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Diffusion of Solar Cosmic Rays and the Power Spectrum of the Interplanetary Magnetic Field *

by

K.V.S.K. Nathan and J. A. Van Allen
Department of Physics and Astronomy
University of Iowa, Iowa City, Iowa 52240

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ABSTRACT

Van Allen and Krimigis [1965] find that solar electrons diffuse through the interplanetary medium at a rate which is similar to that of protons of the same velocity, despite the disparity by a factor of 2×10^3 in their respective magnetic rigidities, and they suggest that this fact serves to extend knowledge of the structure of the interplanetary magnetic field to much smaller scale than has been measured [Coleman, 1966]. This suggestion is developed quantitatively in the framework of Roelof's [1965] theory. It is found that the power spectrum of the interplanetary magnetic field varies inversely as the square of the frequency f in the range $2.7 \times 10^{-4} < f < 0.5$ cps.

INTRODUCTION

The propagation of solar cosmic rays through the interplanetary medium has been described--with a fair measure of success [Parker, 1963] [Krimigis, 1965]--in terms of a diffusion process characterized by an equation of the form

$$\frac{\partial n}{\partial t} = \text{div}\,(D \text{ grad } n) \tag{1}$$

where n is the number density of particles and D is the diffusion coefficient, which may be a function of position. Observed values of the time dependence of the number density have been fitted into solutions of (1) by various authors to extract values of D. One of the striking results of such work is that 75 MeV protons and 40 keV electrons exhibit diffusion-type time histories characterized by similar values of the diffusion coefficient [Van Allen and Krimigis, 1965]. The two classes of particles have the same velocity v but the magnetic rigidity R of the protons is 2×10^3 times as great as that of the electrons.

If diffusion is due to irregularities in the interplanetary magnetic field, as is commonly supposed, it is clear that the foregoing fact makes possible an extension of knowledge of the spectrum of irregularities to much smaller scale than has been observed thus far [Coleman, 1966].

DIFFUSION CONSTANTS AND THE POWER SPECTRUM OF THE INTERPLANETARY MAGNETIC FIELD

Magnetic field lines in interplanetary space trace in rough outline [Wilcox and Ness, 1965] the Archimedian spiral form predicted by Parker [1958]. Superimposed on this average field are random irregularities [Coleman, 1966]. Solar protons and electrons describe simple helical paths around smooth segments of the lines of force. But they are deflected from such trajectories by magnetic irregularities whose dimensions are of the order of their respective gyro-radii [Parker, 1964]. It is the cumulative effect of a large number of such random deflections that results in a diffusion-like process. The diffusion coefficient D can therefore be related to the spectrum of magnetic irregularities [Roelof, 1965] [Gloeckler and Jokipii, 1966].

Roelof [1965] obtains the relation

$$D = \frac{v^2 T}{6} \tag{2}$$

where v is the velocity of the particle and T is a measure of the relaxation time in velocity space. T is given by

$$T = \left(\frac{mc}{q}\right)^2 \frac{2v}{\pi P(k_o)} \tag{3}$$

where

$$k_o = \frac{qB_o}{mvc}, \tag{4}$$

m and q are the mass and charge of the particle, c is the velocity of light, B_o is the mean magnetic field, and $P(k_o)$ is the power spectrum of the field fluctuations at the wave number k_o defined by (4). It is seen that k_o is the reciprocal of the gyro radius of the particle, and that T depends explicitly on the velocity v and also implicitly on it through the wave number k_o.

We will assume that the magnetic field is not explicitly a function of time but that the time dependence arises implicitly from spatial variations of the magnetic field (fixed in the solar wind) as it is convected past a given, fixed point by the solar wind streaming with velocity V. Then the power p(f) at any frequency f is related to P(k) by

$$p(f) = \frac{2\pi}{V} P(k) \qquad (5)$$

since

$$f = \frac{V}{2\pi} k . \qquad (6)$$

Combining equations (2) through (5):

$$D = \left(\frac{mc}{q}\right)^2 \frac{2v^3}{3V} p(f_o) \qquad (7)$$

and

$$f_o = \frac{VqB_o}{2\pi mvc} . \qquad (8)$$

INTERPRETATION OF OBSERVATIONAL DATA

The observations of Van Allen and Krimigis [1965] require that

$$D\ (40\ \text{keV electron}) \approx D\ (75\ \text{MeV proton})$$

or by equations (7) and (8), using $B_o = 5 \times 10^{-5}$ gauss and $V = 400$ km/sec, that

$$p\ (0.50\ \text{cps}) = \left(\frac{m_e}{m_p}\right)^2 p\ (2.7 \times 10^{-4}\ \text{cps}) \qquad (9)$$

where m_e and m_p are the masses of the electron and proton, respectively, and cps denotes the frequency in cycles per second.

Thus,

$$p\ (0.50\ \text{cps}) = 0.30 \times 10^{-6}\ p\ (2.7 \times 10^{-4}\ \text{cps})\ . \qquad (10)$$

Coleman observes that

$$p(f) \propto \frac{1}{f} \quad \text{for} \quad 1.3 \times 10^{-5} < f < 6 \times 10^{-4}\ \text{cps} \qquad (11)$$

and falls off more rapidly with increasing frequency above 6×10^{-4} cps.

Our results (9) and (10) apply strictly only to the two specified, widely-different frequencies. However, in view of Coleman's observations and the physical implausibility of a power spectrum which is other than a smooth, monotonic function of frequency, we offer the tentative generalization that

$$p(f) \propto \frac{1}{f^2} \text{ for } 2.7 \times 10^{-4} < f < 0.5 \text{ cps} . \qquad (12)$$

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