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INTRODUCTION

Whistler data (Carpenter 1963, 1966) and ion spectrometer measurements (Taylor et al., 1965) have shown, that the electron density decreases abruptly and very rapidly above L values of 3 to 5. Recent observations from OGO (Taylor, private communication) confirm this feature of the magnetosphere which is referred to as the plasmapause.

In a paper by Mayr, Brace and Dunham (1967), a study was conducted with the aim of describing and understanding the latitudinal variation of the ion composition (Thomas et al., 1966, Taylor et al., private communication) for the two major ion constituents O^+ and H^+ during solar minimum. It was found that up to 45° geomagnetic latitude, the latitudinal increase of the electron temperature T_e (Brace, Reddy, Mayr 1967) can properly account for the increase of the ion composition transition level. Towards higher latitudes, however, the observed ion composition shows a rapid increase of the transition level which was in contrast to the theoretical result, which indicated a slight decrease of this level. In order to understand this discrepancy Mayr, Brace and Dunham postulated that there exist latitudinal increasing upward fluxes of ionization into protonospheric field tubes, which increase with latitude and which reduce the proton density thus raising the H^+ - O^+ transition level. In order to maintain these fluxes, it was concluded, that ionization has to be removed out of the field tubes through diffusion across field lines, as the effect of a density gradient across field lines maintained through an escape of protons at higher latitude, where the field line configuration permits it. This density gradient across high latitude field lines was considered to be the density plateau constituting the plasmapause. Thus a consistent concept was described, relating plasmapause and ion compositional features at high latitudes.

In this paper the above described concept is investigated. It will be shown that a certain diffusion mechanism postulated by Bohm (1949) can quantitatively account for the above described density plateau and its relation to the ion composition at high latitudes.

THEORY

In the following section a theoretical formulation of our concept is presented. The basic equations are that of particle continuity for H^+ and O^+. Because protons populate primarily the magnetosphere their continuity equation (1) is discussed

$$\frac{\partial [H^+]}{\partial t} = \frac{9}{8} K [H] [O^+] - K [H^+] [O] - \mathrm{div}\, j_{H^+} = 0 \qquad (1)$$

including charge exchange between O^+ and H^+ as the major source of protons. K = is the charge exchange coefficient, j_{H^+} the diffusion flux of H^+.

If diffusion across field lines is included, the diffusion term in Eq. (1) becomes

$$\operatorname{div} j_{H^+} = \nabla_{\perp} j_{\perp} + \nabla_{\parallel} j_{\parallel} \tag{2}$$

where, omitting the subscript H^+, the subscripts $\perp$ and $\parallel$ indicate flux components and derivatives perpendicular and parallel to field lines. $j_{\perp}$ and $j_{\parallel}$ are related through motion equations, to the density gradients across, respectively parallel field lines. Thus Eq. (1) is a partial second order differential equation describing the two dimensional proton distribution along and across field lines.

As boundary conditions we can assume that the diffusion flux and density gradient along the field line is zero at the equator implying an equinox condition. Through the charge exchange reaction, $[H^+]$ is given by an algebraic relation to $[O^+]$ in the charge exchange region at low altitudes, thus providing the second boundary condition for the distribution along field lines.

For the distribution across field lines we can assume that at a field line along which an escape of ionization is possible, the density distribution is very low. Along some inner field line we can assume that the diffusion flux across it is negligibly small. Thus, theoretically the problem is formulated. A rigorous treatment would involve very extensive numerical calculations, so a simplifying approach is made with the aim to get a quantitative insight into the importance of diffusion across field lines.

Due to the lack of chemical sinks and source in the protonosphere, the continuity equation [Eq. (1)] reduces to

$$\operatorname{div} j_H = \nabla_{\perp} j_{\perp} + \nabla_{\parallel} j_{\parallel} = 0 \quad . \tag{4}$$

Without considering the divergence of field tubes Eq. (4) becomes

$$\frac{\partial}{\partial z} j_{\perp} + \frac{\partial}{\partial s} j_{\parallel} = 0 \tag{5}$$

where s is the distance of field lines along and z is the distance of lines across the magnetic field.

Furthermore we assume that $\frac{\partial}{\partial z} j_{\perp}$ is constant along s because of the large scaleheight of the protons. As a consequence of this simplifying assumption Eq. (5) can easily be integrated over s, thus leading to

$$(s_e - s) \frac{\partial}{\partial z} j_{\perp} = j_{\parallel}(s) \tag{6}$$

when s_e is the distance to the equator where $j_{\parallel}(s_e) = 0$.

For $j_{\perp}$ we have employed two different formulas, each of them describing different kinds of diffusion mechanisms. Their implications are discussed.

1. Regular Diffusion

With this notation we understand the drift of plasma across field lines due to collisions between electrons and ions. Referring to Spitzer (1956) we find

$$j_{\perp} = - \frac{1.29 \times 10^{13} \ \ln\Delta \, N}{T_e^{3/2} B^2} \frac{\partial}{\partial z} p \tag{7}$$

when neglecting the effect of the gravitational acceleration which seems justified for higher L values. Employing for p the pressure of the electron-ion gas $p = 2kTN$ (N is electron concentration) and assuming $\ln\Delta = 16$ and $T = 5 \times 10^3$, yields

$$j_{\perp} = -4 \times 10^{-56} \, z^6 N \frac{\partial N}{\partial z} \tag{8}$$

where we adopted

$$B = \frac{B_0 z_e^3}{z^3}$$

with $B_0 = 0.3$ gauss and z_e the earth radius.

Employing Eq. (8) in Eq. (6) leads to

$$\frac{\partial}{\partial z}\left(z^6 N \frac{\partial N}{\partial z}\right) = - j_{\parallel} \frac{2.5 \times 10^{55}}{(s_e - s)} . \tag{9}$$

Up to this point we did not pay any attention to $j_{\parallel}$, the ionization flux parallel to the magnetic field line. It is essential for our argument to discuss this quantity in some detail.

The upward flux of ionization into the protonosphere, the flux $j_{\parallel}$, can depending on its magnitude, very strongly decrease the proton concentration above the charge exchange region. This is the case if the amount of protons flowing upwards is comparable to the net production rate of protons in the charge exchange region. Geisler and Bowhill (1965) have described this dependence in some detail for the proton density in the diffusion barrier. At this level of our investigation we limit our argumentation by stating that the proton density in the protonosphere is generally speaking

$$N = N\left(j_{\parallel},\ [O],\ [H],\ [O^{+}],\ K\right) \tag{10}$$

indicating its dependence on neutral composition, oxygen ion density, charge exchange rate coefficient and above all, indicating its dependence on the transport flux $j_{\parallel}$. By concentrating our attention on the dependence between $j_{\parallel}$ and N we can write

$$j_{\parallel} = j_{\parallel}(N). \tag{11}$$

Going with (11) into (9) yields

$$\frac{\partial}{\partial z}\left(z^6 N \frac{\partial N}{\partial z}\right) = - j_{\shortparallel}(N)\,\frac{2.5 \cdot 10^{55}}{(s_e - s\ )}. \qquad (12)$$

Equation (12) describes the coupling between the density variation across the magnetic field lines and the density on the field line.

To put it in a more illustrative and physical picture one could interpret (12) by saying that the net diffusion flux out and across a field tube according to a density variation, has to be balanced by an upward flux of ions into the tube, which in turn effects the density in the tube itself.

We have solved Eq. (12) by assuming that within the range of our solution $j_{\shortparallel}$ is constant. This assumption is justified by the fact, that according to Geisler and Bowhill (1965), the density is a very sensitive function of $j_{\shortparallel}$ in the vicinity of the critical flux. Thus as a consequence of this assumption, our results will be valid only for fluxes which are in the order of the critical flux. This essential limitation is to be stressed. The implications are of importance for a later argument.

With the above assumption the integration of Eq. (12) leads to

$$N^2 = N_1^2 + j_{\shortparallel} \cdot \frac{2.5 \times 10^{55}}{(s_e - s\ )}\,\frac{1}{10}\left\{\frac{1}{z^5}(5z - 4z_0) - \frac{1}{z_1^5}(5z_1 - 4z_0)\right\}$$

$$+ \left(\frac{\partial N^2}{\partial z}\right)_0 \frac{z_0^6}{5}\left(\frac{1}{z_1^5} - \frac{1}{z^5}\right). \qquad (13)$$

N_1 is assumed as the proton density at high latitudes where an escape of ionization is possible.

A very simple estimation of the escape effect may help to determine an approximate value of N_1. Following Jastrow and Rasool (1965) we find that the density at the escape level is in the order of 10^5/cc. Thus we learn, that the escape of H^+ becomes effective immediately above the charge exchange region and so we may adopt for the escape level (somewhere between 500 and 1000 km) a temperature of 3.10^{3}°K. This leads according to Jastrow and Rasool to the escape flux

$$j_e = .8 \times 10^5 \cdot N_1$$

which is in principle equivalent to the previously defined flux $j_{\parallel}$.

A very simple argument allows to evaluate the above relation. Obviously, the upper limit of N_1 is determined by the critical flux, which represents an upper limit of the proton flux. So we find, that for an assumed critical flux of $10^6/\mathrm{cm}^2\mathrm{sec}$, N_1 is in the order of 10/cc, which is an extremely low value. On the other side, if we allow the flux j_e to be appreciably smaller than the critical flux, it would not effect the H^+ density and thus N_1 would be a sort of diffusive equilibrium value which is at the escape level much larger than the permitted maximum value of 10/cc.

From this we conclude that j_e has to be assumed to be in the vicinity of the critical flux and thus N_1 may be as low as 10/cc due to escape of ionization.

Returning to Eq. (13) we can now define our boundary conditions. The following boundary conditions were employed which are considered to represent typical features of the plasmapause.

$$\text{for} \qquad z_0 = 4\,r_e\,, \quad \left(\frac{\partial N^2}{\partial z}\right)_0 = 0$$
$$\text{and for} \qquad z_1 = 6\,r_e\,, \quad N_1 = 10/\text{cc} \tag{14}$$

This means that we assume, that at $L = 6$ escape of ionization reduces the proton density to 10/cc and that below $L = 4$ this escape of ionization is almost ineffective thus reducing the diffusion flux across field lines to a negligible small amount.

With Eq. (14), Eq. (13) becomes

$$N^2 = 100 + j_{\parallel} \frac{2.5 \times 10^{55}}{(s_e - s\ )} \frac{1}{10} \left\{ \frac{1}{z^5} (5z - 4z_0) - \frac{1}{z_1{}^5} (5z_1 - 4z_0) \right\} . \tag{15}$$

which can be interpreted as a relation between N and $j_{\parallel}$.

Postulating that according to satellite observations $N(z_0)$ is in the order of 10^3/cc, we get a density distribution as shown in Fig. 1 (dashed line), that has the shape of distributions shown from OGOA satellite. However it is essential, that the flux $j_{\parallel}$ has to be assumed in the order of $2/cm^2sec$, which is by several orders of magnitudes smaller than the critical flux, lieing in the range from 10^5 to $10^8/cm^2sec^{-1}$. Such a small flux, being only a fraction of the critical flux has a negligible effect on the proton density.

The quite reasonable density decrease shown in Fig. 1 causes such a small diffusion flux across field lines, that the balancing proton flux into the field tubes is ineffective in reducing the density consistently with the density decrease across field lines. The diffusion mechanism employed here is inappropriate in describing the plasmapause.

2. Irregular Diffusion

Because of this failure, the question arises whether another diffusion mechanism could possibly be responsible for the observed density distribution.

Spitzer (1956) also refers to a different diffusion mechanism, which was suggested by Bohm (1949). This diffusion is produced by a sort of turbulence or plasma oscillations with randomly varying electrostatic fields. Braginskii (1965) points out, that when $\omega\tau \gg 1$, plasma turbulence can in principle cause a strong enhancement of perpendicular (to the magnetic field) transport processes. Though little knowledge exists on the conditions under which plasma oscillations are excited, we have employed Bohm's diffusion coefficient. As will be shown, it offers an appropriate mean in describing features of the plasmapause.

With Bohm's diffusion coefficient the flux perpendicular to field lines is

$$j_{\perp} = -\frac{3.9 \times 10^{18}}{B} \frac{\partial p}{\partial z} . \tag{16}$$

Employing formula (16) instead of (7), we proceed like in the previous section, adopting again the boundary conditions (14). The result is the relation

$$N = 10 + \frac{j_{\parallel}\, 1.4 \times 10^{+19}}{(s_e - s)} \cdot \frac{1}{2} \left\{ \frac{1}{z_1^2} (2 z_1 - z_0) - \frac{1}{z^2} (2z - z_0) \right\} . \tag{17}$$

Postulating again, that at $L = 4$, $N = 10^3$/cc, we get a distribution also shown in Fig. 1 (solid line). The main difference to the preceding result is, that now $j_{\parallel}$ is in the order of 10^6/cm sec. This is a flux which is in the vicinity of the critical flux, thus it can effectively decrease the proton density and thus the density variation shown in Fig. 1 (solid line) can be consistent to the density variation along the corresponding field lines, which was not the case employing the regular diffusion mechanism.

As pointed out previously, Eq. (15) and Eq. (17) contain the assumption, that the flux $j_{\parallel}$ is constant. This is valid if $j_{\parallel}$ is close to the critical flux, because then the proton concentration is a very sensitive function of $j_{\parallel}$ and thus, in turn, this flux is nearly constant for a wide range of $[H^+]$. Obviously this condition was not fulfilled employing the regular diffusion mechanism with (15) and consequently the dashed distribution in Fig. 1 is invalid. With Bohm's diffusion coefficient in (17) however the resulting flux $j_{\parallel}$ comes into the range of the critical flux and thus can satisfy the above condition. Therefore the solid distribution in Fig. 1 may be considered as reasonable.

Comparing our calculated density variation with those seen from OGO-A (Taylor et al., 1965) shows relatively good agreement. However our result appears to be in contrast to IMP 11 results by Serbu and Maier (1966) which indicate a density decrease with z^{-3}.

In addition, the decrease of $[H^+]$ due to escape and diffusion across field lines increases the O^+ - H^+ transition level, thus providing a possible explanation for the discrepancy between the cited ion composition measurements (Taylor, Thomas) and theoretical results (Mayr et al.) on the ion composition at higher latitudes.

The conditions under which Bohm's diffusion mechanism is appropriate are not well understood. It seems plausible, that under magnetospheric conditions of low density and magnetic field strength, plasma instabilities and oscillations may relatively easily excited, so for example by the interaction with electron beams. Yet, the applicability of this diffusion mechanism in the magnetosphere is not established. Our phenomenological approach merely points at the possibility that it might be valid there.

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REFERENCES

1. Bohm, D. — The characteristics of electrical discharges in magnetic fields, Ed. A. Guthrie and R. K. Wakerling, 62-65, McGraw-Hill Book Co., 1949.

2. Braginskii, S. I., Transport processes in a plasma, Ed. M. A. Leontovich, 235-236, Consultants Bureau, New York, 1965.

3. Brace, L. H., B. M. Reddy and H. G. Mayr — Global behaviour of the inosphere at 1000 km altitude, J. Geophys. Res., 72, 1967.

4. Carpenter, D. L. — Whistler evidence of a "knee" in the magnetospheric ionization density profile, J. Geophys. Res., 68, 1675-1682, 1963.

5. Carpenter, D. L. — Whistler studies of the plasmapause in the magnetosphere, 1, Temporal variation in the position of the knee and some evidence on plasma motions over the knee, J. Geophys. Res., 71, 693-710, 1966.

6. Geisler, J. E. and S. A. Bowhill — An investigation of ionosphere-protonosphere coupling, Aeronomy Report No. 5, University of Illinois, Urbana, 1965.

7. Jastrow R. and S. I. Rasool, Introduction to Space Science, Ed. W. N. Hess, Gordon and Breach Science Publishers, 1965.

8. Mayr, H. G., L. H. Brace, and G. S. Dunham — On the ion composition and temperature in the topside ionosphere, NASA X-621-66-573, 1966.

9. Serbu, C. P., and E. J. R. Maier — Low-energy electrons measured on IMP 2, J. Geophys. Res., 71, 3755-3766, 1966.

10. Spitzer, L., Jr., — Physics of fully ionized gases, Interscience Publishers, Inc., New York, 1956.

11. Taylor, H. A., Jr., H. C. Brinton, and C. R. Smith — Positive ion composition in the magnetosphere obtained from OGOA satellite, J. Geophys. Res., 70, 5769-5791, 1965.

12. Thomas, J. O., M. J. Rycroft, L. Colin, and K. L. Chan — Electron density profiles in ionosphere and exosphere, proceedings of the Nato Advanced Study Institute held at Finse, Norway, Edit. Jon Frihagen, North-Holland Publishing Company, Amsterdam, 1966.

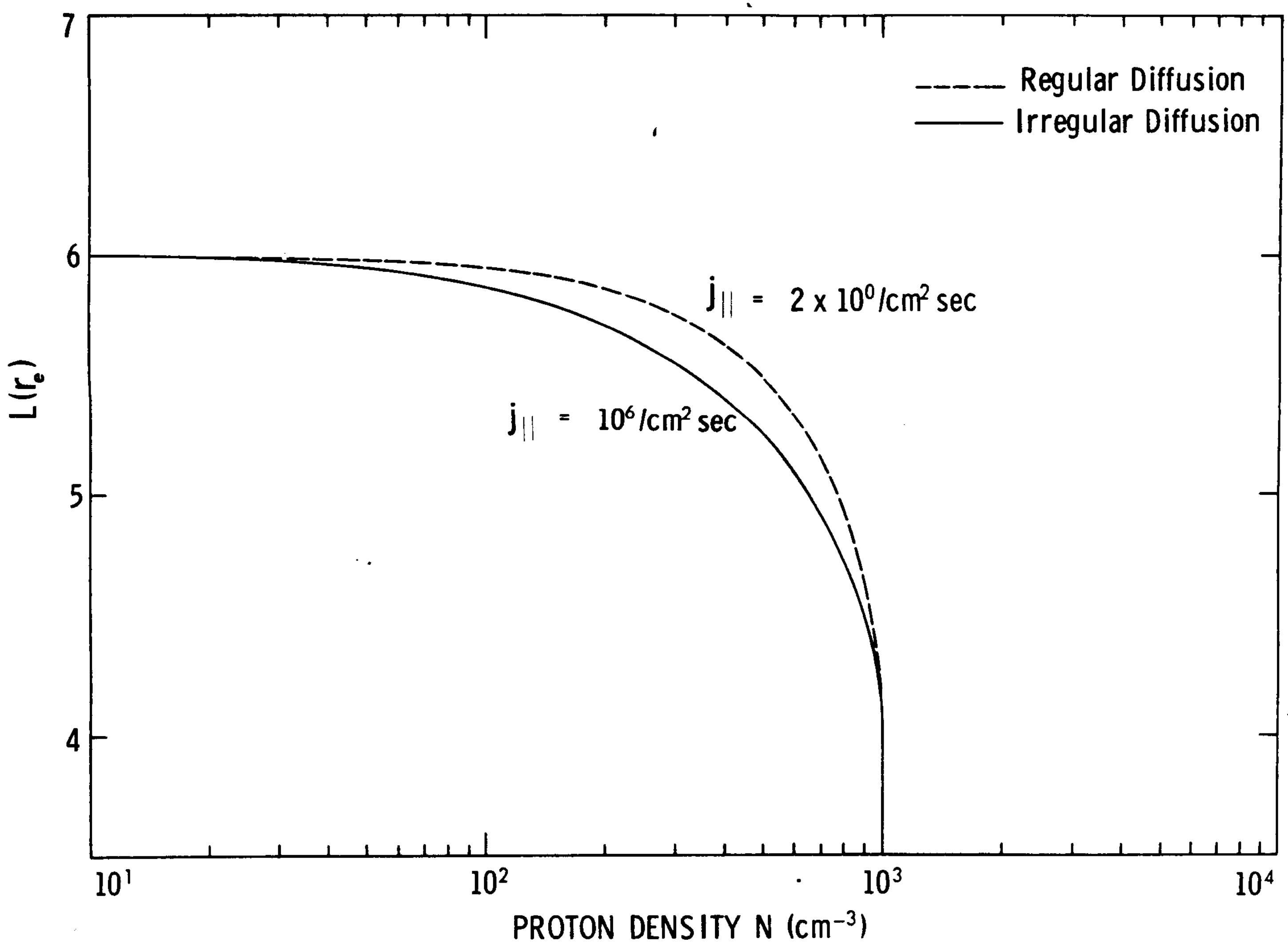

Figure 1. Density distribution between L values 4 and 6 for different diffusion mechanisms. Note the very different values of the proton flux $j_{||}$, on which account clearly the irregular diffusion has to be considered as appropriate.