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THE HELICAL INSTABILITY OF THE POSITIVE COLUMN IN CROSSED FIELDS AND IN ANNULAR PLASMA CONFIGURATIONS

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SUMMARY

The helical instability typical of collision dominated plasmas has been investigated in the presence of transverse magnetic fields. In all cases the critical value of the magnetic field is very sensitive to an applied transverse component. A rectilinear transverse component destabilizes the discharge whilst a minimum B configuration has a strong stabilizing effect. A magnetic field with shear will destabilize the plasma if the shear is in the same sense as the incipient helical plasma perturbation but a shear of the opposite sense stabilizes the plasma.

A coaxial plasma has been investigated in regard to its stability in a solenoidal field. It is found that improved agreement between theory and experiment for the prediction of the critical field value can be obtained if the radial form of the plasma density is taken into account in evaluating the stability criterion. When the coaxial plasma is immersed in a magnetic field with shear it is found that the consequent change in the radial form of the plasma density accounts for the major part of the change in the value of the critical magnetic field.

INTRODUCTION

The well known helical instability of the positive column occurs when the plasma is immersed in a magnetic field which is oriented parallel to

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the electric field which maintains the discharge. However many plasmas including plasma accelerators are subjected to crossed fields where the magnetic field is orthogonal to the electric field. We have studied the effect of superimposing several forms of orthogonal or transverse magnetic fields upon a plasma subjected simultaneously to a strong longitudinal magnetic field, with particular reference to the onset of the helical instability.

A further characteristic of some crossed field plasma devices is that the plasma has an annular or coaxial form. Our work has included the study of a hollow positive column and we have extended the work of Gierke and Wohler (ref. 1) on the formation of the helical instability in such a plasma configuration.

We have studied three forms of orthogonal magnetic field. The first was produced very simply by inclining the discharge tube axis at a small angle to the axis of an applied solenoidal magnetic field. The resultant magnetic field on the positive column included a small rectilinear component which was orthogonal to the electric field driving the column. The second was produced by adding a set of four Ioffe bars running parallel to the axis of the plasma immersed in a solenoidal magnetic field. In this case the magnetic field has a minimum B configuration with the field strength increasing with the radial distance from the axis of the discharge. The third was produced by passing a current carrying conductor through the center of a hollow plasma immersed in a solenoidal magnetic field. In this way an azimuthal component of magnetic field was added and the behavior of the helical instability could be studied in a magnetic field with shear.

THE ONSET OF INSTABILITY IN A TRANSVERSE MAGNETIC FIELD

The sensitivity of the critical value of the longitudinal magnetic field, which marks the onset of the helical instability, to each of these transverse magnetic field configurations is shown in Fig. 1. In all cases the application of a transverse magnetic field of a given amplitude causes a considerably larger change in the critical value of the longitudinal magnetic field. This effect is seen to be greatest for the minimum B transverse field where the given change in the transverse field causes an order of magnitude greater change in the critical value of the longitudinal magnetic field. The shallow maxima at about 3 degrees in Fig. 1(a) have been previously identified with a switch from the single helix mode to the double helix mode of the instability (ref. 2).

THE STABILITY CRITERION FOR A COAXIAL PLASMA

The use of an annular or hollow discharge in the case of the azimuthal transverse magnetic field raises the question of the relationship between the behavior of such a discharge and that of the normal positive column. Belousova (ref. 3) has made a theoretical study of the helical instability in the hollow discharge. In order to derive the stability criterion he assumed the forms for the zero order density distribution (n_o) and the perturbed (helical) form of the density distribution (n_1) which are shown in Fig. 2(b). However, reasoning that the density perturbation would grow preferentially in the region of the discharge where the gradient of the zero order density distribution is directed towards the outer wall we have adopted the mathematical forms for the density distribution shown in Fig. 2(d).

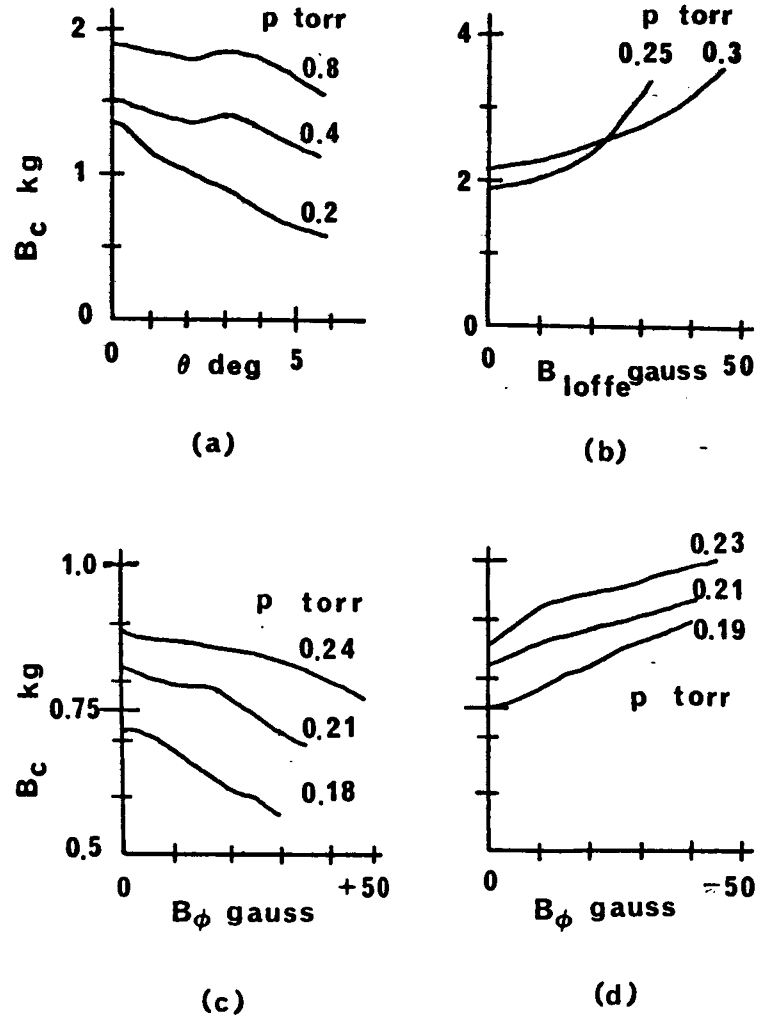

Figure 1. Critical longitudinal magnetic field, B_c, for onset of the helical instability in helium discharges plotted as a function of the transverse magnetic field component for three configurations.

(a) Rectilinear transverse field generated by inclining discharge cell with respect to solenoid axis. $B_T = B_L \tan \Theta$. Discharge diameter 3.75 cm.

(b) Cusped transverse field generated by current carrying Ioffe bars. Discharge diameter 3.6 cm.

(c) and (d) Azimuthal transverse field generated by a current (I_ϕ) flowing through a conductor located on the discharge tube axis. In (c) I_ϕ is parallel to the discharge current I_D. In (d) I_ϕ is antiparallel to I_D. Annular discharge: inner diameter 1.9 cm., outer diameter 11 cm.

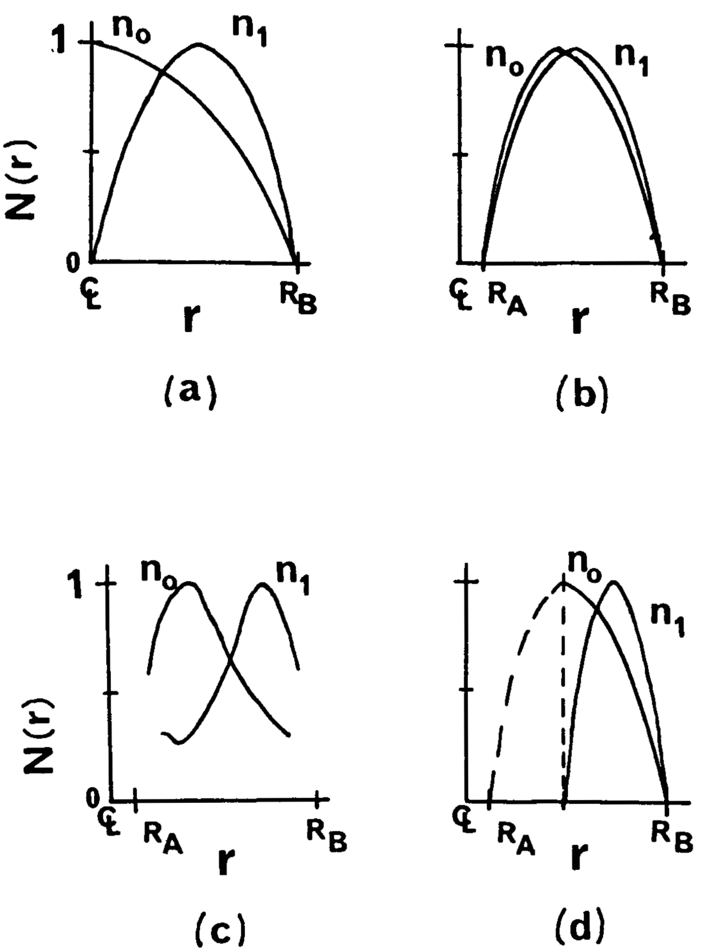

Figure 2. Radial form of zero-order density (n_o) and perturbed density (n_1). Theoretical models use Bessel functions as indicated below.

(a) Forms used by Kadomtsev for the regular positive column of radius R_B $n_o \propto J_o$, $n_1 \propto J_1$.

(b) Forms used by Belousova for hollow column of inner radius R_A and outer radius R_B. $n_o \propto J_o + A\,Y_o$, $n_1 \propto J_1 + B\,Y_1$.

(c) Experimentally measured forms, for the hollow column described in Fig. 1.

(d) Forms used by present authors for hollow column (solid lines) $n_o \propto J_o$ $n_1 \propto J_1$.

These can be seen to have the closer resemblance to the density distribution used by Kadomtsev and Nedospasov (ref. 4) for the ordinary positive column shown in Fig. 2(a). Their validity is further born out by the measured radial density distributions which we have taken using a movable probe and which are shown in Fig. 2(c).

The theoretically predicted curves for the onset of the instability are shown in Fig. 3. together with experimental data. The break in the slope of the experimental curve marks the onset of the instability. The agreement of experiment with our theory in preference to that of Belousova is evident.

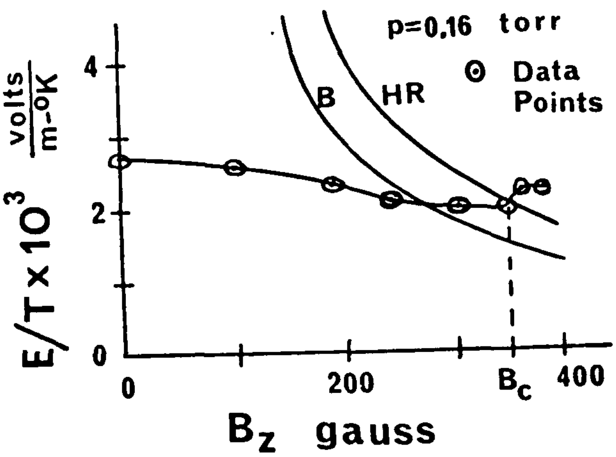

Figure 3. Comparison of theoretical curves for the hollow column of authors (HR) and Belousova (B) plotted in the E/T vs. B plane. (Plasma is stable below the curve, unstable above it.) Experimental results show the onset of instability for the hollow column ($B_c = 350$ g).

THE HELICAL INSTABILITY IN A

MAGNETIC FIELD WITH SHEAR

When a current carrying conductor is inserted down the hollow column the data of Fig. 1(c) and (d) show that the discharge is destabilized when the conductor current is parallel to the discharge current and stabilized when the conductor current is anti-parallel to the discharge current. This phenomenon may be qualitatively understood by considering the change in the zero order density distribution caused by the imposition of the azimuthal

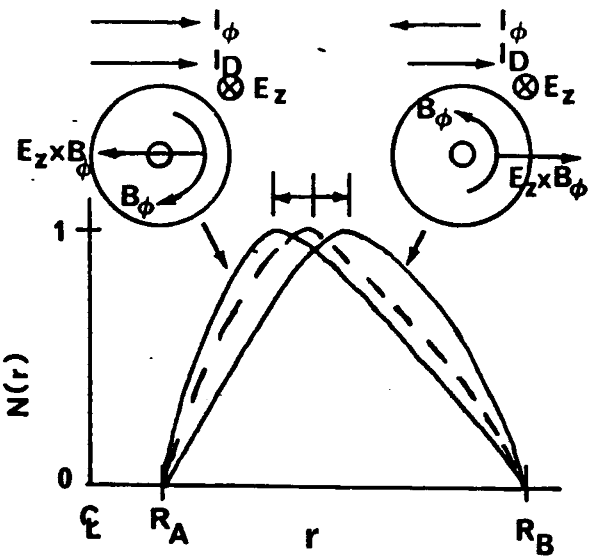

Figure 4. Effect of B_ϕ on zero-order radial density distribution of the hollow column. Direction of E_Z x B_ϕ drift, B_ϕ and I_ϕ are shown for both cases. Dotted curve represents B_ϕ = 0 condition.

magnetic field component. Figure 4 shows how the $E_Z = B_\phi$ plasma drift changes the density gradient towards the outer wall. Now this density gradient determines the ambipolar diffusion of the plasma particles to the wall and hence the radial electric field (E_r) due to this diffusion. The E_r x B_Z azimuthal plasma drift acts to inhibit the natural rotation of the helical perturbation and is therefore a stabilizing effect. The destabilizing effect of the parallel current case is associated with the weakening of the plasma density gradient and the radial electric field.

When the theory is modified to include the azimuthal component of the magnetic field the discharge is predicted to stabilize or destabilize in agreement with experiment. Figure 5 shows this effect for a particular value of the azimuthal magnetic field. The relative importance of the shift in the zero order density due to the azimuthal magnetic field is shown by plotting theoretical curves for which all other effects of the azimuthal magnetic field were ignored. It is clear that the form of the zero order density distribution plays a dominant role in determining the onset of the instability.

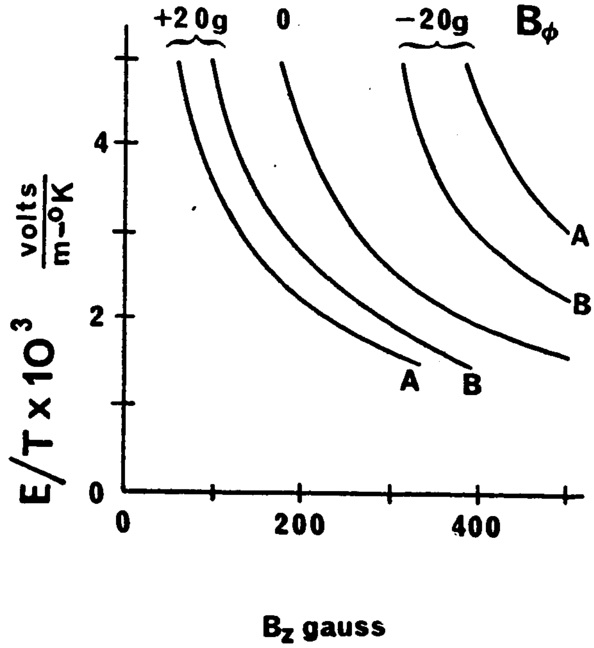

Figure 5. Effect of B_ϕ on stability criterion of the hollow column. Curves marked A represent the full stability criterion. Curves marked B indicate the change in the criterion due to an applied B_ϕ when only the effect of the zero order density distribution is taken into account.

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ACKNOWLEDGEMENTS

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