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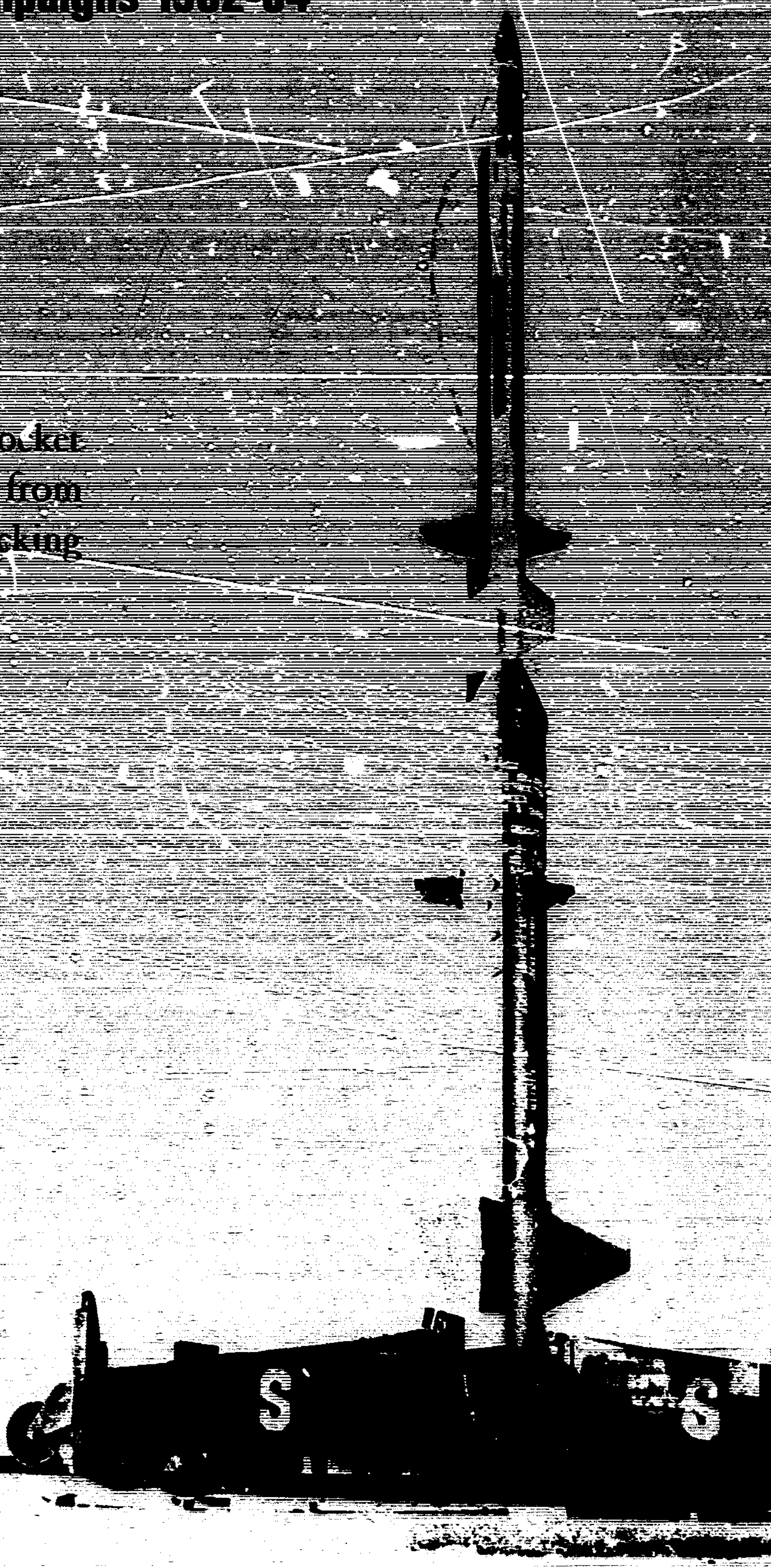
Johan Martin-Löf



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NUMERICAL METHOD FOR ROCKET TRAJECTORY DETERMINATION FROM MULTISTATION DOPPLER TRACKING DATA

Johan Martin-Löf

ABSTRACT

A numerical method for calculation of rocket coordinates from roll-corrected doppler data is described. Tropospheric refraction correction is included as well as a routine for obtaining a starting point. A new method to optimize the choice of starting point is also described. The computer program has been successfully used to reduce data from firings in Sweden 1964. Error estimates are given.

FOREWORD

The Kronogård reports

During the summer of 1962, 1963 and 1964 a series of sounding rocket experiments were performed at Kronogård in northern Sweden under a cooperative agreement between the US National Aeronautics and Space Administration (NASA) and the Swedish Space Research Committee. The main experimenter on the Swedish side was the Institute of Meteorology, University of Stockholm and on the US side groups from USAF Cambridge Research Laboratories (AFCRL) and NASA Goddard Space Flight Center.

The Swedish Space Research Committee set up a technical group to take care of the technical and operational parts of the experiments. While the scientific results from the experiments have been and will be published by the experimenters, this group is preparing a special series of reports covering its acitivites during the campaigns.

The group is since the 1st of July 1965 a division of TUAB, Teleutredningar AB under the name of Space Technology Group.

CONTENTS

1. INTRODUCTION

Continous wave radio tracking systems have for more than 20 years been used for rocket and spacecraft trajectory determination. A large number of different designs are described in the literature. They can be grouped in three major categories according to the basic quantity measured:

1) range
2) range rate
3) angle

In many systems there is a combination of these methods. (1) (2).

Most systems have a ground-based CW transmitter, a transponder in the vehicle to be tracked and one or more receiving stations on the ground.

Range measuring systems employ phase comparison between a number of phase locked harmonically related subcarriers in order to resolve ambiguities. At interplanetary ranges it is advantageous to use cyclic binary codes for the same purpose.

Range rate measuring systems are simpler, they measure the frequency difference due to doppler shift between the emitted signal and the one returned from the vehicle.

Angle measuring systems rely on phase comparison between signals received at antennas placed a few wave lenghts apart. They require complicated receiving equipment to resolve angle ambiguities and are rather sensitive to drifts in the circuitry.

For the tracking of small sounding rockets two basic designs have been used. The first and the oldest system is the multistation DOVAP (Doppler Velocity and Position) in which range rate is measured from at least three receiving stations (3). The second one is the Seddon SSD-system (Single Station Doppler) which combines a range rate measurement and two interferometers at right angles to each other (4). DOVAP has been used since the second world war at White Sands and also at Fort Churchill and at Woomera. The SSD has mostly been used at Wallops Island and also at Ascension Island and Point Barrow. A mobile SSD was operated on a loan basis in the NASA-Swedish rocket grenade experiments in Sweden 1963 with great success.

A similar series of experiments was done in 1964. It was no longer possible to have the mobile SSD and it was necessary to develop a new system. The only system that could be built in the available time and meet the stringent accuracy requirements of the rocket grenade experiment (± 10 m in altitude) was a multi-station DOVAP-system. A number of four stations was chosen to provide some redundancy.

With such a system it is possible to obtain an internal check of the consistency of the solution or alternately to allow a drop-out of one station without loosing the tracking capability. The main disadvantage with the system is an operative one, it requires personnel at four widely separated stations, the activities of which have to be synchronized.

It was considered desirable also to develop a data reduction method for the system. The only previous method that was available (5) is based on works by Garfinkel (6) and suffers from two disadvantages. First, it can only solve the problem for three receivers which makes necessary an evaluation of every possible three station combination. The number of combinations is $n!/(n-3)!3!$ which rises quickly with the number of receivers n. Second, the method cannot solve the problem if the transmitter and the three receivers are in the same plane. As a computer has a limited accuracy numerical difficulties can be encountered also if the stations are close to a common plane. There is a definite risk for this as all stations are necessarily placed on the ground.

This report describes the numerical method and the computer program that was written to perform the calculations. In order to give a better understanding of the problem there is a brief description of the system components and their operation in section 2.

Complications arising from rocket roll movements, atmospheric effects and the problem of finding a starting point for the trajectory are discussed in section 3. The numerical method employed in the computer program is described in section 4. The errors in the method are discussed in section 5 together with practical experience obtained from reducing the data collected in 1964. An iterative method to eliminate the need for a starting point is outlined in section 6. A brief description of the computer programs is given in section 7.

2. GENERAL PRINCIPLES OF OPERATION

The tracking system used in Sweden consists of a CW transmitter of 100 W output at 36.8 MHz, a rocket transponder, four van mounted receiving stations and a playback unit.

A sketch of the location of the different stations on the range is shown in fig. 1. The distance between stations should be of the same order of magnitude as the rocket peak altitude to obtain optimum accuracy. The need for reliable communications between stations led to a closer spacing which in turn leads to greater accuracy in the vertical than in the horizontal direction. As altitude is the most interesting quantity this is tolerable.

In the sounding rocket experiments the vehicle velocity is negligible in comparison with the velocity of light and the travel time for radio waves to and from the rocket is also negligible. These facts justify the use of the following quasistationary approach to the tracking problem.

The transmitter signal is continously transmitted to the rocket where it is received, doubled in frequency and retransmitted by the transponder. See fig. 2. The rocket signal is received at the ground stations and heterodyned with twice the transmitter frequency which is received by a reference receiver. When the rocket moves so that the range sum transmitter-rocket-receiver changes, a beat frequency is obtained, which has a frequency equal to the doppler shift of the rocket signal. Each cycle of this doppler frequency corresponds to an increase in the range sum by one wavelength of the received frequency. If the doppler frequency is integrated and the initial rocket position is known, the range sum for each station can be computed for all consecutive times. A known range sum means that the locus of all possible rocket positions is an ellipsoid of revolution with its foci in the transmitter and the receiver. With three receivers the rocket position can be uniquely determined. More than three receivers give a redundancy that can be used to check internal consistency and estimate system errors.

3. COMPLICATIONS

There are three complicating factors that have to be taken into account in the data reduction process:

1) the roll of the rocket
2) the atmosphere is a slightly nonhomogenous medium
3) the fact that a known starting point on the trajectory is needed.

The roll of the rocket is compensated in the hardware of the playback unit while the other two effects are compensated in the software of the computer program.

3.1. Roll correction

To understand the roll correction we have to study the antenna systems. The transmitter antenna is a crossed dipole emitting a lefthand circularly polarized wave. The electric vector has a left-hand rotation seen in the direction of propagation. The rocket has loop antennas connected so that the receiving and the transmitting antennas essentially behave like electrical dipoles with their axes at right angles to each other and to the rocket longitudinal axis. The receiving stations have crossed dipole antennas and four-port hybrid networks that permit the separation of the received signal into a right- and a left-hand polarized component. There is a definite advantage in using crossed dipole antennas in that they permit reception of the two components in exactly the same point.

3.1.1. Normal case

Suppose that the transmitter and the receiver are both behind the rocket and that the vehicle has a right-hand rotation of r rps as seen from the tail of the rocket. As the transmitter is left-hand polarized and the rocket receiving antenna is linearly polarized this means that the signal received in the rocket has the frequency

$$f' = f_t - f'_d + r$$

f_t is transmitter frequency

f'_d is the doppler shift on the upleg introduced by the motion.

The rocket retransmits the frequency $2f'$ on the plane polarized rocket transmitter antenna. The plane of polarisation rotates with the rocket and the received signal at the ground is separated by the hybrid network in two circularly polarized components of slightly different frequencies. These are:

$$f_L = 2f' - f''_d + r \qquad \text{and}$$

$$f_R = 2f' - f''_d - r$$

where f''_d is the doppler shift introduced on the downleg. Inserting the expression for f' one obtains:

$$f_L = 2f_t - f_d + 3r \quad \text{and}$$

$$f_R = 2f_t - f_d + r$$

where $f_d = f'_d + f''_d$ is the total doppler shift of the signal.

For reference the transmitter signal is also received and fed through a frequency doubler to obtain a reference frequency

$$f_{ref} = 2f_t$$

A simplified block diagram of the receiving stations is shown in fig. 3.

The three signals are fed through identical superheterodyne receivers with common local oscillators. The receivers subtract a common constant frequency from the three signals. After the receivers, the frequencies are about 10 kHz and they are fed into a 4-track tape recorder along with a range time signal. At playback the right-hand signal frequency is tripled and mixed with the left-hand signal. The resulting signal is mixed with twice the reference frequency to obtain a signal of twice the doppler frequency with the roll effect eliminated. This frequency is counted and printed once per second. A signal of twice the roll frequency is obtained by simply mixing the right- and left-hand signals. The roll direction can be determined by observing these two signals on a dual-trace oscilloscope.

It should be noted that the described analysis is valid for the case that both transmitter and receiver are behind the rocket. This is true for all ground stations for a normal trajectory up to the time when the rocket reenters into the atmosphere and starts tumbling. A possible exception is the very first second(s) of flight, but then usually no data is obtained from the distant receivers anyhow, as discussed below in section 3.5.

3.1.2. Anomalous case

If the transmitter is behind and the receiver in front of the rocket we get:

$$f_L = 2f_t - f_d + r$$

$$f_R = 2f_t - f_d + 3r$$

If the transmitter is in front and the receiver behind we get:

$$f_L = 2f_t - f_d - r$$

$$f_R = 2f_t - f_d - 3r$$

Usually these anomalous conditions appear as intermittent disturbances of the normal case due to the fact that the vehicle has both spin and precession. At the transition moments the signal strength drops to zero twice per rocket revolution due to antenna nulls, a fact which creates additional recording difficulties.

Thus when anomalous roll effects appear, automatic roll correction is very difficult and it is necessary to make playbacks of the different channels, then smooth out errors due to signal strength variations and finally make a roll correction in the computations.

3.2. Tropospheric effects

The doppler system measures distances with the wave length of the transmitter signal as yardstick. As the velocity of propagation of the radio waves varies with pressure, temperature and humidity, the length of the yardstick varies with altitude. To obtain maximum accuracy a correction should be applied.

Introduce the refractive index

$$n = \frac{c_o}{c} .$$

and the refractive modulus

$N = (n-1) \cdot 10^6$ where

c_o is the velocity of radiowaves in vacuum
c is the velocity of radiowaves in air

The dependence of the refractive modulus on the meteorological paramters is (7)

$$N = \frac{77.6\ p}{T} + \frac{37 \cdot 10^4\ p_{wv}}{T^2} \quad \text{where}$$

T is the absolute temperature
p is the total pressure in millibars
p_{wv} is the partial pressure of water vapour in millibars.

It can be noted that refractive index is slightly greater than 1 and that it is independent of frequency.

From an ordinary radio sonde measurement it is possible to calculate the refractive index through the significant part of the atmosphere. In order to include a correction into a computer program it is desirable to make a mathematical model of the variation of the refractive index with altitude.

It has been shown that with an accuracy that is sufficient for our purposes the refractive modulus decreases exponentially with the altitude z (8):

$$N = N_o\ e^{-bz}$$

The parameters N_o and b can be determined from a plot of N versus altitude z. The model has proved to be a very good representation of the actual conditions in the Swedish experiments in the summer of 1964.

For the analysis we need to know the number of wavelengths along a given path.

Introduce the following symbols:

$\underline{r}(t)$:	rocket position
$\underline{R}_i$:	receiver position $i = 1, \ldots, n$
u_i:	range sum transmitter-rocket-receiver
$C_i(t)$:	the number of wavelengths of twice the transmitted frequency on the path transmitter-rocket-receiver
f_U, f_D:	frequencies on the up- and downleg
λ_U, λ_D:	wavelengths on the up- and downleg
k_U, k_D:	wavenumbers on the up- and downleg

The rocket position

$$\underline{r}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$$

is given in a cartesian coordinate system where x is north, y is west and z is up. The origin is located in the transmitter antenna.

We have

$$C_i(t) = \int_0^{\underline{r}} 2 \frac{1}{\lambda_U} \, dr + \int_{\underline{r}}^{\underline{R}_i} \frac{1}{\lambda_D} \, dr$$

The integration paths are taken to be two straight lines as it has been shown that we can neglect the curving of the ray paths (9).

$$C_i(t) = \int_0^{\underline{r}} 2\,\frac{f_U}{c_o}\, n(z)\,dr + \int_{\underline{r}}^{\underline{R}_i} \frac{f_D}{c_o}\, n(z)\,dr$$

We can safely neglect the fact that the rocket moves a little bit while the radio waves are on the way and also the fact that the frequency on the downleg is not exactly twice the frequency on the upleg due to the dopplershift.

Thus we can put

$f_D = 2f_U = 2f_t$ and get

$$C_i(t) = \frac{2f_t}{c_o}\left\{\int_0^{\underline{r}} n\,dr + \int_{\underline{r}}^{\underline{R}_i} n\,dr\right\} =$$

$$= \frac{2f_t}{c_o}\left\{\int_0^{\underline{r}} (1 + 10^{-6} N_o e^{-bz})\,dr + \int_{\underline{r}}^{\underline{R}} (1 + 10^{-6} N_o\, e^{-bz})\,dr\right\} =$$

$$= \frac{2f_t}{c_o}\left\{u_i + \frac{N_o 10^{-6}}{\cos\alpha_U}\int_0^z e^{-bz}\,dz + \frac{N_o 10^{-6}}{\cos\alpha_D}\int_0^z e^{-bz}\,dz\right\}$$

where α_U and α_D are the zenith angles of the ray paths and where we have put the receiver altitude to zero in the last integral. Integration gives:

$$C_i(t) = \frac{2f_t}{c_o}\, u_i\left\{1 + \frac{N_o 10^{-6}}{bz}\,(1 - e^{-bz})\right\}$$

$$= u_i\,\frac{2f_t}{c_o}\,\bar{n}\,(z) = u_i\,\bar{k}\,(z)$$

where

$\bar{n}$ is the average refractive index

$\bar{k}$ is the average wave number

3.3. Ionospheric effects

At altitudes above 60 km the refractive index can seriously deviate from 1 under disturbed conditions.

The refractive index is given by (7)

$$n = 1 - \frac{1}{2} \frac{81 N_e}{f^2} \quad \text{where}$$

N_e is the electron concentration in particles/cm^3 and f the frequency in kc/s.

We see that the refractive index is less than one and that it depends on the frequency. Under very disturbed circumstances the deviation from 1 can be as large as 10^{-2}. The situation is further complicated by the earth's magnetic field which causes additional rotation of the polarisation of the tracking signal.

A compensation for ionospheric effects would require complete knowledge of the concentration of charged particles up to the peak of the trajectory. To measure this would be quite a complicated experiment in itself.

However, under normal conditions the ionospheric disturbances are quite small and the effects can be completely neglected. Compensation is not included in the program described in this report.

3.4. Starting point

As the doppler measurements do not give the range sums directly but only the increase in the range sums it is necessary to know one point on the trajectory. The best starting point would be the launcher but unfortunately it is usually impossible to pick up the rocket signal at the distant receiving stations when the vehicle is on the launcher. These stations normally acquire a good signal after about 1 second when the rocket has reached a few hundred meters altitude.

One receiver (number 1) is placed near the launcher and it has a good signal from lift off. Assuming that the first part of the trajectory is a straight line in the direction of the launcher we can get a starting point by solving for the intersection between this straight line and the ellipsoid that is defined be the range sum for station number 1.

Introduce

t_o: time at starting point
$\underline{R}_L$: launcher position
$\underline{n}_L$: launcher direction (unit vector)

t_o is chosen so that all stations have good signal from that time onwards.

The range sum $u_1(t_o)$ for the number 1 receiver is obtained as follows. The range sum at lift-off is

$u_1(0) = |\underline{r}(0)| + |\underline{r}(0) - \underline{R}_1|$ where

$\underline{r}(0) = \underline{R}_L$

The corresponding number of cycles is

$C_1(0) = \bar{k}(z(0)) \cdot u_1(0)$

and the number of cycles at the starting point

$C_1(t_o) = C_1(0) + D_1(0, t_o)$

where the $D_1(0, t_o)$ is the number of the doppler cycles recorded by receiver number 1 between lift-off and the time t_o. Finally

$$u_1(t_o) = \frac{1}{\bar{k}(z(t_c))} \; C_1(t_o)$$

The value of the average wave number can be calculated for an approximate altitude as the variation with altitude is very slow.

The trajectory is approximated by the straight line

$$\underline{r} = \underline{R}_L + p\,\underline{n}_L$$

where p is a parameter. We want to find the intersection with the ellipsoid

$$u_i(t_o) - |\underline{r}| - |\underline{r} - \underline{R}_1| = 0$$

Inserting the straight line equation we get (T denotes transposition)

$$u_1(t_o) - (p^2 + 2p\underline{R}_L^T\,\underline{n}_L + \underline{R}_L^2)^{1/2} -$$

$$(p^2 + 2p\,(\underline{R}_L - \underline{R}_1)^T\underline{n}_L + (\underline{R}_L - \underline{R}_1)^{1/2} = 0$$

Let us write this

$$f(p) = 0$$

This non-linear equation can be solved by means of the Newton-Raphson method. Thus starting from an approximation $p^{(o)}$ a better one is obtained by

$$p^{(j+1)} = p^{(j)} - f(p^{(j)})/f'(p^{(j)}) \qquad j = 0, 1 \ldots$$

we need the derivative

$$f'(p) = \frac{df(p)}{dp} = \frac{p + \underline{R}_L^T\underline{n}_L}{(p^2 + 2p\underline{R}_L^T\underline{n}_L + \underline{R}_L{}^2)^{1/2}} +$$

$$+ \frac{p + (\underline{R}_L - \underline{R}_i)^T\underline{n}_L}{(p^2 + 2p\,(\underline{R}_L - \underline{R}_1)^T\,\underline{n}_L + (\underline{R}_L - \underline{R}_1)^2)^{1/2}}$$

The starting approximation $p^{(o)}$ can be chosen as the predicted altitude at t_o. This choice is not critical but some care should be exercised to avoid obtaining the other possible solution which is below ground level.

The iteration is carried on till the desired accuracy is reached and the starting point is obtained by

$$\underline{r}(t_o) = \underline{R}_L + p\,\underline{n}_L$$

From this we find the range sums for the other stations by

$$u_i(t_o) = |\underline{r}(t_o)| + |\underline{r}(t_o) - \underline{R}_i| \qquad i = 2, \ldots, n$$

4. COMPUTATION OF ROCKET COORDINATES

Having calculated the number of wave lengths on the range sums at the starting point and using the playback results it is easy to obtain the range sums for any later time in the flight:

$$C_i(t) = C_i(t_o) + D_i(t_o, t)$$

$$u_i(t) = \frac{1}{\bar{k}(z(t))}\, C_i(t)$$

where $D_i(t_o, t)$ is the number of doppler cycles recorded between t_o and t.

Let us define a set of errors:

$$e_i(\underline{r}(t)) = u_i(t) - |\underline{r}(t)| - |\underline{r}(t) - \underline{R}_i| \qquad i = 1, \ldots, n$$

If we have only three receivers it is possible to find an $\underline{r}(t)$ that makes all errors zero, but if the number of receivers is greater this is normally impossible due to inevitable errors in the observed quantities. Let us therefore find the vector $\underline{r}$ that minimizes the error function.

$$\sum_i e_i^2(t)$$

and take that value as our estimate of the rocket position at the time t. The problem to find the estimate is a non-linear one so we will linearize it and use the method of steepest descent in an iteration process. Starting from an approximation $\underline{r}^{(j)}$ expand the errors in a truncated Taylor series

$$e_i(\underline{r}^{(j+1)}) = e_i(\underline{r}^{(j)} + \underline{dr}^{(j)}) \approx e_i(\underline{r}^{(j)}) + (\text{grad } e_i(\underline{r}^{(j)}))^T \cdot \underline{dr}^{(j)}$$

$j = 0, 1 \ldots$

To find the vector $\underline{dr}^{(j)}$ that minimizes the simplified expression is a standard linear problem.

Introduce the matrices

$$\underline{e}^{(j)} = \begin{bmatrix} e_1(\underline{r}^{(j)}) \\ \vdots \\ e_n(\underline{r}^{(j)}) \end{bmatrix} \quad \text{and } G^{(j)} = \begin{bmatrix} \text{grad } e_1(r^{(j)})^T \\ \vdots \\ \text{grad } e_n(r^{(j)})^T \end{bmatrix}$$

Our problem is then to minimize the magnitude of the vector

$$\underline{e}^{(j)} + G^{(j)} \cdot \underline{dr}^{(j)}$$

The solution is well-known:

$$\underline{dr}^{(j)} = -(G^TG)^{-1}G^T\underline{e}^{(j)}$$

Thus we get the next approximation of the rocket position

$$\underline{r}^{(j+1)} = \underline{r}^{(j)} + \underline{dr}^{(j)}$$

and repeat the process until the magnitude of the improvement is small enough

$\underline{dr}^{(j)} < \delta$

where δ is a suitably small number.

In the calculations we need

$$\text{grad } e_i(\underline{r}) = - \text{grad } |\underline{r}| - \text{grad } |\underline{r} - \underline{R}_i| = \frac{\underline{r}}{|\underline{r}|} - \frac{\underline{r} - \underline{R}_i}{|\underline{r} - \underline{R}_i|}$$

We can thus obtain an estimate of the rocket position for any desired time. Usually an evaluation is done for every second through the significant part of the trajectory. The calculations are sped up by a good choice of starting point $\underline{r}^{(o)}$. Most easily this is obtained by extrapolation of the movement during the previous second.

5. ERRORS

In the iterative process described in the preceeding paragraph it is usually not possible to reduce the errors e_i to zero. The magnitude of these residual errors give a good measure of the accuracy of the obtained rocket coordinates. This paragraph will be devoted to a discussion of the nature and origin of these errors.

The errors have two sources: imperfections in the mathematical model and imperfections in the recorded doppler data. This paragraph will be mainly devoted to a discussion of the first source while a detailed discussion of the other one is planned in a following report about the doppler equipment.

The refractive index on the ground can be determined with an accuracy of about 10^{-5}. This corresponds to about 0.5 meters on the range sums computed at the starting point for the distant receivers. The average of the refractive index for the upper reaches of the trajectory can be determined with an accuracy of about 10^{-6}. Thus the total error in the range sums due to refractive index can be estimated to be less than 0.6 meters corresponding to an altitude error of 0.3 meters. As an error can be expected to have an equal effect on all range sums the horizontal error is negligible.

Position coordinates for transmitter and receivers can generally be determined with an accuracy of better than 0.1 meter in areas where a good geodetic survey exists so that errors of this origin can be neglected.

The most serious source of error in the mathematical model is the starting point determination. It is very probable that the rocket does deviate from the initial line of firing, especially in the lowest part of the atmosphere where wind influence is at its greatest. The starting point is obtained from the intersection between the ellipsoid defined by the measured range sum at the receiver close to the launcher and the initial line of firing. If the transmitter and the receiver are both located in the vicinity of the launcher the ellipsoid surface is nearly horizontal over the launcher. Thus the altitude of the starting point is determined with an accuracy of the order of the resolution of the system, in this case one meter. The horizontal position of the starting point has an error equal to the deviation of the flight path from the firing direction. This deviation can normally be expected to be within ± 20 meters for starting points at about 300 meters altitude. The horizontal error directly affects the initial range sums calculated for the outer stations and will depend on the acutal system geometry. For the system used in Sweden the errors at about 100 km altitude are within ± 20 meters vertically and ± 100 m horizontally. It should be pointed out that these errors are the systematic total errors and that differential erros between points in the same altitude regime are approximately one order of magnitude smaller.

In section 6 below will be described a method for obtaining a better starting point through an optimization process.

Finally a brief discussion of the errors in the doppler data will be given. For the case that the recording and playback process is perfect there is only a round-off error due to the fact that the counter in the playback unit records only integer cycles. These errors are essentially of random nature and are distributed at random between 0 and 2 meters (measured on the range sums). They will result in a minor dispersion of points around the trajectory. It is possible to reduce them to a completely unsignificant level by introducing additional phase resolution in the counter.

A more serious problem is the fact that there are recording difficulties especially in the beginning and the end of flight. In the first part the distant receivers almost look into the nulls of the antenna pattern of the rocket when the latter rotates. Thus the received signal will have superimposed on it large amplitude and phase fluctuations resulting in rapid phase variations of the doppler signal. Under such circumstances it is possible to loose or gain a few cycles during playback.

In the experiments mentioned errors of this kind were insignificant for rockets with low roll rate (1-2 rps) where the residual range sum errors were about 10 meters or less during flight. Rockets with high roll rate ($\sim$10 rps) had range sum errors of the order of 20 meters. This increase was probably due to playback difficulties.

6. OPTIMIZATION OF THE STARTING POINT

For every time

$$t = t_k \qquad k = 1, 2 \ldots, m$$

an estimate of the position $\underline{r}(t_k)$ has been obtained by minimizing the squaresum of the errors

$$e_i(t_k) \qquad i = 1, \ldots, n$$

The errors are also functions of the initial position $\underline{r}(t_o)$ which is determined with the approximate method described above. A better value of $\underline{r}(t_o)$ can be found by finding the value that minimizes the squaresum of the average of all errors in the trajectory.

To simplify calculations let us study the slightly modified errors e'_i: (the average refractive index $\bar{n}$ is very close to 1)

$$e'_i(t_j) = \bar{n}(z(t_j))e_i(r(t_j)) =$$

$$= \bar{n}(z(t_j))\left\{u_i(t_j) - |\underline{r}(t_j)| - |\underline{r}(t_j) - \underline{R}_i|\right\} =$$

$$= \bar{n}(z(t_o)) \left\{ |\underline{r}(t_o)| + |\underline{r}(t_o) - \underline{R}_i| \right\} + D_i(t_o, t_j) \frac{c_o}{2f_t} -$$

$$- \bar{n}(z(t_j)) \left\{ |\underline{r}(t_j)| + |\underline{r}(t_j) - \underline{R}_i| \right\}$$

The gradient with respect to $\underline{r}(t_o)$ is

$$\text{grad } e'_i(t_j) = \bar{n}(z(t_o)) \left\{ \frac{\underline{r}(t_o) - \underline{R}_i}{|\underline{r}(t_o) - \underline{R}_i|} \right\}$$

which is independent of t_j, a fact which is the reason for introducing these modified errors.

Thus it is simple to minimize the squaresum of the mean of the modified errors. Using the modified errors gives a slightly higher weight to the lower points but as $\bar{n}$ is very close to 1 this is insignificant.

Introduce the matrices

$$\underline{\bar{e}}' = \begin{bmatrix} \frac{1}{m} \sum_{j=1}^{m} e'_1(t_j) \\ \vdots \\ \frac{1}{m} \sum_{j=1}^{m} e'_n(t_j) \end{bmatrix} = \begin{bmatrix} \bar{e}'_1 \\ \vdots \\ \bar{e}'_n \end{bmatrix} \quad \text{and}$$

$$G' = \begin{bmatrix} \text{grad } \bar{e}'_1{}^T \\ \vdots \\ \text{grad } \bar{e}'_n{}^T \end{bmatrix}$$

We want to minimize the magnitude of the vector

$$\underline{\bar{e}}' + G' \underline{dr}(t_o)$$

As before the solution is

$$dr(t_o) = - (G'^T G')^{-1} G'^T \underline{\bar{e}}'$$

A better approximation of the starting point is now

$\underline{r}(t_o) + \underline{dr}(t_o)$

From this point the whole tracking process is repeated. A still better starting point is then obtained and so on until no significant improvement can be made.

This method has been successfully tested on the Swedish 1964 data. When the input data are of good quality the systematic errors can be reduced to 1 meter or less. Convergence is slow, however, so computer running time can be rather considerable.

The greatest advantage with the method is that it is possible to obtain accurate tracking data for the significant part of the trajectory where the doppler data quality is normally very good even if the quality on the first part is poor due to influences from rocket burning and rapid movements.

7. COMPUTER PROGRAMS

A Fortran IV program called TRACK utilizing the described tracking method has been written and successfully used on an IBM 7090 computer to reduce data collected during the Kronogård experiments in 1964.

The program consists of three subroutines. The first one performs the starting point computation. From this point the second subroutine performs the main tracking calculations and produces rocket coordinates both in printed form and on punched cards for further data processing. The third subroutine produces a magnetic tape for an off-line digital plotter that gives an accurate plot of the coordinates versus time.

Execution time for the object program is modest. The reduction of 200 seconds of flight data takes 1.6 minutes when coordinates are computed for every second.

The optimization of the starting point has been implemented in a program called OPTRAC. As this program repeats the essential computations of TRACK a large number of times the running time can be rather long. Furthermore, in some cases the program has been found to converge towards a solution below ground level when the data quality was poor. These facts limit the applicability of those cases where good data are available for the essential part of the trajectory.

8. SUMMARY

Multistation doppler tracking is capable of giving trajectory determination for sounding rockets to within a few meters.

It has been shown that it is possible to reduce the tracking data by computer with a fairly fast and simple numerical method which does also give an estimate of the accuracy of the obtained results.

Starting from roll-corrected playback the program compensates for tropospheric effects and obtains a starting point from which the tracking starts.

The program is written in Fortran IV and has successfully processed data from the NASA-Swedish Space Research Committee experiments in northern Sweden during the summer of 1964.

9. ACKNOWLEDGEMENTS

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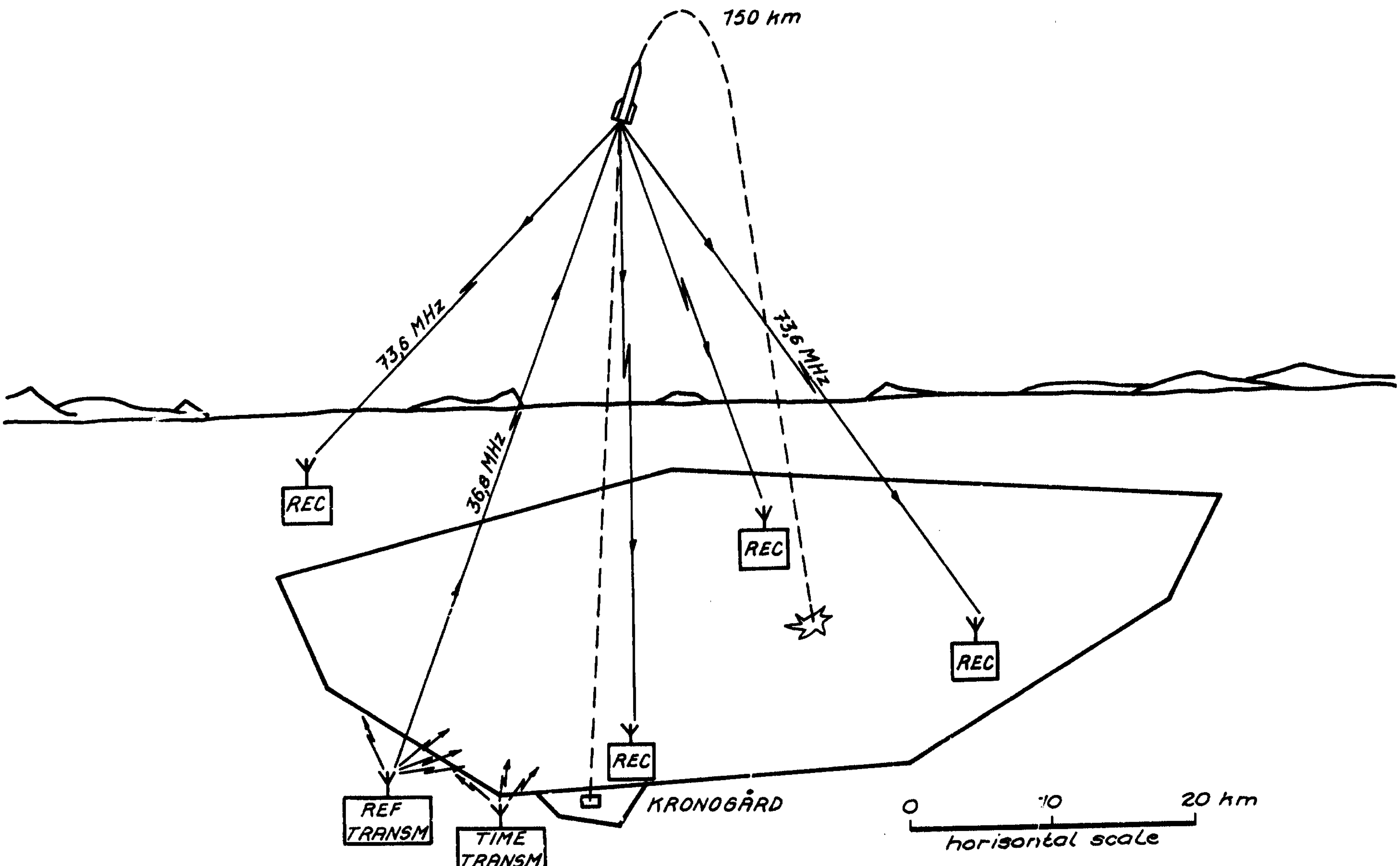


FIG. 1 K-64 4-STATION DOPPLER TRACKING SYSTEM

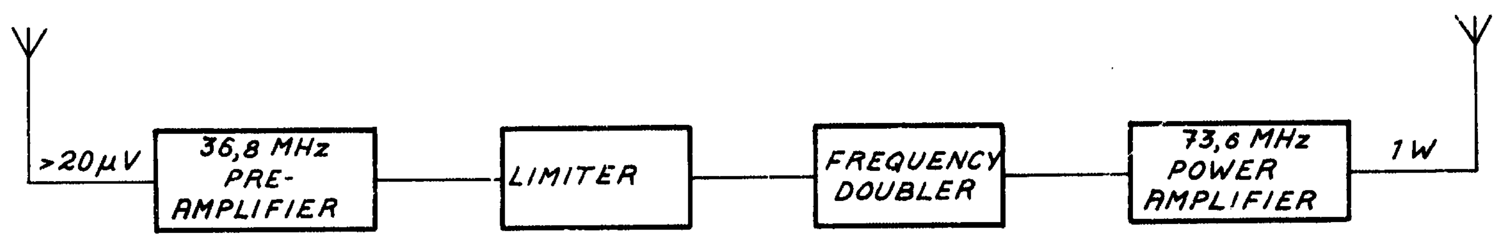
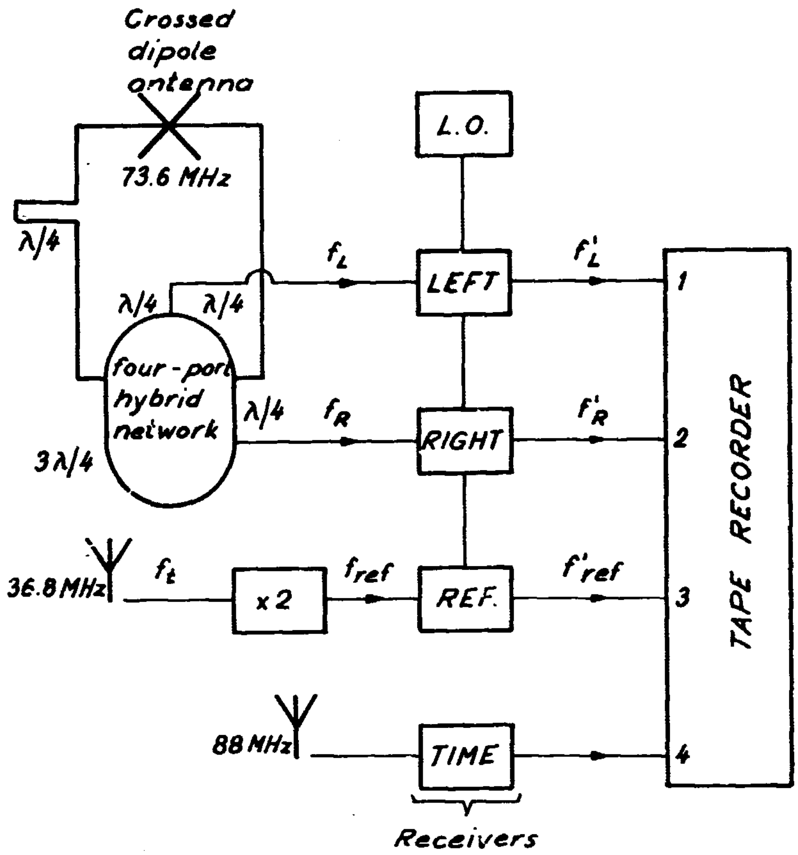
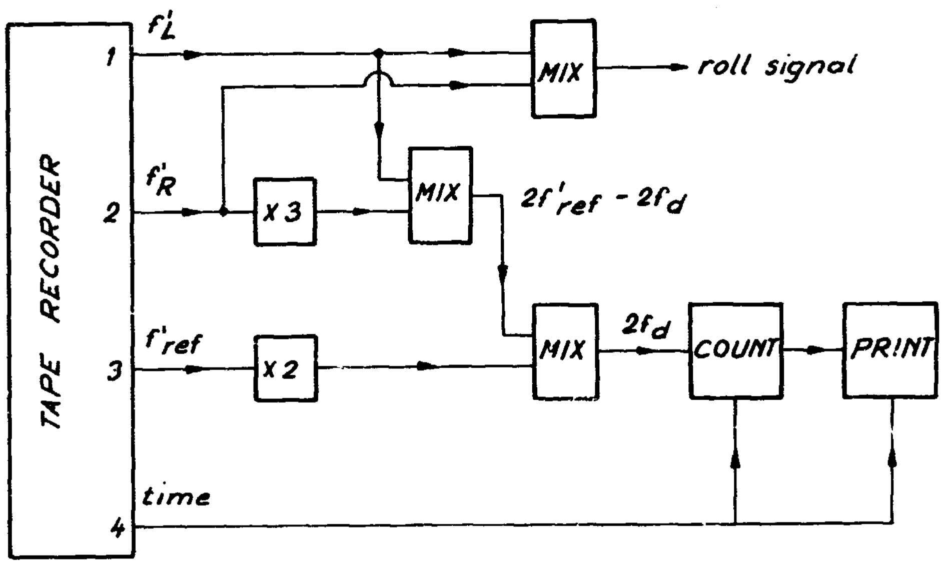


FIG. 2 TRANSPONDER BLOCK DIAGRAM



a. Doppler signal recording



b. Doppler signal playback

Fig. 3 Simplified block diagrams for doppler equipment