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RELATIONSHIP BETWEEN MAGNITUDE OF APPLIED SPIN RECOVERY MOMENT AND ENSUING NUMBER OF RECOVERY TURNS

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SUMMARY

An analytical study has been made to investigate the relationship between the magnitude of the applied spin recovery moment and the ensuing number of turns made during recovery from a developed spin with a view toward determining how to interpolate or extrapolate spin recovery results with regard to determining the amount of control required for a satisfactory recovery. Five configurations were used which are considered to be representative of modern airplanes: a delta-wing fighter, a stub-wing research vehicle, a boostglide configuration, a supersonic trainer, and a sweptback-wing fighter.

The results obtained indicate that there is a direct relationship between the magnitude of the applied spin recovery moments and the ensuing number of recovery turns made and that this relationship can be expressed in either simple multiplicative or exponential form. Either type of relationship was adequate for interpolating or extrapolating to predict turns required for recovery with satisfactory accuracy for configurations having relatively steady recovery motions. Any two recoveries from the same developed spin condition can be used as a basis for the predicted results provided these recoveries are obtained with the same ratio of recovery control deflections. No such predictive method can be expected to give satisfactory results for oscillatory recoveries.

INTRODUCTION

Spin research experience has shown that the effect of any control in bringing about a recovery from a developed spin depends upon the moments that control provides and upon the effectiveness of those moments in producing a change in angular velocity and thus an upsetting of the spin equilibrium. (See ref. 1.) The relative effectiveness of pitching, rolling, and yawing moments depends upon the mass distribution of the airplane; in particular, it depends on whether the vehicle is mass loaded more heavily along its fuselage or its wing, and by the degree of difference between the wing and fuselage loading. (See ref. 1.) With each particular mass loading, there is a particular combination of control deflections which are considered to be the optimum control deflections for recovery.

For a particular combination of control deflections, the larger the magnitude of these deflections, the larger will be the applied recovery moment, the larger will be the corresponding change produced in the angular velocity, and the more rapid should be the ensuing recovery. (For example, see ref. 2.) Part of the results presented in reference 2 was analyzed by assuming there was a linear relationship between the amount of applied recovery moment used and the ensuing number of turns made during recovery. However, an examination of the results shown in reference 2 indicates that this relationship is not linear.

The present investigation was therefore made to determine analytically a more accurate relationship between the magnitude of the applied recovery moment and the number of turns made during recoveries from a developed spin. Such a relationship would be expected to be of value in interpolating or extrapolating the recovery characteristics of experimental spin tests and spin analyses. The relationship might also afford a building block toward generalizing or simplifying empirical or theoretical spin analysis.

SYMBOLS

The body system of axes was used in the calculations. This system of axes, related angles, and positive directions of corresponding forces and moments are illustrated in figure 1. The International System of Units is used throughout, with the U.S. Customary Units included in the parentheses. The International Units were obtained by using the conversion factors of reference 3.

b wing span, meters (feet)

C_l rolling-moment coefficient, $\dfrac{M_X}{\frac{1}{2}\rho V_R^2 Sb}$

C_m pitching-moment coefficient, $\dfrac{M_Y}{\frac{1}{2}\rho V_R^2 S\bar{c}}$

C_n yawing-moment coefficient, $\dfrac{M_Z}{\frac{1}{2}\rho V_R^2 Sb}$

C_X longitudinal-force coefficient, $\dfrac{F_X}{\frac{1}{2}\rho V_R^2 S}$

C_Y side-force coefficient, $\dfrac{F_Y}{\frac{1}{2}\rho V_R^2 S}$

C_Z normal-force coefficient, $\dfrac{F_Z}{\frac{1}{2}\rho V_R^2 S}$

$C_{l_p} = \dfrac{\partial C_l}{\partial \frac{pb}{2V_R}}$

$C_{l_r} = \dfrac{\partial C_l}{\partial \frac{rb}{2V_R}}$

$C_{l_\beta} = \dfrac{\partial C_l}{\partial \beta}$

$C_{l_{\delta_a}}$ rolling-moment coefficient due to aileron deflection per degree

$C_{l_{\delta_r}}$ rolling-moment coefficient due to rudder deflection per degree

$C_{m_q} = \dfrac{\partial C_m}{\partial \frac{q\bar{c}}{2V_R}}$

$C_{m_{\delta_e}}$ pitching-moment coefficient due to elevator deflection per degree

$C_{n_p} = \dfrac{\partial C_n}{\partial \frac{pb}{2V_R}}$

$C_{n_r} = \dfrac{\partial C_n}{\partial \frac{rb}{2V_R}}$

$C_{n_\beta} = \dfrac{\partial C_n}{\partial \beta}$

$C_{n_{\delta_a}}$ yawing-moment coefficient due to aileron deflection per degree

$C_{n_{\delta_r}}$ yawing-moment coefficient due to rudder deflection per degree

$C_{X_{\delta_e}}$ longitudinal-force coefficient due to elevator deflection per degree

$C_{Y_\beta} = \dfrac{\partial C_Y}{\partial \beta}$

$C_{Y_{\delta_a}}$ side-force coefficient due to aileron deflection per degree

$C_{Y_{\delta_r}}$ side-force coefficient due to rudder deflection per degree

$C_{Z_{\delta_e}}$ normal-force coefficient due to elevator deflection per degree

$\Delta C, \Delta C_b, \Delta C_c$ total recovery-moment coefficient (yawing or rolling) due to ailerons and rudder applied for recovery (for instance, $\Delta C_n = \Delta C_{n,r} + \Delta C_{n,a}$); subscripts b and c refer to total coefficient applied for corresponding recovery number in table III, and where two subscripts are included in same equation, those subscripts are not equal

$\Delta C_{n,a} = C_{n_{\delta_a}} \delta_a$

$\Delta C_{n,r} = C_{n_{\delta_r}} \delta_r$

ΔC_x total recovery-moment coefficient (yawing or rolling) used in obtaining predicted number of turns made during recovery N_x; subscript refers to recovery number in table III

$\bar{c}$ mean aerodynamic chord, meters (feet)

F_X longitudinal force acting along X body axis, newtons (pounds)

F_Y side force acting along Y body axis, newtons (pounds)

F_Z normal force acting along Z body axis, newtons (pounds)

g acceleration due to gravity, meters/second^2 (taken as 32.17 feet/second^2)

h altitude, meters (feet)

h_o initial altitude, meters (feet)

I_X, I_Y, I_Z moment of inertia about X-, Y-, and Z-axis, respectively, kilogram-meter^2 (slug-foot^2)

$\frac{I_X - I_Y}{mb^2}$ inertia yawing-moment parameter

K multiplicative recovery factor, factor relating magnitude of total coefficient applied for recovery and number of turns made during recovery

M_X rolling moment acting about X body axis, newton-meters (foot-pounds)

M_Y pitching moment acting about Y body axis, newton-meters (foot-pounds)

M_Z yawing moment acting about Z body axis, newton-meters (foot-pounds)

m mass, $\frac{W}{g}$, kilograms (slugs)

N,N_b,N_c number of turns made during recovery; subscripts b and c refer to calculated number of turns for the corresponding recovery number in table III, and where two subscripts are included in the same equation, those subscripts are not equal

N_x predicted number of turns made during recovery; subscript refers to recovery number in table III

p,q,r component of resultant angular velocity about X, Y, and Z body axis, respectively, radians/second

R exponential recovery factor, factor relating the magnitude of total coefficient applied for recovery and the number of turns made during recovery

S wing area, meter^2 (foot^2)

t time, sec

u,v,w component of resultant linear velocity V_R along X, Y, and Z body axis, respectively, meters/second (feet/second)

V_R resultant linear velocity, meters/second (feet/second)

W weight, newtons (pounds)

X,Y,Z body axes

α angle of attack, angle between relative wind V_R projected into XZ-plane of symmetry and X body axis, positive when relative wind comes from below XY body plane, degrees

β angle of sideslip, angle between relative wind V_R and projection of relative wind on XZ-plane, positive when relative wind comes from right of plane of symmetry, degrees

δ_a aileron deflection with respect to chord line of wing, left or positive when trailing edge of right aileron down, degrees

δ_e elevator deflection with respect to fuselage reference line, positive with trailing edge down, degrees

δ_r — rudder deflection with respect to fin, left or positive when trailing edge to left, degrees

ρ — air density, kilograms/meter^3 (slugs/foot^3)

θ_e — total angular movement of X body axis from horizontal plane measured in vertical plane, positive when airplane nose is above horizontal plane, radians or degrees

ϕ_e — total angular movement of Y body axis from horizontal plane measured in YZ body plane, positive when clockwise as viewed from rear of airplane (if X body axis is vertical, ϕ_e is measured from reference position in horizontal plane), radians or degrees

ϕ — angle between Y body axis and horizontal measured in vertical plane, positive for erect spins when right wing downward and for inverted spins when left wing downward, radians or degrees

ψ_e — horizontal component of total angular deflection of X body axis from reference position in horizontal plane, positive when clockwise as viewed from vertically above airplane, radians or degrees

A dot over a symbol represents derivative with respect to time; for example, $\dot{u} = \frac{du}{dt}$.

METHODS AND CALCULATIONS

The developed spin and spin recovery motions were calculated by using a high-speed digital computer which solved the equations of motion and associated formulas given in the appendix. These equations of motion represent six degrees of freedom along and about the body system of axes. (See fig. 1 for illustration of body axes.)

A sketch showing the planforms of the configurations used is presented in figure 2. Configuration A represents a delta-wing fighter, configuration B represents a stub-wing research vehicle, configuration C represents a boostglide configuration, configuration D represents a supersonic trainer, and configuration E represents a sweptback-wing fighter. The mass and dimensional characteristics of these configurations are given in table I.

The aerodynamic data inputs used in the digital computer calculations are presented in figures 3 to 8. The data for configurations A, B, C, D, and E were obtained from references 4, 5, 6, 7, and 8, respectively. Values of the derivative C_{m_q} used in the pitching equation of motion (appendix) were constant for all angles of attack and were -0.45 for configuration A, -10.0 for configuration B, -0.8 for configuration C, -6.6 for configuration D, and -2.0

for configuration E. The bases for the selection of these values are given in references 4 to 8.

The developed spins were calculated in a manner similar to the spin tunnel testing technique. (See ref. 4.) That is, the initial conditions used assumed a very high angle of attack with applied rotation about a vertical spin axis. From this condition, the spin parameters will undergo some intermediate motions until the configuration eventually achieves its own equilibrium developed spin (turning toward the pilot's right). The initial conditions used for these calculations are shown in table II.

Spin recovery attempts were made by deflecting the rudder against the direction of rotation and the ailerons with the direction of roll (left rudder and right ailerons when in an erect spin to the pilot's right). These are the optimum control deflections for recovery from developed spins for airplanes loaded relatively heavily along the fuselage (ref. 1), as are the configurations investigated herein. The elevators remained in the original up position at all times. A spin is normally considered terminated when either the spin rotation ceases or the angle of attack becomes and remains less than the stall angle. Usually when the angle of attack becomes less than the stall angle, the airplane enters a steep dive without significant rotation ($\dot{\psi}_e \approx 0$). In some cases, however, the airplane may be turning or rolling in a spiral glide or an aileron roll. Also, sometimes the airplane may roll or pitch to an inverted attitude from the erect spin and may still have some rotation, but it is considered to be out of the original erect spin.

For each of the configurations investigated, a single erect developed spin was obtained. From each developed spin, several recoveries were made by varying the magnitudes of the applied recovery moments. For convenience, this was accomplished herein by multiplying the control effectiveness data shown in figures 5 and 6 by varying magnitudes of control deflections. These control deflections were made in such a manner that the ratio of δ_r to δ_a remained the same to insure that the same ratio of incremental rolling and yawing recovery moments were acting at any time during the recovery.

The recovery results for the configurations having steady recovery motions were then utilized to determine whether some simple relationship between the applied recovery moment and the ensuing number of recovery turns could be devised. Any relationship thus obtained is to be used to predict the recovery turns for any magnitude of applied recovery moment desired based on an interpolation or extrapolation of two points. However, spin research has shown that, in general, no such simple relationship based on an interpolation or extrapolation of two points can be expected to predict oscillatory recovery results. For example, even several recoveries obtained from a single oscillatory spin using the same magnitude of applied recovery moment but initiated during different phases of the oscillation, would show a significant variation of turns for recovery.

RESULTS AND DISCUSSION

Developed Spin and Spin Recovery Calculations

Sample results of the developed spin and spin recovery calculations for each configuration are presented in figures 9 to 13 in the form of time histories. Each calculation allowed time for any oscillatory disturbances caused by the specific combinations of initial conditions used to damp out and for the configuration to achieve as steady a developed spin condition as possible; then recovery controls were applied. Several recoveries were obtained from the developed spin for each configuration by varying the magnitudes of the applied recovery moment, represented by the magnitudes of the recovery control deflections. The magnitudes of the recovery control deflections used and the ensuing number of turns made during the recovery calculations for each configuration are presented in figure 14. The sample recovery for configurations A, B, and C presented in figures 9 to 11 corresponds to recovery 1 for that configuration in table III.

The results of the calculations for configurations A, B, and C show that these configurations had relatively flat and steady developed spins and that, after the application of recovery controls, the angle of attack and rate of rotation decreased steadily until recovery was achieved. On the other hand, the results of the calculations show that configuration D also had a relatively flat and steady developed spin but that the motions during recovery became oscillatory. Configuration E had both an oscillatory developed spin and an oscillatory recovery motion.

Analysis of Calculated Recoveries

A summary of the recovery calculations is presented in figure 14 where recovery turns are shown as a function of the applied recovery moment, represented by the magnitude of the aileron deflections used for recovery. This plot shows that the recovery turns are not a linear function of the magnitude of applied recovery moment. Furthermore, figure 14 shows that a progressively increasing increment of applied recovery moment is required to obtain the same incremental improvement in recovery turns as the number of recovery turns decreases. Figure 14 also shows that the curves for configurations A, B, and C, which have relatively steady recovery motions, also have a smooth regular variation of turns required for recovery with the applied recovery moment. On the other hand, configurations D and E, which had relatively more oscillatory recoveries, exhibit a more erratic variation of turns for recovery with applied recovery moment. These trends are as would be expected for oscillatory spins based on spin research experience, and the variations shown serve to indicate the relatively greater error which might occur if any relationship between the magnitude of recovery moment and recovery turns were to be used as a predictive relationship. If enough results were available, a smooth curve could probably be drawn to eliminate these more erratic variations, and a general trend would be obtained similar to that encountered for the steady recoveries. This approach would give a more accurate prediction of recovery turns for the oscillatory recoveries than would a linear prediction. However, it would require so many results to give a reasonable basis for the predictive method that it would probably have been as easy to have used that same number of results to obtain the information desired in a more direct manner. In fact, no simple method of extrapolation or interpolation based on calculations for two points can be relied on for cases involving oscillatory recoveries. For example, an extrapolation based on recoveries 3 and 4 for configurations D and E would have been grossly in error. Consequently, no attempt has been made to show a correlation of actual results with any method of extrapolation for the cases of the oscillatory recoveries of configurations D and E.

Presentation of the multiplicative relationship.- One method of relating the magnitude of the recovery moment and the number of turns made during recovery was devised based on the assumption that some multiplicative relationship existed. Obviously, this relationship cannot be of the simple form $N_b - N_c = K(\Delta C_c - \Delta C_b)$, since the results shown in figure 14 were not linear.

The magnitudes of the recovery moments (represented in table III by the magnitude of the deflection of δ_a) and the turns made during recovery were nondimensionalized in the form $\frac{N_b}{N_c}$ and $\frac{\Delta C_c}{\Delta C_b}$, respectively. An example of these nondimensionalized results, where $N_b = N_1$, are shown by the symbols presented for each configuration in figure 15. An examination of figure 15 indicates that the results for configurations A, B, and C (the configurations which had relatively steady recovery motions) show an approximately linear relationship of $\frac{N_1}{N_c}$ with $\frac{\Delta C_c}{\Delta C_1}$. However, by virtue of the nondimensionalizing method used, the minimum values of $\frac{N_1}{N_c}$ or $\frac{\Delta C_c}{\Delta C_1}$ will be 1.0. The multiplicative relationship will therefore be of the form $\left(\frac{N_1}{N_c} - 1\right) = K\left(\frac{\Delta C_c}{\Delta C_1} - 1\right)$, where K represents the slope of the linear relation of $\frac{N_1}{N_c}$ with $\frac{\Delta C_c}{\Delta C_1}$ and is referred to as the multiplicative recovery factor.

Values of K were obtained for each possible combination of pairs of recoveries for each configuration. The range of values for K thus obtained for each configuration is presented in table III.

Each value of K obtained from a particular pair of calculated recoveries was used to predict the recovery turns for each of the five remaining magnitudes of recovery controls used. These predicted recovery turns were computed by use of the following equations:

if $\Delta C_x < \Delta C_b$

$$N_x = \left[K\left(\frac{\Delta C_b}{\Delta C_x} - 1\right) + 1\right]N_b$$

if $\Delta C_x > \Delta C_b$

$$N_x = \frac{N_b}{K\left(\frac{\Delta C_x}{\Delta C_b} - 1\right) + 1}$$

where ΔC_b and N_b are from one of the particular pair of recoveries used to determine the value of K used.

The range of predicted recovery turns thus obtained is presented in table III and is also shown in figure 16 as the band between the pair of curves for each configuration. The average percent error of the predicted recovery turns compared to each of the calculated recovery turns is also shown in table III. An examination of these average percent errors indicates that, in general, for a configuration having steady recovery motions (configurations A, B, and C), the multiplicative recovery factor is capable of predicting the calculated turns for recovery with an accuracy in the order of 1 percent error.

Presentation of the exponential relationship.- Another method of relating the magnitude of the recovery moment and the number of turns made during recovery was devised based on the assumption that some exponential relationship existed. Again, the magnitudes of the recovery moments (represented in table III by the magnitude of the deflection of δ_a) and the turns made for recovery were nondimensionalized in the form $\frac{N_b}{N_c}$ and $\frac{\Delta C_c}{\Delta C_b}$, respectively. The exponential relationship was then assumed to be of the form $\left(\frac{N_b}{N_c}\right) = \left(\frac{\Delta C_c}{\Delta C_b}\right)^R$ where R is referred to as the exponential recovery factor.

Values of R were obtained for each possible combination of pairs of recoveries for each configuration. The range of values of R thus obtained is presented for each configuration in table III.

Each value of R obtained from a particular pair of calculated recoveries was used to predict the recovery turns for each of the five remaining magnitudes of recovery controls used. These predicted recovery turns were computed by the use of the following equations:

if $\Delta C_x < \Delta C_b$

$$N_x = N_b\left(\frac{\Delta C_b}{\Delta C_x}\right)^R$$

if $\Delta C_x > \Delta C_b$

$$N_x = \frac{N_b}{\left(\frac{\Delta C_x}{\Delta C_b}\right)^R}$$

where ΔC_b and N_b are from one of the particular pair of recoveries used to determine the value of R used.

The range of predicted recovery turns thus obtained is presented in table III and is also shown in figure 17 as the band between the pair of curves for each configuration. The average percent error of the predicted recovery turns compared to each of the calculated recovery turns is also shown in table III. An examination of these average percent errors indicates that, in general, the exponential recovery factor is capable of predicting the calculated turns for recovery with an accuracy of approximately 1 percent error for configurations having steady recovery motions.

General Remarks

A comparison of the predicted recovery results for both the multiplicative and exponential relationships indicates that the use of either method will give excellent interpolation or extrapolation of recovery characteristics and that both methods have about the same average percent error when compared with the calculated recoveries. The two methods are about equally easy to apply. Hence, at the present time, there does not seem to be any basis for preferring one method or the other.

When using either of the relationships presented herein, a total recovery moment or moment coefficient (in the form ΔC) should normally be used rather than a recovery control deflection angle. The control effectiveness coefficients for most configurations probably vary somewhat with the control deflection angle. (For instance, see the variations of rudder effectiveness of ref. 7.) In some cases, the control effectiveness coefficients may vary to such an extent that increasing a control deflection angle by as much as twice its original setting may not even increase the total recovery-moment coefficient. (For instance, see the variations of $\Delta C_{n,a}$ in ref. 7.)

The values of K or R presented herein were determined on the basis of a single developed spin for each configuration. Different combinations of prospin control deflections or different simulated mass loadings or altitudes will result in different developed spins, each of which will require a separate recovery analysis. For any single developed spin, different combinations of recovery control deflections will probably result in different values of K or R and will therefore also require separate recovery analysis. Additional analyses must be made to determine whether the K or R values can be made to apply to a configuration in general instead of only to an individual pair of spin recoveries for a single developed spin as was done herein and to determine whether either K or R can be broken down into some more detailed algebraic relationship involving aerodynamic factors and/or mass and dimensional characteristics.

CONCLUSIONS

Based on the results of an analytical study in which five configurations were used to investigate the relationship between the magnitude of the applied recovery moment and the number of turns made during recovery from a developed spin, the following conclusions are drawn:

1. The number of turns required for recovery is not a linear function of the magnitude of the applied recovery moment. Instead, a progressively increasing increment of applied recovery moment is required to obtain the same incremental improvement in recovery turns as the number of recovery turns decreases.

2. There are two simple relationships between the magnitude of the applied recovery moment and the ensuing number of recovery turns made, and these relationships were expressible in either simple multiplicative or exponential form. With either of these relationships, any two recoveries from the same developed spin condition can be used as a basis for the predicted interpolations or extrapolations provided those two recoveries were obtained with the same ratio of recovery control deflections.

3. Both the multiplicative and the exponential relationships were shown to be adequate for interpolating or extrapolating to predict the calculated turns for recovery with satisfactory accuracy for configurations having relatively steady recovery motions.

4. No such simple predictive method of interpolation or extrapolation based on two points can be expected to give satisfactory results for oscillatory recoveries.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., September 12, 1966,
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APPENDIX

EQUATIONS OF MOTION AND ASSOCIATED FORMULAS

The equations of motion used in calculating the spinning motions were

$$\dot{p} = \frac{I_Y - I_Z}{I_X} qr + \frac{\rho V_R^2 Sb}{2I_X}\left[C_{l_\beta}\beta + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r + \frac{b}{2V_R}\left(C_{l_p}p + C_{l_r}r\right)\right]$$

$$\dot{q} = \frac{I_Z - I_X}{I_Y} pr + \frac{\rho V_R^2 S\bar{c}}{2I_Y}\left(C_m + C_{m_{\delta_e}}\delta_e + \frac{\bar{c}}{2V_R} C_{m_q}q\right)$$

$$\dot{r} = \frac{I_X - I_Y}{I_Z} pq + \frac{\rho V_R^2 Sb}{2I_Z}\left[C_{n_\beta}\beta + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r + \frac{b}{2V_R}\left(C_{n_p}p + C_{n_r}r\right)\right]$$

$$\dot{u} = -g \sin \theta_e + vr - wq + \frac{\rho V_R^2 S}{2m}\left(C_X + C_{X_{\delta_e}}\delta_e\right)$$

$$\dot{v} = g \cos \theta_e \sin \phi_e + wp - ur + \frac{\rho V_R^2 S}{2m}\left(C_{Y_\beta}\beta + C_{Y_{\delta_a}}\delta_a + C_{Y_{\delta_r}}\delta_r\right)$$

$$\dot{w} = g \cos \theta_e \cos \phi_e + uq - vp + \frac{\rho V_R^2 S}{2m}\left(C_Z + C_{Z_{\delta_e}}\delta_e\right)$$

In addition, the following auxiliary formulas were used:

$$\alpha = \tan^{-1} \frac{w}{u}$$

$$\beta = \sin^{-1} \frac{v}{V_R}$$

$$V_R = \sqrt{u^2 + v^2 + w^2}$$

APPENDIX

$$\dot{h} = u \sin \theta_e - v \cos \theta_e \sin \phi_e - w \cos \theta_e \cos \phi_e$$

$$h = h_o + \int \dot{h} \, dt$$

$$\dot{\theta}_e = q \cos \phi_e - r \sin \phi_e$$

$$\dot{\psi}_e = \frac{\dot{\phi}_e - p}{\sin \theta_e}$$

$$\phi = \sin^{-1}(\sin \phi_e \cos \theta_e)$$

$$N = \frac{\int \dot{\psi}_e \, dt}{2\pi}$$

$$\dot{\phi}_e = p + r \tan \theta_e \cos \phi_e + q \tan \theta_e \sin \phi_e$$

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TABLE I.- MASS AND DIMENSIONAL CHARACTERISTICS

	Configuration A		Configuration B		Configuration C		Configuration D		Configuration E	
m, kg (slugs)	11 254	(771.1)	5704	(390.8)	3538	(242.4)	4554	(312.0)	10 782	(738.8)
S, m^2 (ft^2)	64.57	(695.05)	18.58	(200)	31.47	(338.72)	15.79	(170)	35.80	(385.33)
b, m (ft)	11.62	(38.12)	6.82	(22.36)	6.00	(19.7)	7.70	(25.25)	10.87	(35.67)
$\bar{c}$, m (ft)	7.24	(23.755)	3.13	(10.27)	6.36	(20.875)	2.36	(7.73)	3.61	(11.83)
Center of gravity, percent $\bar{c}$		30.0		19.5		43		21.5		33
I_X, kg-m^2 (slug-ft^2)	18 438	(13 600)	5814	(4288)	3806	(2807)	2305	(1700)	15 875	(11 709)
I_Y, kg-m^2 (slug-ft^2)	173 539	(128 000)	99 492	(73 384)	20 943	(15 447)	39 995	(29 500)	112 060	(82 654)
I_Z, kg-m^2 (slug-ft^2)	187 096	(138 000)	101 502	(74 867)	23 460	(17 304)	40 809	(30 100)	120 985	(89 237)
$\frac{I_X - I_Y}{mb^2}$		-1021×10^{-4}		-3536×10^{-4}		-1343×10^{-4}		-1397×10^{-4}		-825×10^{-4}
δ_e, deg		-25		-30		-20		-15		-30
δ_a, deg		± 7		$\pm 7\frac{1}{2}$		± 20		± 60		± 15
δ_r, deg		± 25		$\pm 7\frac{1}{2}$		± 20		± 6		± 6

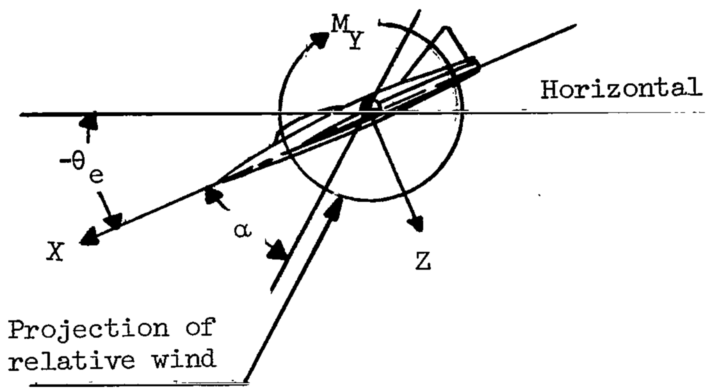
TABLE II.- INITIAL CONDITIONS USED IN CALCULATIONS

	Configuration A		Configuration B		Configuration C		Configuration D		Configuration E	
α, deg		70.1		55		75		68		85
β, deg		-0.3		-2		0		-4		0
θ_e, deg		-19.9		-35		-15		-22		-5
ϕ_e, deg		0		0		0		-2		0
V_R, m/sec (ft/sec)	98.97	(324.7)	97.30	(319.22)	77.82	(255.3)	92.31	(302.87)	103.53	(339.66)
$\dot{\psi}_e$, rad/sec		1.07		0.45		1.93		1.79		2.5
h_o, m (ft)	12 192	(40 000)	12 192	(40 000)	12 192	(40 000)	12 192	(40 000)	12 192	(40 000)
δ_e, deg		-25		-30		-20		-15		-30
δ_r, deg		21 right		7.5 right		20 right		6 right		6 right
δ_a, deg		7 left		7.5 left		20 left		60 left		12 left

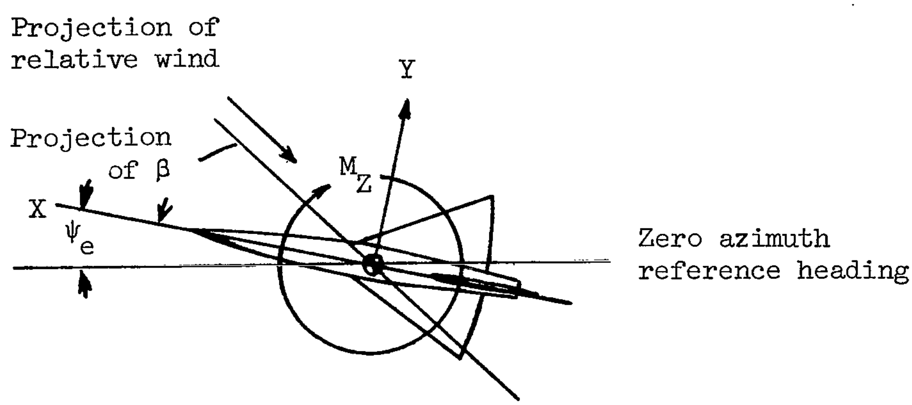
TABLE III.- COMPARISON OF CALCULATED AND PREDICTED SPIN RECOVERY RESULTS

Recovery	Calculated results			Predicted results					
	Recovery controls			Multiplicative method			Exponential method		
	δ_a, deg right	δ_r, deg left	Calculated turns for recovery (a)	K	Predicted turns for recovery (a)	Average percent error in predicted recovery turns	R	Predicted turns for recovery (a)	Average percent error in predicted recovery turns
Configuration A									
1	4	12	5.228		5.227 to 5.758	3.70		5.246 to 5.467	1.80
2	5	15	4.465		4.370 to 4.723	1.75		4.432 to 4.584	.89
3	6	18	3.914	0.64706	3.797 to 4.033	1.00	0.70699	3.872 to 3.970	.54
4	7	21	3.489	to	3.389 to 3.540	.73	to	3.455 to 3.520	.44
5	8	24	3.157	0.78021	3.082 to 3.198	.62	0.78910	3.130 to 3.203	.48
6	9	27	2.883		2.819 to 2.932	.66		2.868 to 2.947	.67
7	10	30	2.653		2.581 to 2.716	.85		2.658 to 2.735	1.11
Configuration B									
1	8	8	6.195		6.127 to 6.210	0.31		6.109 to 6.210	0.36
2	9	9	5.502		5.461 to 5.519	.18		5.453 to 5.519	.20
3	10	10	4.957	0.96424	4.929 to 4.968	.13	0.96556	4.925 to 4.967	.14
4	11	11	4.513	to	4.493 to 4.521	.13	to	4.492 to 4.519	.13
5	12	12	4.136	1.0076	4.119 to 4.147	.14	1.0072	4.118 to 4.146	.14
6	13	13	3.823		3.800 to 3.831	.15		3.799 to 3.830	.15
7	14	14	3.559		3.527 to 3.558	.28		3.526 to 3.559	.27
Configuration C									
1	14	14	4.275		4.301 to 4.565	2.75		4.295 to 4.427	1.75
2	16	16	3.949		3.886 to 4.097	1.18		3.913 to 4.022	.81
3	18	18	3.655	0.57787	3.583 to 3.733	.72	0.59403	3.619 to 3.695	.54
4	20	20	3.416	to	3.340 to 3.451	.51	to	3.375 to 3.459	.50
5	22	22	3.193	0.71054	3.142 to 3.246	.45	0.71875	3.169 to 3.268	.54
6	24	24	3.005		2.977 to 3.064	.49		2.991 to 3.104	.76
7	26	26	2.837		2.816 to 2.905	.63		2.837 to 2.960	1.14

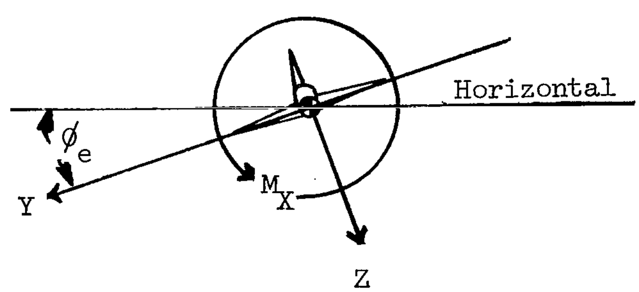
[a]Three-decimal-place numbers were used for the calculated and predicted turns for recovery solely to allow a more accurate computation of the average percent error for the predicted recoveries.



(a) Φ_e and $\psi_e = 0$.



(b) θ_e and $\Phi_e = 0$.



(c) θ_e and $\psi_e = 0$; $\Phi = \Phi_e$.

Figure 1.- Body system of axes and related angles.

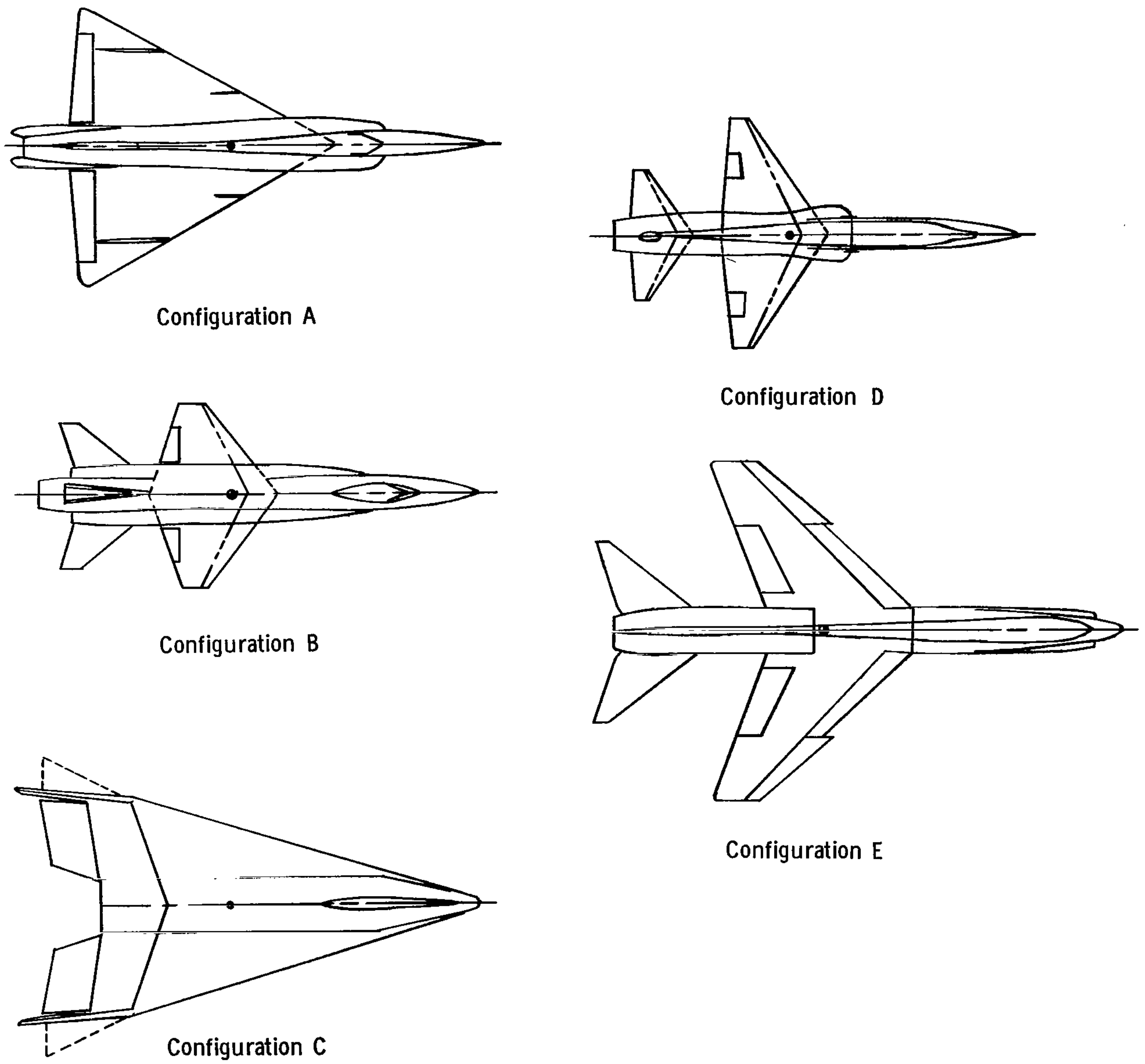


Figure 2.- Plan views of configurations.

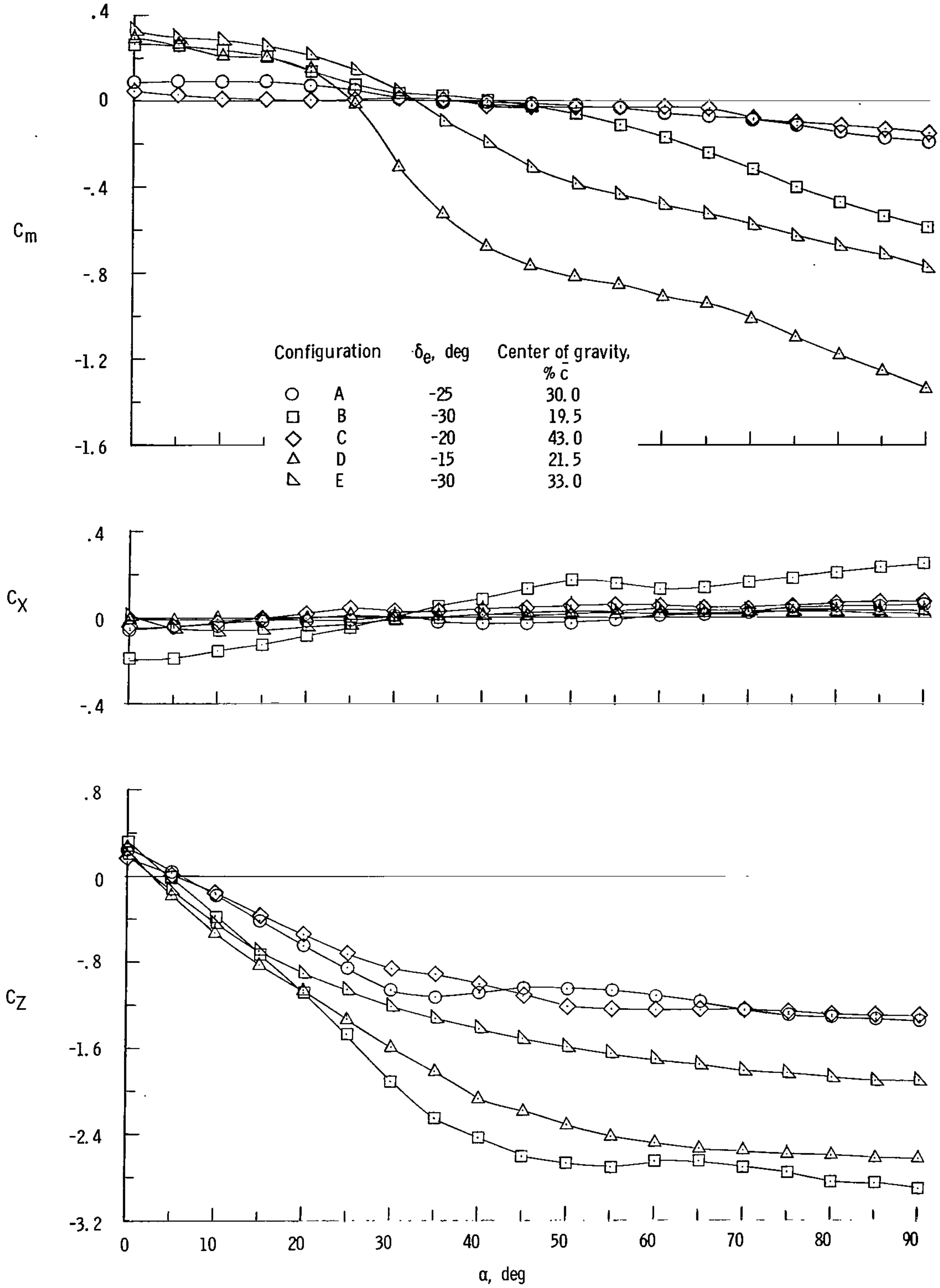


Figure 3.- Variation of static longitudinal stability characteristics with angle of attack.

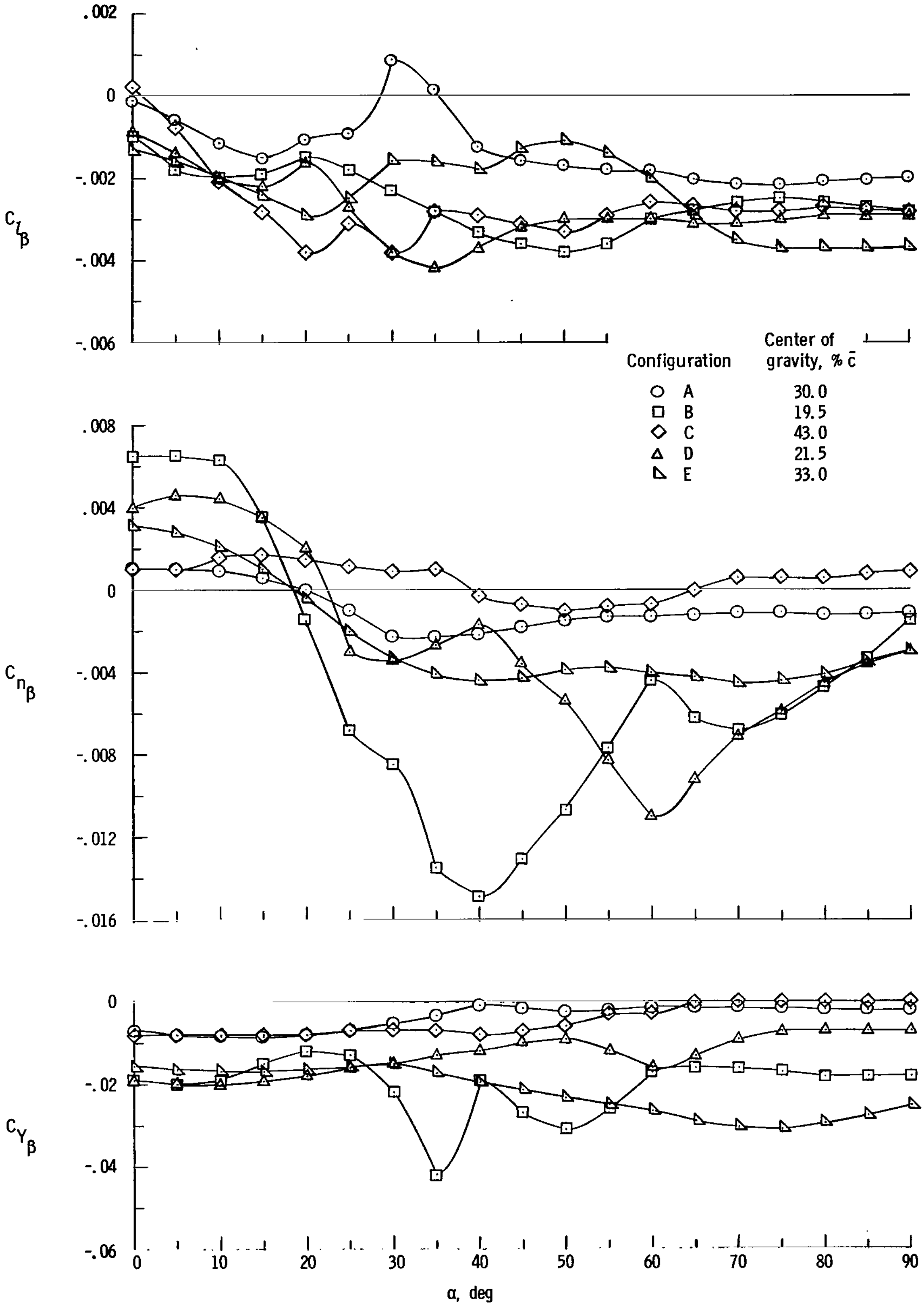


Figure 4.- Variation of static lateral stability characteristics with angle of attack.

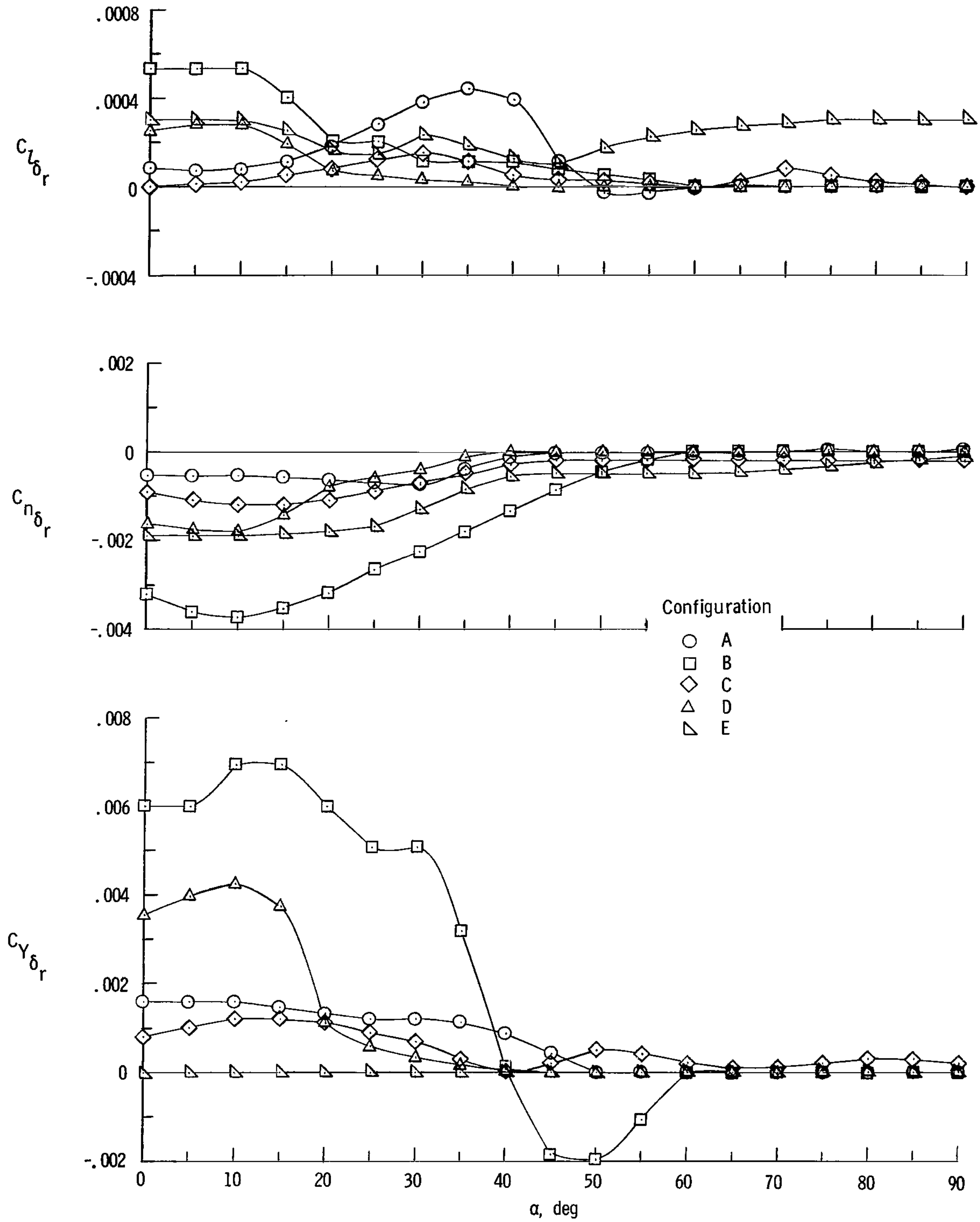


Figure 5.- Variation of rudder effectiveness with angle of attack.

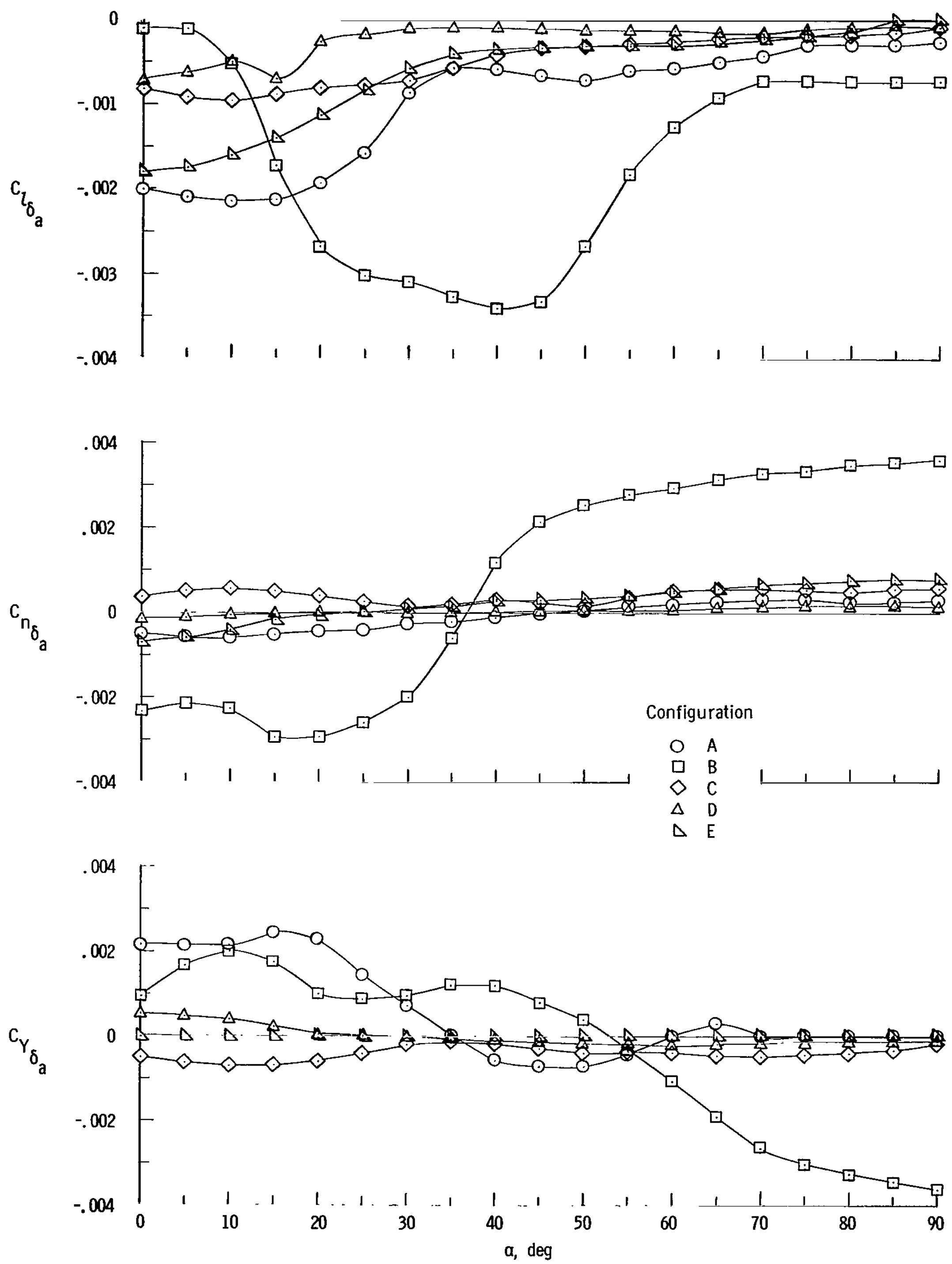


Figure 6.- Variation of aileron effectiveness with angle of attack.

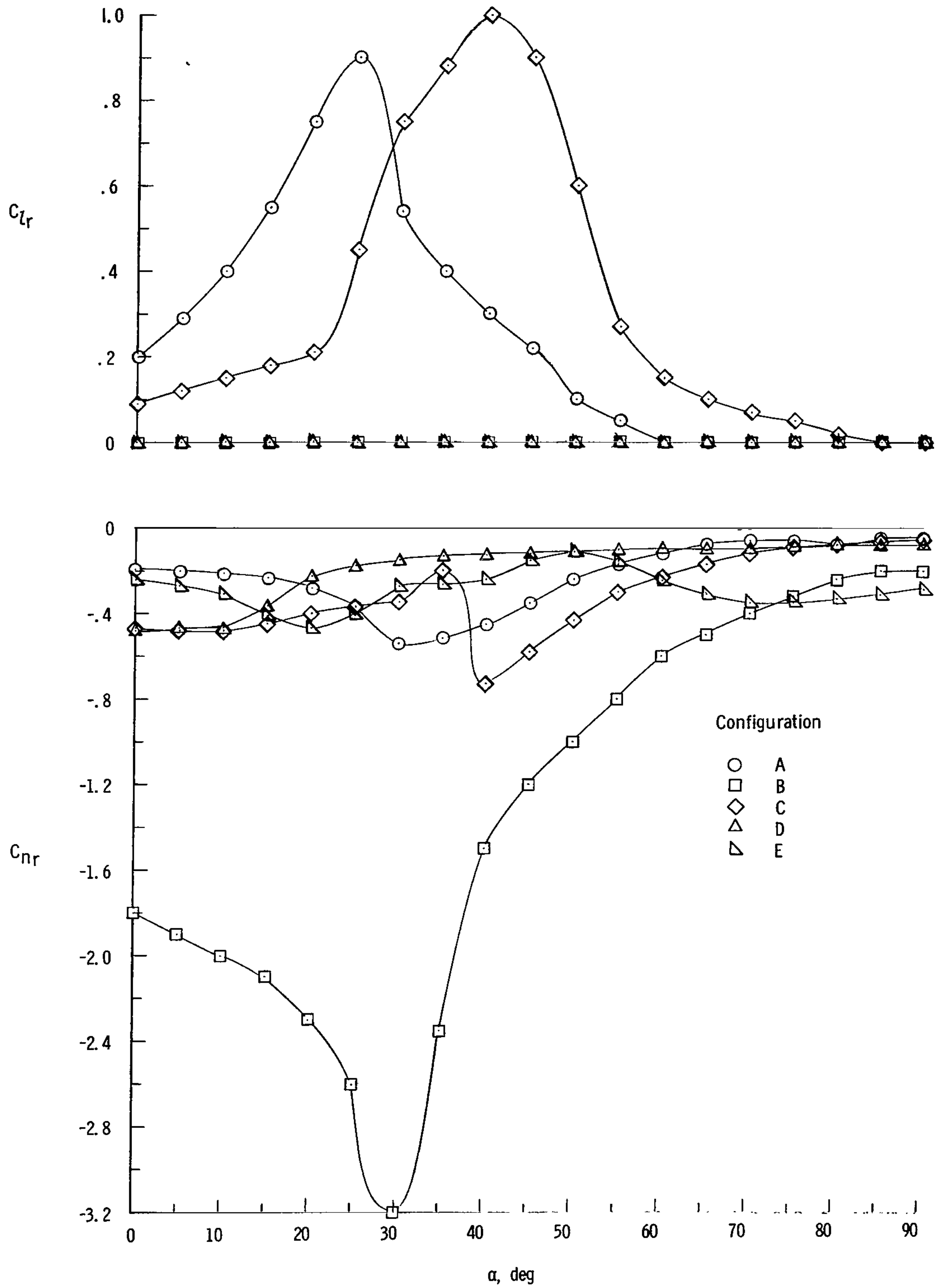


Figure 7.- Variation of C_{l_r} and C_{n_r} with angle of attack.

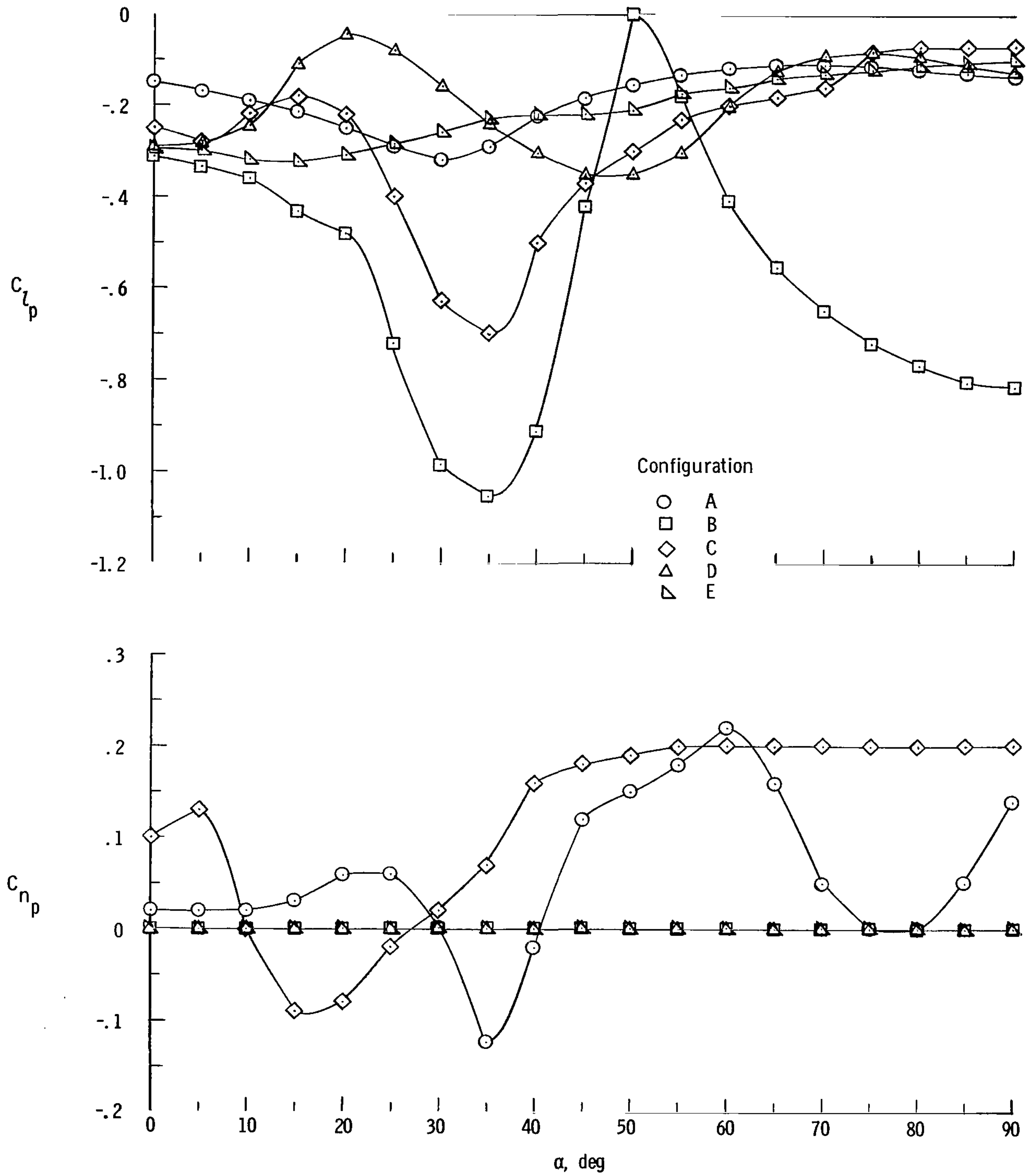


Figure 8.- Variation of C_{l_p} and C_{n_p} with angle of attack.

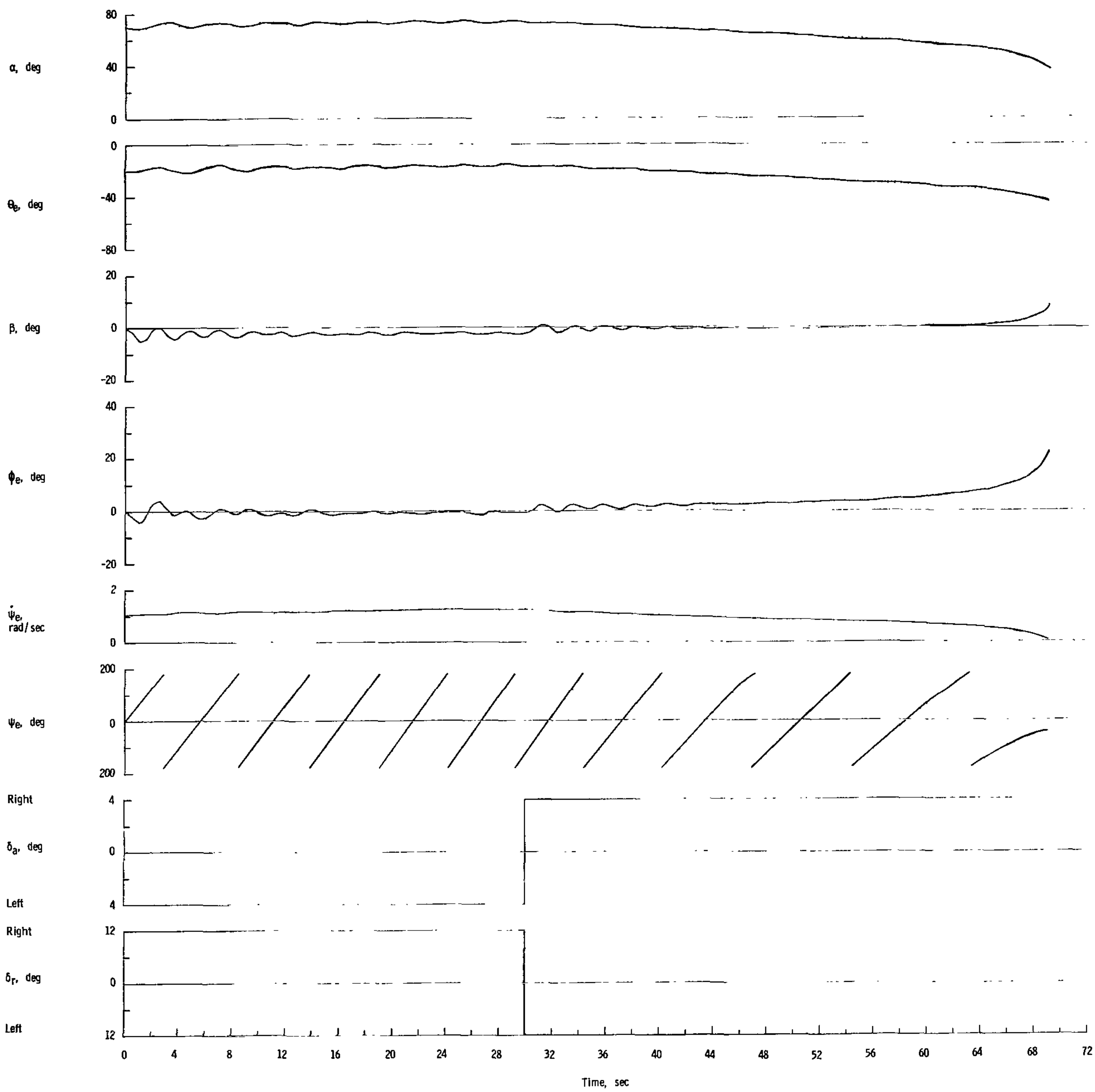


Figure 9.- Calculated developed spin and spin recovery motions for configuration A. $\delta_e = -25^0$.

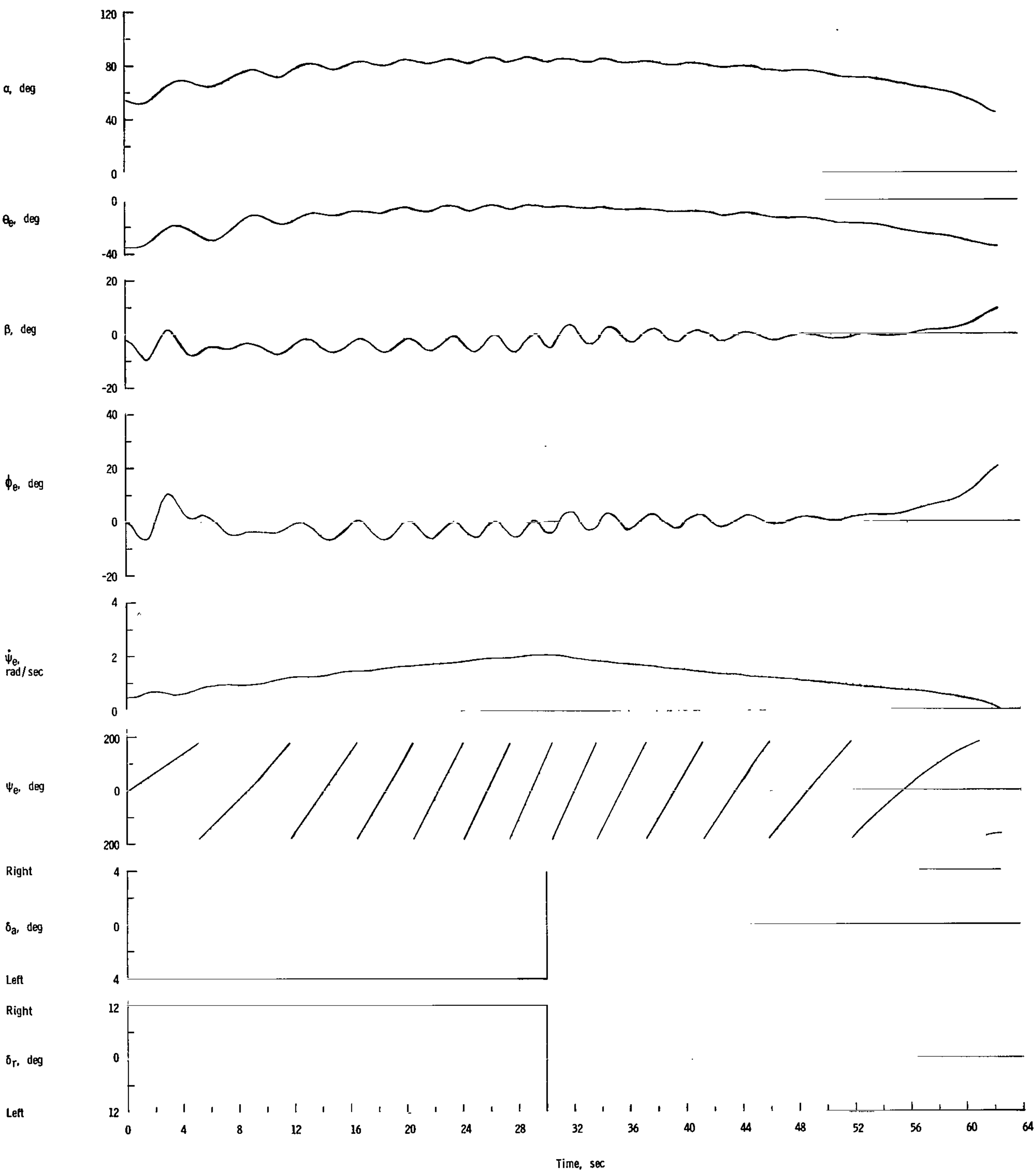


Figure 10.- Calculated developed spin and spin recovery motions for configuration B. $\delta_e = -30^0$.

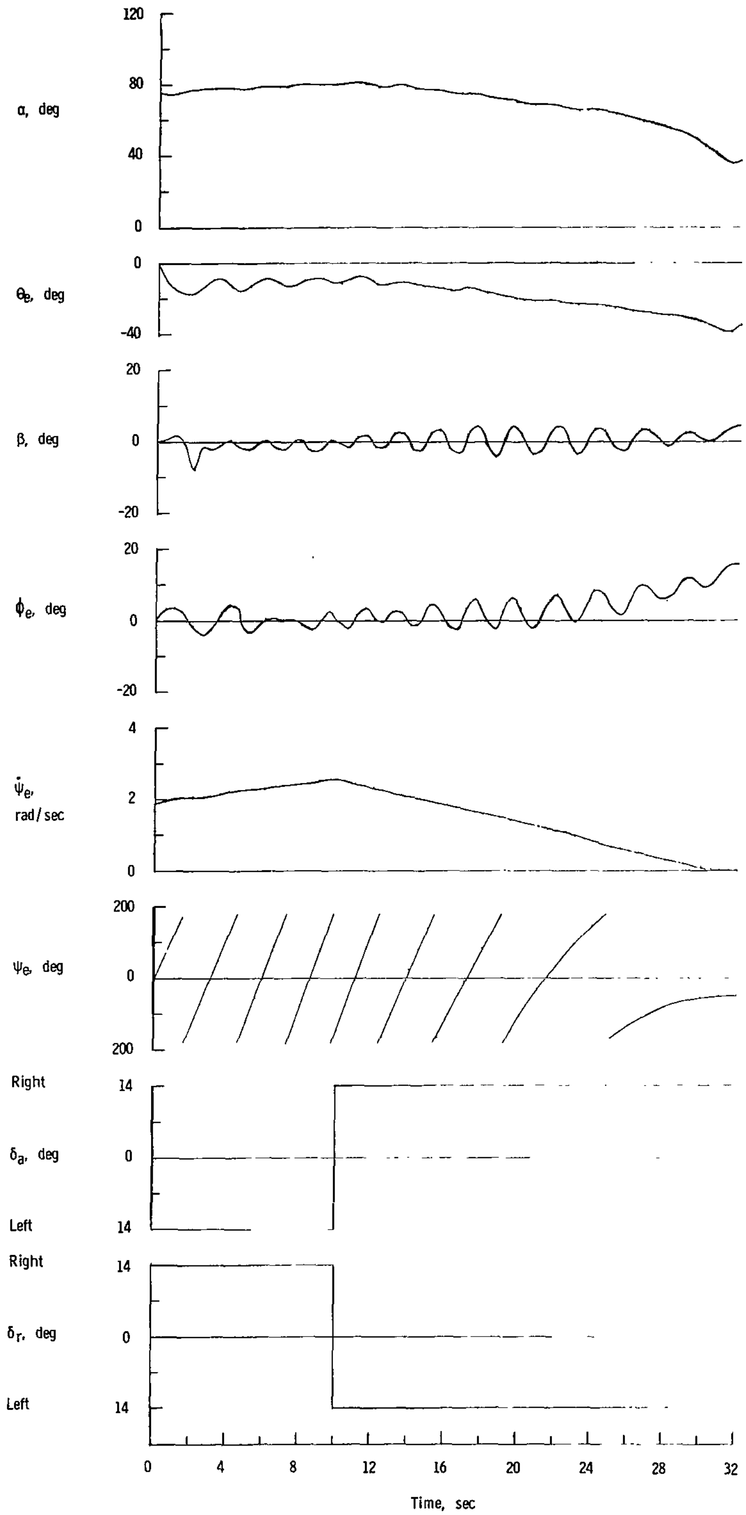


Figure 11.- Calculated developed spin and spin recovery motions for configuration C. $\delta_e = -20^{\circ}$.

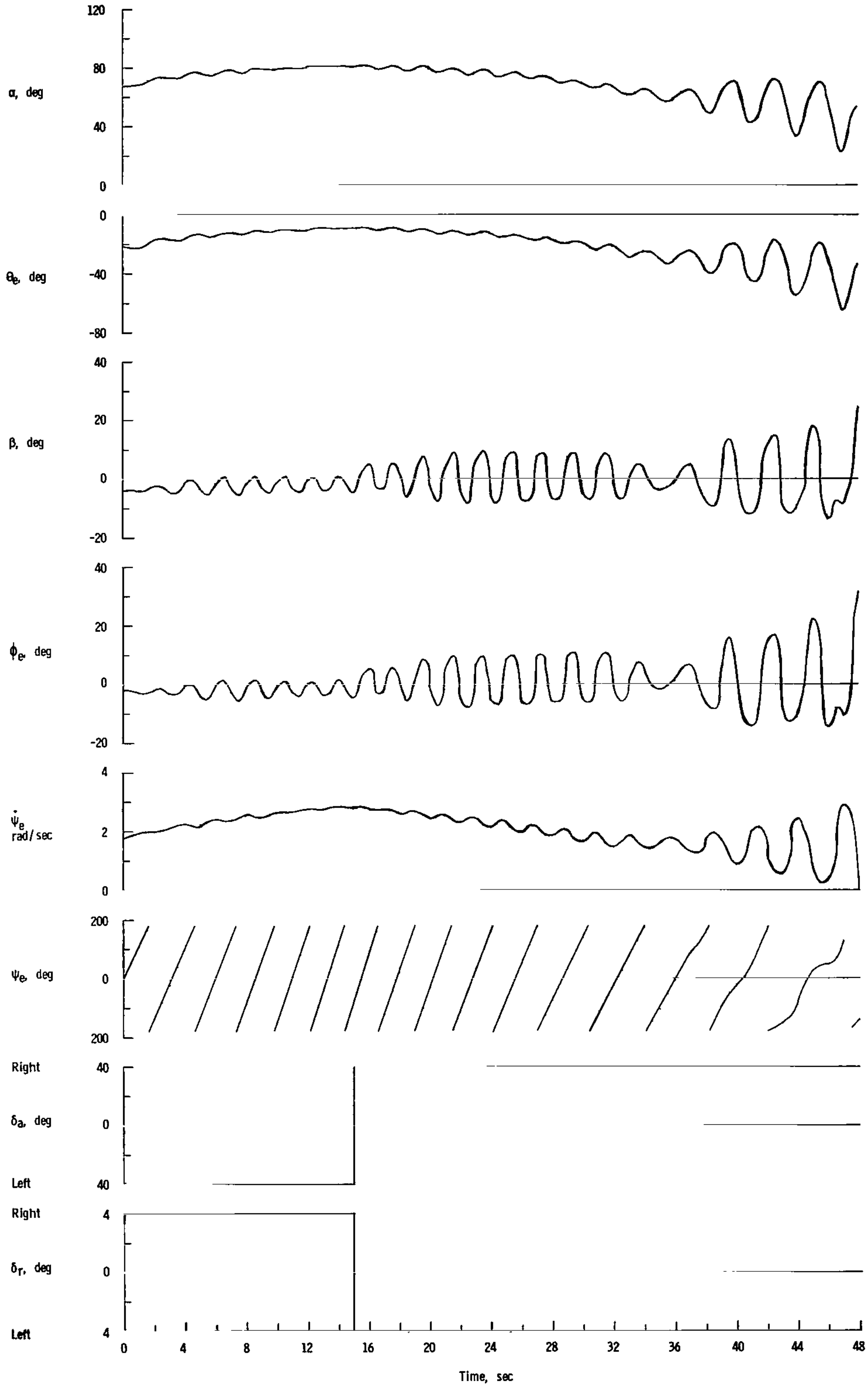


Figure 12.- Calculated developed spin and spin recovery motions for configuration D. $\delta_e = -15^o$.

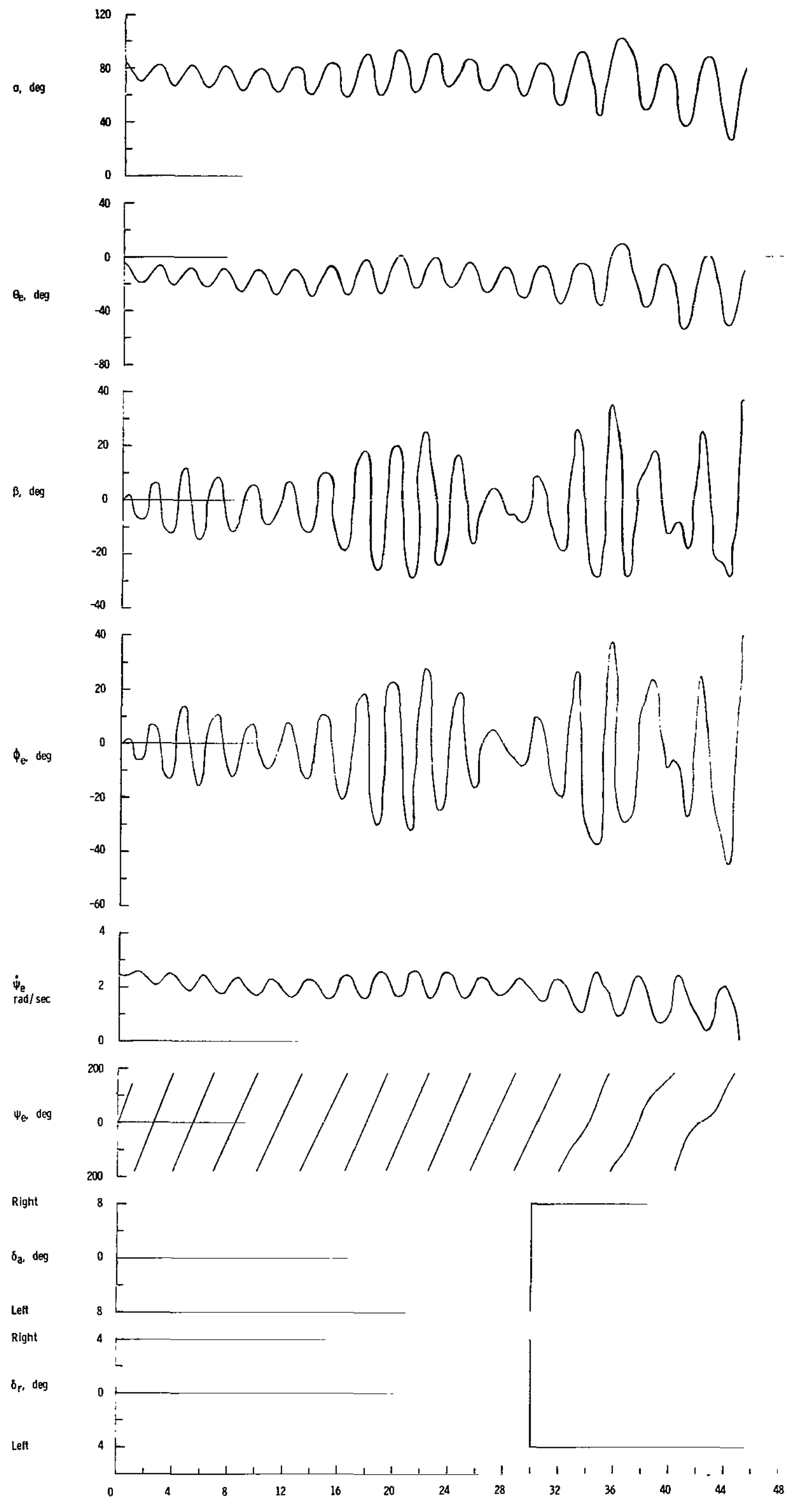


Figure 13.- Calculated developed spin and spin recovery motions for configuration E. $\delta_e = -30^0$.

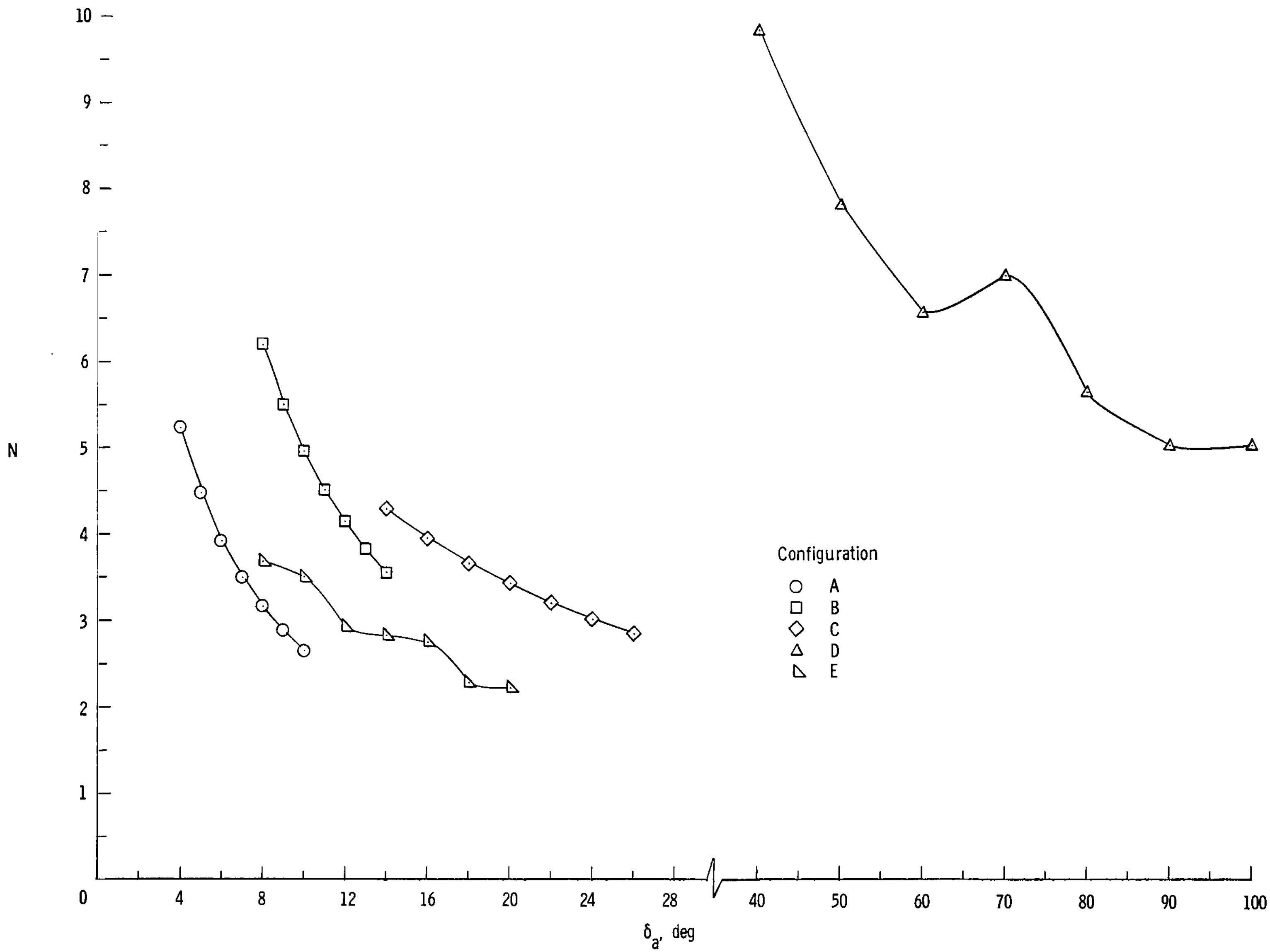


Figure 14.- Variation of spin recovery turns with the δ_a deflections used for recovery for all configurations. Note change of scale in δ_a.

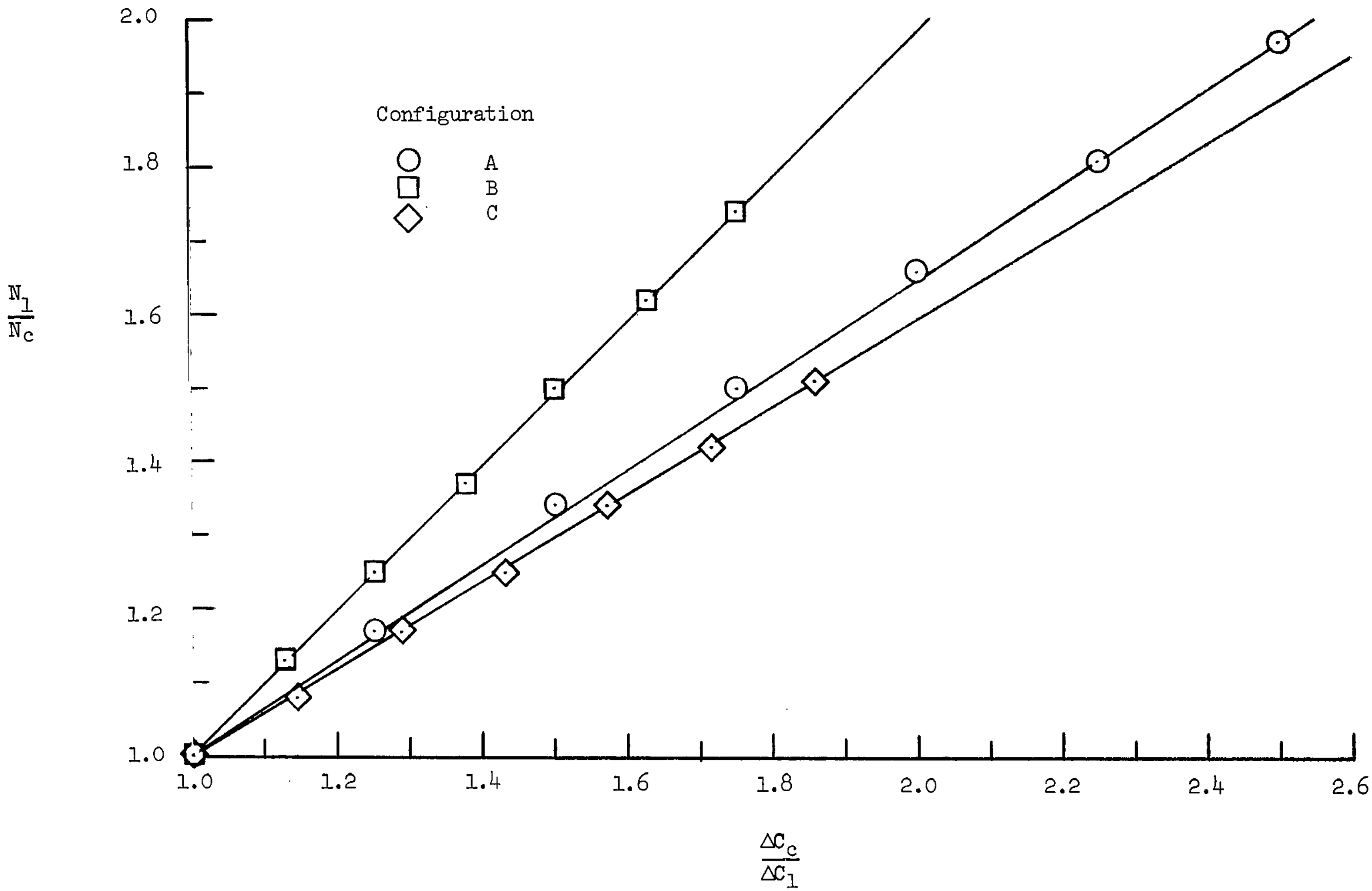


Figure 15.- Variation of $\frac{N_l}{N_c}$ with $\frac{\Delta C_c}{\Delta C_l}$ for configurations A, B, and C.

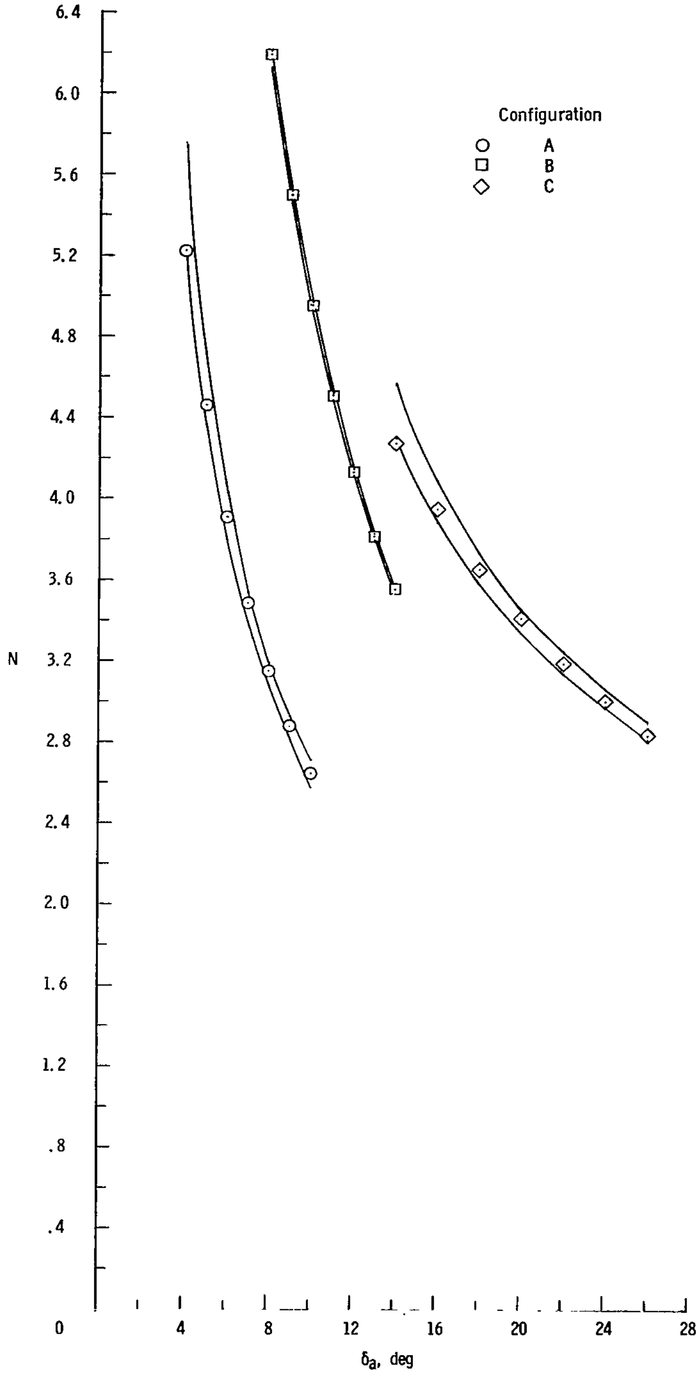


Figure 16.- Range of recovery turns predicted by multiplicative method as a function of δ_a used for recovery, represented for each configuration by area between appropriate pair of curves. Calculated recovery results, shown by symbols, are presented for comparison.

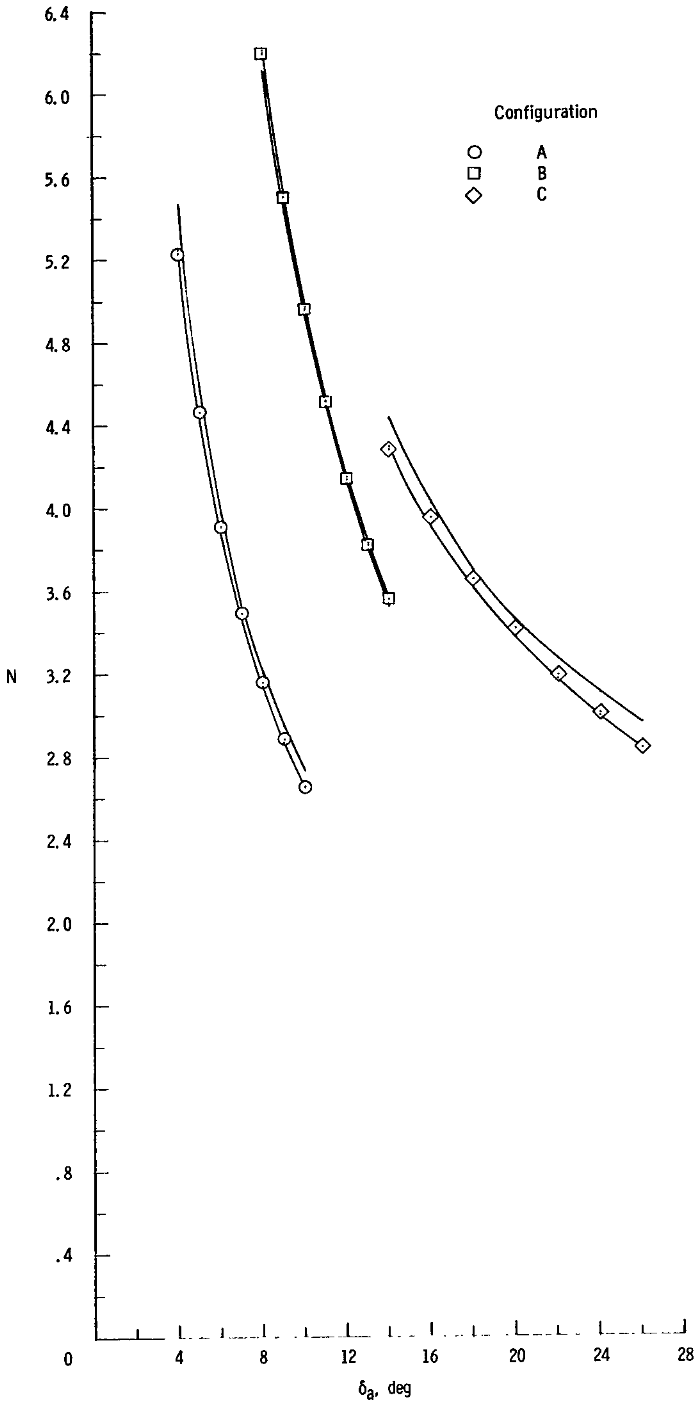


Figure 17.- Range of recovery turns predicted by exponential method as a function of δ_a used for recovery, represented for each configuration by area between appropriate pair of curves. Calculated recovery results, shown by symbols, are presented for comparison.

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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