

A THEORY OF GEOMAGNETIC MICROPULSATIONS

By

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ABSTRACT

A theory of one of several possible mechanisms responsible for micropulsations is presented. A source mechanism at the boundary of the magnetosphere and its strong effect on the bandwidth and relatively small effect on the spectrum are discussed. The important role of the transmission path, including that in the outer magnetosphere in determining the spectrum of micropulsation signals received at the surface of the earth, is analyzed. The crucial role of the slightest variation of the path length in the outer magnetosphere (due to variations in the solar wind) on the spectrum is discussed. Sharp peaks in the amplitude spectrum are predicted at time periods $T \sim 935, 311, 134, 103, 71, 39, 14, 9, 7$ and 4 . On the other hand a crude analysis of power spectral density reveals spectral peaks at $T \sim 2.5, 6, 11$ and especially 20 with a slope of approximately 6db/octave at large time periods only.

1. INTRODUCTION

The first evidence of geomagnetic micropulsations was probably reported by Birkeland (1913). Since then single station observations have been reported by Rolf (1931), Harang (1936), Sucksdorff (1936), Holmberg (1953), Duffus and Shand (1958), Veldkamp (1960), Campbell (1959, 1960a, 1960b), Jungmeister (1960), Tepley (1961) and McPherron and Ward (1965). Extensive data from several stations and their analyses have been made available by Wright, Shand and Duffus (1959), Benioff (1960), Troitskaya (1961), Duncan (1961), Tepley and Wentworth (1962) and Smith (1964). Jacobs and Sinno (1960) present an analysis of the data obtained during the IGY from a world-wide network of stations. Several review articles by Aarons (1960), Kato (1962), Campbell (1963) and Heirtzler (1964) have appeared.

Several theoretical models have been proposed to explain the phenomenon. Scholte (1960) studies the propagation of waves in a compressible plasma and shows that the only mode which propagates without geometrical attenuation is that of vorticity about a line of force. This is also found by MacDonald (1961). Scholte believes that this mode is responsible for micropulsations and argues that the fundamental wavelength is four times the distance between the earth's surface and exosphere. This is the fundamental wavelength of the free vibration of a line of force of the earth's magnetic field, considered to be fixed at the earth's surface (a good conductor) and free at the exospheric end. This will be a good

estimate, if the source mechanism in the exosphere does not play any role in determining the frequency range of disturbance and the transmission path is homogeneous. Hydromagnetic waves of frequencies covering most of the micropulsation spectrum are very strongly attenuated in the lower ionosphere (Dessler, 1959; Francis and Karplus, 1960; Akasofu, 1960; Watanabe, 1962). Presumably, the absorption and reflection may produce secondary sources, the waves from which may be received at the surface of the earth (Akasofu, 1960). Bomke (1962) suspects that the 1-cps micropulsations are electric-dipole radiation from the ionosphere and are connected with the absorption of auroral particles there. According to him, micropulsations in the range 0.1 to .01 cps are produced by magnetic-dipole radiation and are the result of absorption and reflection of hydromagnetic waves in the lower ionosphere. Campbell and Matsushita (1962) surmise that micropulsation storms can be attributed to an ionospheric current system set up at times of primary electron bombardment from solar plasma. Jacobs and Watanabe (1962) considered the transmission of hydromagnetic waves in the lower exosphere up to a height of 2000 km and demonstrated for the first time the possibility of pronounced resonances in the transmission. These resonances are in the Pc1 and Pc2 ranges. Prince and Bostick (1964) considered the transmission through the path from the outer boundary of the geomagnetic field to the surface of the earth. Greifinger and Greifinger (1965) have discussed the transmission problem in the ionosphere up to a height of 500 km and found a resonance in the Pc4 range. Extending their analysis to about 2000 km, Field and Greifinger (1965) found resonances in Pc1 and Pc2 ranges.

In all the work mentioned above the problem of a sustained generation mechanism of micropulsations is not considered. Also, Jacobs and Watanabe (1962) incorrectly imply that signal amplification is tantamount to power amplification. Signal amplification is accomplished essentially by phase shift through the transmission path, while it is evident that a passive transmission medium cannot provide any power amplification. The power involved in micropulsations, including the loss in lower ionosphere, must be provided by the source mechanism. A source mechanism at the boundary of the magnetosphere is proposed in this paper. It is shown that the effect of long inhomogeneous transmission path from the ionosphere to the boundary of the magnetosphere which has been neglected by Jacobs and Watanabe (1962), Greifinger and Greifinger (1965) and Field and Greifinger (1965) is of profound importance in determining the spectrum of micropulsations. Though Prince and Bostick (1964) considered the entire transmission path, they as well as the other authors mentioned above, did not discuss the crucial effect of variation of lengths of the various parts of the path. These variations, especially in the outer magnetosphere, due to the variation in the solar wind conditions, can be always expected. It is shown here that even the slightest variation in the length of the path in the outer magnetosphere can completely annihilate resonances of the lower exosphere found by the authors mentioned above. Longer period resonances with larger amplitudes, originating in the outer magnetosphere have also been found. These have not been found by Greifinger and Greifinger (1965) and Field and Greifinger (1965). The occurrence frequency is thought to be related to the sensitivity of the transmission on the path length in the outer magnetosphere. Ration-

alized MKS units are used throughout the paper.

It should also be mentioned that micropulsations in the range 1-2 cps have been explained to be hydromagnetic emissions by electron bunches oscillating along field lines in the exosphere. (Wentworth and Tepley, 1962). Yanagihara (1963) discusses pulsations in the range 30 to .1 cps in the auroral zones. Most of these higher frequency oscillations fall in the Pcl range and beyond. Those which have frequencies higher than Pcl, should be called hydromagnetic emissions and may have their origin along the lines suggested by Wentworth and Tepley (1962) and Yanagihara (1963). The theory presented here excludes the consideration of these higher frequency emissions.

Lastly, spatial coherence of micropulsations and the perturbing effects of geological structures and land-sea boundaries have been investigated by Duffus, Shand and Wright (1962) and Duffus, Kinneer, Shand and Wright (1962). Some symmetry relations in magnetic oscillations at a pair of conjugate points are explained by Sugiura and Wilson (1964). Bursts of irregular pulsations in the auroral zone, which appear at the onset of a sharp negative bay are explained in terms of a hydromagnetic instability of an electron beam passing through magnetospheric plasma by Nishida (1964). A thorough discussion of the phase characteristics of micropulsations and their implications has been presented by Herron (1966). Some characteristics of Pcl micropulsations have been explained in terms of the effects on its propagation of a duct in the F region of the ionosphere (Manchester, 1966).

2. GENERATION OF MICROPULSATIONS

2.1 Source Mechanism

The source mechanism at the interface between solar wind and the magnetosphere discussed here was considered in rudimentary form by Sen (1962). In this reference some work by Parker (1958) which may be relevant to the source mechanism and Dungey's (1955) work are compared and contrasted with that of the author. It has been shown by Sen (1965) that substantial portions of the magnetospheric boundary are unstable under all conditions of solar wind. It is suggested here that the hydromagnetic surface waves generated by this instability may leave the magnetospheric boundary along the lines of force of the geomagnetic field and travel toward the surface of the earth as micropulsations.

Let us discuss in crude physical terms the consequence of such an instability. The growth of the unstable surface waves will ultimately be stopped by non-linear interactions. It is reasonable to assume that the resulting finite amplitude waves will not be restricted to the regions of the magnetospheric boundary which gave rise to the instability, because the various modes of wave motion on the curved bounded surface will be coupled with one another. That is, the regions of the magnetospheric boundary designated as "stable" and "unstable" by Sen (1965) will communicate with each other through mode coupling and as a result the entire boundary will be covered with non-linear surface waves. The mode which may reasonably easily leave the boundary surface

and enter the magnetosphere, is the one which propagates parallel to the magnetic field. This is because magnetic lines of force can guide these waves along the surface, around the bend and then into the magnetosphere toward the surface of the earth. In what follows we will show through a simplified model that some characteristic frequencies are associated with this mode.

2.2 Bandwidth of Mircopulsations at Source

The shape of the dayside of the magnetospheric boundary is not far from being hemispherical. For our purpose it is sufficient to consider it hemispherical with geocentric radius R , which is also the equatorial penetration distance of the solar wind. This is calculated as usual by equating the solar wind pressure and the geomagnetic field pressure. Hence we have

$$R = [2B_0^2 / (\mu_0 \rho V_s^2)]^{1/6} r_0 \quad (1)$$

where ρ is the density and V_s is the undisturbed speed of the solar wind, B_0 is the surface value of the earth's dipole magnetic field at the equator (equal to $.315\gamma$), r_0 is the radius of the earth and μ_0 is the permeability of free space. The nightside of the boundary is a long tail in the anti-solar direction. Let us consider this as a conical surface of half angle of 15° , flaring out from the circular base of radius R on the meridional plane perpendicular to the earth-sun line. The geometry of this model is shown in Fig. 1 (Sen, 1965). It can be deduced (Sen, 1965) that the dispersion relation for the waves on the boundary is

$$\begin{aligned}
& c^4 - 2c^3 V_0 \cos \psi' + c^2 (V_0^2 \cos^2 \psi' - 2V_a^2) + 2cV_a^2 V_0 \cos \psi' \\
& + V_a^4 [2 - \cos^2(\phi - \psi')] \cos^2(\phi - \psi') - V_a^2 V_0^2 \cos^2 \psi' = 0. \quad (2)
\end{aligned}$$

Where $c = \omega/k$ is the phase velocity, V_0 is the tangential component of solar wind velocity, V_a is the Alfvén speed, ϕ is the angle between V_0 and the magnetic field and ψ' is the angle between V_0 and the direction of propagation of waves. For propagation parallel to the magnetic field $\psi' = \phi$ and after discarding two trivial roots for this case, we find

$$c = V_0 \cos \phi \pm V_a \quad (3)$$

In Eq. (3), V_0 should be expressed in terms of the undisturbed solar wind speed V_s . We do this by using spherical co-ordinates r, θ, ψ where ψ is measured from the earth-sun line as shown in Fig. 1. By assuming specular reflection of the solar corpuscles from the magnetospheric boundary, one finds

$$V_0 = V_s \sin \alpha = V_s (1 - \sin^2 \theta \cos^2 \psi)^{\frac{1}{2}}, \quad (4)$$

where α is defined in Fig. 1.

Now from geometrical considerations ϕ can be expressed as

$$\sin \phi = \sin \psi / \sin \alpha = \sin \psi [1 - \sin^2 \theta \cos^2 \psi]^{-1} \quad (5)$$

Furthermore, from the fact that the solar wind pressure equals the geomagnetic field pressure on the boundary, it follows that

$$\mu_0 H^2 = 2\rho V_s^2 \sin^2 \theta$$

Hence

$$V_a = 1.42 V_s \sin \theta \quad (6)$$

Substituting Eqs. (4), (5) and (6) in Eq. (3), we find

$$C^+ = V_s (\cos \theta \cos \psi + 1.42 \sin \theta) \quad (7)$$

$$C^- = V_s (-\cos \theta \cos \psi + 1.42 \sin \theta)$$

where C^+ and C^- denote the phase velocity parallel and anti-parallel, respectively, to the direction of streaming of solar wind on the boundary surface. The farthest point (either north or south of the equator on the boundary surface) the waves can possibly travel to, and be reflected, is the point where C^- vanishes. Denoting the latitude of this point on the boundary surface by θ'_c , we find

$$\theta'_c = \tan^{-1}(1.42/\cos \psi) \quad (8)$$

where $\theta' = 90^\circ - \theta$ is the latitude. The average velocity between these two extreme points is found as

$$c_{av} = \int_{-\theta'_c}^0 c^- d\theta' + \int_0^{\theta'_c} c^+ d\theta'$$

Substituting Eqs. (7) in this and integrating, one obtains

$$c_{av} = 1.42 v_s \sin \theta'_c / \theta'_c$$

An upper bound on the period T of fundamental mode is obtained by considering $\pm \theta'_c$ as the two reflection points of the waves. This is easily calculated as

$$T = 2(2R\theta'_c)/c_{av} = 2.82 R\theta'^2_c / (v_s \sin \theta'_c) \quad (9)$$

In Fig. 2 we show in a polar plot, the domain on the boundary surface where the waves propagating parallel to the geomagnetic field will set up standing surface waves, upon reflection between the points $\pm \theta'_c$. The solid heavy curve is the plot of θ'_c (plotted as latitudinal angle) vs ψ (plotted as local time) as given by Eq. (8). The light radial lines of constant ψ (local time) denote the geomagnetic field lines on the surface and the numbers next to them indicate the period of fundamental mode calculated from Eq. (9). The values $v_s = 400$ km/sec and $R = 10.3 \times 6.4 \times 10^3$ km corresponding to the quiet day conditions of the solar wind have been chosen.

Another upper bound on the period of fundamental mode is obtained by the following argument. The surface waves on the boundary discussed here must leave the interface along the geomagnetic field lines and propagate as Alfvén waves towards the earth. The surface waves will be trans-

formed into Alfvén waves in the regions where the geomagnetic lines of force leave the interface between the solar wind and magnetosphere. These regions will produce reflections of the surface waves, as the velocity of the surface waves is different from that of the Alfvén waves near the boundary. Considering the path length between these reflection points to be of the order of the radius of curvature of the boundary surface, we find the latitude of either reflection point as

$$|\theta'_{\text{reflection}}| \sim .5 \text{ radian.}$$

Replacing θ'_c in Eq. (9) by the above value, we find another upper bound on T (for quiet day conditions) as 242 sec. This is the least upper bound because the reflections will be far from total reflections. Comparing this number with the values shown in Fig. 2, we conclude that the upper limit on the period of micropulsations, dictated by the source mechanism, probably lies between 250 and 1100 sec.

As the surface waves are hydromagnetic in nature, their wavelengths must be greater than the thermal ion gyro-radius. This latter quantity can be taken to be several hundred km in the vicinity of the magnetospheric boundary. The corresponding time period can be calculated to be approximately $\frac{1}{2}$ to 1 sec for typical values of the average surface wave velocity obtained from the formula given in Sec. 2.2. Hence this can be taken to be a lower bound on the time period of micropulsations. Using this lower bound and the upper bound derived above, one can estimate the bandwidth of micropulsation signal at the source to be approximately a few tenths of a second to hundreds of seconds.

3. Propagation of Micropulsations

The question of the transmission of micropulsations from their source in the outer magnetosphere to the surface of the earth is of great importance, because it will be shown that this determines the spectrum to a very large degree. Jacobs and Watanabe (1962) have discussed the transmission of hydro-magnetic waves in the lower exosphere up to a height of 2000 km. Prince and Bostick (1964) considered the transmission from the outer boundary of the geomagnetic field to the surface of the earth. Greifinger and Greifinger (1965) discussed the transmission problem in the ionosphere up to a height of 500 km. Field and Greifinger (1965) extended this to about 2000 km. Jacobs and Watanabe (1962) and Greifinger and Greifinger (1965) and Field and Greifinger (1965) did not include in their studies the path in the outer magnetosphere, which has a crucial effect in the transmission characteristics. The inclusion of this path in the present theory yields two important new features. First, it is shown here that the transmission of micropulsations is extremely sensitive to the variation in various path lengths encountered in the propagation from the boundary of the magnetosphere to the surface of the earth. As the distance of the magnetospheric boundary is quite variable with variable solar wind conditions, the path length in the outer magnetosphere will be subject to considerable variation. It will be shown that as little as 1 per cent variation in this path length will completely annihilate a resonance in the transmission. Hence resonances in the lower exosphere and ionosphere, or even in the outer exosphere, may not necessarily imply the reception of those micropulsation frequencies on the surface of the earth. It will also be shown that some resonances are more

sensitive to this "detuning" than others and hence their occurrence is less probable. Secondly, the inclusion of the path in the outer magnetosphere yields resonances at longer time periods with much larger amplitudes. This is so because it is shown that the amplitude of these resonances is inversely proportional to the resistive loading of the dissipative ionosphere, which decreases substantially with increasing time period.

3.1 Model of Propagation Medium

We shall consider Alfvén mode of one-dimensional propagation along the geomagnetic field, supposed to be vertical. The propagation path from the boundary of the magnetosphere to the surface of the earth is highly inhomogeneous. Figure 3a shows a layered model of the entire path. For most of the path from the magnetospheric boundary down to the ionosphere, the medium is essentially inhomogeneous, dissipationless and dispersionless to hydromagnetic waves. Hence this region is characterized by a variable Alfvén velocity. The profile of Alfvén velocity given by Dessler et al (1960) is used to obtain a three-layered approximation of this path. The important feature of the Alfvén velocity profile in terms of its maximum in the lower exosphere is incorporated in this three-layered approximation. Below this region is the dissipative and dispersive ionosphere. Here the waves propagate more like electromagnetic waves in a highly conductive medium. (Watanabe, 1962). We model this region by a layer, 60 km thick with an average conductivity of $\sigma = 10^{-4}$. Lastly, there is a short path in ionized atmosphere just above the surface of the earth. Here the waves propagate as electromagnetic waves in vacuum. This path is modeled as an 80 km thick layer of vacuum. The earth is taken to be a perfect conductor.

3.2 Method of Analysis

As Alfvén waves are transverse waves, one can formulate this one-dimensional propagation problem in analogy with electromagnetic wave transmission line. The analog in our case will be called hydromagnetic transmission line. The two parameters characterizing a transmission line are propagation constant k and intrinsic impedance η , which is the ratio of electric field E to magnetic field H . For hydromagnetic transmission line carrying Alfvén waves we have

$$k = 2\pi/(V_a T) \quad ; \quad \eta = \mu_0 V_a \quad (10)$$

where V_a is the Alfvén speed, T is the time period of the wave and μ_0 is the permeability of free space. The second relation above follows directly from Maxwell's equation

$$\nabla \times \bar{E} = - \mu_0 \frac{\partial \bar{H}}{\partial t} .$$

Figure 3b shows the cascaded hydromagnetic transmission line consisting of several sections, corresponding to the layered model of the propagation path of Fig. 3a. In the dissipative layer both k and η are complex and can be obtained from Maxwell's equations in the usual manner as follows:

$$k_2 = (1 + j) (\pi \mu_0 \sigma / T)^{\frac{1}{2}} \triangleq (1 + j) k_d \quad ; \quad \eta_2 = (1 + j) (\pi \mu_0 / \sigma T)^{\frac{1}{2}} \triangleq (1 + j) \eta_d \quad (11)$$

where $\sigma = 10^{-4}$. The values of k and η for the other sections of the transmission line are obtained directly from Eq. (10).

We now define the transmission coefficient T_i of the i^{th} section of the line as

$$T_i = H_{i-1}/H_i$$

which is the ratio of the magnetic field at the output to that of the input of the i^{th} section. The input impedance Z_i of the i^{th} section is defined as

$$Z_i = E_i/H_i$$

which is the ratio of the electric field and magnetic field at the input of that section. We shall set

$$Z_i = R_i + jx_i$$

where R_i and x_i are the input resistance and reactance, respectively. In Fig. 4a these various quantities are shown for each section. As we are interested only in the amplitude of the micropulsation signal on the surface of the earth, we find the magnitude of overall transmission coefficient $|T_0|$ as

$$|T_0| = |H_{\text{source}}| / |H_{\text{earth}}| = |T_5| |T_4| |T_3| |T_2| |T_1| \quad (12)$$

Because of the extremely large wave lengths of electromagnetic waves at such low frequencies, the transmission coefficient $|T_1|$ through the vacuum section l_1 , will be very close to unity. Hence we set $|T_1| \approx 1$. It is clear that the transmission $|T_i|$ is dependent on all the sections following it and none preceding it. It will be shown later that the dependence of $|T_i|$ on all the sections following it can be

summarized through its dependence on Z_{i-1} , the input impedance of the cascaded sections following it. Hence though the transmission through the vacuum section is unity, it cannot be neglected because its impedance Z_0 will influence all the transmission coefficients $|T_i|$, $i > 1$. This is unlike Jacobs and Watanabe (1962), Greifinger and Greifinger (1965) and Field and Greifinger (1965), who discard this section from their discussion entirely on the basis that the transmission through the section is unity.

It can be easily shown that the transmission through the dissipative section l_2 is a monotonically decreasing function of T . Therefore the resonances of $|T_0|$ will be those of

$$|T_5| |T_4| |T_3| \triangleq |T'_0|, \text{ with } |T_1| \approx 1. \quad (13)$$

As we are primarily interested in the resonances in the overall transmission coefficient, we may merely find the resonances of $|T'_0|$ from the reduced transmission line shown in Fig. 4b, where section l_3 is terminated by the impedance Z_2 of the cascaded sections l_2 and l_1 . It is pointed out now that after the resonances have been found by this technique, the overall transmission coefficient $|T_0|$ can be simply found as

$$|T_0| = |T'_0| |T_2| \quad (14)$$

where $|T_2|$ is the monotonically decreasing transmission coefficient through the dissipative section.

The impedance Z_1 of the shorted l_1 section of the line is easily found as

$$Z_1 = j\eta_1 \tan k_1 l_1 \quad \therefore R_1 = 0, \quad x_1 = .632/T. \quad (15)$$

For l_2 section we find through usual transmission line calculations,

$$T_2 = \eta_2 [\eta_2 \cosh k_2 l_2 + Z_1 \sinh k_2 l_2]^{-1} ;$$

$$Z_2 = T_2 [Z_1 \cosh k_2 l_2 + \eta_2 \sinh k_2 l_2] = R_2 + jx_2 .$$

Substituting the values of η_2 , k_2 , Z_1 and l_2 from Eqs. (11), (15) and Fig. 4 in this we find,

$$|T_2| = \sqrt{2}\eta_d [(\eta_d \cosh a \cos a - \eta_d \sinh a \sin a - x_1 \cosh a \sin a)^2 + (\eta_d \cosh a \cos a + \eta_d \sinh a \sin a + x_1 \sinh a \cos a)^2]^{-\frac{1}{2}} ;$$

$$R_2 = \eta_d [\eta_d^2 (\sinh 2a - \sin 2a) + 2\eta_d x_1 (\sinh^2 a + \sin^2 a) + \frac{1}{2} x_1^2 (\sinh 2a + \sin 2a)] D^{-1} ; \quad (16)$$

$$x_2 = \eta_d [\eta_d^2 (\sinh 2a + \sin 2a) + \eta_d x_1 (\cosh 2a + \cos 2a) + \frac{1}{2} x_1^2 (\sinh 2a - \sin 2a)] D^{-1} ;$$

where

$$D = 2\eta_d^2 (\sinh^2 a + \cos^2 a) + x_1^2 (\sinh^2 a + \sin^2 a) + \eta_d x_1 (\sinh 2a - \sin 2a) ;$$

and

$$\eta_d = .2T^{-\frac{1}{2}} \quad \text{and} \quad a = 1.6T^{-\frac{1}{2}}.$$

For the non-dissipative sections we can establish the following relations:

$$|T_i| = \left[\left(\cos \frac{a_i l_i}{T} - \frac{x_{i-1}}{\eta_i} \sin \frac{a_i l_i}{T} \right)^2 + \frac{R_{i-1}^2}{\eta_i^2} \sin^2 \frac{a_i l_i}{T} \right]^{-\frac{1}{2}} \quad (17)$$

$$R_i = R_{i-1} |T_i|^2 ;$$

$$x_i = |T_i|^2 \left[x_{i-1} \cos \frac{2a_i l_i}{T} + \frac{\eta_i}{2} \left(1 - \frac{x_{i-1}^2 + R_{i-1}^2}{\eta_{i-1}^2} \right) \sin \frac{2a_i l_i}{T} \right] \quad (18)$$

where

$$i = 3, 4, 5 \quad \text{and} \quad a_3 = 1.1 \times 10^{-5}, \quad \eta_3 = .754 ; \quad (19)$$

$$a_4 = 3.5 \times 10^{-6}, \quad \eta_4 = 2.26 ; \quad a_5 = 3.5 \times 10^{-5}, \quad \eta_5 = .226 ;$$

and $|T_2|$, R_2 and x_2 are given by Eq. (16).

It is clear from Eqs. (17) through (19) that the only way $|T_i|$ depends on the sections l_j , $j < i$ is through its dependence on R_{i-1} and x_{i-1} . Hence the effects of all sections following the i^{th} section on $|T_i|$ are summarized through the appearance of the input resistance and reactance of the $(i-1)$ th section in the expression for $|T_i|$. The calculation is done in step-by-step manner using the results of $(i-1)$ th section to obtain the result of i^{th} section.

The resonances of $|T'_0|$ will be given by the relative maxima of $|T_5|$, $|T_4|$, $|T_3|$. It is obvious that in general the relative maxima of $|T_5|$, $|T_4|$, $|T_3|$ will not be the same as the individual resonances of $|T_5|$, $|T_4|$ and $|T_3|$.

Equations (17) through (19) show that the input resistance R_i is directly proportional to the square of the transmission coefficient $|T_i|$. Hence, if the i^{th} section is in resonance, then clearly R_i will be maximum. This maximum in R_i will cause the transmission coefficient $|T_{i+1}|$ of the preceding section to be drastically reduced. Hence, resonance of an individual section may, in general, result in a decrease in the overall transmission coefficient $|T'_0|$.

3.3 An Illustrative Example

The important features of our results of the propagation problem have been mentioned in the introductory lines of Sec. 3. In order to demonstrate some of these very simply through an example, we further simplify the model described above. For this purpose one needs to consider only one more section for the outer magnetosphere, beyond the section l_3 for lower exosphere, where Jacobs and Watanabe (1962), Greifinger and Greifinger (1965) and Field and Greifinger (1965) stop. Hence, in reference to the model of Fig. 4, we retain l_3 section and combine l_4 and l_5 sections into one section, say l_{5-4} and obtain the simplified model shown in Fig. 5. For the l_{5-4} section we have used a weighted mean value of Alfvén speed of 2.40×10^5 m/sec. For simplicity, let us take some fixed typical values of $R_2 = .02$ and $x_2 = .05$.

(i) Masking of resonances of lower exosphere and ionosphere

The resonances of $|T_3|$ are found by minimizing the denominator of the expression in Eq. (17). This is done by the usual procedure of differentiating the quantity which is raised to the $-\frac{1}{2}$ power in Eq. (17) with respect to l_3/T

and setting it to zero and then determining the sign of the second derivative. This yields the time periods of resonance given by the condition

$$\tan \frac{2a_3 l_3}{T} = - \frac{2\eta_3 x_2}{\eta_3^2 - x_2^2 - R_2^2} , \quad (20)$$

and the amplitude of resonance as

$$|T_3|_{\max} = \sqrt{2\eta_3} \left\{ (\eta_3^2 + x_2^2 + R_2^2) - [(\eta_3^2 - x_2^2 - R_2^2)^2 + (2\eta_3 x_2)^2]^{\frac{1}{2}} \right\}^{-\frac{1}{2}} \quad (21)$$

In deriving Eqs. (20) and (21) we have used the fact that $\eta_3^2 - x_2^2 - R_2^2 > 0$. As R_2 and $x_2 \ll \eta_3$, Eqs. (20) and (21) can be reduced to

$$\tan \frac{a_3 l_3}{T} \approx \frac{\eta_3}{x_2} , \quad \text{i.e. } T \approx 10, 3.3, 2, 1.4, \dots; \quad (22)$$

and

$$|T_3|_{\max} \approx \eta_3 / R_2 , \quad \text{i.e. } |T_3|_{\max} \approx 38.$$

Though we have not chosen the same parameters, these numerical results obtained from the drastically simplified model for the resonances of l_3 section only, are not very dissimilar to those obtained by Greifinger and Greifinger (1965) and Field and Greifinger (1965).

Now under the condition of resonance for l_3 section, say when $T = 10$, let us calculate $|T_{5-4}|$ from

$$|T_{5-4}| = \left[\left(\cos \frac{a_{5-4} l_{5-4}}{T} - \frac{x_3}{\eta_{5-4}} \sin \frac{a_{5-4} l_{5-4}}{T} \right)^2 + \frac{R_3^2}{\eta_{5-4}^2} \sin^2 \frac{a_{5-4} l_{5-4}}{T} \right]^{-\frac{1}{2}} \quad (23)$$

where

$$R_3 = R_2 |T_3|_{\max}^2 \approx 28.8 \quad \text{and} \quad x_3 \approx -x_2 |T_3|^2 \approx -72 . \quad (24)$$

With the values of R_3 and x_3 given by Eq. (24), we find from Eq. (23),

$$|T_{5-4}| \approx \eta_{5-4} [(x_3^2 + R_3^2) \sin^2 (a_{5-4} \ell_{5-4} / T)]^{-\frac{1}{2}} \approx 4.5 \times 10^{-3}$$

$$\therefore |T'_0| = |T_3|_{\max} |T_{5-4}| \approx .17$$

which shows that the resonance of the lower exosphere is completely annihilated by the path in the outer magnetosphere.

(ii) New long period resonances

Due to the inclusion of the path ℓ_{5-4} in the outer magnetosphere, our model will yield new resonances. We expect these resonances to have much longer period than those of ℓ_3 , because the path length $\ell_{5-4} \gg \ell_3$ and the intrinsic and terminal impedances of the two sections are not greatly different. An examination of Eq. (17) reveals that for such long periods

$$R_3, x_3 \ll \eta_{5-4} .$$

With these approximations the resonance condition is given by

$$\tan (a_{5-4} \ell_{5-4} / T) \approx \eta_{5-4} / x_3 \quad (25)$$

and the corresponding resonance amplitude $|T_{5-4}|$ is given

by

$$|T_{5-4}|_{\max} \sim \eta_{5-4}/R_3 . \quad (26)$$

For these time periods R_3 , x_3 and $|T_3|$ can be approximated from Eq. (17) as

$$R_3 \sim |T_3|^2 R_2, \quad x_3 \sim |T_3|^2 x_2, \quad |T_3| \sim 1 . \quad (27)$$

Using the approximations of Eq. (27) in Eq. (25), we obtain the resonance condition as

$$\tan(a_{5-4} l_{5-4}/T) \approx \eta_{5-4}/x_2, \quad \text{i.e. } T \approx 1080, 334, 198, 141, \dots$$

The corresponding resonance amplitude can be obtained from Eq. (26) by using the approximations of Eq. (27) as

$$|T_{5-4}|_{\max} \sim \eta_{5-4}/R_2 = 15.1 .$$

These longer period resonances have not been found by Jacobs and Watanabe (1962), Greifinger and Greifinger (1965) and Field and Greifinger (1965). Furthermore, as R_2 is in fact not a constant but a strongly decreasing function with increasing T , these resonances have much larger amplitude than those of lower exosphere with shorter time periods. Hence, the value of the resonance amplitude given above for constant $R_2 = .02$ is misleading for the real situation.

Now we examine the transmission of these resonances through the lower exosphere. As we have already noted in Eq. (27) that $|T_3| \sim 1$, we find

$$|T'_0| = |T_{5-4}| |T_3| \sim |T_{5-4}|_{\max} = 15.1$$

This shows that these longer period resonances of the outer magnetosphere are not at all masked by the lower exosphere. This is the exact reverse of the situation with the resonances of lower exosphere and ionosphere which are completely masked by the effect of outer magnetosphere.

(iii) Coincident resonance

When section l_3 is in resonance, R_3 and x_3 are very large and consequently the transmission $|T_{5-4}|$ through section l_{5-4} is much less than one. If the path length l_{5-4} is adjusted to produce a maximum of $|T_{5-4}|$, it is shown below that this maximum is approximately equal to unity, because of very large x_3 and R_3 . Only under these conditions will the resonance of l_3 survive in the overall transmission. That is, a maximum in overall transmission $|T'_0|$ can be achieved by simultaneously maximizing $|T_3|$ with respect to T , yielding a "temporal" resonance of lower exosphere and maximizing $|T_{5-4}|$ with respect to l_{5-4} , yielding a "spatial" resonance of the outer magnetosphere. The conditions for the coincidence of the "temporal" resonance of l_3 and "spatial" resonance of l_{5-4} are:

T determined from

$$\tan(a_3 l_3 / T) \approx \eta_3 / x_2, \quad \text{i.e. } T \approx 10, 3.3, 2.3, \dots$$

and l_{5-4} determined from

$$\tan(a_{5-4} l_{5-4} / T) \approx \eta_{5-4} x_3 (x_3^2 + R_3^2 - \eta_{5-4}^2)^{-1} \quad (28)$$

Now as section l_3 is in resonance, x_3 and R_3 are both very large and one can approximately obtain from Eq. (28)

$$l_{5-4} \approx \text{----}, 5.4 \times 10^7, 5.7 \times 10^7, 5.9 \times 10^7, \text{----} \quad (29)$$

The probability that l_{5-4} has a value exactly as given by Eq. (29) is very small, but non-zero. Under these conditions we find

$$|T_3|_{\max} = \eta_3/R_2 \approx 38 \quad \text{and} \quad |T_{5-4}|_{\text{sp. res}} \approx 1,$$

where the subscript "sp. res" stands for "spatial" resonance. This obviously produces a maximum in the overall transmission

$|T'_0|_{\max}$ as

$$|T'_0|_{\max} = |T_3|_{\max} |T_{5-4}|_{\text{sp. res}} \approx 38 .$$

This demonstrates that a resonance of the lower exosphere or ionosphere can appear on the surface of the earth, if and only if the outer magnetosphere is in "spatial" resonance simultaneously with the "temporal" resonance of the lower exosphere or ionosphere. As pointed out before, this concurrence is a rather improbable but not impossible event. This statement obviously does not apply when the source of micropulsations is located in the lower exosphere or ionosphere. This is the case for micropulsations caused by energetic particle precipitation (see Sec. 5.2).

3.4 Detailed Analysis of Transmission Problem

(i) Masking of resonances of middle and lower exosphere

We now return to the formulation of section 3.2 for detailed analysis. $|T_3| |T_4|$ is calculated from Eqs. (17) and (18) with the parameter values given by Eqs. (15) and (16). Figure 6 shows the result of this calculation by displaying a plot of $|T_3| |T_4|$ vs T . The resonances of $|T_3| |T_4|$ are seen to be at $T = 4, 7, 9, 14$ and 39 . At these time periods of resonance and with the corresponding amplitudes of $|T_3| |T_4|$, the overall transmission coefficient $|T'_0|$ can be calculated from Eqs. (13) and (19). These are found to be

$|T'_0| \approx 1.3, .1, .3, 1$ and $.9$ at $T = 4, 7, 9, 14,$ and $39,$

respectively. It is clear that all the resonances of middle and lower exosphere are completely masked out in the overall transmission due to the effect of the path in the outer magnetosphere.

(ii) Significant maxima of overall transmission

The path lengths of the various sections are widely different from one another and their intrinsic and terminal impedances are not greatly different. This results in widely separated resonances of various sections. Furthermore, the resonances can be shown to be very sharp. Hence, a resonance of any section will correspond to a relative maximum of the overall transmission. But we have demonstrated above that the resonance of section l_3 is masked by section l_4 and that of section l_4 is masked by section l_5 . Hence the relative maxima of the overall transmission, at the

resonances of all the sections below the outer magnetosphere, will have very small amplitudes and these will not be present in the micropulsation signals received on the surface of the earth. The only relative maxima of the overall transmission which will have large amplitudes are those at the resonances of the outer magnetospheric path ℓ_5 . These are the only relative maxima of the overall transmission which are of any significance from the point of view of the reception of micropulsation signals on the surface of the earth. In what follows we shall discuss only this and designate it by the symbol $|T'_0|_{\max}$.

We calculate $|T_5|$ from Eq. (19), which is transformed into the following form for easier manipulation.

$$|T_5| = \sqrt{2\eta_5} \left\{ (x_4^2 + R_4^2 + \eta_5^2) + [(x_4^2 + R_4^2 + \eta_5^2)^2 - (2\eta_5 R_4)^2]^{\frac{1}{2}} \sin(2a_5 \ell_5 / T + \psi) \right\}^{-\frac{1}{2}} \quad (30)$$

where $\psi = \tan^{-1} [(\eta_5^2 - R_4^2 - x_4^2) / (2\eta_5 x_4)]$

It is clear from Eq. (30) that $|T_5|$ has a resonance, when the sine function is -1. Hence the condition for resonance is

$$\tan(2a_5 \ell_5 / T) = -2\eta_5 x_4 (\eta_5^2 - x_4^2 - R_4^2)^{-1} \quad (31)$$

and the amplitude of resonance is

$$|T_5|_{\max} \approx (x_4^2 + R_4^2 + \eta_5^2)^{\frac{1}{2}} / R_4 \quad (32)$$

As these resonances have periods much longer than those of the resonances of l_3 and l_4 , we approximately have

$$R_4 = |T_3|^2 |T_2|^2 R_2, \quad x_4 \approx |T_3|^2 |T_2|^2 x_2, \quad |T_2| \sim 1, \quad |T_3| \sim 1.$$

With these approximations Eqs. (31) and (32) can be simplified as

$$\tan(a_5 l_5 / T) \approx \eta_5 / x_2, \quad \text{i.e. } T = 935, 311, 134, 103 \text{ and } 71 \quad (33)$$

and

$$\begin{aligned} |T_5|_{\max} &\approx \eta_5 / R_2 \\ &= 4.2 \times 10^4, 4.7 \times 10^3, 8.7 \times 10^2, 5.1 \times 10^2 \text{ and } 2.4 \times 10^2, \end{aligned}$$

respectively.

The amplitudes of these resonances are much larger than those of the resonances of lower exosphere and ionosphere with shorter time periods. The reason for this is that according to Eq. (16), R_2 is a strongly decreasing function with increasing T . Hence, for such long period resonances, R_2 is drastically reduced and Eq. (33) yields very large amplitudes.

Now the maxima of the overall transmission $|T'_0|_{\max}$ are given by

$$|T'_0|_{\max} = |T_3| |T_4| |T_5|_{\max} \approx |T_5|_{\max};$$

the values given in Eq. (33). Hence the maxima of the overall transmission corresponding to the resonances of outer magnetosphere are much larger than those at shorter time periods. Specifically, the estimates of time periods 935,

311 and 134 are higher than the longest period resonances found in all other theoretical work by Prince and Bostick (1964), Greifinger and Greifinger (1965), Field and Greifinger (1965) and Field and Greifinger (1966). Their amplitudes are also much larger than those calculated by other authors. It is believed that these may account for the giant pulsations of periods more than 2 minutes and amplitudes of several hundred gammas observed by Veldkamp (1960) and Nagata, Kokburn and Iijima (1963). The resonances in the vicinity of $T \sim 103$ and 71 have been found by the previously mentioned authors.

Significant maxima of overall transmission can also be achieved through a coincidence of "temporal" resonance of lower and middle exosphere (sections l_3 and l_4) and "spatial" resonance of outer magnetosphere (section l_5), according to the notion developed in section 3.3. As a matter of fact one can find "spatial" resonances of section l_5 at any T by adjusting the path length l_5 , under any conditions of sections l_3 and l_4 , when these may or may not be in resonance. We find the "spatial" resonance condition from Eq. (31) as

$$l_5 = \frac{T}{2a_5} \tan^{-1} \left[- \frac{2\eta_5 x_4}{\eta_5^2 - x_4^2 - R_4^2} \right], \quad (34)$$

where the solution for l_5 which is of the order of 4.2×10^7 is chosen. The corresponding amplitude of overall transmission $|T'_0|_{\text{sp. max}}$ is found by using Eq. (32) as

$$|T'_0|_{\text{sp. max}} = |T_3| |T_4| (x_4^2 + R_4^2 + \eta_5^2)^{\frac{1}{2}} / R_4, \quad (35)$$

where the value of $|T_3| |T_4|$ is taken from Fig. 6.

Hence we find these coincident resonances at $T \sim 39, 14, 9, 7,$ and 4 with the amplitudes of 108, 50, 18, 22 and 29, respectively. We note that these amplitudes do not include the effect of attenuation through the lower ionosphere. Prince and Bostick (1964), Greifinger and Greifinger (1965), and Field and Greifinger (1965) have also found resonances in the neighborhood of time periods 14, 7 and 4.

Because the value of l_5 given by Eq. (34) is very close to its nominal value of 4.2×10^7 in our model, it is of interest to determine the small difference directly. In order to do this, we find the variation in $|T_0|$ due to a very small fractional variation ϵ in the path length l_5 by setting

$$l_5 = L_5(1 - \epsilon); \quad \epsilon \ll 1 \quad \text{and} \quad L_5 \sim 4.2 \times 10^7 .$$

We can now linearize the perturbation of the sinusoidal term in Eq. (30) as

$$\sin(2a_5 l_5 / T + \psi) = \sin(2a_5 L_5 / T + \psi) - (2\epsilon a_5 L_5 / T) \cos(2a_5 L_5 / T + \psi)$$

With this substitution, Eq. (30) can be reduced to

$$|T_5| \approx |T_5|_{L_5} \times \left\{ 1 + \frac{|T_5|_{L_5}^3 \epsilon a_5 L_5 [(x_4^2 + R_4^2 + \eta_5^2)^2 - (2\eta_5 R_4)^2]^{\frac{1}{2}}}{2\eta_5^2 T} \cos\left(\frac{2a_5 L_5}{T} + \psi\right) \right\} \quad (36)$$

where $|T_5|_{L_5}$ is the transmission coefficient when $l_5 = L_5$

and is given by

$$|T_5|_{L_5} = \sqrt{2}\eta_5 \times$$

$$\left\{ (x_4^2 + R_4^2 + \eta_5^2) + [(x_4^2 + R_4^2 + \eta_5^2)^2 - (2\eta_5 R_4)^2]^{\frac{1}{2}} \sin(2a_5 L_5 / T + \psi) \right\}^{-\frac{1}{2}}$$

and if L_5 is not near a resonance. On the other hand

$$|T_5| = \sqrt{2}\eta_5 \times$$

$$\left\{ [(x_4^2 + R_4^2 + \eta_5^2)^2 - (2\eta_5 R_4)^2]^{\frac{1}{2}} (2\epsilon a_5 L_5 / T) \cos(2a_5 L_5 / T + \psi) \right\}^{-\frac{1}{2}} \quad (37)$$

if L_5 is near a resonance. By taking $L_5 = 4.2 \times 10^7$ as given in Fig. 4 and using Eqs. (36) and (37), $|T_5|$ is calculated for a range of ϵ . Then $|T_0| = |T_5| |T_4| |T_3|$ is obtained from this and Fig. 6 at the resonances of $|T_4| |T_3|$ and plotted vs $\epsilon\%$ in Fig. 7. In Fig. 8, $|T_0|$ is plotted vs $\epsilon\%$ for several values of time period T for which $|T_4| |T_3|$ is off-resonance. It is apparent from Figs. 7 and 8 that an extremely small variation in the path length in the outer magnetosphere can cause a "spatial" resonance. The required variation is as little as a fraction of a per cent at low values of T , progressively increasing to a few per cent at moderate values of T . The occurrence of this "spatial" resonance is plausible because of the variable solar wind conditions. But the probability of having this exact length l_5 in the outer magnetosphere is very small. In spite of this very low but non-zero probability, this result is of some significance, because it shows that at any time period whatsoever, a small variation of the path length in the outer magnetosphere due to variable solar wind conditions can produce a resonance of transmission.

Figures 7 and 8 also demonstrate the extreme sensitivity of the overall transmission on the variation of the path length in the outer magnetosphere. It is of great interest to note that this sensitivity is considerably smaller in the situation when the lower and middle exosphere are off-resonance than when these are in resonance. We note that at the resonances of $|T_4| |T_3|$, R_4 is very large and we have from Eq. (35)

$$|T_0'|_{\text{sp. max}} \approx [|T_3| |T_4|]_{\text{resonance}}$$

We immediately recognize this as the coincidence of a "temporal" resonance of lower and middle exosphere and a "spatial" resonance of the outer magnetosphere which has been discussed above. Now Fig. 7 can be interpreted as showing the variation in the path length necessary to produce the coincidence of a "temporal" resonance of lower and middle exosphere and a "spatial" resonance of the outer magnetosphere. As indicated before, the above arguments are inapplicable when the source of micropulsations is located in the lower exosphere or ionosphere.

(iii) Sensitivity of transmission on the path length in the outer magnetosphere

We have already demonstrated in Figs. 7 and 8 the extreme sensitivity of the overall transmission on the variation of the path length in the outer magnetosphere. Now to be quite specific, let us calculate a definite measure of the sensitivity of overall transmission $|T_0'|$ on l_5 as follows. The percentile variation $\delta\%$ in l_5 required to reduce $|T_0'|$ by a factor of N can be deduced from Eqs. (30), (34) and (35) to be

$$\delta\% = (T/14.6) \cos^{-1} [1 + 2(1 - N^2)(\eta_5 R_4)^2 / (x_4^2 + R_4^2 + \eta_5^2)] \quad (38)$$

Figure 9 shows a plot of $\delta\%$ vs T for $N = 10$. It is clear from this figure that a variation of the order of only 1 per cent in the path length of the outer magnetosphere can completely quench the reception of micropulsation signals on the surface of the earth. Furthermore, the smaller the value of $\delta\%$, the higher will be the probability of quenching of the micropulsation signal of the corresponding time period. In other words $\delta\%$ is a measure of the persistence or occurrence probability of micropulsations. With this interpretation of $\delta\%$, the sharp peaks in Fig. 9 clearly indicate that the occurrence probability of micropulsations of time periods 2.5, 6, 11 and especially 20 should be much higher than others. The peaks of $\delta\%$ corresponding to these time periods are so sharp as to suggest almost single frequency sinusoidal signals, which is a characteristic of micropulsation reception. Because of sharply higher occurrence frequency, one should expect spectral lines or peaks at time periods 2.5, 6, 11 and especially 20 in the power spectrum of micropulsations. This is in striking agreement with Davidson (1964) who finds a first group of spectral peaks at $T \sim 5$ and 10, and a second group at $T \sim 17$ to 70. In the second group, the daytime peaks occur mostly at $T \sim 26$ and sometimes at $T \sim 45$. As our calculations are based on the dayside of the magnetosphere, we can only compare the daytime peaks at $T \sim 26$ and 45 with our estimate of $T \sim 20$.

(iv) Effect of ionospheric and other attenuation

At this point we want to include the attenuation $|T_2|$ through the ionosphere and other factors in the

estimation of the overall transmission. $|T_2|$ can be calculated from Eq. (16), but for the sake of simplicity we will make a drastic approximation of this as

$$|T_2| \approx T/100 \quad \text{for } 1 \ll T \ll 100$$

$$\approx 1 \quad \text{for } T > 100$$

The approximation is correct in the order of magnitude sense. Furthermore, we will consider an additional attenuation, say of 1/3, for loss through mode conversion at all T . For $T > 100$ there will be another additional attenuation because of loss of wave energy due to lack of guidance along the geomagnetic field, as the wave lengths tend to be comparable with the radius of curvature of field lines. We shall roughly estimate this to be inversely proportional to T . Hence the total effective attenuation $|T_{att}|$ can be very crudely depicted as in Fig. 10. It is recalled that Eq. (35) gives the amplitude of transmission at any time period under the condition of "spatial" resonance of the path in the outer magnetosphere. Hence it represents the maximum possible transmission at any time period, which can be achieved by proper adjustment of the path length in the outer magnetosphere. As such Eq. (35) gives the greatest upperbound on the transmission and consequently the transmission obtained from the attenuation of Fig. 10 and Eq. (35) will be designated

$|T_o|_{gub}$ and expressed as

$$|T_o|_{gub} = |T_{att}| |T_3| |T_4| (X_4^2 + R_4^2 + \eta_5^2)^{\frac{1}{2}} / R_4 \quad (39)$$

This is plotted in Fig. 11 as a function of time period T .

As explained above, this quantity is quite different from the transmission coefficient discussed by other authors like Greifinger and Greifinger (1965), Field and Greifinger (1965) and Prince and Bostick (1964). One of the possible meanings which can be attached to this quantity is that it is the cut-off point at the high end of the probability distribution function of the transmission coefficient.

4. On the Spectrum of Micropulsations

4.1 Amplitude Spectrum

Now we will attempt to estimate the spectrum of micropulsations at the surface of the earth. We note that the transmission path of the micropulsations discussed in Section 3 is a linear system. Hence, using the concept of transfer function of a linear system, we can write in terms of Fourier Transforms

$$H_{\text{earth}}(j\omega) = T_o(j\omega) H_{\text{source}}(j\omega) \quad (40)$$

where

$$H_{\text{earth}}(j\omega) = \int_{-\infty}^{+\infty} H_{\text{earth}}(t) e^{-j\omega t} dt; \quad H_{\text{source}}(j\omega) = \int_{-\infty}^{+\infty} H_{\text{source}}(t) e^{-j\omega t} dt;$$

$T_o(j\omega)$ = Transfer function of the transmission path =
Transmission coefficient T_o defined in section 3.

The quantity of interest is the amplitude spectrum $|H_{\text{earth}}(j\omega)|$ at the surface of the earth, which is easily found from Eq. (40) as

$$|H_{\text{earth}}(j\omega)| = |T_o| |H_{\text{source}}(j\omega)| \quad (41)$$

We have discussed $|T_o|$ in section 3.4 in great detail. It

now remains to specify $|H_{\text{source}}(j\omega)|$, before we can determine the amplitude spectrum at the surface of the earth.

In section 2 we have discussed the unstable waves on the magnetospheric surface as the source of micropulsations. There we have indicated that the entire curved bounded surface will be covered with surface waves of a variety of modes through mode coupling. Hence, the spectra at the source, though discrete, may be quite dense. Furthermore, due to strong mode coupling the spectra may not be far from uniform. Lastly, the bandwidth of the spectrum at the source has been estimated to be a few tenths of a second to hundreds of seconds. Hence, it is not unreasonable to assume that the spectrum $|H_{\text{source}}(j\omega)|$ is discrete, dense, essentially uniform and band-limited. We now recall from section 3.4 that the overall transmission coefficient $|T_{\circ}|$ has widely separated, extremely narrow and very high resonances. In comparison, the off-resonance values of $|T_{\circ}|$ are very low. Therefore the right hand side of Eq. (41) can be looked upon as dense uniform spectra $|H_{\text{source}}(j\omega)|$ modulated by slowly varying $|T_{\circ}|$. It essentially picks out a few of the spectral components of the source and elevates these to the levels of the resonances of the transmission path. $|T_{\circ}|$ is off-resonance for most of the spectral components of the source and these will not appear in the spectrum at the surface of the earth. Hence, we conclude that the basic structure of the spectrum of micropulsations received on the surface of the earth will be that of the transmission coefficient $|T_{\circ}|$ within the bandwidth of a few tenths of a second to hundreds of seconds. The fine structure of this spectrum will be obviously due to the detailed distribution of the dense spectra at the source

(and also possibly various sources of noise in the transmission path). In view of our discussion in section 3.4 (iii), we would expect a predominance of micropulsations of a few frequencies in the vicinity of each time period $T \sim 935, 311, 134, 103$ and 71 , and less probably in the vicinity of $T \sim 39, 14, 9, 7$ and 4 . The last statement does not, of course, apply when the source is located in the lower exosphere or ionosphere. This will be so for micropulsations caused by energetic particle precipitation (see section 5.2). In that case the resonances at $T \sim 39, 14, 9, 7$ and 4 will be dominant.

It is pointed out that the amplitude spectrum discussed above is the Fourier Transform of the received signal, whereas power spectral density is the squared magnitude of the Fourier Transform of the auto-correlation function of the signal. All the authors on the subject including Prince and Bostick (1964), Greifinger and Greifinger (1965), and Field and Greifinger (1965, 1966) present the squared amplitude spectrum (obtained as the squared transmission coefficient) as the power spectral density. This is strictly correct if and only if the source is white noise, and the transmission path has no statistical variations. The assumption about the source may not be bad for some classes of micropulsations. But it will be shown below that a certain slight statistical variation in the transmission path will have profound influence on the power spectral density, causing it to be far different from the squared magnitude of the transmission coefficient. Hence, the comparisons between the theoretical resonances of transmission and the observed spectral peaks made by Prince and Bostick (1964), Greifinger and Greifinger (1965), Field and Greifinger (1965, 1966) are somewhat misleading. All one could expect

at the resonances of transmission is a measure of relative prevalence, but not necessarily spectral peaks. In this restricted sense one can say that our estimate of micropulsations at $T \sim 4, 7$ and 9 are consistent with the observation of Yanagihara (1963), those at $T \sim 7, 9, 14, 71$ and 103 with the findings of Maple (1959), those at $4, 7, 9, 14, 71, 103,$ and 134 with the observation of Benioff (1960). Our estimate of giant pulsations at $T \sim 134, 311$ and 935 are corroborated by the observations of Veldkamp (1960) and Nagata, Kokburn and Iijima (1963).

4.2 Power Spectral Density

Power spectral density gives the statistical distribution in frequency of the power of the signal. Consequently, in general it is impossible to determine it without very complex statistical analysis involving the stochastic behavior of the source mechanism and the transmission path. Without embarking on such a nearly impossible task, we can nevertheless demonstrate a significant feature of the power spectral density by the following simple analysis. The signal at the source in our theory is essentially deterministic, although admittedly with some statistical fluctuations due to some randomness inherent in solar wind conditions. Leaving this aside, we intend to show the strong effect of the slight statistical behaviour of the transmission path on the power spectral density on the surface of the earth. The statistical behaviour in question is the fluctuation of the path length in the outer magnetosphere caused by the fluctuations in the solar wind conditions. We have seen in section 3.4(iii) that the transmission of micropulsations is extremely sensitive to the slightest variation of this path length. Hence, the statistical behaviour and specifically

the power spectral density of micropulsation signal at the surface of the earth will be strongly affected by this. We have already indicated in section 3.4(iii) that sharp peaks in persistence probability or occurrence probability as reflected by dominant peaks in δ % may correspond to spectral lines at $T \sim 2.5, 6, 11$ and 20 in the power spectral density at the surface of the earth. Now we wish to find the broad structure as a function of time period. We note that the value of power spectral density at a certain frequency is proportional to the product of occurrence probability of that frequency (of which δ % is a measure) and the square of the amplitude of transmitted signal at that frequency. Hence the power spectral density $P(\omega)$ can be very roughly written as proportional to the product

$$P(\omega) \sim T_{ogub}^2 \delta \quad (42)$$

Substituting Eqs. (38) and (39) in the above equation, we obtain some rough features of $P(\omega)$ as shown in Fig. 12. The slope of 7 db/octave of the power spectral density for large T is remarkable close to the observed value of 6 db/octave (Santirocco and Parker, 1963 and Davidson, 1964). For small T the slope is 42 db/octave and the agreement is poor.

A better measure of the occurrence probability for the present purpose will be the variation in the path length related to the probable amplitude of the received signal. That is, we define a new measure δ' %, the percentile variation in the path length (ℓ_5) in the outer magnetosphere required to reduce the overall transmission coefficient to a maximum possible value of A from its greatest upper bound. A is an appropriately small quantity

for which the reception of micropulsations can be considered negligible. From Eqs. (30), (34), (35) and (38), this can be deduced to be

$$\delta' \% = \frac{T}{14.6} \cos^{-1} \left[1 - \frac{2\eta_5^2}{X_4^2 + R_4^2 + \eta_5^2} \left(\frac{|T_d|^2 |T_4|^2 |T_{att}|^2}{A^2} - \frac{R_4^2}{X_4^2 + R_4^2 + \eta_5^2} \right) \right] \quad (43)$$

where $|T_{att}|$ is taken from Fig. 10. A plot of $\delta' \%$ versus time period T is shown in Fig. 13 for $A = .1$. Now the power spectral density $P(\omega)$ is roughly written as proportional to the product

$$P(\omega) \sim |T_o|^2_{gub} \delta' \quad (44)$$

Using Eqs. (39) and (43) in the above equation, we obtain the broad structure of $P(\omega)$ versus time period T as shown in Fig. 14. Again the slope of the power spectral density of Fig. 14 for large T is 6db/octave which is exactly the same as that observed by Santirocco and Parker (1963) and Davidson (1964) although for shorter T the slope is 28 db/octave and the agreement is poor. As pointed out at the outset of this section, Figs. 12 and 14 are not intended to be plots of theoretical power spectral density in any sense. Nevertheless, our crude analysis makes some general points about its trend with increasing time period, especially at large T .

5. Other Characteristics of Micropulsations

5.1 Nighttime Micropulsations

The theory presented here relies on the unstable waves generated on the dayside of the magnetospheric boundary

and consequently their propagation to the dayside of the earth. Hence, the detailed calculations are directly relevant to the daytime micropulsations. In principle the nighttime micropulsations can be explained by simple extensions of the mechanisms proposed here. These are caused by the unstable waves generated on the surface of the magnetospheric tail (Sen, 1965) and/or the same from the dayside of the magnetospheric boundary propagating through mode coupling partly in the east-west directions to the nightside and then descending along the geomagnetic field lines to the surface of the earth.

5.2 Other Mechanisms Responsible for Micropulsations

It is pointed out that the theory proposed here is only one of several possible mechanisms responsible for micropulsations. The others are: high frequency hydromagnetic emissions by charged particle bunches oscillating along geomagnetic field lines (Wentworth and Tepley, 1962, Jacobs and Watanabe, 1963); hydromagnetic whistlers (Obayashi, 1965) which includes the effect of wave dispersion in the hydromagnetic emission mechanism mentioned above; the effect of pulsating current systems in the ionosphere set up by periodic precipitation of particles initiated by the periodic agitation of the geomagnetic field by solar disturbance (Campbell and Matsushita, 1962); and lastly, the result of transient disturbances of the geomagnetic field. The first two mechanisms listed above are for high frequency emissions whose frequencies fall in the Pc 1 range and beyond. We have excluded the consideration of these emissions, which fall anyway at the high frequency end of the bandwidth of micropulsations estimated in sec. 2.2, and some of it may properly be part of the ELF noise spectrum.

As discussed by Campbell and Matsushita, the third mechanism presupposes a sustained periodic oscillation of the distant geomagnetic field whose origin and nature are left unspecified. Furthermore, as the particle precipitation in this theory is connected with the auroral zone electrons, one should use the recent idea of auroral phenomena in terms of particle acceleration by an instability of the neutral sheet in the geomagnetic tail. As a matter of fact, instabilities of the neutral sheet may set up characteristic oscillations on the geomagnetic lines leading to the auroral zones and provide a mechanism for periodic precipitation of particles required by the theory of Campbell and Matsushita (1962). This possibility should be looked into.

A wide range of micropulsations may be due to the last mechanism. Some of the variety of causes of disturbance to the geomagnetic field are sudden commencement, substantial precipitation of high energy particles due to natural causes, and man-made nuclear explosions in the magnetosphere. The characteristics of the micropulsations due to these transient causes will be different from those of our steady state theory in two important respects. The first is that the spectrum of the transient source will influence the spectrum of the received signal much more strongly than that of our steady state theory or the common assumption of white noise at source (Jacobs and Watanabe, 1962; Prince and Bostick, 1964; Greifinger and Greifinger, 1965 and Field and Greifinger, 1965). The second is the manner in which the transfer function of the transmission path influences the spectrum of the received signal in contrast to that in our steady state theory or the consideration of steady state transmission coefficient

(Jacobs and Watanabe, 1962; Prince and Bostick, 1964; Greifinger and Greifinger, 1965 and Field and Greifinger, 1965, 1966). We can briefly indicate these differences in principle. An examination of Eq. (40) immediately reveals that in a steady state theory the frequencies of the received signal are the same as those of the source, only the magnitudes and phases of various spectral components are altered by the transfer function of the transmission path. In a transient situation, the frequencies of the received signal are determined both by the spectral content of the source and the transfer function. That is, the frequencies of the received signal contain the frequencies of the transient disturbance as well as the natural modes of the transmission path. The natural modes of the transmission path appeared to play a significant role in our steady state theory only because these were the highly discriminating pass bands of the effective filtering action of the transmission path and the nature of our source as discrete, dense and essentially uniform. In the case of general transient source, this will not be the case as explained above. In a future paper we would discuss this situation analytically.

Lastly, the suggestion of Herron (1966) that most micropulsations are interference signals that propagate with a phase velocity that increases with decreasing period, is an important one and should be further explored.

5.3 Correlation With Various Geophysical Phenomena

It has been shown in section 2 and elsewhere (Sen, 1965) that the spatial extent of the instability on the dayside of the magnetospheric boundary is almost completely independent of the speed and density of the solar wind. (But the same is critically dependent on the angle

between the geomagnetic field and the compressed interplanetary field at the boundary.) The stable regions on the nightside of the boundary are somewhat dependent on solar wind conditions, but parts of it are always unstable. Hence, it is clear that the source of micropulsations in this theory is hardly correlated with K or K_p indices. Furthermore, in section 3.4 (iii) we have shown that the reception of micropulsations on the surface of the earth can be completely quenched by the slightest variation of the path length in the outer magnetosphere. As the slight variations in the path length necessary for such quenching can always be caused by the slight fluctuations normally expected in the solar wind at all times, it appears that the transmission probability or occurrence probability of micropulsations are not dependent on K or K_p indices. Hence in our theory neither the source mechanism nor the transmission characteristics (with one exception described below) are substantially correlated with K or K_p indices.

Only in one respect will there be a relationship between K and K_p indices and micropulsations in our theory. Under highly enhanced solar wind, the path length in the outer magnetosphere will be considerably shortened. In section 3 we have seen that the longest period resonances of the transmission path are determined by this path length, and these decrease with decreasing value of the path length. Hence one can expect a somewhat negative correlation of very long period micropulsations with K or K_p indices. In section 5.2 we have noted that most shorter period micropulsations may have their origin in a mechanism different from the one proposed in this paper. Therefore we must exclude these from our consideration at the moment. Hence, in conclusion, one can surmise that long

period micropulsations have essentially no correlation with K or K_p indices, with the exception of the very long period oscillations which can have a somewhat negative correlation. This is in general agreement with the observations of Maple (1959).

Now we can discuss the case of shorter period pulsations. In section 5.2 we have indicated the possibility that most shorter period micropulsations may have their origin in the transient disturbance of the geomagnetic field due to precipitation of energetic particles. The events of energetic particle precipitation are obviously related to K and K_p indices. Furthermore, auroral events are certainly caused by energetic particle precipitation. Hence, the class of shorter period micropulsations generated by energetic particle precipitation should be correlated with not only K and K_p indices, but also with auroral zone bremsstrahlung, ionospheric absorption of cosmic noise and negative bay disturbance. This positive correlation has been observed by Benioff (1960), Sechrist (1962), Chen et al (1962), and Campbell and Matsushita (1962).

5.4 Fine Structure of Micropulsations

In the theory presented here we have attempted to derive only the broad features of the received micropulsation signals. That is, we have found the dominant frequencies and some general behaviour of the power spectral density. The variety of fine structures found in micropulsations have not been considered here. Nevertheless, from our observations in section 4.1 we can state that the fine structure will be determined by the dense spectra of our source and also possibly various sources of noise in the transmission path. For the shorter period micropulsations caused by energetic particle precipitation, the

spectrum of the source will play the decisive role in determining the fine structure.

5.5 Latitudinal, Diurnal and Seasonal Variations

We have considered these variations as manifestations of detailed behaviour of micropulsations dependent on a variety of geophysical phenomena and have not attempted to explain these in the framework of our theory. Nevertheless one can make the following conjectures.

The geomagnetic lines of force on the boundary of the magnetosphere descend to the surface of the earth at higher latitudes, specifically in the auroral zones. Recalling that the source of micropulsation in our theory is at the boundary of the magnetosphere, it appears reasonable that both the occurrence frequency and amplitude of pulsations will be maximum in the auroral zones. For various reasons this has been suspected by other authors and is in accordance with the findings of Jacobs and Sinno (1959, 1960). The variation of the period of micropulsation with latitude is probably a more complex phenomenon. The realistic transmission problem will certainly involve mixed mode latitude-dependent propagation and the resulting transmission resonances will reflect latitude dependence. Moreover, in our theory the source mechanism itself dictates a certain latitude dependence of the time period. The unstable waves on the boundary of the magnetosphere, discussed in section 2, will exist in a certain layer whose thickness we have ignored so far. From our discussion in section 2.2 it is obvious that the period of the longest period wave generated will decrease as one moves deeper into the layer toward the earth. As the geomagnetic lines of force here connect to latitudes lower than the auroral zone, it appears

that the time period of pulsations may decrease with decreasing latitude. This is in accordance with the observations of Obayashi and Jacobs (1958).

The diurnal variation in terms of the difference between daytime and nighttime micropulsations is probably due to the day-night asymmetry of the magnetosphere and the day to night variation in the ionization level of the ionosphere. The continuous dependence of the occurrence frequency on the geomagnetic local time can be explained in the framework of our theory. We have indicated in section 2 that the mode on the boundary of the magnetosphere which may account for micropulsations is the one parallel to the local geomagnetic field. On the other hand the most unstable mode is the one parallel to the local tangential streaming velocity of the solar wind on the boundary surface. Hence the most favorable situation for the occurrence of micropulsation will be the one when the tangential streaming velocity of the solar wind is nearly parallel to the local geomagnetic field. This is so in the vicinity of the local noon meridian of the hemispherical boundary surface shown in Fig. 1. Hence, one may expect that the occurrence frequency of pulsations on the surface of the earth is maximum near the local noon. This is not contrary to the observations of Troitskaya (1953) and specially Jacobs and Sinno (1959).

Lastly, the seasonal variation may be attributed to the variation in the angle of attack of the solar wind with respect to the dipole axis of the earth and the variation in the ionization level of the ionosphere. It is understood that in any season there will be some fluctuation in the angle of attack, but with changing season there

will be a systematic change of the average angle of attack. It is this latter change which may play a role in the seasonal variation in micropulsations.

5.6 Polarization

All the authors on this subject including the present one have considered single mode propagation of circularly polarized longitudinal or transverse wave. In reality the propagation will be in mixed mode, oblique to the geomagnetic field and hence the polarization of the signal at the surface of the earth will be, in general, elliptical. The polarization would be quite variable due to variability of mode coupling in the transmission path and abrupt changes in the conductivity at the surface of the earth. (Dawson and Sugiura, 1963; Pope et al, 1963; Santirocco and Parker, 1963.)

5.7 Probable Origin of Class Distinctions of Pulsations

Some of the various classes of micropulsations may have somewhat different origins. The classes Pc's and Pi's (Jacobs et al, 1964) are certainly due to different mechanisms. In view of our discussion in section 5.2 we can only conjecture the following. Pc 1 may be due to hydromagnetic emissions (Wentworth and Tepley, 1962; Jacobs and Watanabe, 1963) or hydromagnetic whistlers (Obayashi, 1965). Some of Pc2 and Pc 3 may be related to pulsating current systems in the ionosphere postulated by Campbell and Matsushita (1962), or to transient disturbance of the geomagnetic field due to sudden commencement, or to precipitation of energetic particles through other mechanisms. Most of Pc 4 and Pc 5 and at least some of Pc 2 and Pc 3 may be due to the mechanism proposed in the present theory. Lastly,

Pi 1 and Pi 2 are probably due to transient disturbance of the geomagnetic field initiated by precipitation of energetic particles through a variety of geophysical causes.

6. Conclusions and Discussion

6.1 Conclusions

A theory of one of several possible mechanisms responsible for micropulsations is presented in this paper. A source mechanism, its strong effect on the bandwidth and relatively small effect on the spectrum, the important role of the transmission path in determining the spectrum and the crucial role of the variation of the path length in the outer magnetosphere on the spectrum of the received signal are discussed. Specifically, the following conclusions are drawn.

(i) A source mechanism at the interface between solar wind and the magnetosphere is proposed. It is shown that the hydromagnetic surface waves generated by an instability on this interface are transmitted through the magnetosphere and received on the surface of the earth as micropulsations.

(ii) The surface waves suffer strong reflections due to the difference in their velocity and the Alfvén velocity in the regions where the geomagnetic lines leave the interface, and/or the strong variation of the surface wave velocity on the interface. These set-up characteristic wave lengths and the resulting bandwidth is found to be a few tenths of a second to hundreds of seconds. The spectral lines at the source may be very dense and have the relatively small effect of probably controlling the fine structure of the spectrum at the surface of the earth.

(iii) The amplitude spectrum of micropulsation signal at the surface of the earth is essentially determined by the resonances of the transmission path. The transmission of micropulsations through the entire path in the magnetosphere from its outer boundary to the surface of the earth is discussed in terms of hydromagnetic transmission lines. It is found that the most likely resonances of transmission are due to the path in the outer magnetosphere and are approximately at $T \sim 935, 311, 134, 103$ and 71 . The resonances of the lower exosphere and upper ionosphere are usually marked out by the effect of the path in the outer magnetosphere, because the overall transmission is extremely sensitive to the variation in the path length in the outer magnetosphere. This variation is always to be expected due to the variable solar wind conditions. In the exceptional situation when the "temporal" resonances of the lower exosphere and upper ionosphere coincide with the "spatial" resonances of the outer magnetosphere, we again find significant resonances of the overall transmission. Under those improbable conditions, we find resonances at approximately $T \sim 39, 14, 9, 7$ and 4 .

For the class of micropulsations caused by a source mechanism in the lower exosphere or ionosphere, the group of resonances at $T \sim 39, 14, 9, 7$ and 4 will be dominant in determining the amplitude spectrum.

(iv) Giant pulsations of very long periods and very high amplitudes are well explained by the theory.

(v) The persistence or occurrence probability is a function of the sensitivity of the overall transmission on the variation of the path length in the outer magnetosphere. Hence the sharp peaks in this sensitivity at

$T \sim 2.5, 6, 11$ and specially 20 correspond to spectral peaks in the power spectral density. The power spectral density depends both on the occurrence probability and the squared amplitude of transmission. Hence the broad features of the spectrum are reflected in the product of this sensitivity of the overall transmission and the squared amplitude of transmission. At large T this reveals a slope of approximately 6 db/octave.

(vi) The long period micropulsations due to the mechanism proposed here should have essentially no correlation with K or K_p indices, with the exception of the very long period pulsations which can have a somewhat negative correlation.

6.2 Discussion

Now we discuss the limitations and reservations of the theory presented in this paper.

(i) The source mechanism proposed here is only one of several possible mechanisms responsible for various classes of micropulsations. The other possibilities have been discussed in section 5.2.

(ii) The coupled modes of wave motion caused by the instability on the curved bounded surface of the magnetosphere are not analytically discussed with any rigour. Furthermore, the finite amplitude of these waves needs to be determined through non-linear analysis. Only then the complete spectrum at source can be found and consequently the actual spectrum and its fine structure at the surface of the earth can be determined.

(iii) Our discrete layered model of the transmission path is obviously a drastic simplification of the

real continuous problem. The latter, however, necessitates extensive numerical solutions.

(iv) All the authors on this subject including the present one have considered single mode propagation of circularly polarized longitudinal or transverse wave. In our case it is a little more justified, because our source mechanism will tend to direct hydromagnetic energy mostly in the Alfvén mode (linearly polarized) which is the low frequency limit of the circularly polarized transverse wave. Even then, in reality, the propagation will be somewhat oblique to geomagnetic field in mixed mode. It is clear that only a mixed mode analysis will reveal latitude dependence and polarization of micropulsation received at the surface of the earth.

(v) A rigorous and comprehensive derivation of power spectral density is lacking in all work on the subject including the present one.

(vi) The explanations of correlations with various geophysical phenomena discussed in section 5.3 are probably reasonable conjectures. But the discussions on fine structure, latitudinal, diurnal and seasonal variations, polarization and distinctions of various classes of micropulsations in sections 5.4 through 5.7 are quite incomplete and hypothetical.

Since the preparation of this paper, Parks, McPherron and Anderson (1966) have demonstrated correlations between micropulsations and energetic electron precipitation and auroral substorms, as briefly mentioned in section 5.3. Heacock (1966) has found a band of micropulsations at $T \sim 4$, which is in remarkable agreement with our estimate of exactly the same period in sections 3.4

and 4. Lastly, Fernando and Kannangara (1966) have found pulsations of time period 38 ± 5 and 60 ± 5 , which can be compared with our estimates of $T \sim 39$ and 71, respectively, in sections 3.4 and 4.

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CAPTIONS FOR FIGURES

- Fig. 1 Model of the magnetospheric boundary. Cross section in the plane containing the earth-sun line and the dipole axis of the earth. V_o is the component tangential to the boundary and V_s is the undisturbed velocity of the solar wind. Spherical coordinates γ, θ, ψ are used. (Sen, 1965).
- Fig. 2 An upper bound of the time period of the fundamental mode of the waves on the magnetospheric boundary surface. The heavy curve is the polar plot of θ'_c (plotted as latitudinal angle) vs ψ (plotted as local time), as calculated from Eq. (8). The numbers next to these indicate the period of the fundamental mode calculated from Eq. (9). The figure is symmetrical about the equator of the hemispherical boundary.
- Fig. 3 A layered model of the propagation path of micropulsations from the magnetospheric boundary to the surface of the earth. Alfvén speed V_a , conductivity σ and length l_i of each layer is shown.
- Fig. 4 (a) Hydromagnetic transmission line analog of the layered model of the propagation path of Fig. 3. The length l_i , transmission coefficient $|T_i|$, intrinsic impedance η_i , propagation constant k_i , magnetic field H_i and impedance Z_i of each section are shown. All the quantities are defined in Sec. 3.2. (b) The reduction of the cascaded transmission line for the calculation of resonances.

- Fig. 5 Transmission line analog for the simplified model of illustrative example 3.3. Fixed typical values of R_2 and X_2 have been taken.
- Fig. 6 Transmission of micropulsations through middle and lower exosphere and upper ionosphere. $|T_3|$ and $|T_4|$, the transmission coefficient through the middle exosphere l_4 and the lower exosphere and upper ionosphere l_3 , is plotted vs time period T . Equations (15) through (18) have been used to obtain this plot.
- Fig. 7 Variation in the overall transmission of micropulsations with the variation in the path length in the outer magnetosphere. The overall transmission coefficient $|T_0'|$ is plotted vs the percentile variation $\epsilon\%$ of the path length l_5 in the outer magnetosphere for various values of the time period T noted on the curves. $|T_0'|$ is the overall transmission coefficient through the entire path down to the lower ionosphere. The values of time period T chosen are those for which the lower exosphere and upper ionosphere l_3 and the middle exosphere l_4 are in resonance. Equations (36) and (37) and Fig. 6 have been used.
- Fig. 8 Variation in the overall transmission of micropulsations with the variation in the path length in the outer magnetosphere. The overall transmission coefficient $|T_0'|$ is plotted vs the percentile variation $\epsilon\%$ of the path length l_5 in the outer mag-

netosphere for various values of the time period T noted on the curves. $|T_o'|$ is the overall transmission coefficient through the entire path down to the lower ionosphere. The values of time period T chosen are those for which the lower exosphere and upper ionosphere, l_3 ; and the middle exosphere, l_4 , are off resonance. Equations (36) and (37) and Fig. 6 have been used.

Fig. 9 Sensitivity of overall transmission on the path length in the outer magnetosphere. Percentile variation $\delta\%$ of the path length l_5 in the outer magnetosphere required to reduce the overall transmission $|T_o'|$ by a factor of 10, is plotted vs time period T . Equations (15) through (18) and Eq. (38) have been used.

Fig.10 Approximate total attenuation due to ionospheric and other factors. The total attenuation $|T_{att}|$ is plotted vs time period T .

Fig.11 The greatest upperbound of the overall transmission of micropulsations. The greatest upperbound $|T_o|_{\text{abs max}}$ of the overall transmission through the entire path down to the surface of the earth is plotted vs time period T . Figures 6 and 10 and Eqs. (15) through (18) and Eq. (39) have been used.

Fig.12 $|T_o|^2_{\text{gub}} \delta$, a quantity roughly indicative of the power spectral density $P(\omega)$, is plotted vs. time period T . Eqs. (38) and (39) have been used.

Fig. 13 Sensitivity of the amplitude of micropulsation signal received on the surface of the earth, on the path length in the outer magnetosphere. Percentile variation $\delta\%$ of the path length l_5 in the outer magnetosphere required to reduce the overall transmission coefficient to a maximum possible value of $1/10$ from its greatest upper bound given in Fig. 11, is plotted vs time period T . Figures 6 and 10 and Eqs. (15) through (18) and Eq. (40) are used.

Fig. 14 $|T_o|^2_{gub} \delta'$, another quantity roughly indicative of the power spectral density $P(\omega)$, is plotted vs. time period T . Eqs. (39) and (43) have been used.

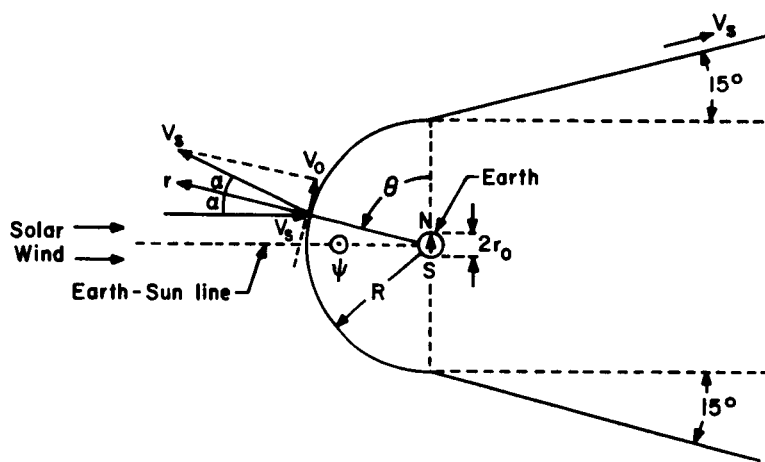


FIG. 1

7 00-LE-UT/7

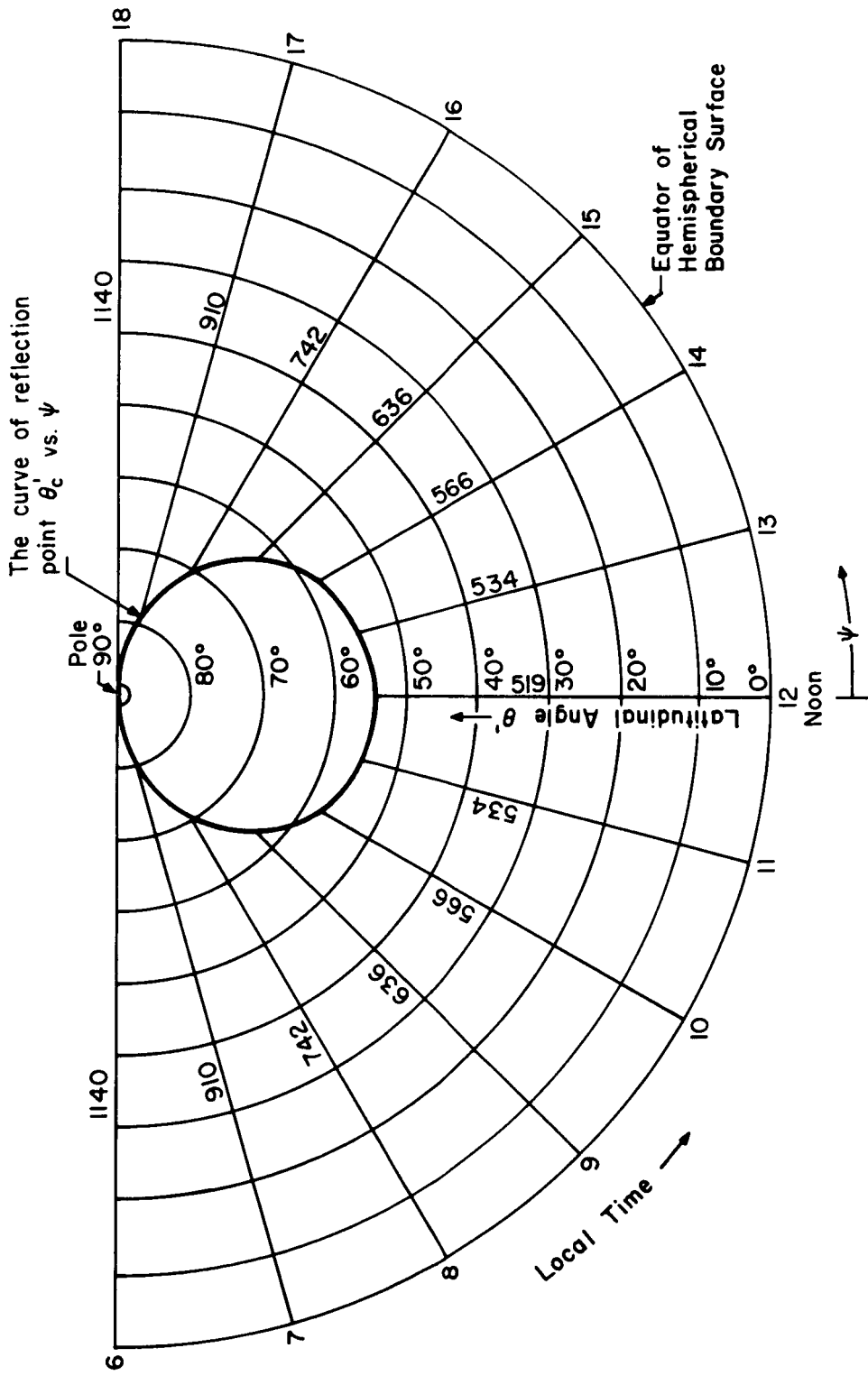


FIG. 2

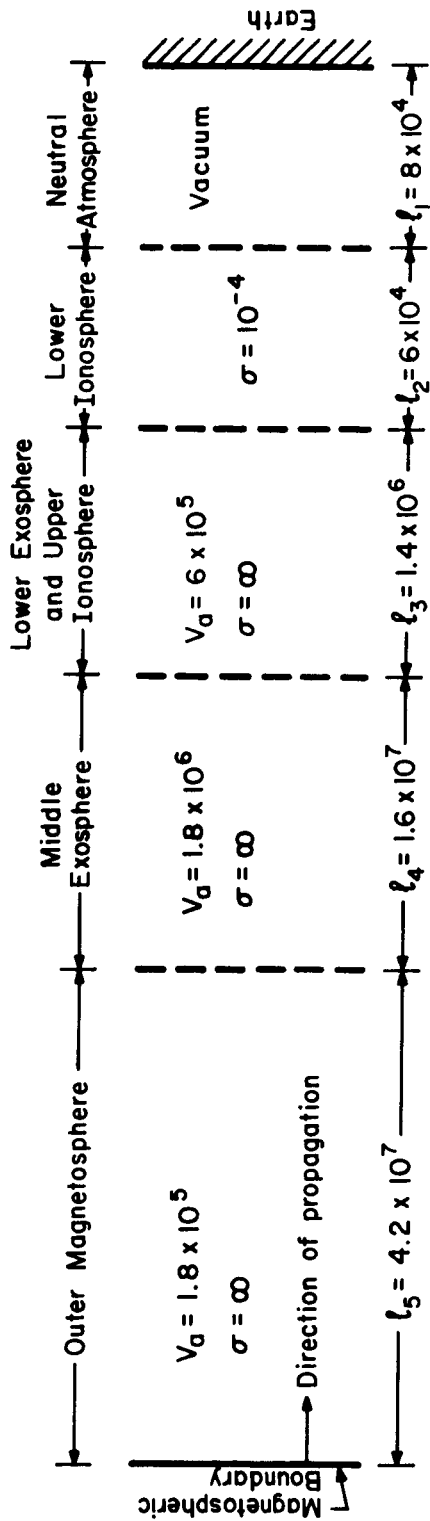
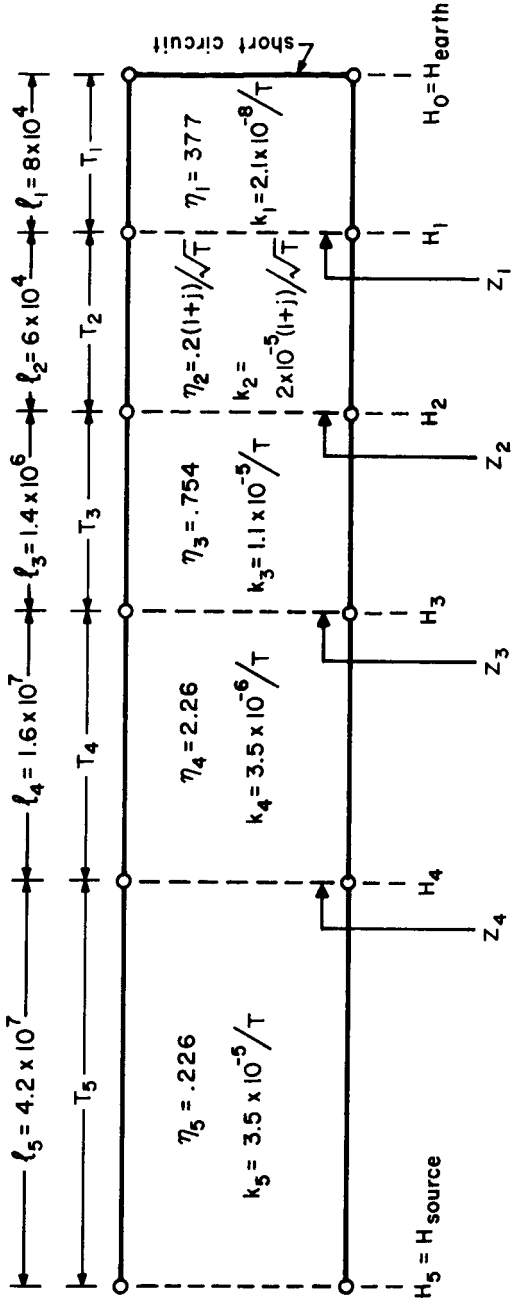
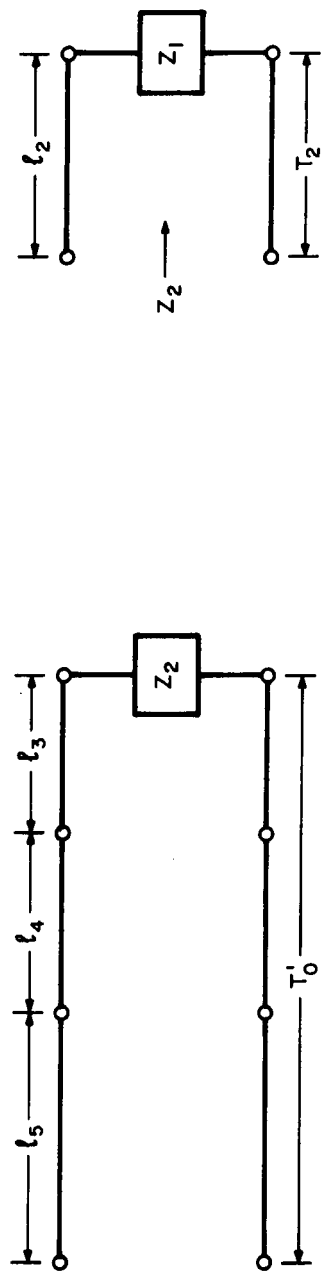


FIG. 3



(a)



(b)

FIG. 4

l_{5-4} is somewhat variable with
the nominal value of 5.8×10^7

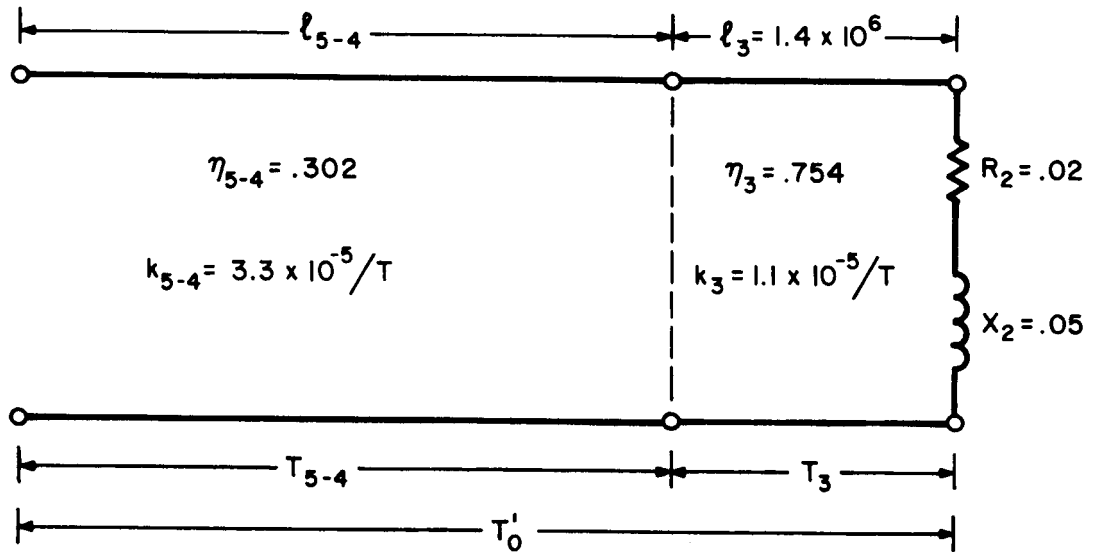


FIG. 5

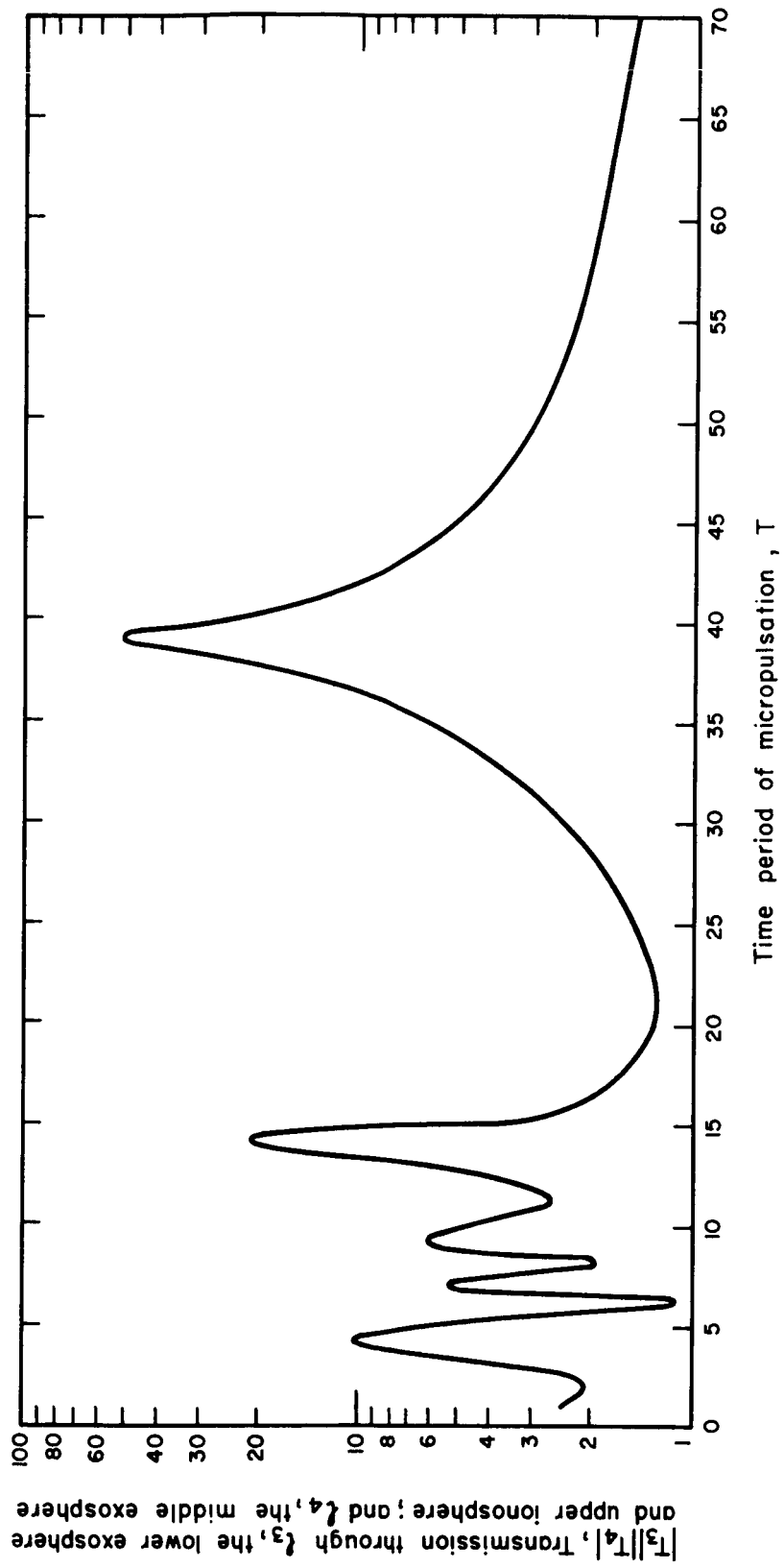


FIG. 6

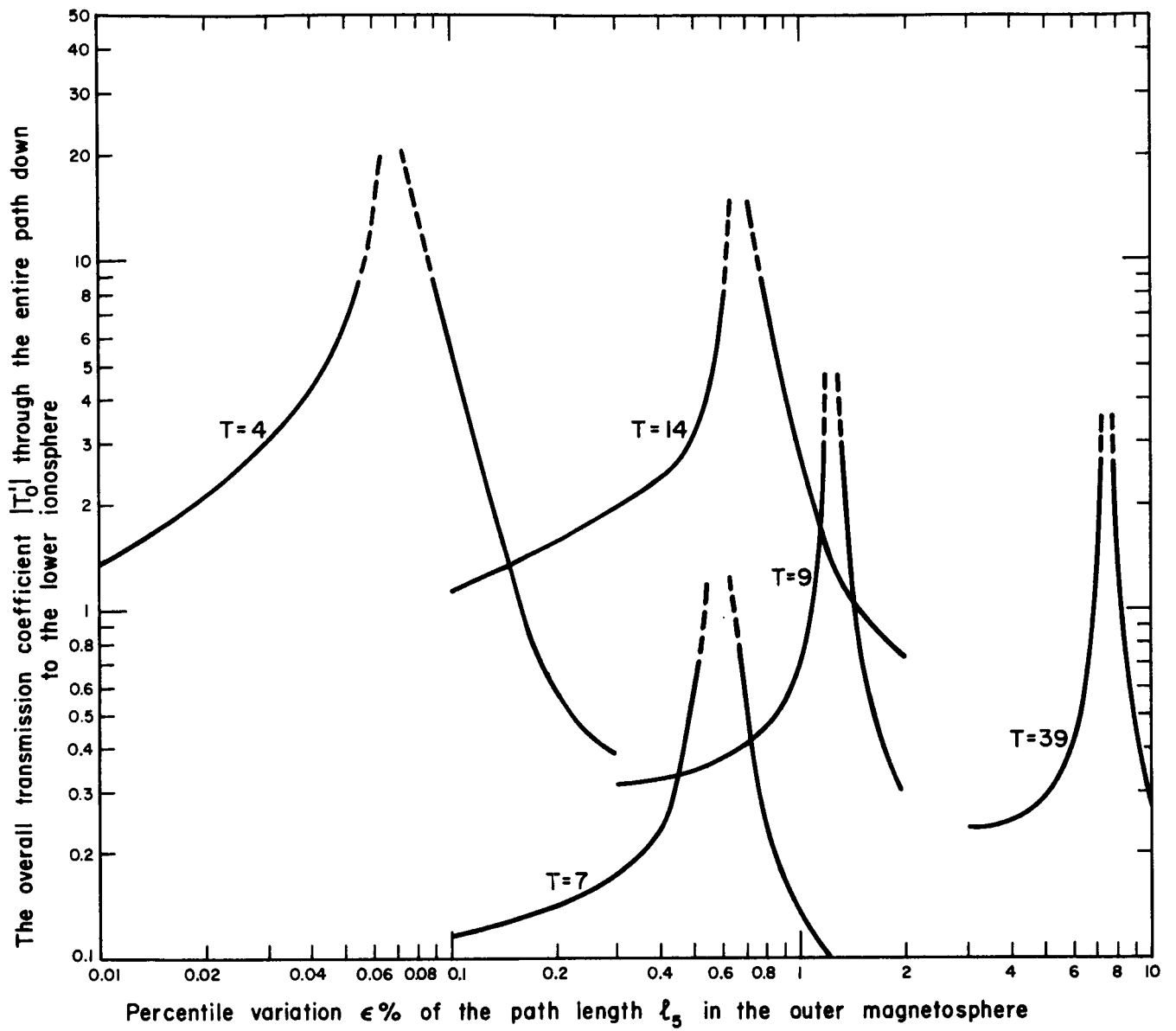


FIG. 7

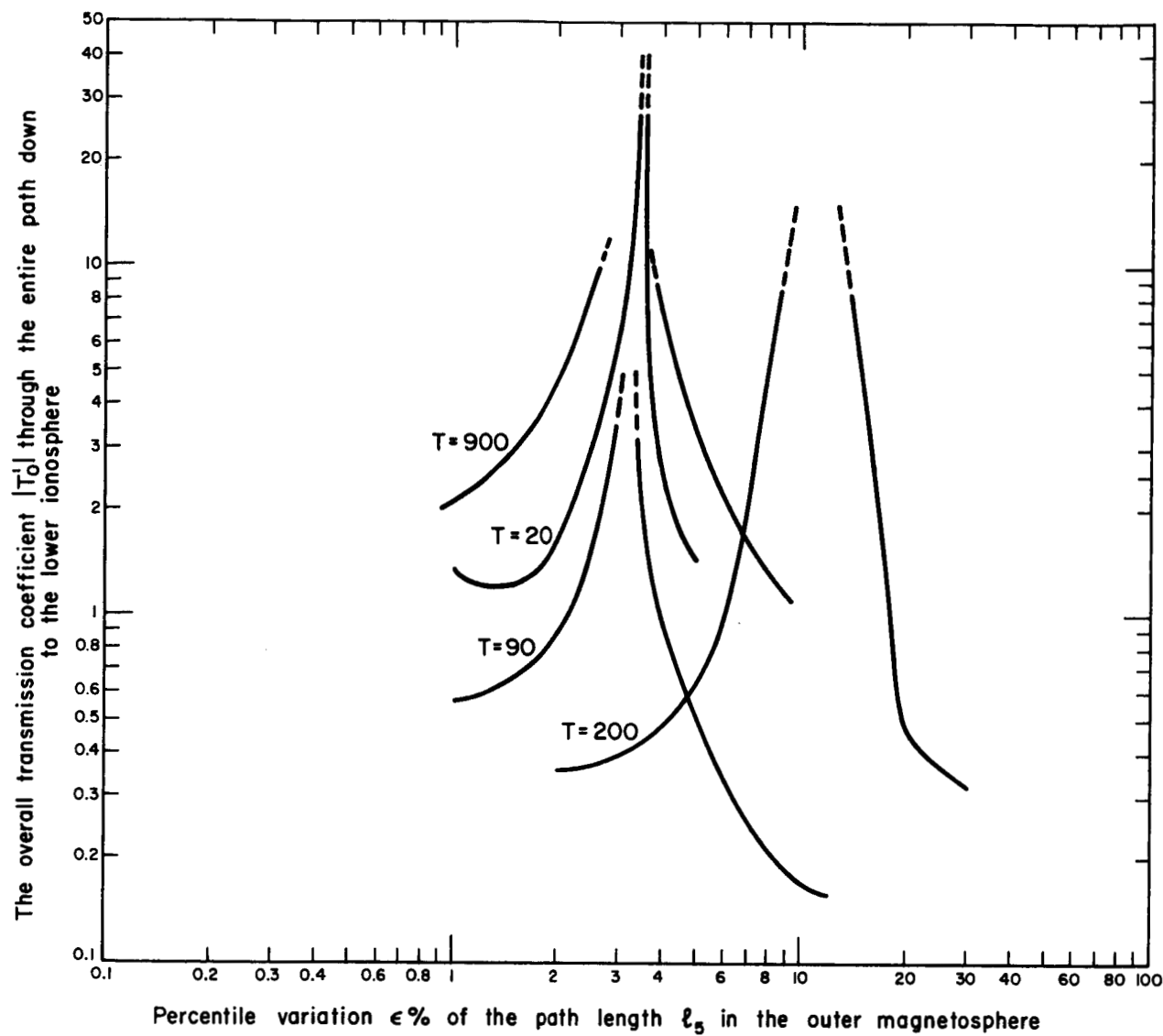


FIG. 8

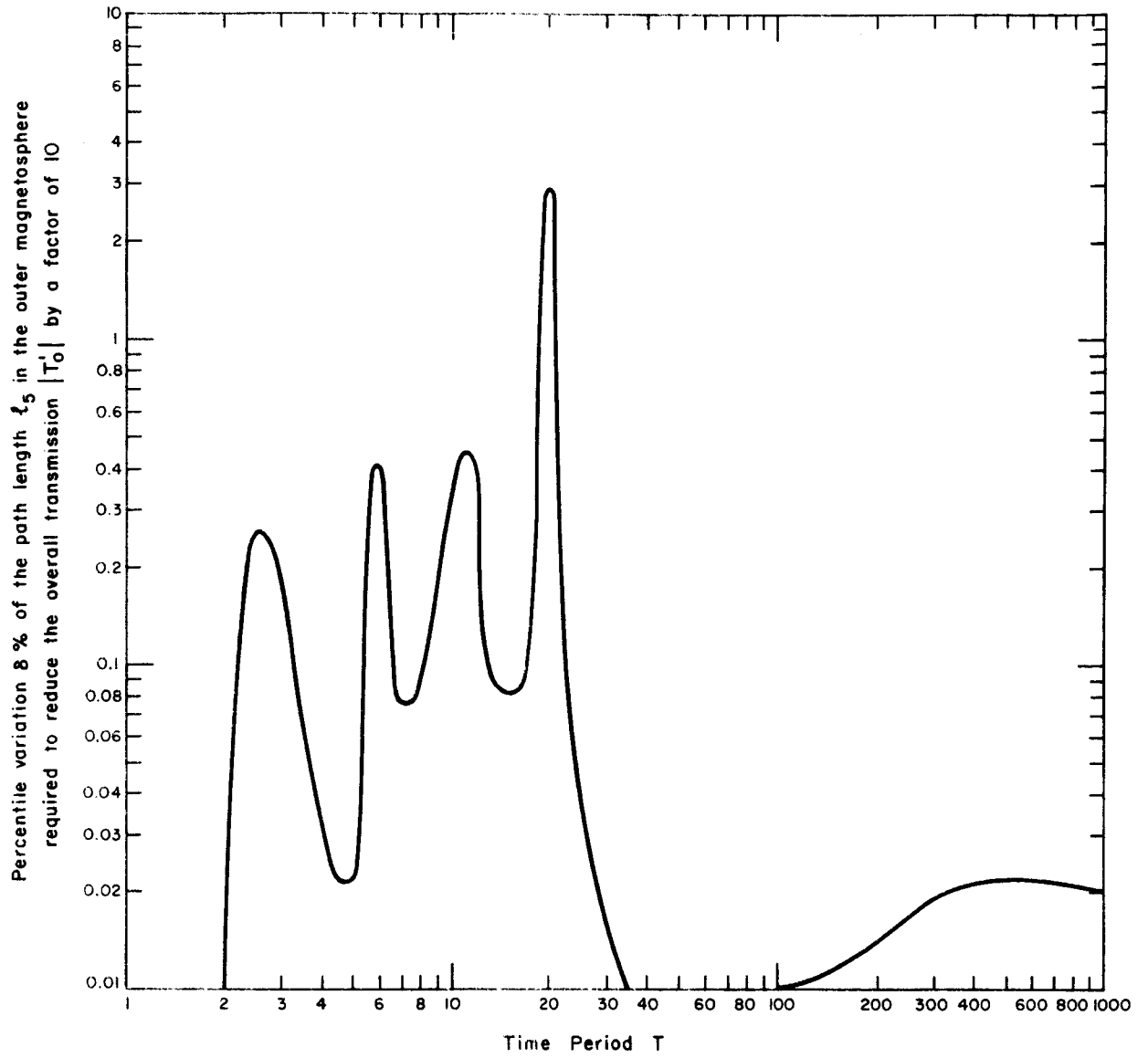


FIG. 9

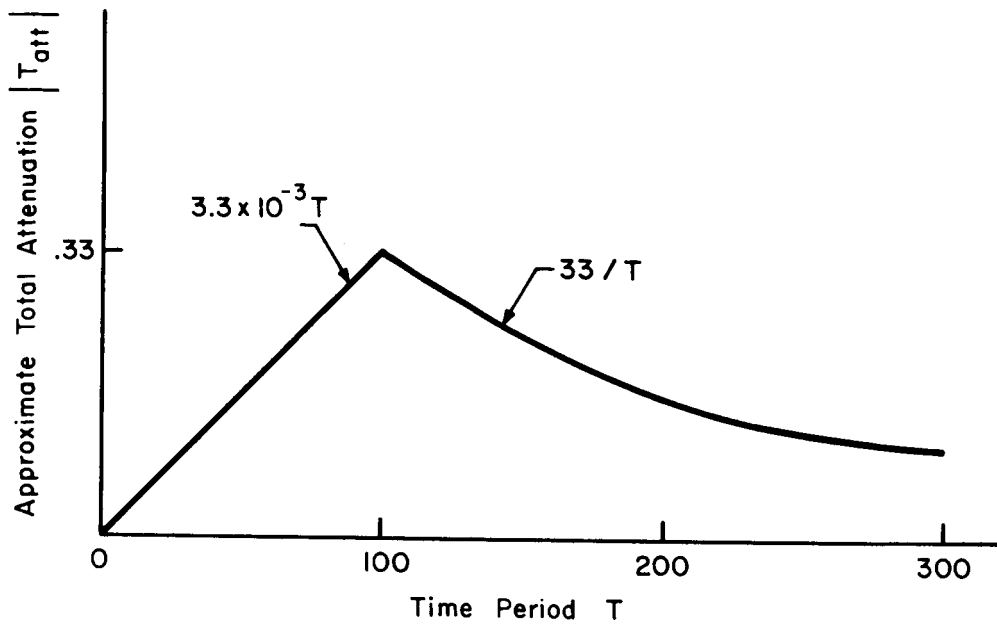


FIG. 10

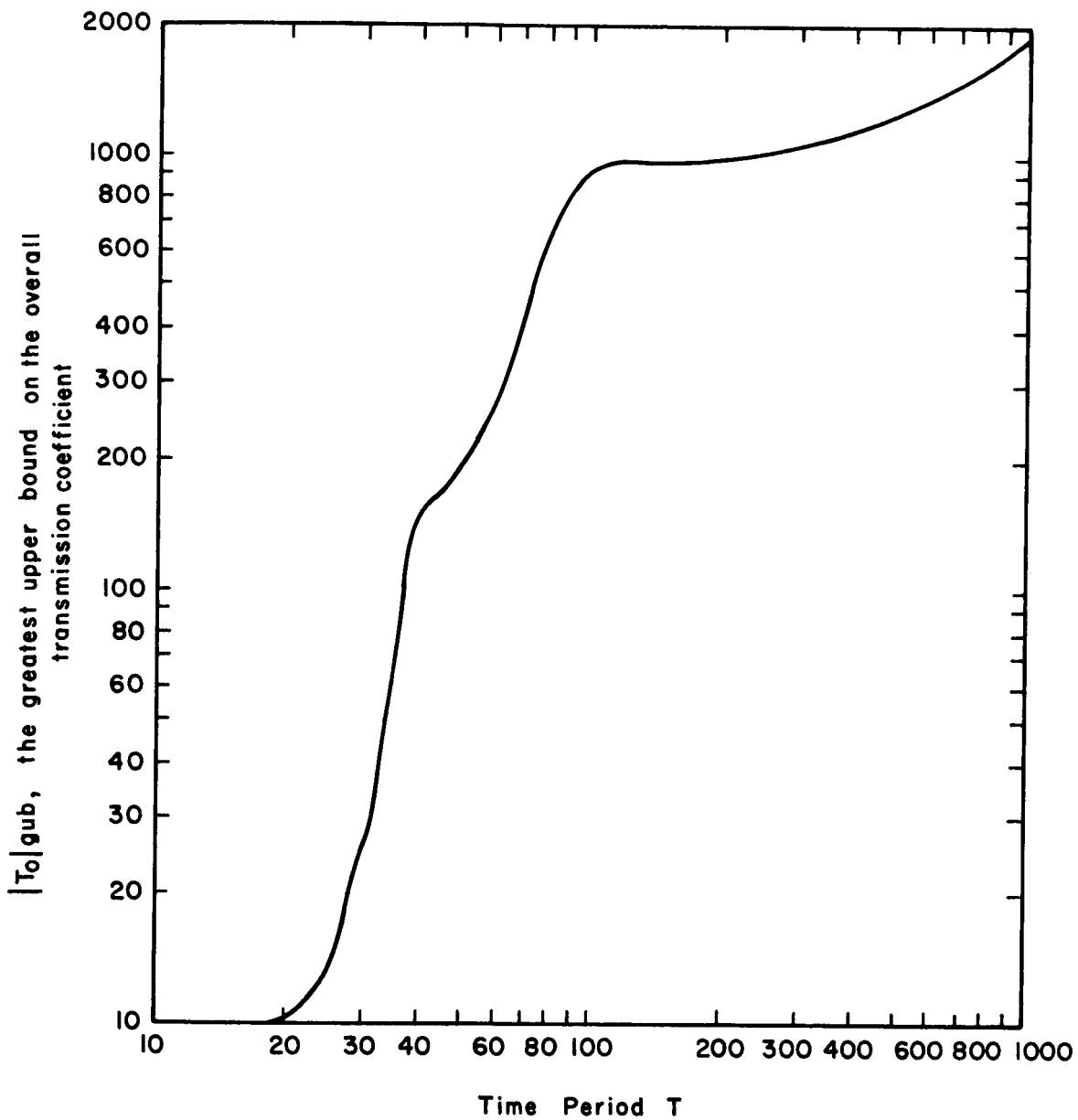


FIG. 11

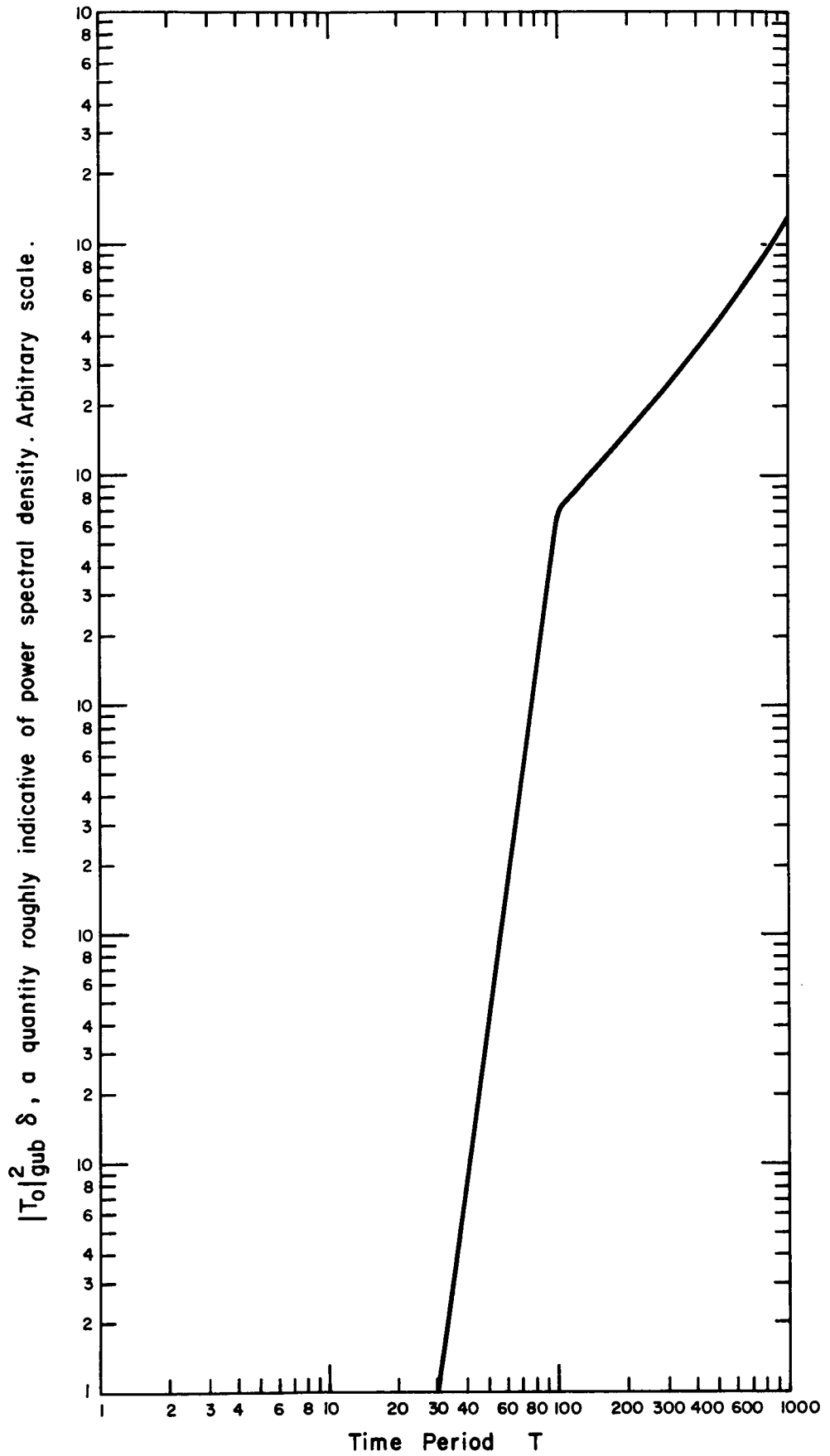


FIG. 12

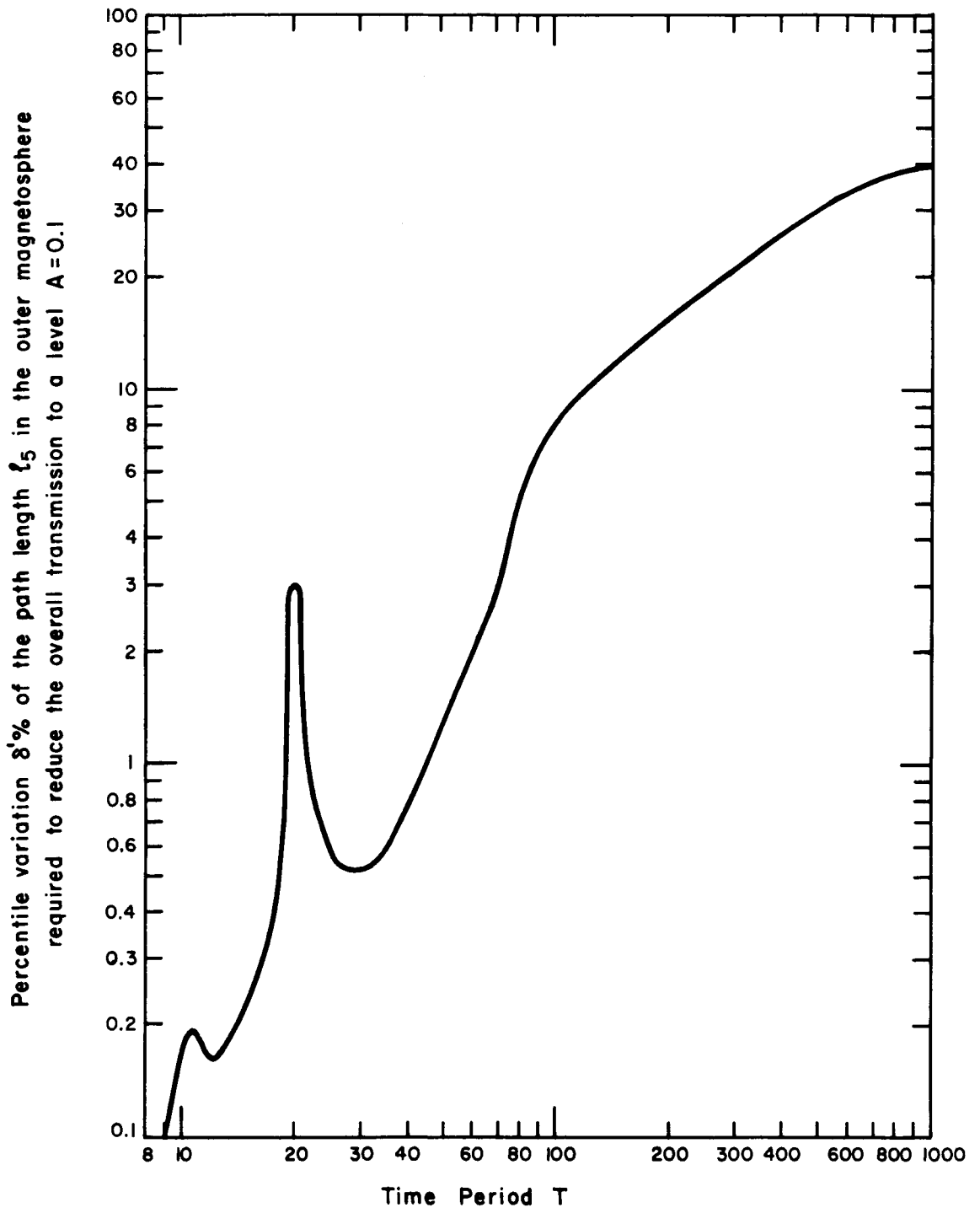


FIG. 13

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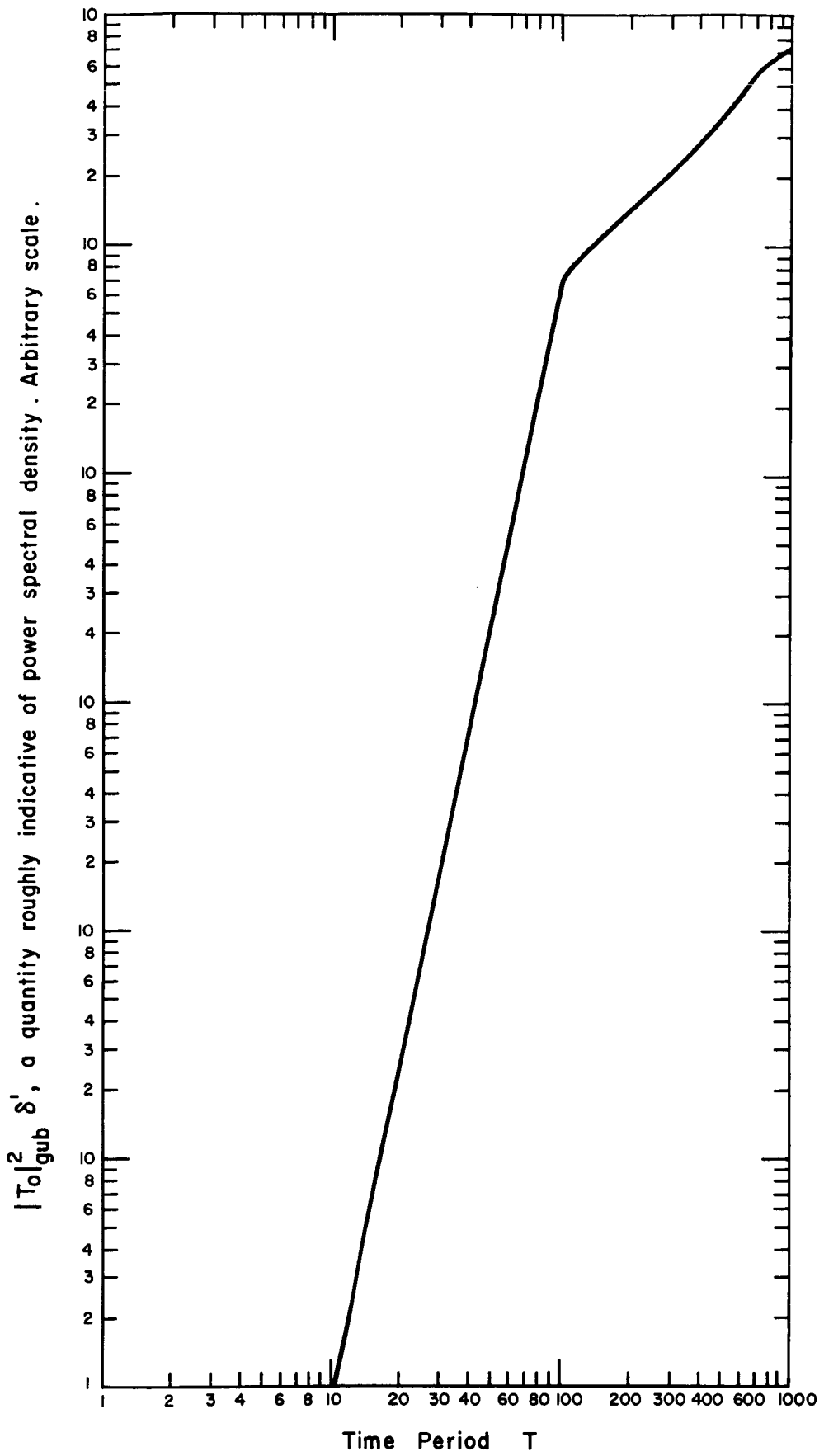


FIG. 14