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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

A propellant utilization (PU) system manages the outflow of propellants in order to reduce residuals (the masses of liquid oxygen and hydrogen remaining in the tanks at engine shutdown). The difference bridge in the Centaur PU system performs the function of comparing the masses of fuel and oxidizer so that the system can correct for a deviation from the desired mass ratio. An analysis was performed on the difference bridge to determine bridge sensitivity, factors that affect bridge balance, and effects of telemetry circuit malfunctions on the bridge. The results of the analysis show the following:

(1) The output sensitivity of the bridge is as specified.

(2) The bridge balance is not affected significantly by specified variation of parameters.

(3) Telemetry circuit malfunctions, with the exception of loss of prime power, do not significantly affect the bridge balance.

With the exception of the effect on the main bridge due to loss of 400-hertz power to the quantity-readout circuits, the design of the bridge is adequate.

INTRODUCTION

A propellant utilization (PU) system manages the outflow of propellants in order to reduce residuals, that is, the masses of liquid oxygen (LO_2) and hydrogen (LH_2) which remain in the tanks at the time of engine shutdown.

Without PU control, dispersions on variables such as tanked propellant weights and engine operating mixture ratio greatly increase residuals, with a resulting pound-for-pound reduction in payload. The net payload gain attributed to PU control on Centaur is in excess of 220 pounds (100 kg). The tankage of the Centaur is approximately 25 000 pounds (11 350 kg) of liquid oxygen and 5000 pounds (2270 kg) of liquid hydrogen.

The Centaur propellant utilization system employs capacitance probes that sense the propellant mass remaining in the main tanks. The electrical current through these sensors is proportional to the sensed propellant mass. The currents are compared in an electronics package and their imbalance, which represents an off-nominal propellant mass ratio, is amplified and used to drive a positioning device that controls flow valves in each rocket engine. These valves control engine mixture ratio to regain the desired propellant mass relation in the tanks. (See ref. 1 for a detailed description of the PU system.)

Although results of four Centaur flights that incorporated this PU system showed less than the equivalent of 15 pounds (6.82 kg) error in the desired burnable liquid-hydrogen residual, an analysis was conducted to determine the adequacy of the difference-bridge design with respect to parameter variations and telemetry readout circuit malfunctions. Specifically, the design areas investigated were

(1) The actual sensitivity of the bridge in terms of the error current that flows into the bridge error amplifier as the result of a deviation from the required mass ratio of fuel and oxidizer

(2) The magnitude of the quadrature component of error signal (the error signal 90° out of phase with the desired error signal) as a function of vehicle tank level (A quadrature signal above a certain level would saturate the error amplifier and mask the actual error signal.)

(3) The effects of variation of stray wiring and cable capacitances upon the balance of the main bridge

(4) The adequacy of the specified value for minimum probe-lead insulation resistance as measured before flight

(5) The effect of a malfunction of the quantity telemetry circuits upon the main bridge

Other critical factors that are important but have not as yet been investigated in detail are the effects of temperature and 400-hertz harmonics upon bridge balance.

DESCRIPTION OF PROPELLANT UTILIZATION SYSTEM

The Centaur PU system consists of five major components: liquid-oxygen and liquid-hydrogen propellant sensors, an electronics package, and two servopositioners. A block diagram of the system is shown in figure 1. Figure 2 shows the location of the probes within the vehicle. The liquid-hydrogen probe is mounted on the cylindrical tank wall. The liquid-oxygen probe is canted at a 7° angle (relative to the vehicle longitudinal axis) and mounted on the thrust support structure in the aft tank of the vehicle. The probes are positioned in each tank to cover a region from approximately 4 to 80 percent of the full tank. Interconnecting cables carry capacitance bridge current to and from each probe to the electronics package and also carry the bridge error signal to the servopositioners. The liquid-oxygen-probe cables pass through the vehicle aft bulkhead. The liquid-hydrogen-probe cables run from the electronics package through a cableway external to the tanks to the forward bulkhead tank access door, and then through this door to the fuel probe.

The term pounds of liquid-oxygen error is used to relate all system errors to a common base. A pound of liquid-oxygen error is the deviation of any system parameter representing the equivalent liquid-oxygen mass imbalance of 1 pound from the desired mass ratio of liquid oxygen and liquid hydrogen.

DESCRIPTION OF BRIDGE OPERATION

The difference bridge compares the masses of liquid hydrogen and liquid oxygen as represented by electrical capacitances; any deviation from a standard oxidizer-fuel ratio causes the bridge to be unbalanced. This imbalance produces an error voltage that is used to control the engine oxidizer valves to restore the bridge to balance. In the calibration procedure, the bridge is balanced at two points of simulated probe capacitance by means of a wet-voltage trim adjustment and a dry-voltage trim adjustment. One point represents the capacitance of fully covered hydrogen and oxygen tank probes, while the other point represents the capacitances of fully uncovered probes. Since the probe capacitances are linear, with these initial voltage adjustments the bridge will be balanced in flight for any values of hydrogen and oxygen probe capacitances that represent a 5 to 1 mass ratio of liquid oxygen to liquid hydrogen.

From the hydrogen and oxygen legs of the difference bridge, current transformers are used to couple signal currents that flow in the hydrogen and oxygen probes to quantity-indicating circuits. The signal current causes an error signal current to be developed at the input of a servoamplifier. The output of the servoamplifier positions the arm of a motor-driven feedback potentiometer to reduce the input error current to zero. Voltage from the wiper arm of a potentiometer mechanically coupled to the feedback potentiometer indicates the quantity of fuel or oxygen remaining.

Figure 3 shows the circuit schematic of the difference bridge with the loading on the quantity-readout transformers. A simplified equivalent circuit is shown in figure 4.

Two bias voltages may be set into the main bridge if desired. One is the error bias voltage which causes the bridge to be balanced at an engine mixture ratio that gives a residual quantity of either liquid hydrogen or liquid oxygen at the completion of burn. The other bias voltage that may be set in is the coast bias voltage which is used for two-burn missions. This coast bias voltage causes more liquid oxygen than normal to be used during the first burn to compensate for the expected greater evaporation rate of liquid hydrogen over that of liquid oxygen during the coast phase. The error bias factors are 476 pounds (216 kg) of residual oxygen per volt of 0-phase bias setting and 95 pounds (43.2 kg) of residual hydrogen per volt of π-phase bias setting. (All symbols are given in appendix A.) The coast bias factor is 47.6 pounds (21.6 kg) of excess liquid hydrogen per volt of bias setting. (See appendix B for the derivation of these factors.)

There is also a quadrature voltage adjustment to balance out the quadrature component of error voltage. This quadrature component is attenuated by at least a factor of 15 in the last half of the error amplifier because of the quadrature rejection circuit. If the quadrature error voltage at the output of the error amplifier is equivalent to 100 pounds (45.4 kg) or more of liquid-oxygen error after being attenuated by the quadrature rejection circuit, the amplifier is saturated. If the amplifier is saturated, the real error signal is masked. Because of attenuation by the quadrature rejection circuit, a quadrature error voltage at the output of the difference bridge of at least 300 pounds (136 kg) of liquid-oxygen equivalent error would be necessary to saturate the amplifier. The output of the first half of the amplifier prior to the quadrature rejection network does not saturate with an error signal equivalent to 300 pounds (136 kg) of liquid-oxygen or less.

MATHEMATICAL ANALYSIS OF DIFFERENCE BRIDGE

Using the equivalent circuit of the difference bridge as shown in figure 4, mathematical expressions were written for output error voltage, bridge balance conditions, quadrature component of error voltage, and sensitivity of the bridge. The effects of shorts across the primaries and secondaries of the quantity transformers, loss of 400-hertz power to the quantity circuits, and an electrical or mechanical failure that prevents the quantity rebalance potentiometers from functioning were analyzed. The adequacy of the specified value of minimum probe-lead insulation resistance was also checked. Values for the capacitors and resistors are listed in table I.

Bridge Sensitivity

As shown in appendix B, the expression for bridge sensitivity is given as

$$\frac{\partial \epsilon}{\partial C_o} \cong \frac{\psi}{C_a} + j\frac{\psi}{\omega R_7 C_a^2} \tag{1}$$

In equation (1), the real term is the inphase sensitivity and the imaginary term is the

TABLE I. - VALUES FOR RESISTORS AND CAPACITORS IN MAIN BRIDGE

Capacitances	
Main-bridge error amplifier input, C_a, μF	0.08
Liquid-hydrogen-tank probe,[a] C_h, pF	510 to 626
Liquid-oxygen-tank probe,[b] C_o, pF	174 to 258
Hydrogen-quantity circuit, C_{qh}, μF	>1
Oxygen-quantity circuit, C_{qo}, μF	>1
Hydrogen-quantity compensating, C_{sh}, pF (selected)	~2600
Oxygen-quantity compensating, C_{so}, pF (selected)	~1000
C_1, pF	5100
Hydrogen-probe low-impedance-lead stray, C_2, pF	585 ± 74
Hydrogen-probe high-impedance-lead cable, C_3, pF	1537 ± 93
C_4, pF	200
C_5, pF	100
C_6, pF	510
Oxygen-probe high-impedance lead cable, C_7, pF	245 ± 16
Oxygen-probe low-impedance-lead stray, C_8, pF	97 ± 15
C_9, pF	3300
Resistance, ohms	
Wet-trim potentiometer equivalent resistor, R_1	50
R_2	390
R_3	30
Error bias potentiometer equivalent resistor, R_4	50
R_5	30
Dry-trim potentiometer equivalent resistor, R_6	50
R_7	2 MΩ
R_8	510
Coast bias potentiometer equivalent resistor, R_9	50

[a]ΔC_h of 1 pF = 34.5 lb (15.7 kg) liquid-hydrogen error.
[b]ΔC_o of 1 pF = 238 lb (108 kg) liquid-oxygen error.

quadrature sensitivity. Substituting the values for ω, R_7, C_a, and ψ in the preceding equations yields

$$\frac{\partial \epsilon}{\partial C_o} \cong 1250 + j\ 3.11 \ \frac{\mu V}{pF}$$

The preceding value of sensitivity is divided by the factor 238 pounds (108 kg) of error per pF of ΔC_o to obtain the sensitivity in terms of pounds of liquid-oxygen error.

$$\frac{\partial \epsilon}{\partial C_o} = 5.26 + j\ 0.013 \ \frac{\mu V}{\text{lb (0.454 kg) of } LO_2 \text{ error}}$$

Since the 400-hertz error is developed across an error amplifier equivalent input capacitance C_a of 0.08 microfarad, the sensitivity is

$$\frac{\partial i}{\partial C_o} = 0.003 + j\ 1.06 \ \frac{nA}{\text{lb (0.454 kg) of } LO_2 \text{ error}}$$

The real term is the quadrature sensitivity and the imaginary term is the inphase sensitivity.

Calculations for Quadrature Adjust Voltage and Quadrature Component of Error Voltage

If there are no coast bias and error bias voltages set in, then the quadrature adjust voltage δ for balance is given in appendix B by equation (B13) as

$$\begin{aligned} \delta &= -10^7 C_o(-100) + 6.2\times10^{-3}(-38.3) - 1.92\times10^7\ C_h(72.4) \\ \delta &= 10^9 C_o - 1.39\times10^9\ C_h - 0.237 \end{aligned} \tag{2}$$

Equation (2) shows that δ can balance out the quadrature component of error voltage for C_{od} and C_{hd}; however, for C_{ol} and C_{hl}, δ has to be changed to another value. For $C_{od} = 174$ pF and $C_{hd} = 510$ pF,

$$\delta = -0.772 \text{ volt}$$

For $C_{ol} = 258$ pF and $C_{hl} = 626$ pF,

$$\delta = -0.849 \text{ volt}$$

Therefore, δ would have to be changed 0.077 volt to keep the quadrature component of error voltage equal to zero as the oxygen and fuel probe capacitances change from dry to wet values.

If the quadrature adjust voltage δ is set to -0.772 volt to balance out the quadrature component of error voltage at dry values of oxygen and hydrogen probe capacitances, a quadrature component of error voltage will exist at wet values of the probe capacitances. Appropriate values were substituted into the imaginary part of equation (B11) to find the magnitude of this quadrature component; the coast bias and error bias voltages were still assumed to be equal to zero. The magnitude of the quadrature component of error voltage at the wet values of probe capacitances was -0.199 millivolt. This quadrature voltage is equivalent to 37.8 pounds (17.2 kg) of liquid-oxygen error since 5.26 microvolts equals 1 pound (0.455 kg) of liquid-oxygen error. Insignificant amounts of quadrature voltage to be cancelled out are added by τ and λ; for example, if $\tau = -2.46$ volts and $\lambda = -2.23$ volts, their maximum values, δ would only have to be increased 0.008 volt to cancel the effects of τ and λ.

Errors Due to Changes in Cable and Stray Capacitances

The partial derivatives of ϵ with respect to C_2, C_8, C_3, and C_7 were taken to find numerical values for the sensitivity of the error voltage ϵ to variations in stray capacitances C_2 and C_8 and cable capacitances C_3 and C_7. The derivations for these sensitivities are contained in appendix B.

To be meaningful the values for sensitivities should be expressed in the equivalent of pounds of liquid-oxygen error per picofarad change of capacitance. The sensitivity is given as 5.26 microvolts of error voltage per pound of liquid-oxygen error. Dividing equations (B27), (B30), (B33), and (B36) by 5.26 and setting the voltages δ, τ, and λ equal to zero result in the following equivalent sensitivities expressed in pounds of liquid-oxygen error per picofarad change in capacitance (kg of LO_2 error/pF of ΔC):

$$C_2 \text{ sensitivity} = -1.42\times10^{-4} - j\,4.32\times10^{-2} = -0.645\times10^{-4} - j\,1.96\times10^{-2}$$

$$C_8 \text{ sensitivity} = -2.26\times10^{-1} + j\,5.65\times10^{-4} = -0.103 + j\,2.57\times10^{-4}$$

$$C_3 \text{ sensitivity} = -3.61\times10^{-2} - j\,2.09\times10^{-5} = -1.64\times10^{-2} - j\,0.95\times10^{-5}$$

$$C_7 \text{ sensitivity} = 4.10\times10^{-1} + j\,4.60\times10^{-3} = 0.186 + j\,2.09\times10^{-3}$$

Assuming the lead and stray capacitances change their values by 5 percent because of temperature changes during flight (tests indicate a far lower percentage) results in the following equivalent errors at the output of the difference bridge expressed in pounds (kg) of liquid oxygen:

Inphase error due to change in $C_2 = 4.17\times10^{-3}$ lb (1.89×10^{-3} kg)

Quadrature error due to change in $C_2 = 1.27$ lb (0.578 kg)

Inphase error due to change in $C_8 = 1.1$ lb (0.5 kg)

Quadrature error due to change in $C_8 = 2.74\times10^{-3}$ lb (1.25×10^{-3} kg)

Inphase error due to change in $C_3 = 2.8$ lb (1.27 kg)

Quadrature error due to change in $C_3 = 1.61\times10^{-3}$ lb (0.73×10^{-3} kg)

Inphase error due to change in $C_7 = 5$ lb (2.27 kg)

Quadrature error due to change in $C_7 = 5.61\times10^{-2}$ lb (2.55×10^{-2} kg)

Adequacy of Specified Value of Minimum Probe-Lead Insulation Resistance

The specified value for the minimum probe-lead insulation resistance as measured just before flight is 1000 megohms. This specified value pertains to the insulation resistance between the two signal output leads of a probe as well as to that between a signal lead and ground. Normally, the value measured is between 15 000 and 16 000 megohms.

During the calibration procedure before flight, the quadrature adjust potentiometer is adjusted to balance out the quadrature component of error current at dry values of probe capacitances. This potentiometer is capable of balancing out up to ±1120 nanoamperes of quadrature current.

The magnitude of this leakage-resistance problem can be determined by considering the worst-case effects. Based on the calculated values of leakage currents (see appendix B), the worst case would arise when the bridge was balanced prior to flight with the minimum allowable leakage resistance across the liquid-oxygen probe and infinite resistance across the liquid-hydrogen probe and, during the flight, the liquid-oxygen-probe leakage resistance changed to an infinite value. This change in resistance would produce a change in quadrature current of 100 nanoamperes or the equivalent of 94.5 pounds (43 kg) of liquid-oxygen error.

The first half of the error amplifier does not saturate with less than 300 pounds (136 kg) of liquid-oxygen error. Therefore, a worst-case situation producing a change in quadrature current equivalent to 300 pounds (136 kg) of liquid-oxygen error would arise from a minimum insulation resistance of 314 megohms. The specified value of

1000 megohms is the minimum desired to allow an adequate safety margin.

Effect of Malfunction of Quantity-Indicating Circuits

It is desirable to know the magnitude of error introduced into the main bridge as the result of a failure in the quantity-readout circuits. The loss of 400-hertz power to the quantity circuits and any electrical or mechanical failure that prevents the quantity-rebalance potentiometers from functioning was investigated. Figures 6 and 7 show in detail the circuits for the quantity-telemetry bridges and their connections to the main bridge.

Nonfunctioning of Hydrogen Rebalance Potentiometer

As shown in appendix B, the worst-case inphase error current introduced into the main bridge as the result of the hydrogen rebalance potentiometer not functioning is equivalent to 9.55 pounds (4.34 kg) of liquid oxygen.

Nonfunctioning of Oxygen Rebalance Potentiometer

As shown in appendix B the worst-case inphase error current introduced into the main bridge as the result of the oxygen rebalance potentiometer not functioning is equivalent to 7 pounds (3.18 kg) of liquid oxygen.

Simultaneous Nonfunctioning of Oxygen and Hydrogen Rebalance Potentiometers

For this case, the error currents into the main amplifier subtract since the error currents, as the result of nonfunctioning potentiometers, are of opposite sign. The equivalent inphase error is 2.55 pounds (1.16 kg) of liquid oxygen.

Loss of Power to Hydrogen-Quantity Circuit

As shown in appendix B, the worst-case inphase error current introduced into the main bridge as the result of loss of power to the oxygen-quantity circuit is equivalent to 199 pounds (90.5 kg) of liquid oxygen.

Loss of Power to Oxygen-Quantity Circuit

As shown in appendix B, the worst-case inphase error current introduced into the main bridge as the result of loss of power to the hydrogen-quantity circuit is equivalent 86.2 pounds (39.2 kg) of liquid oxygen.

Loss of Power to Both Quantity Circuits

In the event of simultaneous loss of power to both circuits, the equivalent error is equal to the difference of the individual errors at full tank. Therefore, the equivalent inphase error for this case is 113 pounds (51.4 kg) of liquid oxygen.

Effect of Shorting Quantity-Transformer Secondaries

Shorting the secondaries of the quantity transformers would have no effect on the main bridge because in operation the quantity circuit provides a rebalance voltage which keeps the error voltage developed across the transformer secondary extremely close to zero. Thus, in effect, the transformer secondaries are shorted. In the equivalent circuit of the bridge (fig. 4), the capacitors C_{qo} and C_{qh} would be very large and would produce the equivalent of a transformer short circuit across the primary or secondary.

CONCLUSIONS

The design of the difference bridge is adequate. However, loss of 400-hertz power to the liquid-hydrogen-quantity telemetry-readout circuit between full-tank and half-tank conditions would introduce enough inphase error current into the main bridge to saturate the main loop. The chance of this occurring in flight is extremely remote. The following are the conclusions for each factor analyzed:

1. The output sensitivity of the bridge is 1.06 nanoamperes per pound (2.33 nA/kg) of liquid-oxygen error.

2. The quadrature voltage adjustment cannot balance out the quadrature component of error voltage over the full range of tank levels. However, the maximum quadrature equivalent error of 37.8 pounds (17.2 kg) of liquid oxygen is far below the value of 300 pounds (136 kg) or more necessary to saturate the amplifier.

3. Variations in stray and cable capacitances of 5 percent, which is more than is expected in flight, introduce less than 10 pounds (4.55 kg) of liquid-oxygen equivalent in-

phase error and less than 2 pounds (0.908 kg) of liquid-oxygen equivalent quadrature error.

4. The specified value of minimum probe-lead insulation resistance measured before flight is adequate.

5. Loss of 400-hertz power to the quantity-readout circuits would introduce very large inphase errors into the main bridge.

   a. Loss of 400-hertz power to the liquid-hydrogen-quantity circuit alone at full tank results in 199 pounds (90.5 kg) of liquid-oxygen equivalent inphase error.

   b. Loss of 400-hertz power to the liquid-hydrogen-quantity circuit alone at half tank results in 90.5 pounds (41.1 kg) of liquid-oxygen equivalent inphase error.

   c. Loss of 400-hertz power to the liquid-oxygen-quantity circuit alone at full tank results in 86.2 pounds (39.2 kg) of liquid-oxygen-equivalent inphase error.

   d. Loss of 400-hertz power to both quantity circuits at full tank results in 113 pounds (51.4 kg) of liquid-oxygen-equivalent inphase error.

6. Any electrical or mechanical failure that prevents either or both quantity-readout rebalance potentiometers from functioning introduces less than 10 pounds (4.55 kg) of equivalent inphase error into the main bridge.

7. A short across a quantity-readout-transformer primary or secondary winding has an insignificant effect upon the main bridge.

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APPENDIX A

SYMBOLS

C capacitance

C_a input capacitance of main-bridge error amplifier

C_h capacitance of liquid-hydrogen-tank probe

ΔC_h capacitance change of liquid-hydrogen-tank probe

C_{hd} capacitance of uncovered hydrogen probe

C_{he} capacitance of empty voltage leg of hydrogen-quantity bridge

C_{hf} capacitance of rebalance leg of hydrogen-quantity bridge

$C_{h\ell}$ capacitance of covered hydrogen probe

C_o capacitance of liquid-oxygen-tank probe

ΔC_o capacitance change of liquid-oxygen-tank probe

C_{od} capacitance of uncovered oxygen probe

C_{oe} capacitance of empty voltage leg of oxygen-quantity bridge

C_{of} capacitance of rebalance leg of oxygen-quantity bridge

$C_{o\ell}$ capacitance of covered oxygen probe

C_q total capacitance across secondary of quantity transformer

C_{qh} capacitance of hydrogen-quantity circuit referred to hydrogen-quantity transformer primary

C_{qo} capacitance of oxygen-quantity circuit referred to oxygen-quantity transformer primary

C_{sh} hydrogen-quantity compensating capacitance

C_{so} oxygen-quantity compensating capacitance

$C_{1,4,5,6,9}$ capacitances

C_2 stray capacitance of hydrogen-probe low-impedance lead

C_3 cable capacitance of hydrogen-probe high-impedance lead

C_7 cable capacitance of oxygen-probe high-impedance lead

C_8 stray capacitance of oxygen-probe low-impedance lead

e voltage node

F_c	coast bias factor
F_h	liquid-hydrogen error bias factor
F_o	liquid-oxygen error bias factor
I_{oh}	main amplifier error current due to malfunction of hydrogen-quantity circuit
I_O	current through liquid-oxygen probe
I_{OO}	main amplifier error current due to malfunction of oxygen-quantity circuit
I_{sh}	current through C_{sh}
I_{so}	current through C_{so}
i_h	current through liquid-oxygen probe
i_o	current through liquid-oxygen probe
i_6	current through dry-adjust leg of bridge
$R_{2,3,5,7,8}$	resistors
R_1	wet-trim potentiometer equivalent resistor
R_4	error bias potentiometer equivalent resistor
R_6	dry-trim potentiometer equivalent resistor
R_9	coast bias potentiometer equivalent resistor
S_h	sensitivity of hydrogen probe
S_o	sensitivity of oxygen probe
V_{he}	hydrogen-quantity empty-adjust voltage
V_{hf}	hydrogen-quantity full-adjust voltage
V_{oe}	oxygen-quantity empty-adjust voltage
V_{of}	oxygen-quantity full-adjust voltage
β	dry adjust voltage
δ	quadrature adjust voltage
ϵ	error voltage at output of difference bridge
ϵ_h	error voltage at input to hydrogen-quantity error amplifier
ϵ_o	error voltage at input to oxygen-quantity error amplifier
η	main-bridge wet adjust voltage
λ	error bias adjust voltage
π phase	voltage 180° out of phase with 400-Hz reference voltage
τ	coast bias adjust voltage
ψ	100-volt, π-phase voltage of difference bridge
0 phase	reference phase of 400-Hz voltage

APPENDIX B

MATHEMATICAL ANALYSIS

Detailed herein are derivations of equations for the bridge error voltage, bridge sensitivity, bridge balance, quadrature adjust voltage, wet and dry adjust voltages, bias factors, cable and stray capacitance sensitivity, quadrature error current due to insulation resistance, quantity readout bridge adjust voltages, and error currents due to malfunction of quantity indicating circuits.

Derivation of Equation for Error Voltage

The nodal equations for voltage nodes ϵ, e_3, e_4, e_1, and e_2 of figure 4 are

$$\left(j\omega C_a + j\omega C_{qh} + j\omega C_{qo} + \frac{1}{R_7} + \frac{1}{R_6 + \frac{1}{j\omega C_6}}\right)\epsilon - j\omega C_{qh}e_3 - j\omega C_{qo}e_4 - \frac{\beta}{R_6 + \frac{1}{j\omega C_6}} - \frac{\delta}{R_7} = 0 \tag{B1}$$

$$-j\omega C_{qh}\epsilon - j\omega C_h e_1 + (j\omega C_{qh} + j\omega C_h + j\omega C_3)e_3 = 0 \tag{B2}$$

$$-j\omega C_{qo}\epsilon - j\omega C_o e_2 + \left(j\omega C_{qo} + j\omega C_7 + j\omega C_o + \frac{1}{R_5 + R_9 + \frac{1}{j\omega C_5}} + \frac{1}{R_3 + R_4 + \frac{1}{j\omega C_4}}\right)e_4$$

$$- \frac{\tau}{R_5 + R_9 + \frac{1}{j\omega C_5}} - \frac{\lambda}{R_3 + R_4 + \frac{1}{j\omega C_4}} = 0 \tag{B3}$$

$$\left(\frac{1}{R_1 + R_2} + j\omega C_1 + j\omega C_2 + j\omega C_h\right)e_1 - j\omega C_h e_3 - \frac{\eta}{R_1 + R_2} = 0 \tag{B4}$$

$$\left(\frac{1}{R_8} + j\omega C_9 + j\omega C_8 + j\omega C_o\right)e_2 - j\omega C_o e_4 - \frac{\psi}{R_8} = 0 \tag{B5}$$

In writing equations (B1) to (B5), the equivalent series resistance of the capacitors was neglected since the capacitors are glass dielectric capacitors with extremely low series resistance. Also, the insulation resistance of probe leads was considered to be infinite. If

$$A = j\omega C_a + j\omega C_{qh} + j\omega C_{qo} + \frac{1}{R_7} + \frac{1}{R_6 + \frac{1}{j\omega C_6}}$$

$$D = j\omega C_{qo} + j\omega C_7 + j\omega C_o + \frac{1}{R_5 + R_9 + \frac{1}{j\omega C_5}} + \frac{1}{R_3 + R_4 + \frac{1}{j\omega C_4}}$$

$$B = \frac{\beta}{R_6 + \frac{1}{j\omega C_6}}$$

$$F = \frac{\tau}{R_5 + R_9 + \frac{1}{j\omega C_5}} + \frac{\lambda}{R_3 + R_4 + \frac{1}{j\omega C_4}}$$

$$G = \frac{1}{R_1 + R_2} + j\omega(C_1 + C_2 + C_h)$$

$$K = \frac{1}{R_8} + j\omega(C_9 + C_8 + C_o)$$

$$M = j\omega(C_{qh} + C_h + C_3)$$

then solving the preceding equations simultaneously gives the following as the expression for the error voltage ϵ at the input to the error amplifier:

$$\epsilon = \frac{\dfrac{D}{j\omega C_{qo}}\left(\dfrac{\delta}{R_7} + B\right) + F + \dfrac{j\omega C_o\psi}{KR_8} - \dfrac{j\omega C_o^2}{KC_{qo}}\left(\dfrac{\delta}{R_7} + B\right) + \dfrac{j\omega C_{qh}C_h\eta\left(D + \dfrac{\omega^2 C_o^2}{K}\right)}{C_{qo}\left(MG + \omega^2 C_h^2\right)(R_1 + R_2)}}{\dfrac{j\omega G C_{qh}^2\left(-D - \dfrac{\omega^2 C_o^2}{K}\right)}{C_{qo}\left(MG + \omega^2 C_h^2\right)} + \dfrac{A}{C_{qo}}\left(\dfrac{D}{j\omega} - \dfrac{j\omega C_o^2}{K}\right) - j\omega C_{qo}} \tag{B6}$$

Derivation of Simplified Expression for Bridge Error Voltage

The terms of A, B, D, and F in the error voltage expression of equation (B6) can be simplified by neglecting terms three orders of magnitude less than terms to which they are added and the terms in the numerator and denominator can be simplified by neglecting quantities two orders of magnitude less than quantities to which they are added.

Combining the simplified terms for the numerator and denominator and neglecting terms two orders of magnitude less than terms to which they are added give the following simplified expression for the error voltage:

$$\epsilon = \frac{\dfrac{\delta}{R_7} + \omega^2\left\{\left[C_6R_6 + \dfrac{C_o^2R_8C_6}{C_{qo}}\right]\beta + C_5^2(R_5 + R_9)\tau + C_4^2(R_3 + R_4)\lambda\right\}}{\dfrac{1}{R_7} + \dfrac{C_{qh}}{C_{qo}}\left\{\omega^2\left[2C_o^2R_8 - C_5^2(R_5 + R_9)\right] - C_4^2(R_3 + R_4)\right\} + j\omega(C_a + C_6)}$$

$$+ \frac{\omega^2\left[C_o(C_9 + C_o)R_8\psi + C_h(C_1 + C_2 + C_h)(R_1 + R_2)\eta\right]}{\dfrac{1}{R_7} + \dfrac{C_{qh}}{C_{qo}}\left\{\omega^2\left[2C_o^2R_8 - C_5^2(R_5 + R_9)\right] - C_4^2(R_3 + R_4)\right\} + j\omega(C_a + C_6)}$$

$$+ \frac{j\omega(C_6\beta + C_5\tau + C_4\lambda + C_o\psi + C_h\eta)}{\dfrac{1}{R_7} + \dfrac{C_{qh}}{C_{qo}}\left\{\omega^2\left[2C_o^2R_8 - C_5^2(R_5 + R_9)\right] - C_4^2(R_3 + R_4)\right\} + j\omega(C_a + C_6)} \tag{B7}$$

Derivation for Sensitivity of Bridge

Equation (B7) may be simplified for the purpose of calculating the sensitivity of the bridge to changes in the hydrogen or oxygen probe capacitances. Assume that the quadrature adjust voltage has been adjusted to balance out the numerator quadrature terms in equation (B7) at one tank level and that $C_{qh} = C_{qo}$. The term $1/R_7$ in the denominator of equation (B7) is three orders of magnitude larger than the sum of the other three real terms in the denominator. Therefore, the following simplified equation is valid for calculating bridge sensitivity:

$$\epsilon = \frac{j\omega(C_6\beta + C_5\tau + C_4\lambda + C_o\psi + C_h\eta)}{\frac{1}{R_7} + j\omega(C_a + C_6)} \tag{B8}$$

The partial derivative of ϵ with respect to C_o gives an expression for the sensitivity of the bridge:

$$\frac{\partial\epsilon}{\partial C_o} = \frac{\omega^2(C_a + C_6)\psi + j\omega\frac{\psi}{R_7}}{\frac{1}{R_7^2} + \omega^2(C_a + C_6)^2}$$

Since $C_a >> C_6$ and $\omega^2(C_a + C_6)^2 >> 1/R_7^2$

$$\frac{\partial\epsilon}{\partial C_o} \cong \frac{\psi}{C_a} + j\frac{\psi}{\omega R_7 C_a^2} \tag{B9}$$

Derivation of Equations for Bridge Balance and Quadrature Adjust Voltage

Equation (B7) may be simplified to find the inphase and quadrature components of the error voltage. Using the simplified denominator of equation (B8) and substituting actual values for the resistors and capacitors in the numerator of equation (B7) results in the following simplified equation:

$$\epsilon = \frac{11.2\ C_o\psi + 8.24\times10^{-11}\ \beta + 5.06\times10^{-12}\ \tau + 2.02\times10^{-11}\ \lambda + 15.8\ C_h\eta}{\frac{1}{R_7} + j\omega C_a}$$

$$+ \frac{5\times10^{-7}\ \delta + j\omega(C_6\beta + C_5\tau + C_4\lambda + C_o\psi + C_h\eta)}{\frac{1}{R_7} + j\omega C_a} \tag{B10}$$

Multiplying the numerator and denominator of equation (B10) by the complex conjugate of the denominator and dividing the numerator by the denominator result in the following equation:

$$\epsilon = \frac{C_6\beta + C_5\tau + C_4\lambda + C_o\psi + C_h\eta}{C_a} - j\left(2.5\times10^{4}\ C_o\psi - 1.55\times10^{-5}\ \beta\right)$$

$$+ j\left(0.31\times10^{-5}\ \tau + 0.61\times10^{-5}\ \lambda - 4.8\times10^{4}\ C_h\eta - 2.5\times10^{-3}\ \delta\right) \tag{B11}$$

The real and imaginary parts of equation (B11) must equal zero for the bridge to be balanced, that is, for the error voltage ϵ to be equal to zero. Equating the real and imaginary terms of equation (B11) to zero gives as expressions for the inphase bridge balance condition and the quadrature adjust voltage, respectively,

$$C_6\beta + C_5\tau + C_4\lambda + C_o\psi + C_h\eta = 0 \tag{B12}$$

and

$$\delta = -10^{7}C_o\psi + 6.2\times10^{-3}\ \beta + 1.24\times10^{-3}\ \tau + 2.44\times10^{-3}\ \lambda - 1.92\times10^{7}\ C_h\eta \tag{B13}$$

Equation (B12) is used to calculate the values of the wet and dry adjust voltages. With the proper values of these voltages, the bridge will be balanced for a selected mass ratio at any tank level. With given values of wet and dry voltages, equation (B13) can be used to calculate the value of the quadrature adjust voltage that will cancel out the quadrature component of error voltage at one tank level.

Derivation of Values for Wet and Dry Adjust Voltages

To find the values of the wet adjust voltage η and the dry adjust voltage β the following equations are written for the covered and uncovered probes, respectively:

$$C_{ol}\psi + C_{hl}\eta + C_6\beta + C_5\tau + C_4\lambda = 0 \qquad \text{(B14a)}$$

$$C_{od}\psi + C_{hd}\eta + C_6\beta + C_5\tau + C_4\lambda = 0 \qquad \text{(B14b)}$$

Substracting equation (B14b) from equation (B14a) gives

$$\psi(C_{ol} - C_{od}) + \eta(C_{hl} - C_{hd}) = 0$$

and

$$\eta = -\frac{\Delta C_o}{\Delta C_h}\psi \qquad \text{(B15)}$$

Since ψ = 100 volts at π phase or -100 volts

$$\eta = -\frac{84}{116}(-100)$$

$$= 72.4 \text{ volts}$$

Substituting this value of η in equation (B14b), with the coast bias voltage τ and the error bias voltage λ set equal to zero, yields

$$\beta = \frac{-C_{od}\psi - C_{hd}\eta}{C_6}$$

$$= \frac{-174(-100) - 510(72.4)}{510}$$

$$= -38.3 \text{ volts} \qquad \text{(B16)}$$

Error bias factor. - The equation for bridge balance of the inphase component of error voltage is given by equation (B12) as

$$C_o\psi + C_6\beta + C_5\tau + C_4\lambda + C_h\eta = 0$$

The values of η and β were found to be 72.4 and -38.3 volts, respectively. The value of C_o at the instant $C_h = C_{hd}$ can be found as a function of the error bias adjust voltage λ by substituting the appropriate values in equation (B12) and letting the coast bias voltage τ equal zero. The result is given by

$$C_o = 174 + 2\lambda \tag{B17}$$

where C_o is expressed in picofarads. If $\lambda = 1$ volt, then $C_o = 176$ picofarads.

Therefore, when an error bias of 1 volt is set into the system, the capacitance of the liquid-oxygen probe is 2 picofarads more than its uncovered or dry value for bridge balance at the instant the fuel probe is uncovered. The sensitivity of the oxygen probe is given by

$$S_o = \frac{\text{total lb (kg) of } LO_2}{C_{o\ell} - C_{od}}$$

$$= \frac{20\,000}{84}$$

$$= \frac{238 \text{ lb (108 kg) } LO_2}{\text{pF}} \tag{B18}$$

Therefore, the liquid-oxygen error-bias factor can be expressed as

$$F_o = S_o(\Delta C_o)_{\lambda=1}$$

$$= 238(2)$$

$$= \frac{476 \text{ lb (216 kg) of residual } LO_2}{\text{0-phase volt of bias setting}} \tag{B19}$$

Similarly, C_h can be expressed as a function of the error bias voltage by

$$C_h = 510 - 2.76\ \lambda \tag{B20}$$

If $\lambda = -1$volt, then $C_h = 512.76$ picofarads.

Therefore, when an error bias of -1 volt is set into the system, the capacitance of the hydrogen probe is 2.76 picofarads more than its dry value for bridge balance at the instant the liquid-oxygen probe is uncovered. The sensitivity of the hydrogen probe is given by

$$S_h = \frac{\text{total lb (kg) of } LH_2}{C_{hl} - C_{hd}}$$

$$= \frac{4000}{116}$$

$$= \frac{34.5 \text{ lb (15.7 kg) of } LH_2}{\text{pF}} \tag{B21}$$

Therefore, the liquid-hydrogen error-bias factor can be expressed as

$$F_h = S_h(\Delta C_h)_{\lambda=-1}$$

$$= 34.5(2.76)$$

$$= \frac{95.2 \text{ lb (43.2 kg) of residual } LH_2}{\pi\text{-phase volt of bias setting}} \tag{B22}$$

Coast bias factor. - The value of C_o at the instant $C_h = C_{hl}$ can be found as a function of the coast bias adjust voltage τ by substituting the appropriate values in equation (B12) and letting the error bias voltage λ equal zero. The result is

$$C_o = 258 + \tau \tag{B23}$$

where C_o is expressed in picofarads. If $\tau = -1$ volt, $C_o = 257$ picofarads.

Therefore, as a result of a coast bias of -1 volt being set into the system, the capacitance of the liquid-oxygen probe is 1 picofarad less than its covered or wet value for bridge balance with a wet value of fuel probe capacitance. The coast bias factor can be expressed as

$$F_c = S_o(\Delta C_o)_{\tau=-1}$$

$$= 238(1)$$

$$= \frac{238 \text{ lb (108 kg) of } LO_2}{\pi\text{-phase volt of bias setting}} \tag{B24}$$

Since the bridge is balanced for a 5 to 1 mass ratio,

$$F_c = \frac{47.6 \text{ lb (21.6 kg) of } LH_2}{\pi\text{-phase volt of bias setting}}$$

Derivation of Sensitivity of Error Voltage to Changes in Cable and Stray Capacitances

The general procedure to find the sensitivity of the error voltage ϵ to variations in stray capacitances C_2 and C_8 and lead capacitances C_3 and C_7 is to express ϵ as a function of the desired capacitance and then to take the partial derivative of ϵ with respect to the desired capacitance. The values for C_o and C_h are dry tank values. It was assumed that C_{qh} is equal to C_{qo} and has a value of 0.1 microfarad for the worst case condition. Sensitivities are expressed in microvolts per picofarad.

Sensitivity to C_2 variations. - Equation (B6) expressed as a function of C_2 is

$$\epsilon = \frac{a + cC_2 + j(b + mC_2)}{e + fC_2 + lC_2^2 + j\left(g + hC_2 + nC_2^2\right)} \tag{B25}$$

where

$$a = 0.5\times10^{-6}\,\delta + 8.23\times10^{-11}\,\beta + 5.05\times10^{-12}\,\tau + 2.02\times10^{-11}\,\lambda + 2\times10^{-9}\,\psi + 7.95\times10^{-9}\,\eta$$

$$b = 1.28\times10^{-7}\,\beta + 2.52\times10^{-7}\,\tau + 5.04\times10^{-7}\,\lambda + 4.38\times10^{-7}\,\psi - 1.94\times10^{-13}\,\delta + 1.28\times10^{-6}\,\eta$$

$$c = 1.42\,\eta$$

$$e = 5\times 10^{-7}$$

$$f = -1.68\times 10^{-6}$$

$$g = 4.54\times 10^{-4}$$

$$h = -3.48$$

$$l = -1.5\times 10^{2}$$

$$m = -6.96\times 10^{-17}\,\eta$$

$$n = -3.1\times 10^{8}$$

The partial derivative of ϵ with respect to C_2 is expressed by the following equation:

$$\frac{\partial \epsilon}{\partial C_2} = \frac{\left[ce - gm - af + bh + 2C_2(bn - al) + C_2^2(mn - cl)\right]}{\left[e + fC_2 + lC_2^2 + j\left(g + hC_2 + nC_2^2\right)\right]^2} + \frac{j\left[gc + me - bf - ah - 2C_2(bl + an) - C_2^2(ml + cn)\right]}{\left[e + fC_2 + lC_2^2 + j\left(g + hC_2 + nC_2^2\right)\right]} \tag{B26}$$

Substituting appropriate values and multiplying by 10^{-6} to convert from volts per farad to microvolts per picofarad of change in C_2 result in

$$\frac{\partial \epsilon}{\partial C_2} = -7.5\times 10^{-4} + 4.7\times 10^{-6}\,\tau + 9.42\times 10^{-6}\,\lambda + 1.67\times 10^{-8}\,\delta - j(0.227 + 7.6\times 10^{-6}\,\delta + 1.03\times 10^{-8}\,\tau + 2.12\times 10^{-8}\,\lambda) \tag{B27}$$

where the real term is the inphase sensitivity and the imaginary term is the quadrature sensitivity.

Sensitivity to C_8 variations. - Equation (B6) as a function of C_8 is

$$\epsilon = \frac{a + cC_8 + j(b + mC_8)}{e + fC_8 + j(g + hC_8)} \tag{B28}$$

where

$$a = 5.05\times10^{-12}\,\tau + 2.02\times10^{-11}\,\lambda + 2\times10^{-9}\,\psi + 5\times10^{-7}\,\delta + 8.23\times10^{-11}\,\beta + 8.81\times10^{-9}\,\eta$$

$$b = 2.52\times10^{-7}\,\tau + 5.04\times10^{-7}\,\lambda + 4.38\times10^{-7}\,\psi - 1.94\times10^{-13}\,\delta + 1.28\times10^{-6}\,\beta + 1.28\times10^{-6}\,\eta$$

$$c = 0.56\,\psi + 4.37\times10^{-9}\,\eta$$

$$e = 5\times10^{-7}$$

$$f = -2.48\times10^{-7}$$

$$g = 2.02\times10^{-4}$$

$$h = -2.27\times10^{-4}$$

$$m = -6.27\times10^{4}\,\beta - 6.4\times10^{-7}\,\eta$$

$$\frac{\partial\epsilon}{\partial C_8} = \frac{(ce - gm - af + bh) + j(gc + me - bf - ah)}{\left[e + fC_8 + j(g + hC_8)\right]^2} \tag{B29}$$

$$\frac{\partial\epsilon}{\partial C_8} = -1.19 - 3.04\times10^{-16}\,\delta - 1.4\times10^{-13}\,\tau + 2.8\times10^{-13}\,\lambda$$

$$- j(-2.97\times10^{-3} + 2.78\times10^{-13}\,\delta + 1.56\times10^{-16}\,\tau + 3.18\times10^{-16}\,\lambda) \tag{B30}$$

Sensitivity to C_3 variations. - Equation (B6) expressed as a function of C_3 is

$$\epsilon = \frac{a + cC_3 + j(b + mC_3)}{e + fC_3 + j(g + hC_3)} \tag{B31}$$

where

$$a = 0.5\times10^{-13}\,\delta + 8.23\times10^{-18}\,\beta + 5.05\times10^{-19}\,\tau + 2.02\times10^{-18}\,\lambda + 2\times10^{-16}\,\psi + 8.82\times10^{-16}\,\eta$$

$$b = 12.85\times10^{-14}\,\beta + 2.52\times10^{-14}\,\tau + 5.04\times10^{-14}\,\lambda + 4.38\times10^{-14}\,\psi$$

$$- 1.94\times10^{-20}\,\delta + 3.22\times10^{-13}\,\eta$$

$$c = 5\times10^{-7}\,\delta + 8.23\times10^{-11}\,\beta + 2\times10^{-9}\,\psi + 5.05\times10^{-12}\,\tau + 2.02\times10^{-11}\,\lambda$$

$$e = 5\times10^{-14}$$

$$f = 5\times10^{-7}$$

$$g = 2.02\times10^{-11}$$

$$h = 4.54\times10^{-4}$$

$$m = 2.52\times10^{-7}\,\tau + 5.04\times10^{-7}\,\lambda + 12.85\times10^{-7}\,\beta + 4.38\times10^{-7}\,\psi - 1.94\times10^{-13}\,\delta$$

$$\frac{\partial\epsilon}{\partial C_3} = \frac{ce - gm - af + bh + j(gc + me - bf - ah)}{\left[e + fC_3 + j(g + hC_3)\right]^2} \tag{B32}$$

$$\frac{\partial\epsilon}{\partial C_3} = -0.19 - 1.45\times10^{-4}\,\tau - 2.9\times10^{-4}\,\lambda - 2.54\times10^{-6}\,\delta$$

$$+ j(-1.1\times10^{-4} - 0.708\times10^{-6}\,\tau - 1.41\times10^{-6}\,\lambda + 2.88\times10^{-4}\,\delta) \tag{B33}$$

Sensitivity to C_7 variations. - Equation (B6) expressed as a function of C_7 is

$$\epsilon = \frac{a + cC_7 + j(b + mC_7)}{e + fC_7 + j(g + hC_7)} \tag{B34}$$

where

$$a = 0.5\times10^{-6}\,\delta + 8.23\times10^{-11}\,\beta + 5.05\times10^{-12}\,\tau + 2.02\times10^{-11}\,\lambda + 2\times10^{-9}\,\psi + 8.81\times10^{-9}\,\eta$$

$$b = 1.28\times10^{-6}\,\beta - 1.94\times10^{-13}\,\delta + 2.52\times10^{-7}\,\tau + 5.04\times10^{-7}\,\lambda + 4.38\times10^{-7}\,\psi + 1.28\times10^{-6}\,\eta$$

$$c = 5\delta + 8.23\times10^{-4}\,\beta + 8.81\times10^{-2}\,\eta$$

$$e = 5\times10^{-7}$$

$$f = 5$$

$$g = 2.02\times10^{-4}$$

$$h = -0.119$$

$$m = 12.8(\beta + \eta)$$

$$\frac{\partial\epsilon}{\partial C_7} = \frac{ce - gm - af + bh + j(gc + me - bf - ah)}{\left[e + fC_7 + j(g + hC_7)\right]^2} \tag{B35}$$

$$\frac{\partial\epsilon}{\partial C_7} = 2.16 + 1.23\times10^{-4}\,\delta + 5.82\times10^{-7}\,\tau + 1.16\times10^{-6}\,\lambda$$

$$+ j(2.42\times10^{-2} + 3.09\times10^{-5}\,\tau + 6.19\times10^{-5}\,\lambda - 2.48\times10^{-2}\,\delta) \tag{B36}$$

Calculations for Quadrature Component of Currents Due to Insulation Resistance of Probes

The equivalent circuit of the difference bridge shown in figure 4 was further simplified to that in figure 5, to determine whether the minimum specified value of lead insulation resistance is adequate. This simplified circuit is justified by

(1) Assuming that C_{qh} equals C_{qo} and is very much larger in value than other capacitors in the circuit

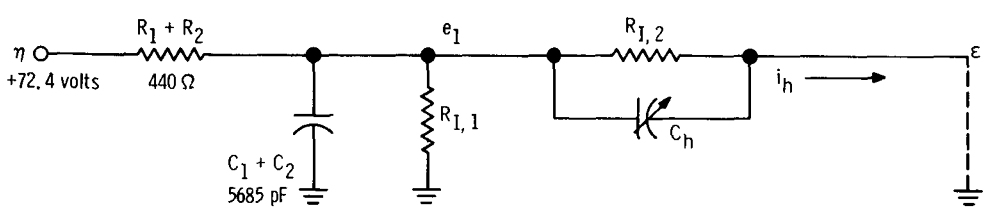
(2) Assuming that the error and coast bias voltages are set equal to zero, and neglecting the impedances in the error and coast bias legs since they are much larger than the input impedance to the error amplifier

(3) Neglecting the insulation resistance between a probe high impedance lead to ground since it is in parallel with the 5000 ohms input impedance to the error amplifier

(4) Neglecting C_3 and C_7 (the cable capacitances of the probe high-impedance leads) since they are small compared with the 80 000-picofarad input capacitance of the error amplifier with which they are in parallel

For analysis purposes, the error voltage ϵ at the input to the error amplifier may be assumed to be at ground potential.

Quadrature current in the liquid-hydrogen leg of difference bridge. - The circuit of the liquid-hydrogen leg in figure 5 is shown in the following sketch:



$R_{I,1}$ is the probe low-impedance lead to ground insulation resistance; $R_{I,2}$ is the insulation resistance between the two signal leads from the probe.

Solving for i_h gives:

$$i_h = \frac{\dfrac{\eta}{R_1 + R_2}\left(\dfrac{1}{R_{I,2}} + j\omega C_h\right)}{\dfrac{1}{R_1 + R_2} + \dfrac{1}{R_{I,1}} + \dfrac{1}{R_{I,2}} + j\omega(C_1 + C_2 + C_h)}$$

Since $1/(R_1 + R_2)$ is nine orders of magnitude larger than $1/R_{I,1} + 1/R_{I,2}$,

$$i_h \cong \frac{\dfrac{\eta}{R_1 + R_2}\left(\dfrac{1}{R_{I,2}} + j\omega C_h\right)}{\dfrac{1}{R_1 + R_2} + j\omega(C_1 + C_2 + C_h)}$$

Multiplying the numerator and denominator of the preceding equation by the complex conjugate of the denominator gives

$$i_h \cong \frac{\dfrac{\eta}{(R_1 + R_2)^2 R_{I,2}} + \dfrac{\omega^2 \eta C_h(C_1 + C_2 + C_h)}{R_1 + R_2} + j\omega \dfrac{\eta}{R_1 + R_2}\left(\dfrac{C_h}{R_1 + R_2} - \dfrac{C_1 + C_2 + C_h}{R_{I,2}}\right)}{\left(\dfrac{1}{R_1 + R_2}\right)^2 + \omega^2(C_1 + C_2 + C_h)^2}$$

Since $C_h/(R_1 + R_2)$ is six orders of magnitude larger than $(C_1 + C_2 + C_h)/R_{I,2}$ and $\left[1/(R_1 + R_2)\right]^2$ is four orders of magnitude larger than $\omega^2(C_1 + C_2 + C_h)^2$,

$$i_h \cong \frac{\eta}{R_{I,2}} + \omega^2 \eta C_h(C_1 + C_2 + C_h)(R_1 + R_2) + j\omega\eta C_h \tag{B37}$$

The real term in equation (B37) is the quadrature component of the current in the liquid-hydrogen leg. The $\eta/R_{I,2}$ term is the contribution of the probe-lead insulation resistance to the quadrature current.

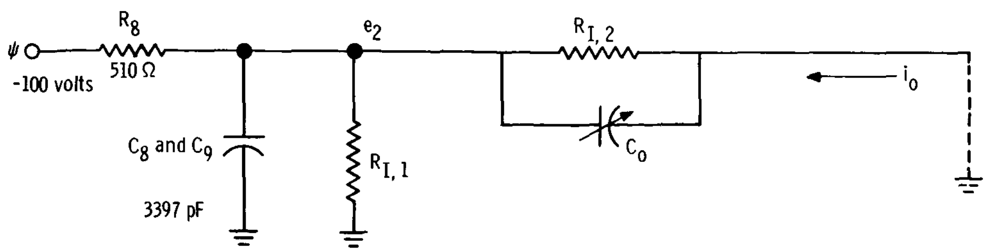
If appropriate values are substituted into the real terms of equation (B37), the magnitude of the quadrature current can be found for full and empty tank conditions and for a 1000-megohm insulation resistance. The magnitudes of the quadrature currents are as follows:

(1) The component of quadrature current due only to a resistance $R_{I,2}$ of 1000 megohms is 72.4 nonoamperes.

(2) The quadrature current at the full tank condition when $R_{I,2}$ is infinite is 795 nanoamperes.

(3) The quadrature current at the empty tank condition when $R_{I,2}$ is infinite is 635 nanoamperes.

Quadrature current in liquid-oxygen leg of difference bridge. - The circuit of the liquid-oxygen leg in figure 5 is shown in the following sketch:



Using the same reasoning as in the preceding section gives

$$i_o \cong \frac{\psi}{R_{I,2}} + \omega^2 \psi C_o (C_8 + C_9 + C_o) R_8 + j\omega\psi C_o \tag{B38}$$

If appropriate values are substituted into the real terms of equation (B38), the magnitude of the quadrature current can be found for full and empty tank conditions and for a 1000-megohm insulation resistance. The magnitudes of the quadrature currents are as follows:

(1) The component of quadrature current due only to a resistance $R_{I,2}$ of 1000 megohms is 100 nanoamperes.

(2) The quadrature current at the full tank conditions when $R_{I,2}$ is infinite is 304 nanoamperes.

(3) The quadrature current at the empty tank condition when $R_{I,2}$ is infinite is 200 nanoamperes.

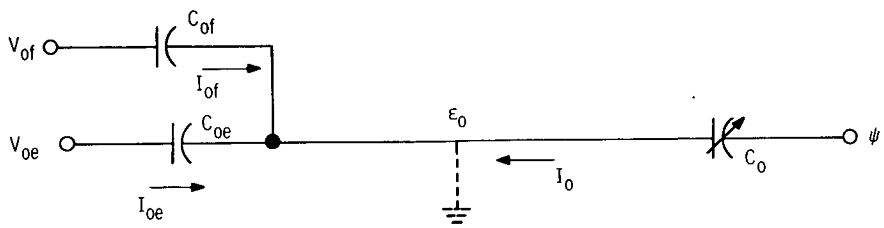
Quadrature current in dry adjust leg. - The current i_6 in the dry adjust leg is given as

$$i_6 = \frac{\beta}{R_6 - j\frac{1}{\omega C_6}} \tag{B39}$$

Substituting appropriate values into equation (B39) gives a value of 3.15 nanoamperes for the quadrature component of i_6.

Derivation of Quantity Readout Bridges Full and Empty Adjust Voltages

Liquid-oxygen-quantity bridge. - The simplified representation of the liquid-oxygen-quantity bridge is shown in the following sketch:



The values of the full adjust voltage V_{of} and empty adjust voltage V_{oe} can be calculated from the following relation:

$$I_{of} + I_{oe} + I_o = 0 \tag{B40}$$

For the full-tank condition

$$V_{of}C_{of} + V_{oe}C_{oe} = -\psi C_{ol} \tag{B41}$$

For the empty-tank condition

$$V_{oe}C_{oe} = -\psi C_{od} \tag{B42}$$

$$V_{oe} = -(-100)\,\frac{174}{3000}$$

$$= 5.8 \text{ volts}$$

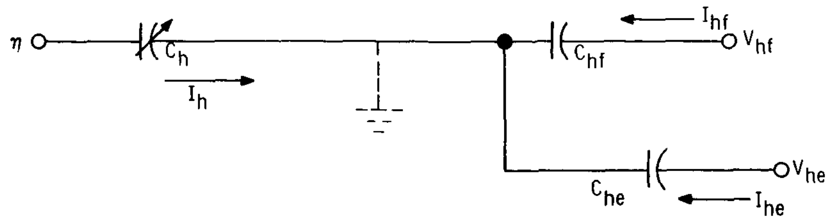
Subtracting equation (B42) from equation (B41) yields

$$V_{of}C_{of} = -\psi(C_{ol} - C_{od}) \tag{B43}$$

$$V_{of} = -(-100)\,\frac{84}{270}$$

$$= 31.1 \text{ volts}$$

Liquid-hydrogen-quantity bridge. - The simplified representation of the liquid-hydrogen-quantity bridge is presented in the following sketch:



The values of the full-adjust voltage V_{hf} and the empty-adjust voltage V_{he} can be calculated from the following relation:

$$I_h + I_{hf} + I_{he} = 0 \tag{B44}$$

For the full-tank condition

$$\eta C_{hl} = -V_{hf}C_{hf} - V_{he}C_{he} \tag{B45}$$

For the empty-tank condition

$$\eta C_{hd} = -V_{he}C_{he} \tag{B46}$$

$$V_{he} = -\frac{72.41(510)}{3000}$$

$$= -12.3 \text{ volts}$$

Subtracting equation (B46) from equation (B45) results in

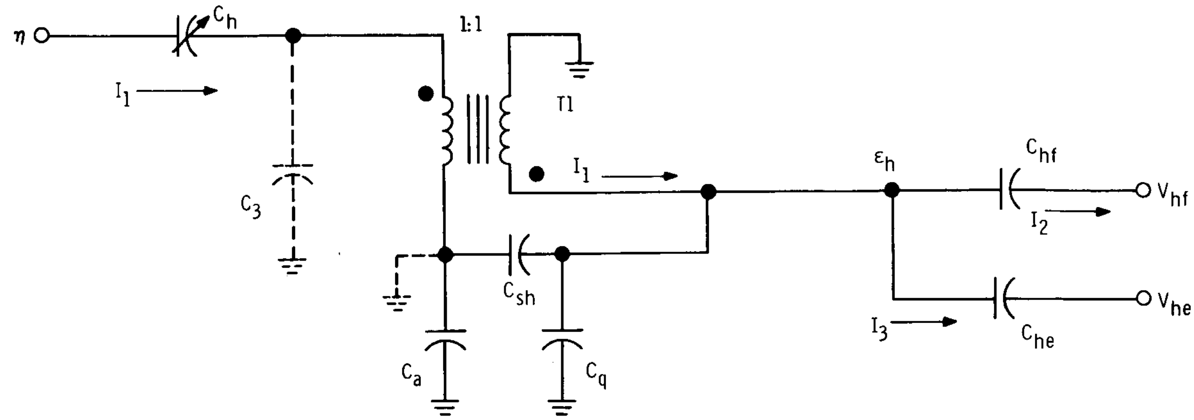
$$\eta(C_{hl} - C_{hd}) = -V_{hf}C_{hf} \tag{B47}$$

$$V_{hf} = -\frac{72.41(116)}{270}$$

$$= -31.1 \text{ volts}$$

Derivation of Errors Due to Malfunction of Quantity Indicating Circuits

Nonfunctioning of hydrogen rebalance potentiometer. - The simplified hydrogen quantity readout circuit and its connection to the main bridge are shown in the following sketch:



For quantity bridge balance, $I_1 = I_2 + I_3$, no current flows through C_{sh} or C_q, and $\epsilon_h = 0$. The hydrogen-quantity bridge was balanced for a full tank and the rebalance potentiometer wiper arm was assumed to remain at the full adjust voltage position. For the condition that $C_h = C_{hd}$ (the dry value of the hydrogen probe), ϵ_h is no longer zero since I_1 cannot supply the current demanded by $I_2 + I_3$. Therefore ϵ_h is reflected into the transformer primary and causes a current $\Delta I_1 = \epsilon_h j\omega(C_{hd} + C_3)$. If $C_{sh} = C_{hd} + C_3$, ΔI_1 flows through C_{sh} and no error current flow into the main bridge error amplifier. Hence, C_{sh} can be selected to compensate for the nonfunctioning of the rebalance potentiometer at one value of $C_h + C_3$. In practice, C_{sh} is selected to compensate at the dry value of the liquid-hydrogen probe capacitance. The worst case for maximum error current into the main amplifier with the rebalance potentiometer remaining at the full adjust voltage is for the half tank condition. The expressions for ΔI_1 and I_{sh} (the current through C_{sh}) at half tank are

$$\Delta I_1 = \epsilon_h j\omega\left[C_{hd} + C_3 + \left(\frac{C_{hl} - C_{hd}}{2}\right)\right] \tag{B48}$$

$$I_{sh} = \epsilon_h j\omega(C_{hd} + C_3) \tag{B49}$$

main amplifier error current is

$$I_{oh} = \Delta I_1 - I_{sh}$$

$$= \epsilon_h j\omega\left(\frac{C_{hl} - C_{hd}}{2}\right) \tag{B50}$$

and the error voltage is

$$\epsilon_h = \frac{\eta C_h + V_{hf}C_{hf} + V_{he}C_{he}}{C_q + C_{sh} + C_h + C_{hf} + C_{he} + C_3} \tag{B51}$$

By substituting appropriate values into equation (B51), ϵ_h can be calculated for the half tank condition.

$$\epsilon_h = \frac{[72.41(568) - 31.1(270) - 12.3(3000)]10^{-12}}{(55\,000 + 2047 + 568 + 270 + 3000 + 1537)10^{-12}}$$

$$= -69 \text{ millivolts}$$

The magnitude of error current into the main amplifier can be found by substituting the preceding value for ϵ_h into equation (B50).

$$I_{oh} = -j\ 69\times10^{-3} \times 2.52\times10^{3} \times 58\times10^{-12}$$

$$= -j\ 10.1 \text{ nA}$$

This error current can be expressed in equivalent pounds (kg) of liquid oxygen by dividing the value of I_{oh} by 1.06 nanoamperes per pound (2.33 nA/kg) of liquid-oxygen error. The equivalent inphase error is 9.55 pounds (4.34 kg) of liquid oxygen.

Nonfunctioning of oxygen rebalance potentiometer. - In a manner similar to that described in the previous section, the equivalent error can be found at the half-tank condition as the result of the oxygen-quantity rebalance potentiometer remaining at the full adjust voltage. The pertinent equations are

$$\Delta I_1 = \epsilon_o j\omega\left[C_{od} + C_7 + C_5 + C_4 + \left(\frac{C_{ol} - C_{od}}{2}\right)\right] \tag{B52}$$

$$I_{so} = \epsilon_o j\omega(C_{od} + C_7 + C_5 + C_4) \tag{B53}$$

$$I_{oo} = \epsilon_o j\omega \frac{C_{ol} - C_{od}}{2} \tag{B54}$$

$$\epsilon_o = \frac{\psi C_o + V_{of}C_{of} + V_{oe}C_{oe}}{C_q + C_{so} + C_o + C_{of} + C_{oe} + C_7 + C_5 + C_4}$$

$$= 70.4 \text{ millivolts} \tag{B55}$$

$$I_{OO} = j\ 7.43 \text{ nA}$$

Equivalent inphase error is 7 pounds (3.18 kg) of liquid oxygen.

Simultaneous nonfunctioning of oxygen and hydrogen rebalance potentiometers. - For this case, the error currents into the main amplifier subtract since the error currents, as the result of nonfunctioning potentiometers, are of opposite sign. The equivalent inphase error is 2.55 pounds (1.16 kg) of liquid oxygen.

Loss of Power to Hydrogen Quantity Circuit

As the result of loss of 400-hertz power to the liquid-hydrogen-quantity circuit the equivalent main bridge error can be found in a manner similar to that used in the previous sections.

However, the worst case is at the full-tank condition if it is assumed that the compensating capacitor $C_{sh} = C_{hd} + C_3$. The pertinent equations are as follows

$$\Delta I_1 = \epsilon_h j\omega \left[C_{hd} + C_3 + (C_{hl} - C_{hd})\right] \tag{B56}$$

$$I_{sh} = \epsilon_h j\omega (C_{hd} + C_3) \tag{B57}$$

$$I_{oh} = \epsilon_h j\omega (C_{hl} - C_{hd}) \tag{B58}$$

$$\epsilon_h = \frac{\eta C_{hl}}{C_q + C_{sh} + C_{hl} + C_{hf} + C_{he} + C_3}$$

$$= 725 \text{ millivolts} \tag{B59}$$

$$I_{oh} = j\ 211 \text{ nA}$$

The equivalent inphase error is 199 pounds (90.5 kg) of liquid oxygen. When the tank is half full, the error is 90.5 pounds (41.1 kg).

Loss of Power to Oxygen-Quantity Circuit

In a similar manner, the equivalent error due to loss of power to the oxygen-quantity circuit can be found. It is assumed that $C_{so} = C_{od} + C_7 + C_5 + C_4$. The relevant equations are

$$\Delta I_1 = \epsilon_o j\omega \left[C_{od} + C_7 + C_5 + C_4 + (C_{ol} - C_{od})\right] \tag{B60}$$

$$I_{so} = \epsilon_o j\omega (C_{od} + C_7 + C_5 + C_4) \tag{B61}$$

$$I_{OO} = \epsilon_o j\omega (C_{ol} - C_{od}) \tag{B62}$$

$$\epsilon_o = \frac{\psi C_o}{C_q + C_{so} + C_o + C_{of} + C_{oe} + C_7 + C_5 + C_4}$$

$$= -433 \text{ millivolts} \tag{B63}$$

$$I_{OO} = -j\ 91.5 \text{ nA}$$

Equivalent inphase error is 86.2 pounds (39.2 kg) of liquid oxygen.

Loss of Power to Both Quantity Circuits

In the event of simultaneous loss of power to both circuits, the equivalent error is equal to the difference of the individual errors at full tank. Therefore, the equivalent inphase error for this case is 113 pounds (51.4 kg) of liquid oxygen.

Effect of Shorting Quantity-Transformer Secondaries

Shorting the secondaries of the quantity transformers would have no effect on the main bridge because in operation the quantity circuit provides a rebalance voltage which keeps the error voltage developed across the transformer secondary extremely close to zero. Thus, in effect, the transformer secondaries are shorted. In the equivalent circuit of the bridge (fig. 4), the capacitors C_{qo} and C_{qh} would be very large and would produce the equivalent of a transformer short circuit across the primary or secondary.

In the expression for the error voltage (eq. (B7)), C_{qo} and C_{qh} appear only in the denominator. If $C_{qo} = C_{qh}$, the term $\omega^2\left[2C_o^2R_8 - C_5^2(R_5 + R_9) - C_4^2(R_3 + R_4)\right]$ is three orders of magnitude less than the term $1/R_7$ and thus does not change the value of the real term in the denominator. If C_{qo} is larger than C_{qh}, the term added to $1/R_7$ is less than when $C_{qo} = C_{qh}$. If C_{qh} equals $10C_{qo}$, the term added to $1/R_7$ is still two orders of magnitude less than $1/R_7$ and will not significantly change the value of the real term in the denominator.

REFERENCE

1. Judge, John F.: Centaur Propellant Control Succeeds. Missiles and Rockets, vol. 17, no. 17, Oct. 25, 1965, pp. 28, 31-32.

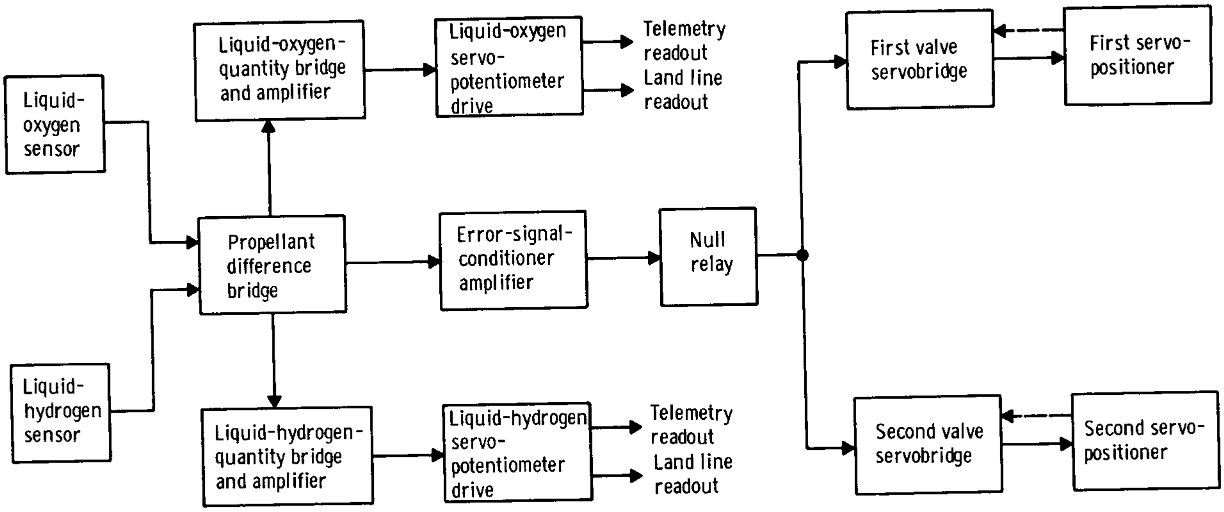


Figure 1. - Block diagram of propellant utilization system.

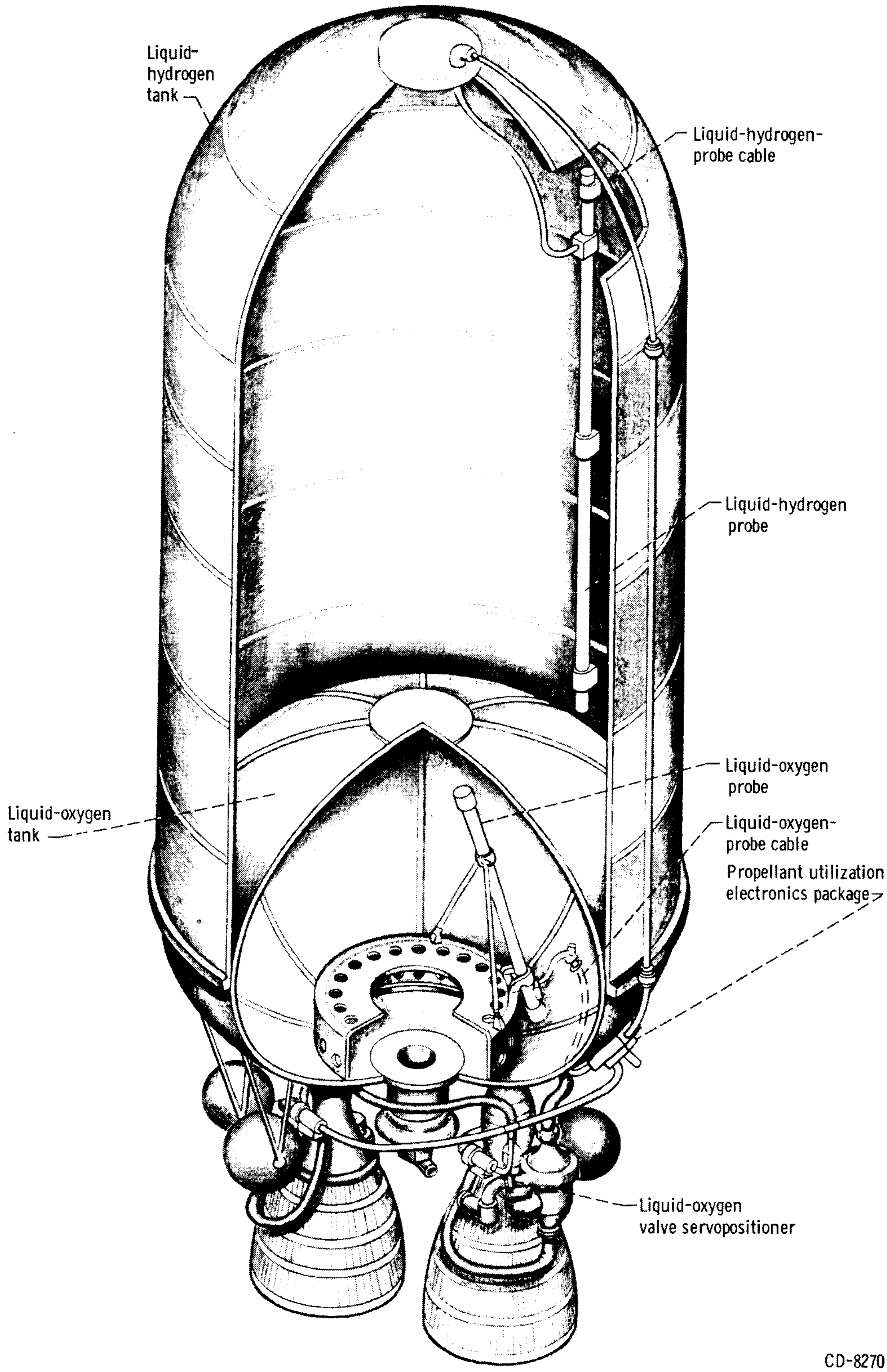


Figure 2. - Location of probes within vehicle.

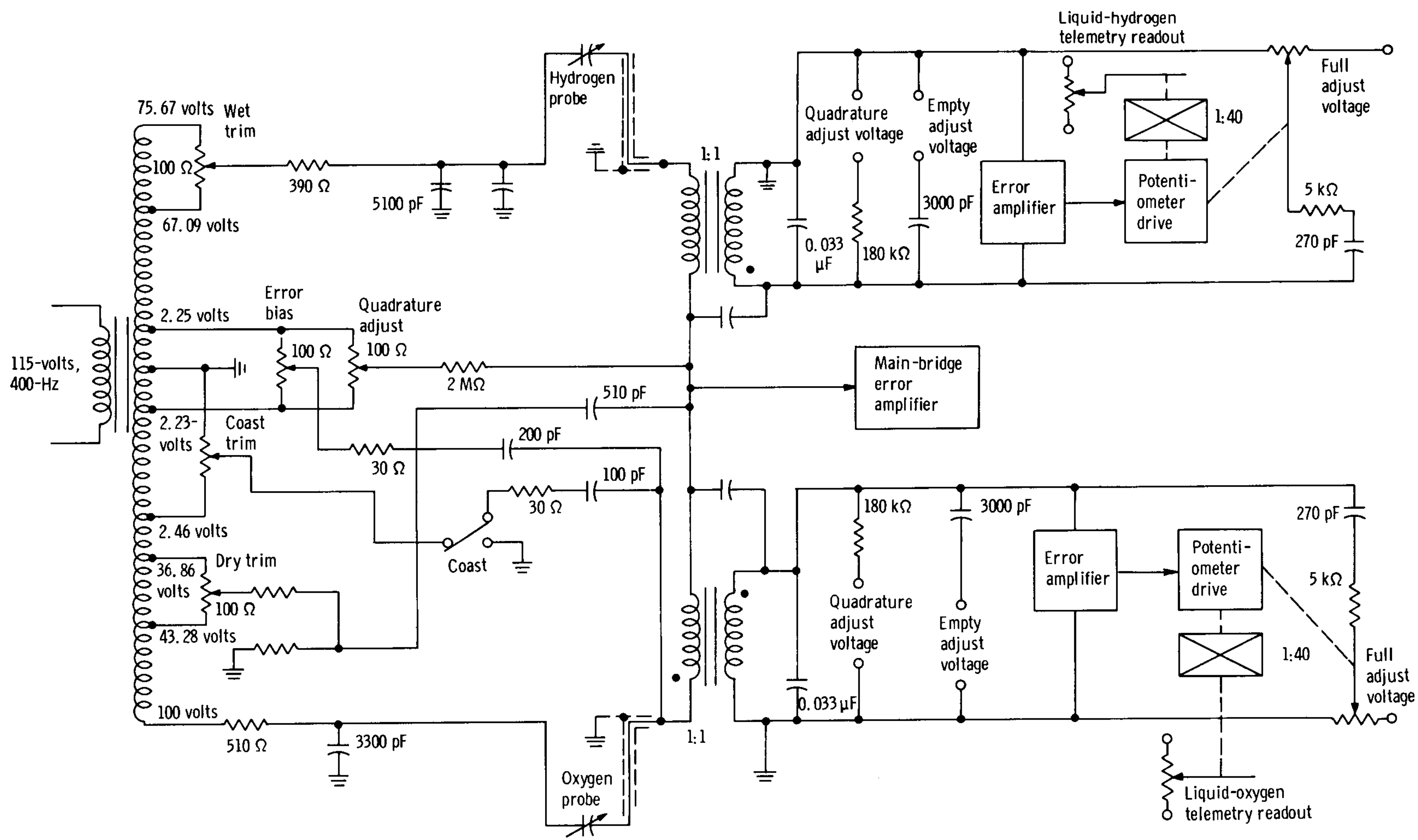


Figure 3. - Simplified circuit schematic of difference and quantity bridges.

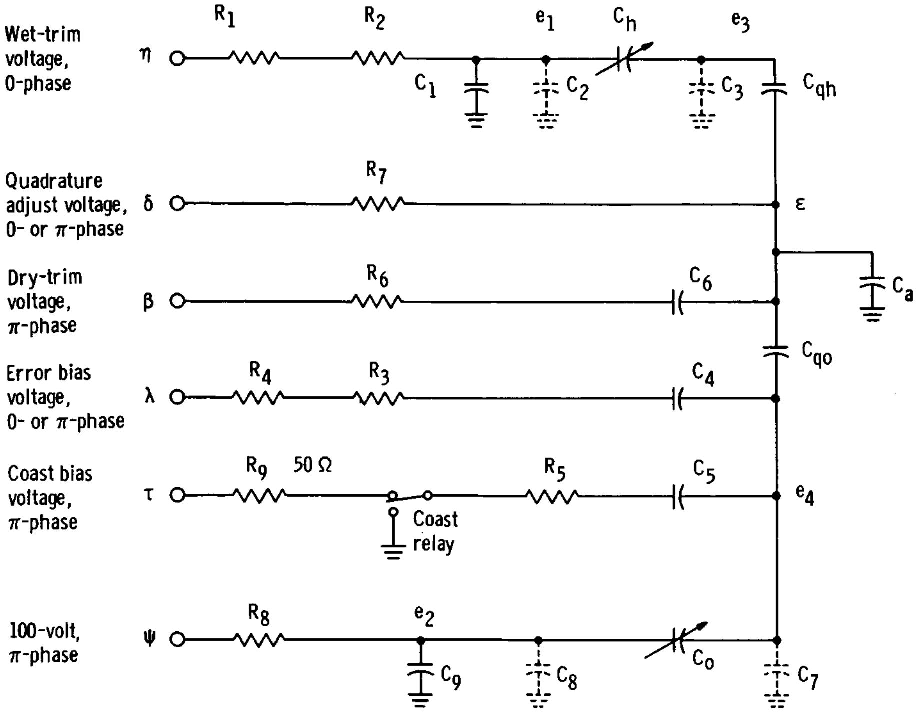


Figure 4. - Equivalent circuit of difference bridge.

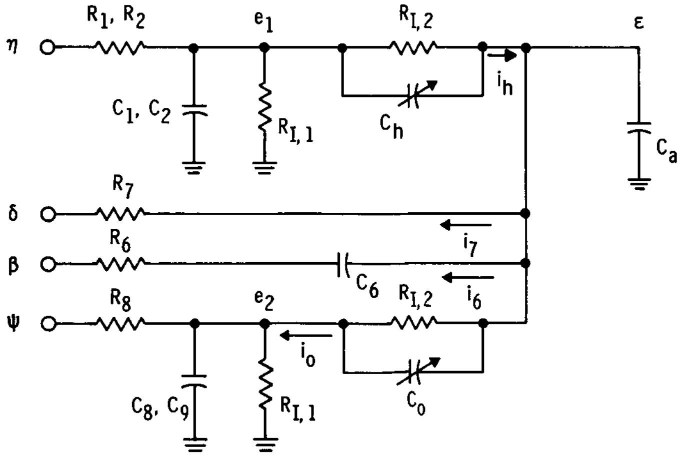


Figure 5. - Simplified equivalent circuit of difference bridge.

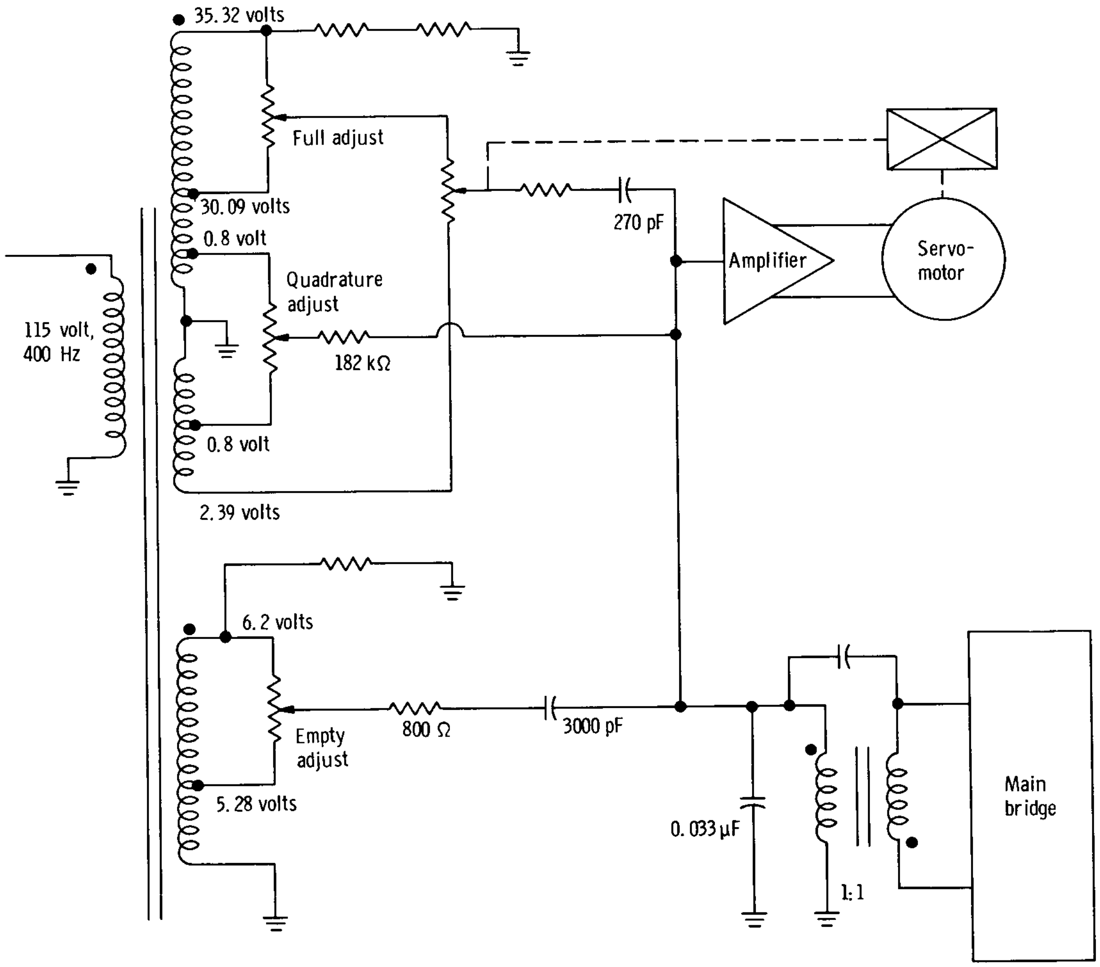


Figure 6. - Oxygen-quantity-bridge circuit.

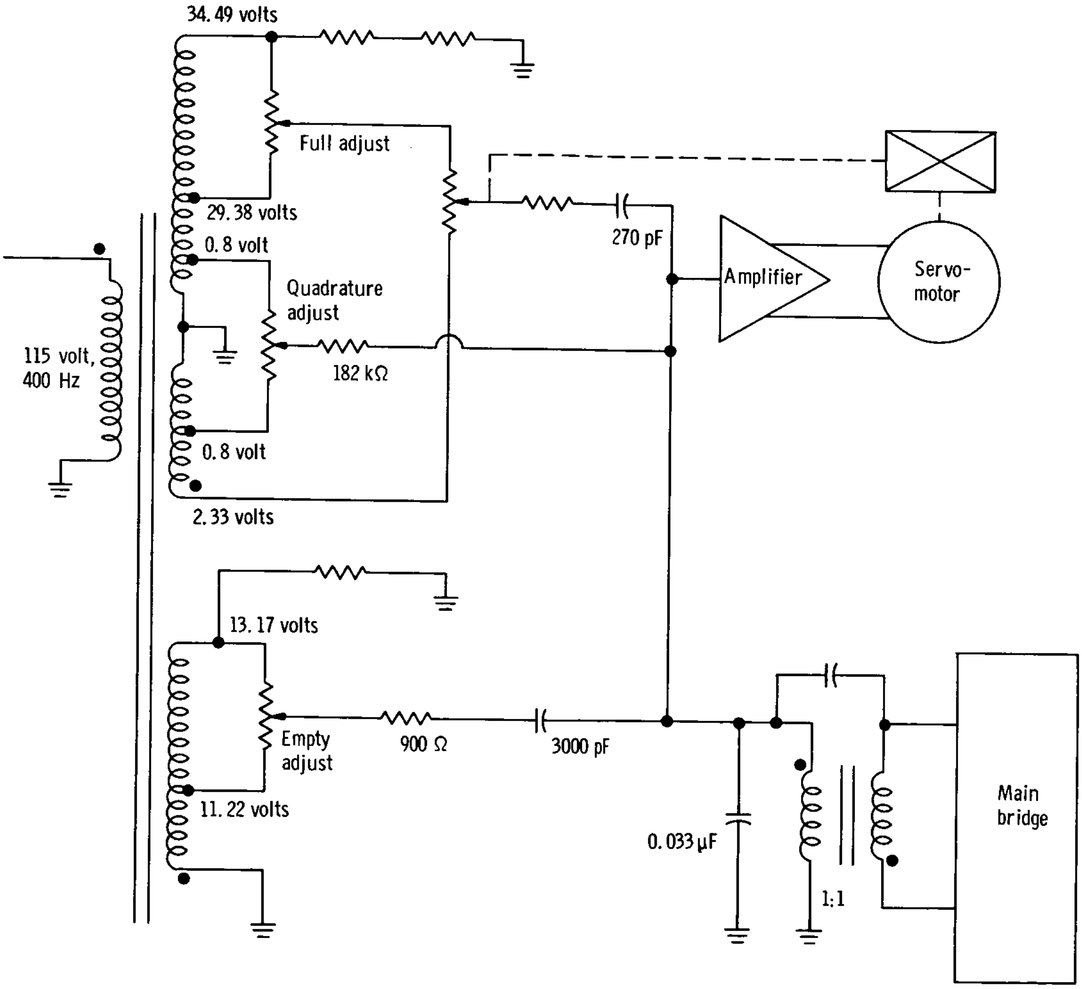


Figure 7. - Fuel-quantity-bridge circuit.