the Midcourse Maneuver Program (Table 3) is the only option that has the capability of computing a maneuver. The post-launch input (Table 3) provides considerable flexibility. The following alternatives are provided:

(1) Specify the nominal aiming point or an alternate aiming point in **B • R**, **B • T** coordinates and time of closest approach.

(2) Proceed with the Mission Capability Study, if necessary, or do not proceed with the Mission Capability Study.

(3) Attitude Control normal or Canopus sensor locked on some other body (such as Achenar, or the Earth).

(4) Plot or do not plot dispersion ellipses.

Operational Option B, the Mission Capability Study, will be used to evaluate the effects of the various spacecraft and trajectory constraints on trajectories which are within the propulsion capability of the system. The output from the Capability Ellipse Generator is fed directly into the Plotting Program. These two programs constitute the Mission Capability Study.

Operational Option C, Spacecraft and Trajectory Constraint Evaluation, is used to evaluate the suitability of various areas in the R, T plane as potential aiming points.

Section III presents a detailed discussion of the program links.

III. Discussion of Program Links

A. Midcourse Decision Program

1. Introduction. The Midcourse Decision Program has two primary functions:

(1) To determine whether there is a midcourse maneuver that satisfies all of the spacecraft constraints and still results in a trajectory with the desired miss vector **B**, and an acceptable time of flight.

(2) To choose an optimum maneuver if such maneuvers exist. Once the maneuver has been determined, the program exits to the Command Generation Program. If there is no acceptable maneuver, a statement to this effect is printed out and a Mission Capability Study initiated.

The decision sequence operates roughly as follows: First compute, by a homing scheme, the exact maneuver required to obtain the desired miss vector and the desired time of flight. Using a linear approximation, estimate the magnitude of the maneuver required to correct miss only; if this maneuver requires more propellant than is provided, enter the Mission Capability Study sequence. Similarly, if the propellant provided is insufficient to correct the miss and still yield an *acceptable* absolute time of flight, print a statement to this effect, compute the residual miss associated with an acceptable flight time, and enter the Mission Capability Study sequence. If the maneuver satisfies the two absolute constraints, test for violation of the constraint imposed by the presence of nulls in the pattern of the low-gain antenna; namely, that the maneuver must be made so as to avoid losing telemetry data during the maneuver. If the maneuver does not pass this test, search for a maneuver that does not lose telemetry communication and minimizes the time-of-flight error that is introduced. A maneuver that results in the desired terminal conditions can always be found if enough propellant is available; however, care must be taken to ensure that the antenna constraint will not be violated during the pitch turn. Violation of this constraint may mean the loss of phase lock in the communications loop. The time needed to re-acquire phase lock once the signal strength is restored is indeterminate, and an unpredictable amount of data may be lost.

2. Calculation of unmodified maneuver. For operational convenience, the time at which the midcourse motor is to be ignited, $T1$, will be specified in the postlaunch input. This time, $T1$, is related to the time at which the midcourse maneuver execute command is transmitted, TX, by the following equation:

$$TX = T1 - TM - TT - \frac{ALT}{C} \tag{1}$$

Where TM is the time from receipt of the execute signal by the Central Computer and Sequencer aboard the spacecraft to ignition of the midcourse rocket motor. TT is the time delay from start of the command transmission to initiation of the midcourse maneuver sequence, but contains no light-time correction (i.e., from start of receipt by CC&S). The ALT/C term is for light-time correction. C, TM, and TT are given in the prelaunch input.

The Orbit Determination Program will provide a best estimate of the six injection coordinates and the injection time. These coordinates will specify the best-fit trajectory. The desired aiming point will be specified in the postlaunch input as a $\mathbf{B}\cdot\mathbf{R}$, $\mathbf{B}\cdot\mathbf{T}$ coordinate. The desired GMT of arrival time will also be specified. The initial conditions from the Orbit Determination Program are integrated to closest approach. The associated linearized flight time is then adjusted by the difference between time of closest approach and desired time of closest approach.[1] The adjusted linearized flight time is used as the desired linearized flight time, TL. The same adjustment is used to compute TL_{max} and TL_{min} from TCA_{max} and TCA_{min}.

[1] In practice, a bias is usually introduced by the approximation that $\Delta T_L = \Delta T_F$ (typically, for a planetary flyby mission, these times may differ as much as 6 hr). The maneuver analyst will observe this bias and compensate by introducing a bias of opposite sign in the desired linearized flight time, T_L.

The premaneuver miss is then

$$\Delta \mathbf{M} = \begin{bmatrix} \mathbf{B}\cdot\mathbf{R}\,(\text{O.D.}) - \mathbf{B}\cdot\mathbf{R}\,(\text{desired}) \\ \mathbf{B}\cdot\mathbf{T}\,(\text{O.D.}) - \mathbf{B}\cdot\mathbf{T}\,(\text{desired}) \\ T_L\,(\text{O.D.}) \quad - T_L\,(\text{desired}) \end{bmatrix} \tag{2}$$

The ideal midcourse maneuver, **V**, will be defined as that midcourse velocity increment which, when added to the best-fit trajectory at time $T1$, will modify it so that it satisfies the specified terminal conditions. This ideal **V** must be calculated with an iterative procedure.

With the desired terminal location specified in $\mathbf{B}\cdot\mathbf{R}$, $\mathbf{B}\cdot\mathbf{T}$ and T_L coordinates, search these coordinates until an accuracy of BTOL in position and TTOL in time is obtained. The detailed equations and procedure are given in the flow sequence (Section IV). The geometry is shown in Fig. 2.

An analysis of the errors caused by ignoring the position translation during the rocket motor burning period shows that the errors from this source are negligible compared to other errors in the system. Therefore, the position translation effect can be ignored during the computation and any effects checked after choosing a maneuver.

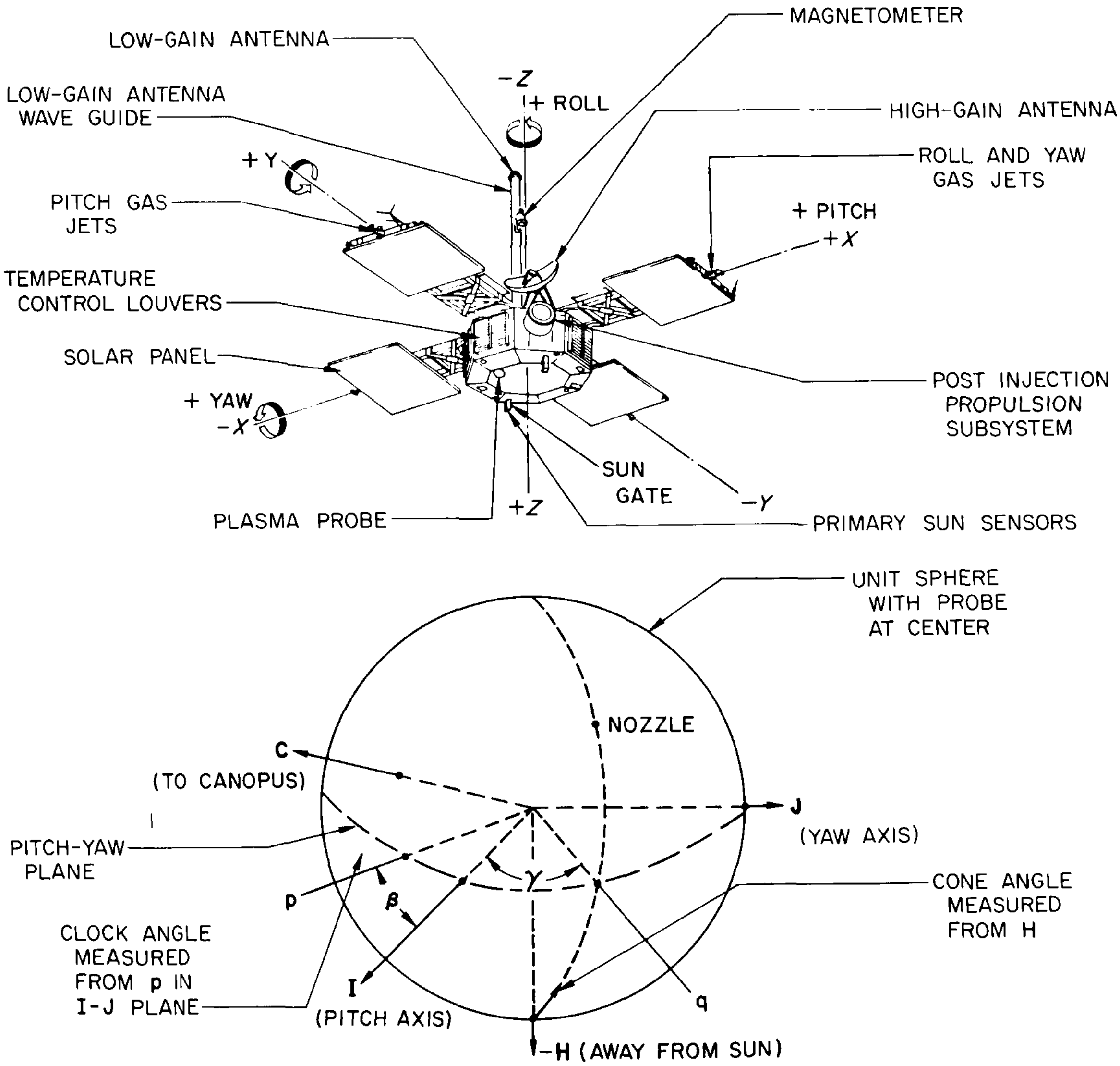

Fig. 2. Spacecraft coordinate system

3. Modification of the ideal or unmodified maneuver for propulsion, time of flight, and antenna constraint. The unmodified maneuver, **V**, described above, is chosen to null the miss and the time-of-flight variation. However, the magnitude of the unmodified maneuver may exceed the available propulsion capability *VCAP*. The presence of nulls in the pattern of the low-gain antenna used for telemetry transmission during the midcourse maneuver may also impose a constraint: namely, that the angle between the negative direction[2] of the spacecraft roll axis ($-z$) and the probe to Earth line shall not be greater than a specified value *at any time* during the pitch turn, or during the burning period. This limiting angle, denoted by *ACA*, is nominally 90 deg for *Mariner*. Its value depends on the performance of the spacecraft communication system.

In the absence of other constraints, it is always possible to choose a maneuver that nulls errors in the miss components ($\mathbf{B} \cdot \mathbf{R}$, $\mathbf{B} \cdot \mathbf{T}$) and satisfies the antenna constraint during the burning period at the cost of some time-of-flight variation and propellant.

The maneuver calculated according to this procedure may require more impulse than is provided, or may introduce an unacceptable time-of-flight variation which could cause failure of the mission through loss of communications during the terminal phase. These conditions will cause a mission capability study to be initiated and an alternate aiming point to be chosen. The maneuver may violate the antenna constraint during the pitch turn, in which case it will be modified, consistent with the absolute constraints. If this cannot be done, the antenna constraint may be violated.

Since the spacecraft should be in the proper attitude for communication with the Earth via the low-gain antenna before the midcourse sequence is started, satisfaction of the antenna constraint should be guaranteed if a maneuver were made with no-pitch turn, i.e., a roll turn only. Such a no-pitch maneuver having the proper critical plane component, if it exists, will be calculated and printed out, as well as the minimum maneuver (critical plane maneuver) to correct the miss distance only. These maneuvers correct the spatial miss components but do result in flight-time errors.

A procedure has been devised which constructs the maneuver to be applied (denoted by **VM**) according to the following rules:

(1) If the ideal maneuver satisfies all the constraints, apply that maneuver.

(2) If the miss can be corrected, and a modification of the ideal maneuver exists which satisfies the antenna constraint without violating the time-of-flight or propulsion constraints, modify the maneuver to correct the miss and satisfy the antenna constraint, and, if a choice exists, to minimize the time-of-flight variation subject to the propulsion constraint.

(3) If the miss components can be corrected, but the time of flight is unacceptable (unlikely situation), the operation is nonstandard; enter the Mission Capability Study sequence through the residual miss subroutine.

(4) If the miss components cannot be corrected without violating the propulsion constraint, the operation is nonstandard; enter the Mission Capability Study sequence.

The decision sequence to be described depends upon linearization of the miss components and the time of flight. For purposes of the decision sequence, the three quantities to be corrected are $\mathbf{B} \cdot \mathbf{R}$, $\mathbf{B} \cdot \mathbf{T}$, and the linearized time of flight T_L. The constraint situation is evaluated using the linear approximation; if a finite modification of the maneuver is required, higher-order errors introduced in the desired terminal conditions are removed by an iterative procedure. During successive steps of the iterations, the virtual aiming point is biased negatively by the amount of the miss computed in the last iteration. The test for the propulsion constraint is applied at each iteration. A count of the number of iterations is kept.

The mathematical basis of the decision sequence will now be described. If it is assumed that the dependence of the change $\Delta\mathbf{M}$ in the three-component miss vector[3] $\mathbf{M}^T = (\mathbf{B} \cdot \mathbf{R}, \mathbf{B} \cdot \mathbf{T}, T_L)$ upon the midcourse velocity impulse vector **VM** can be approximated by a linear relation.

$$\Delta\mathbf{M} = K\,\mathbf{VM}$$

(this implies that a change in velocity **VM** introduces a change in the miss components of $\Delta\mathbf{M}$).

[2]For the *Mariner* spacecraft, this corresponds to the positive direction of the antenna axis (defined as **M1** Eq. 19).

[3]The superscript T indicates the transpose of a vector or matrix, i.e., $\binom{a}{b}^T = (a, b)$.

Where

$$K = \begin{bmatrix} \frac{\partial \mathbf{B}\cdot\mathbf{R}}{\partial \dot{X}}, & \frac{\partial \mathbf{B}\cdot\mathbf{R}}{\partial \dot{Y}}, & \frac{\partial \mathbf{B}\cdot\mathbf{R}}{\partial \dot{Z}} \\ \frac{\partial \mathbf{B}\cdot\mathbf{T}}{\partial \dot{X}}, & \frac{\partial \mathbf{B}\cdot\mathbf{T}}{\partial \dot{Y}}, & \frac{\partial \mathbf{B}\cdot\mathbf{T}}{\partial \dot{Z}} \\ \frac{\partial T_L}{\partial \dot{X}}, & \frac{\partial T_L}{\partial \dot{Y}}, & \frac{\partial T_L}{\partial \dot{Z}} \end{bmatrix} \quad \mathbf{VM} = \begin{pmatrix} \Delta\dot{X} \\ \Delta\dot{Y} \\ \Delta\dot{Z} \end{pmatrix} \tag{3}$$

It will prove convenient to consider the rows of the K matrix as transposed gradient vectors in a velocity space as follows:

$$\boldsymbol{\lambda}_1^T = \nabla_v (\mathbf{B}\cdot\mathbf{R})^T = \left(\frac{\partial \mathbf{B}\cdot\mathbf{R}}{\partial \dot{X}}, \frac{\partial \mathbf{B}\cdot\mathbf{R}}{\partial \dot{Y}}, \frac{\partial \mathbf{B}\cdot\mathbf{R}}{\partial \dot{Z}}\right) \tag{4}$$

$$\boldsymbol{\lambda}_2^T = \nabla_v (\mathbf{B}\cdot\mathbf{T})^T = \left(\frac{\partial \mathbf{B}\cdot\mathbf{T}}{\partial \dot{X}}, \frac{\partial \mathbf{B}\cdot\mathbf{T}}{\partial \dot{Y}}, \frac{\partial \mathbf{B}\cdot\mathbf{T}}{\partial \dot{Z}}\right) \tag{5}$$

$$\boldsymbol{\lambda}_3^T = \nabla_v (T_L)^T = \left(\frac{\partial T_L}{\partial \dot{X}}, \frac{\partial T_L}{\partial \dot{Y}}, \frac{\partial T_L}{\partial \dot{Z}}\right) \tag{6}$$

The change in a given miss component produced by a vector maneuver **VM** is

$$\Delta M_i = \boldsymbol{\lambda}_i \cdot \mathbf{VM} \tag{7}$$

so that the first-order change is maximal if **VM** is parallel to λ_i, and zero if **VM** is perpendicular to $\boldsymbol{\lambda}_i$. It follows that a maneuver **VM** which lies in the direction

$$\boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2$$

produces no first-order change in the miss components $\mathbf{B}\cdot\mathbf{R}$, $\mathbf{B}\cdot\mathbf{T}$. This direction is noncritical and the plane of the vectors $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ is critical. The unit normal **N** to the critical plane is defined as follows:

$$\mathbf{N} = \frac{\boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2}{|\boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2|} \quad \text{if } \boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2 \cdot \boldsymbol{\lambda}_3 \geq 0 \tag{8}$$

$$\mathbf{N} = -\frac{\boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2}{|\boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2|} \quad \text{if } \boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2 \cdot \boldsymbol{\lambda}_3 < 0 \tag{9}$$

It should be noted that $\boldsymbol{\lambda}_1 \times \boldsymbol{\lambda}_2 \cdot \boldsymbol{\lambda}_3$ is the determinant of the matrix $\mathbf{K}$. The choice of sign ensures that a maneuver in the $+\mathbf{N}$ direction causes an increase in the time of flight, a convenient choice for later purposes.

After the maneuver **V** has been computed the component in the critical plane, **VCP**, can then be computed as follows:

$$\mathbf{VCP} = \mathbf{V} - (\mathbf{V}\cdot\mathbf{N})\,\mathbf{N} \tag{10}$$

If the component of the unmodified maneuver in the critical plane is held constant, the component in the noncritical direction, $\mathbf{VM}\cdot\mathbf{N}$, can be adjusted to satisfy the various constraints. The noncritical velocity components, as well as the quantities $\mathbf{VM}\cdot\mathbf{N}$ and $\mathbf{VM}\cdot\lambda_3$, will be printed out for each velocity vector examined by the program. The decision is reduced to the problem of determining the existence and, if it exists, the value of the single scalar parameter $\mathbf{VM}\cdot\mathbf{N}$ such that the resultant maneuver $\mathbf{VM} = \mathbf{VCP} + (\mathbf{VM}\cdot\mathbf{N})\,\mathbf{N}$ satisfies the constraints.

The propulsion and time-of-flight constraints produce absolute limits on the allowable value of $\mathbf{VM}\cdot\mathbf{N}$. To satisfy the propulsion constraint, with a velocity capability of $VCAP$, the following must be true:

$$VCAP \geq VCP \tag{11}$$

$$-VNMX \leq \mathbf{VM}\cdot\mathbf{N} \leq +VNMX \tag{12}$$

where

$$VNMX = [(VCAP)^2 - (VCP)^2]^{1/2} \tag{12a}$$

A mission capability study is initiated if Eq. (11) is not satisfied or if it is satisfied and Eq. (12) cannot be satisfied while simultaneously satisfying the time-of-flight constraint.

To have an accessible cone maneuver, the following should be tested:

$$|\,\mathbf{I}\cdot\mathbf{VM}/VM\,| \leq \sin\xi \tag{13}$$

where **I** is a unit vector in the direction of the pitch axis before turning, and ξ is the angle between the roll axis and the motor nozzle axis. If this inequality is not satisfied, the maneuver must be modified.

Similarly, if the earliest allowable linearized time of flight, TL_{min}, and the latest allowable linearized time of flight, TL_{max}, represent the specified limits on the time of closest approach, the following is required

$$VNTL \leq \mathbf{VM} \cdot \mathbf{N} \leq VNTU \tag{14}$$

where

$$VNTL = \mathbf{V} \cdot \mathbf{N} - \frac{TL - TL_{min}}{\boldsymbol{\lambda}_3 \cdot \mathbf{N}} \tag{14a}$$

$$VNTU = \mathbf{V} \cdot \mathbf{N} + \frac{TL_{max} - TL}{\boldsymbol{\lambda}_3 \cdot \mathbf{N}} \tag{14b}$$

if TL is the linearized time of flight corresponding to the desired time of closest approach. The choice of sign for the normal-to-critical plane specified earlier guarantees the proper ordering of the inequality in Eq. (14).

Define the quantities L_1 and L_2:

$$L_1 = min\,(+\,VNMX, VNTU) \tag{15}$$

$$L_2 = max\,(-\,VNMX, VNTL) \tag{16}$$

(1) If $L_1 \geq L_2$, both propulsion and flight-time constraints can be satisfied.

(2) If $L_1 < L_2$, enter mission capability study.

As a result of each constraint, propulsion and time, there is an interval of values of $\mathbf{VM} \cdot \mathbf{N}$ which satisfies those constraints. The values of $\mathbf{VM} \cdot \mathbf{N}$ chosen must fall in the intersection of the two intervals of values which satisfy the propulsion constraint and the time constraint.

If a finite interval of allowed values of $\mathbf{VM} \cdot \mathbf{N}$ exists, set $\mathbf{VM} = \mathbf{V}$ if $\mathbf{V} \cdot \mathbf{N}$ is in that interval. If $\mathbf{V} \cdot \mathbf{N}$ is not in that interval (i.e., $V > VCAP$), choose $\mathbf{VM}$ at the closest end of that interval:

$$\mathbf{VM} = \mathbf{VCP} + \frac{VNMX\,(\mathbf{V} \cdot \mathbf{N})\,\mathbf{N}}{|\mathbf{V} \cdot \mathbf{N}|} \tag{17}$$

This satisfies the propulsion and flight-time constraints. If the intersection is empty, there are no maneuvers which satisfy both constraints. In this event, the minimum residual miss that would result if the flight-time constraint were satisfied must be computed (Appendix C) and the mission capability study sequence initiated.

The antenna constraint is tested next. If $\mathbf{E}$ is the unit probe-Earth vector and $\mathbf{M1}$ the unit vector along the negative roll axis, the maneuver must satisfy the following condition:

$$\mathbf{E} \cdot \mathbf{M1} \geq \cos ACA \tag{18}$$

Moreover, the roll axis of the spacecraft should not enter the null cone during the pitch turn, when the roll axis turns from the probe-Sun line, as shown in Fig. 3. Let $\mathbf{M1}\ (t)$ be a unit vector in the instantaneous direction of the axis of symmetry of the low-gain antenna pattern. Set up the following unit vectors (Fig. 2):

$\mathbf{H}$ = unit probe-to-Sun vector

$\mathbf{C}$ = unit probe-to-star vector with nominal values for star being Canopus

$$\mathbf{q} = \frac{\mathbf{C} \times \mathbf{H}}{|\mathbf{C} \times \mathbf{H}|}$$

$$\mathbf{p} = \mathbf{H} \times \mathbf{q}$$

$\mathbf{I} = \mathbf{q} \sin \beta + \mathbf{p} \cos \beta$ = a unit vector in the direction of the pitch axis before turning, where β is a prelaunch input, nominally equal to -45 deg

$\mathbf{J} = \mathbf{K} \times \mathbf{I}$ = unit vector in direction of the yaw axis before turning

Then

$$\mathbf{M1}[P(t)] = \mathbf{J} \sin P(t) - \mathbf{K} \cos P(t) \tag{19}$$

where $P(t) = \dot{\theta}_{cp} t$, the instantaneous pitch angle. The pitch rate $\dot{\theta}_{cp}$ is nominally ± 0.18 deg/sec.

The motor is mounted as shown in Fig. 2. To attain the required midcourse velocity, the unit vector along the motor axis, $\mathbf{T}$, must be pointed so that

$$\mathbf{T} = -\frac{\mathbf{VM}}{VM} \tag{20}$$

The motor axis makes an angle, ξ, nominally 88.5 deg with the roll axis, and an angle, γ, nominally 45 deg with the pitch axis (both values are prelaunch input quantities), and

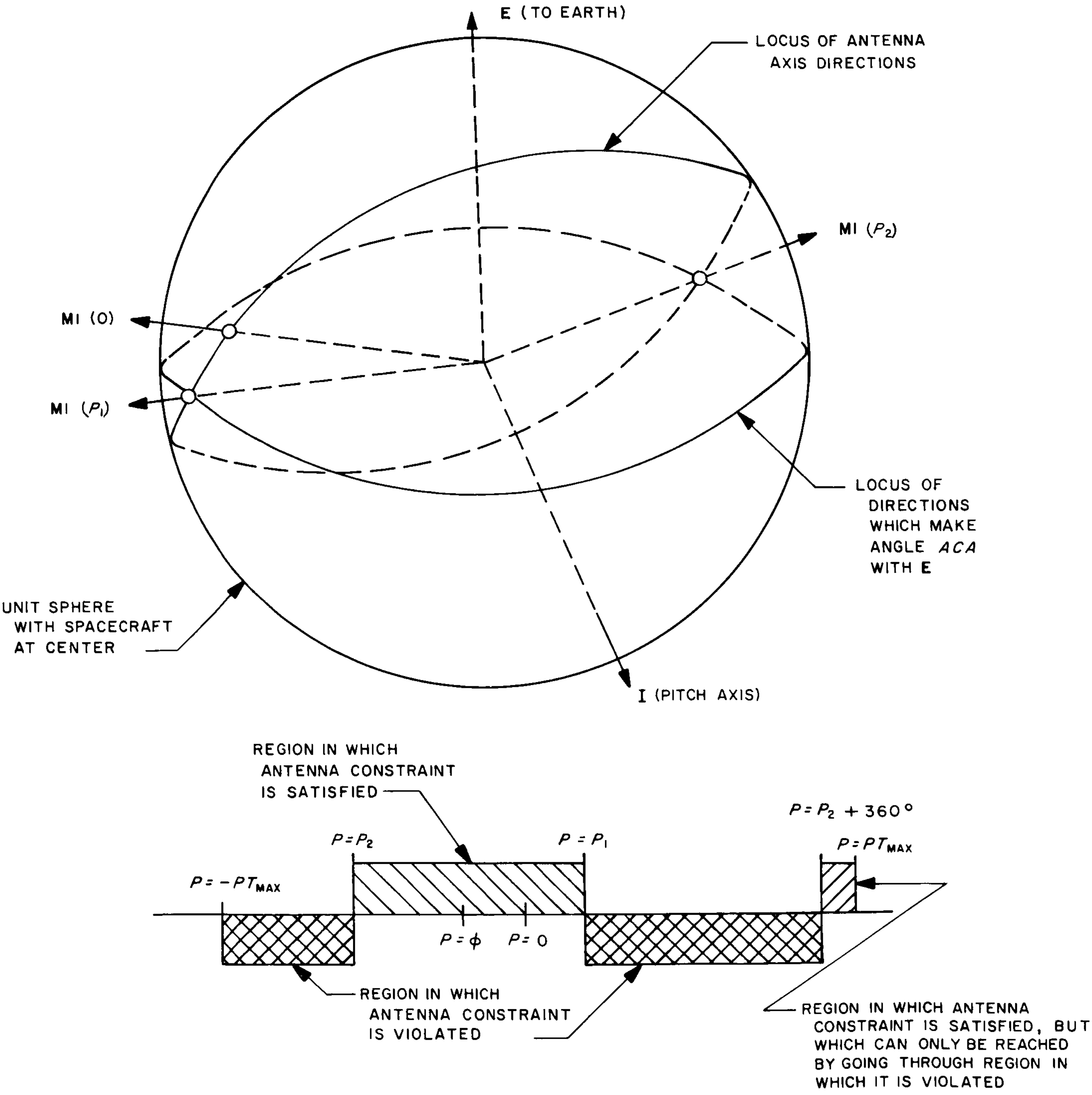

Fig. 3. Pitch-turn geometry showing antenna constraint regions

so has components in spacecraft body fixed coordinates, **I**, **J**, **K**:

$$\mathbf{T}_0 = \begin{pmatrix} \sin\xi \cos\gamma \\ \sin\xi \sin\gamma \\ \cos\xi \end{pmatrix} \tag{21}$$

After the pitch turn, the motor axis may be expressed in **I**, **J**, **K** coordinates by:

$$\mathbf{T}_p = \Theta_p^T \mathbf{T}_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos PT & -\sin PT \\ 0 & \sin PT & \cos PT \end{pmatrix} \mathbf{T}_0$$

$$= \begin{pmatrix} \sin\xi \cos\gamma \\ \sin\xi \sin\gamma \cos PT - \cos\xi \sin PT \\ \sin\xi \sin\gamma \sin PT + \cos\xi \cos PT \end{pmatrix} \tag{22}$$

In the new spacecraft coordinates:

$$\mathbf{I}_p = \mathbf{I}$$

$$\mathbf{J}_p = \mathbf{J} \cos PT + \mathbf{K} \sin PT$$

$$\mathbf{K}_p = -\mathbf{J} \sin PT + \mathbf{K} \cos PT$$

$$\mathbf{T}_p' = \begin{pmatrix} \sin\xi \cos\gamma \\ \sin\xi \sin\gamma \\ \cos\xi \end{pmatrix} \tag{23}$$

After the roll turn, the motor axis may be expressed in $\mathbf{I}_p$, $\mathbf{J}_p$, $\mathbf{K}_p$ coordinates by:

$$\mathbf{T}_R' = \Theta_R^T \mathbf{T}_p' = \begin{pmatrix} \cos RT & -\sin RT & 0 \\ \sin RT & \cos RT & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{T}_p'$$

$$= \begin{pmatrix} \sin\xi \cos\gamma \cos RT - \sin\xi \sin\gamma \sin RT \\ \sin\xi \cos\gamma \sin RT + \sin\xi \sin\gamma \cos RT \\ \cos\xi \end{pmatrix} \tag{24}$$

and in **I**, **J**, **K** coordinates by:

$$\mathbf{T}_R = \begin{pmatrix} \sin\xi \cos(\gamma + RT) \\ \sin\xi \cos PT \sin(\gamma + RT) - \cos\xi \sin PT \\ \sin\xi \sin PT \sin(\gamma + RT) + \cos\xi \cos PT \end{pmatrix} \tag{25}$$

To attain the desired maneuver, the motor axis must point in exactly the opposite direction of the required velocity vector, so that:

$$\mathbf{T}_R = -\frac{\mathbf{VM}}{VM} = -\frac{1}{VM} \begin{pmatrix} \mathbf{VM} \cdot \mathbf{I} \\ \mathbf{VM} \cdot \mathbf{J} \\ \mathbf{VM} \cdot \mathbf{K} \end{pmatrix} \tag{26}$$

and equating components, the pitch and roll turns are:

$$PT = -\tan^{-1}\left(\frac{\mathbf{VM} \cdot \mathbf{J}}{\mathbf{VM} \cdot \mathbf{K}}\right) \pm \left|\cos^{-1}\left(-\frac{\cos\xi}{\sqrt{1 - (\mathbf{VM} \cdot \mathbf{I})^2 / VM^2}}\right)\right| \tag{27}$$

$$RT = -\gamma \pm \left|\cos^{-1}\left(-\frac{\mathbf{VM} \cdot \mathbf{I}}{VM \sin\xi}\right)\right| \tag{28}$$

The following three conditions are possible:

(1) Two values of PT and RT if $\sin\xi > |\mathbf{I} \cdot \mathbf{VM}/VM|$.

(2) One value of PT and RT if $\sin\xi = |\mathbf{I} \cdot \mathbf{VM}/VM|$.

(3) No real values of PT and RT if $\sin\xi < |\mathbf{I} \cdot \mathbf{VM}/VM|$.

If the third condition occurs, the maneuver is to be modified by the smallest possible change in the noncritical velocity component, ΔVN, so that there is one set of turns. To calculate the required ΔVN, the following must be defined:

$$A = (\mathbf{K} \cdot \mathbf{N})^2 + (\mathbf{J} \cdot \mathbf{N})^2 - \cos^2\xi$$

$$B = (\mathbf{VM} \cdot \mathbf{K})(\mathbf{N} \cdot \mathbf{K}) + (\mathbf{VM} \cdot \mathbf{J})(\mathbf{N} \cdot \mathbf{J}) - (\mathbf{VM} \cdot \mathbf{N}) \cos^2\xi$$

$$D = (\mathbf{VM} \cdot \mathbf{K})^2 + (\mathbf{VM} \cdot \mathbf{J})^2 - VM^2 \cos^2\xi$$

$$\Delta VN = -\frac{1}{A}\left[B \pm \sqrt{B^2 - AD}\right] \tag{29}$$

and

$$\mathbf{VM}' = \mathbf{VM} + (1 + \epsilon)(\Delta VN)\,\mathbf{N} \tag{30}$$

where the factor $(1 + \epsilon)$ moves the velocity vector just past the inaccessible cone boundary and avoids the numerical difficulties of placing the vector exactly on the boundary. Nominally, $\epsilon = 0.01$. (It should be noted that $B^2 < AD$ implies that both the desired maneuver and the noncritical direction are in the inaccessible cone. A maneuver would not be made under these conditions.) Of the two solutions, the one with the smallest absolute value should be used, unless it violates one of the constraints and the other solution does not.

If the second condition listed above occurs, there is one set of turns. This is really a special case of the first condition with both sets of turns identical.

The antenna constraint is to be investigated for each possible pitch angle. If the constraint is violated, a message to this effect must be printed out. In situation (1), the most probable situation, if only one solution satisfies the antenna constraint, that solution is to be used. If both solutions satisfy the antenna constraint, the solution which maximizes the quantity $\mathbf{E} \cdot \mathbf{M1}(TP)$ must be used. The quantity which determines the status of the antenna constraint is the scalar product:

$$\mathbf{M1}\,(t) \cdot \mathbf{E} = \mathbf{E} \cdot \mathbf{M1}\,(0) \cos P\,(t) + \mathbf{E} \cdot \mathbf{J} \sin P\,(t) \qquad (31)$$

whose value must be greater than cos ACA at all times during the turn. It is useful to rewrite the equation as

$$\mathbf{M1}(t) \cdot \mathbf{E} = AM \cos\,[P(t) - \phi] \qquad (32)$$

where

$$(AM)^2 = [\mathbf{E} \cdot \mathbf{M1}(0)]^2 + (\mathbf{J} \cdot \mathbf{E})^2$$

$$\tan \phi = \frac{\mathbf{J} \cdot \mathbf{E}}{\mathbf{E} \cdot \mathbf{M1}(0)}$$

At the beginning of the turn, the scalar product $\mathbf{M1} \cdot \mathbf{E}$, whose value is at that time $AM \cos \phi = \mathbf{E} \cdot \mathbf{M1}(0)$, should satisfy the constraint if telemetry is being successfully received before the maneuver. Therefore, it must be true that $AM \geq \cos ACA$. During the turn, the value of the scalar product oscillates sinusoidally, with the zero-to-peak amplitude AM. The constraint will be violated if the condition

$$AM \cos\,[P(t) - \phi] < \cos ACA \qquad (33)$$

obtained at any time during the turn. The turn may be tested for constraint violation by finding the values of P at which

$$AM \cos\,(P - \phi) = \cos ACA \qquad (34)$$

There are two solutions to this equation:

$$P_1 = \phi + \left| \cos^{-1}\left(\frac{\cos ACA}{AM}\right) \right| \qquad (35)$$

and

$$P_2 = \phi - \left| \cos^{-1}\left(\frac{\cos ACA}{AM}\right) \right| \qquad (36)$$

To accommodate the possibility that the turns which are computed orient the spacecraft such that the Sun shines on a bay which may be at a higher temperature than anticipated, the allowable range of pitch turns, $P_2 \leq PT \leq P_1$, has the capability of arbitrary modification such that $PT_{\text{ARB-min}} \leq PT \leq PT_{\text{ARB-max}}$. Since the arbitrary allowable pitch range will not be known until after the computed turns have been examined by the cognizant spacecraft engineers, the midcourse program will proceed as usual and compute the command message and will then be capable of restart at a point that can re-enter the decision program without repetition of the search routine. If the absolute constraints are violated by the maneuver computed after the program is re-entered, message No. 12 in Table 6 is to be printed, and the program will stop, but will be capable of re-entering as previously stated. P_1 and P_2 must be compared to the magnitude of the maximum allowable pitch turn, $PT_{\max}$, an input quantity

if

$$P_1 > PT_{\max}, \text{ set } P_1 = PT_{\max} \qquad (37)$$

if

$$P_2 < -PT_{\max}, \text{ set } P_2 = -PT_{\max} \qquad (38)$$

The pitch turn must satisfy the inequality

$$P_2 \leq PT \leq P_1 \qquad (39)$$

Figure 3 illustrates the geometry involved.

If the antenna constraint is not satisfied, the maneuver is modified by adding a noncritical velocity component such that the pitch turn is right at the limits of the above inequality. The angle between the low-gain antenna axis and Earth direction:

$$LGE_{\pm} = \cos^{-1} [\mathbf{E} \cdot \mathbf{M1}(PT_{\pm})] \tag{40}$$

is computed and printed out so that the extent of antenna constraint violation may be examined.

To determine the limit at which to put the pitch turn, the following quantities are formed:

$$\epsilon_{+} = \min\Big\{ |PT_{+} - P_1|, |PT_{-} - P_1|, |PT_{+} + 360° - P_1|, |PT_{-} + 360° - P_1| \Big\} \tag{41}$$

$$\epsilon_{-} = \min\Big\{ |P_2 - PT_{+}|, |P_2 - PT_{-}|, |P_2 + 360° - PT_{+}|, |P_2 + 360° - PT_{-}| \Big\} \tag{42}$$

if

$$\epsilon_{+} > \epsilon_{-}$$

set

$$PT = P_2$$

if

$$\epsilon_{+} \leq \epsilon_{-}$$

set

$$PT = P_1$$

This determines the low-gain antenna position as follows:

$$\mathbf{M1}(PT) = \mathbf{M1}(0) \cos PT + \mathbf{J} \sin PT \tag{43}$$

The direction of the velocity vector that has the same critical plane component is found by the intersection of the small circle centered at **M1** with arc radius ξ and the arc between **N** and $\mathbf{VCP}/VCP \triangleq \omega$, as shown on the unit sphere in Fig. 4. The unit vector in the direction of the desired maneuver is $\mathbf{v}_d$ and of the resultant modified maneuver is $\mathbf{v}$.

From spherical triange No. 1

$$\cos \psi = \frac{\omega \cdot \mathbf{M1}}{[1 - (\mathbf{N} \cdot \mathbf{M1})^2]^{1/2}} \tag{44}$$

and from spherical triange No. 2

$$\cos \xi = (\mathbf{N} \cdot \mathbf{M1}) \cos x + (\omega \cdot \mathbf{M1}) \sin x \tag{45}$$

and

$$x = \sin^{-1} \left(\frac{\cos \xi}{[(\mathbf{N} \cdot \mathbf{M1})^2 + (\omega \cdot \mathbf{M1})^2]^{1/2}} \right) - \tan^{-1} \left(\frac{\mathbf{N} \cdot \mathbf{M1}}{\omega \cdot \mathbf{M1}} \right) \tag{46}$$

The following two cases are possible:

(1) $$\cos^2 \xi < (\mathbf{N} \cdot \mathbf{M1})^2 + (\omega \cdot \mathbf{M1})^2 \tag{47}$$

There are two solutions, corresponding to the double-valued arc sine function. Calling the modified maneuver **VMM**

$$\mathbf{VMM} = \mathbf{VCP} + \frac{VCP}{\tan x} \mathbf{N} \tag{48}$$

$$|\mathbf{VMM}| = \frac{VCP}{\sin x} \tag{49}$$

Use the value of x that maximizes

$$\mathbf{v} \cdot \mathbf{v}_d = \frac{\mathbf{VM}}{VM} \cdot \frac{\mathbf{VMM}}{VMM} \tag{50}$$

(2) $$\cos^2 \xi = (\mathbf{N} \cdot \mathbf{M1})^2 + (\omega \cdot \mathbf{M1})^2 \tag{51}$$

There is only one solution (again, a special case of Item (1), with two identical solutions):

$$\mathbf{VMM} = \mathbf{VCP} + \frac{VCP}{\tan x} \mathbf{N} \tag{52}$$

If these logical branches have been entered after setting P_1 and P_2 equal to $PT_{\text{ARB-max}}$ and $PT_{\text{ARB-min}}$, respectively, when the pitch turn is computed, only the value which

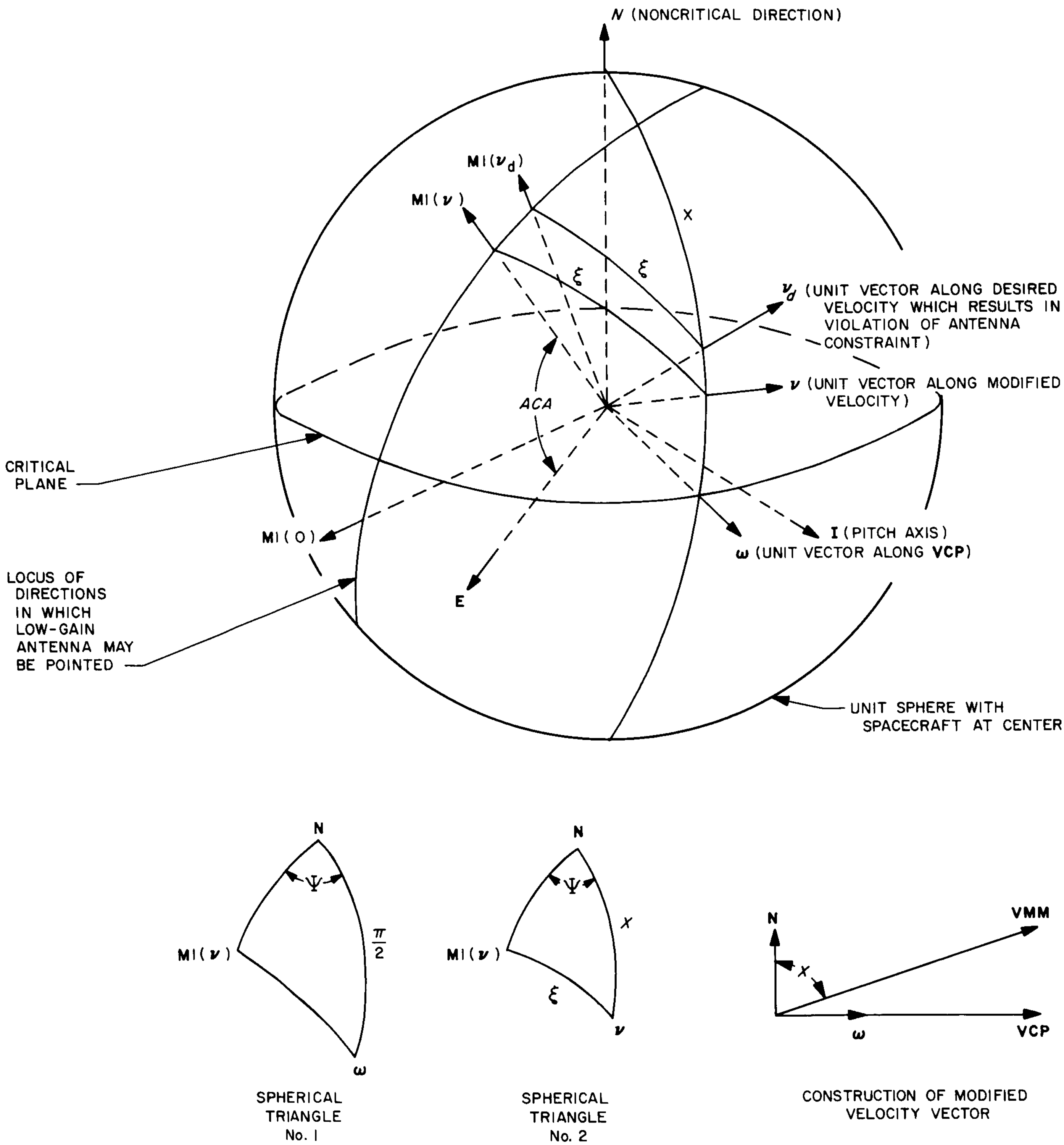

Fig. 4. Maneuver modification for antenna constraint

is closest to that chosen by the test described in Section III-B, Midcourse Command Generation Program, is to be considered.

It is of some interest to compute the NO-PITCH maneuver, *VNP*, which has the desired critical plane component and is made by a roll turn only. This guarantees that the low-gain antenna orientation does not change relative to the Earth, and also allows the Sun to shine normally on the solar panels during the maneuver. The motor axis direction, after the roll turn, may be expressed in **I**, **J**, **K** coordinates:

$$\mathbf{T}_R = \Theta_R^T \mathbf{T}_0 = \begin{pmatrix} \sin\xi \cos(\gamma + RT) \\ \sin\xi \sin(\gamma + RT) \\ \cos\xi \end{pmatrix} \tag{53}$$

Then

$$\mathbf{VNP} = \mathbf{VM} + \Delta V\mathbf{N} = -|VNP|\,\mathbf{T}_R = -\mathbf{T}_R\,(VM^2 + \Delta V^2 + 2\Delta V\,\mathbf{VM}\cdot\mathbf{N})^{1/2} \tag{54}$$

Equating the **K** components

$$-\mathbf{VNP}\cos\xi = \mathbf{VM}\cdot\mathbf{K} + \Delta V\,\mathbf{N}\cdot\mathbf{K} \tag{55}$$

and solving for ΔV

$$\Delta V = \frac{(\mathbf{VM}\cdot\mathbf{N})\cos^2\xi - (\mathbf{VM}\cdot\mathbf{K})(\mathbf{N}\cdot\mathbf{K})}{(\mathbf{N}\cdot\mathbf{K})^2 - \cos^2\xi} \pm \frac{\cos\xi}{(\mathbf{N}\cdot\mathbf{K})^2 - \cos^2\xi}\left[(\mathbf{VM}\cdot\mathbf{K})^2 - 2(\mathbf{VM}\cdot\mathbf{N})(\mathbf{VM}\cdot\mathbf{K})(\mathbf{N}\cdot\mathbf{K}) + VM^2(\mathbf{N}\cdot\mathbf{K})^2 - VCP^2\cos^2\xi\right]^{1/2} \tag{56}$$

and

$$\mathbf{VNP} = \mathbf{VM} + \mathbf{N}\Delta V \tag{57}$$

compute

$$A = \cos^{-1}\left(-\mathbf{K}\cdot\frac{\mathbf{VC}}{VC}\right) \tag{58}$$

If $A - \xi < 90°$, there may be two solutions. Use the solution that satisfies the relationship:

$$\cos^{-1}\left(-\mathbf{K}\cdot\frac{\mathbf{VNP}}{VNP}\right) = \xi \tag{59}$$

If both solutions satisfy Eq. (59), use the one of smaller magnitude.

If $A - \xi > 90°$, print: NO *VNP* AVAILABLE.

The required roll turn may be computed from the **I** and **J** components:

$$\mathbf{VNP}\cdot\mathbf{J} = -|\mathbf{VNP}|\sin\xi\sin(\gamma + RT) \tag{60}$$

$$\mathbf{VNP}\cdot\mathbf{I} = -|\mathbf{VNP}|\sin\xi\cos(\gamma + RT) \tag{61}$$

$$RT = -\gamma + \tan^{-1}\left(\frac{-\mathbf{J}\cdot\mathbf{VNP}}{-\mathbf{I}\cdot\mathbf{VNP}}\right) \tag{62}$$

To solve for the required pitch and roll turns for the critical plane maneuver, assume that **VM** = **VCP**, and solve for the turns, as above, for **VM**.

If the Canopus sensor is locked on some other body, the components of the unit vector from the probe to that body should be substituted for **C** when calculating *PT* and *RT*.

When the maneuver to be applied has been obtained by modifying the component of the maneuver in the non-critical direction, an error will ordinarily exist in the miss components at the terminal point. This error results from having modified the maneuver with a scheme developed from a linear model. The following iteration scheme is to be used to eliminate this error:

(1) Add the maneuver impulse to the velocity on the best-fit orbit at time $T1$, and run the trajectory to the desired terminal condition; find the deviations $\Delta\mathbf{B}\cdot\mathbf{R}$, $\Delta\mathbf{B}\cdot\mathbf{T}$, from the desired values, where $\Delta\mathbf{B}\cdot\mathbf{R} = \mathbf{B}\cdot\mathbf{R}_{achieved} - \mathbf{B}\cdot\mathbf{R}_{desired}$, etc.

(2) If the terminal value of the miss does not satisfy the following tolerance conditions:

$\max(|\Delta\mathbf{B}\cdot\mathbf{R}|, |\Delta\mathbf{B}\cdot\mathbf{T}|) < BTOL$ ($BTOL$ nominally 75 km)

$\Delta T_L < TTOL$ ($TTOL$ nominally 0.005 day)

the homing process should be repeated, biasing the virtual aiming point as follows:

$$(\mathbf{B}\cdot\mathbf{R})_{virtual} = (\mathbf{B}\cdot\mathbf{R})_{desired} - \Delta\mathbf{B}\cdot\mathbf{R} \tag{63}$$

$$(\mathbf{B}\cdot\mathbf{T})_{virtual} = (\mathbf{B}\cdot\mathbf{T})_{desired} - \Delta\mathbf{B}\cdot\mathbf{T} \tag{64}$$

$$(T_L)_{virtual} = (T_L)_{desired} - \Delta T_L \tag{65}$$

The tolerance $BTOL$ and $TTOL$ will be given in the prelaunch input. Table 5 contains a list of quantities required as output from the Midcourse Decision Program.

B. Midcourse Command Generation Program

The primary purpose of the Midcourse Command Generation Program is to convert the midcourse maneuver velocity vector **VM** into the three stored commands required by the CC&S to perform the maneuver. The initial step in converting this velocity vector to a form which is suitable for command transmission to the spacecraft is to translate it into a pitch turn and a roll turn in spacecraft coordinates. This has been done in the Midcourse Decision Program.

The polarities are stored and the turn magnitudes are now converted to turn durations. The pitch turn duration is

$$TP = \frac{PT}{\dot{\theta}_{cp}} \tag{66}$$

and the roll turn duration is

$$TR = \frac{RT}{\dot{\theta}_{cr}} \tag{67}$$

In Eqs. (66) and (67), $\dot{\theta}_{ci}$ is the spacecraft turn rate.

The required turn duration commands are now formulated by adjusting the turn duration for the CC&S computation lag, if any, and converting the resulting turn duration command to a form usable by the CC&S after dividing by the command resolution.

The required commands are:

$$TPC = \frac{TP - DT1P}{DT2P} \tag{68}$$

$$TRC = \frac{TR - DT1R}{DT2R} \tag{69}$$

It should be noted that TRC and TPC are dimensionless.

Burn duration, TVM, is computed from theoretical motor performance compared to extensive test data (Ref. 2). The pulse train is such that discrete burn times may be commanded as follows: the first pulse calls for 0.06 sec, followed by 4 pulse cycles, the first three of which call for 0.04-sec increments and the fourth calls for an 0.08-sec increment (i.e., 0.06, 0.10, 0.14, 0.18, 0.26, 0.30, 0.34, 0.38, 0.46, 0.50, 0.54, 0.58, 0.66, 0.70, etc.). The burn duration command, $TVMC$, is the number of pulses with the least time difference between computed and commanded burn times. $TVMC$ is dimensionless.

TPC, TRC, and $TVMC$ are rounded off to the nearest integer, and are then converted to binary form for command transmission to the spacecraft. A subroutine already exists for performing this function. Output in octal form is also required. The command word format is illustrated in Appendix A.

The three stored quantitative commands for the midcourse maneuver will be written on tape. During the space flight operation, the midcourse maneuver command message will be transmitted to the appropriate DSIF station. The tape format is specified in the *Mariner* Ground Command Subsystem functional specification.

After these computations are performed, print the quantities given in Table 6 for the Midcourse Command

Generation Program. Some additional computations are required for this printout.

(1) Compute the GMT of the following events:

Event	GMT
(a) TX, transmit command time	$T1 - TT - TM - ALT/C$
(b) Begin pitch time	$T1 - TR_{max} - TP_{max}$
(c) End pitch turn	$T1 - TR_{max} - TP_{max} + TP$
(d) Begin roll turn	$T1 - TR_{max}$
(e) End roll turn	$T1 - TR_{max} + TR$
(f) Ignite motor (an input, simply list)	$T1$
(g) Cut off motor	$T1 + TVM$

The constants in these equations TT, TP_{max}, TM, TR_{max}, and C are specified by prelaunch input.

(2) The rate of change of the impact parameter vector **B**:

$$\dot{\mathbf{B}} = \frac{\mathbf{B}(\mathbf{VM} \text{ at } T1 + TLAM) - \mathbf{B}(\mathbf{VM} \text{ at } T1)}{TLAM}$$

$$= \frac{d}{dt}\begin{pmatrix} \mathbf{B}\cdot\mathbf{R} \\ \mathbf{B}\cdot\mathbf{T} \end{pmatrix} \tag{70}$$

where $TLAM$ is a prelaunch input.

$\mathbf{B}(\mathbf{VM}$ at $T)$ is the vector with components $\mathbf{B}\cdot\mathbf{R}$ and $\mathbf{B}\cdot\mathbf{T}$ resulting from a maneuver **VM** at time T. The value of $\dot{\mathbf{B}}$ allows the determination of how long an interval may safely elapse before sending the execute signal without changing the quantitative commands, if the signal cannot be sent at the nominal time.

(3) Quantities which describe the accuracy with which the location of the terminal point can be predicted, given the uncertainties in orbit determination and in the performance of the midcourse maneuver. These quantities depend on the moment matrices of the components $\mathbf{B}\cdot\mathbf{R}$, $\mathbf{B}\cdot\mathbf{T}$, TL of the generalized miss vector **M**. The description of the miss from orbit determination errors was already printed once as part of the Midcourse Introductory Printout. The formulae for the computations are given in Tables 1 and 6. The 3×3 moment matrix N_g of miss caused by guidance component errors is computed from the formula

$$N_g = K(LX)K^T \tag{71}$$

where

$$K_{ij} = \frac{\partial M_i}{\partial V_j} \qquad \begin{matrix} i = 1, 2, 3 \\ j = 1, 2, 3 \end{matrix}$$

$$LX = \left[\sigma_1^2 - \sigma_2^2 + \frac{\sigma_3^2 - \sigma_4^2}{(VM)^2}\right] LV + [\sigma_2^2 (VM)^2 + \sigma_4^2]\, I$$

I is the 3×3 identity matrix. LV is described in Appendix B and σ_1, σ_2, σ_3, and σ_4 represent standard deviations of shutoff, pointing, resolution, and autopilot errors, respectively (Ref. 3).

A detailed trajectory with fine printout is always run automatically following the Command Generation Program. The fine printout is designed to explore the mission and target-dependent characteristics.

(4) Because of the critical nature of the pitch-turn and roll-turn calculation, a special check computation was added to the Command Generation Program. The unit vector in the direction of the thrust axis of the spacecraft after the spacecraft has executed a pitch turn, PT, and a roll turn, RT, will be calculated with the following equation:

$$VMT = -\Theta \mathbf{T}_0 \tag{72}$$

where

$$\Theta = \begin{pmatrix} \cos RT & -\sin RT & 0 \\ \cos PT \sin RT & \cos PT \cos RT & -\sin PT \\ \sin PT \sin RT & \sin PT \cos RT & \cos PT \end{pmatrix}$$

$$\mathbf{T}_0 = \mathbf{K} \cos \xi + (\mathbf{I} \cos \gamma + \mathbf{J} \sin \gamma) \sin \xi$$

The angular deviation of VMT from VM will be denoted by $ADVMT$.

$$ADVMT = \cos^{-1} \frac{\mathbf{VMT}\cdot\mathbf{VM}}{VM} \tag{73}$$

This calculation will show the effects of command quantization and call attention to any gross error.

(5) The probability of the spacecraft impacting the planet will be computed by integrating the probability density function in the R, T plane.

Define the 2×2 upper left-hand submatrix of $(N + N_g)$ as the M matrix (Appendix B).

$$P_{\text{impact}} = \frac{1}{2\pi \mid M \mid^{1/2}} \int_{-RC}^{+RC} \int_{-a(y)}^{+a(y)} \exp \times \left\{ - \tfrac{1}{2} [x - x_0, y - y_0] M^{-1} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix} \right\} dx\, dy \tag{74}$$

where

$x_0 = \mathbf{B} \cdot \mathbf{R}$ on the best-fit post maneuver trajectory

$y_0 = \mathbf{B} \cdot \mathbf{T}$ on the best-fit post maneuver trajectory

$a(y) = \sqrt{RC^2 - y^2}$

$RC = RA \left(1 + \frac{2A}{RA}\right)^{1/2}$

RA = radius of target planet

A = magnitude of semimajor axis of approach hyperbola $= \mu / C_3$

(6) To assist in redetermining the orbit after the midcourse maneuver has been performed, the following information will be provided to the Orbit Determination Group:

(a) The best estimate of the midcourse motor shutoff time in *GMT*, *TMI*.

(b) The 3×3 covariance matrix of velocity execution errors in trajectory coordinates.

(c) The three velocity components of the intended maneuver in trajectory coordinates.

The midcourse injection time, *TMI*, can be obtained from the following equation:

$$TMI = T1 + TVM \tag{75}$$

(7) The 40, 60, 80, and 95% dispersion ellipses in the R, T plane will be computed from the M matrix discussed in Paragraph (5) above. Equations for λ_1, the semimajor axis, for λ_2, the semiminor axis, and θ, the orientation angle, are given in Table 1. These ellipses will be plotted to the same scale used in the plotting program. The equation for each ellipse is:

$$(m_1, m_2)\, M^{-1} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = k^2 \tag{76}$$

where $(m_1, m_2) = (\mathbf{B} \cdot \mathbf{T}, \mathbf{B} \cdot \mathbf{R})$ and the probability, $P(A)$, of being within the ellipse depends on the value of k:

$P(A)$	40%	60%	80%	95%
k	1.02	1.35	1.80	2.45

In polar coordinates (ρ, ψ), the equation of the ellipse is:

$$\rho = k \left[\frac{\cos^2 (\psi - \theta)}{\lambda_1^2} + \frac{\sin^2 (\psi - \theta)}{\lambda_2^2} \right]^{-1/2} \tag{77}$$

The center of this ellipse is to be at $\mathbf{B} \cdot \mathbf{R}_{\text{desired}}$, $\mathbf{B} \cdot \mathbf{T}_{\text{desired}}$.

(8) The change in range rate, which gives immediate verification of maneuver performance:

$$\Delta \dot{r} = - \mathbf{E} \cdot \mathbf{VM} \tag{78}$$

C. Capability Ellipse Generator

The maximum capability, an ellipse in the R, T plane, will be obtained by applying maximum maneuvers in various directions in the critical plane at a guidance point $T1$. The data in Table 7 will be printed for each trajectory thus obtained. The $\mathbf{B} \cdot \mathbf{R}$, $\mathbf{B} \cdot \mathbf{T}$ coordinates of these trajectories will be available as input to the contour plotting program so that the maximum capability ellipse can be plotted.

Table 7. Output quantities—capability ellipse generator

Item	Description
1	Trajectory No
2	Miss component $\mathbf{B} \cdot \mathbf{R}$
3	Miss component $\mathbf{B} \cdot \mathbf{T}$
4	Distance of closest approach to target planet
5	Date and GMT of closest approach

It is convenient at this point to define a right-hand coordinate system with respect to the critical plane. Let $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ be mutually perpendicular unit vectors in the critical plane (Fig. 5) such that

$$\boldsymbol{\mu}_1 = \boldsymbol{\lambda}_1 / \lambda_1 \tag{79}$$

$$\boldsymbol{\mu}_2 = \frac{\boldsymbol{\lambda}_2 - (\boldsymbol{\lambda}_2 \cdot \boldsymbol{\mu}_1)\, \boldsymbol{\mu}_1}{\sqrt{\lambda_2^2 - (\boldsymbol{\lambda}_2 \cdot \boldsymbol{\mu}_1)^2}} \tag{80}$$

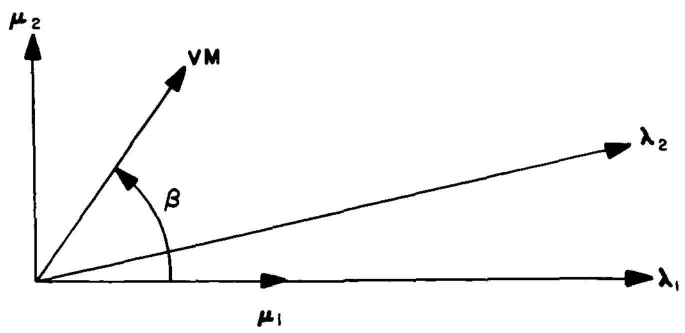

Fig. 5. Base vectors for capability ellipse generation

Let

$$|\mathbf{VM}| = VCAP$$

Set

$$\beta = IA$$

where β is an angle measured in the critical plane from the $+\boldsymbol{\mu}_1$ axis and IA is the initial angle, an input quantity, nominally zero. The set of flyby trajectories will be generated by holding VM constant while the angle β is incremented by 72 deg four times to obtain five trajectories which define the capability ellipse.

For each value of β, compute the midcourse maneuver

$$\mathbf{VM} = VCAP\,(\cos\beta\,\boldsymbol{\mu}_1 + \sin\beta\,\boldsymbol{\mu}_2) \tag{81}$$

Apply this maneuver to the best-fit orbit at the guidance time $T1$ and run the trajectory. The required output quantities, which are given in Table 8, are available in the trajectory program.

For each value of β, calculate

$$\Delta T = TFI - TF \tag{82}$$

where

TFI = flight time on the uncorrected trajectory

TF = flight-time on the modified trajectory

For each of the five values of $\mathbf{B}\cdot\mathbf{R}$, $\mathbf{B}\cdot\mathbf{T}$ coordinates from the integrated trajectories the value of ΔT should be printed, beside the point, in hours to two decimal places.

The five points:

$$x_i = \mathbf{B}\cdot\mathbf{R}_i - \mathbf{B}\cdot\mathbf{R}_{\text{uncorrected}} \tag{83}$$

$$y_i = \mathbf{B}\cdot\mathbf{T}_i - \mathbf{B}\cdot\mathbf{T}_{\text{uncorrected}} \tag{84}$$

$i = 1, 2, 3, 4$, and 5, are used to fit an ellipse through those points corresponding to $\beta = IA + (i-1)\times 72$ deg:

$$a_1 x_i^2 + a_2 y_i^2 + a_3 x_i y_i = 1 \tag{85}$$

where

$$Q^T = \begin{bmatrix} x_1^2 & x_2^2 & x_3^2 & x_4^2 & x_5^2 \\ y_1^2 & y_2^2 & y_3^2 & y_4^2 & y_5^2 \\ x_1y_1 & x_2y_2 & x_3y_3 & x_4y_4 & x_5y_5 \end{bmatrix}$$

$$A^T = (a_1, a_2, a_3)$$

$$U^T = (1, 1, 1, 1, 1)$$

Then

$$QA = U \tag{86}$$

and

$$A = (Q^TQ)^{-1}\,Q^TU \tag{87}$$

In polar coordinates:

$$x = r\cos\theta \tag{88}$$

$$y = r\sin\theta \tag{89}$$

$$r = (a_1\cos^2\theta + a_2\sin^2\theta + a_3\sin\theta\cos\theta)^{1/2} \tag{90}$$

Generate a table by stepping θ by 1-deg increments, but including the five values corresponding to $\theta_i = \tan^{-1}(y_i/x_i)$

$$\theta \qquad r \qquad \Delta\mathbf{B}\cdot\mathbf{R} = x \qquad \Delta\mathbf{B}\cdot\mathbf{T} = y \qquad \mathbf{B}\cdot\mathbf{R} = x + \mathbf{B}\cdot\mathbf{R}_{\text{uncorrected}} \qquad \mathbf{B}\cdot\mathbf{T} = y + \mathbf{B}\cdot\mathbf{T}_{\text{uncorrected}} \tag{91}$$

The last two columns contain the points to be plotted. After the table, print: Sum of square of residuals = X, where

$$X = \sum_{i=1}^{5} [(x_{ic} - x_i)^2 + (y_{ic} - y_i)^2] \qquad (92)$$

where x_{ic}, y_{ic} are the values corresponding to $\theta_i = \tan^{-1} (y_i/x_i)$ in the above equations.

IV. Subprogram Flow Sequences

In this section, flow sequences are given for the Midcourse Decision Program, Midcourse Command Generation Program, and Capability Ellipse Generator Program.

A. Flow Sequence—Midcourse Decision Program

1. Input. The following are inputs to the Midcourse Decision Program:

(1) Best estimate of injection condition from orbit determination.

(2) $T1$ = time of motor ignition.

(3) $VCAP$ = velocity capability of midcourse propulsion system.

(4) Target description, desired $\mathbf{B \cdot R}$, $\mathbf{B \cdot T}$, and time of closest approach.

(5) Access to Prelaunch Input.

2. Operation. The following steps operate the Midcourse Decision Program:

(1) Compute TX, the transmission time of the Execute Command, the execute maneuver command

$$TX = T1 - TM - TT - \frac{ALT}{C} \qquad (93)$$

(2) Run trajectory from injection to $T1$; record coordinates $X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}$ at $T1$. Run to the encounter, record $\mathbf{B \cdot T}$, $\mathbf{B \cdot R}$, T_L.

(3) Search for the required maneuver, $\mathbf{V}$, as follows:

First, using the trajectory program with initial conditions described in Item (2) above, integrate variational equations to obtain the K matrix, with elements $K_{ij} = \partial M_i / \partial V_j$.

Second, compute $\mathbf{\Delta B}$, the difference between the specified values of $\mathbf{B \cdot R}$, $\mathbf{B \cdot T}$ and the linearized time of flight and the values obtained on the best-fit orbit, $\Delta B_1 = \mathbf{B \cdot R}$ (velocity at $T1$) $- \mathbf{B \cdot R}$ desired, etc.

Third, compute an estimate of the maneuver

$$\mathbf{V} = -\mathbf{K}^{-1}\, \mathbf{\Delta B}$$

Fourth, add the maneuver $\mathbf{V}$ to the velocity at time $T1$ and change the spacecraft mass to $\mathrm{MASS}_{\mathrm{initial}} - kV$, where k is an input quantity, and integrate a new trajectory, obtaining new terminal conditions. When the search converges, the powered flight program is to integrate the burn and compute initial conditions for a trajectory integration. Test the terminal conditions for convergence: require that $\mathbf{B \cdot R}$, $\mathbf{B \cdot T}$ coordinates differ by at most $BTOL$ and that time of closest approach differ by at most $TTOL$. When the search converges, the powered flight program is to integrate the burn and compute initial conditions for a trajectory integration. If convergence is not achieved, return to the second step under Item (3) above and consider the new integrated trajectory as the best-fit orbit. Proceed to the third step under Item (3) using the same K matrix $ITER$ times, where $ITER$ is an input quantity, nominally 3. If convergence is not achieved after $ITER$ attempts, return to the first step listed under Item (3) and form a new K matrix. If NPR K matrices are computed, where NPR is an input quantity, nominally 6, and convergence has not been achieved, print: NO CONVERGENCE AFTER *NPR* SEARCHES.

(4) Run trajectory and record terminal conditions.

(5) Compare terminal conditions with desired values: If the greater of the errors $\mathbf{\Delta B \cdot R}$, $\mathbf{\Delta B \cdot T}$ exceeds $BTOL$, or ΔTL exceeds $TTOL$, continue. Otherwise, proceed to Item (7).

(6) Bias the virtual aiming point negatively by the amount of the differences found in Step (5) by replacing the desired miss-components $\mathbf{B \cdot R}$, $\mathbf{B \cdot T}$ by ($\mathbf{B \cdot R} - \mathbf{\Delta B \cdot R}$, $\mathbf{B \cdot T} - \mathbf{\Delta B \cdot T}$). Go to the third step listed under Item (3) above.

(7) From the K matrix last used, calculate the unit vector in the noncritical direction, $\mathbf{N}$, and the constants $\mathbf{N \cdot E}$, $\boldsymbol{\lambda}_3 \cdot \mathbf{N}$.

(8) Form the projection $\mathbf{VCP}$ of the optimum maneuver $\mathbf{V}$ in the critical plane $\mathbf{VCP} = \mathbf{V} - (\mathbf{V \cdot N})\, \mathbf{N}$.

(9) Print the data in Group I output.

(10) Test the magnitude of VCP:
If $VCP > VCAP$, print: AVAILABLE PROPELLANT INSUFFICIENT TO NULL MISS COMPONENTS:

ENTER MISSION CAPABILITY STUDY. Exit to the MISSION CAPABILITY STUDY.

If $VCP < VCAP$, continue.

(11) Using the Eqs. (12a), (14a), and (14b), compute $VNMX$, $VNTU$, and $VNTL$.

(12) Form L_1, L_2 such that

$$\min(+VNMX, VNTU) \tag{94}$$

$$\max(-VNMX, VNTL) \tag{95}$$

(13) Compare L_1, L_2.

If $L_1 < L_2$,

print: TIME-OF-FLIGHT CONSTRAINT CANNOT BE SATISFIED WITH AVAILABLE PROPELLANT: ENTER MISSION CAPABILITY STUDY, and compute and plot the RESIDUAL MISS, as described in Appendix C.

If $L_1 \geq L_2$, continue

(14) Test the unmodified maneuver V for violation of the antenna constraint. If no violations occur, proceed to Item (15). If a violation occurs, compute modified maneuver as described in Section III-B. Test modified maneuver for violation of absolute constraints.

(15) When the iteration loop has converged, print: MANEUVER HAS BEEN MODIFIED TO ALLOW FOR CONSTRAINTS, and Group III output (Table 5).

(16) Turn Computation

Input

VM.

Access to prelaunch input.

Operation

(a) Compute

$$TEST = |\mathbf{I}\cdot\mathbf{VM}/VM| \tag{96}$$

(b) Test

If $\sin\xi < \text{TEST}$

$$A = (\mathbf{K}\cdot\mathbf{N})^2 + (\mathbf{J}\cdot\mathbf{N})^2 - \cos^2\xi \tag{97}$$

$$B = (\mathbf{VM}\cdot\mathbf{K})(\mathbf{N}\cdot\mathbf{K}) + (\mathbf{VM}\cdot\mathbf{J})(\mathbf{N}\cdot\mathbf{J}) \tag{98}$$

$$D = (\mathbf{VM}\cdot\mathbf{K})^2 + (\mathbf{VM}\cdot\mathbf{J})^2 - (VM\cos\xi)^2 \tag{99}$$

$$\Delta VN = -\frac{1}{A}[B \pm (B^2 - AD)^{1/2}] \tag{100}$$

$$\mathbf{VM}_{(new)} = \mathbf{VM}_{(old)} + (1+\epsilon)(\Delta VN)\mathbf{N} \tag{101}$$

Of the two possible $\mathbf{VM}_{(new)}$ values, use the one with the smallest absolute value. Return to Item (a) above.

(c) If $\sin\xi = TEST$

$$PT = \pi - \tan^{-1}\left(\frac{\mathbf{VM}\cdot\mathbf{J}}{\mathbf{VM}\cdot\mathbf{K}}\right) \tag{102}$$

$$RT = -\gamma - \cos^{-1}\left(\frac{\mathbf{VM}\cdot\mathbf{I}}{VM\sin\xi}\right) \tag{103}$$

(d) If $\sin\xi > TEST$

$$PT_{(+)} = \cos^{-1}\left(-\frac{\cos\xi}{[1-TEST^2]^{1/2}}\right) - \tan^{-1}\left(\frac{\mathbf{VM}\cdot\mathbf{J}}{\mathbf{VM}\cdot\mathbf{K}}\right) \tag{104}$$

$$RT_{(+)} = -\gamma - \cos^{-1}\left(-\frac{\mathbf{VM}\cdot\mathbf{I}}{VM\sin\xi}\right) \tag{105}$$

$$PT_{(-)} = -\cos^{-1}\left(-\frac{\cos\xi}{[1-TEST^2]^{1/2}}\right) - \tan^{-1}\left(\frac{\mathbf{VM}\cdot\mathbf{J}}{\mathbf{VM}\cdot\mathbf{K}}\right) \tag{106}$$

$$RT_{(-)} = -\gamma + \cos^{-1}\left(-\frac{\mathbf{VM}\cdot\mathbf{I}}{VM\sin\xi}\right) \tag{107}$$

Form

$$\mathbf{E}\cdot[\mathbf{M1}(PT_{(+)}) - \mathbf{M1}(PT_{(-)})] \tag{108}$$

and set $PT = PT_{(+)}$ if this quantity is positive, set $PT = PT_{(-)}$ if this quantity is negative. If antenna constraint is violated by the pitch turn chosen, test the alternate turn before modifying the maneuver.

Computations in Test Maneuver Subroutine

(a) Input data

Maneuver vector **VM**.

Unit vectors **H**, **E**, **N**, **K**.

Scalars $\mathbf{E} \cdot \mathbf{M1}(0)$, cos ACA.

(b) Computations

Form the unit vector **q**, normal to the Sun-probe-Canopus plane:

$$\mathbf{q} = \frac{\mathbf{C} \times \mathbf{H}}{\mathbf{C} \times \mathbf{H}} \tag{109}$$

Form the unit vector **p** in the Sun-probe-Canopus plane:

$$\mathbf{p} = \mathbf{H} \times \mathbf{q} \tag{110}$$

Form the unit vector **I** in the direction of the pre-maneuver spacecraft pitch axis

$$\mathbf{I} = \mathbf{q} \sin \beta + \mathbf{p} \cos \beta$$

$$\text{nominally } \beta = -45^\circ \tag{111}$$

Form the unit vector **J** in the direction of the pre-maneuver spacecraft yaw axis

$$\mathbf{J} = \mathbf{K} \times \mathbf{I} \tag{112}$$

Compute

$$AM = \{(\mathbf{J} \cdot \mathbf{E})^2 + [\mathbf{E} \cdot \mathbf{M1}(0)]^2\}^{1/2} \tag{113}$$

$$\phi = \tan^{-1} \frac{\mathbf{J} \cdot \mathbf{E}}{\mathbf{E} \cdot \mathbf{M1}(0)} \tag{114}$$

Find two roots such that $AM \cos (P_i - \phi) = \cos ACA$ i.e.:

$$P_2 = \phi - \cos^{-1}\left(\frac{\cos ACA}{AM}\right) \tag{115}$$

$$P_1 = \phi + \cos^{-1}\left(\frac{\cos ACA}{AM}\right) \tag{116}$$

If $P_2 < -PT_{\max}$, set $P_2 = -PT_{\max}$

If $P_1 > PT_{\max}$, set $P_1 = PT_{\max}$

Arbitrary values of P_1 and P_2 may be input to reduce the range, if desired.

(c) Choosing pitch turn

Determine whether PT lies between P_1 and P_2. If there are two values of PT and only one lies in that range, use that one. If both values lie in that range they both satisfy the antenna constraint, then use the one that maximizes $\mathbf{E} \cdot \mathbf{M1}(PT)$. If neither value lies in that range, the maneuver is to be modified by the process described in Eqs. (44) through (52).

B. Midcourse Command Generation Program

1. Input. The following are inputs to the Midcourse Command Generation Program:

(1) **VM**.

(2) Access to prelaunch input.

2. Calculations. The following are calculations for the Midcourse Command Generation Program:

(1) Compute

(a) $$TP = \frac{PT}{\dot{\theta}_{CP}} \tag{117}$$

(b) $$TR = \frac{RT}{\dot{\theta}_{CR}} \tag{118}$$

(c) TVM is obtained from motor burn routine, *PRPLS*.

(d) The sign of PT or RT determines the polarity for the turns, the motor burn always has (+) polarity.

(2) Compute the following three dimensionless quantities, to nearest integer:

$$TCP = \frac{|TP| - DT1P}{DT2P} \tag{119}$$

$$TRC = \frac{|TR| - DT1R}{DT2R} \tag{120}$$

$TVMC$ by counting to the nearest value in the series given in Section III-B.

(3) Code *TPC, TRC, TVMC* in binary form, with proper parity and polarity bits.

(4) Miscellaneous calculations; event times, error analysis.

(5) Print out quantities presented in Table 6.

*3. **Output.*** The output consists of quantities listed in Table 6.

C. Capability Ellipse Generator

*1. **Input.*** The following are input into the Capability Ellipse Generator Program:

(1) Best estimate of injection conditions from orbit determination.

(2) $T1$: motor start time.

(3) *VCAP*.

(4) K matrix.

(5) Access to prelaunch input.

(6) Set counter equal to zero.

*2. **Operation.*** Operation of the Capability Ellipse Generator Program as follows:

(1) Set $|\mathbf{VM}| = VCAP$.

(2) Set up unit vectors $\mathbf{u}_1$, and $\mathbf{u}_2$.

(3) Set $\beta = IA$.

(4) Compute maneuver:

$$VM = VM\,(\cos\beta\,\mu_1 + \sin\beta\,\mu_2) \qquad (121)$$

(5) Apply maneuver computed in Step (4) at $T1$ and run trajectory to closest approach.

(6) Printout information in Table 7.

(7) Increase β by 72°, increase counter by 1.

(8) Test

If Counter ≥ 5, proceed with Step (9).

If Counter < 5, proceed with Step (4).

(9) Fit the ellipse through the five points obtained, tabulate and plot the points.

*3. **Output.*** Output quantities for the Capability Ellipse Generator are listed in Table 7.

V. Concluding Remarks

A. Program Tests

As a part of the operational guidance loop, the program, like other spacecraft elements, must fulfill certain test specifications that are generated during the shakedown period. The exact tests used to determine the acceptability of the program included:

(1) The program must not stop or halt for unknown reasons, i.e., without printing an error message. Random input were applied to check this feature.

(2) The specified maneuvers must cause the spacecraft to reach the desired terminal point within the given tolerances.

(3) All output were checked against the output of the usual trajectory printout for a wide range of trajectory conditions.

B. Convenience of Use in Real Time

Careful attention must be given to the problems imposed by operating the program in real time. Manual operations must be minimized; however, it should be possible to modify any parameters during operation, even those given purely numeric values, and to obtain a check print of all input parameters at any time. The user of the program must not only be able to understand the output personally, but must also be able to explain his decisions in real time to people who may not be familiar with the details of the problem, but who bear the responsibility for success of the operation. For this reason, all output must be adequately labeled with appropriate English words, and the units must be given.

Appendix A

Inaccessible Maneuver Directions

The mechanization of the *Mariner* spacecraft is such that there is a cone in space inside of which the midcourse motor cannot be pointed. This inaccessible cone of maneuver directions exists because the motor is mounted so that it does not lie in the plane containing the pitch and yaw axes. This appendix derives this relationship and shows the implications of being unable to point the midcourse motor in a specific direction.

To avoid an unbalanced torque, the thrust vector must pass through the spacecraft's center of gravity. When the motor mounting is adjusted accordingly the thrust axis will make an angle less than 90° with the roll axis. The spacecraft coordinate system is shown in Fig. A-1. The midcourse motor thrust axis is also shown, mounted with nominal values of clock angle (longitude), $\lambda = 45°$; and cone angle (co-latitude), $\phi = 90° - \alpha$.

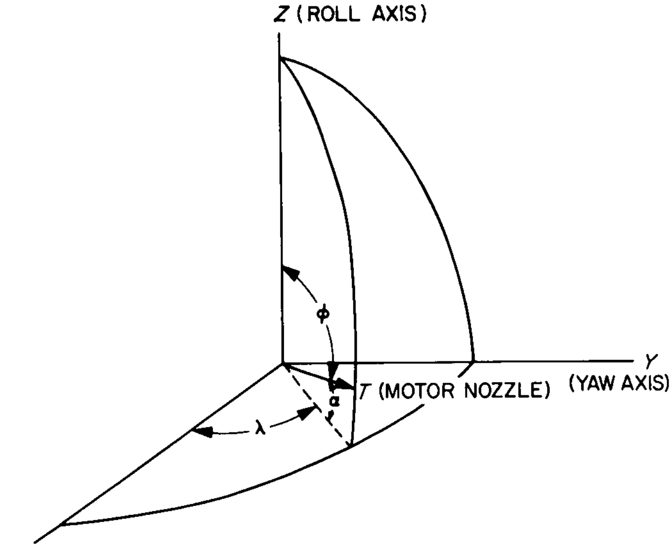

Fig. A-1. Spacecraft coordinate system and motor nozzle

To point the motor with a pitch-roll sequence of turns, the pitch turn must move the roll axis so that it makes an angle ϕ with the desired thrust direction. The roll turn can then align the motor thrust axis with this direction.

Consider the spacecraft coordinate system before the pitch turn to be the inertial system XYZ and, after the pitch turn, to be $X'Y'Z'$. Then, any body-fixed vector, such as the roll axis direction, is changed by the pitch turn so that:

$$\mathbf{k}' = \Theta_p^T \mathbf{k} \qquad \text{(A-1)}$$

where the pitch turn is expressed by the rotation matrix Θ_p which is defined in the main body of the text.

To have the new roll axis make an angle ϕ with the desired thrust direction represented by a unit vector:

$$T_D = \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix} = \begin{pmatrix} \cos \beta_x \\ \cos \beta_y \\ \cos \beta_z \end{pmatrix}$$

We must have:

$$\mathbf{k}' \cdot \mathbf{T}_D = \cos \phi = T_z \cos \theta_p - T_y \sin \theta_p \qquad \text{(A-2)}$$

The solution of this equation for the cosine of the required pitch turn, θ_p, is

$$\cos \theta_p = (1 - T_x^2)^{-1} \{ T_z \cos \phi \pm T_y (\sin^2 \phi - T_x^2)^{1/2} \qquad \text{(A-3)}$$

The behavior of the solution depends on the nature of the discriminant:

$$D = \sin^2 \phi - T_x^2 \qquad \text{(A-4)}$$

If $D < 0$, no real solution is obtained

If $D = 0$, one solution is obtained

If $D > 0$, two solutions are obtained

For each value of $\cos \theta_p$ there are two values of θ_p which satisfy Eq. (A-4). The spurious values may be eliminated by using Eq. A-2. At this time, however, only the first case, where there are no solutions, is of interest.

Remembering that the components of the unit vector $\mathbf{T}_D$ are the direction cosines of the thrust direction, it follows that the condition for no solution is:

$$T_x^2 = \cos^2 \beta_x > \sin^2 \phi = \cos^2 \alpha$$

or

$$|\beta_x| < \alpha$$

and

$$|180° - \beta_x| < \alpha \qquad \text{(A-5)}$$

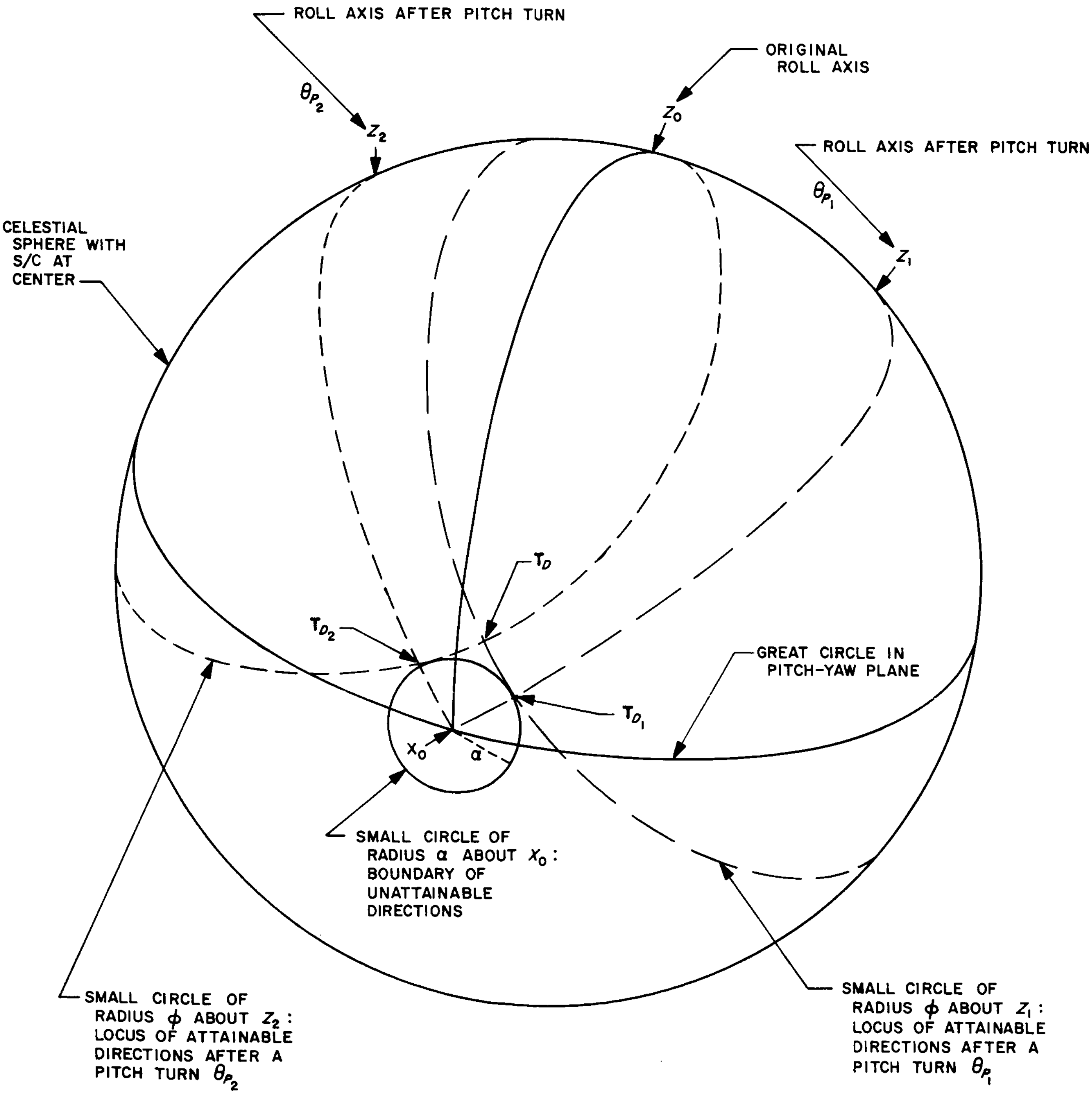

Fig. A-2. Geometry of inaccessible direction

Thus the inaccessible cone is about the initial pitch axis (X_0) direction with half angle α. Figure A-2 illustrates the geometry by showing a unit sphere about the spacecraft with the various directions indicated. If the desired thrust direction is $\mathbf{T}_D$, as shown, then either of the two pitch turns θ_{p_1} or θ_{p_2} allows a roll turn to point the motor in that direction. The directions $\mathbf{T}_{D_1}$ and $\mathbf{T}_{D_2}$ are those for which only one solution exists ($D = 0$ in Eq. A-4) and that solution is θ_{p_1} or θ_{p_2}, respectively. It is clear that the small circle about X_0 of radius α cannot be crossed by any other small circle of radius $\phi = 90° - \alpha$ with center 90° from X_0 (corresponding to $D < 0$ in Eq. (A-4)).

So now the logical question to ask is "What penalty may be incurred due to this situation?" There is no problem if the desired maneuver is not in the inaccessible cone, and this is the most probable situation. The worst case would be for the desired maneuver to be along the original pitch axis. How then should the maneuver be performed? Clearly the critical plane (Ref. 4) component (V_c) of the attainable maneuver should equal that of the desired maneuver, and only the non-critical component (V_n) should be changed. Thus, if

$$\mathbf{V}_0 = \mathbf{V}_c + \mathbf{V}_n \tag{A-6}$$

where

$$V_c = V_0 \sin\gamma$$
$$V_n = V_0 \cos\gamma$$

if γ is the angle between $\mathbf{V}_0$ and the noncritical direction. The maneuver to be attained then should be:

$$\mathbf{V}_A = \mathbf{V}_c + \mathbf{V}_n + \Delta\mathbf{V}_n = \mathbf{V}_a + \Delta\mathbf{V}_n \tag{A-7}$$

where

$$\Delta V_n = V_0 \sin\gamma\,[\cot\gamma - \cot(\alpha + \gamma)]$$

This pointing geometry is shown in Fig. 3. The residual miss is then:

$$\left.\begin{aligned}\Delta\mathbf{M}_A &= \mathbf{M}_0 + K\,\mathbf{V}_A \\ &= \mathbf{M}_0 + K\,\mathbf{V}_0 + K\,\Delta\mathbf{V}_n \\ &= K\,\Delta\mathbf{V}_n\end{aligned}\right\} \tag{A-8}$$

and this is only a flight-time error, since, to first order, the desired maneuver $\mathbf{V}_0$ is such that it nulls the miss $\mathbf{M}_0$:

$$K\,\mathbf{V}_0 = -\mathbf{M}_0 \tag{A-9}$$

where the K matrix is made up of three rows, each one being the velocity gradient of one of the miss components, as developed in the main body of the text.

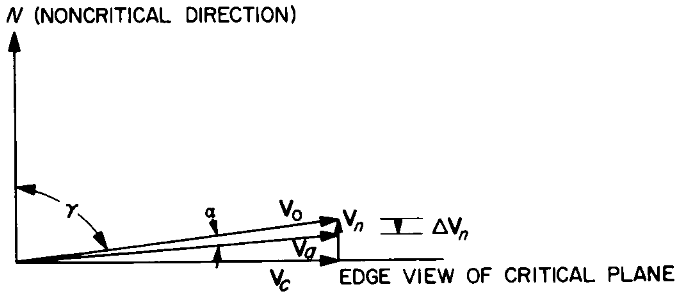

Fig. A-3. Critical coordinates and attainable velocity

The noncritical direction, $\mathbf{n}$, is that direction in which a change of velocity does not affect $\mathbf{B} \cdot \mathbf{R}$ or $\mathbf{B} \cdot \mathbf{T}$ so that Eq. A-8 becomes.

$$K\,\Delta\mathbf{V}_n = \begin{pmatrix} 0 \\ 0 \\ \boldsymbol{\lambda}_3 \cdot \mathbf{n}\,\Delta V_n \end{pmatrix} \tag{A-10}$$

Assume a typical *Mariner* trajectory (going to Venus in 1967) which requires a 50 m/sec maneuver (this is very large) to be performed precisely along the pitch axis seven days after launch. With an inaccessible cone and half angle $\alpha = 2°$, the flight-time error due to adjusting the velocity vector from $\mathbf{V}_0$ to $\mathbf{V}_a$ would be approximately 1 hr.

This penalty does not appear to be too severe. Another solution to the problem would be to wait until the geometry changes, as the spacecraft rotates around the Sun, so that the required maneuver is no longer inaccessible. However, this might require waiting several days and such a delay hardly seems justified.

Appendix B
Mapping of Orbit Determination and Midcourse Guidance Errors into Miss at the Target

I. Introduction

In choosing the midcourse maneuver, it is necessary to consider the dispersion at the target that may result from the errors which are inevitably associated with the performance of the maneuver and the determination of the orbit. It must be ensured that the probability of the spacecraft hitting the planet is less than some specified value. Further, it is necessary to maximize the probability that the post-maneuver trajectory will satisfy the experimental requirements.

Uncertainty in the miss is produced both by orbit determination errors and the errors in the midcourse guidance system. The uncertainty is described by a 3×3 covariance matrix of the following variables:

(1) Error in $\mathbf{B} \cdot \mathbf{R} = M_1$.

(2) Error in $\mathbf{B} \cdot \mathbf{T} = M_2$.

(3) Error in linearized time of flight $T_L = M_3$.

A description of the computation of these covariance matrices follows.

A. Dispersion of Miss Components Caused by Uncertainties in Orbit Determination

The uncertainty in determining the orbit is expressed by the covariance matrix Λ of injection conditions. If q^i represents the i^{th} position or velocity coordinate at injection, the ij element of Λ is the mathematical expectation of the covariance $q^i q^j$:

$$\Lambda_{ij} = E(q^i q^j)$$

A statistical estimate of Λ is furnished by the tracking program. The following transformation maps the 6×6 Λ matrix into the 3×3 N matrix of miss components:

$$N = U \Lambda U^T$$

where

$$N_{ij} = E(M_i M_j)$$

and

$$U_{ij} = \frac{\partial M_i}{\partial q^j} \qquad \begin{array}{l} i = 1, 2, 3 \\ j = 1, 2, 3, 4, 5, 6 \end{array}$$

This computation is performed in the orbit determination program and the results are transmitted to the present program (Table 1). A number of quantities such as the semiaxes of the dispersion ellipse are computed from the matrix N. The formulas for these computations are given in Table 1.

B. Dispersion in Miss Components Caused by Midcourse Guidance Errors

There are four different sources that can give rise to errors in the midcourse velocity impulse. Assume that the effects of these errors can be estimated sufficiently well by a linear approximation. The statistical dispersion in velocity with a given applied maneuver $\mathbf{VM}$, which has components VM_x, VM_y, and VM_z, is given by the 3×3 covariance matrix LX:[4]

$$LX = E(\Delta VM_i \, \Delta VM_j) = \left[\sigma_1^2 - \sigma_2^2 + \frac{\sigma_3^2 - \sigma_4^2}{(VM)^2} \right] LV + \left[(VM)^2 \sigma_2^2 + \sigma_4^2 \right] I$$

where

$$LV = \begin{bmatrix} (VM_x)^2 & (VM_x)(VM_y) & (VM_x)(VM_z) \\ (VM_y)(VM_x) & (VM_y)^2 & (VM_y)(VM_z) \\ (VM_z)(VM_x) & (VM_z)(VM_y) & (VM_z)^2 \end{bmatrix}$$

and I is the 3×3 identity matrix. The values of the four execution error variances are given by Prelaunch Input. This covariance matrix of velocity errors is mapped into the 3×3 covariance matrix N_g of errors in the miss components at the target by the following transformation:

$$N_g = K(LX) K^T$$

where

$$(K)_{ij} = \frac{\partial M_i}{\partial VM_j}$$

[4]A special case of the generalization developed in Ref. 3 is used.

In summary, the following computation should be accomplished:

(1) Having obtained the maneuver vector **VM**, form the LV matrix.

(2) The K matrix is available from the search for **VM**.

(3) Form LX, the covariance matrix of velocity errors ΔV_x, ΔV_y, ΔV_z in trajectory coordinates.

(4) Form $N_g = K(LX)\,K^T$.

(5) Compute the same functions of the elements of N_g that were computed for the orbit determination matrix N: namely, the semiaxes and orientations of the dispersion ellipses in $\mathbf{B} \cdot \mathbf{R}$, $\mathbf{B} \cdot \mathbf{T}$ and the RMS time and position error.

(6) Compute the same functions of the elements of $M = N + N_g$ that were computed for N. This represents the dispersions caused by both orbit determination uncertainty and midcourse execution errors.

Appendix C

Residual Miss Calculation

If the propulsion and time-of-flight constraint cannot be satisfied with the available propellant ($L_1 < L_2$), it is of considerable interest to evaluate the residual miss which would occur if the time-of-flight constraint were satisfied. The flight-time constraint demands that:

$$TL_{\min} \leq TL \leq TL_{\max}$$

Therefore, if the linearized flight time of the best-fit orbit is TL_{OD}:

If $\quad TL_{OD} < TL_{\min}, \quad$ set $TL'_{\text{desired}} = TL_{\min}$

If $\quad TL_{OD} > TL_{\max}, \quad$ set $TL'_{\text{desired}} = TL_{\max}$

or TL'_{desired} may be input if some other specific value is desired. The adjusted premaneuver miss is then:

$$\Delta\mathbf{M}' = \begin{bmatrix} \mathbf{B}\cdot\mathbf{R}_{OD} - \mathbf{B}\cdot\mathbf{R}_{\text{desired}} \\ \mathbf{B}\cdot\mathbf{T}_{OD} - \mathbf{B}\cdot\mathbf{T}_{\text{desired}} \\ TL_{OD} - TL'_{\text{desired}} \end{bmatrix} = \begin{bmatrix} \Delta M_1 \\ \Delta M_2 \\ \Delta M'_3 \end{bmatrix}$$

To satisfy the flight-time constraint, the projection, p, of $\Delta\mathbf{VM}$, which will be of maximum magnitude, $VCAP$, must be such that

$$\Delta\mathbf{VM}\cdot\boldsymbol{\lambda}_3 = -\Delta M'_3$$

$$\therefore p = -\frac{\Delta M'_3}{\lambda_3}$$

The component of $\Delta\mathbf{VM}$ perpendicular to $\boldsymbol{\lambda}_3$ must then be adjusted to minimize the residual miss. The magnitude of this component will be:

$$r = (VCAP^2 - p^2)^{1/2}$$

Define:

$$\mathbf{f} = \frac{\mathbf{N}\times\boldsymbol{\lambda}_3}{|\mathbf{N}\times\boldsymbol{\lambda}_3|}$$

$$\mathbf{g} = \frac{\mathbf{f}\times\boldsymbol{\lambda}_3}{\lambda_3}$$

These two unit vectors are perpendicular to $\boldsymbol{\lambda}_3$ and to each other. Construct the vector:

$$\Delta\mathbf{VM} = p\,\frac{\boldsymbol{\lambda}_3}{\lambda_3} + r(\mathbf{f}\cos\alpha + \mathbf{g}\sin\alpha)$$

The residual miss components are:

$$\Delta M_{1R} = \Delta M_1 + \Delta\mathbf{VM}\cdot\boldsymbol{\lambda}_1$$

$$\Delta M_{2R} = \Delta M_2 + \Delta\mathbf{VM}\cdot\boldsymbol{\lambda}_2$$

To minimize the residual miss, the following function must be minimized:

$$F(\alpha) = \{[\Delta M_{1R}(\alpha)]^2 + [\Delta M_{2R}(\alpha)]^2\}^{1/2}$$

by setting $\alpha = \alpha + \Delta\alpha$ and computing $F(\alpha)$ until α has reached 360°. The value of α, $\alpha_{\min}$, and the corresponding minimum value, $F(\alpha_{\min})$ is to be output as well as $\Delta M_{1R}(\alpha_{\min})$, $\Delta M_{2R}(\alpha_{\min})$, $\Delta\mathbf{VM}(\alpha_{\min})$, and the turns to accomplish $\Delta\mathbf{VM}(\alpha_{\min})$. The computed values of $F(\alpha)$ are automatically plotted vs α to show the behavior of the residual miss near the minimum value.

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