THEORETICAL RESEARCH ON STELLAR ATMOSPHERES

C. A. WHITNEY
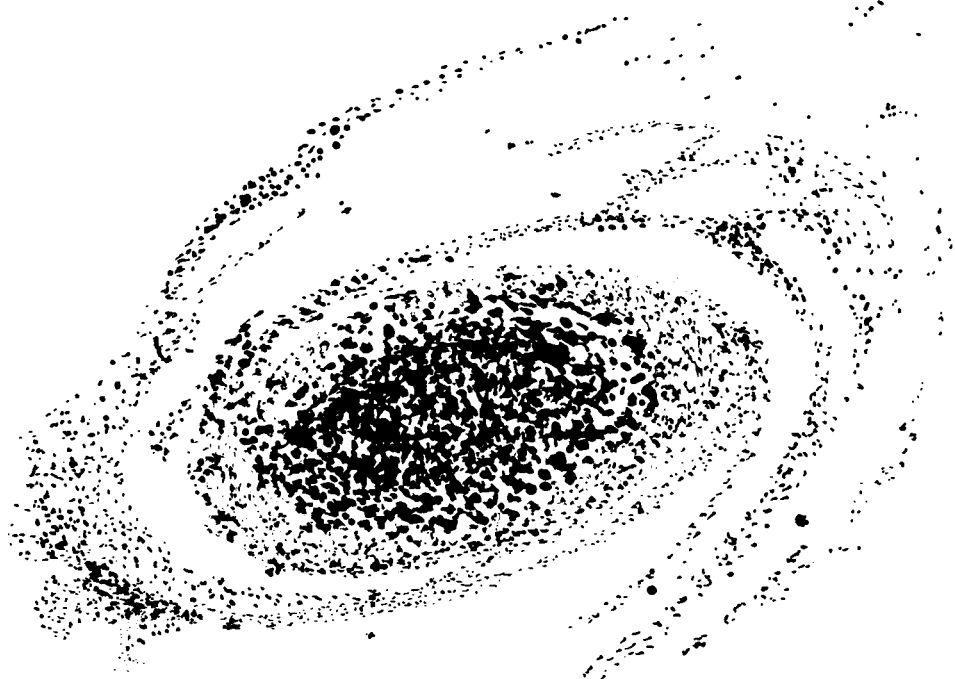
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Charles A. Whitney

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ABSTRACT

A brief sketch of the development of the theory of stellar atmospheres since 1920 is followed by a discussion of problems that may be central to this theory during the next five years.

The second section provides an annotated bibliography of theoretical work performed at Harvard College Observatory and Smithsonian Astrophysical Observatory during the decade 1957-1967.

RÉSUMÉ

Un bref exposé du développement, depuis 1920, de la théorie des atmosphères stellaires est suivi d'une discussion des problèmes qui peuvent être centraux à cette théorie pendant les cinq prochaines années.

La seconde partie fournit une bibliographie annotée du travail théorique accompli à l'observatoire du Harvard College et à l'observatoire d'astrophysique du Smithsonian, au cours des dix dernières années, de 1957 à 1967.

КОНСПЕКТ

Краткий очерк развития с 1920 года теории звездных атмосфер сопровождается обсуждением проблем, которые могут быть основными для этой теории втечение будущих пяти лет.

Второй отдел дает аннотированную библиографию теоретической работы выполненной в обсерватории Гарвардского колледжа и в Смитсониан астрофизической обсерватории втечение десяти лет с1957 до 1967.

PREFACE

This report is intended as a bibliographical guide to theoretical work done by the stellar atmospheres group at the Harvard and Smithsonian observatories during the past decade. Although the abstracts are so brief that they cannot actually convey the contents of the published work, an attempt has been made to place these papers within the context of the broader field of stellar physics and evolution.

THEORETICAL RESEARCH ON STELLAR ATMOSPHERES

Charles A. Whitney

1. INTRODUCTION

The theory of stellar atmospheres is imbedded in several branches of astrophysics, and to set the context for the present report it is useful to trace briefly the recent history of stellar physics.

By the end of 1920, the mathematical tools for delving deeply into stellar interiors had been provided, but two essential features of the theory were still missing. The opacity of stellar material was still an unknown quantity, because the quantum theory had not yet been used to calculate the rates of interaction between radiation and matter. Also, the source of the energy that is carried away in starlight was unknown. These elements were still missing when Eddington's Internal Constitution of the Stars was published in 1926; yet he was able to circumvent ignorance to an amazing extent by an ingenious combination of assumption, physical reasoning, and mathematical analysis.

The first book dealing solely with stellar atmospheres, Unsöld's Physik der Sternatmosphären, appeared in 1935 and summarized substantial progress in the quantum calculation of opacity.

Within a decade of the publication of Unsöld's book, the first of a sequence of massive computations of model atmospheres appeared. Strömgren and his students, and later Pecker and his students, constructed these mathematical models with desk calculators, and in magnitude these efforts remind one of Johannes Kepler's attempts to fit an orbit to the motions of Mars.

In principle, the conditions of hydrostatic and radiative equilibrium were imposed on these models. In practice, however, the computational scheme was insufficient to ensure strict conservation of the radiative flux. The assumption of local thermodynamic equilibrium (LTE), introduced by K. Schwarzschild, was adopted, because any other assumption would have required prohibitively laborious calculations.

These so-called classical models of stellar atmospheres provided a useful basis for a qualitative understanding of spectral classification and for the approximate determination of stellar temperatures and surface gravities. But they were not capable of treating many intriguing aspects of stellar spectra. Struve and Swings, for example, starting in the 1930's, had described a number of "peculiar" features such as bright lines of complex and often variable structure and dark lines with anomalous intensities. These features and abundant data on the spectra of variable stars were kept outside the domain of quantitative theoretical analysis to await the development of "nonclassical" models. They are still there, outside our grasp, but continued progress in solar and stellar physics promises to provide insight into these nonclassical problems.

In 1950, Chandrasekhar's mathematical investigations of stellar and planetary atmospheres culminated in the publication of his Radiative Transfer, a treatise that developed some powerful techniques for handling monochromatic radiative equilibrium (scattering) in finite atmospheres. Kourganoff's Basic Methods in Transfer Problems (1952) summarized the mathematical progress of the preceding two decades, while the textbooks of Wooley and Stibbs (The Outer Layers of a Star, 1953) and of Ambartsumian (Theoretical Astrophysics, 1952, 1958) summarized the physical theory. The second, considerably expanded, edition of Unsöld's book appeared in 1955.

By 1960, the electronic computer had been applied to the construction of model stellar atmospheres and to the synthesis of stellar spectra. The last decade, and in particular the last few years, has thus seen the exploitation of the classical theory of stellar atmospheres and the calculation of numerous precise models. The calculations are now being carried to high precision, not because there is faith in the relevance of the classical models but because a lack of faith motivates a search for a failure of the theory. The dynamical state of the solar atmosphere and the presence of emission lines in stellar spectra are adequate evidence of the need to extend the theoretical development.

Use of the computer has already permitted a relaxation of the assumption of LTE, and it has allowed a systematic study of the range of validity of this assumption. Thomas and Athay, in their Physics of the Solar Chromosphere (1961), emphasized the need to reject LTE as an initial assumption in studying the solar chromosphere, and several lines of subsequent work have indicated its failure even in the deeper atmospheric layers.

In anticipation of future developments, one point is clear. Observations from balloons, rockets, and satellites will expose wide stretches of the red and violet spectra. We look ahead with keen anticipation, because these spectral regions contain the bulk of radiation from hot stars and from cool protostars; these regions may also reveal extended, tenuous envelopes carrying matter away from aging stars or into unformed stars.

In looking ahead, we can take either of two approaches:

A. We can attempt to guess what will be the most exciting problems revealed by new observations.

B. We can try to foresee the areas of theory that will flourish and be fruitful for the interpretation of new data.

The point of view adopted for these comments is along the second line, toward an extrapolation or "analytical continuation" of present work. These extrapolations will inevitably broaden as new results appear.

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Let us first consider the question: Where do we stand now? It is clear from the bibliographical summary given below that the mathematical techniques for the successful construction of classical or static atmospheric models are now available. Although the opacity problem is still serious, the exploitation of shock tubes and atomic beams, the empirical analysis of the solar atmosphere, and the development of theory are all contributing to its solution. The calculation of collision cross sections is in essentially the same situation as that of opacities. Specifically, we urgently need cross sections for collisional transitions among the upper levels of neutral hydrogen.

The next 5 years should see a final codification of classical models, and the succeeding 5 should put us well on the way toward developing dynamical models.

As a basis for anticipating the 5 years immediately ahead of us, let us survey the current theoretical problems relating to three particular classes of main-sequence stars: A) the hot stars of temperature greater than 10,000° K; B) the extremely cool stars of temperature below 4000° K; and C) the sun.

A. The Hot Stars

The greatest current interest is in departures from LTE, in the influence of line blanketing, and in the determination of the helium abundance.

Since departures from LTE are probably detectable in the visible and red continua, we need not await the definitive ultraviolet data to check this aspect of the calculations. Preliminary results seem to be consistent with the physical theory, but final confirmation of this portion of the theory awaits the determination of collisional rates among the upper levels of hydrogen.

It appears that uncertainties in the calculation of continuous spectra are now at the level of 10 to 15%, and a large portion of this uncertainty stems from the influence of line blanketing.

Thus, for stars of moderately high temperature, reliable photospheric models have evidently been achieved, but, as we shall see below, this is only part of the study.

B. Cool Stars

Among the very cool stars, the complexity of line spectra and the inaccessibility of the infrared have made progress very slow. The continuum opacity of these atmospheres is almost as low as that in the earth's atmosphere; hence the pressure is high and the influence of pressure on the calculation of opacity substantially increases the subtlety of the calculations. The important sources of opacity are H_2O, the ionization continua of negative ions, and Rayleigh scattering.

Models computed without a consideration of convection may be quite unrealistic, but they are interesting. They are the most nongray, or the most "colorful," models ever computed; the continua show opacity variations by a factor of 100. Enhanced opacity in the infrared continuum of these models, and even in the sun itself, promises the direct detectability of chromospheres at wavelengths greater than about 50 µ.

Also, these models show purely radiative temperature inversions at small optical depths, arising from the strong depth dependence of the opacity and from departures from LTE. Although we see here the possibility of a purely radiative chromosphere, or temperature rise, there is little doubt that convection penetrates to great heights in the photospheres of these stars.

C. The Sun

Solar photospheric models are still interesting because they reveal persistent and substantial discrepancies with observed near-ultraviolet continua. Even the construction of a self-consistent empirical model has not yet been achieved.

The role of solar observations will continue to be particularly crucial in the development of the theory of stellar atmospheres, because the sun provides immediate clues to models for the origin of peculiar stellar features. This point is exemplified by the H + K emission features, which certainly point to the universality of chromospheres among the cooler stars.

1.1 Atmospheric Velocity Fields

The velocity field of the solar atmosphere is a suggestive source of models for the line broadening that is known to exist among stellar giants. But the solar velocity field has four distinct components, each predominating at a different geometrical level:

    A.   The turbulent subphotospheric convection.

    B.   The gravitational oscillations of the upper photosphere.

    C.   The chaotic field of the chromosphere.

    D.   The steady outward flow of the solar wind whose energy source is the high temperature of the corona.

These four regimes of motion are of a distinctive physical nature, and it is essential for us to distinguish their relative roles in explaining certain symptoms of velocity fields seen in stellar spectra. To distinguish them empirically, we require high spectral purity for selected lines at a wide range of wavelengths. We probably also require time resolution on the order of minutes.

1.2 Expanding Shells

These comments on velocity fields lead naturally into the next phase of theoretical development. A variety of spectral data indicates the existence of expanding shells around giants and supergiants. In some cases these shells produce emission and absorption lines in the visible spectrum (P Cygni, Wolf-Rayet stars, α Herculis, Be stars), and recent work by Spitzer and

Morton indicates that some B giants have shells that produce no effects in the visible at all. Rocket spectra have revealed ultraviolet resonance lines of C IV and Si IV that display a violet displacement of 2000 km/sec and are associated with emission lines showing little or no displacement.

These observations imply stellar mass loss, which must be significant from the cosmological standpoint.

1. 3 Stellar Rotation

Spectroscopic effects of rotation, apart from the Doppler broadening of lines, occur only at the high rotational velocities found among hot stars. Unfortunately, the influence of rapid rotation is easily mimicked by small changes of temperature stratification or by a shift in the mean temperature of the atmosphere. So far, the only substantial atmospheric calculations are based on simplified interior models and von Zeipel's theorem.

If more reliable calculations can be executed, there exists the possibility of determining both the equatorial velocity of rotation and the angle of inclination for individual stars. But it is quite clear that the empirical exploitation of these results will require observations in the ultraviolet.

1. 4 Continuum Photometry

Bolometric corrections for hot stars, an essential type of data for studies of stellar evolution and cosmology, can only be obtained from ultraviolet photometry. Also, the ultraviolet continuum will provide an important check on two aspects of theoretical models — the opacity and the anticipated departures from LTE.

These continuum studies cannot be interpreted without line-blanketing data obtained from high-resolution spectra of representative stars. (There are several historical instances in which the interpretation of "continuum" data was vitiated by the failure to correct for the superposed line spectrum.)

1. 5 Line Spectra

There are several other types of high-resolution studies of line spectra that, if performed in the ultraviolet, may have a profound influence on the theory of stellar atmospheres:

A. Study of the helium problem, which has proved very difficult in the visible. This may be soluble with data from the ultraviolet.

B. Use of resonance lines to detect shells and determine abundances without the uncertainties of calculating excitation conditions.

C. Search for peculiarities of B, A, and F stars in the ultraviolet.

D. Determination of isotope ratios.

2. ANNOTATED BIBLIOGRAPHY

2.1 Mathematical Techniques

The Proceedings of the First Harvard-Smithsonian Conference on Stellar Atmospheres (Proc. H.-S. C.S.A. No. 1) and of the Second Conference (Proc. H.-S. C.S.A. No. 2) contain a number of papers dealing with mathematical and computational techniques. These conferences were published as SAO Special Reports No. 167, December 21, 1964, and No. 174, May 17, 1965, respectively.

Convenient schemes for integrating the equation of hydrostatic equilibrium are discussed by:

O. Gingerich
    1963. Studies in non-gray stellar atmospheres. I. A basic computer program. Astrophys. Journ., vol. 138, pp. 576-586.
    1964. Hamming's method for integrating the equation of hydrostatic equilibrium. Proc. H.-S. C.S.A. No. 1, pp. 56-57.

Numerical techniques for the evaluation of integrals associated with the transfer equation and for the solution of Milne's integral equation are developed and demonstrated on the basis of fitting quadratic segments in:

E. H. Avrett and R. Loeser
    1963. A simple and accurate method for the evaluation of the Milne integrals. Journ. Quant. Spectrosc. Radiat. Transfer, vol. 3, pp. 201-209.

and they are modified by Avrett to incorporate trigonometric segments in:

E. H. Avrett
    1964. Evaluation of flux and intensity integrals. Proc. H.-S. C.S.A. No. 1, pp. 63-73.

O. Gingerich

    1964. A procedure for evaluating the flux integrals. Proc. H.-S.

         C.S.A. No. 1, pp. 77-79.

A systematic discussion of the <u>integral equations</u> describing the source function for various idealized atoms is given in:

E. H. Avrett and R. Loeser

    1966. Kernel representations in the solution of line-transfer problems.

         Smithsonian Astrophys. Obs. Spec. Rep. No. 201, 114 pp.

Although the integral-equation and differential-equation approaches to the transfer problem are mathematically equivalent, the differential-equation method promises to be the more effective method for treating inhomogeneous media. A new approach, which promises numerical stability, utilizes the <u>generalized Riccati transformation</u> to convert two-point differential equations into equivalent sets of one-point systems. The method was developed independently by G. B. Rybicki and P. D. Usher and is described, with several examples, in:

G. B. Rybicki

    1965. A new differential equation approach to transfer problems.

         Proc. H.-S. C.S.A. No. 2, pp. 149-161.

G. B. Rybicki and P. D. Usher

    1966. The generalized Riccati transformation as a simple alternative

         to invariant imbedding. Astrophys. Journ., vol. 146, pp. 871-

         879.

G. B. Rybicki and D. G. Hummer[*]

    1967. Spectral line formation in variable-property media. (in press)

A summary review of all methods that had been applied to the non-LTE line formation problem is given in:

D. G. Hummer and G. B. Rybicki

> 1967. Computational methods for non-LTE line-transfer problems. In Methods of Computational Physics vol. 7, ed. by B. Alder, S. Fernbach, and M. Rotenberg, Academic Press, New York, pp. 53-127.

2.2 Temperature-Correction Procedures

The structure of a stellar atmosphere in static equilibrium is, in principle, determined once the radiative flux, surface gravity, and chemical composition are specified. In a gray atmosphere the radiation field and the matter field can be decoupled by use of the optical depth as the independent variable. The temperature distribution can be found by solving a linear integral equation (Milne's), and then the pressure can be obtained by integration of a nonlinear, first-order differential equation. But for a nongray atmosphere, the process of deriving the structure, given the parameters, involves the solution of a highly complex and nonlinear integral equation because the matter and radiation problems cannot be decoupled. Iteration techniques are indispensable, and the first such technique was devised by Strömgren and employed by Miss Underhill and J. -C. Pecker. The use of such a technique represented a fundamental step in the development of model atmospheres, but the process converged slowly because the iterations did not adequately exploit information contained in the moments of the radiation field. (In current terminology, it was a pure "Lambda," or intensity, iteration process.)

Krook initiated a systematic search for more efficient iteration procedures in the following three papers:

M. Krook

> 1955. On the solution of equations of transfer. I. Astrophys. Journ., vol. 122, pp. 488-497.

M. Krook

1959. Structure of stellar atmospheres. II. Astrophys. Journ., vol. 129, pp. 724-733.

1959. Structure of stellar atmospheres. III. Astrophys. Journ., vol. 130, pp. 286-295.

The first of these papers elucidates the intimate relationships between various techniques that had been applied to the gray case; the second establishes a _moment method_; and the third describes a _double moment method_. An alternative formulation in terms of the _integral equations_ is given in:

M. Krook and J. -C. Pecker

1958. Sur le calcul de modèles d'atmosphère en équilibre radiatif (cas non-gris). Comptes Rendus, vol. 247, pp. 1177-1179.

The _numerical test_ of moment methods is carried out in:

P. H. Stone and J. E. Gaustad

1961. The application of a moment method to the solution of non-gray radiative-transfer problems. Astrophys. Journ., vol. 134, pp. 456-468.

A related technique employing approximate representation of the kernels in the Milne integral equations is tested in:

P. H. Stone

1963. Approximate integral equations for the temperature in non-gray model atmospheres. Astrophys. Journ., vol. 137, pp. 628-640.

A _Poincaré-Lighthill perturbation procedure_, in which both the optical depth and the temperature are perturbed, as developed by Krook, was applied to the nongray problem in:

O. Gingerich

1961. A study of non-gray stellar atmospheres. Thesis, Harvard University.

M. Krook

> 1963. A perturbation method for non-gray stellar atmospheres. Astrophys. Journ., vol. 137, pp. 863-873.

Krook's initial treatment was based on the gray solution in the zeroth order, and an iterative form was developed shortly after and described in:

E. H. Avrett and M. Krook

> 1963. The temperature distribution in a stellar atmosphere. Astrophys. Journ., vol. 137, pp. 874-881.

An extension to include scattering and radiation pressure is provided in:

E. H. Avrett and M. Krook

> 1963. A rapidly convergent iterative procedure for the calculation of the temperature-pressure relation in a stellar atmosphere. Journ. Quant. Spectrosc. Radiat. Transfer, vol. 3, pp. 107-113.

Early applications of the Krook-Avrett technique to models for the solar atmosphere are described in:

O. Gingerich

> 1962. Krook's iterative procedure for the temperature distribution in model stellar atmospheres. (abstract) Astron. Journ., vol. 67, pp. 272.
> 1963. Studies in non-gray stellar atmospheres. I. A basic computer program. Astrophys. Journ., vol. 138, pp. 576-586.

The method is rediscussed and its relationship to other methods is briefly indicated in:

E. H. Avrett

> 1964. Temperature-correction procedures. Proc. H.-S. C.S.A. No. 1, pp. 83-91.

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2.3 Influence of Departures from LTE

The assumptions of radiative equilibrium and of hydrostatic equilibrium are not sufficient for us to specify a complete set of equations for a model atmosphere. An additional assumption is needed concerning the microscopic state of the gas. For a static atmosphere, the most general assumption, denoted <u>local kinetic equilibrium (LKE)</u>, requires that the <u>net</u> rate of all transitions into every microscopic state vanishes. The following series of papers examines the differences between atmospheric models computed with the LKE assumption and those computed with the more restrictive assumption of <u>local thermodynamic equilibrium (LTE)</u>, which adopts the Boltzmann-Saha distribution for the populations of microscopic states.

The possible importance of the distinction between LTE and the more general LKE is demonstrated in:

W. Kalkofen

1964. Departures from LTE implied by bound-free and free-free transitions. Proc. H.-S. C.S.A. No. 1, pp. 175-183.

This paper considered an atmosphere in radiative equilibrium and composed of pure hydrogen, and the author pointed out the slow convergence of iteration schemes incorporating line radiation. Neglecting radiative and collisional bound-bound transitions in an adopted five-level model of hydrogen led to a computed overpopulation of the ground state by a factor of 2 and to an under-population of the second and third levels by a similar factor near the surface.

The influence of bound-bound radiation is investigated by including the Lyman-α line in:

W. Kalkofen and E. H. Avrett

1964. Departures from LTE in a model atmosphere. (abstract) Astron. Journ., vol. 69, p. 546.

This paper also indicates that departures from LTE influence the continuum in such a way that the use of LTE models to interpret empirical spectra will lead to an underestimate of the effective temperature (or the bolometric correction) of hot stars.

These investigations are carried ahead with more complete computations of opacity and a set of model atmospheres in:

S. E. Strom and W. Kalkofen

1966. The effect of departures from LTE on the stellar continuum fluxes in the spectral-type range B5-A0. Astrophys. Journ., vol. 144, pp. 76-95; abstract in Astron. Journ., vol. 70, pp. 693-694, 1965.

The authors adopted detailed balancing for the hydrogen lines and assumed all levels except the three lowest levels of hydrogen to be populated according to LTE. The models showed level 2 to be underpopulated and level 3 to be overpopulated by about 10% in the photosphere. These population changes, with respect to the LTE models, depressed the temperatures inferred from the Balmer discontinuity by about 1000° K in the range 10000 to 15000° K.

A basic assumption in the calculations reported in these papers was that detailed balancing exists among the radiative transitions in the hydrogen lines and in the Lyman continuum. The basis of this assumption of "line saturation" is discussed in detail in:

W. Kalkofen

1966. Deviations from LTE in stellar atmospheres. Journ. Quant. Spectrosc. Radiat. Transfer, vol. 6, pp. 633-651.

This paper also describes a formal development of the transfer equation for the general LKE case and indicates the nature of the results to be expected in the low-density surface layers of an atmosphere in radiative equilibrium.

The influence of bound-bound collisional transitions was studied for two models of T_{eff} = 15000 and 10000° K and for a model of T_{eff} = 7500° K in which LKE is adopted for the H⁻ ion as well as for hydrogen. The results are reported in:

W. Kalkofen and S. E. Strom

    1966. The effects of deviations from LTE and line blanketing on stellar atmospheres in the range B5 to A5. Journ. Quant. Spectrosc. Radiat. Transfer, vol. 6, pp. 653-660.

S. E. Strom and W. Kalkofen

    1966. Deviations from LTE and their effect on stellar spectra in the range B5 to F0. (abstract) Astron. Journ., vol. 71, pp. 873-874.

The authors conclude that collisional transitions may reduce departures from LTE in main sequence stars, but definitive calculations will not be possible until cross sections are more reliable.

A direct empirical test for departures from LTE is suggested and discussed in detail in:

S. E. Strom and W. Kalkofen

    1967. An observational test for departures from LTE in the spectral range B5 to A0. Astrophys. Journ., in press.

The method requires empirical evaluation of the ratio of the Balmer discontinuity D_B, to the Paschen discontinuity D_p. For LTE models, the radio D_B/D_p is nearly independent of temperature and gravity, while it depends on gravity for the non-LTE models. A trial with published data indicates that while the LTE models predict too low a value of D_p for a given D_B, the non-LTE models provide an improved fit.

An <u>extension of those studies to cooler stars</u> will be found in:

S. E. Strom

    1967. Departures from LTE in G-star and K-star atmospheres.

        Astrophys. Journ., in press.

In this paper, models computed for $4000°\,K < T_{eff} < 6000°\,K$ and for various assumed rates of the associative detachment of H^- are described. The interplay between collisional and radiative transitions under the requirement of radiative equilibrium leads, in some cases, to the prediction of <u>purely radiative temperature inversion</u>, first discussed by Cayrel. Unfortunately, a wide uncertainty in the relevant cross section for H^- prevents a definitive analysis.

2.4 Properties of Model Stellar Atmospheres

The temperature distribution in a nongray atmosphere can be widely different from that in a gray atmosphere at the same temperature. The nongray atmosphere will usually, but not necessarily, have a lower surface temperature than the gray one. On the other hand, at large depths such that $\tau_\nu > 1$ for <u>all</u> frequencies, the temperature gradient in corresponding gray and nongray atmospheres can be made to coincide through use of the depth scale defined by the Rosseland mean of opacity.

The temperature distribution at small optical depths in a schematic <u>atmosphere with a step-function opacity</u> is treated as a boundary-layer problem in:

G. F. Carrier and E. H. Avrett

    1961. A non-gray radiative transfer problem. Astrophys. Journ., vol.

        134, pp. 469-481.

The essential step in this procedure is to consider the atmosphere to be divided into two regions: 1) a boundary-layer in which $\tau \ll 1$, and 2) a deeper region in which higher derivatives of certain functions of the temperature are negligible.

Numerical solutions for this step function of opacity are obtained to high accuracy using the appropriately modified Milne integral equation in:

E. H. Avrett and R. Loeser

1964. Boundary temperatures for models with a step-function absorption coefficient. Proc. H.-S. C.S.A. No. 1, pp. 196-198.

This paper mapped the domain of parameters in which the surface temperature was increased by departures from grayness.

An approximate treatment of surface temperatures is provided by:

M. Lecar

1964. Estimates of the boundary temperature of a nongray stellar atmosphere. Proc. H.-S. C.S.A. No. 1, pp. 185-194.

This paper provides analytical solutions that also show the possibility of a temperature rise for slightly nongray atmospheres.

The calculation of continuum opacities is discussed in:

O. Gingerich

1961. Polynomial approximation for the negative hydrogen-ion absorption coefficient. Astron. Journ., vol. 134, pp. 653-656.

1964. Review of opacity calculations. Proc. H.-S. C.S.A. No. 1, pp. 17-41.

The following sequence of papers discusses the structure and spectra of model atmospheres in radiative equilibrium:

S. E. Strom and E. H. Avrett

1964. Calculation of early-type model stellar atmospheres. (abstract) Astron. Journ., vol. 69, pp. 150-151.

1964. Detailed examination of a non-gray stellar atmosphere. (abstract) Astron. Journ., vol. 69, p. 559.

S. E. Strom and E. H. Avrett

    1964.   The temperature structure of early-type model stellar atmos-
            pheres.   I.   Detailed analysis of a 10000° K model. Astrophys.
            Journ., vol. 140, pp. 1381-1390.

    1965.   The temperature structure of early-type model stellar atmos-
            pheres.   II. A grid of stellar models. Astrophys. Journ.,
            Suppl. No. 103, vol. 12, pp. 1-98.

One purpose of the First Harvard-Smithsonian Conference on Stellar Atmospheres was the <u>comparison of independent attempts to compute an accurate model</u> in radiative equilibrium and with the assumption of LTE. The comparison provided satisfactory confirmation of the numerical accuracy of the computer program, and the results are published in:

O. Gingerich, D. Mihalas,[*] S. Matsushima,[**] and S. Strom

    1965.   An archetype non-gray stellar atmosphere. Astrophys. Journ.,
            vol. 141, pp. 316-319.

Owing to the wide range of opacities and associated wavelength intervals, the <u>influence of absorption lines</u> on atmospheric structure is difficult to assess even with the simplest assumptions concerning the physical proc-esses of line formation. A recent attempt to gain insight into this problem can be found in:

O. Gingerich

    1966.   Blanketing approximations for solar models. Journ. Quant.
            Spectrosc. Radiat. Transfer, vol. 6, pp. 609-617.

The formulation of a "direct" attack on the problem is described in:

S. E. Strom and R. Kurucz

    1966.   Statistical procedure for computing line-blanketed model stellar
            atmospheres. (abstract) Astron. Journ., vol. 71, p. 181.

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[*]Princeton University Observatory

[**]University of Iowa

S. E. Strom and R. Kurucz

1966. A statistical procedure for computing line-blanketed model
stellar atmospheres with applications to the F5 IV star
Procyon. Journ. Quant. Spectrosc. Radiat. Transfer, vol. 6,
pp. 591-607.

This work verified the importance of lines in determining the atmospheric
temperature near the surface, under the assumption of LTE, and indicated
a substantial improvement in the fit of theoretical fluxes to those observed in
stars similar to the sun.

2.5 Syntheses and Analyses of Stellar Spectra

Direct analyses of spectra are limited to one practical case — that of
determining the temperature structure of the solar atmosphere through anal-
ysis of limb-darkening and monochromatic-intensity (or flux) data. Virtually
all other studies are based on syntheses of spectra through the construction
of model atmospheres with arbitrary, but judiciously chosen, parameters.
Those values of the parameters giving a best fit with observed spectra are
adopted as "representing" the particular star.

As implied by the introductory remarks, each regime of stellar temper-
atures has its associated set of peculiar problems. In the hot and warm
stars the problems are departures from LTE, the determination of the helium
abundance, line blanketing, and stellar rotation. In the warm stars they are
convection, line blanketing, and departures from LTE. In the cool stars,
the principal difficulties are in opacity calculations and in the incorporation
of molecular bands in the radiative-transfer problem.

Progress in all these areas depends heavily on successive syntheses
and comparison with observed spectra.

2.5.1 The hot stars

Synthetic continuum and line spectra have been computed and compared with observations in:

S. E. Strom

　1964. A comparison between model atmospheres and observations of early-type stars. Proc. H.-S. C.S.A. No. 1, pp. 293-311.

　1964. Stellar effective temperatures in the spectral range B6 to A0. (abstract) Astron. Journ., vol. 69, p. 559.

F. M. Stienon and S. E. Strom

　1964. Observational and predicted contours of Hγ. (abstract) Astron. Journ., vol. 69, p. 559.

E. H. Avrett and S. E. Strom

　1964. Comparison between model atmospheres and spectra from early-type stars. Ann. d'Astrophys., vol. 27, pp. 781-796.

Determinations of effective temperature, gravity, and metal abundances for selected A stars are reported in:

S. E. Strom. O. Gingerich, and K. M. Strom

　1965. Metal abundance determinations for Vega and Sirius. (abstract) Astron. Journ., vol. 70, p. 148.

S. E. Strom and K. M. Strom

　1966. Abundance analysis of the A stars α Lyr, γ Gem, 63, 64, and 68 Tau. (abstract) Astron. Journ., vol. 71, pp. 181-182.

S. E. Strom, O. Gingerich, and K. M. Strom

　1966. Studies in non-gray stellar atmospheres. III. The metal abundances of Sirius and Vega. Astrophys. Journ., vol. 146, pp. 880-913.

This last paper concludes that Sirius is similar in composition to metallic-line A stars, while Vega is similar to the sun. The synthetic spectra agree well with observations, and the values of T_{eff} and g are "in excellent agreement with independent measures of these parameters."

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Tables of monochromatic limb-darkening for a grid of model atmospheres in the temperature range 10000 to 7000° K are computed for radiative equilibrium models and are published in:

O. Gingerich
    1966.   Limb darkening for a grid of model stellar atmospheres.
        Astrophys. Journ., vol. 144, pp. 1213-1215.

2.5.2 Solar-type models

The computation of a <u>theoretical</u> model that successfully predicts the solar-continuum radiation in the near infrared, visible, and ultraviolet has not yet been accomplished. The <u>central difficulty</u> appears to be in the calculation of the continuous opacity and in the treatment of line blanketing. Departures from LTE and the uncertainties associated with the theory of photospheric convection do not appear to be important in the visible and ultraviolet continuum.

These problems are discussed in:

O. Gingerich
    1964.   Investigations of a model solar atmosphere. (abstract) Astron.
        Journ., vol. 69, p. 139.

O. Gingerich and J. C. Rich
    1966.   Metallic continuous absorption coefficients in the solar ultra-
        violet. (abstract) Astron. Journ., vol. 71, pp. 161-162.
    1967.   The far ultraviolet spectrum of the sun. Solar Physics
        (in press).

D. Carbon, O. Gingerich, and R. Kurucz
    1967.   Effects of line blanketing on solar windows. Solar Physics
        (in press).

The use of infrared data for determining the mean thermal structure of the low chromosphere is discussed in:

R. W. Noyes, O. Gingerich, and L. Goldberg
    1966. On the infrared continuum of the sun and stars. Astrophys.
        Journ., vol. 145, pp. 344-347.

A search for chromospheric limb-brightening at 23.4 μ is described in:

R. W. Noyes, J. M. Beckers, F. J. Low, and A. W. Davidson
    1966. Center-to-limb variations of the solar continuum in the far
        infrared and millimeter wavelength regions. (abstract)
        Astron. Journ., vol. 71, p. 866.

A numerical experiment testing a proposed method of inferring micro-turbulent velocities in the solar atmosphere is described in:

J. Allen and S. E. Strom
    1967. Remarks on the Goldberg-Unno procedure for determining
        microturbulent velocities. (abstract) Astron. Journ.,
        in press.

2.5.3 Cool stars

The computation of model atmospheres for the temperature range
T = 4000 to 2500° K is complicated by the formation of numerous molecules and negative ions, and by the need to incorporate line and band absorptions, as well as convective transport, in the models.

Preliminary investigations of these problems are reported in:

O. Gingerich and S. S. Kumar
    1964. Calculations of low-temperature model stellar atmospheres.
        (abstract) Astron. Journ., vol. 69, p. 139.

O. Gingerich, D. Latham, J. Linsky, and S. S. Kumar[*]

1966. Model atmospheres for late-type stars. Presented at IAU
Symposium at Trieste, June. Smithsonian Astrophys. Obs.
Spec. Rep. No. 240, in press, 1967.

O. Gingerich

1967. Model atmospheres for cool stars. Smithsonian Astrophys.
Obs. Spec. Rep. No. 240, in press.

2.6 Line Formation

The earliest discussions of the formation of spectrum lines employed
either of two extreme models to describe the fate of a "lost" photon:

A. Scattering, in which the photon re-emerges at precisely the same
frequency (monochromatic radiative equilibrium).

B. Absorption, in which the photon energy is converted to thermal
kinetic energy, and in which the emission processes are uncorrelated with
the absorptions.

The development of the quantum theory provided the formalism for a
kinetic theory of line formation, and the 1920's saw the first discussions of
the competition between radiative and collisional transitions among excited
and ionized levels. Interest in this problem resurged when Thomas, Jefferies,
and collaborators indicated its importance for chromospheric spectra and
for the formation of strong lines on the solar disk.

Mathematically, the problem can be formulated to produce a coupled set
of nonlinear integral equations, and the major progress of the past few years
has been in the numerical solution of idealized problems.

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[*]Leander McCormick Observatory

A review of earlier work and a discussion of the concept of <u>thermalization</u> <u>length</u> for a two-level atom are given by:

E. H. Avrett and D. G. Hummer

> 1965. Non-coherent scattering. II. Line formation with a frequency independent source function. Mon. Not. Roy. Astron. Soc., vol. 130, pp. 295-331.

<u>The Formation of Spectrum Lines</u> was the title of the Second Harvard-Smithsonian Conference held in January 1965. The table of contents, given in the appendix to this report, lists the following papers, which give solutions to several line-transfer problems:

W. Kalkofen and E. H. Avrett

> Solution of the line and continuum transfer problem for a three-level atom. Pp. 249-273.

E. H. Avrett

> Solutions of the two-level line transfer problem with complete redistribution. Pp. 101-140.

R. W. Noyes

> Dielectronic recombination and the solar H and K lines.
> Pp. 405-409.

The formal relationship between the <u>source function for a multilevel</u> <u>atom</u> and the two-level atom is developed in:

W. Kalkofen

> 1965. Radiative transfer in lines for media in statistical equilibrium. Proc. H.-S. C.S.A. No. 2, pp. 187-214; Scientific Rept. No. 2, Project 8647, Contract AF19(628)-3877 AFCRL 65-347, 32 pp., June.

The conditions for equality of the source functions in the components of a multiplet are established in:

E. H. Avrett

1966. Source function equality in multiplets. Astrophys. Journ., vol. 144, pp. 59-75.

This paper and

E. H. Avrett and W. Kalkofen

1966. Transfer of line radiation by multilevel atoms. Paper presented at the Symposium on Interdisciplinary Aspects of Radiative Energy Transfer, February.

provide numerical solutions for homogeneous atmospheres and discuss the physical theory of multiplet formation.

Two recent papers that have attacked the problem of inhomogeneous atmospheres employing differential rather than integral equations are:

G. B. Rybicki and D. G. Hummer

1966. Non-LTE line formation with spatial variations in the Doppler width. Journ. Quant. Spectrosc. Radiat. Transfer, vol. 6, pp. 661-671.

1967. Spectral line formation in variable-property media. I. The Riccati method. (in press)

2.7 Dynamical Atmospheres

The atmospheres of pulsating stars present a number of theoretical and empirical problems that remain largely unsolved.

Earlier discrepancies between various techniques for determining the
radius variations of classical cepheids are explained on the basis of variable
line blanketing in:

C. A. Whitney
1955. The radii of δ Cephei and η Aquilae. I. Astrophys. Journ.,
vol. 121, pp. 682-689.

1955. The radii of δ Cephei and η Aquilae. II. Astrophys. Journ.,
vol. 122, pp. 385-389.

The hydrodynamics of atmospheric pulsation is discussed for an iso-
thermal atmosphere and applied to an analysis of the variations of W
Virginis in:

C. A. Whitney
1956. Stellar pulsation. I. Momentum transfer by compression waves
of finite amplitude. Ann. d'Astrophys., vol. 19, pp. 34-43.

1956. Stellar pulsation. II. A kinematic model for the atmosphere of
W Virginis. Ann. d'Astrophys., vol. 19, pp. 142-164.

The implications of this work for the distinction between the continuous
velocity curves of classical Cepheids and the discontinuous curves of W
Virginis stars are developed in:

C. A. Whitney
1955. Note on the discontinuous velocity-curves of population II
Cepheids. Ann. d'Astrophys., vol. 18, pp. 375-377.

1962. The duration of line-splitting in W Virginis. Astrophys. Journ.,
vol. 136, pp. 674-677.

The structure of shock fronts and the influence of shocks on the spectra
of variable stars are analyzed in:

A. Skalafuris and C. A. Whitney
1961. Radiative cooling behind shock fronts in stellar atmospheres.
Ann. d'Astrophys., vol. 24, pp. 420-424.

C. A. Whitney and A. Skalafuris

1963. The structure of shock fronts in atomic hydrogen. I. The effects of precursor radiation in the Lyman continuum. Astrophys. Journ., vol. 138, pp. 200-215.

C. A. Whitney

1965. Physical basis for the interpretation of the continuous spectra of pulsating variable stars. Presented at Fifth IAU Symposium on Cosmical Gas Dynamics, Nice, France; IAU Symp. No. 28, pp. 198-206.

The dynamical structure of the solar atmosphere has been explored in:

C. A. Whitney and P. H. Stone

1957. Numerical integration for nonadiabatic compression waves in the solar atmosphere. (abstract) Astron. Journ., vol. 62, p. 149.

C. A. Whitney

1958. Granulation and oscillations of the solar atmosphere. Smithsonian Contr. Astrophys., vol. 2, p. 365.

1963. Thermal response of the solar atmosphere. Astrophys. Journ., vol. 138, pp. 537-551.

The general problem of the temperature minimum separating stellar photospheres from overlying chromospheres is discussed in:

C. A. Whitney

1959. Stellar thermopauses. In Modèles d'Étoiles et Évolution Stellaire. Mem. Soc. Roy. Sci. Liège, Ser. 5, vol. 3, pp. 436-438.

APPENDIX

AGENDA OF THE HARVARD-SMITHSONIAN
CONFERENCES ON STELLAR ATMOSPHERES

SAO Special Report No. 167

PROCEEDINGS

FIRST HARVARD-SMITHSONIAN CONFERENCE

ON STELLAR ATMOSPHERES

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