RESONANCE RECTIFICATION EFFECTS IN WARM MAGNETOPLASMAS

by

F. W. Crawford, R. S. Harp and T. D. Mantei

NASA Research Grant NGR 05-020-077

SU-IPR-Report No. 177

July 1967

Institute for Plasma Research
Stanford University
Stanford, California

RESONANCE RECTIFICATION EFFECTS IN WARM MAGNETOPLASMAS*

by

F. W. Crawford, R. S. Harp and T. D. Mantei

Institute for Plasma Research
Stanford University
Stanford, California

ABSTRACT

This paper extends the study of resonance rectification phenomena in plasmas to the case where a static magnetic field is present. Parallel wire probe geometry has been chosen so that the rf electric field is primarily perpendicular to the magnetic field. Under these conditions, experiments show that there are two distinct series of resonance peaks, one occurring between successive cyclotron harmonics, the other precisely at the cyclotron harmonics. Cold plasma theory is inadequate to account for the existence of these resonances. An explanation is proposed, based on the form of the warm plasma permittivity component perpendicular to the magnetic field, which explains satisfactorily all of the experimental observations.

*This work was supported by the NASA.

CONTENTS

LIST OF FIGURES

I. INTRODUCTION

Resonance rectification effects have been much studied in the last few years, principally because of their potential diagnostic applications to electron density measurement in laboratory and space plasmas. The basic phenomenon, which was first reported by Takayama et al,[1] occurs when a dc potential and a superimposed rf signal, of frequency ω, are applied to a metallic probe immersed in a plasma. There is a rectification effect due to the nonlinear response of the system, in particular the plasma sheath, which causes the dc component of the probe current to increase above its value in the absence of an rf signal. As the frequency of the modulating signal is increased from zero, the incremental dc component increases to a maximum which occurs somewhat below the electron plasma frequency, ω_p. For higher frequencies, it decreases towards zero.

A simplified explanation of the effect can be based on considerations of the cold plasma permittivity. If it is assumed that the sheath region close to the probe is free from electrons, it may be considered at all frequencies as a capacitive reactance. For $\omega < \omega_p$, the equivalent plasma permittivity $[= 1 - (\omega_p^2/\omega^2)]$ is negative, so that the plasma reactance is effectively inductive. At some frequency, there is consequently a series resonance at which the rf electric field across the sheath rises to a high value, and causes a resonance peak to occur in the rectified current. This theory, proposed originally by Mayer and Harp[2] independently, can be justified on more rigorous grounds from numerical solution of the full plasma/sheath equation.[3] An additional phenomenon introduced by microscopic theory, based on the Vlasov equation, is that of collisionless damping of the resonance. This is predicted to occur strongly due to phase mixing, if the resonance frequency approaches ω_p, and is observed experimentally.

The work referred to so far has all been carried out in the absence of magnetic field. The question now arises of what extensions can be made to situations where a static magnetic field is present. Some experiments were carried out in 1963 by Uramoto et al,[4] using a planar probe parallel to the magnetic field. The results showed rectification peaks at the cyclotron frequency and its second harmonic. A resonance close to the fundamental cyclotron frequency is consistent with a simple extension of the theory to take into account the differing plasma permittivity components parallel and perpendicular to the magnetic field. Detailed analytical treatments have been given,[5] but no further experiments seem to have been reported.

The purpose of the work to be described in this paper has been to study resonance rectification in a plasma immersed in a magnetic field, using probe geometry such that the perpendicular component of the plasma permittivity dominates the rectification characteristics. The experimental results to be presented in Section III exhibit two distinct series of resonances, which cannot be explained from cold plasma permittivity considerations. They can, however, be elucidated by reference to warm plasma theory, i.e. by taking into account nonzero electron temperature. This theory is outlined in Section II. The characteristics of the resonances compare well experimentally with predictions based on it.

II. THEORY

It is convenient to consider first the resonance rectification effects that might be expected for a cold magnetoplasma, and then to describe the differences introduced by taking account of nonzero electron temperature. The discussion will be restricted to geometries where the rf electric field is perpendicular to the static magnetic field. These conditions may be approached experimentally in parallel plate, parallel wire, or coaxial cylinder electrode configurations.

(A) Cold Plasma.

The cold plasma permittivity tensor component appropriate to an electric field applied perpendicular to the static magnetic field is[6]

$$\epsilon_{\perp} = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2} , \tag{1}$$

where the electron plasma and cyclotron frequencies, ω_p and ω_c, are given by

$$\omega_p^2 = \frac{\rho e}{\epsilon_0 m} , \qquad \omega_c = \frac{eB}{m} , \tag{2}$$

and where ρ is the electronic charge density; e and m are the electronic charge and mass respectively, and B is the magnetic field strength.

The reactance of an arbitrary volume of plasma normalized to its value with free space as the dielectric is

$$X_p = \frac{1}{\epsilon_{\perp}} . \tag{3}$$

This function is sketched in Fig. 1(a). It will be noted that as ω increases from zero, the reactance decreases from a positive value to zero at the cyclotron frequency. Between this point and the upper hybrid frequency $(\omega^2 = \omega_p^2 + \omega_c^2)$ the reactance is negative, implying

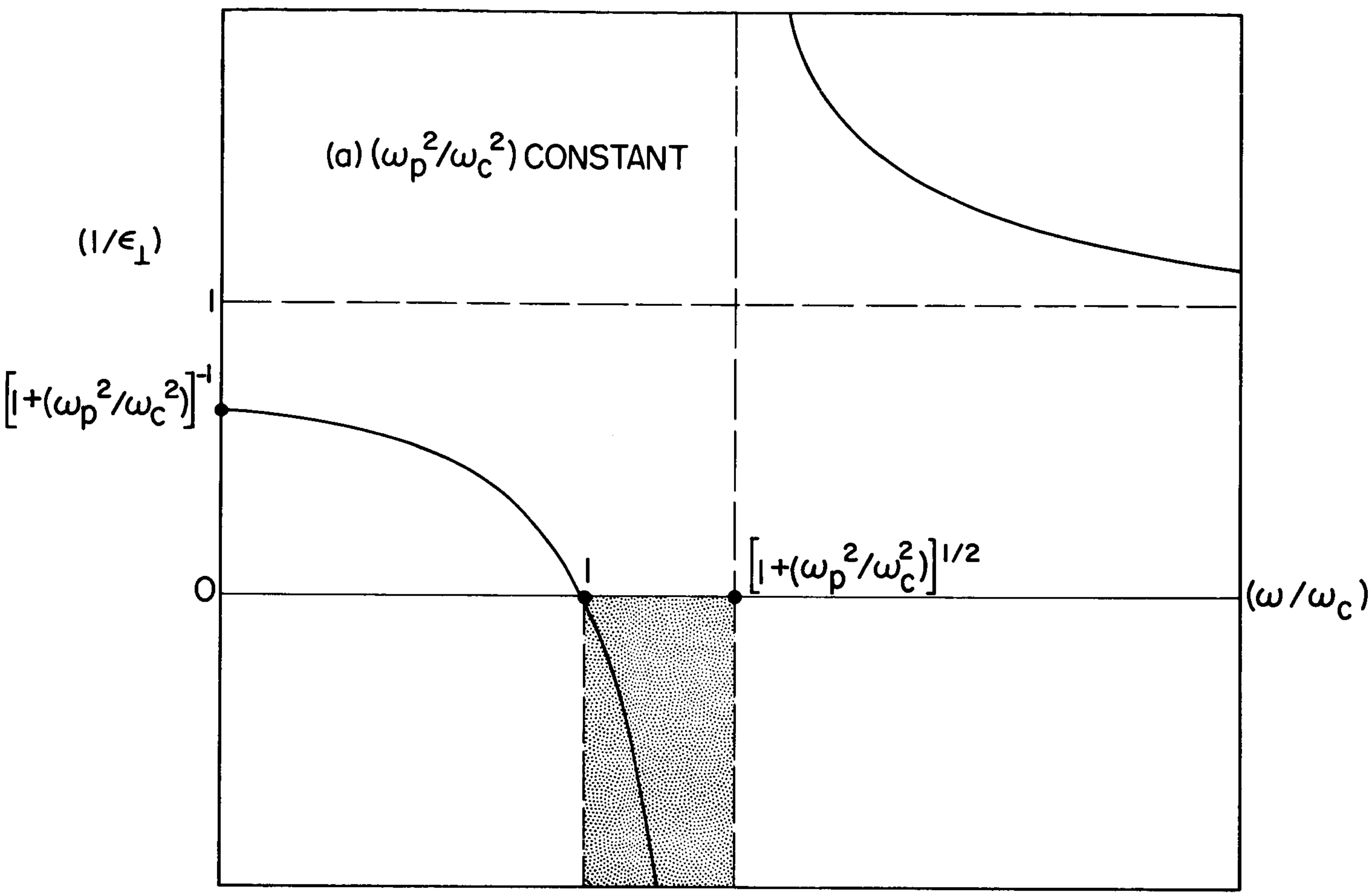

FIG. 1(a). Normalized reactance of cold magnetoplasma (RF electric field perpendicular to static magnetic field).

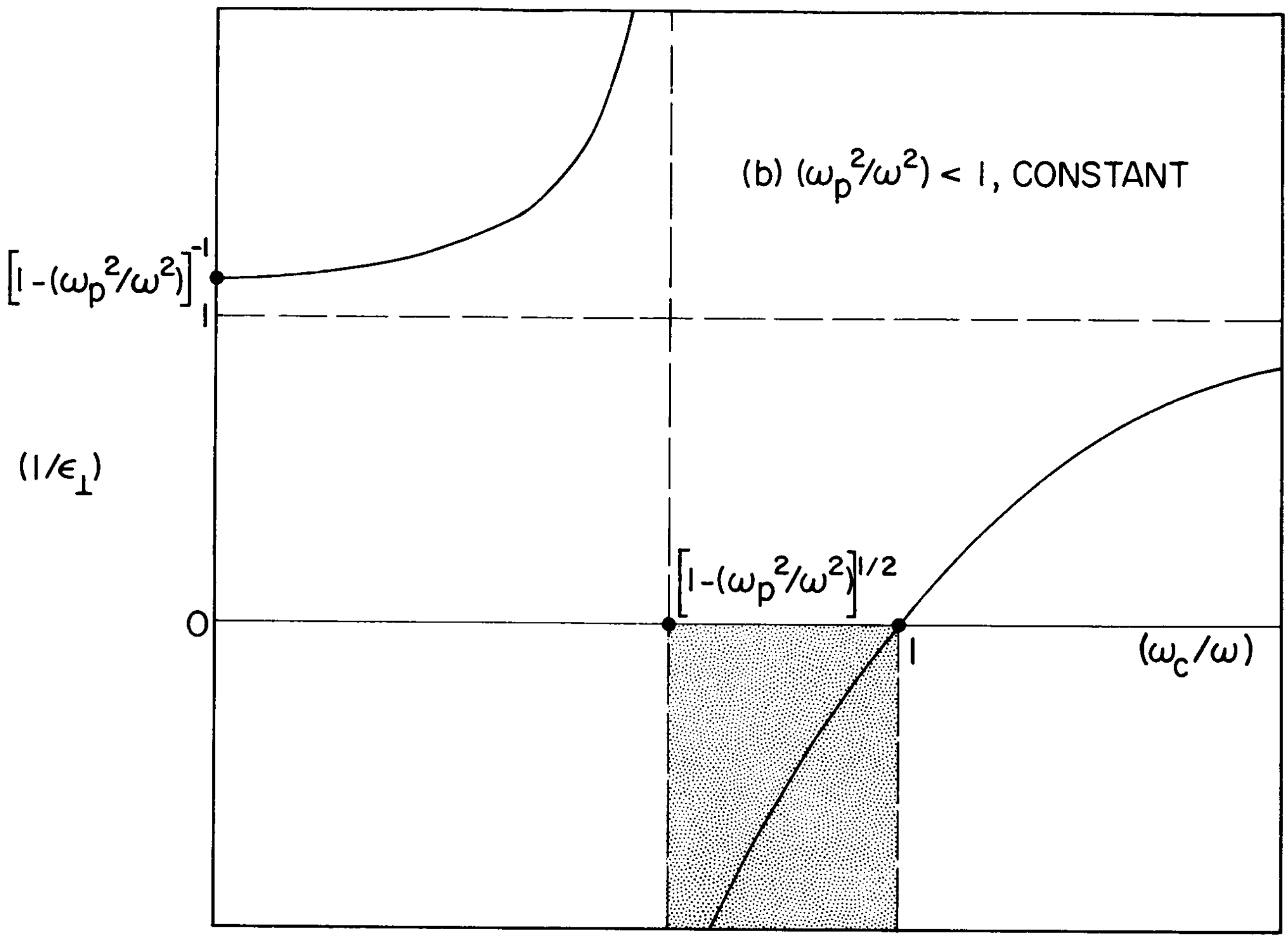

FIG. 1(b). Normalized reactance of cold magnetoplasma (RF electric field perpendicular to static magnetic field).

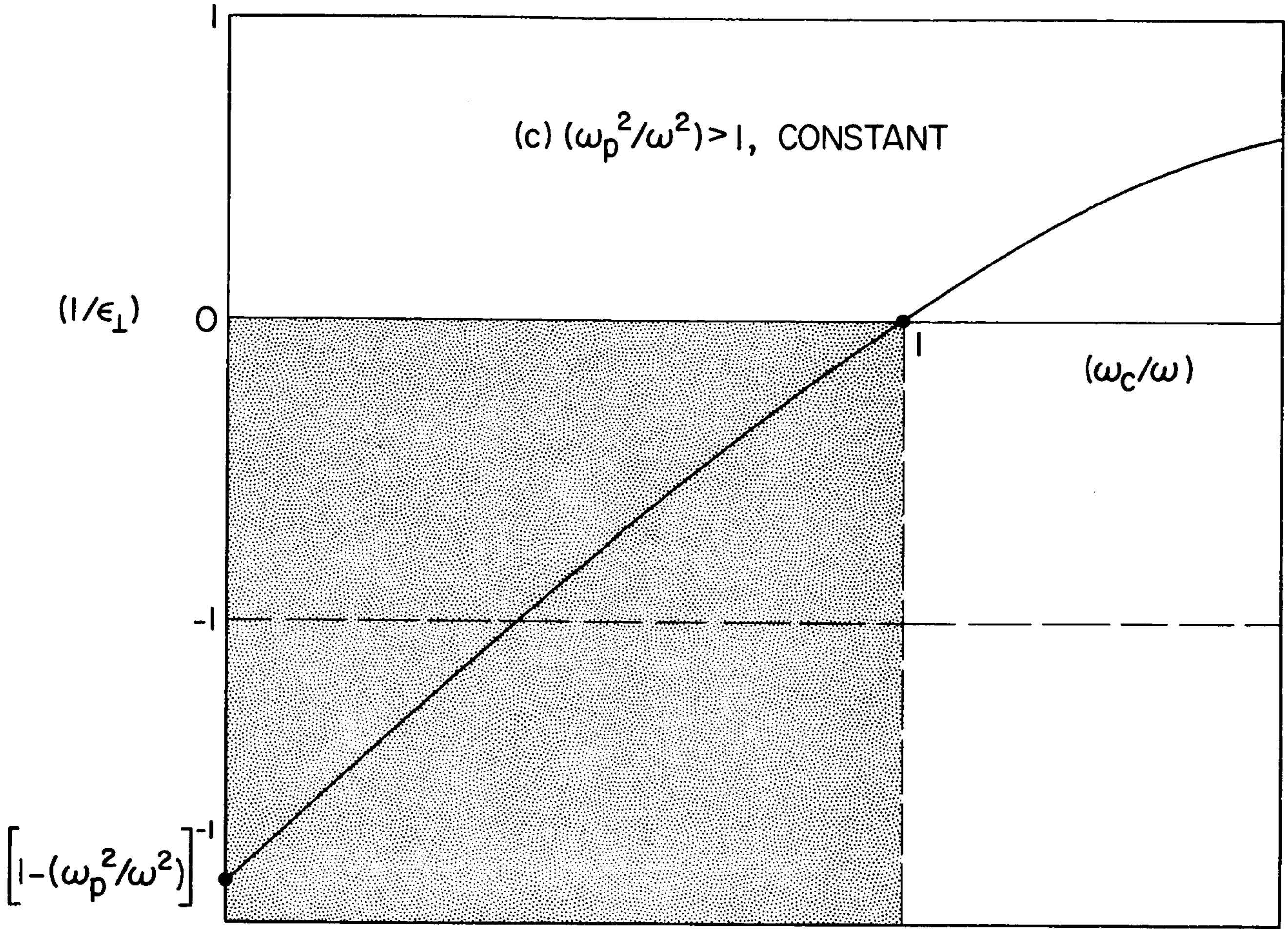

FIG. 1(c). Normalized reactance of cold magnetoplasma (RF electric field perpendicular to static magnetic field).

that the plasma is inductive. For frequencies above the upper hybrid the reactance is again capactive, decreasing with increasing ω towards the free space value. Since the sheath reactance is always capacitive, it follows that there will be a series resonance at some frequency lying between the cyclotron frequency and the upper hybrid. At this point, the electric field across the sheath will rise to a high value causing a resonance in the rectified current component.

In laboratory experiments, it is most convenient to vary ω_c while keeping ω and ω_p constant. Under these conditions, two possibilities can be distinguished for the behavior of the plasma reactance. The first is shown in Fig. 1(b), for $(\omega_p^2/\omega^2) < 1$. The series resonance can always occur, since the reactance can take all values between 0 and $-\infty$. The cyclotron frequency must lie between the upper hybrid frequency and the signal frequency. For $(\omega_p^2/\omega^2) > 1$, as in Fig. 1(c), the maximum negative value of the reactance is finite, , so series resonance will not necessarily occur. If it does, however, the cyclotron frequency can lie anywhere between zero and the signal frequency.

(B) Warm Plasma.

When the electron temperature is nonzero, Eq. (1) becomes[6]

$$\epsilon_\perp = 1 - \frac{\omega_p^2}{\omega_c^2} \sum_{n=1}^{\infty} \frac{\exp(-\lambda)\, I_n(\lambda)}{\frac{\lambda}{2}\left[\left(\frac{\omega}{n\omega_c}\right)^2 - 1\right]}, \tag{4}$$

where λ has been written for $(k_\perp R)^2$; $k_\perp$ is the wave number for propagation perpendicular to the magnetic field; $R[= (\kappa T_e/m)^{1/2}/\omega_c]$ is the gyroradius of a particle with thermal energy; κ is Boltzmann's constant and T_e is the electron temperature. The electron velocity distribution is assumed to be Maxwellian. This is a much more complicated expression than for the cold plasma case since it depends on the wavenumber, i.e. on the spatial Fourier components of the electric field in the vicinity of the probe. Strictly $\epsilon_\perp$ is exact, and $k_\perp$ is arbitrary, only for an infinite plasma, though we shall be discussing bounded systems in what follows.

Figure 2(a) shows the behavior of Eq. (4) for a fixed value of (ω_p^2/ω_c^2). It will be seen that, in addition to the shaded area corresponding to that of Fig. 1(a), there is an infinite series of such regions bounded below by the cyclotron harmonics and above by a set of frequencies dependent on the value of λ chosen. Figures 2(b) and (c) correspond to an experimental situation in which ω_c is varied, with ω and ω_p fixed. In both cases, an infinite set of resonances are predicted having the cyclotron harmonics as their upper bounds. The locations of the lower bounds are those points which satisfy the cyclotron harmonic wave dispersion relation $(\epsilon_\perp = 0)$ for a given λ. These are shown in Fig. 3. This figure also serves to emphasize the important distinction between the ranges $0 < (\omega_p^2/\omega^2) < 1$ and $1 < (\omega_p^2/\omega^2)$. In the first case, there are successive passbands and stopbands as (ω_c/ω) varies. In the latter, there is propagation for all values of (ω_c/ω) less than unity.

A number of predictions concerning resonance rectification effects may now be made. First, we note that for an experimental resonance probe of the type used in our experiments, and shown in Fig. 4(a), the electric field will be described by a spectrum of $k_\perp$ values at any given frequency. Since some of the $k_\perp$ values can be close to resonance, a rectification effect should be expected in each passband. For the case of Fig. 2(b), it would be possible for the peaks to be located anywhere in the regions between successive cyclotron harmonics. For the case of Fig. 2(c), however, the peaks must lie inside the frequency ranges defining the passbands. This series of resonances, due to the spectrum of $k_\perp$ values excited, will be referred to as the 'principal' series.

To determine the exact location of the resonance rectification peaks would be an extremely difficult problem. We can, however, make some further qualitative predictions. In general, the form of the plasma impedance will be,[8]

$$Z_p(\omega) = \int_{-\infty}^{+\infty} \frac{F(k_\perp,\omega)}{\epsilon_\perp(k_\perp,\omega)}\, dk_\perp \, , \tag{5}$$

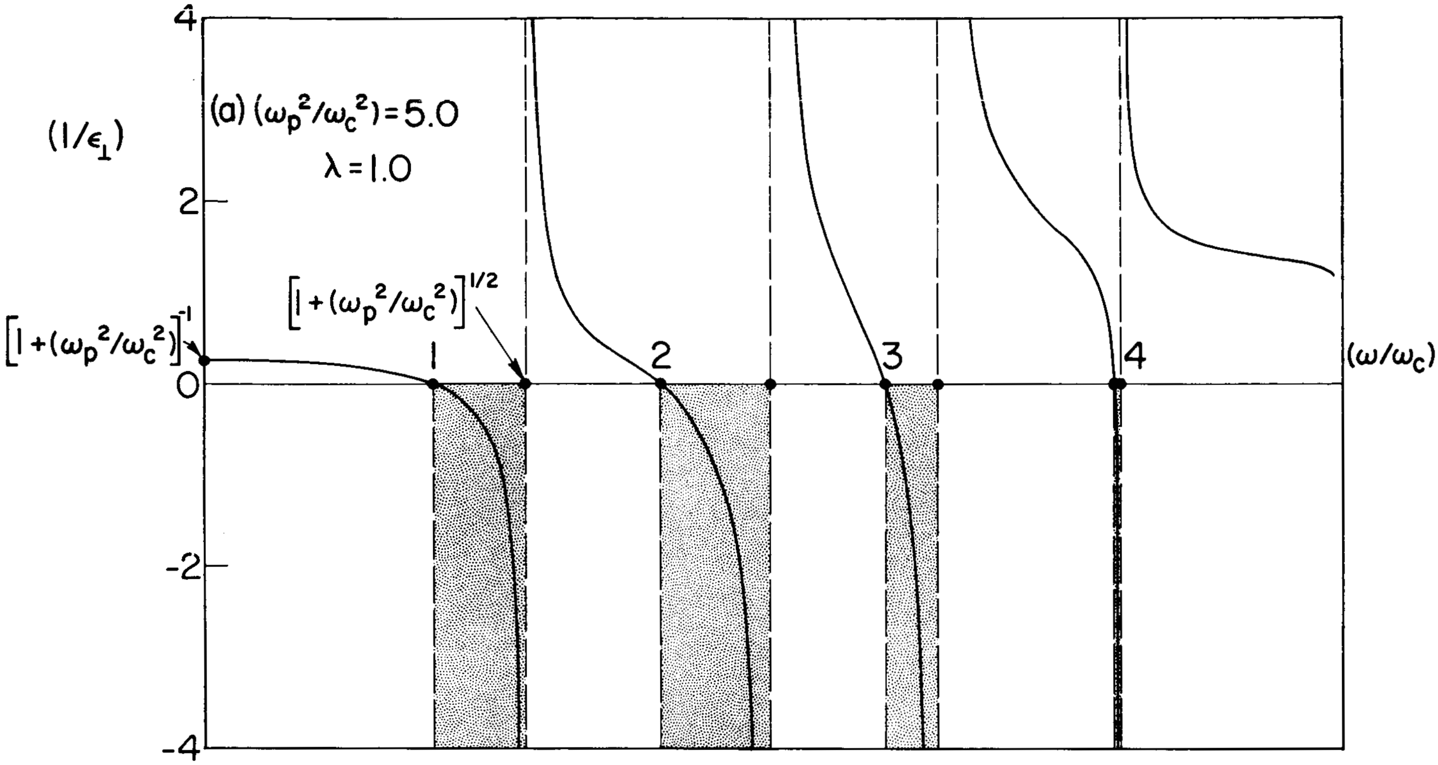

FIG. 2(a). Normalized reactance of warm magnetoplasma (RF electric field perpendicular to static magnetic field).

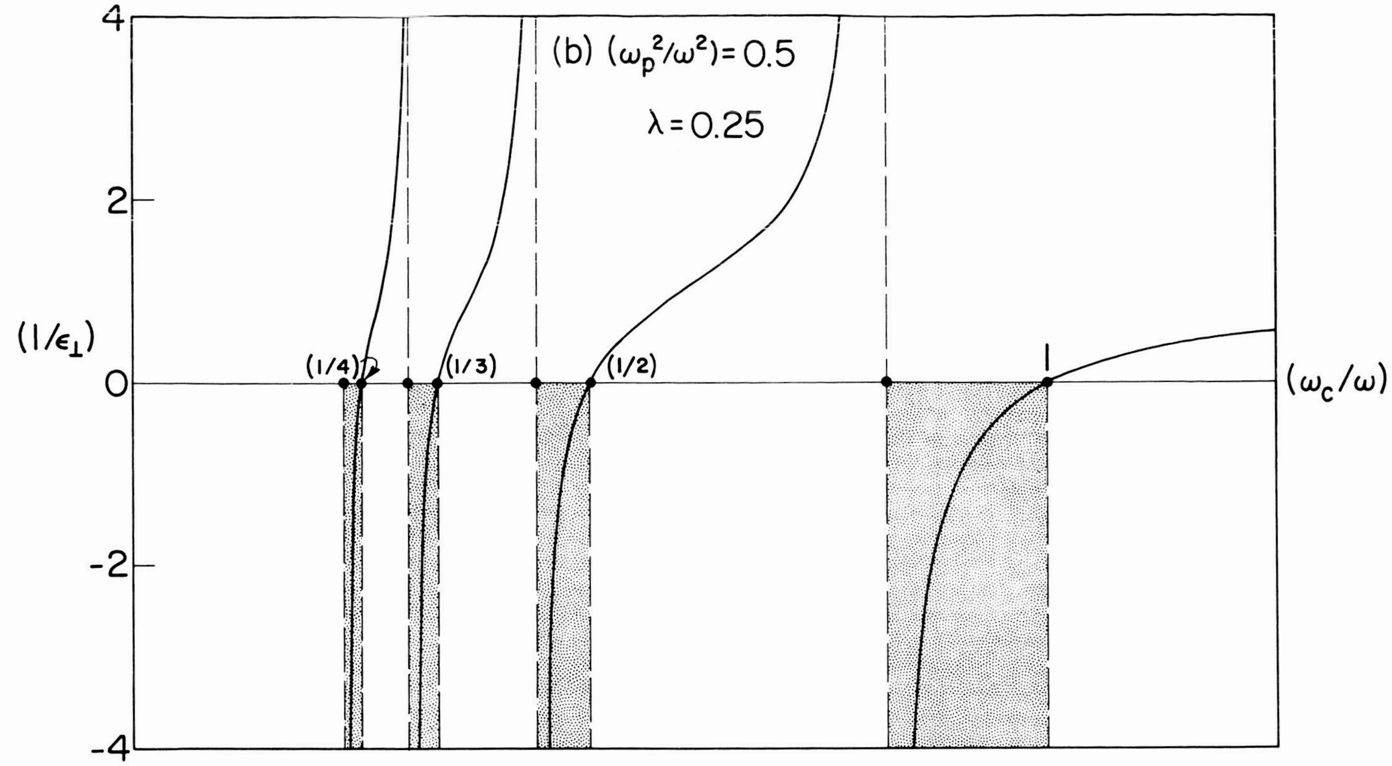

FIG. 2(b). Normalized reactance of warm magnetoplasma (RF electric field perpendicular to static magnetic field).

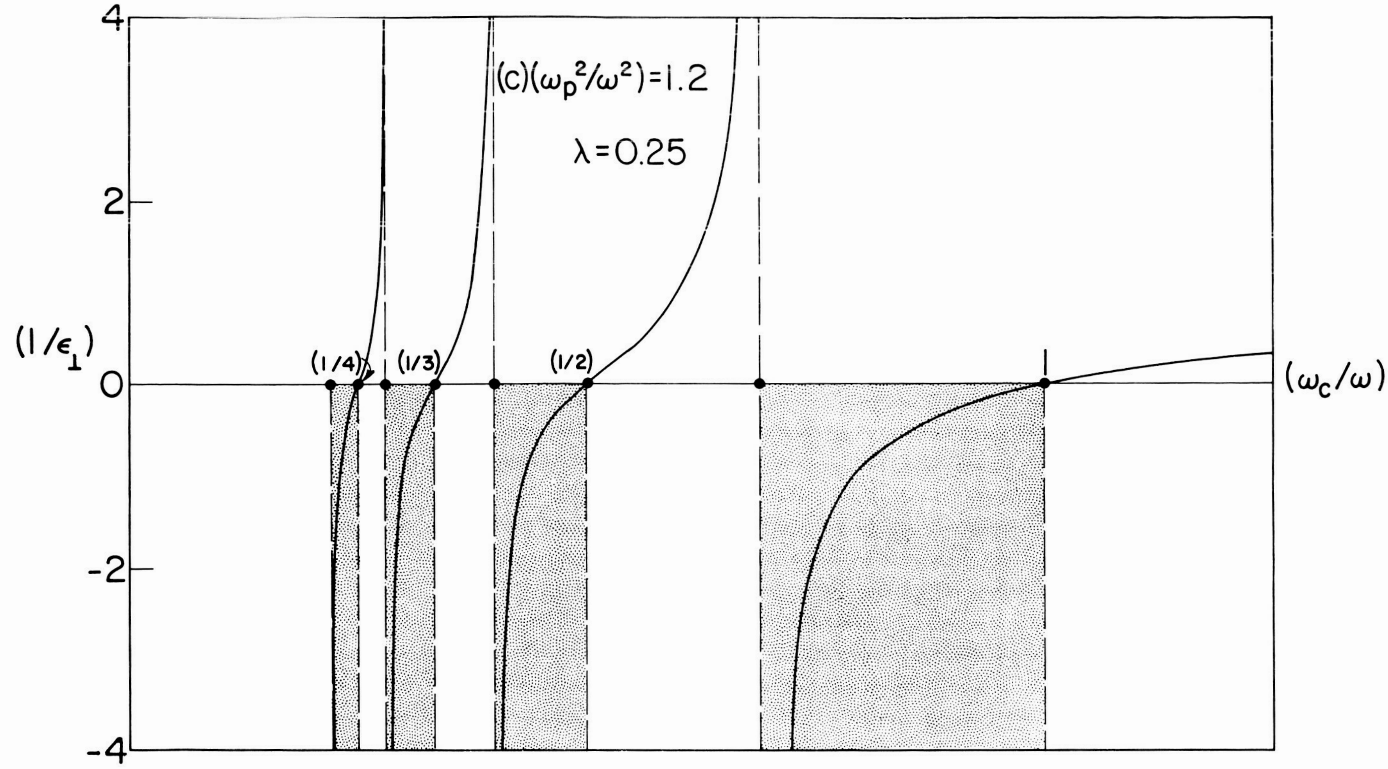

FIG. 2(c). Normalized reactance of warm magnetoplasma (RF electric field perpendicular to static magnetic field)

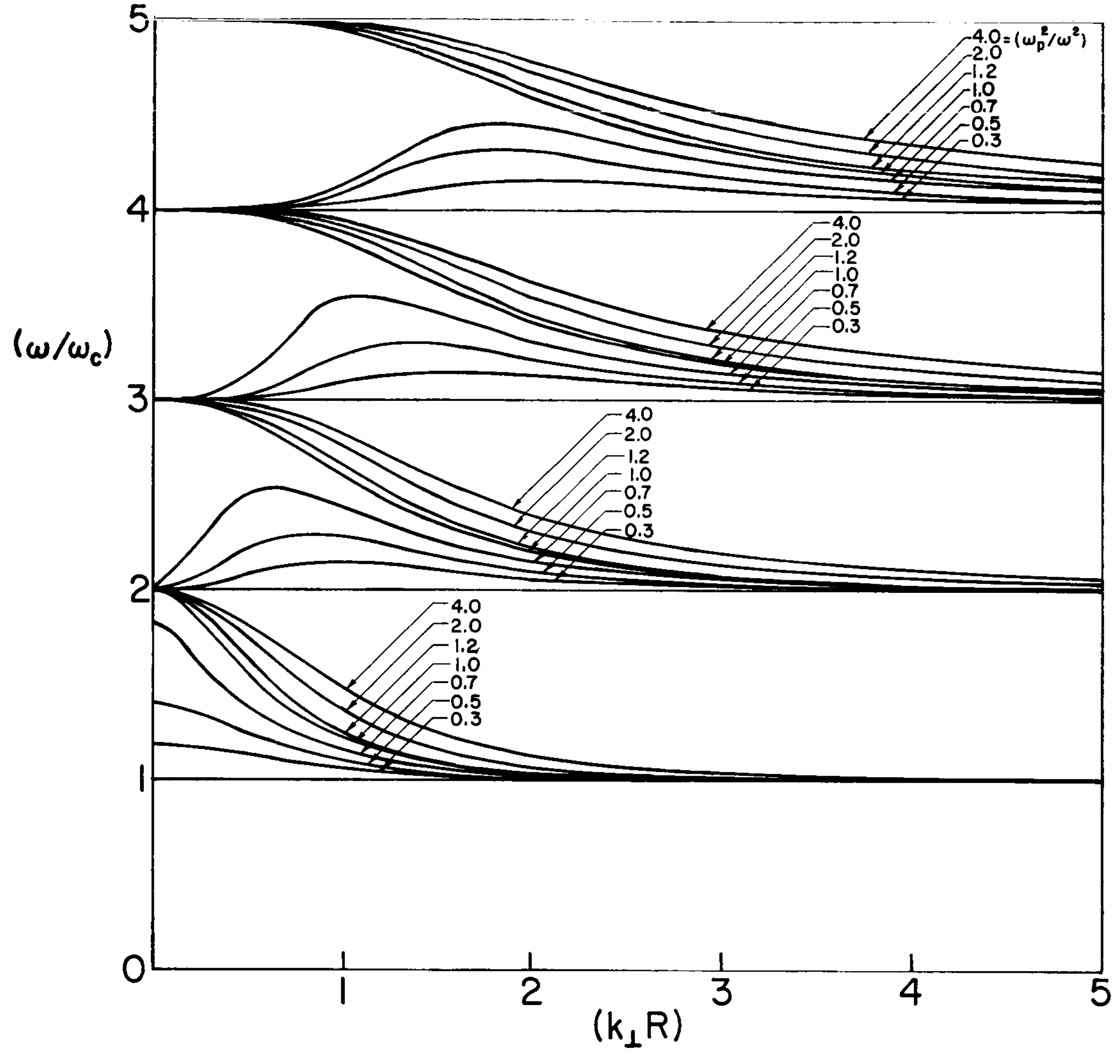

FIG. 3. Cyclotron harmonic wave dispersion characteristics for perpendicular propagation in a warm magnetoplasma: Maxwellian electron velocity distribution.

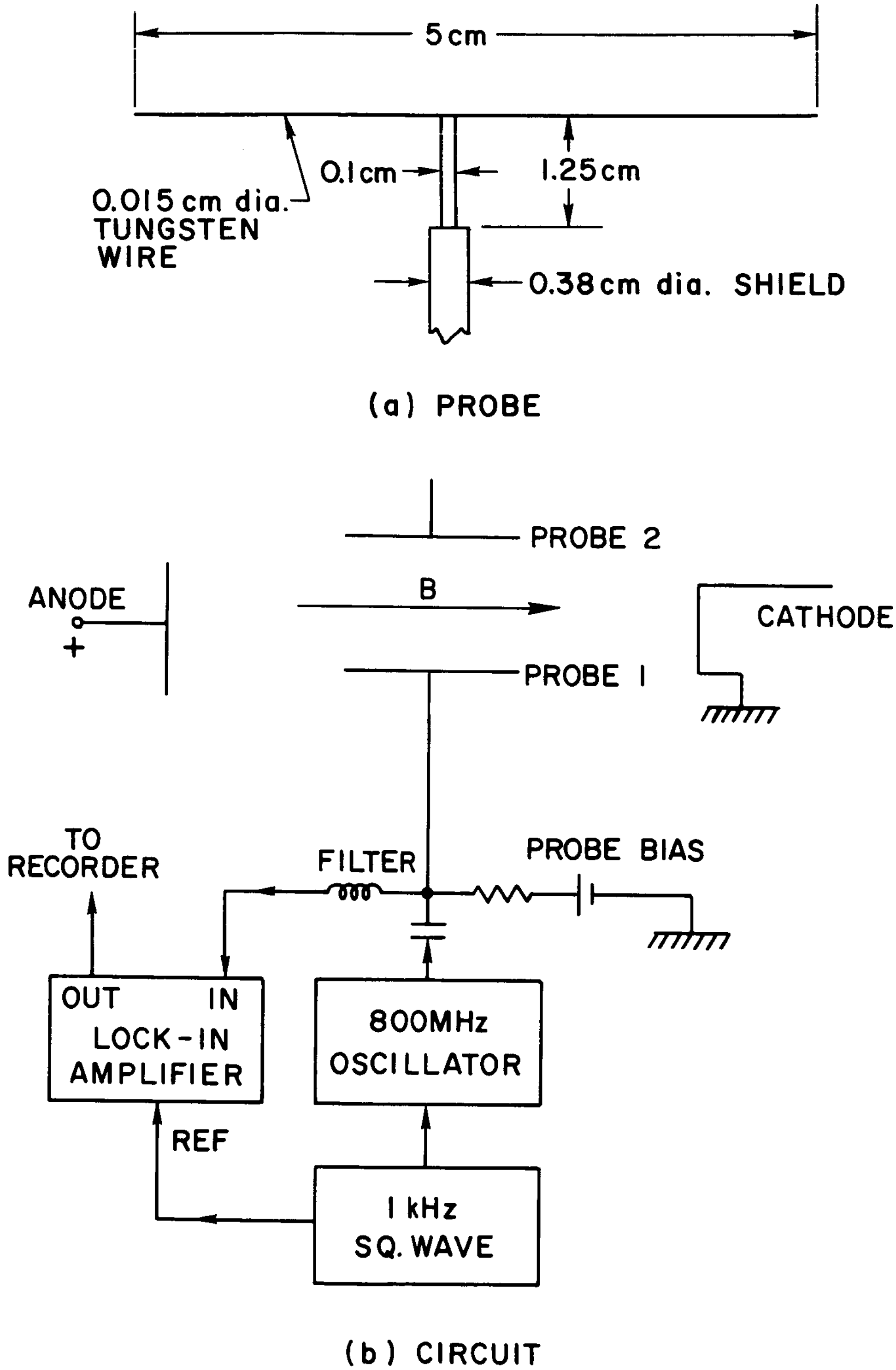

FIG. 4. Set-up for experimental resonance rectification studies.

where the form of F depends on the geometry and the driving sources. The integral represents a weighted area of $(1/\epsilon_\perp)$, for which the range of $k_\perp$ will probably be bounded experimentally by values corresponding to an electronic Debye length, and to the transverse dimensions of the apparatus.

Figure 5 shows the dispersion characteristics and the variation of $(1/\epsilon_\perp)$ with $(k_\perp v_T/\omega)$, for some typical experimental parameters. The shaded area is cut off in each plot at the highest value of $(k_\perp v_T/\omega)$ which was found observable in our experiments. The shaded area is approximately zero near the bottom of the passband, $(\omega/\omega_c) = 2.05$. At a slightly higher value, $(\omega/\omega_c) = 2.15$, the negative shaded area begins to dominate, i.e. the plasma becomes inductive. Nearer the top of the passband, at $(\omega/\omega_c) = 2.25$, the plasma is relatively strongly inductive. Although the weighting function has been neglected, the implication is that the resonance should occur progressively nearer the top of the passband as the capacitive sheath reactance increases, or as the plasma density decreases.

It will be noted from Eqs. (4) and (5) that the plasma reactance is zero at each cyclotron harmonic, since $\epsilon_\perp$ is infinite, and that the full rf source potential is then impressed across the sheath. The question arises as to whether this effect, which is independent of geometry, will give rise to an observable resonance peak. We cannot say a priori whether the effect will be strong compared with the principal resonances, but as we shall see in Section III both effects do occur: Double resonances are observed. The resonances which are located at the cyclotron harmonics, independent of the sheath properties will be referred to as 'harmonic' resonances.

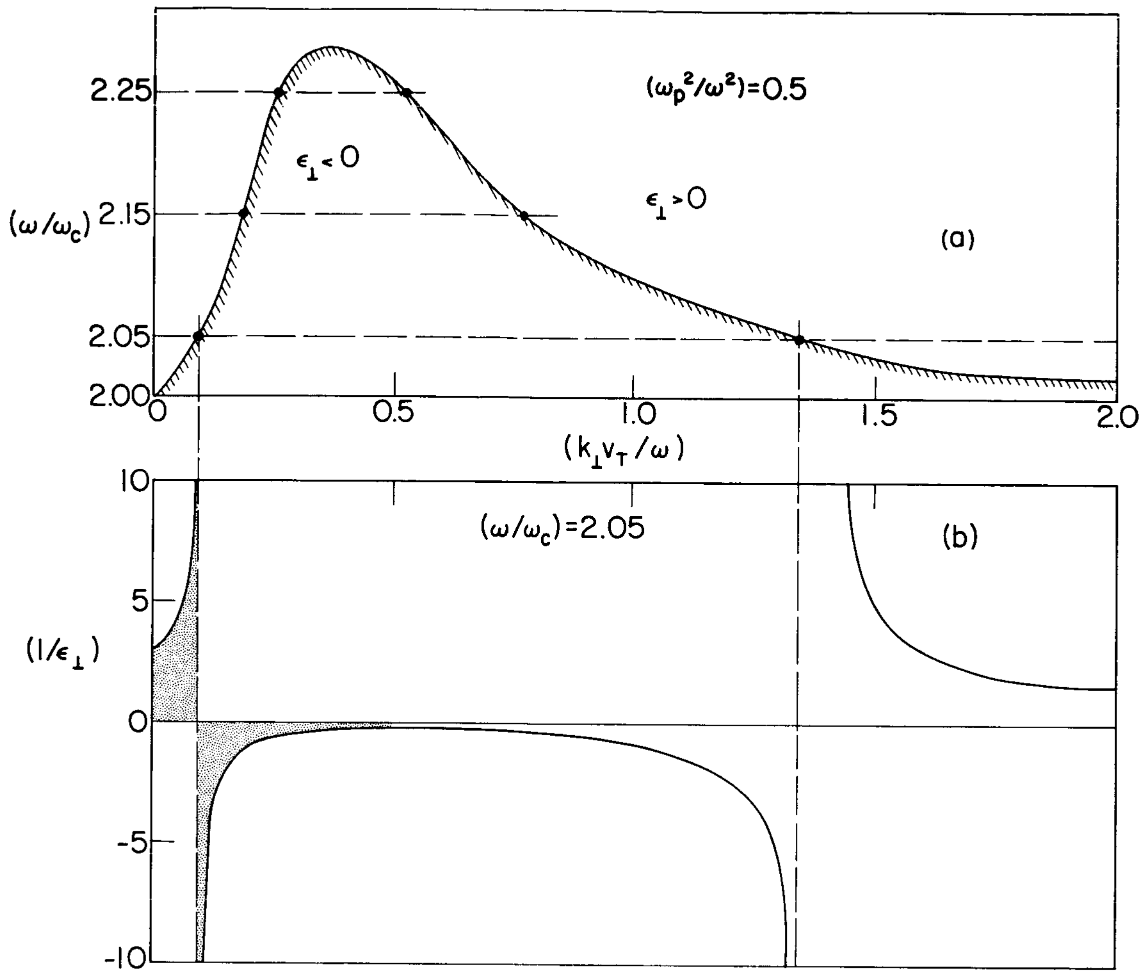

FIG. 5(a) & (b). Variation of $(1/\epsilon_\perp)$ with $(k_\perp v_T/\omega)$ for typical experimental parameters $[v_T = (\kappa T_e/m)^{1/2}]$.

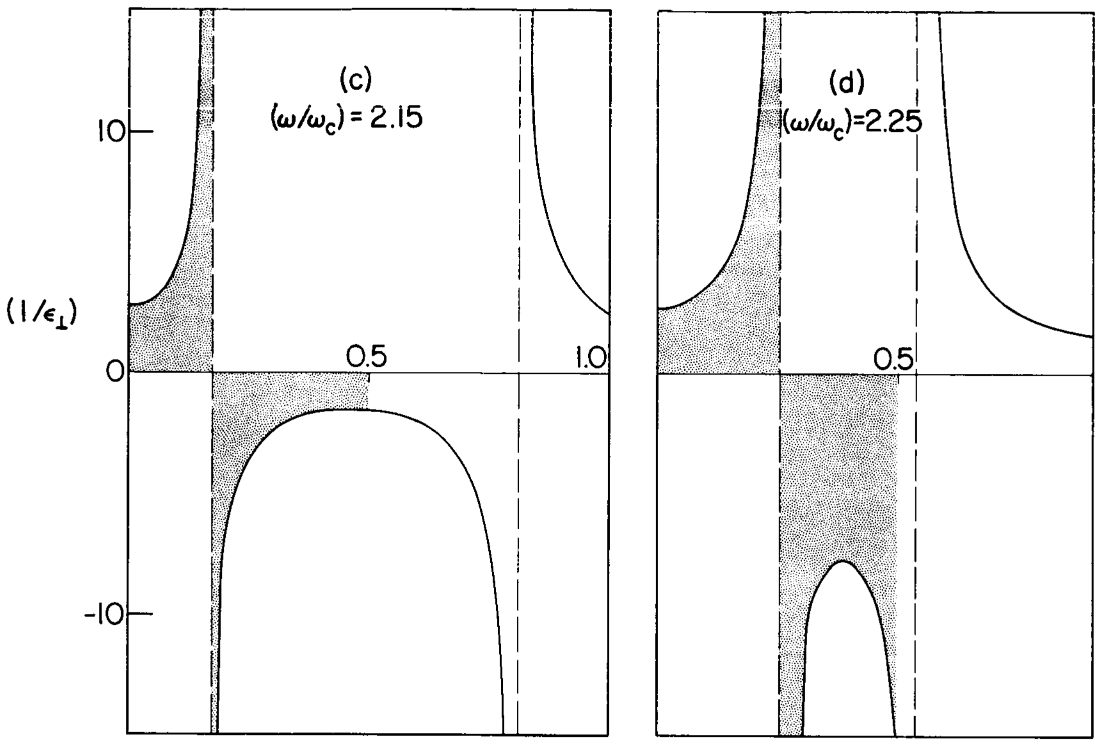

FIG. 5 (c) & (d). Variation of $(1/\epsilon_\perp)$ with $(k_\perp v_T/\omega)$ for typical experimental parameters $[v_T = (\kappa T_e/m)^{1/2}]$.

III. EXPERIMENTS

All experiments were carried out in a 6 cm diameter, positive column discharge in argon, at a pressure of about 0.6 mTorr. Signals were applied to a probe similar to that sketched in Fig. 4. A description of the remainder of the apparatus has been given elsewhere in connection with experiments on cyclotron harmonic wave propagation and need not be repeated here.[7]

(A) Resonance Rectification

Observations of resonance rectification phenomena were made using the circuit of Fig. 4(b). The 1 kHz output signal corresponded to the rectified component of the dc probe current. This signal was amplified, rectified and applied to the vertical deflection input of a recorder. The horizontal deflection was calibrated in terms of the magnetic field strength. Typical results are shown in Fig. 6 for a series of values of (ω_p^2/ω^2). In obtaining these plots, it was found that the plasma density increased somewhat as the magnetic field increased. To obtain precise data, the discharge current was set at the first, second and third cyclotron harmonics. The records presented are consequently for substantially constant values of (ω_p^2/ω^2).

Figure 6 shows strong double peaks corresponding to the two series of resonances. The shaded regions indicate the frequency bands in which the principal resonances should lie, according to the considerations of Section II. As predicted, for low densities they approach the edges of the passbands, while for higher densities, $(\omega_p^2/\omega^2) \geq 0.6$, they move towards the centers of the bands. The harmonic resonances lie close to the cyclotron harmonics except that a slight shift can be detected at low densities, $(\omega_p^2/\omega^2) \leq 0.4$.

Figure 7 shows the effect of varying probe potential, and hence the sheath thickness and reactance. Starting from floating potential, it will be seen that the peaks approach each other as increasing electron current is taken. This occurs, as predicted in Section II, since the sheath reactance is decreasing. The final record in the series shows that if the probe is biased negatively with respect to floating potential,

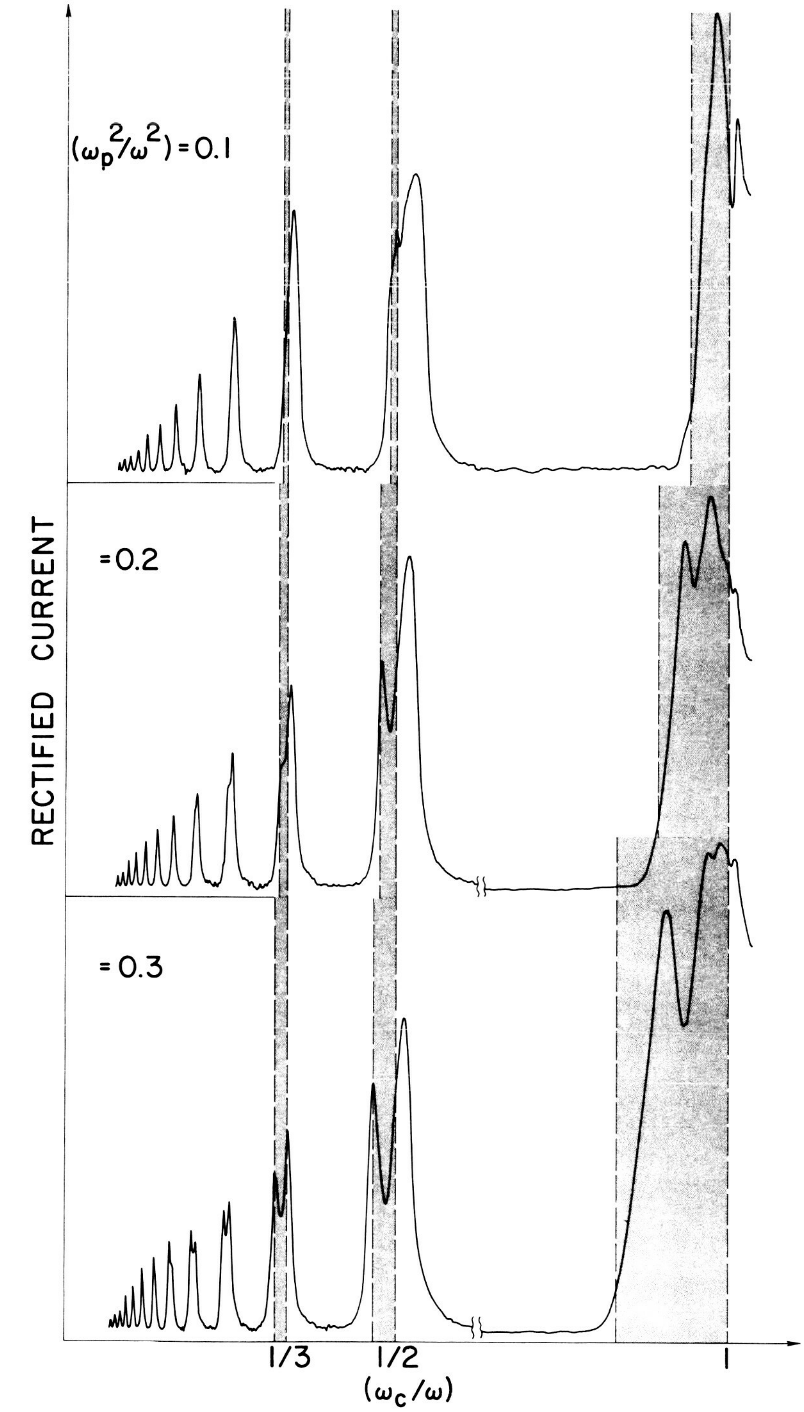

FIG. 6. Resonance rectification records [The curves have been separated vertically for clarity. Portions to the right of a break were taken with reduced sensitivity].

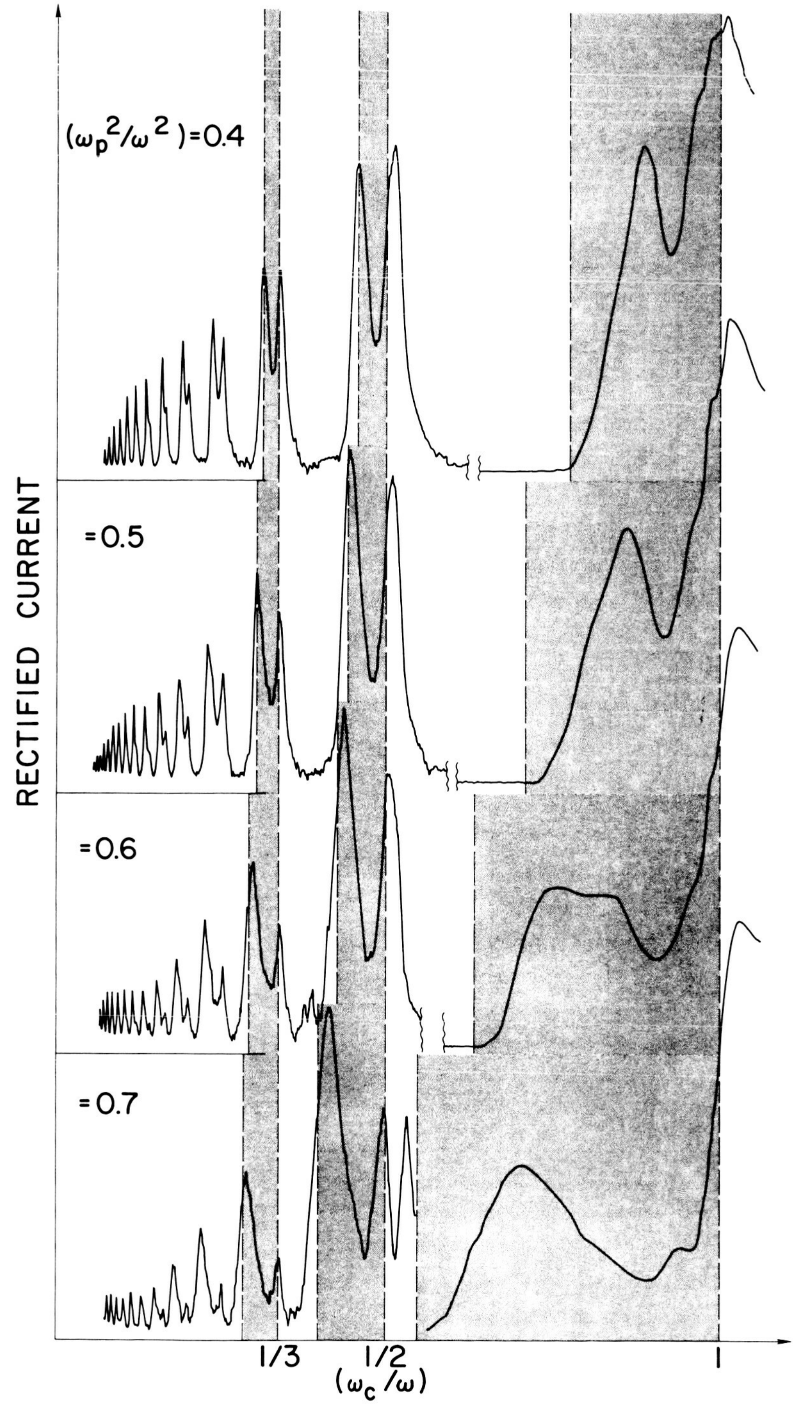

FIG. 6. cont'd. Resonance rectification records [The curves have been separated vertically for clarity. Portions to the right of a break were taken with reduced sensitivity].

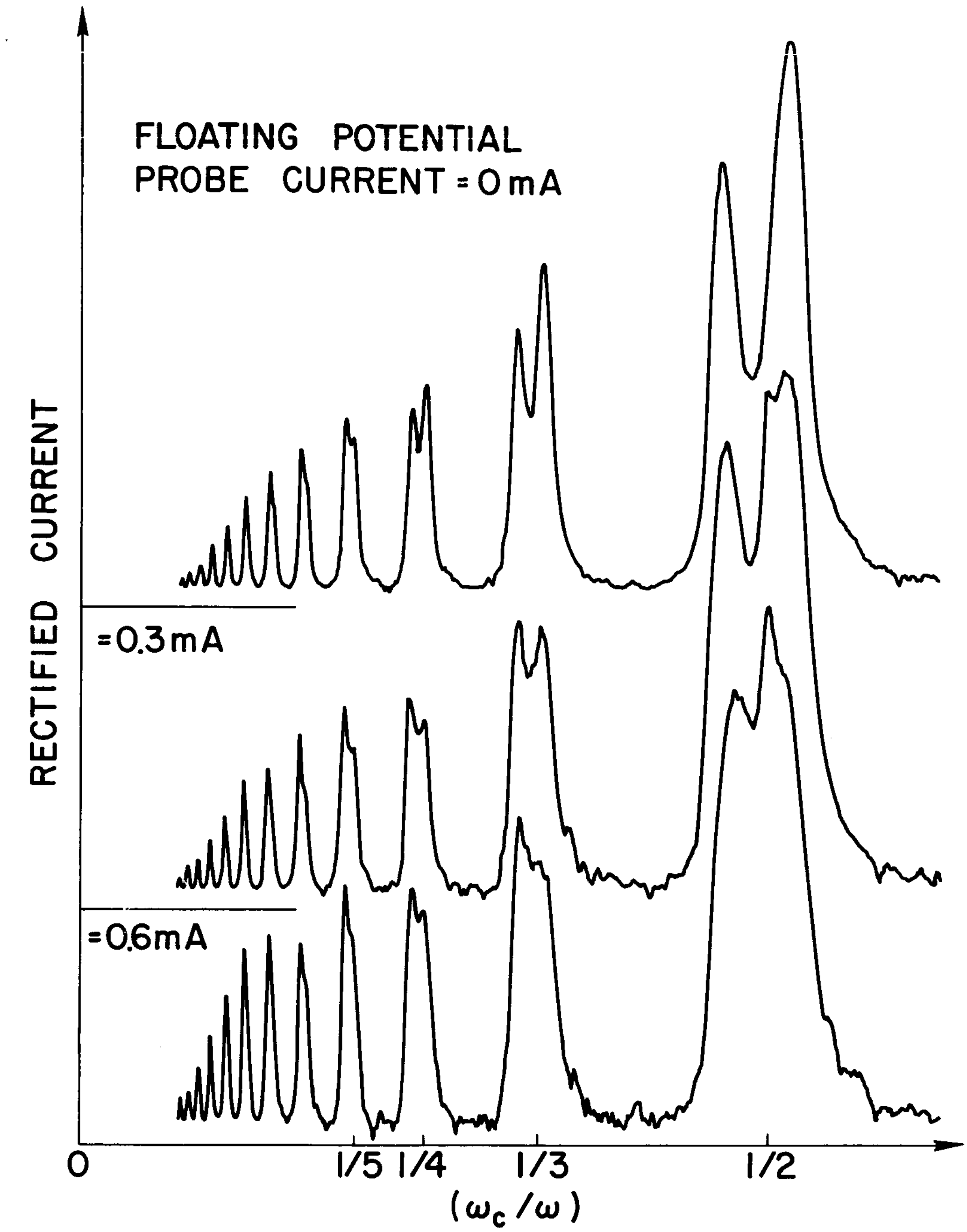

FIG. 7. **Effect of probe bias current on resonance rectification peaks** $[(\omega_p^2/\omega^2) = 0.4$. **The curves have been separated vertically for clarity].**

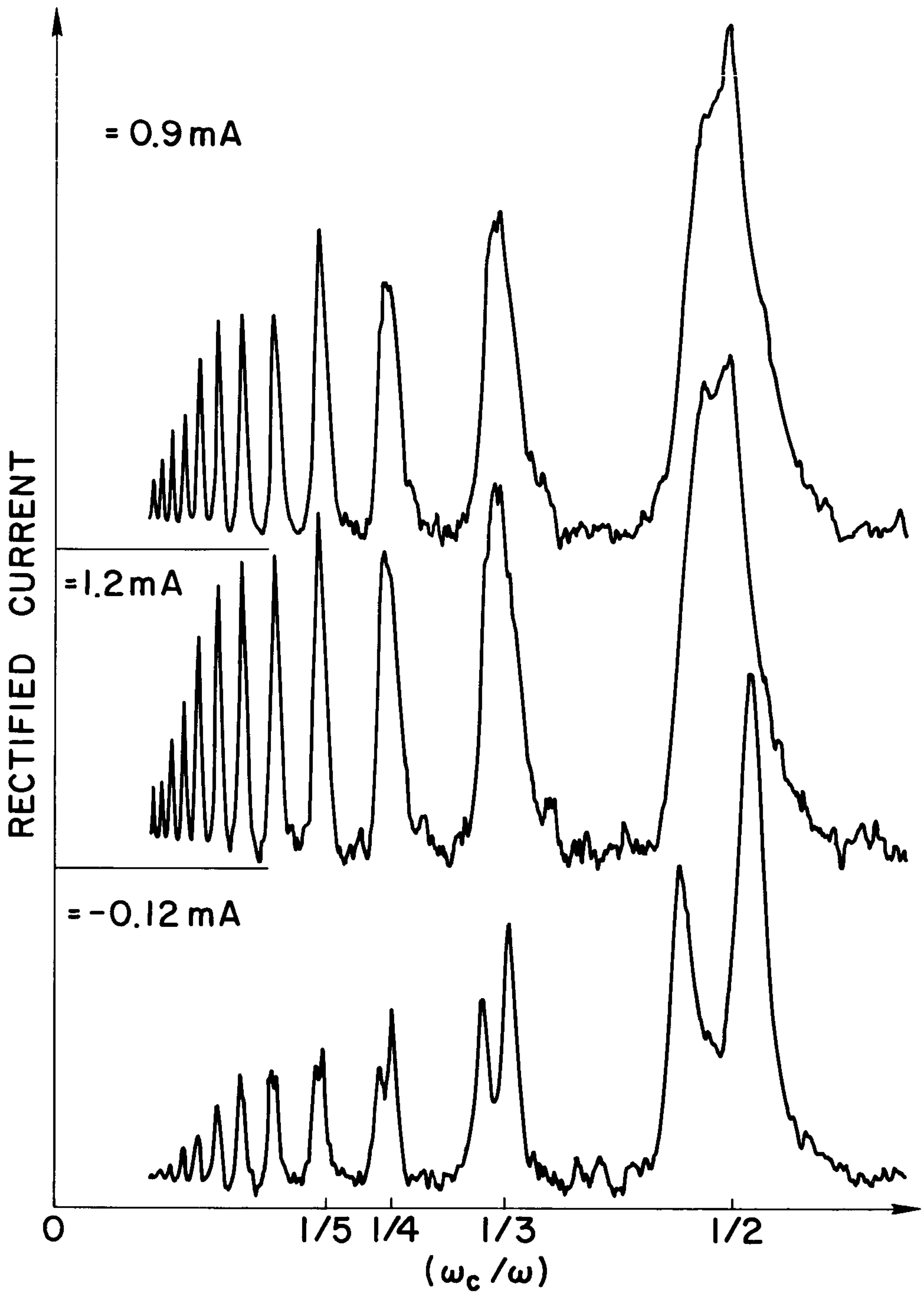

FIG. 7 cont'd. Effect of probe bias current on resonance rectification peaks $[(\omega_p^2/\omega^2) = 0.4$. The curves have been separated vertically for clarity].

the sheath reactance increases and the peak separation increases. As expected, the principal resonances are always found experimentally to remain within the wave passbands.

(B) Transmission Records.

Measurements of transfer admittance were made by transferring the connection to the detector, amplifier and recorder to Probe No. 2. For a constant rf potential on Probe 1, the detected signal, which is a function of current, indicates transfer admittance effectively. The experiments were carried out with two purposes in mind. First, to determine if the admittance extrema corresponded to locations of the resonance rectification peaks. And second to provide an electron density calibration. The latter can be obtained by noting the location of the upper hybrid absolute minimum point in transmission for a given current. This establishes a calibration of plasma frequency in terms of the discharge current.[7]

Specimen records are shown in Fig. 8. The principal resonances are found to lie between the admittance minima and the cyclotron harmonics, as expected. This is also the region in which interference peaks, denoting cyclotron harmonic wave propagation, occur on the admittance curves. It is important to note that in the steady-state transmission measurements, a dc bias applied to the transmitting and/or receiving antenna does not affect the measured width of the passband. This implies that the probe current has a negligible effect on the body of the plasma, and that the results of Fig. 7 cannot be attributed to such spurious effects.

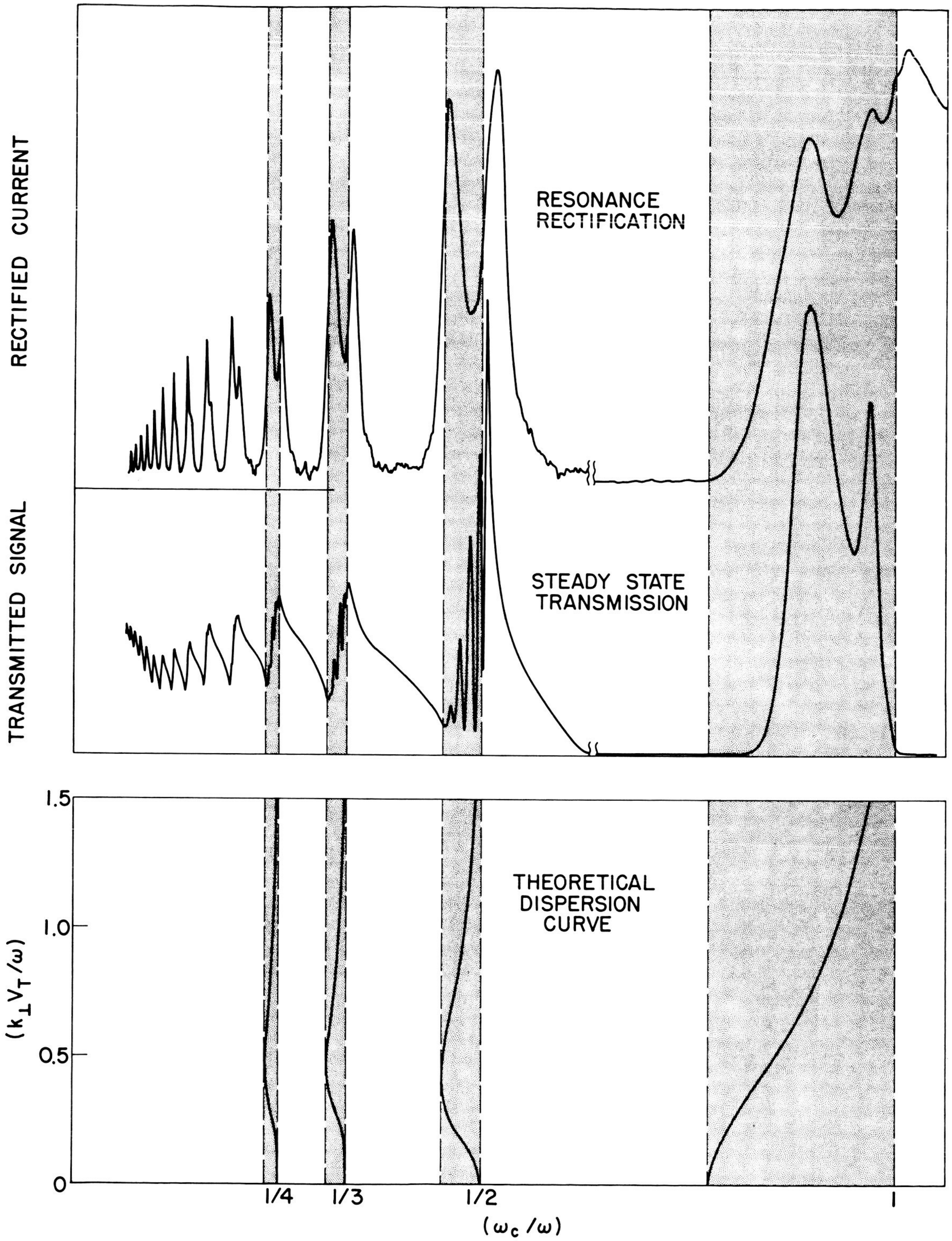

FIG. 8. Comparison of transfer admittance and resonance rectification records with theoretical cyclotron harmonic wave dispersion characteristics $[(\omega_p^2/\omega^2) = 0.4]$.

IV. DISCUSSION

The experiments described extend the conditions under which resonance rectification phenomena have been observed to the case of a plasma immersed in a magnetic field. The results demonstrate clearly that under conditions where the electric field is primarily perpendicular to the static magnetic field, and the perpendicular component of the plasma permittivity can be expected to be most important, warm plasma effects manifest themselves strongly. In particular, series of resonances not predicted by cold plasma theory can be observed. Although the location of the principal series can be predicted semi-quantitatively by the theory expressed in Fig. 2, detailed predictions of the locations of the peaks would require precise knowledge of the sheath structure and of the plasma near the sheath edge. The problem is further complicated by the difficulty of approaching ideal geometry in an experimental situation. For these reasons, the use of the principal series as the basis of a diagnostic technique seems much less promising than was the case for the resonance probe in the absence of magnetic field.

A limited amount of information can be obtained from the harmonic resonance peaks, since they give a measure of the local value of the cyclotron frequency. The experiments indicate, however, that the peaks are not always precisely at the harmonics, and in any case there are many easier ways to measure magnetic field strength. It may be mentioned in this context that, if the warm plasma permittivity properties are to be used in diagnostics, it is probably best to use them in terms of cyclotron harmonic wave propagation $(\epsilon_{\perp} = 0)$, from which the electron density, temperature, and magnetic field strength can all be obtained either from interference measurements, on records such as those of Fig. 9, or from pulse delay measurements for wave-packets transmitted between two probes. These methods have already been shown elsewhere to give excellent results,[7,9] and are independent of electrode sheath effects.

REFERENCES

1. K. Takayama, H. Ikegami and S. Miyasaki, Phys. Rev. Letters 5, 238, (1960).

2. H. M. Mayer, Proc. Sixth International Conference on Ionization Phenomena in Gases, Paris, France, July 1963 (SERMA Publishing Co., Paris 1964) Vol. 4, 129; R. S. Harp, Appl. Phys. Letters 4, 186 (1964).

3. R. S. Harp and F. W. Crawford, J. Appl. Phys. 35, 3436 (1964); W. M. Leavens, J. Res. NBS 69D, 1321 (1965); R. Buckley, Proc. Roy. Soc. A290, 196 (1966).

4. J. Uramoto, H. Ikegami and K. Takayama, Institute of Plasma Physics Report No. 15, Nagoya University, Japan (October 1963).

5. F. W. Crawford, J. Appl. Phys. 36, 3142 (1965); J. R. Wait, J. Appl. Phys. 37, 4905 (1966).

6. T. H. Stix, The Theory of Plasma Waves, (McGraw-Hill Book Co., New York, 1962).

7. F. W. Crawford, R. S. Harp, and T. D. Mantei, J. Geophys. Res. 72, 57 (1967).

8. F. W. Crawford, T. D. Mantei and J. A. Tataronis, Int. J. Elect. 21, 341 (1966).

9. R. S. Harp, Appl. Phys Letters 6, 51 (1965); Proc. Seventh International Conference on Phenomena in Ionized Gases, Belgrade, Yugoslavia, August 1965 (Gradevinska Knjiga Publishing House, Belgrade 1966) Vol. 2, 294.