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PLANETARY QUARANTINE

A SEQUENTIAL DECISION MODEL OF PLANETARY QUARANTINE PRIMARY OBJECTIVES

C. A. Trauth, Jr., 2571

SANDIA CORPORATION

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ABSTRACT

There is apt to be much uncertainty in any program of planetary exploration. This uncertainty leads naturally to possible uncertainty in the time period in which planetary quarantine is desirable and to possible uncertainty in the total number of missions to be launched in the vicinity of any given planet.

A model is developed in this report which makes possible the derivation of mission non-contamination requirements without a priori knowledge of either the time period in which planetary quarantine is to be observed or the total number of missions to be used in exploring the planet in question.

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CONTENTS

I - Introduction

The use of mathematical models in the development and analysis of a program is described in [1] and [2]. It is argued in these documents that:

(i) modeling is useful in relating primary program objectives to actions designed to achieve them with an acceptable associated penalty, and

(ii) modeling is useful in determining reasonable specific primary objectives.

A possible means of accomplishing this in the planetary quarantine program is discussed. In order to relate objectives to actions, an objective hierarchy is constructed in which the objectives at the k^{th} level are analyzed in terms of the "significant factors" affecting their achievement, and the objectives are then related to these significant factors using mathematical models. The k^{th} level objectives are then translated into $(k+1)^{st}$ level objectives involving the model parameters representing the significant factors. This process is continued until objectives which are directly physically realizable are obtained. The selection of both the significant factors and models is largely a matter of judgement.

It is the purpose of this paper to discuss:

(1) the nature of planetary quarantine primary (1^{st} level) objectives,

(2) the significant factors affecting the achievement of planetary quarantine primary objectives, and

(3) a sequential decision model for relating these significant factors to the primary objectives.

Generally speaking, the conclusions to be drawn are these:

- In attempting to develop a national program upon which action may be based, non-contamination objectives should not be considered exclusive of other national objectives related to space exploration.
- There are many sources of uncertainty in any space exploration program, and these will influence the attainment of the planetary quarantine objectives.
- The sequential decision model presented here seems to be capable of allowing these uncertainties to be considered in a planetary quarantine program.

II - Planetary Quarantine Objectives

A. Background and Non-Contamination Objectives

The National Academy of Sciences in 1958 recognized the possibility that, with the advent of space exploration, extraterrestrial bodies might become contaminated with living terrestrial organisms. The Academy expressed a concern over the possible detrimental consequences of such an occurrence [3]. An *ad hoc* committee, the Committee on Contamination by Extraterrestrial Exploration (CETEX), was formed by the International Council of Scientific Unions (ICSU) in 1958 to study this potential problem [3]. In its short life-time (1958-1959), this committee recognized two principles pertinent to the discussion here. The first was that *certain* knowledge that a planet was not contaminated was, in all likelihood, possible only if that planet was avoided by space vehicles.* The second was that exploration of planets would take place, and that the nations involved in such exploration would determine their own time schedules for this exploration.* The committee expressed concern that the time in which to find an acceptable solution to the contamination problem was short, and felt that some immediate action was necessary [4]. Accordingly, the problem was referred to a permanent committee, the Committee on Space Research (COSPAR), of the ICSU.

COSPAR, acknowledging the uncertainty in knowledge about the contamination of extraterrestrial bodies recognized by CETEX states ([6], 1966):

> It is suggested, therefore, that the basic probability of 1×10^{-3} that a planet will be contaminated during the period of biological exploration continues to be accepted as the guiding criterion for the exploration of Mars, or other planets deemed important for the investigation of extra-terrestrial life or precursors or remnants thereof.

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*A liberal interpretation of the CETEX reports [4] and [5].

As yet, there appears to be no specific national objective of non-contamination except general concurrence with that of COSPAR which specifies an objective that must be "shared" by all nations.

The general intent of the COSPAR objective seems reasonable, so that it is assumed in this document that the national primary objective for non-contamination is of the form:

OBJECTIVE 1. The probability that any planet deemed important for study of extraterrestrial life, or precursors or remnants thereof, be contaminated during the next T years shall not exceed $(1 - \hat{P}_{N.C.})$.

The term $\hat{P}_{N.C.}$, in this context, represents the minimum acceptable probability of not contaminating the planet in question during the allotted time period, T. The word "contamination" and the parameters T and $\hat{P}_{N.C.}$ are considered to be variables (see [1]) for any specification of a planet, and, of course, the importance of any planet in the study of extraterrestrial life is a matter for decision. The phrase "planets deemed important.." has been retained from the COSPAR statement. This tacitly excludes the natural satellites of the planets of our solar system and excludes consideration of contaminating meteoroids which might later impact Earth and falsely imply the existence of extraterrestrial life. Whether this is reasonable or not is certainly a matter of opinion.

B. Other Planetary Quarantine Objectives

It is generally recognized that the achievement of Objective 1 for certain specific values of $\hat{P}_{N.C.}$ would involve an unreasonable or impossible cost. Recently, there has been an indication of Congressional concern about this possibility [7]. On the other hand, $\hat{P}_{N.C.}$ should be large enough to insure some meaningfulness from the scientific standpoint. Thus, there is another planetary quarantine objective:

> OBJECTIVE 2. The objective of non-contamination (Objective 1) should be attained in such a manner that the penalty associated with its achievement is acceptable nationally.

In effect, this implies that specifics for the variables in Objective 1 should be chosen so that the cost of attainment is acceptable and the scientific penalty is also acceptable.

There is at least one other type of objective that should be considered in studying planetary quarantine. This is the time constraint imposed upon planetary quarantine activities by flight project activities. Current Voyager project plans call for a Mars landing in 1973 [8] and, of course, flyby missions have already been launched. This leads to a general objective statement of the form:

> OBJECTIVE 3. Means for achieving Objective 1 should be known before the year Y.

III - Other National Objectives Influencing Planetary Quarantine Objectives

A. Reasons for Planetary Quarantine

The statement of a non-contamination objective such as Objective 1 implies the belief that such an objective is needed. For purposes here, such an objective is assumed desirable based on arguments of three types: political, humanistic, and scientific (e.g., [3], [9]). Since the validity of arguments of the first two types is primarily a subjective matter, the emphasis in this document will be upon the scientific need for planetary quarantine.

B. Scientific Objectives

Exploration tends to imply the desire for information. It will be assumed that at any given time the information which seems most relevant, needed and obtainable is being sought, and it will be further assumed that relevancy and need are functions of the knowledge possessed about the subjects being investigated.

Currently, information about the possible existence of extraterrestrial life (or its precursors or remnants) is desired, and, because of this, the need for planetary quarantine has been expressed. Briefly, the argument is that terrestrial contamination might destroy,alter,or make impossible the detection of extraterrestrial life forms if they exist or falsely imply their existence if they do not. Thus, the desire for planetary quarantine is dependent upon the desire for biological information, and the latter is part of the overall desire for scientific information about the solar system. This implies that general scientific space exploration objectives will influence the nature of the planetary quarantine program.

There are several items related to space exploration which seem relevant to any consideration of planetary quarantine. These are:

(a) the current concept of obtaining information involves space probes designed to perform scientific experiments on or near the planets of our solar system,

(b) that part of the scientific exploration period in which biological experimentation is to be performed provides a lower bound for T (see Objective 1),

(c) scientific objectives should help to determine the meaning of "contamination" and reasonable values of $\hat{P}_{N.C.}$,

(d) the exact nature of any exploration program is uncertain because of the uncertainties in:

- scientific information desired as a function of time
- performance of spacecraft and experiments
- knowledge about the planets being explored.

IV - Significant Factors Associated with the Non-Contamination Objective

A. Factors Influencing the Meaning of "Contamination"

The word "contamination", as it appears in Objective 1, is considered to be undefined. There are at least three factors which influence any specification of its meaning.

These are:

(1) The scientific desire for non-contamination. There is much disagreement in the scientific community about the appropriate definition of contamination. Concern ranges from bacterial contamination only to various types of chemical contamination (see [3], [10] and [11]). Any choice is, at best, a guess since the type of information desired, and thus possibly influenced by contamination, is very likely unknown (Section III.B.).

(2) Current technical capabilities. At present, for example, it seems impossible to predict, even statistically, whether viruses are present on a lander capsule prior to its launch. This inability stems primarily from a lack of suitable means for measuring the viral burden of spacecraft surfaces. Inclusion of viruses in the definition of "contamination" would, therefore, seem impractical now.

(3) Possible penalties. Penalties are of two basic types. There may be a scientific information loss if "contamination" is not adequately defined, and the dollar cost for planetary quarantine may prove exhorbitant for some definitions of the word "contamination".

It will be assumed that the word "contamination" is undefined for purposes of this document. However, it will be assumed that, whatever its definition, it is a binary proposition. That is, either a planet is contaminated in a specified time period, or it is not. It is also assumed that uncertainty in the measurement or knowledge about the state of contamination is possible and that reference to the "probability of contamination" is appropriate.

B. Factors Influencing the Value of T

The time period, T, in which Objective 1 is to be observed is influenc by at least four factors. These are:

(1) The nature of the exploration program. The sequencing of all experimentation in the exploration program has a direct bearing on the time period in which biological experimentation is to be performed.

(2) The uncertainties in exploration. The uncertainties listed in Section III.B. lead to uncertainties in desired biological information experiments and, hence, time needed for their performance. The uncertainties in total scientific information desired may have the same effect on T.

(3) Technical capabilities. For example, manned landing on a planet may preclude the attainment of Objective 1. If this is so, then the time period T should include no manned landings. The knowledge of manned landing dates is likely another uncertainty.

(4) Scientific penalties. If T is chosen too short, and an adequate planetary quarantine cannot be maintained for a scientifically desired time, the possibility exists that the risk of information loss is greater than that which is scientifically desirable.

Because of the implied uncertainty in T and the risk associated with estimat it will be assumed that T is unknown. In essence, this requires that some means of attaining Objective 1 be found which admits an unknown T.

C. Significant Factors Associated with $\hat{P}_{N.C.}$

If the desired time period, T, of planetary quarantine were known for a given planet, and if the *total* number of spacecraft, n(T), to be used in exploration of that planet were also known, then $\hat{P}_{N.C.}$ could be expressed in terms of n(T) and $\hat{P}_C(n(T))$, the maximum acceptable probability of contamination from any of the n(T) spacecraft. A simple model doing this is:

$$\hat{P}_{N.C.} = [1 - \hat{P}_C(n(T))]^{n(T)}.$$

It is thus reasonable to suppose that n(T) and $\hat{P}_C(n(T))$ are related to significant factors influencing the attainment of Objective 1. Thus, there are at least four factors which are associated with $\hat{P}_{N.C.}$:

(1) *The time period T*. This is assumed to be unknown (Section IV.B.).

(2) *The total number of spacecraft, n(T), used in exploration of the planet*.

(3) *The maximum acceptable probability of contamination, $\hat{P}_C(n(T))$, from any one of the n(T) spacecraft*.

(4) *The uncertainties*. These arise in T (Section IV.B.). Also, n(T) is uncertain because of the uncertainties in

- the time period T,
- the scientific information desired as a function of time,
- performance of spacecraft and experiments,
- knowledge about the planet being investigated.

Independent of its dependence upon n(T), $\hat{P}_C$ is influenced by uncertainties in:

- the proper definition of the word "contamination",
- scientifically desirable values for $\hat{P}_{N.C.}$
- knowledge about the planet under investigation.

D. Summary of Significant Factors

On the basis of the above discussion, the following assumptions will be made.

(1) The word "contamination" is undefined. It is a binary proposition, knowledge or measurement of which is uncertain. The definition of "contamination" may vary with time as a function of knowledge gained.

(2) The time period, T, in which the non-contamination objective (Objective 1) is applicable, is unknown.

(3) The probability $\hat{P}_{N.C.}$ may be a function of time due to the scientific uncertainties about the definition of "contamination".

The factors which influence the attainment of Objective 1 are assumed to be:

(1) The total number of spacecraft, n(T), to be launched in the vicinity of the planet in question.

(2) The maximum acceptable probability of contamination, $\hat{P}_C(N(T))$, for any of the n(T) spacecraft.

(3) The uncertainty in T.

(4) The uncertainty in scientific information desired as a function of time

(5) The uncertainty in the performance of spacecraft and experiments.

(6) The uncertainty about the planet under investigation.

V - A Sequential Decision Model for Planetary Quarantine

The objective of this section is to relate Ojbective 1 to the factors which have been assumed significant to its attainment in such a fashion that the relationship derived is consistent with the assumptions made in the previous section.

The simple model

$$\hat{P}_{N.C.} = [1 - \hat{P}_C(n(T))]^{n(T)}$$

was used in the preceeding section to illustrate the dependence of $\hat{P}_{N.C.}$, the minimum acceptable probability of not contaminating the planet in question during the allotted time period T, upon n(T), the total number of missions to be launched in the vicinity of the planet during the period T, and upon $\hat{P}_C(n(T))$, the maximum acceptable probability of contamination of the planet from each of the n(T) missions. This model also demonstrates that individual mission criteria may be derived from Objective 1 even though the notion of contamination is not well-defined. This is done by translating program "contamination" requirements into mission "contamination" requirements in such a fashion that the word "contamination", whatever its meaning, is used in the same sense in both cases. Operationally, of course, assumptions regarding the meaning of the word contamination must be made, and it was assumed (Section IV.D that such a definition may change with time.

Because of the assumption that "contamination" is undefined, one of the goals in the development of the model presented here was that this ability of the above model to translate program "contamination" requirements into mission "contamination" requirements, independent of the definition of "contamination", should be retained. The approach taken was to retain terms similar to $\hat{P}_C(\hat{n}(T))$; a slightly different form being necessitated by the lack of knowledge about T and n(T).

Recall that the parameter T was assumed to be unknown (Section IV.D.). This and the other uncertainties listed in the previous section (Section IV.D.) lead to a possible gross uncertainty in n(T), the total number of missions to be sent to the vicinity of the planet in question in the time period T. It is this uncertainty which makes the model above unsuitable in practice. In view of the uncertainty in n(T), it was deemed desirable to develop a model which would yield a mission-oriented requirement such as $\hat{P}_C(n(T))$ and which would, at the same time, be operationally less dependent on the <u>total</u> number of missions to be launched.

In attempting this, it was observed that there seems to be a willingness on the part of responsible parties to make <u>estimates</u> of the total number of missions to be launched toward Mars in the next 20 years. Thus, suppose that N_1 represents an <u>estimate</u> of the total number of missions to be launched in the vicinity of some planet for which Objective 1 is deemed appropriate. Then, if no more than N_1 missions are launched, the model

$$\hat{P}_{N.C.} = (1 - \hat{P}_1)^{N_1} \tag{1}$$

yields a requirement on the probability of contamination, P_1, for each of the N_1 missions, namely,

$$P_1 \leq \hat{P}_1.$$

That is, if no more than N_1 missions are launched, and if $P_1 \leq \hat{P}_1$ for each mission launched, then the probability that the planet is not contaminated during the exploration program, denoted $P_{N.C.}$, satisfies the inequality

$$P_{N.C.} \geq \hat{P}_{N.C.}$$

which, in essence, represents the attainment of Objective 1.

Now, suppose that after M_1 of the originally estimated N_1 missions are launched (with M_1 strictly less than N_1), it is decided that the original estimate, N_1, is incorrect. This means that, instead of launching the remaining N_1-M_1 missions, there will now be an estimate of N_2 further missions to be launched. Each of these N_2 missions will need to satisfy a contamination requirement different from the one satisfied by the first M_1 missions. This new requirement may be derived from the relation

$$\hat{P}_{N.C.} = (1 - \hat{P}_1)^{M_1} (1 - \hat{P}_2)^{N_2} \tag{2}$$

That is, $\hat{P}_1$ defines a requirement (presumably already achieved) on the first M_1 missions that have been launched, and a new requirement on P_2, the probability of contamination for any of the remaining N_2 missions, is generated; namely

$$P_2 \leq \hat{P}_2.$$

Notice that $\hat{P}_2$ is the only unknown appearing in equation (2) (assuming $\hat{P}_{N.C.}$, N_1,N_2, M_1 are given) and thus, $\hat{P}_2$ may be obtained when $M_1 < N_1$, as assumed.

If, then, $P_2 \leq \hat{P}_2$ for each of the remaining N_2 missions and $P_1 \leq \hat{P}_1$ for the M_1 missions already launched, one again has $P_{N.C.} \geq \hat{P}_{N.C.}$; implying the attainment of Objective 1.

Proceeding in this spirit, again suppose that after M_2 of the newly estimated additional N_2 missions have been launched (again $M_2 < N_2$), it is decided that the estimate N_2 is incorrect. Instead, it is estimated that N_3 additional missions will be needed. This makes the estimated total number of missions equal to $M_1 + M_2 + N_2$ with $M_1 + M_2$ having been launched and N_3 additional missions estimated. Then a new requirement on the probability of contamination, P_3, for any of these remaining N_3 missions may be derived from:

$$\hat{P}_{N.C.} = (1 - \hat{P}_1)^{M_1} (1 - \hat{P}_2)^{M_2} (1 - \hat{P}_3)^{N_3}; \tag{3}$$

namely,

$$P_3 \leq \hat{P}_3$$

This is possible since $\hat{P}_1$ and $\hat{P}_2$ are known from solving equations (1) and (2), in that order. Note that if $M_1 < N_1$ and $M_2 < N_2$, it is always possible to solve equation (3) for $\hat{P}_3$.

With this background, we define:

N_1 to be the first estimate of the total number of missions to be launched in the vicinity of the planet in question,

M_1 to be the number of these N_1 missions launched prior to a reestimation of the total number of missions required,

M_1+N_2 to be the second estimate of the total number of missions to be launched in the vicinity of the planet in question,

M_1+M_2 to be the number of these M_1+N_2 missions launched prior to a third estimate of the number of missions required,

and generally,

$\left(\sum_{j=1}^{k-1} M_j\right) + N_k$ to be the k^{th} estimate of the total number of missions to be launched in the vicinity of the planet in question, and

$\left(\sum_{j=1}^{k-1} M_j\right) + M_k$ to be the number of these missions launched prior to the (k+1)st estimate of the number of missions required.

Further, $\hat{P}_k$ is defined to be the maximum acceptable probability of contaminatio from any of the last N_k missions needed to fulfill the k^{th} estimate of the tota number of missions required.

The model, then, is sequential in character:

$\hat{P}_1$ is obtained from equation (1), and generally, if $\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_{k-1}$ are known, then $\hat{P}_k$ is obtained by solving

$$(1 - \hat{P}_k)^{N_k} = \frac{\hat{P}_{N.C.}}{\prod_{j=1}^{k-1} (1 - \hat{P}_j)^{M_j}} \tag{4}$$

Let $P_C^{(i)}$ denote the probability of contamination of the planet in question from the i^{th} mission. Then, if

$$P_C^{(i)} \leq \hat{P}_j, \text{ for } \sum_{s=1}^{j-1} M_s < i \leq M_j$$

$$\text{and } 1 \leq j < k, \text{ and}$$

$$P_C^{(i)} \leq \hat{P}_k, \text{ for } \sum_{s=1}^{k-1} M_s < i \leq N_k$$

then $P_{N.C.}$, the probability of not contaminating the planet in question during its biological exploration, will satisfy the inequality

$$P_{N.C.} \geq \hat{P}_{N.C.}$$

which represents the achievement of Objective 1. This statement assumes that the k^{th} estimate of the number of missions required is the final estimate.

One factor yet to be included in the model is the possible dependence of $\hat{P}_{N.C.}$ upon time (Section IV.D.). This factor is introduced by assuming that $\hat{P}_{N.C.}$ may be changed only when a reestimate of the needed number of missions is undertaken. Such an assumption may be made without any loss of generality since a change in $\hat{P}_{N.C.}$ can always be accompanied by a "no change" reestimation of the total number of missions required. Thus, in equation (4), $\hat{P}_{N.C.}$ is replaced by the $\underline{k^{th}}$ $\underline{\text{estimate}}$ of $\hat{P}_{N.C.}$, denoted $P_{N.C.}^{(k)}$. The k^{th} mission-oriented requirement, $\hat{P}_k$, is then obtained from the expression

$$(1 - \hat{P}_k)^{N_k} = \frac{P_{N.C.}^{(k)}}{\prod_{j=1}^{k-1} (1 - \hat{P}_j)^{M_j}} \tag{5}$$

for k > 1, and $\hat{P}_1$ is obtained from equation (1) as before. In this form, the existence of a nonzero solution for $\hat{P}_k$ depends upon the magnitude of $P^{(k)}_{N.C.}$. That is, there is a nonzero solution for $\hat{P}_k$ if and only if the right hand side of equation (5) is less than 1 (see the Appendix).

It should also be remarked that it may be desirable to treat N_j as a sum of numbers n_{ij}, i = 1, 2, ..., r_j, where the division into r_j numbers is associated with a desire to distinguish between certain "classes" of missions. The dependence of the index, r, upon the estimate number, j, is included to provide for the possibility that the notion of "classes" may change with time. For example, the division may refer to "sterilized" and "unsterilized" or to "lander", "flyby" and "orbiter" missions. In this case, equation (5) is replaced by

$$\prod_{i=1}^{r_k} (1 - \hat{P}_{ik})^{n_{ik}} = \frac{P^{(k)}_{N.C.}}{\prod_{j=1}^{k-1} \left[\prod_{i=1}^{r_j} (1 - \hat{P}_{ij})^{m_{ij}} \right]} \tag{6}$$

where $\hat{P}_{ij}$ is the maximum acceptable probability of contamination of the planet from any mission of the i^{th} class desired additionally after the j^{th} estimate of the total number of missions, in that class, required. Here,

$$M_j = \sum_{i=1}^{r_j} m_{ij}$$

and m_{ij} is the number of the n_{ij} estimated missions actually launched prior to the (j+1)st estimate of the total number of missions required in each of the r_{j+1} classes.

Initially, in such an approach, equation 1 is replaced by

$$\prod_{i=1}^{r_1} (1 - \hat{P}_{i1})^{n_{i1}} = P_{N.C.}^{(1)} \tag{7}$$

Notice that the use of a model based on equations (6) and (7) does not lead to unique solutions for the $\hat{P}_{ik}$ when $r_k > 1$. This may or may not be a disadvantage depending upon the choice of classes to be considered (see the Appendix)

Depending upon the approach chosen, equations (1) and (5) or equations (6) and (7) yield a model which provides a sequential means of deriving mission planetary quarantine requirements from Objective 1. At the same time, no *a priori* specification of the time period, T, or the total number of missions, n(T) is required. In fact, no *a posteriori* knowledge of T is needed if one is willing to admit the possibility of imposing planetary quarantine requirements upon missions for which they are unnecessary.

VI - Discussion of the Model

A. The Ratio $\hat{P}_k/\hat{P}_{k-1}$

The remarks in this section will be directed primarily toward the model defined by equations (1) and (5) of the previous section with the assumption that $P_{N.C.}^{(k)}$ is a constant as k varies. A discussion of the more general model given by equations (6) and (7) may be found in the Appendix.

If it is assumed that the reestimates, N_k, are always such that additional missions are added to the total, i.e.,

$$N_k > N_{k-1} - M_{k-1},$$

then

$$\hat{P}_k < \hat{P}_{k-1}.$$

Thus, the non-contamination criteria for missions continually become more stringent. This is examined mathematically in the Appendix.

The usefulness of such a model is rather obviously dependent upon the nature of the change in $\hat{P}_k$ as a function of k. For example, if $\hat{P}_1 \doteq 10^{-5}$, the implied mission requirement appears to be an attainable goal [10]. But, if $\hat{P}_2 = 10^{-6}\,\hat{P}_1$, the second requirement defined by $\hat{P}_2$ would appear to be unreasonable [10]. Hence, it would seem desirable to know something about the ratio $\hat{P}_k/\hat{P}_{k-1}$ as k increases.

It is shown in the Appendix that

$$\frac{\hat{P}_k}{\hat{P}_{k-1}} \approx \frac{N_{k-1} - M_{k-1}}{N_k} \tag{8}$$

when $P_{N.C.}^{(k)} = P_{N.C.}^{(k-1)}$. The "exact" calculations presented below indicate that approximation (8) is quite accurate. The data in TABLES 1 and 2 is based upon the assumption that $P_{N.C.}^{(k)} = 0.999$ for all k involved. See Page 3.

Again supposing the worst situation, that is, $N_k > N_{k-1} - M_{k-1}$, it seems desirable that the ratio $\hat{P}_k/\hat{P}_{k-1}$ be as large as possible since $\hat{P}_k < \hat{P}_{k-1}$ (see the Appendix). With this in mind, two general conclusions may be drawn from approximation (8). The first is that if N_k is very nearly equal to $(N_{k-1} - M_{k-1})$ then $\hat{P}_k/\hat{P}_{k-1}$ will be nearly equal to one. Thus the desire for as little change as possible in mission requirements implies a desire for accurate estimation,at each stage, of the total number of missions required. This is essentially an observation about the desired nature of N_k. The second conclusion regards M_{k-1}. Suppose that the k^{th} estimation had been made when fewer than M_{k-1} missions had been launched, but that the total number of missions estimated at the k^{th} stage remained the same. Then the new number launched prior to the k^{th} estimation may be represented as $(M_{k-1} - v)$ and the new estimate as $(N_k + v)$ where $v \geq 0$. In this case, the k^{th} requirement, $\hat{P}_k$, is replaced by $\hat{P}_k(v)$. Using estimation (8),

$$\frac{\hat{P}_k(v)}{\hat{P}_{k-1}} \approx \frac{N_{k-1} - (M_{k-1} - v)}{N_k + v}$$

N_1	M_1	N_2	$\hat{P}_1$	$\hat{P}_2$
10	8	5	1×10^{-4}	3.99×10^{-5}
		10	1×10^{-4}	2.00×10^{-5}
		20	1×10^{-4}	9.98×10^{-6}
15	12	5	6.67×10^{-5}	3.99×10^{-5}
		10	6.67×10^{-5}	2.00×10^{-5}
		20	6.67×10^{-5}	9.98×10^{-6}
20	10	15	5.00×10^{-5}	3.33×10^{-5}
		20	5.00×10^{-5}	2.50×10^{-5}
		30	5.00×10^{-5}	1.67×10^{-5}
	15	10	5.00×10^{-5}	2.50×10^{-5}
		15	5.00×10^{-5}	1.66×10^{-5}
		20	5.00×10^{-5}	1.25×10^{-5}
		30	5.00×10^{-5}	8.32×10^{-6}
30	25	10	3.33×10^{-5}	1.66×10^{-5}
		20	3.33×10^{-5}	8.31×10^{-6}
		30	3.33×10^{-5}	5.54×10^{-6}

TABLE 1 - Two Stage Decisions

N_1	M_1	N_2	M_2	N_3	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
10	5	10	5	10	1.00×10^{-4}	5.00×10^{-5}	$2.50 \times 10^{-}$
10	8	10	5	10	1.00×10^{-4}	2.00×10^{-5}	$1.00 \times 10^{-}$
15	10	10	5	10	6.67×10^{-5}	3.33×10^{-5}	$1.67 \times 10^{-}$
20	15	20	15	20	5.00×10^{-5}	1.25×10^{-5}	$3.13 \times 10^{-}$
20	10	15	10	15	5.00×10^{-5}	3.33×10^{-5}	$1.11 \times 10^{-}$
30	25	10	5	10	3.33×10^{-5}	1.66×10^{-5}	8.34×10

TABLE 2 - Three Stage Decisions

It can be shown that

$$\frac{\hat{P}_k(v)}{\hat{P}_{k-1}} > \frac{\hat{P}_k(v-1)}{\hat{P}_{k-1}} \quad \text{if} \quad \frac{N_{k-1} - M_{k-1}}{N_k} < 1.$$

Here v is assumed to be greater than zero. But since $\hat{P}_k/\hat{P}_{k-1} < 1$, it is desirable to choose v so that $\hat{P}_k(v)/\hat{P}_{k-1}$ is maximal, and this clearly occurs when v is chosen as large as possible. Thus, in order that the mission requirements be no more demanding than necessary, the decision stages should occur as early as possible after it is recognized that an increase in the number of missions is needed.

As an example, assume that $(N_{k-1} - M_{k-1})/N_k < 1$ for all k,and that the time for the k^{th} decision was chosen "early" so that $(N_{k-1} - M_{k-1})/N_k = \frac{1}{2}$.

Then approximation (8) allows one to conclude that

$$\hat{P}_k \approx \frac{1}{2^{k-1}} \hat{P}_1.$$

Hence, in particular,

$$\hat{P}_2 \approx \frac{1}{2} \hat{P}_1$$

$$\hat{P}_3 \approx \frac{1}{4} \hat{P}_1$$

$$\hat{P}_4 \approx \frac{1}{8} \hat{P}_1 \quad \text{and}$$

$$\hat{P}_5 \approx \frac{1}{16} \hat{P}_1,$$

so that four decisions were possible without changing the original criterion, $\hat{P}_1$, by a factor of 10. Assuming this might be a reasonable criterion for "attainability", it is possible, under the assumed conditions, to continue to derive "attainable" criteria through at least four decision stages. Similar types of behavior were exhibited in TABLE 2 earlier.

Thus the mission criteria become more demanding with time when re-estimation constantly indicates a need for greater numbers of missions. However, if the estimates are made "reasonably" early and are not too great with respect to the unlaunched remainder of the previously estimated number of missions, the model seems to provide a reasonable means of deriving mission criteria for the newly estimated number to be launched.

If, at any stage, the newly estimated total number of missions is less than the previous total number estimated, one has a choice. The associated lessening of the stringency of mission requirements may be adopted, or one may continue to use the previous requirement. The latter choice might be desirable if the previous requirement was still acceptable and there was concern over possible future increases in the number of missions required (even though such an occurrence was not being contemplated).

The more general model represented by equations (6) and (7) of the previous section has not been discussed primarily for two reasons: the possibility that solutions may not exist if $P_{N.C.}^{(k)}$ varies and, more importantly, the non-uniqueness of solutions. It is safe to assert that if the k^{th} estimate results in an increase in the number, n_{ik}, of missions required and $P_{N.C.}^{(k)}/P_{N.C.}^{(k-1)} \geq 1$, then at least one of the ratios, $\hat{P}_{ik}/\hat{P}_{ik-1}$ must be less than one. It is also true that the ratio $\hat{P}_{ik}/\hat{P}_{ik-1}$ is quite sensitive to changes in $P_{N.C.}$ when $\hat{P}_{ik-1}$ is small. This is discussed further in the Appendix.

B. Possible Compensating Factors

There are at least three factors which may help to compensate for the decrease in mission requirements, $\hat{P}_k$, with increasing k (again assuming a tendency toward increased numbers of missions). These will now be discussed.

1. Use of Estimates for $\hat{P}_k$

In the actual use of the model shown as equations (1) and (5), with $\hat{P}_{N.C.}$ independent of k, it is possible to use the estimates of actual contamination in the expressions $(1 - \hat{P}_i)^{M_i}$. That is, the a posteriori probabilities of contamination for missions launched in the j^{th} stage may be known as a result of measurements taken after to their launch. Thus, $(1 - \hat{P}_i)^{M_i}$ may be replaced by an expression of the form

$$\prod_{j=1}^{M_i} (1 - \tilde{P}_{ij})$$

where, it is assumed that $\tilde{P}_{ij} \leq \hat{P}_i$. The $\tilde{P}_{ij}$ are the estimated a posteriori probabilities of planetary contamination. If any $\tilde{P}_{ij}$ is strictly less than P_i, then there will be a less stringent mission requirement in the next stage. Using this approach, the model becomes

$$
\left.
\begin{aligned}
(1 - \hat{P}_1)^{N_1} &= P_{N.C.}^{(1)}, \text{ and for } k > 1 \\
(1 - \hat{P}_k)^{N_k} &= \frac{P_{N.C.}^{(k)}}{\prod\limits_{i=1}^{k-1} \left[\prod\limits_{j=1}^{M_i} (1 - \tilde{P}_{ij}) \right]}
\end{aligned}
\right\} \qquad (9)
$$

Using this model, one may take advantage of the fact that missions may exceed the planetary quarantine requirements to compensate for the theoretical decrease in $\hat{P}_k$. The same may be done, of course, with the more general model given by equation (6) and (7).

2. Change in Technical Capabilities

In the course of time, it is not unreasonable to suppose that changes in technical capabilities would make possible the achievement of more demanding planetary quarantine mission requirements. However, compensation for decreasing $\hat{P}_k$ from technological change tends to imply a continuing commitment to research in the areas where possible benefit may be derived.

3. Decrease in the Probability of Bias

In [12], the probability of contamination of a planet from a spacecraft was related to the probability that contamination, if deposited on the planet, would "bias" future experimentation. This was, in essence, an analysis of $\hat{P}_k$ solely from the point of view of achieving scientific objectives. Roughly speaking, $\hat{P}_k$ is linearly dependent upon this probability of "biasing" future experimentation. Thus, if information gained as a result of experimentation indicates that deposited contamination is less likely to "bias" further experimentation than was originally assumed, one may need to take no additional action to achieve more demanding planetary quarantine requirements.

C. Relationship to Objectives Two and Three

The sequential decision model developed here seems to allow for the uncertainty in space exploration programs, and include those other factors which were considered significant in Section IV. Thus, it may be useful in helping to derive values for $\hat{P}_{N.C.}$ which are acceptable from a penalty point of view (see [1] or [2]) as well as deriving mission requirements, $\hat{P}_k$, from fixed values of $\hat{P}_{N.C.}$. Therefore it is possible that this model represents a first step toward the attainment of Objective 2 (Section II).

Because only an estimate of the total number of missions required is needed initially, less information about the exploration program is required *a priori*. This need for less information should aid in the attainment of planetary quarantine Objective 3 (Section II).

D. Comparisons with Other Models

The first planetary quarantine"requirements" model [13] attempted to derive individual mission requirements and include the uncertainty arising from the lack of knowledge about spacecraft and experimental performance. In doing so, however, it assumed that

- the probability of mission success did not vary appreciably from mission to mission,
- the total number of experiments to be performed was known, and
- infinitely many missions may be necessary due to the uncertainty in spacecraft performance.

The second of these assumptions implies that little consideration was given, in the model, to uncertainties arising from the lack of knowledge about the scientific information desired. The first assumption may present no difficulty. The last assumption would allow a conservative mission criterion to be derived if the same model with finitely many missions assumed always led to a less stringent mission requirement. It was shown in [12] that this did not occur.

A model presented in [14] attempted to correct the latter deficiency, but did so by equating an expression derived in [13] under the assumption of infinitely many flights with an expression derived on the basis of finitely many flights. This was pointed out in [15].

The original model [13] was extended in [15] to include a means for differentiating between "hard" and "soft" landings. Again the essential features appearing in [13] were retained.

Finally, the model used in COSPAR discussions in 1966 is found in [16]. Essentially the same model appears in [17]. This model assumes that an upper bound for the total number of missions (divided into several "classes") is known. This assumption implies the existence of <u>some</u> knowledge about the time period T and the total number of missions required, n(T). The amount of knowledge required depends upon the degree of realism desired in the upper bounds. Recently [18], this model has been reinterpreted so that the numbers and requirements relate, not to missions, but "sources of contamination". The number of such sources depends, of course, upon the number of missions so that, again, some <u>a priori</u> knowledge about the exploration program is assumed. Furthermore, this model introduces the additional problem of enumerating all the "sources of contamination" and then specifying some proportional "importance" to these sources due to the non-uniqueness of solutions (for requirements for each source) inherent in the model. To accomplish this latter in any optimal fashion would tend to imply much knowledge about the sources of contamination and its control at each source. This type of information may ultimately be needed in any approach to planetary quarantine, but its inclusion in models for possible international use now may present some problems. Finally, "sources of contamination" seem more likely to change with time than do "classes" of spacecraft, and the model makes no allowance for this possibility.

VII - Conclusion

In this document, the non-contamination objective of planetary quarantine was assumed to be of the form:

OBJECTIVE 1. The probability that any planet deemed important for study of extraterrestrial life, or precursors or remnants thereof, be contaminated during the next T years shall not exceed $(1 - \hat{P}_{N.C.})$.

Here $\hat{P}_{N.C.}$ represents the least acceptable probability that a planet under consideration should not be contaminated in the time period T. The word "contamination" and the parameters T and $\hat{P}_{N.C.}$ were considered variable.

It was assumed that the primary desire for a non-contamination objective arises from scientific objectives. In examining scientific objectives, it was found that there appears to be much uncertainty in space exploration programs arising from uncertainties in:

- scientific information desired as a function of time
- performance of spacecraft and experiments
- knowledge about the planets being explored.

It was observed that with complete knowledge about a space exploration program, the time period, T, in Objective 1 could be determined. Also, it would be possible to determine n(T), the total number of missions to be launched in vicinity of the planet in question during the period T. If these are known, then it is possible to derive mission requirements from Objective 1 in a simple fashion using the model

$$\hat{P}_{N.C.} = [1 - \hat{P}_C(n(T))]^{n(T)}$$

where $\hat{P}_C(n(T))$ represents the maximal acceptable probability of contamination from any of the n(T) missions.

The uncertainties occurring in space exploration make certain a priori knowledge of T and n(T) unlikely however, and a model reflecting this uncertainty seems desirable.

The sequential decision model presented in this document includes this uncertainty by allowing estimates of n(T) to be made periodically. At the same time, mission requirements may be derived from these estimates with the use of the model. At any decision stage, these requirements are derived in such a manner that Objective 1 will be attained if the requirements are satisfied by each of the additional missions estimated.

Specifically, the model, in its simplest form is given by

$$(1 - \hat{P}_1)^{N_1} = \hat{P}_{N.C.} \text{ and for } k > 1$$

$$(1 - \hat{P}_k)^{N_k} = \frac{\hat{P}_{N.C.}}{\prod_{i=1}^{k-1} (1 - \hat{P}_i)^{M_i}}$$

where

N_1 is the first estimate of the total number of missions to be launched in the vicinity of the planet in question

M_1 is the number of these N_1 missions launched prior to the second estimate of the number of missions required,

and, in general, for $k > 1$

$\left(\sum_{j=1}^{k-1} M_j\right) + N_k$ is the k^{th} estimate of the **total** number of missions to be launched in the vicinity of the planet in question, and

$\left(\sum_{j=1}^{k-1} M_j\right) + M_k$ is the number of these missions launched prior to the (k+1)st estimate of the number of missions required.

Further, $\hat{P}_k$ is defined to be the maximum acceptable probability of contamination of the planet in question from any of the last N_k missions needed to fulfill the k^{th} estimate of the total number of missions required.

In theory, this model:

- requires no a priori knowledge about T or n(T) or the meaning of the word "contamination",
- but, makes use of any such knowledge available,
- can make use of a posteriori knowledge about mission requirements fulfillment, and
- implies possible penalties for operation without knowledge ($\hat{P}_k$ may decrease as a function of k, implying more demanding mission requirements).

The aforementioned penalties are minimized by

- accurate prediction of the number of missions required, and
- early readjustment of mission numbers when the need for a change is recognized.

These penalties may be compensated for by:

- the use of a posteriori mission knowledge
- the improvement in contamination control technology, and
- improved knowledge about the planet being investigated.

Of all the models now available, this appears to be the only one which makes no a priori assumption about T and n(T). However, this sequential decision model makes use of such information when it is available.

Two other possible planetary quarantine objectives were considered. These were

OBJECTIVE 2. The objective of non-contamination (Objective 1) should be attained in such a manner that the penalty associated with its achievement is acceptable nationally.

and

OBJECTIVE 3. Means for achieving Objective 1 should be known before the year Y.

The sequential decision model presented in this document may aid appreciably in the achievement of Objective 3, due to the lack of need for precise a priori knowledge about the exploration program. It may also provide a foundation for studies aimed at the achievement of Objective 2 (see [1] or [2]).

Thus, generally speaking, the sequential decision model developed in this document seems to possess those attributes which were assumed desirable on the basis of the nature of planetary quarantine objectives as they were envisioned here.

VIII - References

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IX - Appendix

This appendix is devoted primarily to a mathematical examination of the more general model presented in Section V. Not all of the possible relationships are thoroughly examined, the intent being only to examine those which seem basic to an understanding of the model.

The model being examined is given by equations (6) and (7) of Section V, viz.,

$$\prod_{i=1}^{r_1} (1 - \hat{P}_{i1})^{n_{i1}} = P_{N.C.}^{(1)} \tag{A1}$$

and, for k > 1,

$$\prod_{i=1}^{r_k} (1 - \hat{P}_{ik})^{n_{ik}} = \frac{P_{N.C.}^{(k)}}{\prod_{j=1}^{k-1} \left[\prod_{i=1}^{r_j} (1 - \hat{P}_{ij})^{m_{ij}} \right]} \tag{A2}$$

Here, $N_j = \sum_{i=1}^{r_j} n_{ij}$ and $M_j = \sum_{i=1}^{r_j} m_{ij}$.

The model represented by equations (1) and (5) of Section V is a special case obtainable from this model by setting $r_1 = r_2 = \ldots = r_k = 1$.

COMMENT 1. The kth mission requirement defined by $\hat{P}_k$ may be derived from knowledge of the (k-1)st stage only. Specifically,

$$\prod_{i=1}^{r_k} (1 - \hat{P}_{ik})^{n_{ik}} = \frac{P_{N.C.}^{(k)}}{P_{N.C.}^{(k-1)}} \cdot \prod_{i=1}^{r_{k-1}} (1 - \hat{P}_{ik-1})^{w_{ik-1}} \tag{A3}$$

where $w_{ik-1} = n_{ik-1} - m_{ik-1}$. Also, $W_j = \sum_{i=1}^{r_j} w_{ij}$.

To see this, one need only observe that the following equalities are valid. From equation (A2),

$$\prod_{i=1}^{r_k} (1 - \hat{P}_{ik})^{n_{ik}} \cdot \prod_{i=1}^{r_{k-1}} (1 - \hat{P}_{ik-1})^{m_{ik}} = \frac{P_{N.C.}^{(k)}}{\prod_{j=1}^{k-2} \left[\prod_{i=1}^{r_j} (1-\hat{P}_{ij})^{m_{ij}} \right]}$$

$$= \frac{P_{N.C.}^{(k)}}{P_{N.C.}^{(k-1)}} \; \frac{P_{N.C.}^{(k-1)}}{\prod_{j=1}^{k-2} \left[\prod_{i=1}^{r_j} (1-\hat{P}_{ij})^{m_{ij}} \right]} = \frac{P_{N.C.}^{(k)}}{P_{N.C.}^{(k-1)}} \prod_{i=1}^{r_{k-1}} (1 - \hat{P}_{ik-1})^{n_{ik-1}}.$$

The desired relationship follows immediately.

COMMENT 2. A k^{th} stage solution, that is, values of $\hat{P}_{ik}$ in the range 0 to 1, exists if and only if the right hand side of equation (A2) is no greater than one.

This is rather obvious. If a solution exists, then the right hand side must not exceed one since the left hand side does not (assuming, of course, that $n_{ik} \geq 0$). Conversely, if

$$\prod_{i=1}^{r_k} (1 - \hat{P}_{ik})^{n_{ik}} = 1 \text{ then}$$

each of the terms in the product must also equal one (each being less than or equal to 1). Hence, each $\hat{P}_{ik} = 0$ is a solution. If

$$\prod_{i=1}^{r_k} (1 - \hat{P}_{ik})^{n_{ik}} = 1 - \varepsilon$$

with $\varepsilon > 0$, then $\hat{P}_{ik} = 1 - (1 - \varepsilon)^{1/N_k}$ is a solution.

In investigating the ratios $\hat{P}_{ik}/\hat{P}_{ik-1}$, at least three questions arise:

- under what circumstances might these ratios be less than one (indicating a more demanding mission requirement),
- what factors is the magnitude of $\hat{P}_{ik}/\hat{P}_{ik-1}$ most sensitive to, and
- how might these ratios be easily approximated?

It seems reasonable that one's interest in $\hat{P}_{ik}/\hat{P}_{ik-1}$ is greatest when the i^{th} mission "class" is the same in both stages. Thus, it will be assumed that $r_k = r_{k-1} = r$. The assumption $P_{N.C.}^{(k)} \geq P_{N.C.}^{(k-1)}$ in COMMENT 3, below, corresponds to the assumption that the k^{th} overall non-contamination requirement is no less demanding than that occurring at the (k-1)st stage.

COMMENT 3. If $r_k = r_{k-1} = r$, $P_{N.C.}^{(k)} \geq P_{N.C.}^{(k-1)}$ and $n_{ik} \geq n_{ik-1} - m_{ik-1} (= w_{ik-1})$

with $\sum_{i=1}^{r} n_{ik} > \sum_{i=1}^{r} w_{ik-1}$, then

there is some j, $1 \leq j \leq r$, for which $\hat{P}_{jk} < \hat{P}_{jk-1}$.

Since $P_{N.C.}^{(k)} \geq P_{N.C.}^{(k-1)}$, equation (A3) may be rewritten

$$\prod_{i=1}^{r} (1 - \hat{P}_{ik})^{n_{ik}} \geq \prod_{i=1}^{r} (1 - \hat{P}_{ik-1})^{w_{ik-1}}$$

so that

$$\sum_{i=1}^{r} n_{ik} \ln(1 - \hat{P}_{ik}) \geq \sum_{i=1}^{r} w_{ik-1} \ln(1 - \hat{P}_{ik-1}).$$

On the other hand, if $\hat{P}_{ik}/\hat{P}_{ik-1} \geq 1$ for all i, $1 \leq i \leq r$, then

$$\ln (1 - \hat{P}_{ik}) \leq \ln (1 - \hat{P}_{ik-1}) < 0$$

for all i, so that

$$\sum_{i=1}^{r} n_{ik} \ln (1 - \hat{P}_{ik}) \leq \sum_{i=1}^{r} n_{ik} \ln (1 - \hat{P}_{ik-1})$$

$$< \sum_{i=1}^{r} w_{ik-1} \ln (1 - \hat{P}_{ik-1}).$$

comparing this with the above equation leads to the conclusion that at least one of the $\hat{P}_{ik} < \hat{P}_{ik-1}$.

This comment implies that the model given by equations (1) and (5) of Section V when $P_{N.C.}^{(k)} = P_{N.C.}^{(k-1)}$ has the property

$$\hat{P}_k/\hat{P}_{k-1} < 1 \text{ if } W_{k-1}/N_k < 1.$$

The converse is also true so that:

COMMENT 4. If $r = 1$ and $P_{N.C.}^{(k)} = P_{N.C.}^{(k-1)}$, then

$$\hat{P}_k/\hat{P}_{k-1} > 1 \text{ if and only if } W_{k-1}/N_k > 1.$$

It should be remarked that unless each of the "classes" have as many newly estimated missions at the k^{th} stage as remained to be launched in the (k-1)st, i.e., $n_{ik} \geq w_{ik-1}$, the conclusion in COMMENT 3 cannot necessarily be drawn. For example, solving

$$(1 - \hat{P}_{12})^{n_{12}} (1 - \hat{P}_{22})^{n_{22}} = (1 - \hat{P}_{11})^{w_{11}} (1 - \hat{P}_{21})^{w_{21}}$$

with $n_{12} = w_{11} - 1$ and $n_{22} = w_{21} + 2$ for

$$\hat{P}_{12} \text{ and } \hat{P}_{22} \text{ so that } \hat{P}_{i2}/\hat{P}_{i1} \geq 1,$$

$i = 1,2$, is possible if $\hat{P}_{11} \geq 1 - (1 - \hat{P}_{21})^2$. For, if this is the case, and

$$\hat{P}_{12} = \hat{P}_{11} \text{ (or } \hat{P}_{12}/\hat{P}_{11} = 1),$$

then $(1 - \hat{P}_{22})^{n_{22}} = (1 - \hat{P}_{11})(1 - \hat{P}_{21})^{w_{21}}$

so that

$$\left[\frac{(1 - \hat{P}_{22})}{(1 - \hat{P}_{21})}\right]^{n_{22}} = \frac{(1 - \hat{P}_{11})}{(1 - \hat{P}_{21})^2}$$

Thus,

$$\frac{(1 - \hat{P}_{22})}{(1 - \hat{P}_{21})} = \left[\frac{(1 - \hat{P}_{11})}{(1 - \hat{P}_{21})^2}\right]^{1/n_{22}}$$

$$= \frac{-\hat{P}_{21}}{(1 - \hat{P}_{21})} \cdot \frac{\hat{P}_{22}}{\hat{P}_{21}} + \frac{1}{(1 - \hat{P}_{21})}$$

Therefore,

$$\frac{\hat{P}_{22}}{\hat{P}_{21}} = \frac{1}{\hat{P}_{21}} \left\{1 - (1 - \hat{P}_{21}) \left[\frac{(1 - \hat{P}_{11})}{(1 - \hat{P}_{21})^2}\right]^{1/n_{22}}\right\}$$

But if $\hat{P}_{11} \geq 1 - (1 - \hat{P}_{21})^2$, then

$$\left[\frac{(1 - \hat{P}_{11})}{(1 - \hat{P}_{21})^2}\right]^{1/n_{22}} \leq \frac{(1 - \hat{P}_{11})}{(1 - \hat{P}_{21})^2} \leq 1,$$

so that,

$$1 - (1 - \hat{P}_{21}) \left[\frac{(1 - \hat{P}_{11})}{(1 - \hat{P}_{21})^2}\right]^{1/n_{22}} \geq \hat{P}_{21}$$

Thus $\hat{P}_{22}/\hat{P}_{21} \geq 1$ also, and we observe:

COMMENT 5. In the general model presented in equations (A1) and (A2) [and equations (6) and (7) in Section V] it may be possible to increase the total number of missions required at the k^{th} stage and yet have no more demanding mission requirements at that stage. In order to do this one must decrease the number of missions of at least one "class". The possibility of then obtaining no more demanding requirements at the kth stage depends upon the relative magnitudes of the $\hat{P}_{ik-1}$ at the (k-1)st stage and upon the magnitude of $P^{(k)}_{N.C.}/P^{(k-1)}_{N.C.}$

The observation made in COMMENT 5 leads one to consider the possibility of "optimally" selecting the $\hat{P}_{ik}$ at the kth stage. Many possible criteria for "optimality" exist. For example, one may (possibly) solve the mathematical programming problem

$$\text{maximize} \sum_{i=1}^{r} (\hat{P}_{ik}/\hat{P}_{ik-1})$$

subject to equation (A2) [or (7)] and the logical constraints

$$0 < P_{ik} \leq 1, \; i = 1, \ldots, r.$$

Alternatively one might use the expression

$$\text{minimize} \sum_{i=1}^{r} \left(1 - \frac{\hat{P}_{ik}}{\hat{P}_{ik-1}}\right)$$

as an "optimality" criterion. Both of these admit solutions with $\hat{P}_{ik}/\hat{P}_{ik-1} < 1$, even when such is not necessary. Thus, one might consider additional constraints of the form

$$\hat{P}_{ik}/\hat{P}_{ik-1} \geq 1, \; i \in S_\alpha$$

where S_α is an index set; a subset of $\{1,2,\ldots,r\} = I_r$. When $\alpha = 1$, for example, $S_\alpha = I_r$. Then $S_2, S_3, \ldots, S_{r+1}$ may be the subsets of I_r defined by $I_r - \{j\}$, $j = 1, 2, \ldots, r$, and so forth.

Finally, if the cost associated with attaining a given value of $\hat{P}_{ik}$ were known, denoted $c_i(\hat{P}_{ik})$, then one could (possibly) obtain a solution to

$$\text{minimize} \sum_{i=1}^{r} c_i(\hat{P}_{ik})$$

subject to equation (A2) [equation (7), Section V] and the logical constraints

$$0 < \hat{P}_{ik} \leq 1, \; i = 1, \ldots, r.$$

The final comment to be made deals with approximate means of calculating $\hat{P}_{jk}/\hat{P}_{jk-1}$. From the above discussion, it is evident that this ratio depends upon the relative magnitudes of the $\hat{P}_{ik-1}$. It also depends upon the relative magnitudes of the $\hat{P}_{jk}$ themselves, and thus calculations may be made only when these are prescribed. Their prescription may, however, involve some "optimal" solution, as discussed above, so that the following approximation may be useful only in understanding the behavior of the model.

COMMENT 6. If $r = r_k = r_{k-1}$, $\hat{P}_{jk} << 1$, $\hat{P}_{jk-1} << 1$, for all j, all of the $\hat{P}_{ij}$, $1 \leq i \leq r$, $j = k, k-1$, are of approximately the same order, and

$$\hat{P}_{ik} = \alpha_{ik} \hat{P}_{1k}, \hat{P}_{ik-1} = \alpha_{ik-1} \hat{P}_{1k-1}, \text{ then}$$

$$\frac{\hat{P}_{jk}}{\hat{P}_{jk-1}} \approx \frac{\alpha_{jk}}{\alpha_{jk-1}} \left\{ \frac{P^{(k)}_{N.C.}}{P^{(k-1)}_{N.C.}} \frac{\sum_{i=1}^{r} w_{ik-1} \alpha_{ik-1}}{\sum_{i=1}^{r} n_{ik} \alpha_{ik}} + \frac{(1 - P^{(k)}_{N.C.}/P^{(k-1)}_{N.C.})}{\hat{P}_{jk-1}(1/\alpha_{jk-1}) \sum_{i=1}^{r} n_{ik} \alpha_{ik}} \right\} . \quad \text{(A4)}$$

Here, $w_{ik-1} = n_{ik-1} - m_{ik-1}$, as before

The assumptions allow equation (A3) to be written in the approximate form

$$1 - \sum_{i=1}^{r} n_{ik} \hat{P}_{ik} \approx \frac{P^{(k)}_{N.C.}}{P^{(k-1)}_{N.C.}} [1 - \sum_{i=1}^{r} w_{ik-1} \hat{P}_{ik-1}].$$

Thus,

$$1 - \hat{P}_{1k} \sum_{i=1}^{r} n_{ik} \alpha_{ik} \approx \frac{P^{(k)}_{N.C.}}{P^{(k-1)}_{N.C.}} [1 - \hat{P}_{1k-1} \sum_{i=1}^{r} w_{ik-1} \alpha_{ik-1}].$$

This may be solved, to yield

$$\hat{P}_{1k}/\hat{P}_{1k-1}, \text{ and then } \hat{P}_{jk}/\hat{P}_{jk-1} = \frac{\alpha_{jk}}{\alpha_{jk-1}} \frac{\hat{P}_{1k}}{\hat{P}_{1k-1}} .$$

Finally, replacing $\hat{P}_{1k-1}$ by $(1/\alpha_{jk-1})\hat{P}_{jk-1}$ leads to equation (A4).

There are several points of interest in equation (A4). When r = 1, as in the model defined by equations (1) and (5) of Section V, the expression reduces to:

$$\frac{\hat{P}_k}{\hat{P}_{k-1}} \approx \frac{P_{N.C.}^{(k)}}{P_{N.C.}^{(k-1)}} \frac{W_{k-1}}{N_k} + \frac{1}{\hat{P}_{k-1}} \left(1 - \frac{P_{N.C.}^{(k)}}{P_{N.C.}^{(k-1)}} \right) . \tag{A5}$$

If it is further assumed that $P_{N.C.}^{(k)} = P_{N.C.}^{(k-1)}$, this latter expression (equation (A5)) reduces to approximation (8) of Section VI, namely

$$\frac{\hat{P}_k}{\hat{P}_{k-1}} \approx \frac{N_{k-1} - M_{k-1}}{N_k} . \tag{A6}$$

In both equation (A4) and (A5) the possibility exists that the ratios $\hat{P}_{ik}/\hat{P}_{ik-1}$ or $\hat{P}_k/\hat{P}_{k-1}$ will be very sensitive to variations in $P_{N.C.}^{(k)}/P_{N.C.}^{(k-1)}$ whenever $\hat{P}_{ik-1} \ll 1$ (equation (A4)) or $\hat{P}_{k-1} \ll 1$ (equation (A5)). For example, if one desires that $\hat{P}_k/\hat{P}_{k-1} = 1$, then, from (A5), when $\hat{P}_{k-1} \ll 1$,

$$\frac{P_{N.C.}^{(k)}}{P_{N.C.}^{(k-1)}} \approx \frac{1 - (1/\hat{P}_{k-1})}{W_{k-1}/N_k - (1/\hat{P}_{k-1})}$$

$$= 1 - \frac{(W_{k-1}/N_k - 1)}{[W_{k-1}/N_k - (1/\hat{P}_{k-1})]}$$

$$\approx 1 - \hat{P}_{k-1}(1 - W_{k-1}/N_k).$$

Thus, if $P_1 = 10^{-4}$, $W_1/N_2 = 1/2$, and $P_{N.C.}^{(1)} = 0.999 = 1 - 10^{-3}$, then in order that

$\hat{P}_2 = 10^{-4}$ also, one needs to choose $P_{N.C.}^{(2)} \approx (1 - 5 \times 10^{-5})(1 - 10^{-3}) \approx 0.99895$. Such a <u>slight</u> decrease in overall program goals then allows one to maintain the <u>same</u> mission requirements at the second stage. If $P_{N.C.}^{(2)} = P_{N.C.}^{(1)} = 0.999$, on the other hand, then from approximation (8),

$$\hat{P}_2 \approx 0.5 \times 10^{-4}.$$

In this section, some of the behavior of the more general model of Section V has been examined. In particular, there exist two additional ways, in this model, that one may avoid the difficulty of obtaining increasingly demanding mission non-contamination requirements. These are:

- "optimal" solution of equation (A2) [or (7) of Section V] for the $\hat{P}_{ik}$, and
- very <u>slight</u> periodic decreases in $P_{N.C.}^{(k)}$ whenever they seem justifiable.

DISTRIBUTION:

John W. Beakley
Department of Biology
University of New Mexico
Albuquerque, New Mexico

Richard G. Cornell
Associate Professor of Statistics
Department of Statistics
Florida State University
Tallahassee, Florida

Robert Angelotti
Deputy Chief, Milk and Food Res.
Robert A. Taft Sanitary Engineering Center
Cincinnati, Ohio

Gerald Silverman
Department of Nutrition and Food Science
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

Samuel Schalkowsky
Exotech Incorporated
525 School Street, S.W.
Washington, D. C. 20024

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Biology Department
University of Pennsylvania
Philadelphia, Pennsylvania 19104

A. H. Schwichtenberg
Aerospace Medicine
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5200 Gibson Blvd., SE
Albuquerque, New Mexico

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General Electric Company
Missile and Space Division
301 E. Colorado Blvd.
Pasadena, California 91109

Joshua Lederberg
Department of Genetics
Stanford University
School of Medicine
Palo Alto, California

N. H. Horowitz
Division of Biology
California Institute of Technology
Pasadena, California

George S. Michaelsen
Director, Division of Environmental Health and Safety
University of Minnesota
Minneapolis, Minnesota

Keith H. Lewis
Chief, Milk and Food Research
Robert A. Taft Sanitary Engineering Center
Cincinatti, Ohio

Joseph Stern
Jet Propulsion Laboratory
4800 Oak Grove Dr.
Pasadena, California 91103

Richard G. Bond
School of Public Health
College of Medical Science
University of Minnesota
Minneapolis, Minnesota 55455

John H. Brewer
Biological Safety and Control
Becton, Dickinson and Company
P. O. Box 6711
Baltimore, Maryland 21204

Mark A. Chatigny
Research Engineer
Naval Biological Laboratory
Naval Supply Center
University of California, Berkeley
Oakland, California 94625

Frank B. Engley, Jr.
Chairman, Department of Microbiology
School of Medicine
University of Missouri
Columbia, Missouri

Gilbert V. Levin
Hazleton Laboratories, Inc.
Box 30
Falls Church, Virginia

Irving J. Pflug
Department of Food Science
Michigan State University
East Lansing, Michigan

John A. Ulrich
Department of Microbiology
Mayo Clinic
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