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GRATINGS AND NETWORKS OF RIBBONS AS RADIO REFLECTORS

by William M. Robbins, Jr.

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ASTRO RESEARCH CORPORATION
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GRATINGS AND NETWORKS OF RIBBONS AS RADIO REFLECTORS

By William M. Robbins, Jr.
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SUMMARY

The reflecting properties for electromagnetic radiation, at normal incidence, of a surface formed from gratings or networks of conducting ribbons are analyzed and described in terms of two reflection coefficients. The geometrical reflection coefficient expresses the ratio of reflected to incident power for a grating formed of perfectly conducting elements. The ohmic reflection coefficient expresses the ratio of reflected to incident power for a continuous thin sheet of equivalent surface conductivity.

A network of aluminum ribbons, suitable for use in a space environment at a frequency of 4 mc/sec, with a geometrical reflection coefficient of 0.98 and an ohmic reflection coefficient of 0.99 can be made to have a surface mass density of 270 kg/km^2. In comparison, a continuous sheet of aluminum, 0.5 mil thick, has a surface mass density of 34,300 kg/km^2.

INTRODUCTION

Current interest in the design of very large reflectors for orbiting radiotelescopes operating at low frequencies has led to the investigation of methods whereby the surface density of radio reflectors can be made as low as possible. This need for lightness, coupled with the requirements for reflectivity, for operation in a micrometeoroid environment, and for maintaining a geometrically precise surface, has lead the Astro Research Corporation to the investigation of the properties of networks of ribbons. The results of the studies concerned with the reflection characteristics of such networks are described in this report.

THE GEOMETRICAL REFLECTION COEFFICIENT

When a plane electromagnetic wave impinges at normal incidence upon an infinite grating of perfectly conducting circular cylinders, the E vector being parallel to the axes of the cylinders, a certain fraction of the incident power passes through the grating and the remainder is reflected. According to reference 1, the amplitude reflection coefficient is given by

$$R_a = -\frac{1}{1 - \frac{i \cdot 2s}{\lambda} \cdot \ln\left(\frac{s}{\pi d}\right)} \qquad \text{when } \frac{s}{\lambda} \ll 1 \tag{1}$$

where:

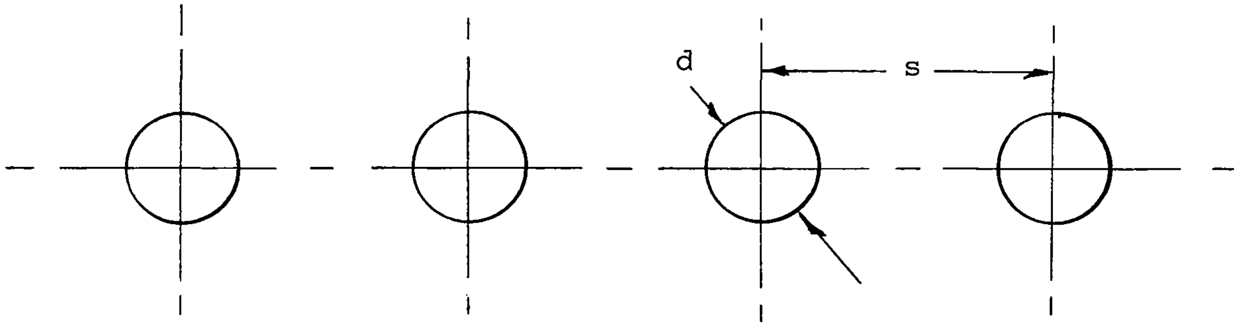
R_a = amplitude reflection coefficient

s = spacing between centers of cylinders

d = diameter of cylinder

λ = wavelength of electromagnetic radiation

i = $\sqrt{-1}$



It follows that the power reflection coefficient, R , is:

$$R = \frac{1}{1 + \frac{4s^2}{\lambda^2}\cdot \ln^2\left(\frac{s}{\pi d}\right)} \qquad (2)$$

The present study is concerned with the use of very light-weight radio reflectors for use in orbiting radiotelescopes, and, therefore, only reflectors made of conductors with cross-sectional dimensions small compared with the conductor spacing are of interest. No analysis of the reflectivity of such networks formed of ribbons has been located. However, in order to obtain any appreciable value of geometrical reflectivity, the spacing of conductors must be only a small fraction of the wavelength of the reflected electromagnetic energy. Therefore, the conductor size will also be very small indeed compared with a wavelength. When a grating of such conductors is reflecting an incident electromagnetic wave, the electromagnetic field at any appreciable distance (compared with conductor dimensions) from the plane of the conductors will depend only upon the current flowing in the conductors and upon the incident radiation, and not upon the conductor shape. The current, in turn, will depend only upon the energy which is stored in the electromagnetic field as a result of a given current and not upon the shape of the conductor. As already stated, only the field in the very near vicinity of a conductor will be affected by the conductor shape and the dimensions of this region will be small enough that the field will be very nearly that which would be caused by a steady current of the same instantaneous magnitude. It is assumed, because of the above reasoning, that two conductors about which there is stored the same energy in the magnetic field when the same current flows, will be interchangeable in the grating insofar as reflectivity is concerned.

It is assumed that the current in each ribbon is small enough that the current distribution in the ribbon is uniform across the ribbon, an assumption which is generally valid except for very large current densities or for conditions where superconductivity occurs. One half of such a ribbon is shown in figure 1 where it is assumed to be carrying a surface current density $\bar{J}_s$. Then:

$$\bar{n} \times \left(\bar{H}_2 - \bar{H}_1\right) = \bar{J}_s \qquad (3)$$

where:

$\bar{n}$ is unit vector normal to surface

$\bar{H}_2$ and $\bar{H}_1$ are the magnetic field intensities

just outside each face of the ribbon. Then by symmetry the component of H parallel to the ribbon must be constant along the ribbon and equal to $J_s/2$. Also the field at a large distance from the strip (but still much less than one wavelength) is the same as that surrounding a circular cylindrical conductor. These two boundary conditions, plus two planes of symmetry (the plane of the ribbon and the plane normal to the ribbon through its center) allow the magnetic field to be plotted by the method of curvilinear squares with the two sets of curves representing lines of constant scalar magnetomotive potential and lines of magnetic flux.

The energy stored in the magnetic field per unit of volume is then $\frac{1}{2}\bar{B}\cdot\bar{H}$ and since the side of a curvilinear square is proportional to $1/B$, each square corresponds to a volume (for unit distance normal to the sheet) which contains the same energy in the magnetic field as every other square. The number of curvilinear squares about the ribbon (in one quadrant) to the outer line of magnetic flux (with an average diameter of 4.22 times the strip width, as indicated by the dotted reference circle) has been counted to be $115\frac{1}{2}$. The circular cylinder which would have the same number of curvilinear squares between it and the reference circle has been computed to have a diameter of 0.458 of the strip width. The ratio of strip width, w , to cylinder diameter, d , for equivalent conductors, is then taken to be

$$\frac{w}{d} = \frac{1}{0.458} = 2.18 \qquad (4)$$

Using (4) to eliminate d from (2) yields

$$R = \frac{1}{1 + \frac{4s^2}{\lambda^2} \cdot \ell n^2 \left(0.695 \cdot \frac{s}{w} \right)} \tag{5}$$

The relationships expressed in equation (5) are shown graphically in figure 2, where the geometrical reflectivity of a grid of perfectly conducting ribbons is plotted against the ratio of width to spacing for various values of the ratio of wavelength to spacing as a parameter.

THE OHMIC REFLECTION COEFFICIENT

A grid or grating which would be a good reflector of radio waves if the elements were perfectly conducting will be a poor reflector if the elements are made of material with sufficiently high resistivity. In order to determine the effects of resistivity in a reflector, the reflection of a plane wave, at normal incidence, by a very thin infinite sheet of finite surface conductivity has been considered.

An infinite plane of surface conductivity, σs, is assumed to have a cartesian coordinate system placed upon it such that the plane is defined by $z = 0$. A plane electromagnetic wave, moving in the positive-z direction impinges upon the plane, a portion of the energy being reflected, a portion transmitted, and a portion being absorbed by the plane. The impinging wave is assumed to be polarized with its E vector oriented in the x-direction.

Letting the subscripts i, t, and r refer to the incident, transmitted, and reflected waves, respectively, the field vectors very close to the plane at $z = 0$ can be expressed as

$$\bar{E}_i = \bar{1}_x E_i \cdot \cos\omega t \tag{6}$$

$$\bar{H}_i = \bar{1}_y H_i \cdot \cos\omega t \tag{7}$$

$$\bar{E}_t = \bar{1}_x E_t \cdot \cos\left(\omega t - \theta_t\right) \tag{8}$$

$$\bar{H}_t = \bar{1}_y H_t \cdot \cos\left(\omega t - \theta_t\right) \tag{9}$$

$$\bar{E}_r = -\bar{1}_x E_r \cdot \cos\left(\omega t - \theta_r\right) \tag{10}$$

$$\bar{H}_r = \bar{1}_y H_r \cdot \cos\left(\omega t - \theta_r\right) \tag{11}$$

where θ_t and θ_r are phase shifts between the incident wave and the transmitted or reflected wave (other than the 180° shift which would occur for the reflected wave for infinite conductivity in the plane).

Letting the subscripts 1 and 2 refer to the negative-z and positive-z sides of the plane

$$\bar{E}_1 = \bar{E}_i + \bar{E}_r \tag{12}$$

$$\bar{H}_1 = \bar{H}_i + \bar{H}_r \tag{13}$$

$$\bar{E}_2 = \bar{E}_t \tag{14}$$

$$\bar{H}_2 = \bar{H}_t \tag{15}$$

Also for each wave

$$E = HZ_o \tag{16}$$

where $Z_o = 377$ ohms is the characteristic impedance of free space.

The boundary conditions at the plane require that

$$\bar{1}_z \times \left(\bar{H}_2 - \bar{H}_1\right) = \bar{J}_s = \sigma_s \bar{E}_1 = \sigma_s \bar{E}_2 \tag{17}$$

Using equations (6) through (16) to substitute into equation (17) and by noting that all terms in either $\cos\omega t$ or $\sin\omega t$ must separately satisfy the equation, the following two expressions are obtained

$$H_t \cdot \cos\theta_t - H_i - H_r \cdot \cos\theta_r = - \sigma_s Z_o H_i + \sigma_s Z_o H_r \cdot \cos\theta_r =$$

$$= - \sigma_s Z_o H_t \cdot \cos\theta_t \qquad (18)$$

$$H_t \cdot \sin\theta_t - H_r \cdot \sin\theta_r = \sigma_s Z_o H_r \cdot \sin\theta_r = - \sigma_s Z_o H_t \cdot \sin\theta_t \qquad (19)$$

Equation (19) can be used to establish that

$$\sin\theta_r = \sin\theta_t = 0 \qquad (20)$$

From equation (18) it can be shown that

$$\frac{H_r}{H_i} = \frac{\sigma_s Z_o}{\left(2 + \sigma_s Z_o\right) \cdot \cos\theta_r} \qquad (21)$$

which, with (20), leads to

$$\frac{H_r}{H_i} = \frac{\sigma_s Z_o}{2 + \sigma_s Z_o} \qquad (22)$$

The magnitudes of the Poynting vectors of the reflected and incident waves are

$$\left| \bar{P}_r \right| = H_r^2 Z_o \qquad (23)$$

$$\left| \bar{P}_i \right| = H_i^2 Z_o \qquad (24)$$

Defining the ratio of incident power to reflected power as the ohmic reflection coefficient, R' ,

$$R' = \frac{|\bar{P}_r|}{|\bar{P}_i|} = \left(\frac{H_r}{H_i}\right)^2 \tag{25}$$

and

$$R' = \left(\frac{\sigma_s Z_o}{2 + \sigma_s Z_o}\right)^2 \tag{26}$$

Now, instead of being a continuous conducting plane, let the surface consist of a grating of ribbons oriented in the direction of the E vector of the incident wave and of such a width and spacing as to result in a high geometrical reflectivity. Further, let the thickness of the ribbons be small compared with the skin depth, where:

$\delta = \left(\dfrac{\lambda}{\pi\sigma\mu c}\right)^{1/2}$ = skin depth

λ = free-space wavelength

σ = conductivity of ribbon

μ = permeability of ribbon = 1.257×10^{-6} henry/meter

c = speed of light = 3×10^{8} m/sec

Under these conditions the grid has an equivalent surface conductivity of

$$\sigma_s = \sigma \cdot \frac{w}{s} \cdot t \qquad \text{for} \quad t \ll \delta \tag{27}$$

If the thickness of the ribbons is considerably greater than the skin depth, then the equivalent surface conductivity is

$$\sigma_s = \sigma \cdot \frac{w}{s} (2\delta) \qquad \text{for} \quad t \gg \delta \tag{28}$$

For $t \simeq \delta$, only a small error is introduced if (27) is used for $t \leq 2\delta$ and if (28) is used for $t \geq 2\delta$.

For aluminum

$$\sigma = 3.20 \times 10^{7} \quad \text{mho/meter}$$

$$\delta = 5.14 \times 10^{-6} \lambda^{\frac{1}{2}} \quad \text{meter}$$

The skin depth for various metals is shown in figure 3 as a function of wavelength.

The ohmic reflection coefficient for a grating of aluminum ribbons is shown in figure 4 as a function of the width to spacing ratio with the thickness, t, or twice the skin depth, 2δ, as a parameter, depending upon which is appropriate.

NETWORKS OF RIBBONS AS RADIO REFLECTORS

For most applications in space, such as a reflector for a radiotelescope, the generation of a desired surface with ribbons requires more geometrical restriction to the surface than can be accomplished with only one family of parallel ribbons which provides in-plane tensile stiffness in only one direction and no shearing stiffness at all. The use of three families of ribbons, joined at the intersections so as to form a system of triangles, provides a surface with properties closely resembling those of a true membrane which generally has only one possible geometric shape (except for elastic deformations) if it is kept everywhere in tension and the appropriate boundary conditions are applied.

Further, a grating can reflect only that portion of an incident wave with a specific orientation of the E vector and the reflection of an arbitrarily polarized wave requires at least two non-parallel families of conductors in the reflecting surface.

It is of interest here to note that when a plane wave is reflected by an infinite grid of more than one family of conductors connected at the junctions, symmetry requires that the current in any conductor into the junction be identical to that out of the opposite side of the junction. Therefore, no current flows from one conductor set to the other and the electrical connections serve no function. However, care should be taken in applying this conclusion, without further consideration, to the regions near the boundary of a non-infinite network or in the vicinity of members that have been severed.

If a plane wave impinges at normal incidence upon a grid of two orthogonal and identical gratings of conductors, the fields and currents can be resolved into two orthogonal and independant sets for any orientation of the E vector and the overall reflection properties of the grid will be independent of the polarization of the incident wave. For other relative orientations of the grating sets, the electromagnetic fields interact and the problem is much more complicated. However, it seems reasonable to assume that when the E vector of the impinging wave lies between two sets of grating conductors whose angular separation is less than 90° the reflection properties are better than that for a single grating, and that if the E vector lies between two sets of grating conductors whose angular separation is greater than 90°, the reflection properties are worse than for a single grating. Thus, for a network of three families of conductors which form a surface of equilateral triangles, the reflectivity will always be better than for a single grating. However, reflectivities will still be defined as those for one of the gratings.

As an example of the application of the two reflection coefficients to a particular surface, a network of aluminum ribbons which cover the surface with equilateral triangles is considered. The thickness of the ribbons is taken to be 0.5 mil (just about the practical minimum) and the width of the ribbons is taken to be 0.1 inch (a value which results in a low probability of fracture by micrometeroids when used in earth orbit). The wavelength of incident radiation is assumed to be 75 meters (a frequency of 4 mc/sec). The spacing of the ribbons is allowed to vary, which then affects the two reflection coefficients and the mass per unit of surface area. For convenience in plotting, two loss factors (which apply to each of the grating sets) are defined as follows:

$$L = \text{geometrical loss factor} = 1 - R$$

$$L' = \text{ohmic loss factor} = 1 - R'$$

The results are shown in figure 5.

As a comparison, a continuous sheet of aluminum, 0.5 mil thick, has a surface density of 34,300 kg/km^2 .

CONCLUDING REMARKS

The reflection properties for electromagnetic radiation, at normal incidence, of a surface formed by gratings or networks of ribbons are analyzed and described in terms of the geometrical and ohmic reflection coefficients.

No attempt has been made herein to establish a required overall reflection coefficient for any given application. However, the attainment of a high overall reflection coefficient, as may be desirable for certain radiotelescope applications, certainly requires that the geometrical and ohmic reflection coefficients each be individually large.

The required mass for attaining a given geometrical coefficient could, theoretically, be made as small as desired but a lower limit is placed by factors such as minimum gage and vulnerability to micrometeoroid damage.

The required mass for attaining a given ohmic reflection coefficient is essentially a matter of having available sufficient conductive material to carry the required reflector currents without undue resistive losses.

Astro Research Corporation
Santa Barbara, California, November 16, 1966.

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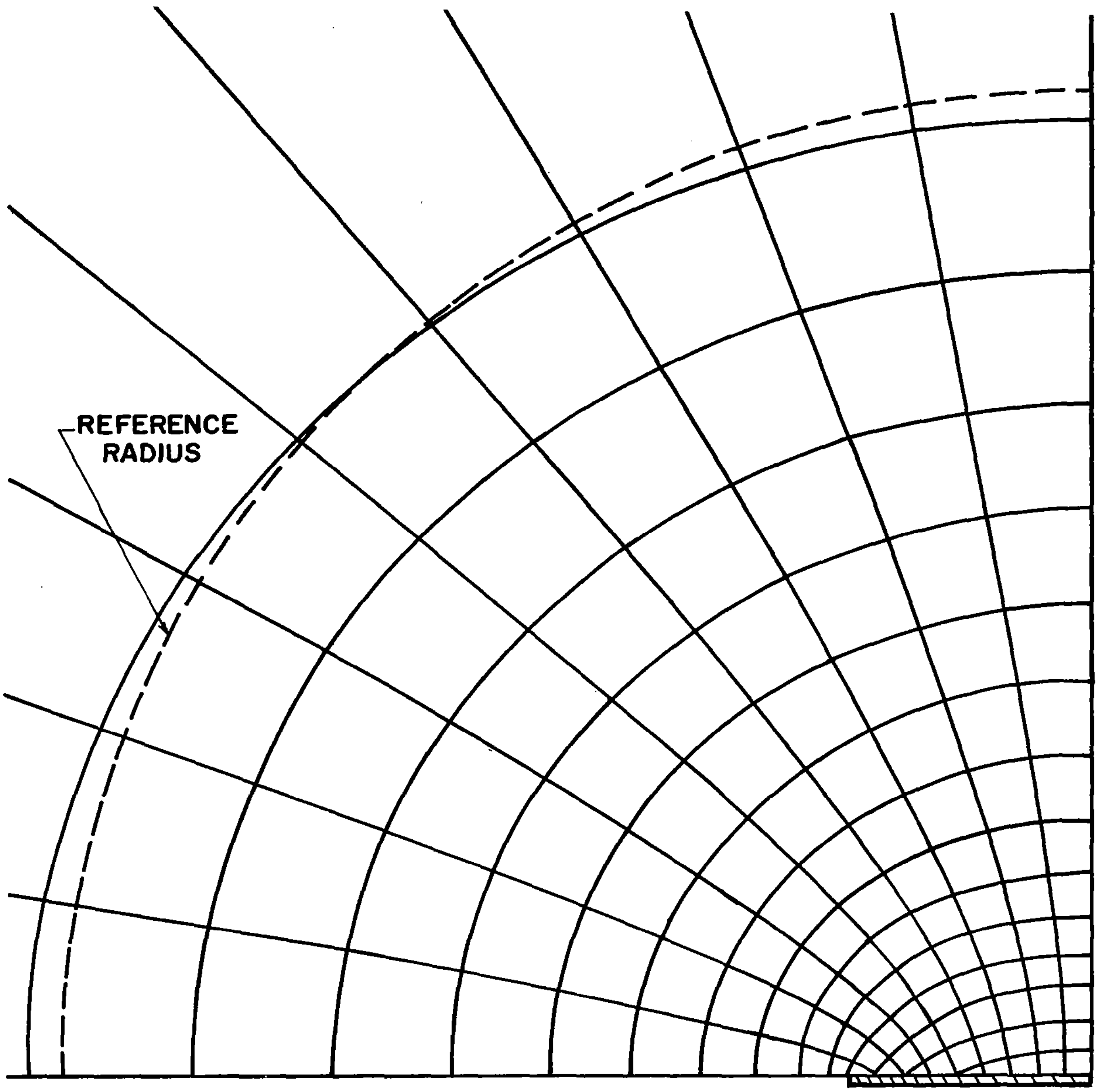

Figure 1. — Magnetic Field about a Conducting Strip

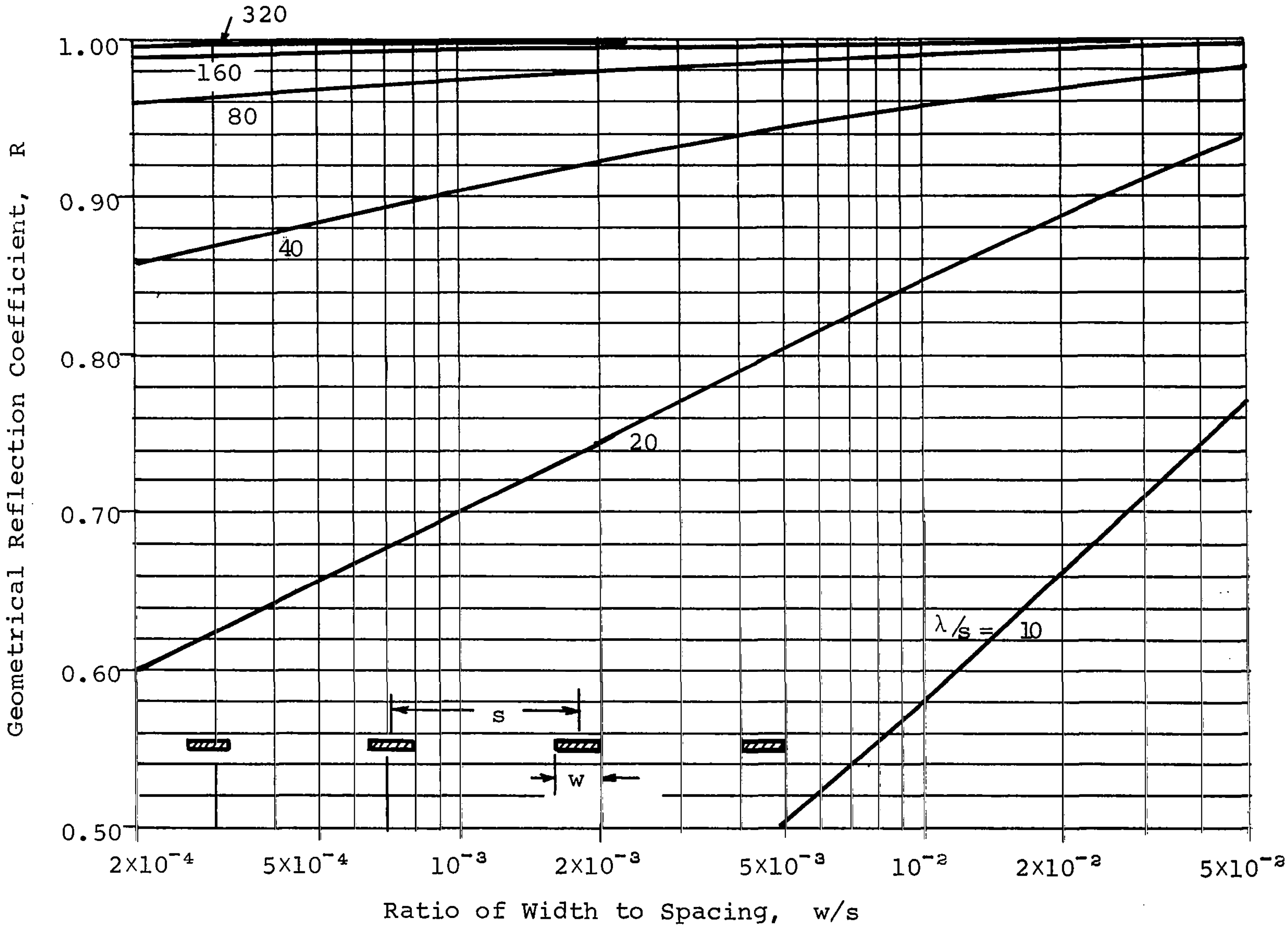

Figure 2. — Geometrical Reflection Coefficient of Grating of Ribbons

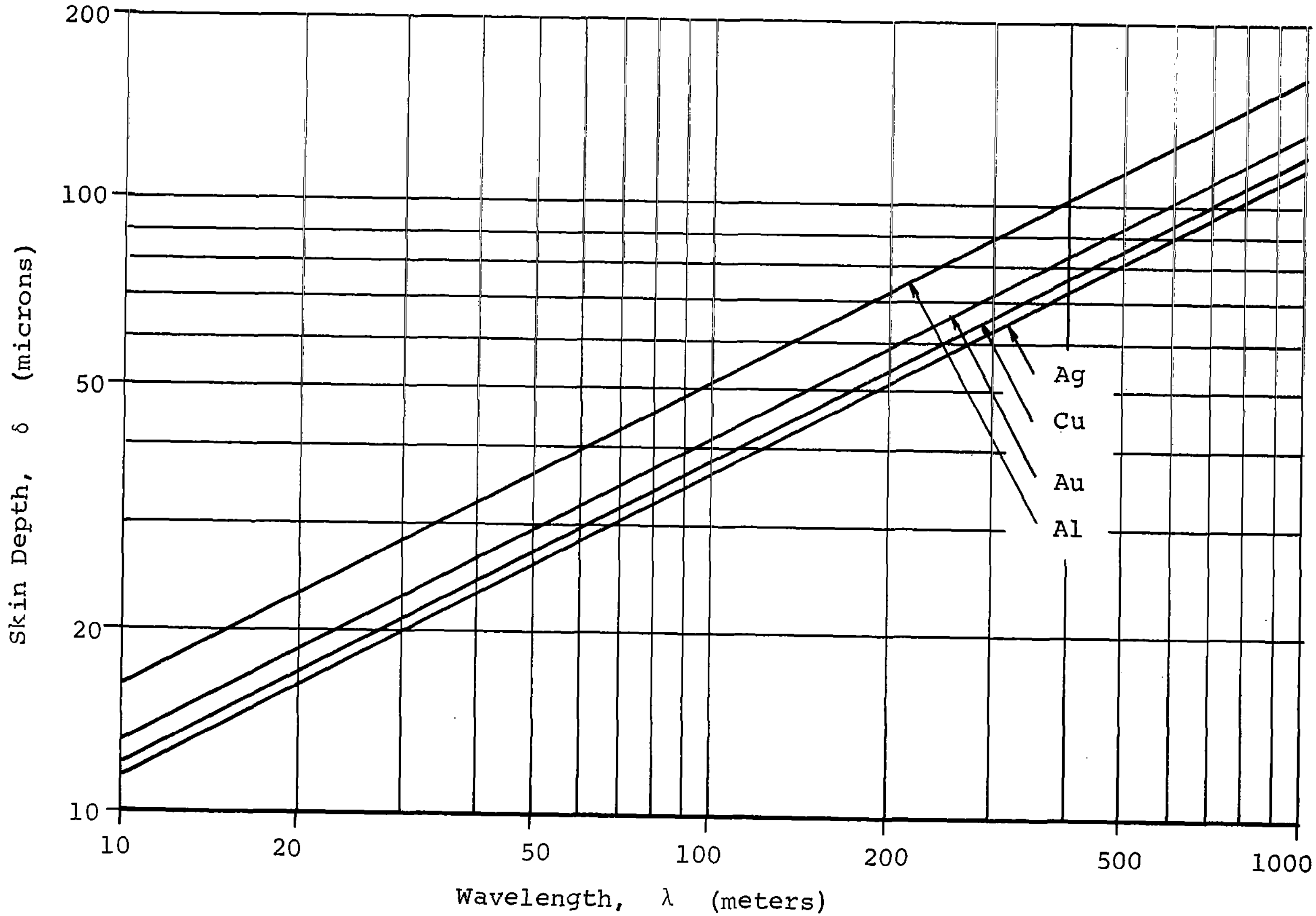

Figure 3. — Skin Depth in Various Metals

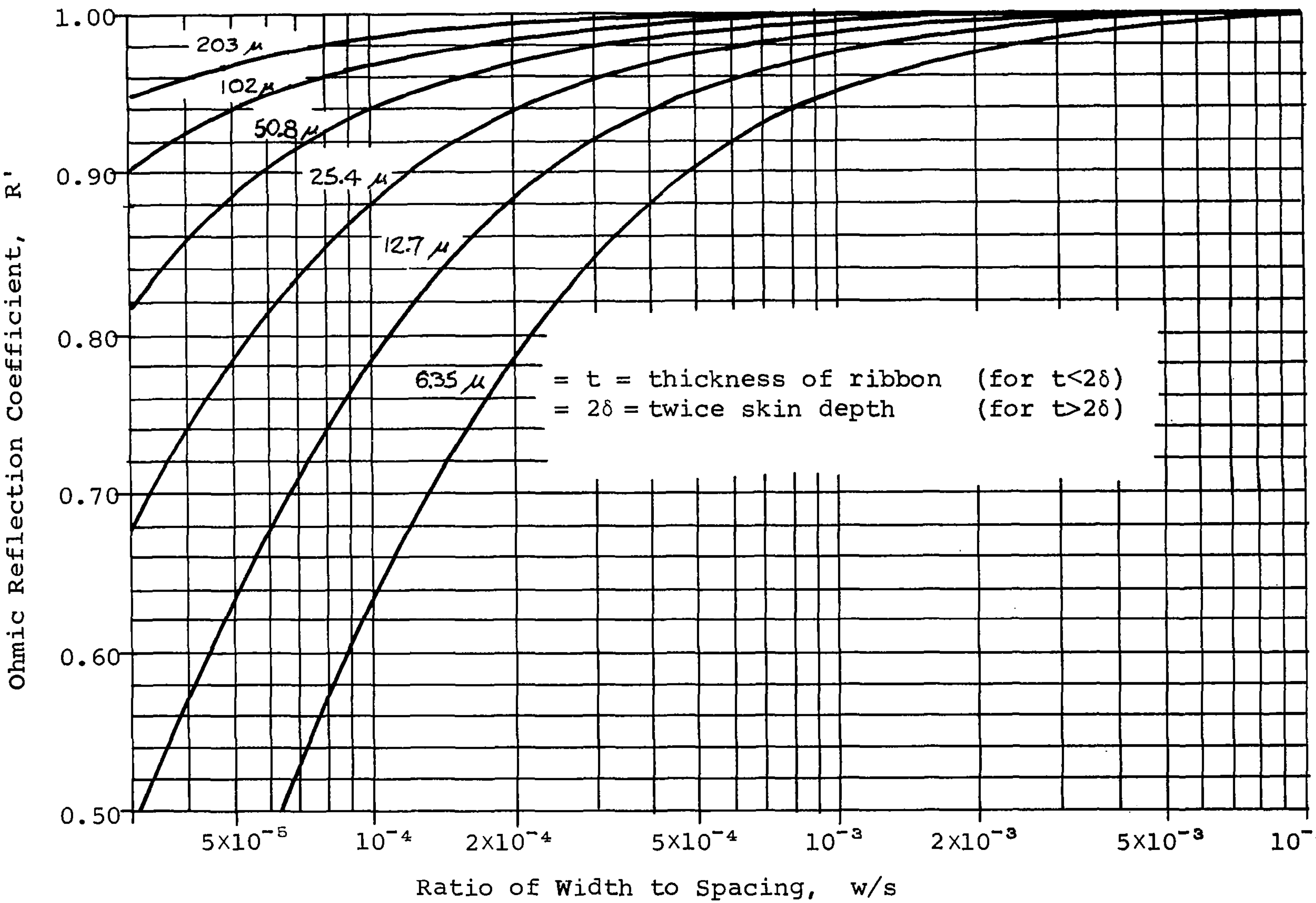

Figure 4. — Ohmic Reflection Coefficient for Grating of Aluminum Ribbons

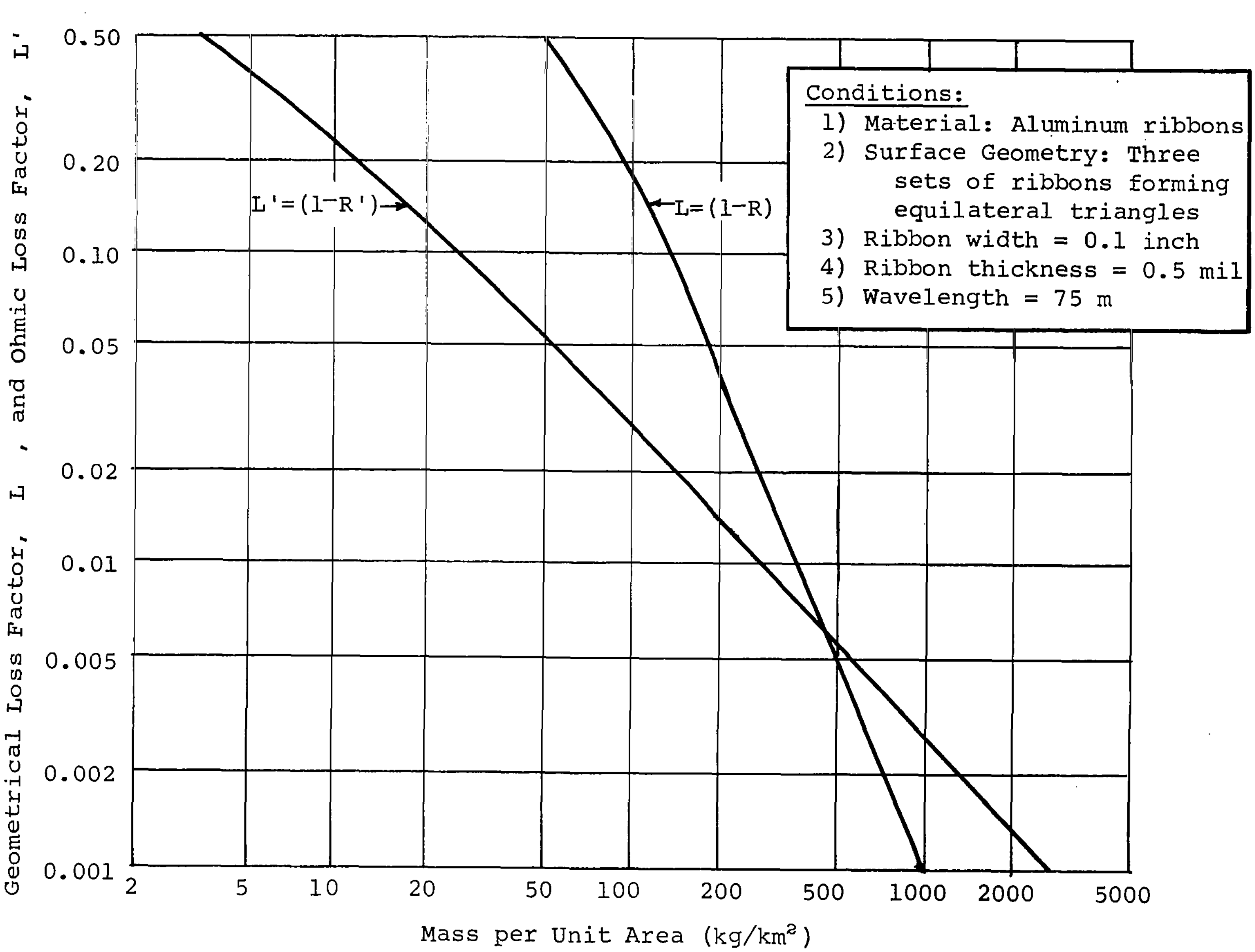

Figure 5. — Geometrical and Ohmic Loss Factors vs Mass per Unit Area

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