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THE LINEARITY EVALUATION OF A DC VOLTAGE TO PULSE-WIDTH CONVERTER

BY

WARREN R. CROCKETT

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Warren R. Crockett

Flight Data Systems Branch
Spacecraft Technology Division

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THE LINEARITY EVALUATION OF A DC VOLTAGE TO PULSE-WIDTH CONVERTER

1. INTRODUCTION

The purpose of this report is to evaluate the linearity and temperature stability of a linear analog dc voltage-to-pulse width converter. The converter is composed of a complimentary monostable flip-flop controlled by the constant-current discharge of a capacitor. Figure 1 shows a schematic of the converter,

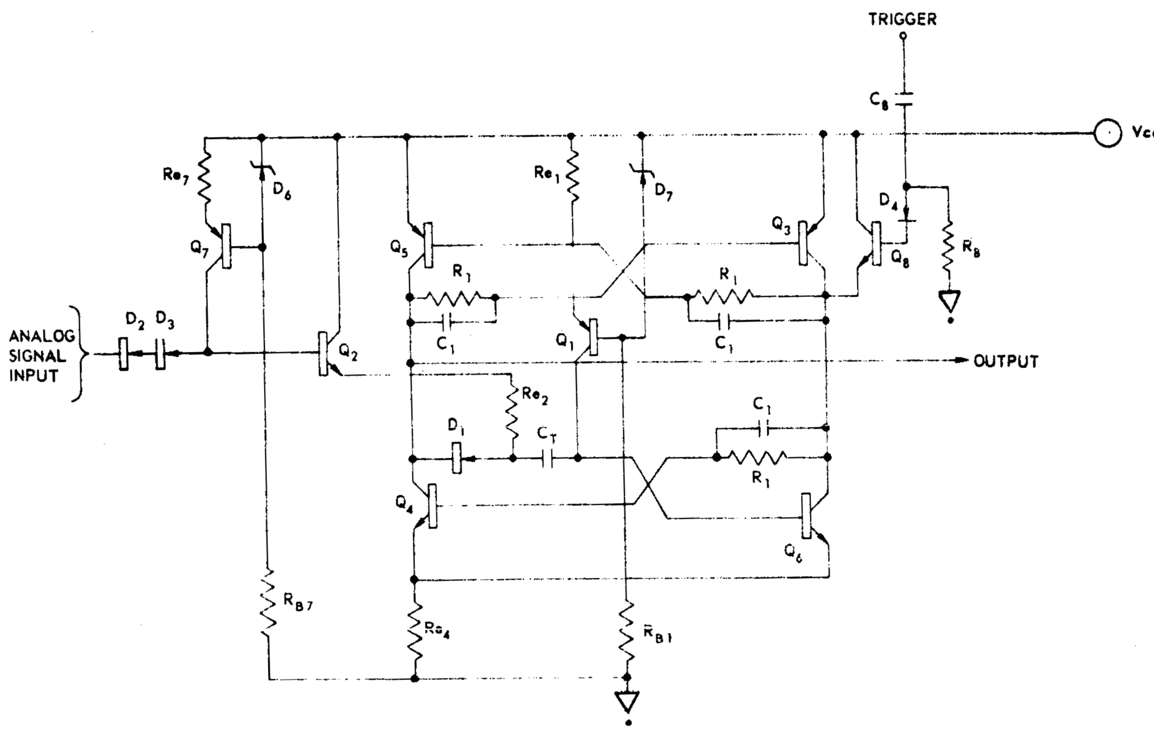

Figure 1. Analog-to-Pulse Width Converter Schematic Diagram

where the input analog dc voltage charges capacitor C_T. A trigger pulse through Q_8 initiates a linear discharge of C_T through the constant current generator Q_1, and the complementary flip-flop operates for one complete cycle with a period determined by the input analog dc voltage.

2. TEST AND EVALUATION

Figure 2 shows a block diagram of the test set up used for checking the converter circuit. The triggering pulse is obtained from the data pulse generator. The analog dc input voltage is provided by a Hewlett Packard dc power supply, and the converter output is displayed on a Beckman Printer.

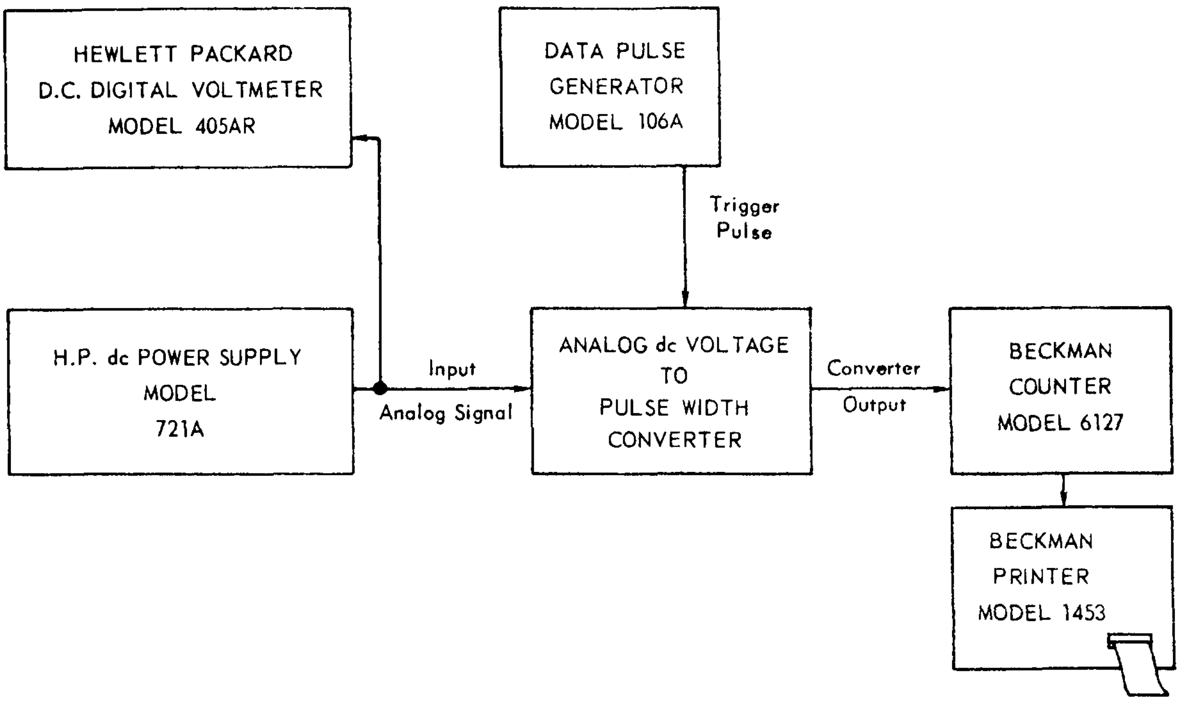

Figure 2. Block Diagram Used to Test the Pulse Width Converter

2.1 Linearity

Linearity is defined as the percentage deviation from the best-fit straight line. Table I shows the output pulse width in microseconds as the analog dc input voltage is varied over a range from zero to 5 v. dc. The equation of the line that best represents this data may be obtained by employing the method of the least square line. From Table I if we let the analog input voltage be along the x-axis, and the input pulse width be along the y-axis, then the least square line approximating the set of points $(X_1\ Y_1)$, $(X_2\ Y_2)$, ... $(X_N\ Y_N)$ has the equation

$$Y = a_0 + a_1 X \tag{1}$$

Table I

INPUT VS. OUTPUT LINEARITY

Input (X) (volts)	Output (Y) Pulse Width (μsec)
0	1.25
.25	3.00
.50	6.25
.75	11.50
1.00	16.50
1.25	22.00
1.50	26.25
1.75	32.00
2.00	36.75
2.25	42.25
2.50	47.00
2.75	52.50
3.00	57.00
3.25	63.00
3.50	67.00
3.75	73.00
4.00	78.00
4.25	82.25
4.50	87.15
4.75	92.80
5.00	97.25

Where the values of Y corresponding to $X = X_1,\ X_2, \ldots X_N$ are $a_0 + a_1X$, $a_0 + a_1X_2. \ldots a_0 + a_1X_N$, and the actual values of the data points are $Y_1\ Y_2$, $\ldots Y_N$ respectively. The sum of the squares of the deviations of data points from the best fit line is

$$\delta = (a_0 + a_1X_1 - Y_1)^2 + (a_0 + a_1x_2 - y_2)^2 + \cdots + (a_0 + a_1x_N - y_N)^2$$

$$= \Sigma(a_0 + a_1x - y)^2 \tag{2}$$

δ is a minimum when the partial derivations of δ with respect to a_0 and a_1 are zero. That is,

$$\begin{aligned} \frac{\partial \delta}{\partial a_0} &= \frac{\partial}{\partial a_0}\left[\Sigma(a_0 + a_1x - y)^2\right] = 0 \\ \frac{\partial \delta}{\partial a_1} &= \frac{\partial}{\partial a_1}\left[\Sigma(a_0 + a_1x - y)^2\right] = 0 \end{aligned} \tag{3}$$

From Equation (3) we obtain the required normal equations.

$$\begin{aligned} Na_0 + a_1\Sigma X - \Sigma Y &= 0 \\ a_0\Sigma x + a_1\Sigma X^2 - \Sigma XY &= 0 \end{aligned} \tag{4}$$

The constants a_0 and a_1 are

$$\begin{aligned} a_0 &= \frac{(\Sigma Y)(\Sigma X^2) - (\Sigma X)(\Sigma XY)}{N(\Sigma X^2) - (\Sigma X)^2} \\ a_1 &= \frac{N(\Sigma XY) - (\Sigma X)(\Sigma Y)}{N(\Sigma X^2) - (\Sigma X)^2} \end{aligned} \tag{5}$$

Where N equals the number of readings.

From the data in Table II and Equation (5) the values for a_0 and a_1 are

$$a_0 = -3.25, \quad a_1 = 20.25$$

Then substituting these values into Equation (1) we have

$$Y = -3.25 + 20.25X$$

Therefore Equation (6) is the equation that best represents the data given in Table I. A plot of this equation and the actual data is depicted in Figure 3. It can be seen from Figure 3 that for analog input signals less than 0.5 v dc, the least square line equation is not in close agreement with the actual data. Let us therefore consider the operating range to be from 0.5 v dc to 5 v dc and discuss the offset below 0.5 v dc in Section 2.2.

For the considered operating range we will determine the maximum percentage deviation of the actual data from the least square line. The equation for maximum percent deviation is

$$\frac{Y - Y_0}{Y_{0\,max}} \times 100 = (\%) \text{ deviation} \tag{7}$$

where Y equals computed data

Y_0 equals actual data

$Y_{0\,max}$ equals full scale reading actual data

By comparing the computed values with the actual data of Table III it can be seen that the maximum deviation occurs when the analog input voltage is 0.5 v dc. This maximum percent deviation is less than one percent.

The root mean square (rms) percent deviation over the range from 0.5 v dc to 5.0 v dc may be obtained from the computed data shown in Table III and the rms equation. The rms equation is

$$\text{rms} = \sqrt{\frac{(\%)^2}{N}} \tag{8}$$

Table II

STATISTICAL ANALYSIS

X	Y	X^2	XY
0.25	3.00	0.064	0.750
0.50	6.25	0.250	3.125
0.75	11.50	0.563	8.625
1.00	16.50	1.00	16.500
1.25	22.00	1.56	27.500
1.50	26.25	2.25	39.375
1.75	32.00	3.06	56.000
2.00	36.75	4.00	73.500
2.25	42.25	5.06	95.062
2.50	47.00	6.25	117.500
2.75	52.00	7.56	144.375
3.00	57.00	9.00	171.000
3.25	63.00	10.56	204.750
3.50	67.00	12.25	234.500
3.75	73.00	14.06	270.000
4.00	78.00	16.00	308.000
4.25	82.25	18.06	349.562
4.50	87.15	20.25	392.175
4.75	92.80	22.56	440.800
5.00	97.25	25.00	485.000
$\Sigma x = 52.50$	$\Sigma y = 991.20$	$\Sigma X^2 = 179.37$	$\Sigma XY = 3438.1$

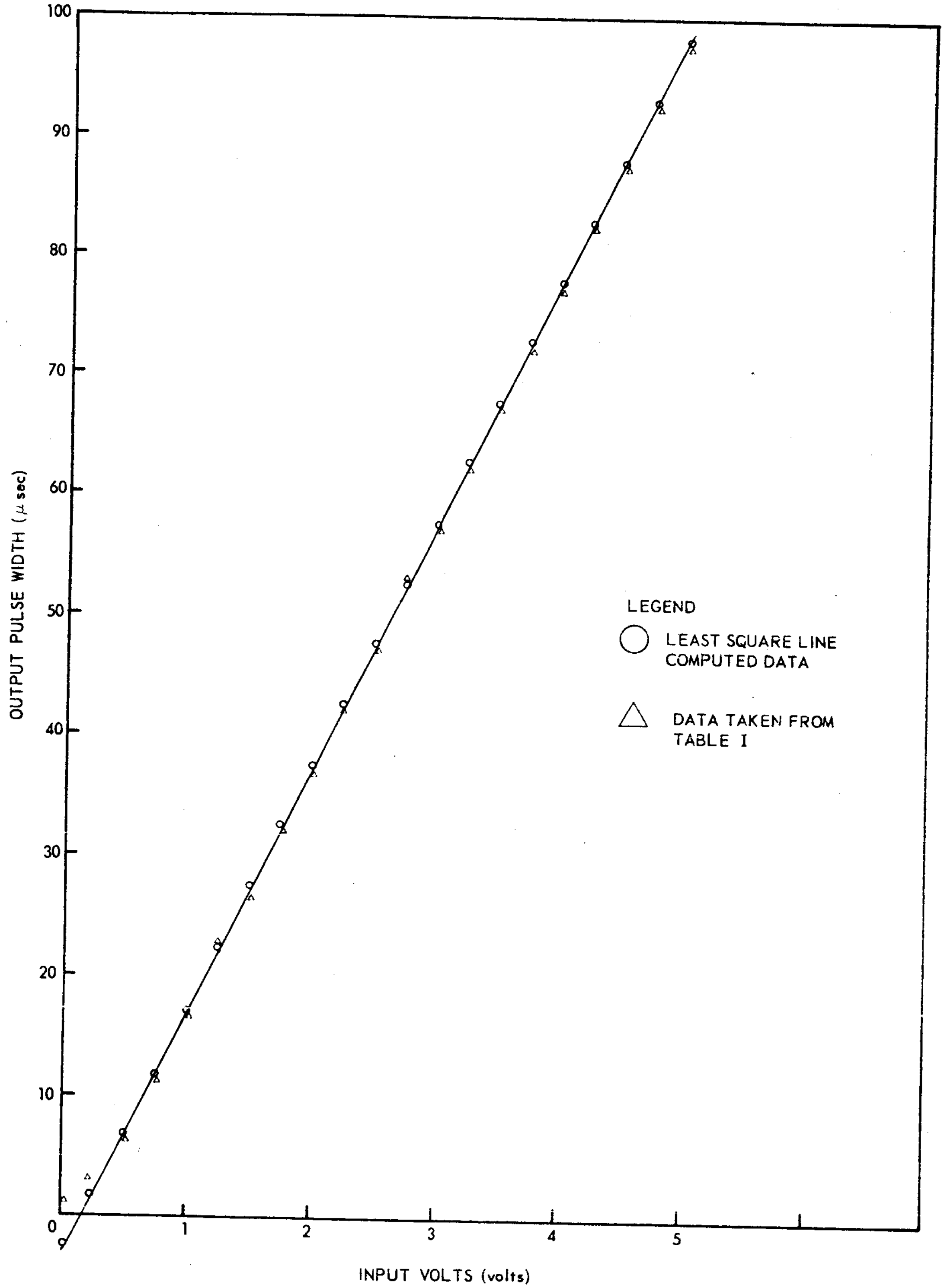

Figure 3. Least Square Line vs Actual Data Points

Table III

(%) AND $(\%)^2$ DEVIATION

Input	Actual Output	Computed Output	Percent Deviation (%)	Percent Deviation $(\%)^2$
X	Y_0	Y	$\frac{Y - Y_0}{Y_{0\,max}} \times 100$	
.50	6.25	6.88	+0.64	0.409
.75	11.50	11.94	+0.45	0.202
1.00	16.50	17.00	+0.51	0.260
1.25	22.00	22.06	+0.06	0.003
1.50	26.25	27.12	+0.89	0.792
1.75	32.00	32.19	+0.19	0.036
2.00	35.75	37.25	+0.51	0.260
2.25	42.25	42.31	+0.06	0.003
2.50	47.00	47.37	+0.38	0.144
2.75	52.50	52.44	-0.06	0.003
3.00	57.00	57.50	+0.51	0.260
3.25	63.00	62.56	-0.45	0.202
3.50	67.00	67.62	+0.63	0.396
3.75	73.00	72.69	-0.31	0.096
4.00	78.00	77.75	-0.25	0.0625
4.25	82.25	82.81	+0.57	0.324
4.50	87.15	87.87	+0.74	0.547
4.75	92.80	92.94	+0.14	0.0196
5.00	97.25	97.40	+0.25	0.0625

$\Sigma\ (\%)^2 = 4.080$

From Table III and Equation (8), the rms = 0.50 percent.

2.2 Temperature Stability

The analog to pulse width converter was environmentally tested over a temperature range of -20°C to +60°C. The data obtained from these tests are plotted in Figure 4.

Figure 5 shows the percentage of pulse width deviation due to temperature. It can be established from this figure that the maximum percent deviation of the output pulse width is 0.6 percent over a temperature range of -20°C to +60°C.

2.3 Control of the Output Slope

Let us consider the non-linear portion of the characteristic below 0.5 v dc input voltage in Figure 3. This nonlinearity can be minimized by adjusting resistor R_{B1} of Figure 1. The variation of this resistor performs the following functions:

(1) It controls the Y-intercept.

(2) It controls the output pulse width at high input analog signals.

Figure 6 shows how the slope of the output is changed by varying resistor R_{B1}. When R_{B1} is set equal to 20k. The output from the converter appears to be linear over the entire range from zero to 5.0 v dc, and then Y-intercept approaches zero. This can be shown by employing the same mathematical procedure used in Section 2.1. From Table IV, and Equation (5) we obtain the constants

$$a_0 = 0.333, \quad a_1 = 12.92$$

Then by substituting these values into Equation (1) we have

$$Y = 0.333 + 12.92X \tag{9}$$

Equation (9) is the least square line equation of the converter when resistor R_{B1} is 20k Ω. It should be noted that the Y-intercept has changed from $a_0 = -3.25$ to $a_0 = 0.333$, and the input voltage range from zero to 5.0 v dc, is linear. From the above calculations it follows that the Y-intercept can be made equal to zero if care is exercised in selecting resistor R_{B1}.

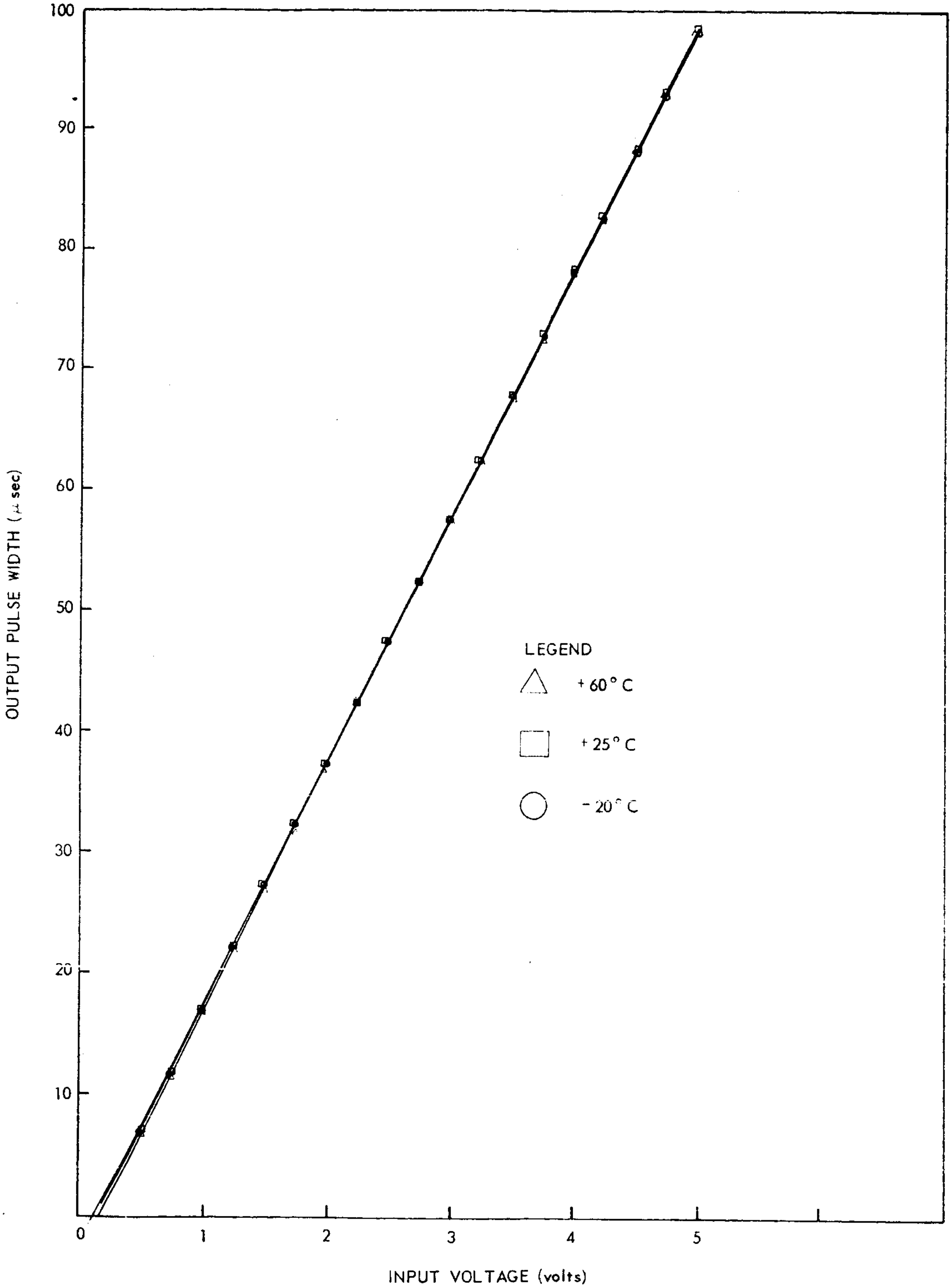

Figure 4. Linearity vs. Temperature Curves

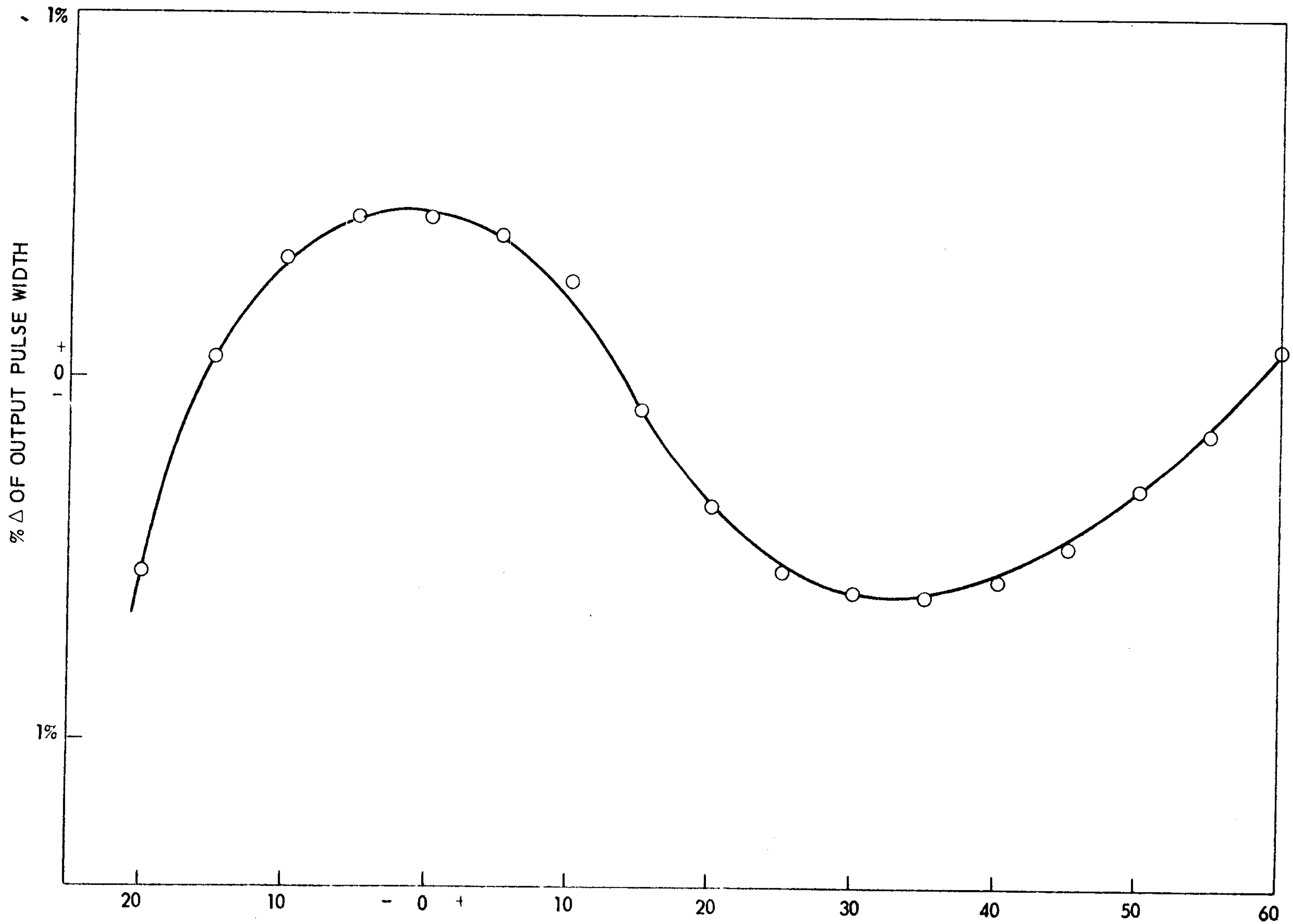

Figure 5. Output Pulse Width Deviation vs. Temperature

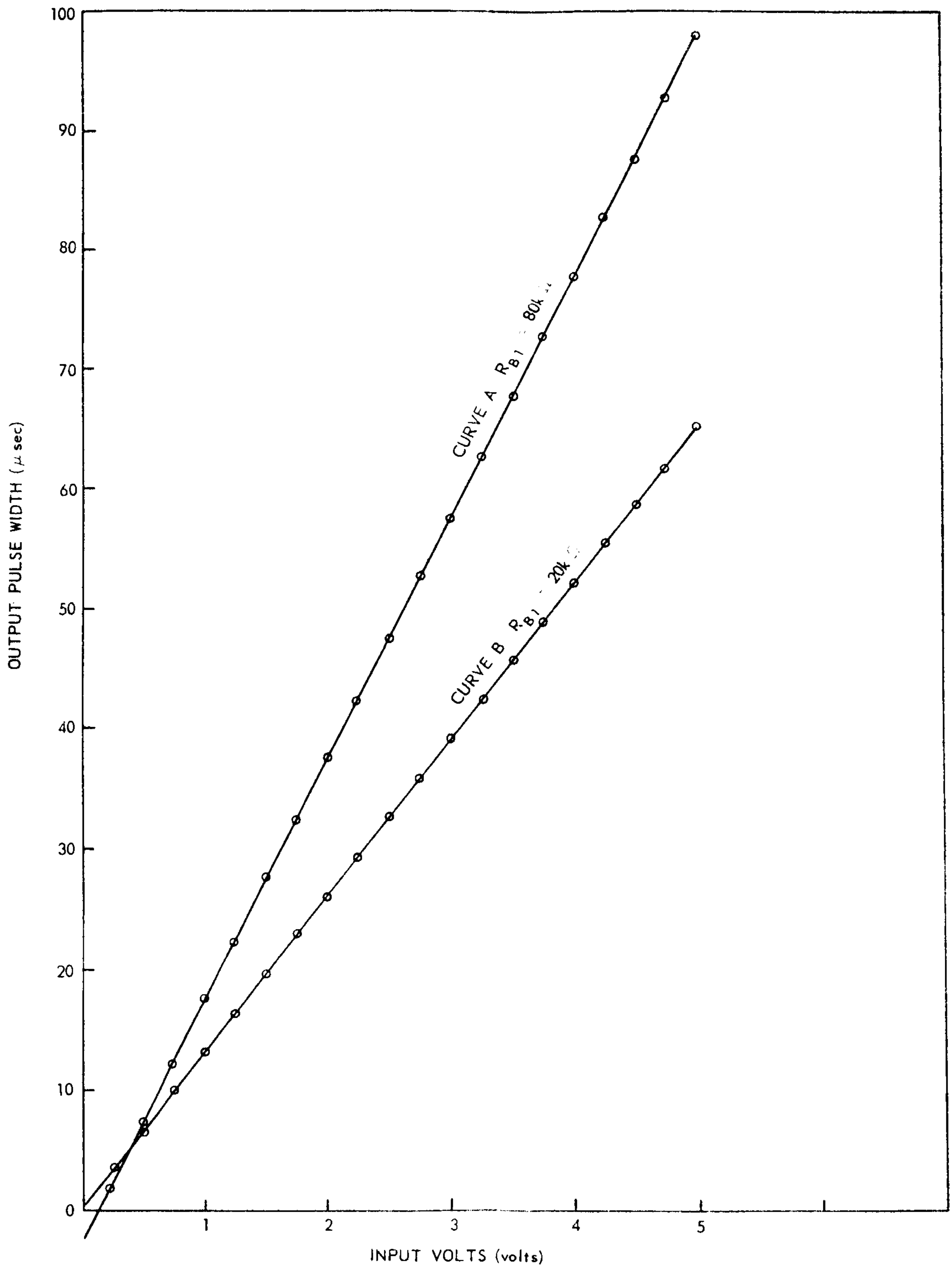

Figure 6. Variation of the Output Slops vs. Variation of R_{B1} Resistor

Table IV

STATISTICAL ANALYSIS WHEN R_{B1} IS 20k

X	Pulse Width Y	X^2	XY
0	0.00	0.00	0.000
.25	3.60	0.064	0.90
.50	6.90	0.250	3.45
.75	10.10	0.563	6.57
1.00	13.25	1.000	13.25
1.25	16.50	1.560	20.62
1.50	19.90	2.25	29.85
1.75	23.00	3.06	40.25
2.00	25.95	4.00	51.90
2.25	29.25	5.06	65.81
2.50	32.70	6.25	81.75
2.75	36.00	7.56	99.00
3.00	39.10	9.00	117.30
3.25	42.25	10.56	137.31
3.50	45.30	12.75	158.55
3.75	48.80	14.06	183.00
4.00	52.10	16.00	208.40
4.25	55.45	18.06	235.66
4.50	58.60	20.25	263.70
4.75	61.80	22.56	293.55
5.00	65.00	25.00	325.0
$\Sigma x = 52.50$	$\Sigma y = 685.55$	$\Sigma x^2 = 179.37$	$\Sigma xy = 2335.82$

With R_{B1} equal to 20K ohms, the worst case and rms deviations remain less than one percent. Therefore, it can be seen that the variation of R_{B1} has little or no effect on the worst case or the rms deviations.

3. CONCLUSIONS

It has been shown that the linearity of this circuit is relatively independent of temperature variations and that the slope of the transfer characteristic can be adjusted to minimize the zero offset by a relatively simple resistor adjustment. The circuit is a combination complementary flip-flop and constant current generator with a power drain less than 100 milliwatts. These features make it particularly useful as an analog to digital converter in scientific satellites where low power, ruggedness, reliability, and good linearity are prime requirements.

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