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A COMPUTER CONTROL SYSTEM FOR AN ADVANCED OAO

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SUMMARY

The purpose of the study covered by this report was to determine the feasibility of using an on-board digital computer and strap-down inertial reference unit for attitude control of an advanced OAO spacecraft. An integrated, three axis control law was assumed which had been previously proven stable in the large. The principal advantage of this type of control system is the ability to complete three axis reorientations over large angles. This system, although apparently complex, has the effect of simplifying the overall system. This is because a reorientation is a simple extension of a hold or point operation, i.e., mode switching may be simplified.

The study indicates that a system of this type is feasible and offers many advantages over present systems. The time for a reorientation using this type of control is considerably shorter than the time required using more conventional methods. The computer and inertial reference unit used in the study had the characteristics of systems presently under development. Feasibility, therefore is within the present state of the art, requiring only continued development of these systems.

The basic system is not limited to OAO but may be adapted for other three axis stabilized spacecraft.

ACKNOWLEDGMENTS

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INTRODUCTION

In recent years, the microminiaturization of electronic components has made possible very small, general purpose (GP) digital computers. It is within the present state of the art to include these computers on-board spacecraft. One of the primary uses for such a computer is data processing. Another important use is in conjunction with the attitude control system of the spacecraft. The advent of the small computer has made it possible to consider control laws for the spacecraft attitude control system that are relatively complex. All the effects of dynamic coupling and large angle nonlinearities may be considered at the start of the design procedure. Small angle limits need not restrict the control system design. Thus an integrated, three axis control system may be designed in place of three, small angle, single axis systems.

Although a great deal of work remains in this area Mortensen (ref. 1) and Meyer (ref. 2) have proposed three axis control laws.

Meyer defines the direction cosine matrix relating the spacecraft axes to a fixed inertial reference as the output of the attitude control system. This matrix is multiplied by a reference matrix, and the control law is formulated from the product or error matrix. The control law he assumes is shown to be asymptotically stable in the large. The attitude matrix is assumed to be known at all times. It may be determined by some type of inertial sensor (star trackers, gyros). The matrix formulation and multiplication, and the control law formulation require a computer.

In an earlier work Mortensen proposed a control law under the assumption of a particular formulation for the spacecraft kinematics. He used the Cayley-Rodrigues parameters. These are also known as Euler-Rodrigues parameters (by Roberson) and the Gibbs vector components (ref. 3). Assuming the parameters defining the body orientation are known, the control using these parameters he shows to be asymptotically stable in the large. The parameters are not easily measured but may be determined by solving the first order differential equations in the parameters and body rates. This method is similar to Meyer's method except the attitude matrix is never formed explicitly.

The control law is formed directly from the kinematic parameters. The computation of the kinematic parameters and the control law formulation require a computer.

Control laws of these types have three characteristics which should prove very valuable in future spacecraft:

- Stability for any attitude. Thus the desired attitude of a spacecraft is unrestricted and may be determined from considerations other than stability.

- Three axis reorientation capability. Slewing all three axes simultaneously results in a rapid reorientation.

- Simplicity. Since a three axis reorientation is simply an extension of a hold or point operation, mode switching may be simplified.

This report is a partial result of a very short conceptual study for an advanced OAO spacecraft. The spacecraft considered was similar to the present OAO but with an extended telescope. The basic assumptions underlying the study with respect to the attitude control system were the presence of a GP, on-board computer and a very precise, strap-down inertial reference unit (IRU)

One of the objectives of the study was the minimization of the number of gimballed star trackers. The results of the study show that with the type of control law covered in this report the control system requires no gimballed star trackers. The trackers assumed are:

- Boresighted star tracker along the roll axis parallel to the experiment telescope.

- Fixed Canopus tracker along the spacecraft yaw axis for initial stabilization.

- Two axis sun tracker, gimballed in pitch, for initial stabilization (See Figure 5).

The primary purpose of the portion of the study covered by this report was to determine the feasibility of a three axis control system for pointing and reorienting the spacecraft. Assuming utilization of the computer and IRU, implementation of the type of control law proposed by Mortensen was considered.

It should be noted that although an advanced OAO is the specific spacecraft considered, many of the principles may be applied to other spacecraft that require a pointing and slew capability.

CONTROL LAWS

As in any spacecraft with a broad operating regime (initial stabilization through fine pointing) a number of different operating modes are required. This is true primarily because of the dynamic range limitations of the sensors and actuators. The control modes may be considered in three categories:

- Initial stabilization. The initial three axis stabilization of the spacecraft requires the acquisition of two celestial references. The two references in this case are the sun and the star Canopus. Initial stabilization takes place in two stages. First, the sun line is acquired and two axis control about this line is maintained with the sun tracker. Acquisition of the sun line with a sun tracker and rate gyros (or IRU) requires a very simple control law. This control law may be shown to be globally stable (ref. 3). No computer is required to implement it. In the second stage of the procedure the spacecraft is slewed around the sun line until the Canopus tracker detects and locks on Canopus.

- Hold or Pointing. Operation in this type of mode, with the sun tracker, the experiment telescope, control system star tracker or gyros (IRU), is essentially small angle control. A linear position plus rate control ($\theta + k\dot{\theta}$), using either a lead network or gyro information, will give satisfactory performance. No computer is required.

- Reorientation. This may be accomplished by three consecutive single axis slews. However, this is time consuming and complex. A large angle, three axis reorientation which is fast and simple may easily be accomplished with a computer, IRU and control law as previously described.

The computer control system encompasses the hold or pointing mode as well as the reorientation mode. Therefore, once the initial acquisition is complete a single control law may be used for holding, pointing and slewing. This control law would use the IRU as the basic sensor during a reorientation and while holding (e.g. when the experiment is occulted), and the optical trackers when pointing.

This control system may also be used in the Canopus search phase of the initial stabilization.

The remainder of the report is concerned primarily with this type of control system.

AN INTEGRATED THREE AXIS CONTROL LAW

Introduction

In order to effect a three axis reorientation it is necessary to know the spacecraft attitude at all times. This requires knowledge of the orthogonal transformation matrix relating the spacecraft body axes to a known, fixed inertial frame. Explicit determination of the entire matrix is not necessary as long as the three independent parameters of the kinematic representation are known.

One method of paramaterizing this matrix is by the Cayley-Rodrigues parameters. See equations A-17 in Appendix A. In terms of these three independent parameters the matrix* is (ref. 3):

$$\bar{\bar{A}} = \frac{1}{1+\alpha^2+\beta^2+\gamma^2}\begin{bmatrix} 1+\alpha^2-\beta^2-\gamma^2 & 2(\alpha\beta+\gamma) & 2(\alpha\gamma-\beta) \\ 2(\alpha\beta-\gamma) & 1-\alpha^2+\beta^2-\gamma^2 & 2(\beta\gamma+\alpha) \\ 2(\alpha\gamma+\beta) & 2(\beta\gamma-\alpha) & 1-\alpha^2-\beta^2+\gamma^2 \end{bmatrix} \tag{1}$$

Continuous knowledge of α, β, γ, determines body attitude at all times with respect to the inertial frame. The parameters are not easily measured directly but may be determined by continuously solving the differential equation relating $\dot{\alpha}, \dot{\beta}, \dot{\gamma}$ to the body rates $\omega_x, \omega_y, \omega_z$.

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1+\alpha^2 & \alpha\beta-\gamma & \alpha\gamma+\beta \\ \alpha\beta+\gamma & 1+\beta^2 & \beta\gamma-\alpha \\ \alpha\gamma-\beta & \beta\gamma+\alpha & 1+\gamma^2 \end{bmatrix}\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{2}$$

*Capital letters with double overbars as square matrices; with single overbars are column matrices.

Using these parameters Mortensen has proposed the following control law (ref. 1):

$$u_x = k_{21}\omega_x + k_1(1 + \alpha^2 + \beta^2 + \gamma^2)\alpha$$

$$u_y = k_{22}\omega_y + k_1(1 + \alpha^2 + \beta^2 + \gamma^2)\beta \tag{3}$$

$$u_z = k_{23}\omega_z + k_1(1 + \alpha^2 + \beta^2 + \gamma^2)\gamma$$

where u_x, u_y, u_z are control torques.

He shows the system with this control law to be asymptotically stable in the large with respect to the origin ($\alpha=\beta=\gamma=0$). The dynamic and control equations are developed in Appendix A. This control law is analogous to a conventional law in that the torque is a function (although non-linear) of rate and position. For small angles (i.e. the spacecraft body axes near the desired attitude) the following approximations hold (ref. 3):

$$\alpha \approx \frac{\theta_1}{2}$$

$$\beta \approx \frac{\theta_2}{2} \tag{4}$$

$$\gamma \approx \frac{\theta_3}{3}$$

where θ_1, θ_2, θ_3 are Euler angles.

Therefore, for small angles the control reduces to a simple linear rate plus position control.

$$u_x \approx k_{21}\omega_x + k_3\theta_1$$

$$u_y \approx k_{22}\omega_y + k_3\theta_2 \tag{5}$$

$$u_z \approx k_{23}\omega_z + k_3\theta_3$$

A control law of this type, globally stable for any attitude, encompasses pointing and slewing without requiring mode switching. The inertial reference is defined as $\alpha=\beta=\gamma=0$. When a new reference is desired e.g. pointing at a different star, the transformation relating the present position to the new position is determined. The corresponding parameters α, β, γ are determined and sent up to the spacecraft. The spacecraft, utilizing this control law, acts to null α, β, γ and thereby moves to the new attitude. This is a reorientation or three axis slew.

Using the strap down IRU and GP computer, α, β, γ may be continuously updated to allow utilization of this control law. See Figure 1.

Attitude Reference Algorithm

The sensor to be used for operational control of the spacecraft during a reorientation is an IRU composed of three inertial gyros. This is a strap-down system with the gyros operating in a pulse rebalance loop. They are continuously nulled by a series of pulses the size and rate of which provide position and rate information. References four and five give a good description of this type of gyro. The information from the IRU must be used in conjunction with the computer to update the parameters α, β, γ throughout a reorientation. This may be done by solving equation 2.

The body rates are not known accurately enough for equation 2 to be solved in its present form. A similar problem was encountered in reference five with respect to the updating of an attitude reference matrix. An integration algorithm based on an incremental angle was developed. A similar procedure may be used here.

A Taylor series expansion may be used to update α, β, γ. The value at time t+h may be determined from the value at time t and the information received from the gyro during the sampling interval h. Only the parameter $\alpha(t)$ will be considered here although similar expressions exist for $\beta(t)$ and $\gamma(t)$.

$$\alpha(t + h) = \alpha(t) + \dot{\alpha}(t)h + \frac{1}{2}\ddot{\alpha}(t)\, h^2 + \frac{1}{6}\dddot{\alpha}(t)\, h^3 + \cdots \tag{6}$$

Using only a first order expansion and substituting for $\dot{\alpha}(t)$ from equation 2:

$$\alpha(t + h) = \alpha(t) + \frac{(1 + \alpha^2(t))\,\Delta_x}{2} + \frac{(\alpha(t)\,\beta(t) - \gamma(t))\,\Delta_y}{2} + \frac{(\alpha(t)\,\gamma(t) + \beta(t))\,\Delta_z}{2} \tag{7}$$

where $\Delta_i = \omega_i(t)h;\quad i = x, y, z.$

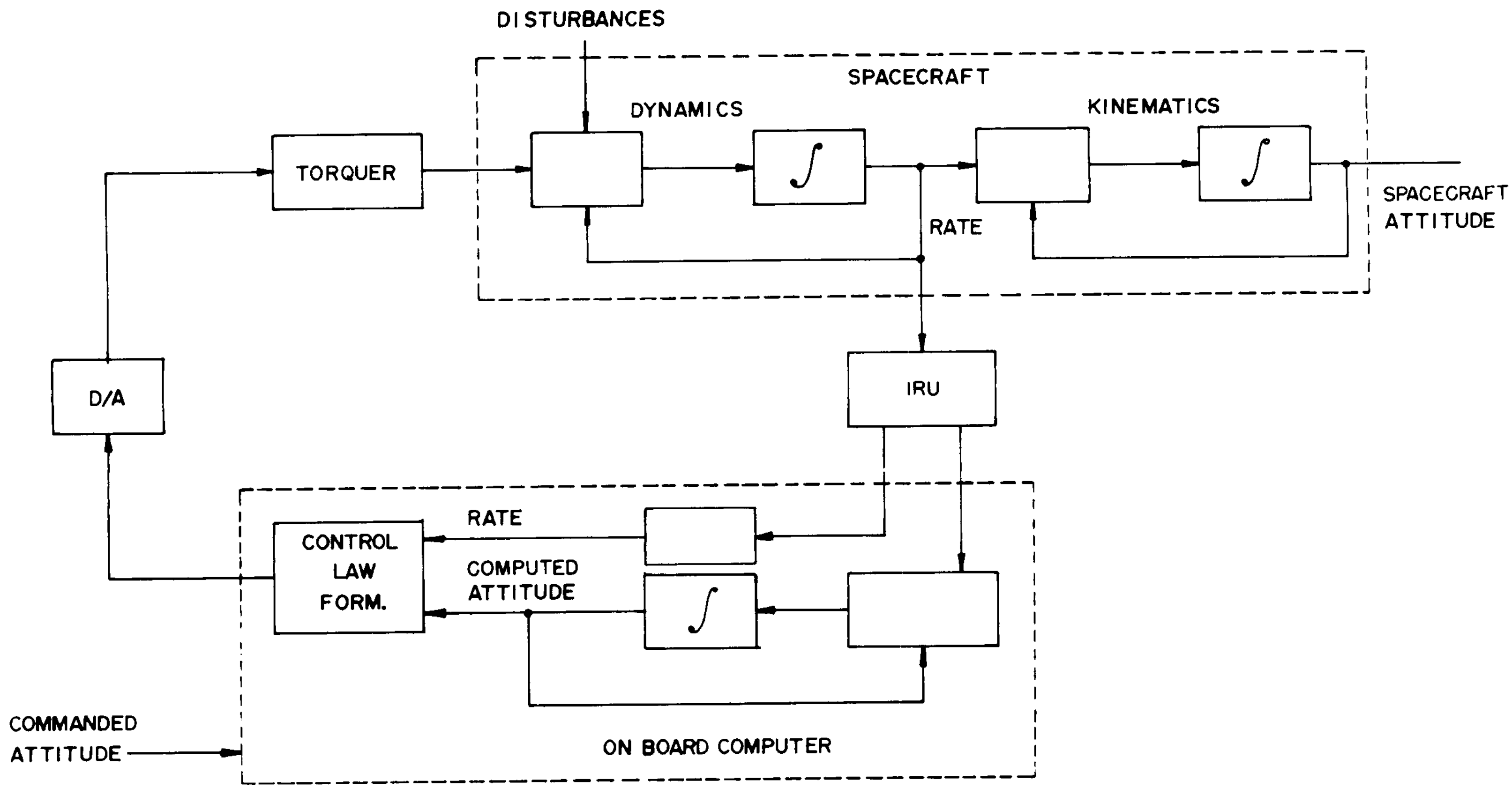

Figure 1–Control System Block Diagram

The Δ_i terms represent the angular output of the i^{th} gyro during the interval h, i.e. a number of pulses of weight q (q is the gyro quantization level, 2.4 arc sec in the IRU slew mode). The updating therefore may be done on an incremental angle basis. If the second order terms of the Taylor series are kept, terms on the order of h^2 appear. Appendix B lists the second order expansions for all three parameters.

Updating the parameters in this manner allows the control law to be updated at each sampling point also. The body rates used in the control law are simply $\Delta i/h$.

A study was conducted to determine the feasibility of this type of algorithm. A digital computer simulation of the system was used to help evaluate the errors. The attitude error was defined as the difference between the true spacecraft attitude and the attitude computed by means of the Taylor expansion. The spacecraft dynamics and true attitude were computed by means of a four point Runge-Kutta routine. The integration step was kept much smaller than the sampling interval h.

The results of the study are as follows:

- The error in the update algorithm does increase with an increase in the sampling interval h. This is not a serious constraint because even relatively long sampling intervals gave good results. Using a second order Taylor expansion and a 5 second sampling interval the errors at the end of a 60° slew were on the order of 14 arc seconds.

- The error in the update algorithm does increase with the magnitude of the slew angle for the longer sampling intervals (2 seconds and up). For a sampling interval of 100 ms the error is apparently independent of the magnitude of the slew angle even for very long slews (165°). It remains on the order of 2.4 arc sec which is the quantization level of the gyros. For a sampling interval of one sec the error for a 165° slew is only about 5.3 arc sec. This was using second order Taylor expansion.

- For moderate sampling intervals and slew angles (up to 2 seconds and 90°) the attitude error is primarily a function of the gyro quantization level and is on this order of magnitude. This also holds true for shorter slews (30°) with longer sampling intervals (5 seconds).

- The computer round-off error may be kept negligible by using a 36 bit, double precision word. The computer considered in the study (see the following section) has the ability to operate single (18 bits) or double precision.

The estimated times for the update computations are shown in Table 1.

Table 1
Estimated Time for the Update Computation

	Single Precision	Double Precision
1st order Taylor	2 ms	7 ms
2nd order Taylor	4 ms	14 ms

These times are based on the computer described in the following section. These computation times are very compatible with the sampling intervals investigated.

These results indicate this type of algorithm for updating the spacecraft attitude parameters is feasible.

Computer

A candidate for the computer required for attitude updating and control law formulation is the "Units" On-Board Processor being developed by the Space Electronics Branch, Information Processing Division, GSFC (ref. 6). This computer is being developed primarily as a data processor but appears to have the capacity and capability for handling the control equations. Some of the features of this computer are:

- Post launch reprogramming capability. The control laws may be changed in flight if desired.

- Plated wire memory unit with 8192 word (18 bits/word) capacity. This may be easily expanded to about 65,000 words.

- Low Power. Total power required is about 10.7 watts with peak memory activity and less than one watt at stand-by.

- Very fast, Duty cycle of less than 0.25 at a 2μs write rate.

- Very small. Memory size is about 3X6X10 in. (For 8192 words).

- Maximum use of monolithic integrated circuits, 80% utilization.

Inertial Reference Unit

The inertial reference unit (IRU) being developed by MIT for OAO will apparently meet the attitude reference requirements. Three operational modes are available: hold, slew and I-stab. The primary concern of this report, three axis reorientation, requires use of the slew mode. For this study the processing being developed for the early models of the IRU was not considered. The only concern was the basic performance of the gyros.

Following are some of the important characteristics of the IRU in its slew mode:

- Resolution, 2.4 arc sec/pulse

- Maximum rate, 480 arc sec/sec

- Pulse rate, 200 pps

- Compensated drift, 20 arc sec/orbit (12 arc sec/hr)

- Torquing error less than 0.01%

Normal Reorientation

The system dynamic, kinematic and control equations were simulated on a digital computer. The primary objective of the simulation was to determine performance with respect to a slew or reorientation. The equations reduce to approximately linear equations for small angles (hold or pointing). Thus it was not necessary to consider the hold or pointing modes of operation.

The angle ϕ (see Appendix A) is a good scaler representation of error, i.e., difference between the actual body orientation and the desired orientation (ref. 2). This is true because α, β, γ are all zero if and only if ϕ is zero. Therefore ϕ was chosen as the primary performance index. For a single axis slew ϕ is equivalent to the Euler angle around the slew axis.

The only requirements for stability are that the position gain (k_p) and rate gains (k_x, k_y, k_z) be positive and non-zero. There are no other requirements with respect to magnitude or linearity. Therefore once the stability criteria are met the designer may set the gains to meet other criteria such as speed of response, damping, etc.

Response. Probably the most important criterion for a normal reorientation is the time for a slew. The shorter the reorientation time, the longer the experimenting time. Another criterion, maintenance of a stable slew axis, is discussed in a following section. Using the time criterion several computer runs were made.

The time for a slew was defined as the time required from the read-in of initial conditions to the on-board computer until the norm reached its minimum value. The norm; defined as

$$\mathrm{NORM} = + (\omega_x^2 + \omega_y^2 + \omega_z^2 + \phi^2)^{1/2} \tag{8}$$

was usually assigned a minimum of 10^{-4}. Since the norm is always positive, the rates were required to be below 10^{-4} rad/sec and ϕ was required to be below 10^{-4} rad (0.34 arc-min) before a slew was considered complete. Therefore slew or reorientation time as defined here includes settling time.

Since a slew is basically a momentum exchange the gains were set to result in a maximum wheel speed during the slew. The rate gains were set to provide good damping at the end of the slew. Both type A and B wheels were used. See Figure 2. The characteristics were as follows:

Stall Torque	0.27	Nm	(0.20 ft-lb)
Max. Momentum	13.6	Nms	(10.0 ft-lb-s)
Time Constant T_m	50	s	

To compare a three axis reorientation with the more conventional method using three single axis slews, a number of runs were made with the slew axis eigenvector aligned with a control axis. This is equivalent to a conventional single axis slew.

Figure 3 shows the response for four single axis and two multi-axis slews (eigenvector 0.5, 0.5, 0.707). The maximum slew rate for the single axis slew is about 0.25×10^{-2} rad/sec. This is determined

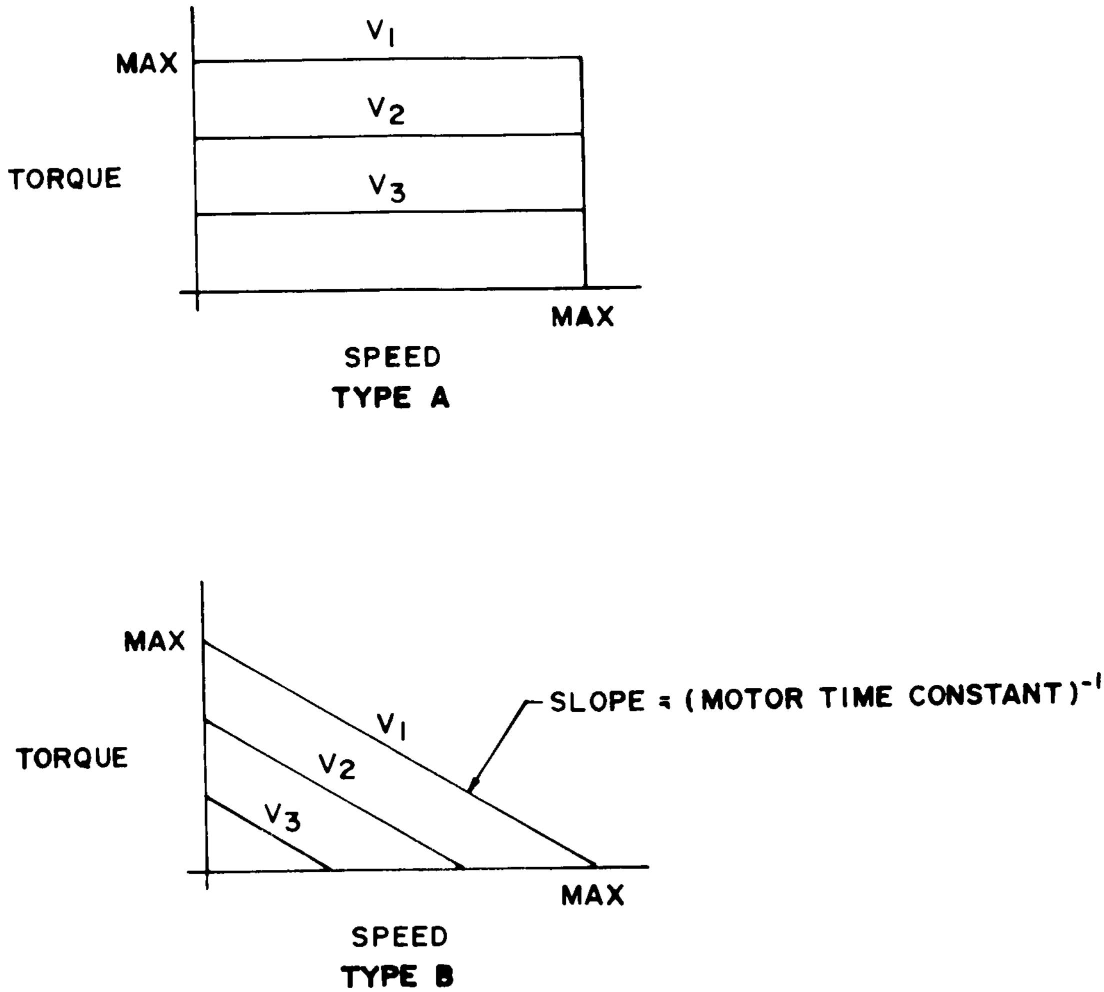

Figure 2—Reaction Wheel Torque-Speed Curves

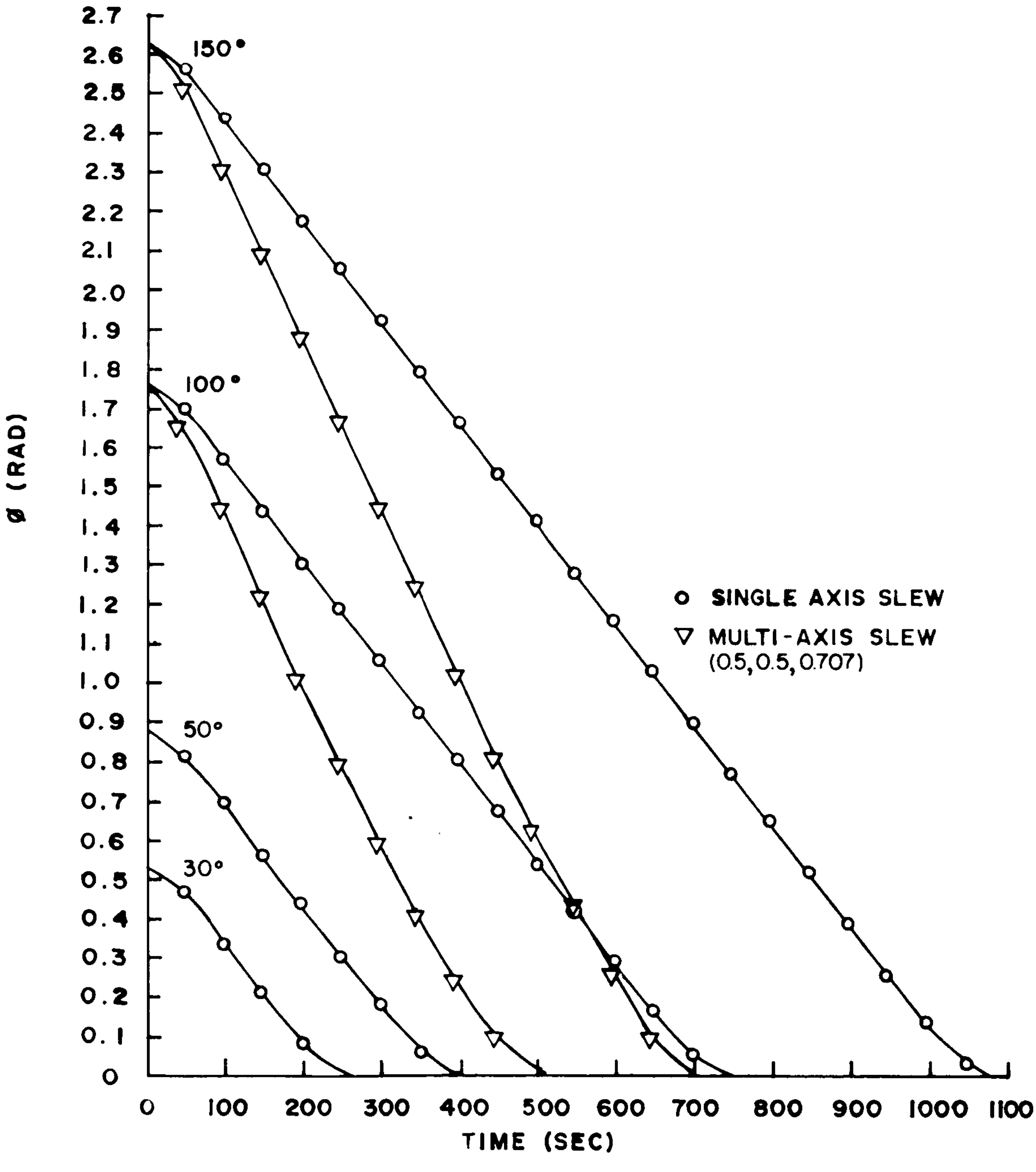

Figure 3–Slew Response

by the 13.6 Nms maximum wheel momentum and the 5420 kg-m^2 body inertia (the same for all axes). The maximum slew rate for the multi-axis slew occurs when all three wheels are saturated. It is about 0.43×10^{-2} rad/sec. Thus all three wheels are being used to reduce ϕ. The single and multi-axis slews in Figure 3 are not directly comparable because the final orientations are different. A comparison will be made later in the report.

Figure 4 is a plot of total slew time for various length slews. The curves all converge to about 160 sec for a 2° slew. This is apparently due to the end effects, i.e., acceleration at the start and settling at the completion of the slew. The slope of the linear portion of the single axis curves is $(0.25 \times 10^{-2})^{-1}$ sec/rad. This is determined by the maximum wheel momentum. All the points checked for multi-axis slews were below the time required for equivalent length single axis slews.

To compare the time required for reorientation by means of a multi-axis slew with the time required for the same reorientation via three single axis slews, three points were checked both ways. See points A, B, C on Figure 4. Table 2 summarizes the results.

Table 2
Reorientation Times

	Euler Angles (rad; deg)			Eigenvector Comp.			ϕ (rad; deg)	T_1 (sec)	T_3 (sec)	T_1/T_3
	θ_1	θ_2	θ_3	C_x	C_y	C_z				
A	0.1745 10	0.2745 15.7	0.1745 10	0.5187	0.6797	0.5187	0.3797 21.8	700	260	2.69
B	0.523 30	0.523 30	0.523 30	0.6546	0.3782	0.6546	0.9722 55.7	1080	420	2.57
C	1.045 60	1.045 60	1.045 60	0.6946	0.1869	0.6946	2.0327 116.5	1680	790	2.13

In the table T_1 is the total time for three single axis slews. This time is determined by using the Euler angles and single axis slew curves of Figure 4. The time to complete the same reorientation using a multi-axis slew is T_3. The ratio T_1/T_3 shows the multi-axis reorientation to be over twice as fast as the conventional slew. This should be expected for at least two reasons:

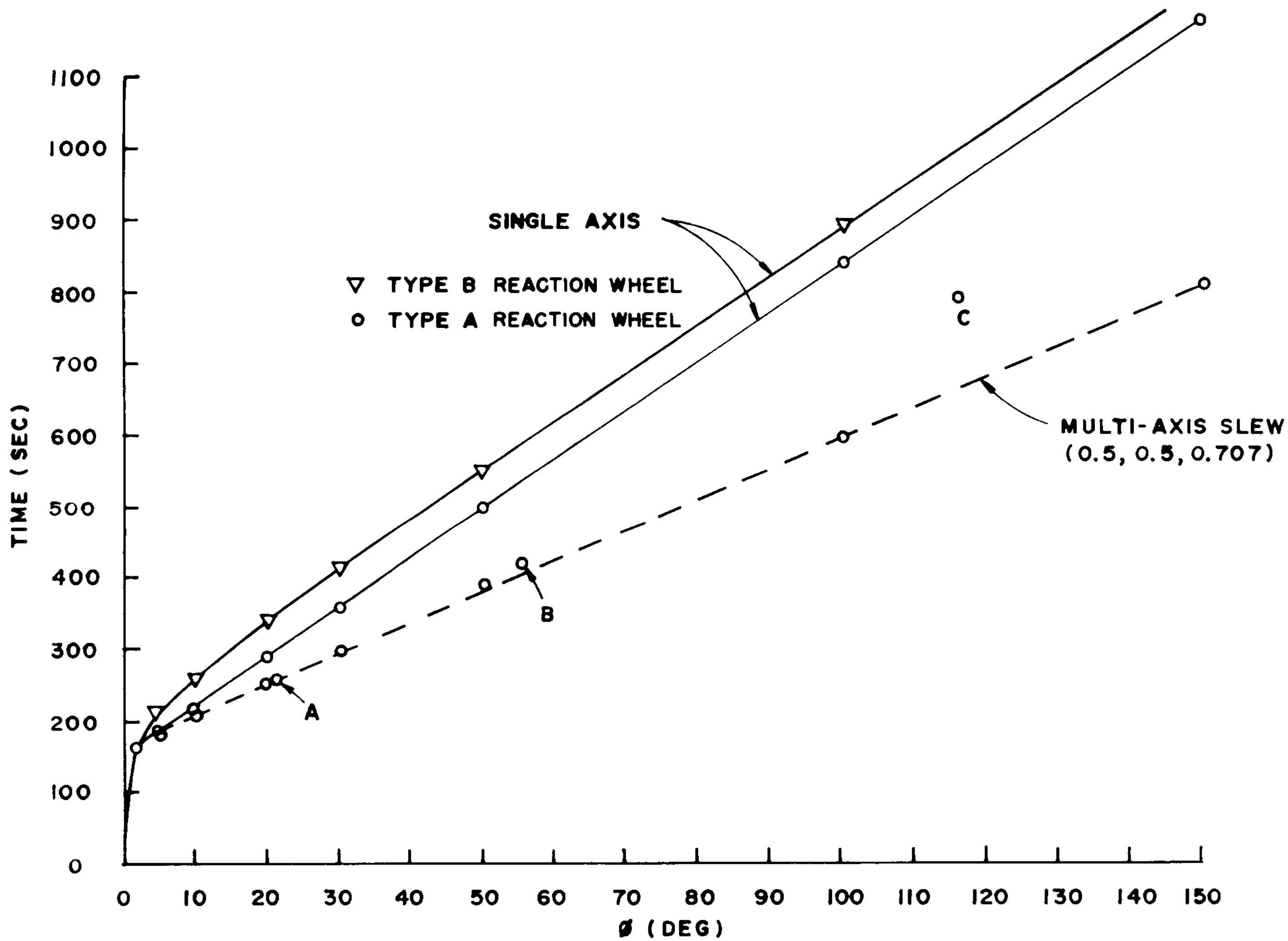

Figure 4–Slew Time

- The end effects which are independent of length of slew occur three times for three slews and only once for a single slew.

- All the wheels contribute simultaneously during a multi-axis slew causing the total rate which the error is being reduced to be higher than for a single axis slew.

The slew time using a type B motor was slightly longer in each case due to the reduced torque available for acceleration and deceleration. The slopes of the linear portions are the same since the maximum wheel speeds are the same.

Accuracy - In a hold or point mode the accuracy is primarily a function of the sensors. When the spacecraft is holding on the experiment telescope or roll axis star tracker there is no problem. When holding on the inertial reference unit (IRU) the primary error will be due to the gyro drift. This is expected to be about 12 arc sec/hr. The desired accuracy determines the update frequency when the spacecraft is holding on the IRU.

There are three sources of error present during a reorientation. They are the attitude updating, gyro drift and the torquing inaccuracy of the gyros.

For a sampling interval of 100 ms the attitude update error may be kept on the order of the gyro quantization level which is 2.4 arc sec.

The reorientation error due to gyro drift will be small because most slews will take less than 20 minutes to complete. The compensated drift during this time is only 3 to 4 arc sec.

There is an error accumulation due to the torque rebalance loop when the gyro is being torqued. The torque rebalance pulses will contain some error in size and shape. This will cause an attitude error build-up over a long slew. The torquing inaccuracy is estimated to be 0.01%. This represents an error of 0.0165° (one arc min) over a 165° slew. Therefore, this is the major attitude error in a large angle reorientation.

The accuracy of the system need not be as high as that required for fine pointing. The objective of the reorientation is the acquisition of a new experiment star. Therefore, the primary factor in determining the required slew accuracy is the field of view of the experiment telescope or the roll star tracker. If the total reorientation error is well within the field of view of the sensor it is feasible to reorient the spacecraft by the method described. With an experiment that has a field of view ±4 arc min (e.g. OAO-Princeton Experiment Package) it does appear feasible to move from fine pointing to fine pointing via a single, large angle, three axis reorientation.

If the experiment field of view is much smaller, it is possible to move to a 6th magnitude or brighter star and lock on this star with the roll star tracker. The IRU is reset and the spacecraft is then moved to the experiment star. This is practical because the average distance from any random point in the sky to the nearest 6th magnitude or brighter star is only about 1.5° (ref. 7). See the section on updating. The error for this magnitude slew will be on the order of 2.5 arc sec. The time for the short slew will be about two minutes. See Figure 4.

Disturbance - The effect of environmental disturbance torques during a slew is to increase the total momentum of the system. The reorientation trajectory with disturbances will in general be different from the undisturbed trajectory. However, the final reference as stored in the computer and IRU is not affected; the system will null at the same reference as the undisturbed system. The time for the reorientation will depend on the magnitude and direction of the disturbances and may be shorter or longer than that required for an undisturbed reorientation. Although an extensive torque disturbance analysis was not conducted some computer runs were made with disturbance torques. No gross changes in system performance were noted. The additional time required for one 165° slew was on the order of one minute with disturbances on the order of 0.0001 to 0.0005 Nm (1000 to 5000 dyne-cm).

Canopus Search

As stated previously the time for a slew is usually an important criterion. It is, however, not the only one. For a Canopus search maintenance of a constant slew is more important.

In the sun, a Canopus search by slewing around the sun line is easily accomplished. Position control with respect to the sun line is maintained by the sun tracker. The roll and yaw wheels are biased as a function of the angle between the sun line and roll axis. See Figure 5. Due to the time of year, orbit inclination, etc. it may not be possible or advantageous to do a Canopus search in the sun. It then becomes

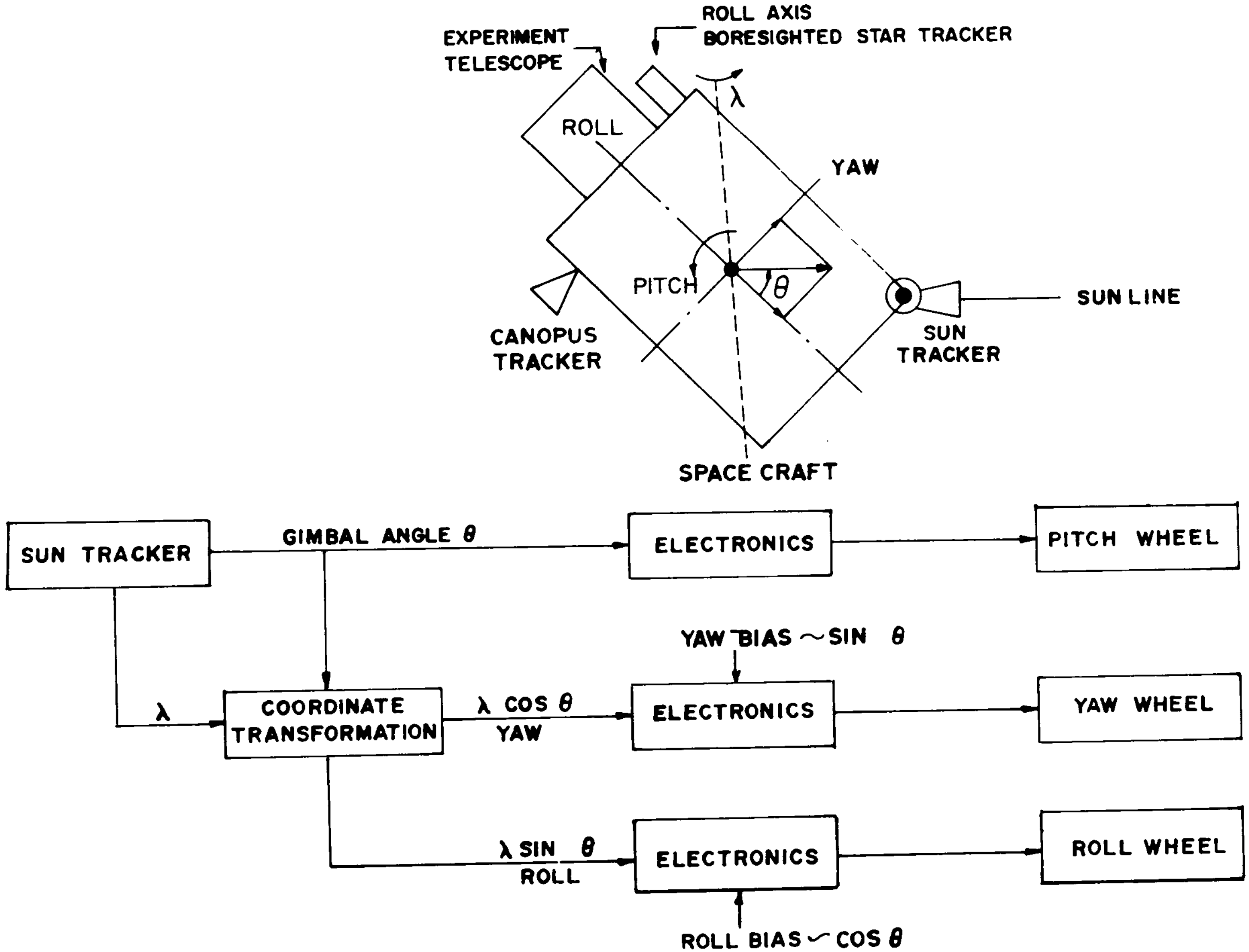

Figure 5–Canopus Search Block Diagram

necessary to slew around an occulted sun line. In order to insure that the fixed Canopus tracker does acquire Canopus, the slew must remain fixed within about 0.2° during the slew. This is equivalent to a three axis reorientation as described previously but with the spacecraft slewing around a fixed eigenvector. This may be accomplished by commanding a reorientation around this eigenvector and limiting the motor torques and maximum momenta by the ratio of the eigenvector components. The body rates are proportional to the eigenvector components and the total velocity vector remains collinear with the slew axis or eigenvector throughout the entire slew. The Canopus tracker trajectory is therefore well defined.

In the normal reorientation the location of the eigenvector is unconstrained. For a significant portion of the slew all three wheels are saturated and the body rate vector is not collinear with the eigenvector. Because of the fixed relationship of the body rates a constrained rate slew will be slower than a normal reorientation. However, if at least one wheel is allowed to saturate, the slew rate will be higher than that for a single axis slew.

A computer simulation confirmed the stability of the eigenvector position for this type of reorientation.

This type of Canopus search is essentially a position reorientation around the occulted sun line. A large slew angle (up to 180°) is commanded around the desired slew axis. When a Canopus presence signal is sensed by the Canopus tracker the IRU is reset and the mode switched to IRU-computer control for settling. After settling, roll and pitch are available from the Canopus tracker and yaw is held by the yaw gyro relative to the sun line. Once in the sun the IRU may be switched from its I-stab mode, reset and retrimmed. The gyro quantization level in the I-stab mode is 38 arc sec. With a 1° FOV Canopus tracker this level should be adequate to insure Canopus comes within the tracker FOV.

Because of the limited maximum slew angle it may be necessary to command a second or at most third slew. However, an attitude reference with respect to the non slew axes is maintained throughout the entire slew.

Updating

The updating of the IRU requires some type of celestial sensor (star and/or sun trackers) to relate the spacecraft axes to the celestial reference. Updating consists of two operations, a reference angle reset and a drift biasing or retrim.

With the boresighted star tracker (BST) along the roll axis, resetting the pitch and yaw gyros is relatively simple. These gyros may be reset any time the spacecraft is holding on the experiment or roll BST.

The updating of the roll axis gyro is slightly more complex because of the tracker locations. A side looking tracker may be used, such as the gimballed sun tracker or an electronically gimballed star tracker. Both methods have advantages and disadvantages with respect to field of view, gimbal reliability, location of stars, etc. These will not be discussed further here.

The relative speed and accuracy of a three axis reorientation makes it possible to reset the roll gyro by another method using only the roll BST. The equations are developed in reference 8. The procedure is as follows:

1. With the spacecraft holding on a known star with experiment or roll BST, the pitch and yaw gyros are reset. The direction of the roll axis in inertial space is now known but some small unknown roll angle exists.

2. Reorient the spacecraft so the roll axis is pointing at a second known star. A two axis pitch/yaw error will exist in the roll BST due to the initial roll error. The initial roll error must not be so large that the second star is out of the field of view of the roll BST at the end of the slew.

3. With the known transformation matrices and errors read from the roll BST compute the roll error and reset the roll gyro. The computation may be done on board or on the ground.

The primary advantage of this method is its independence of a side looking tracker for a roll reset.

Because an extra slew is required with this method, the separation of stars that may be used by the roll BST is important. A study was carried out to determine the distribution of 6th magnitude and brighter stars. Starting at a random point in the sky the distance to the nearest unambigous 6th magnitude or brighter star was determined. Figure 6 is a plot of the data for 1000 random points. An unambiguous star is one that has no other stars of the same magnitude or brighter within 14 arc-min of it. A BST with a 10 arc min square field of view will see only one of these stars. Of 5300 stars (6th magnitude or brighter) about 5000 are unambiguous. The average distance that must be moved is about 1.5°.

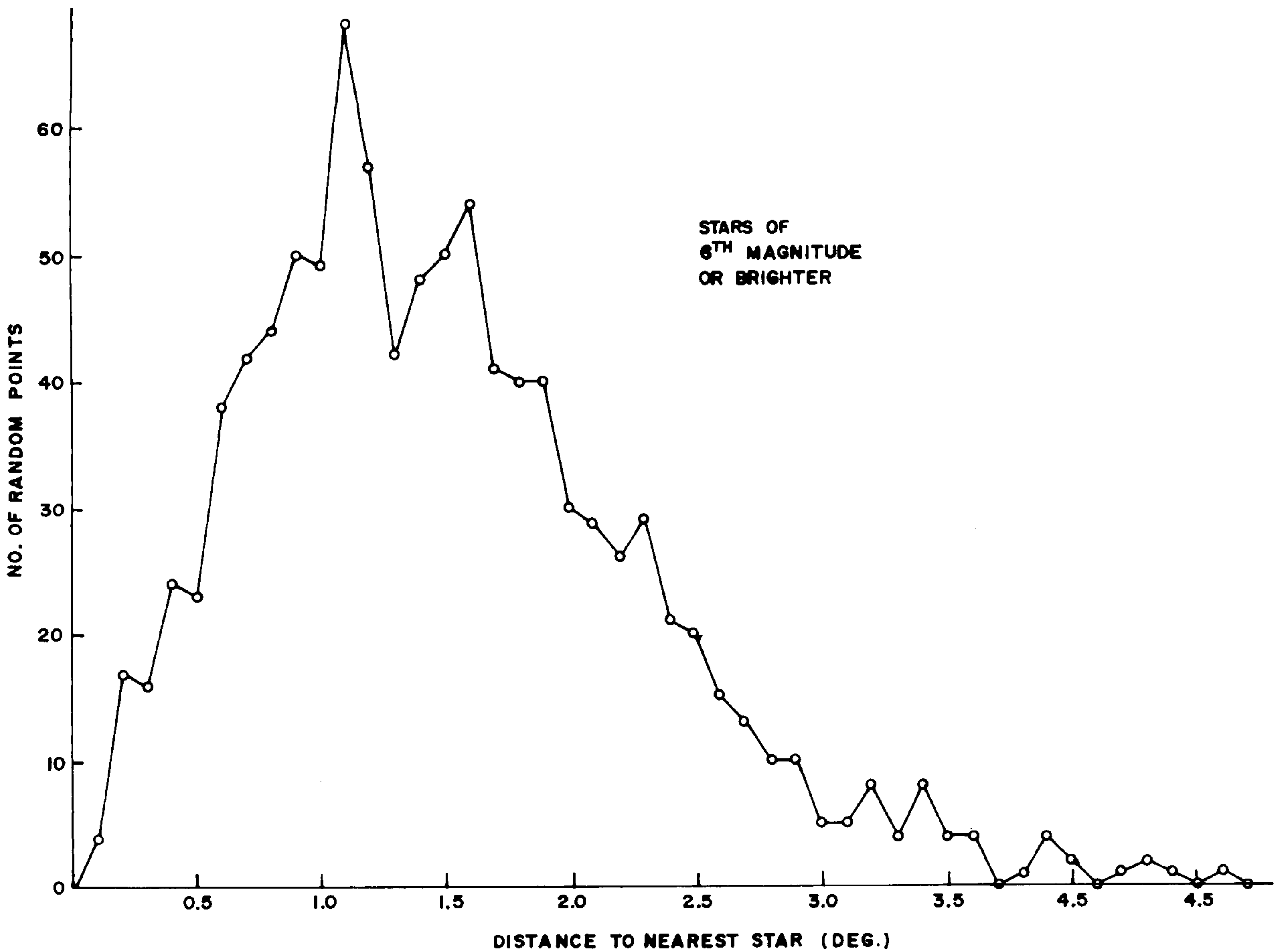

Figure 6–Availability of Reference Stars

The time to complete a slew this short is only about 2 minutes. Therefore, the time for a reset on all three gyros would be short. Even if a reorientation was necessary to acquire a star for the pitch and yaw gyro reset, the total time required for a reset would generally be less than 10 minutes.

A retrim or drift biasing requires a hold on a tracker for the time required to check gyro drift. Retrimming therefore does require a side looking tracker.* Since retrimming would not be required as often as resetting, this method of resetting, independent of the side looking tracker offers systems flexibility.

DISCUSSION

This study was not a general study of various control laws and/or kinematic representations. The choice of Mortensen's control law with the Euler-Rodrigues parameters appeared to be natural for this problem. Further study may show it is advantageous to use Meyer's method where the entire transformation matrix is updated. Basically there is little difference between the two approaches. It may be shown the control laws are very similar.

These types of control laws may be used with other types of sensors. In particular the attitude matrix defined by Meyer as the position output of the system is independent of the sensors used to describe its elements. Star trackers may be used as well as gyros. However it appears that a strap-down inertial reference unit as considered here offers the most advantages for large angle reorientations. The use of gimballed star trackers has two disadvantages. Due to the limited gimbal rotation, large angle reorientations cannot be completed without switching trackers during the slew. This introduces additional constraints on the reorientation maneuver. Secondly, the attitude matrix is more complex because of the presence of the trigonometric functions resulting from the tracker gimbals.

Because of the gyro drift it appears necessary that the spacecraft hold its position during the experimenting time with optical trackers, either the experiment telescope or the roll axis boresighted star tracker. Holding the spacecraft position with an optical tracker requires a

*Roll retrim is theoretically possible with the roll BST by commanding equal and opposite pitch slews separated by the measuring time interval. Using the roll update technique the roll drift during the interval can be calculated.

relatively simple control system and does not require a computer. Therefore, during this time the computer may be freed for processing the experiment data. During a reorientation when no experimenting is being carried out, the computer may be used completely for the control system. Although the computer controlled system may easily handle both the hold and reorientation mode it may be more efficient to use it in this manner.

The in-flight programming capability of the computer may be used to advantage. The control gains may be set initially to carry out the constant eigenvector slew that is required for a Canopus search. Once this is complete the gains may be changed to allow a more rapid slew for normal reorientations.

CONCLUSIONS

1. A computer control system that performs as well during a slew as during pointing is feasible for an advanced OAO. Utilizing an on-board digital computer and strap-down inertial reference unit the system may complete large angle, three axis reorientations with the same control law used for pointing. The computer speed is such that a second order Taylor series may be used to update the spacecraft attitude during a reorientation. The errors due to the attitude updating may be kept on the order of the gyro quantization level. The primary error in a large angle reorientation is due to the gyro torquing inaccuracy. This error, however, is not prohibitive.

2. A general purpose computer now under development appears to meet the computational requirements. This is the "Units" On-Board Digital Processor being developed by the Space Electronics Branch, GSFC.

3. An inertial reference unit being developed for GSFC by MIT appears capable of meeting the sensor requirements.

4. A three axis slew is superior to three single axis slews when time is the reorientation criterion. A three axis slew was shown to be over twice as fast as three single axis slews for the reorientations that were checked. It is also simpler, requiring less commands at the start of the slew.

5. A Canopus search on the dark side of the orbit may be carried out with the same type of control law. Using the proper gain settings a slew may be maintained about a constant, arbitrary axis established by the sun tracker.

6. The inertial reference unit reference may be reset with a single boresighted star tracker. This may be accomplished by successively pointing at two separate stars with the roll axis boresighted star tracker. The complete resetting procedure would take less than 10 minutes.

7. It is possible to use a conventional linear rate and position control when pointing and employ the computer primarily for reorientation. The computer could then be used for data processing when the spacecraft is in a pointing mode.

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APPENDIX A

The spacecraft dynamic and control equations are developed in this appendix. Although Mortensen did not use reaction wheels for control, the method for proving stability closely parallels his development (ref. 1).

Assuming the control torque on the spacecraft is developed by reaction wheels the dynamic equations are (ref. 3).

$$
\begin{aligned}
\dot{H}_x + (I_z - I_y)\,\omega_y\,\omega_z + (\omega_y H_{wz} - \omega_z H_{wy}) &= M_x - \dot{H}_{wx} \\
\dot{H}_y + (I_x - I_z)\,\omega_x\,\omega_z + (\omega_z H_{wx} - \omega_x H_{wz}) &= M_y - \dot{H}_{wy} \\
\dot{H}_z + (I_y - I_x)\,\omega_x\,\omega_y + (\omega_x H_{wy} - \omega_y H_{wx}) &= M_z - \dot{H}_{wz}
\end{aligned}
\tag{A-1}
$$

where

H_x, H_y, H_z – spacecraft momentum about x, y, z (principal axes) less wheel momentum. (Also $H_i = I_i\,\omega_i$; $i = x, y, z$).

I_x, I_y, I_z – spacecraft inertia about x, y, z.

H_{wx}, H_{wy}, H_{wz} – wheel momentum about x, y, z.

M_x, M_y, M_z – external torques about x, y, z.

Assume $M_x = M_y = M_z = 0$. During a slew the external torques are small compared to the control torques and the assumption should be valid. Putting the equation into matrix form:

$$
\begin{bmatrix} \dot{H}_x \\ \dot{H}_y \\ \dot{H}_z \end{bmatrix}
=
\begin{bmatrix} 0 & -H_{Tz}/I_y & H_{Ty}/I_z \\ H_{Tz}/I_x & 0 & -H_{Tx}/I_z \\ -H_{Ty}/I_x & H_{Tx}/I_y & 0 \end{bmatrix}
\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}
-
\begin{bmatrix} \dot{H}_{wx} \\ \dot{H}_{wy} \\ \dot{H}_{wz} \end{bmatrix}
\tag{A-2}
$$

where $H_{Ti} = H_i + H_{wi}$; $i = x, y, z$.

Combining equation A-2 with the kinematic equation 2:

$$\begin{bmatrix} \dot{H}_x \\ \dot{H}_y \\ \dot{H}_z \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} 0 & -H_{Tz}/I_y & H_{Ty}/I_z & 0 & 0 & 0 \\ H_{Tz}/I_x & 0 & -H_{Tx}/I_z & 0 & 0 & 0 \\ -H_{Ty}/I_x & H_{Tx}/I_y & 0 & 0 & 0 & 0 \\ \left(\frac{1+\alpha^2}{2\,I_x}\right) & \left(\frac{\alpha\beta}{2\,I_y}\right) & \left(\frac{\alpha\gamma}{2\,I_z}\right) & 0 & H_z/2I_z & -H_y/2I_y \\ \left(\frac{\alpha\beta}{2\,I_x}\right) & \left(\frac{1+\beta^2}{2\,I_y}\right) & \left(\frac{\beta\gamma}{2\,I_z}\right) & -H_z/2I_z & 0 & H_x/2I_x \\ \left(\frac{\alpha\gamma}{2\,I_x}\right) & \left(\frac{\beta\gamma}{2\,I_y}\right) & \left(\frac{1+\gamma^2}{2\,I_z}\right) & H_y/2I_y & -H_x/2I_x & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \\ \alpha \\ \beta \\ \gamma \end{bmatrix} - \begin{bmatrix} \dot{H}_{wx} \\ \dot{H}_{wy} \\ \dot{H}_{wz} \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{(A-3)}$$

Assuming a control law of the type shown in equations 3 and a reaction wheel with torque-speed curve as shown in Figure 2, Type A, the control equations are:

$$\begin{aligned} \dot{H}_{wx} &= k_x H_x + k_p\left(\frac{1+\alpha^2}{2}\right)\alpha + k_p\left(\frac{\beta\alpha}{2}\right)\beta + k_p\left(\frac{\alpha\gamma}{2}\right)\gamma \\ \dot{H}_{wy} &= k_y H_y + k_p\left(\frac{\alpha\beta}{2}\right)\alpha + k_p\left(\frac{1+\beta^2}{2}\right)\beta + k_p\left(\frac{\beta\gamma}{2}\right)\gamma \qquad \text{(A-4)} \\ \dot{H}_{wz} &= k_z H_z + k_p\left(\frac{\alpha\gamma}{2}\right)\alpha + k_p\left(\frac{\beta\gamma}{2}\right)\beta + k_p\left(\frac{1+\gamma^2}{2}\right)\gamma \end{aligned}$$

where k_x, k_y, k_z – rate gains

k_p – position gain.

The total system is ninth order requiring nine state variables for a complete description of the trajectory in state space. The state variables are the three body momenta, three parameters specifying body position and the three total momenta. The equilibrium position of the spacecraft may be shown to be independent of the system total momenta. Thus there is a three dimensional subspace every point of which is an equilibrium point for the spacecraft. Therefore, for spacecraft stability analysis the six dimensional vector representing the body position and rate may be used in place of the nine dimensional system vector. The total momentum in this formulation may be considered a time varying parameter.

$$\begin{bmatrix} \dot{H}_x \\ \dot{H}_y \\ \dot{H}_z \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} -k_x & -H_{Tz}/I_y & H_{Ty}/I_z & -k_p\left(\frac{1+\alpha^2}{2}\right) & -k_p\left(\frac{\beta\alpha}{2}\right) & -k_p\left(\frac{\alpha\gamma}{2}\right) \\ H_{Tz}/I_x & -k_y & -H_{Tx}/I_z & -k_p\left(\frac{\alpha\beta}{2}\right) & -k_p\left(\frac{1+\beta^2}{2}\right) & -k_p\left(\frac{\beta\gamma}{2}\right) \\ -H_{Ty}/I_x & H_{Tx}/I_y & -k_z & -k_p\left(\frac{\alpha\gamma}{2}\right) & -k_p\left(\frac{\beta\gamma}{2}\right) & -k_p\left(\frac{1+\gamma^2}{2}\right) \\ \frac{1+\alpha^2}{2I_x} & \frac{\alpha\beta}{2I_y} & \frac{\alpha\gamma}{2I_z} & 0 & H_z/2I_z & -H_y/2I_y \\ \frac{\alpha\beta}{2I_x} & \frac{1+\beta^2}{2I_y} & \frac{\beta\gamma}{2I_z} & -H_z/2I_z & 0 & H_x/2I_x \\ \frac{\alpha\gamma}{2I_x} & \frac{\beta\gamma}{2I_y} & \frac{1+\gamma^2}{2I_z} & H_y/2I_y & -H_x/2I_x & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \\ \alpha \\ \beta \\ \gamma \end{bmatrix} \tag{A-5}$$

This is in the form

$$\dot{\overline{X}} = \overline{\overline{F}}(X)\,\overline{X} \tag{A-6}$$

where $\overline{X}$ – $[H_x, H_y, H_z, \alpha, \beta, \gamma]^T$

$\overline{\overline{F}}(X)$ – nonlinear system matrix.

Define the matrices $\bar{\bar{K}}$ and $\bar{\bar{R}}$ as follows

$$\bar{\bar{K}} = \text{diag}\ [-k_x, -k_y, -k_z, 0, 0, 0]$$
$$\bar{\bar{R}} = \bar{\bar{F}} - \bar{\bar{K}} \tag{A-7}$$

Choose as a candidate V function the quadratic form

$$V = \bar{X}^T \bar{\bar{G}} \bar{X} \tag{A-8}$$

where

$$\bar{\bar{G}} = \text{diag.} \left[\frac{1}{I_x}, \frac{1}{I_y}, \frac{1}{I_z}, k_p, k_p, k_p\right]$$

Since $\bar{\bar{G}}$ is a constant it follows that

$$\dot{V} = \dot{\bar{X}}^T \bar{\bar{G}} \bar{X} + \bar{X}^T \bar{\bar{G}} \dot{\bar{X}} \tag{A-9}$$

Substituting equations A-7 into A-6 and then into A-9 it (A-9) becomes

$$\dot{V} = \bar{X}^T \bar{\bar{R}}^T \bar{\bar{G}} \bar{X} + \bar{X}^T \bar{\bar{K}}^T \bar{\bar{G}} \bar{X} + \bar{X}^T \bar{\bar{G}} \bar{\bar{R}} \bar{X} + \bar{X}^T \bar{\bar{G}} \bar{\bar{K}} \bar{X} \tag{A-10}$$

Note that $\bar{\bar{G}}\bar{\bar{R}}$ is skew symmetric and that $\bar{\bar{G}} = \bar{\bar{G}}^T$, therefore

$$\bar{\bar{G}} \bar{\bar{R}} = -(\bar{\bar{G}} \bar{\bar{R}})^T = -\bar{\bar{R}}^T \bar{\bar{G}}^T = -\bar{\bar{R}}^T \bar{\bar{G}} \tag{A-11}$$

This reduces equation A-10 to

$$\dot{V} = \bar{X}^T \bar{\bar{K}}^T \bar{\bar{G}} \bar{X} + \bar{X}^T \bar{\bar{G}} \bar{\bar{K}} \bar{X} \tag{A-12}$$

The following also holds

$$\bar{\bar{K}}^T \bar{\bar{G}} = \bar{\bar{K}}^T \bar{\bar{G}}^T = (\bar{\bar{G}} \bar{\bar{K}})^T = \bar{\bar{G}} \bar{\bar{K}} \tag{A-13}$$

Therefore

$$\dot{V} = 2\,\bar{X}^T\,\bar{\bar{G}}\,\bar{\bar{K}}\,\bar{X}$$
$$= -2\left(\frac{k_x}{I_x}\,H_x^2 + \frac{k_y}{I_y}\,H_y^2 + \frac{k_z}{I_z}\,H_z^2\right) \tag{A-14}$$

Thus if

$$k_x > 0,\ k_y > 0,\ k_z > 0 \tag{A-15}$$

then $\dot{V}$ is negative semidefinite. Since

$$V = \frac{H_x^2}{I_x} + \frac{H_y^2}{I_y} + \frac{H_z^2}{I_z} + k_p\,\alpha^2 + k_p\,\beta^2 + k_p\,\gamma^2 \tag{A-16}$$

is positive definite ($k_p > 0$) the origin is stable. It may be shown that the origin is not only stable but asymptotically stable, i.e., $V \to 0$ as $t \to \infty$.

Since following relations are valid (ref. 3)

$$\alpha = C_x \tan\phi/2$$
$$\beta = C_y \tan\phi/2 \tag{A-17}$$
$$\gamma = C_z \tan\phi/2$$

where

C_x, C_y, C_z – the components of the +1 eigenvector of the orthogonal transformation parameterized by α, β, γ

ϕ – the angle around the +1 eigenvector that the body frame is rotated from the inertial frame.

equation A-16 may be written

$$V = \frac{H_x^2}{I_x} + \frac{H_y^2}{I_y} + \frac{H_z^2}{I_z} + k_p \tan^2\phi/2 \tag{A-18}$$

Therefore, for V to be bounded it is necessary that the body angular momentum be bounded and ϕ must be less than π. The total system momentum must be within the total capacity of the wheels. It will be stored in the wheels at the completion of the reorientation. The latter constraint ($\phi < \pi$) is not a practical restriction on the system since it is easily satisfied. With the boundedness of V established it may be concluded that V is a valid Lyapunov function and the stability conclusions are valid.

Asymptotic stability in the large has not yet been proven in general when a type B reaction wheel is used. However two important special cases may be shown to be asymptotically stable. One is the case in which the total system momentum is zero. The other is the linearized system near the origin.

When a type B reaction wheel is used the additional back emf terms $-H_{wi}/T_m$, $i = x, y, z$ are added to the respective control equations (A-4). When the total system momentum is zero $H_i = -H_{wi}$. Thus the system is reduced to sixth order and it is easily proven stable in a manner analagous to that used in the preceding proof.

When the system equations are linearized near the origin the result is three, uncoupled, linear, second order systems. These are easily proven stable by conventional methods. Any stored momentum represents a shift to a new origin.

APPENDIX B

The equations for updating the parameters α, β, γ using a second order Taylor series are the following:

$$\alpha_{n+1} = \alpha_n + \frac{1}{2}\,[(1+\alpha_n^2)\,\Delta_x + (\alpha_n\beta_n - \gamma_n)\,\Delta_y$$

$$+ (\alpha_n\gamma_n + \beta_n)\,\Delta_z] + \frac{1}{4}\,[\alpha_n\,(1+\alpha_n^2)\,\Delta_x^2$$

$$+ \beta_n\,(\alpha_n\beta_n - \gamma_n)\,\Delta_y^2 + \gamma_n\,(\alpha_n\gamma_n + \beta_n)\,\Delta_z^2$$

$$+ (2\,\alpha_n^2\,\beta_n + \beta_n - \alpha_n\,\gamma_n)\,\Delta_x\Delta_y$$

$$+ (2\,\alpha_n^2\gamma_n + \gamma_n + \alpha_n\,\beta_n)\,\Delta_x\Delta_z$$

$$+ (2\,\alpha_n\,\beta_n\,\gamma_n + \beta_n^2 - \gamma_n^2)\,\Delta_y\Delta_z]$$

$$\beta_{n+1} = \beta_n + \frac{1}{2}\,[(\alpha_n\beta_n + \gamma_n)\,\Delta_x + (1+\beta_n^2)\,\Delta_y$$

$$+ (\beta_n\,\gamma_n - \alpha_n)\,\Delta_z] + \frac{1}{4}\,[\alpha_n\,(\alpha_n\beta_n + \gamma_n)\,\Delta_x^2$$

$$+ \beta_n\,(1+\beta_n^2)\,\Delta_y^2 + \gamma_n\,(\beta_n\,\gamma_n - \alpha_n)\,\Delta_z^2$$

$$+ (2\,\alpha_n\,\beta_n^2 + \alpha_n + \beta_n\,\gamma_n)\,\Delta_x\,\Delta_y$$

$$+ (2\,\alpha_n\,\beta_n\,\gamma_n - \alpha_n^2 + \gamma_n^2)\,\Delta_x\,\Delta_z$$

$$+ (2\,\beta_n^2\,\gamma_n + \gamma_n - \alpha_n\,\beta_n)\,\Delta_y\,\Delta_z]$$

$$\gamma_{n+1} = \gamma_n + \frac{1}{2} [(\alpha_n \gamma_n - \beta_n) \Delta_x + (\beta_n \gamma_n + \alpha_n) \Delta_y$$

$$+ (1 + \gamma_n^2) \Delta_z] + \frac{1}{4} [\alpha_n (\alpha_n \gamma_n - \beta_n) \Delta_x^2$$

$$+ \beta_n (\beta_n \gamma_n + \alpha_n) \Delta_y^2 + \gamma_n (1 + \gamma_n^2) \Delta_z^2$$

$$+ (2 \alpha_n \beta_n \gamma_n + \alpha_n^2 - \beta_n^2) \Delta_x \Delta_y$$

$$+ (2 \alpha_n \gamma_n^2 + \alpha_n - \beta_n \gamma_n) \Delta_x \Delta_y$$

$$+ (2 \beta_n \gamma_n^2 + \beta_n + \alpha_n \gamma_n) \Delta_y \Delta_z]$$