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SOME APPLICATIONS OF A PROGRAMMING LANGUAGE

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August 1967

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CONTENTS

SOME APPLICATIONS OF A PROGRAMMING LANGUAGE

I. SUMMARY

APL is a programming language specifically designed for applied mathematics. It differs from other programming languages which are commonly used in that it is more concise, precise, and economical of symbols. In this paper is shown the APL language for some commonly used transcendental functions. Several improvements of APL are required for such things as program structuring, access to data, presentation of data, i.e., input/output operations and data operations. However, it is felt that this paper gives some indication of the usefulness of APL if one approaches it openmindedly.

A Programming Language, APL is used for certain applications such as:

1. Trigonometric functions and Solution of Kepler's Equation

2. Harmonic Analysis

3. Matrix Operations

The areas of application are chosen primarily for their intrinsic interest to the Mission Trajectory Determination Branch and to indicate the usefulness of the language for Mission Trajectory Determination applications.

II. TRIGONOMETRIC FUNCTIONS

The APL language for computing the Arc cosine, Arc sine, Arc tangent (principal values), Arc tangent (quadrant oriented) Sine and Cosine are presented in Part A. The algorithm for all these computations except for the Arc tangent (principal values) are from reference 2. The algorithm for the Arc tangent (principal values) is presented in the Appendix. Part B is a program which computes the sine and cosine of several groups of data which are typical of some data used in the Mission Trajectory Determination Branch. The program along with the flow chart and the output of the program is presented.

PART A

1. Arc Cosine, Arc Sine
2. Arc tangent (principal values)
3. Arc tangent (quadrant oriented)
4. Sine, Cosine
5. Solution of Kepler's Equation

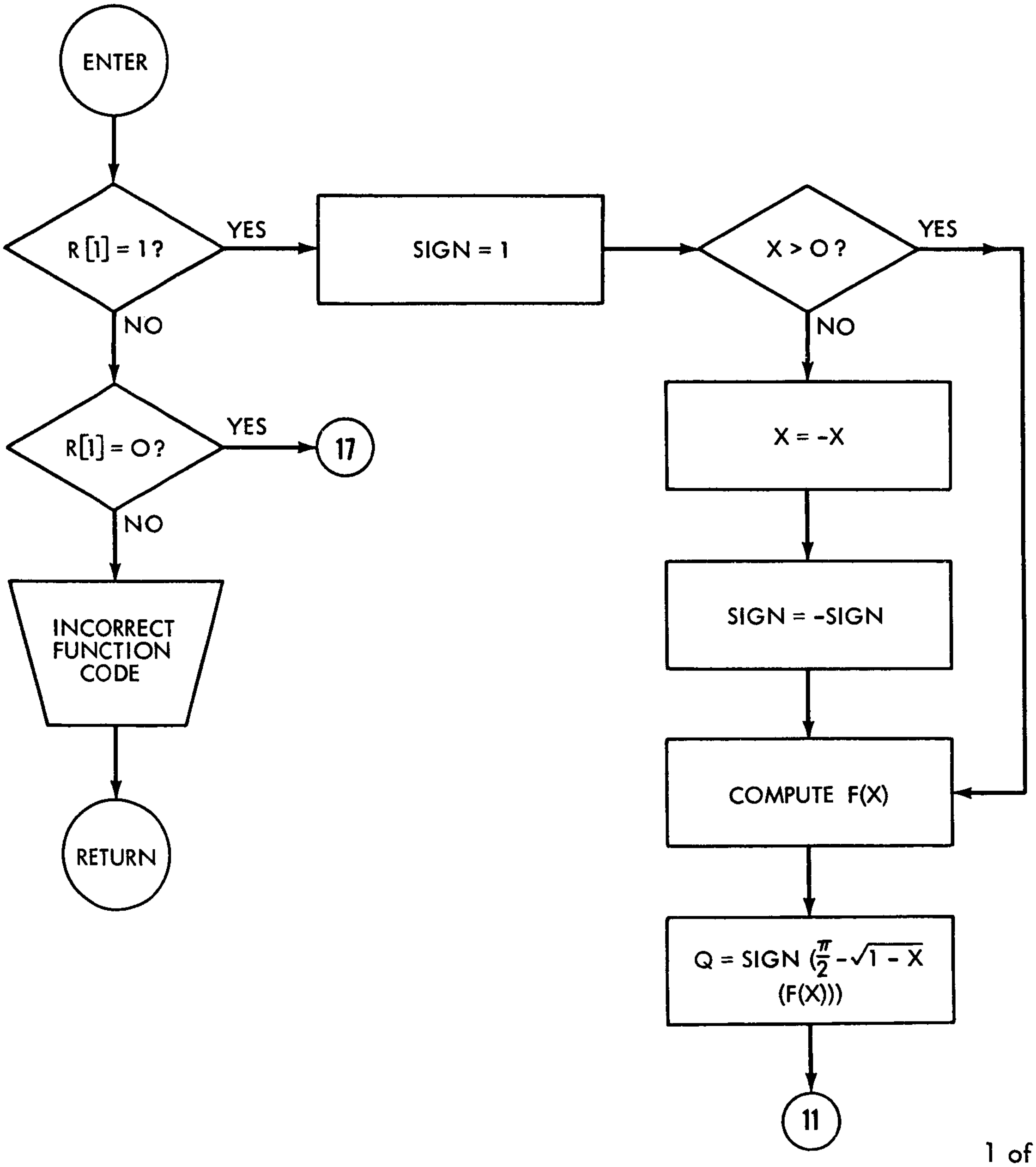
```
      ∇ARCSINCOS[⎕]∇
    ∇ Q←R ARCSINCOS X
[1]   →(R[1]=1)/5
[2]   →(R[1]=0)/17
[3]   'INCORRECT FUNCTION CODE'
[4]   →0
[5]   SIGN←1
[6]   →(X>0)/9
[7]   X←-X
[8]   SIGN←-SIGN
[9]   FX←1.570796305+X×(¯0.2145988016+X×(0.0889789874+X×(¯0.0501743046+X×(0.030891881+X×(¯0.0170881256+X×(
      0.0066700901+X×(¯0.0012624911)))))))
[10]  Q←SIGN×(1.5707963268-((1-X)*0.5)×FX)
[11]  →(R[2]=1)/15
[12]  →(R[2]=0)/0
[13]  'INCORRECT UNITS CODE'
[14]  →0
[15]  Q←57.29577951×Q
[16]  →0
[17]  SIGN←1
[18]  →(X>0)/30
[19]  SIGN←SIGN
[20]  X←-X
[21]  FX←1.570796305+X×(¯0.2145988016+X×(0.0889789874+X×(¯0.0501743046+X×(0.030891881+X×(¯0.0170881256+X×(
      0.0066700901+X×(¯0.0012624911)))))))
[22]  Q1←SIGN×(1.5707963268-((1-X)*0.5)×FX)
[23]  Q←1.5707963268+Q1
[24]  →(R[2]=1)/28
[25]  →(R[2]=0)/0
[26]  'INCORRECT UNITS CODE'
[27]  →0
[28]  Q←Q×57.29577951
[29]  →0
[30]  FX←1.570796305+X×(¯0.2145988016+X×(0.0889789874+X×(¯0.0501743046+X×(0.030891881+X×(¯0.0170881256+X×(
      0.0066700901+X×(¯0.0012624911)))))))
[31]  Q1←SIGN×(1.5707963268-((1-X)*0.5)×FX)
[32]  Q←1.5707963268-Q1
[33]  →(R[2]=1)/37
[34]  →(R[2]=0)/0
[35]  'INCORRECT UNITS CODE'
[36]  →0
[37]  Q←Q×57.29577951
[38]  →0
    ∇
```

Q = R Arcsincos A, 1

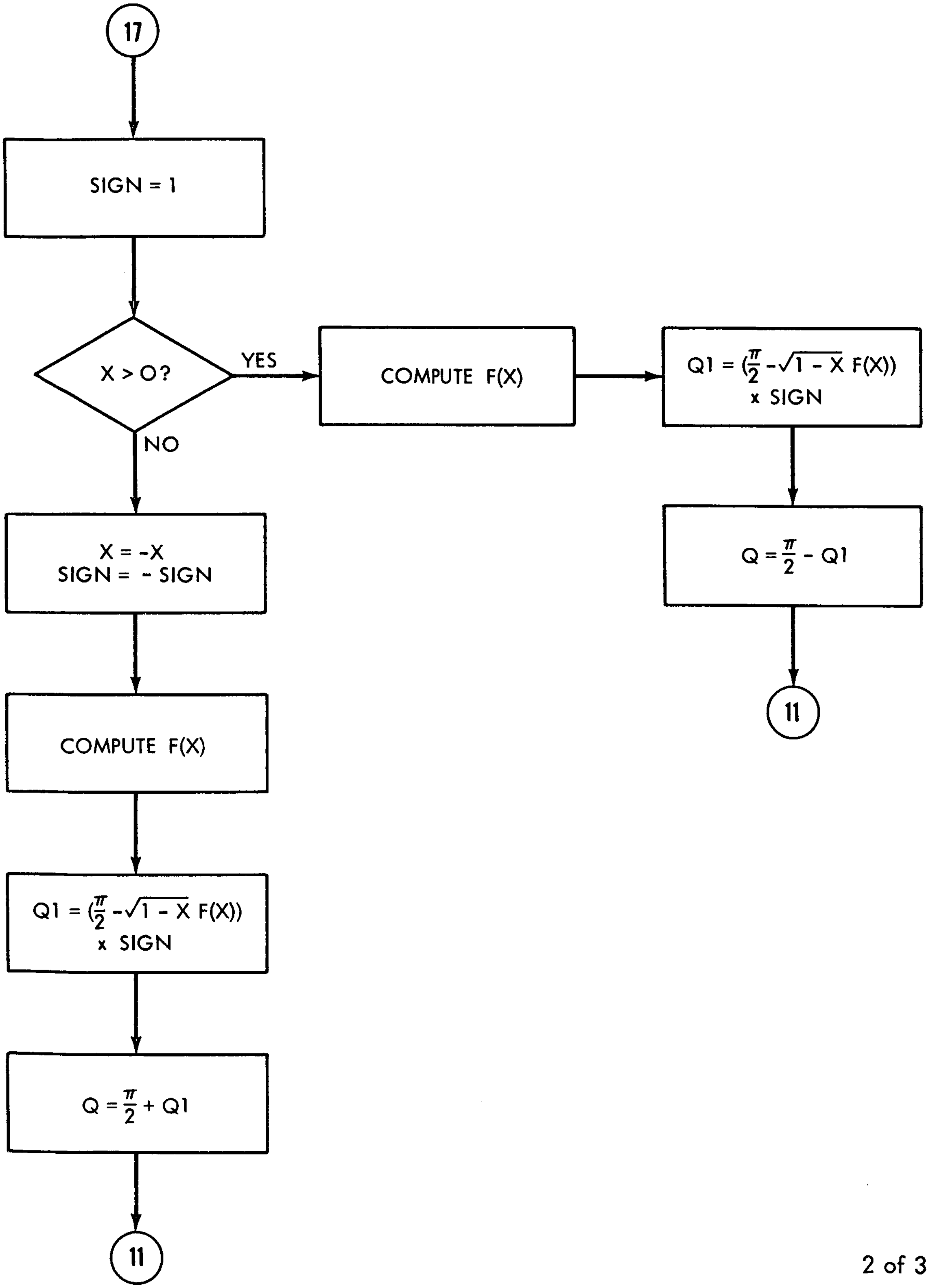
Input: R, two element vector, if first element = 1 compute arcsin, if first element = 0 compute arcos; if second element = 1 answer in degrees, if second element = 0 answer in radians

A, given sine or cosine

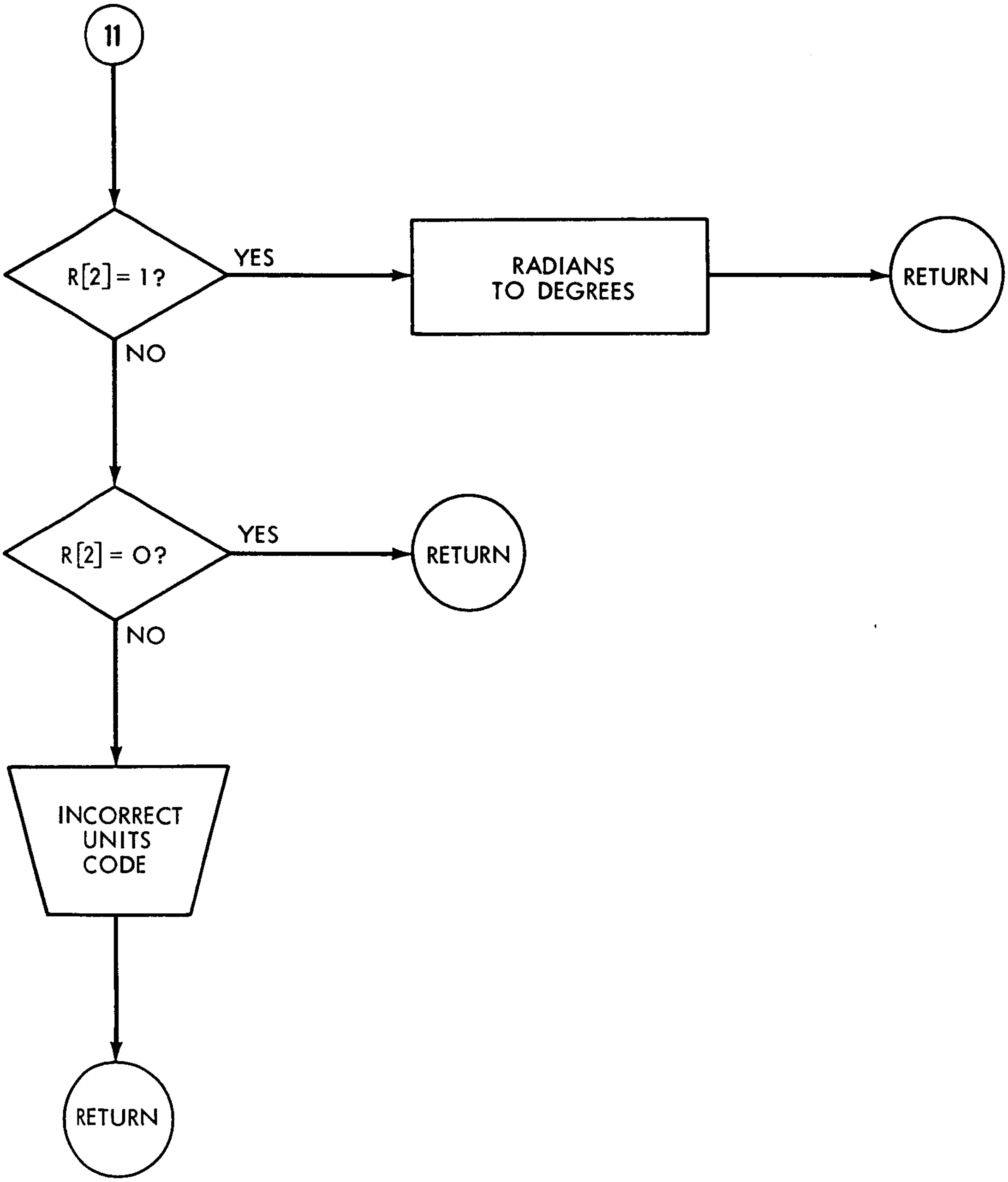
Output: Q, computed angle in degrees or radians



1 of 3


17
SIGN = 1
X > O?
YES
NO
COMPUTE F(X)
$Q1 = (\frac{\pi}{2} - \sqrt{1 - X}\ F(X)) \times SIGN$
$Q = \frac{\pi}{2} - Q1$
11
$X = -X$
$SIGN = -SIGN$
COMPUTE F(X)
$Q1 = (\frac{\pi}{2} - \sqrt{1 - X}\ F(X)) \times SIGN$
$Q = \frac{\pi}{2} + Q1$
11

2 of 3



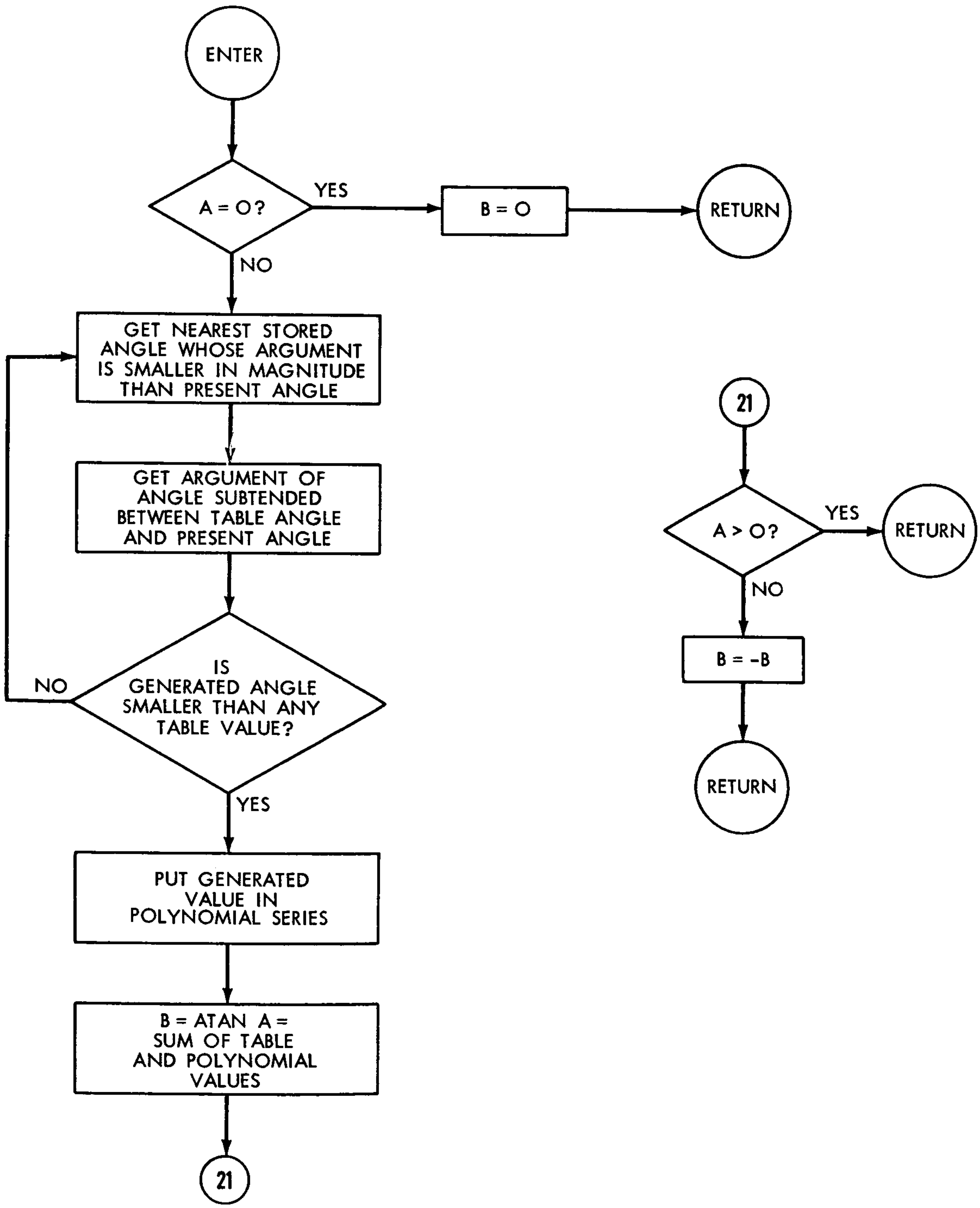
11
R[2] = 1?
YES
RADIANS TO DEGREES
RETURN
NO
R[2] = O?
YES
RETURN
NO
INCORRECT UNITS CODE
RETURN

3 of 3

B ⟵ ATAN A

Input: A - tangent of unknown angle

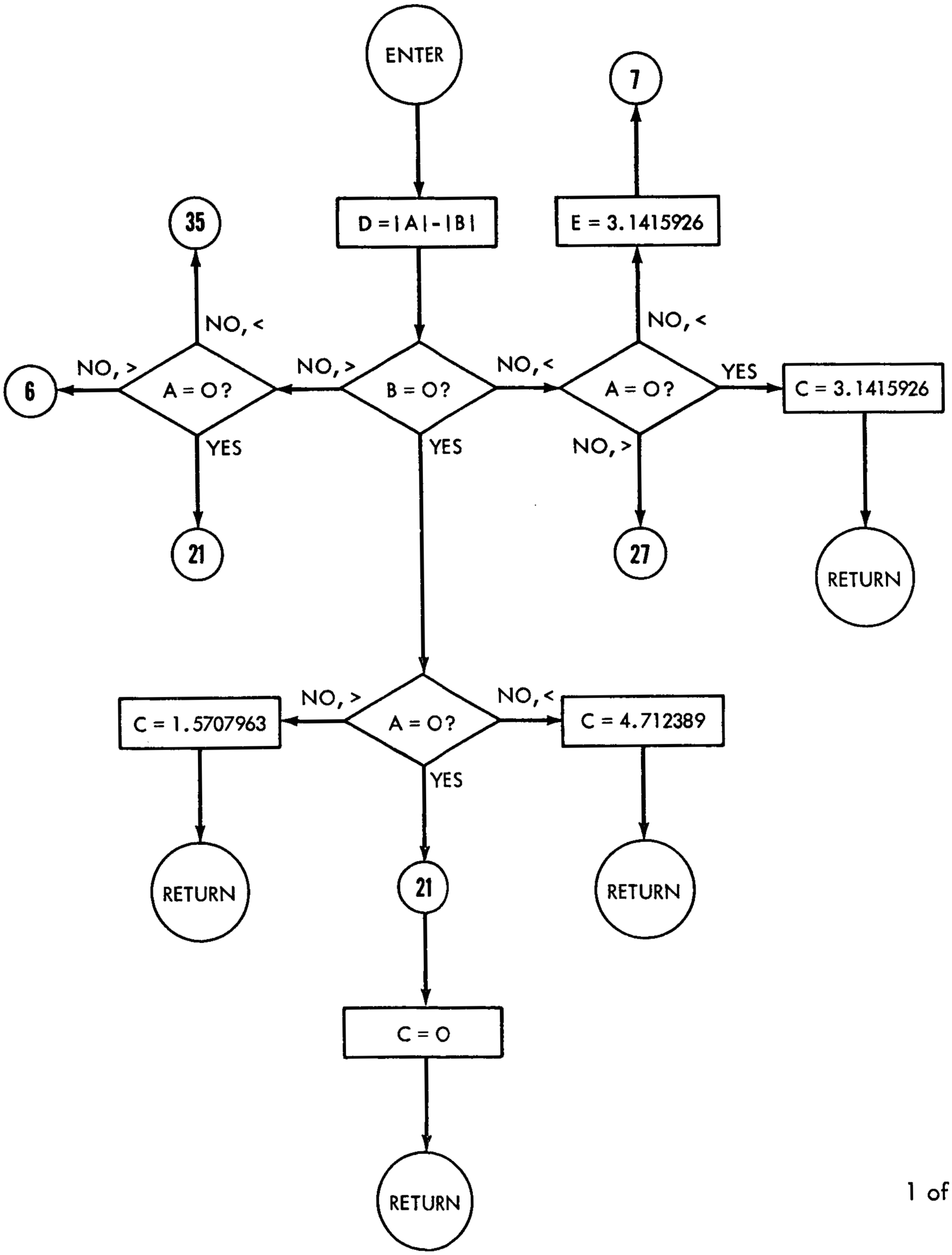
Output: B - value of angle in radians



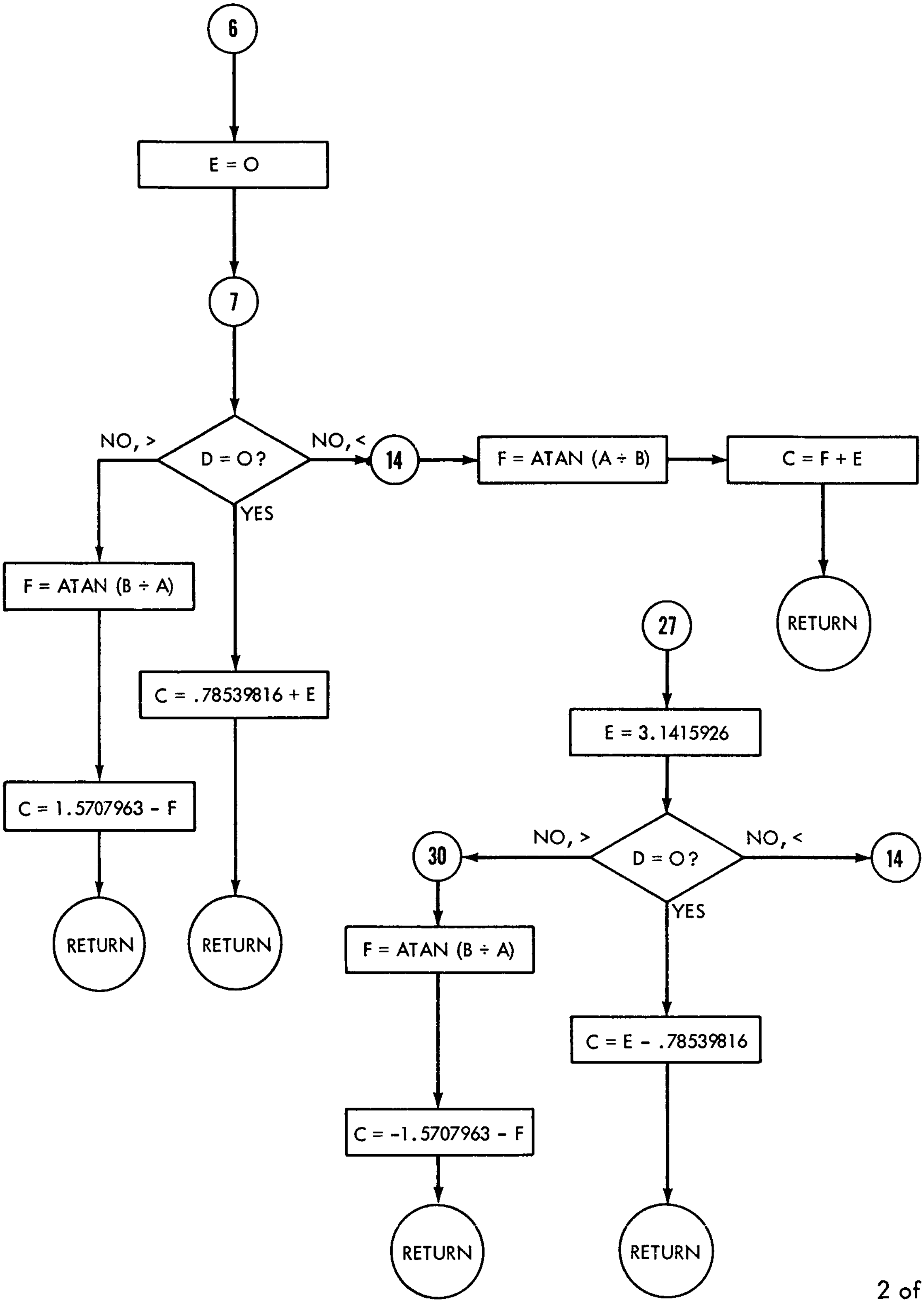
```
      ∇ATAN[⎕]∇
    ∇ B←ATAN A;D;E;F;J;H;C;I;G
[1]   D← 1 0.725 0.5 0.333333 0.25 0.1 0
[2]   E← 0.78539816 0.62730819 0.46364761 0.32175055 0.24497866 0.099668652 0
[3]   F←1,- 0.33333333 0.2 ,- 0.14285714 0.11111111 ,- 0.1 0.09090909
[4]   G←7ρ0
[5]   →(A=0)/24
[6]   J←1
[7]   H←|A
[8]   →(H<D[J])/26
[9]   C←D[J]
[10]  →(H<C)/16
[11]  I←(H-C)÷(1+(H×C))
[12]  G[J]←E[J]
[13]  H←I
[14]  J←J+1
[15]  →9
[16]  →(H<0)/0
[17]  K←I×I
[18]  J←J+1
[19]  G[J]←I×(F[1]-K×(F[2]-K×(F[3]-K×(F[4]-K×(F[5]-K×(F[6]-K×F[7]))))))
[20]  B←+/G
[21]  →(A>0)/0
[22]  B←-B
[23]  →0
[24]  B←0
[25]  →0
[26]  J←J+1
[27]  →8
[28]  →0
    ∇
```

C ⟵ A Qtan B, 3

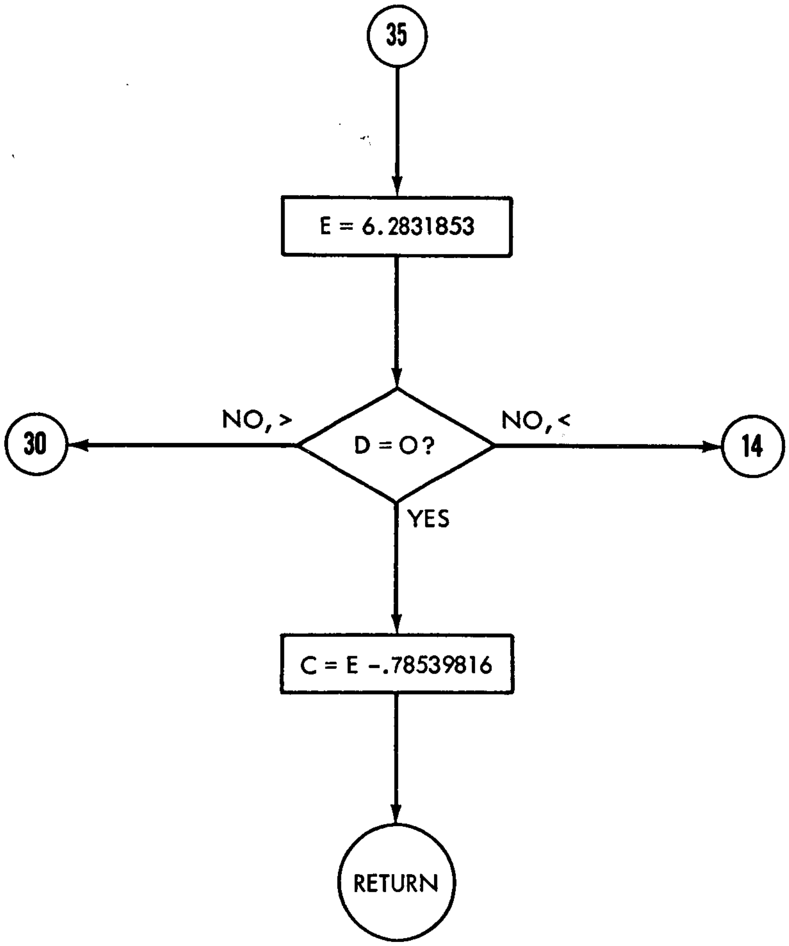
Computes angle C (quadrant oriented) given A = sin C and B = cos C



1 of 3


6
E = O
7
D = O?
NO, >
NO, <
YES
14
F = ATAN (A ÷ B)
C = F + E
RETURN
F = ATAN (B ÷ A)
C = .78539816 + E
C = 1.5707963 - F
RETURN
RETURN
27
E = 3.1415926
D = O?
NO, >
NO, <
YES
30
14
F = ATAN (B ÷ A)
C = E - .78539816
C = -1.5707963 - F
RETURN
RETURN

2 of 3



3 of 3

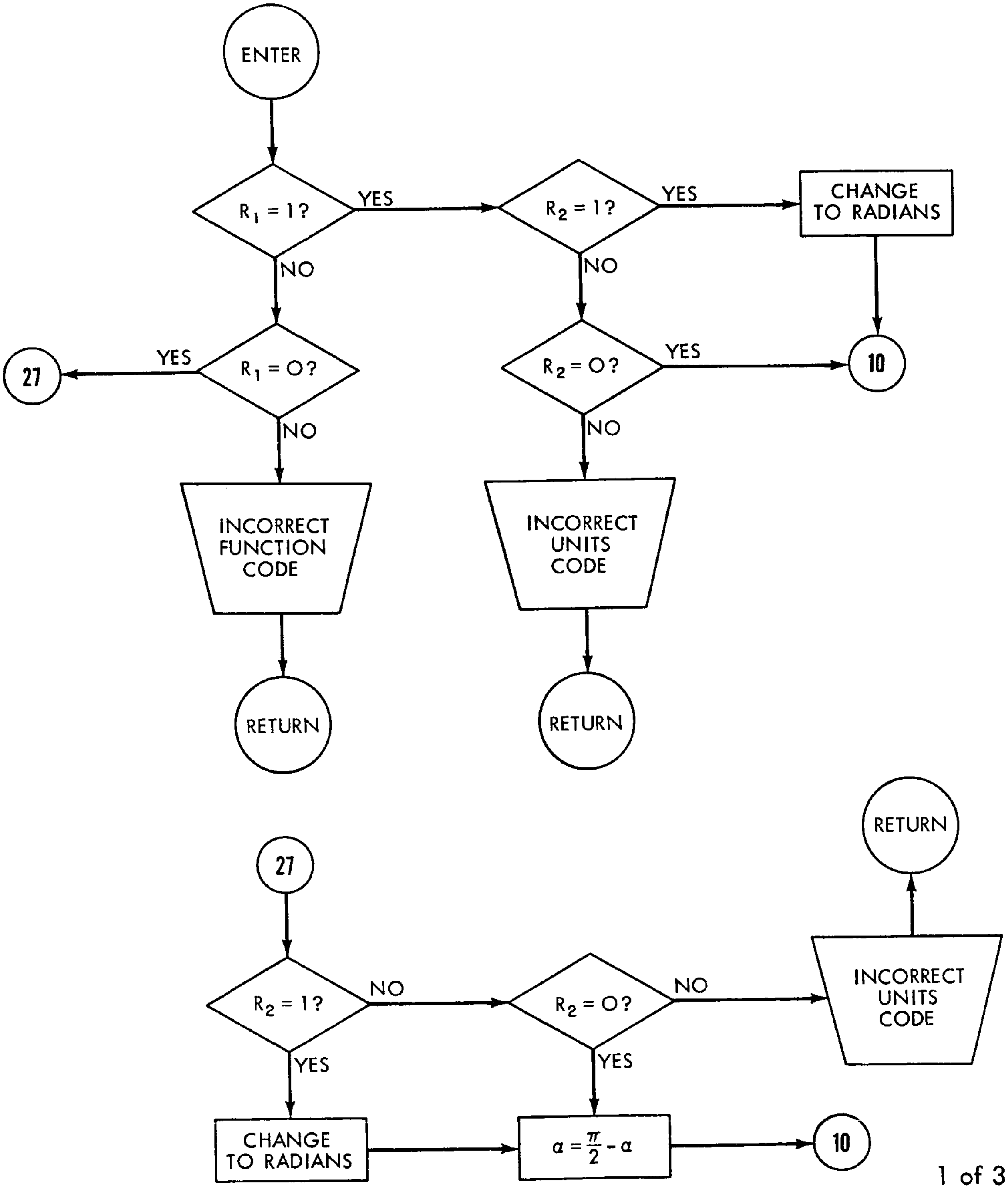
```
      ∇QTAN[⎕]∇
    ∇ C←A QTAN B
[1]   D←(|A)-(|B)
[2]   →(B=0)/17
[3]   →(B<0)/25
[4]   →(A=0)/21
[5]   →(A<0)/35
[6]   E←0
[7]   →(D=0)/12
[8]   →(D<0)/14
[9]   F←ATAN(B÷A)
[10]  C←1.5707963-F
[11]  →0
[12]  C←0.78539816+E
[13]  →0
[14]  F←ATAN(A÷B)
[15]  C←F+E
[16]  →0
[17]  →(A=0)/21
[18]  →(A<0)/23
[19]  C←1.5707963
[20]  →0
[21]  C←0
[22]  →0
[23]  C←4.712389
[24]  →0
[25]  →(A=0)/41
[26]  →(A<0)/43
[27]  E←3.1415926
[28]  →(D=0)/33
[29]  →(D<0)/14
[30]  F←ATAN(B÷A)
[31]  C←¯1.5707963-F
[32]  →0
[33]  C←E-0.78539816
[34]  →0
[35]  E←6.2831853
[36]  →(D=0)/39
[37]  →(D<0)/14
[38]  →30
[39]  C←E-0.48539816
[40]  →0
[41]  C←3.1415926
[42]  →0
[43]  E←3.1415926
[44]  →7
    ∇
```

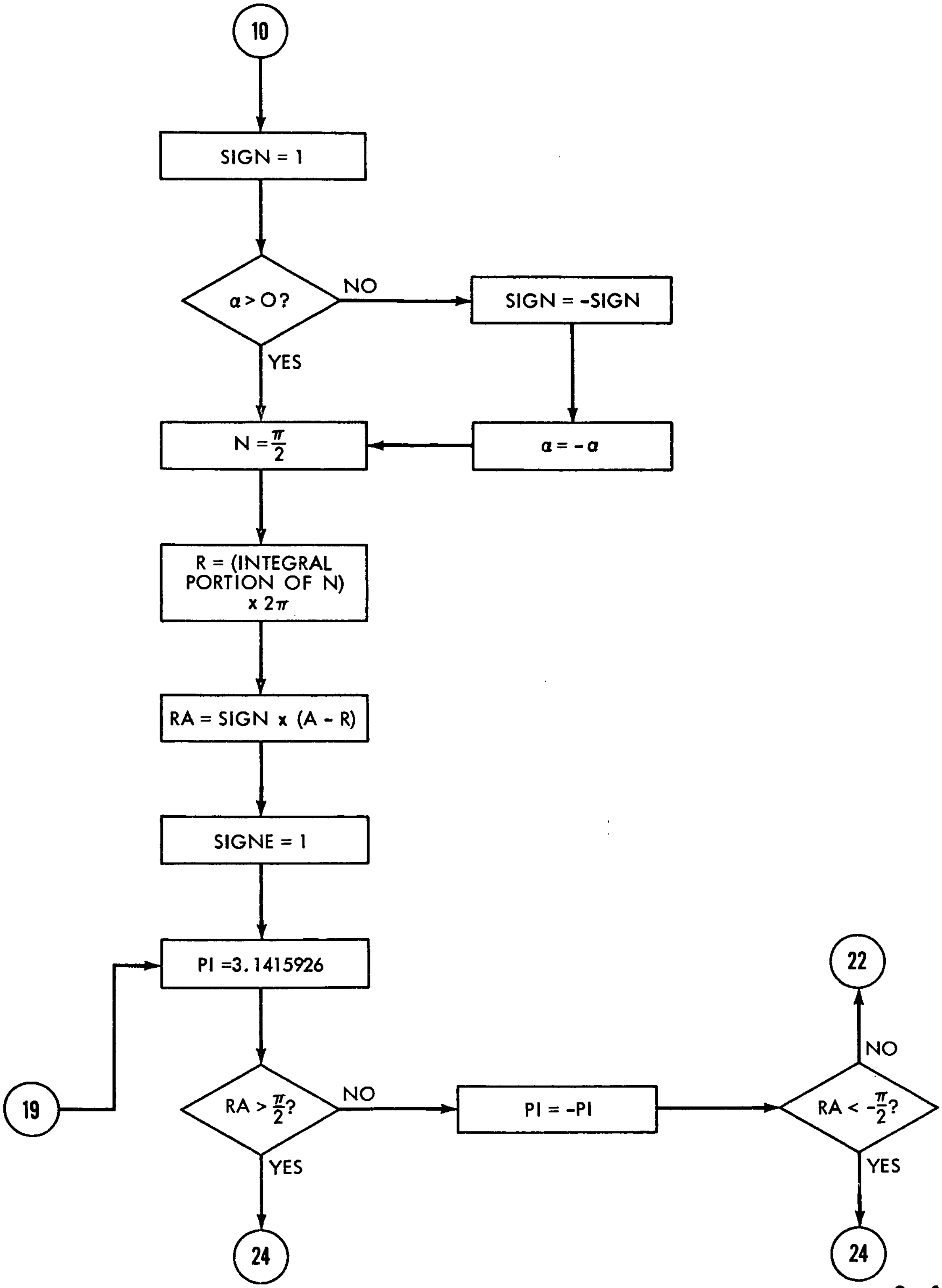
```
      ∇SINCOS[⎕]∇
    ∇ Z←R SINCOS A
[1]    →(R[1]=1)/5
[2]    →(R[1]=0)/27
[3]    'INCORRECT FUNCTION CODE'
[4]    →0
[5]    →(R[2]=1)/9
[6]    →(R[2]=0)/10
[7]    'INCORRECT UNITS CODE'
[8]    →0
[9]    A←A×0.0174532925199
[10]   SIGN←1
[11]   →(A>0)/14
[12]   SIGN←-SIGN
[13]   A←-A
[14]   N←A÷6.2831853172
[15]   R←(⌊N)×6.2831853172
[16]   RA←SIGN×(A-R)
[17]   SIGNE←1
[18]   PI←3.1415926536
[19]   →(RA>1.5707963268)/24
[20]   PI←-PI
[21]   →(RA<¯1.5707963268)/24
[22]   Z←(RA÷1.57079631847)×SIGNE×(1.57079631847+((RA÷1.57079631847)*2)×((¯0.64596371106)+((RA÷
       1.57079631847)*2)×(0.07968967927999999+((RA÷1.57079631847)*2)×((¯0.00467376557)+(0.00015148419)×((RA÷
       1.57079631847)*2)))))
[23]   →0
[24]   RA←RA-PI
[25]   SIGNE←-SIGNE
[26]   →18
[27]   →(R[2]=1)/31
[28]   →(R[2]=0)/32
[29]   'INCORRECT UNITS CODE'
[30]   →0
[31]   A←A×0.0174532925199
[32]   A←1.5707963268-A
[33]   →10
    ∇
```

R Sincos A, 4

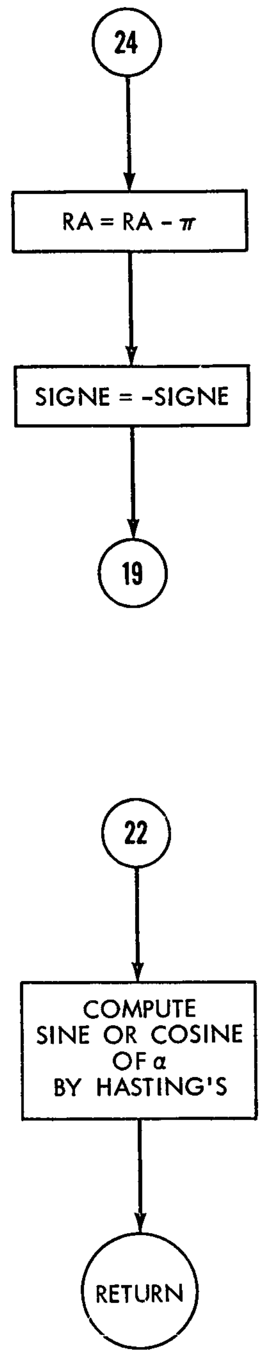
R = 2 element vector, A = angle, α, $0 \leq \alpha \leq 2\pi$

1st element = 1 compute sine, 1st element = 0 compute cosine, 2nd element = 1, α in degrees, 2nd element = 0, α in radians




10
SIGN = 1
α > O?
NO
SIGN = -SIGN
YES
N = π/2
α = -α
R = (INTEGRAL PORTION OF N) x 2π
RA = SIGN x (A - R)
SIGNE = 1
PI =3.1415926
22
NO
19
RA > π/2?
NO
PI = -PI
RA < -π/2?
YES
YES
24
24

2 of 3


24
RA = RA - π
SIGNE = -SIGNE
19
22
COMPUTE
SINE OR COSINE
OF α
BY HASTING'S
RETURN

3 of 3

Solution of Kepler's Equation, 5

```
      ∇ECCENTRICANOM[⎕]∇
    ∇ E←EC ECCENTRICANOM M
[1]   E←M+EC×(1 0 SINCOS M)
[2]   DE←(M+(-E)+EC×(1 0 SINCOS E))÷(1+(-EC)×(0 0 SINCOS E))
[3]   E←E+DE
[4]   NM←E-EC×(1 0 SINCOS E)
[5]   →((|(M-NM))>5E¯9)/2
[6]   →0
    ∇
```

Input is EC = eccentricity
M = Mean anomaly

Output is E = Eccentric anomaly

```
      ∇TRUEANOM[⎕]∇
    ∇ F←EC TRUEANOM E
[1]   AOR←1÷(1+(-EC)×(0 0 SINCOS E))
[2]   S←(AOR×(1-EC*2)*0.5)×(1 0 SINCOS E)
[3]   C←AOR×((0 0 SINCOS E)+(-EC))
[4]   F←S QTAN C
[5]   →0
    ∇
```

Input is EC = eccentricity
E = Eccentric anomaly

Output is F = True anomaly

PART B

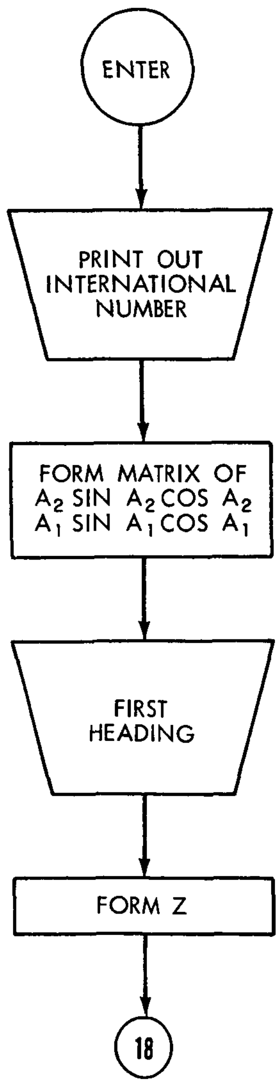
A program to compute the sine and cosine of some angles using APL

A $\leftarrow$ B NTRIGROUT A

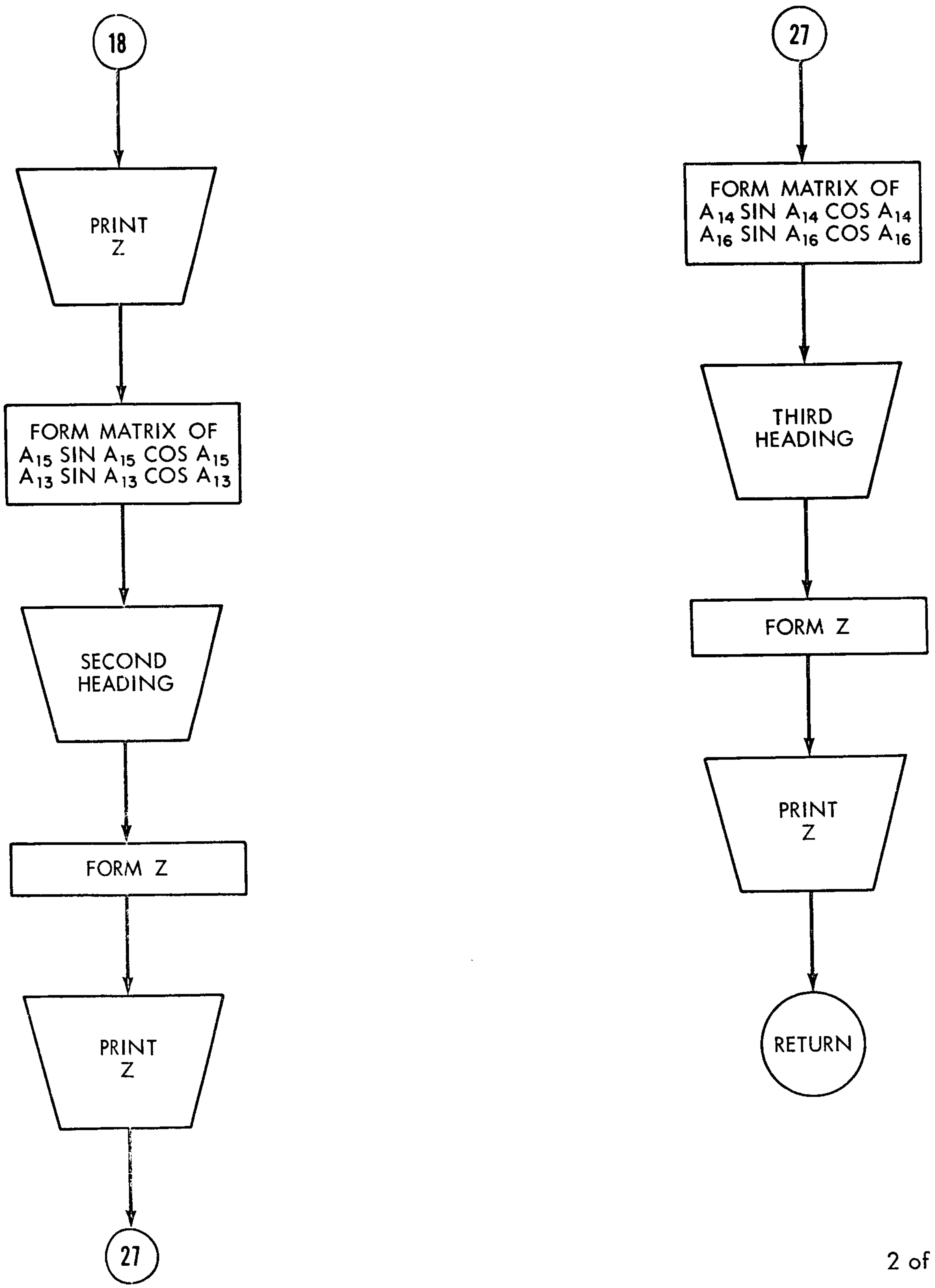
Input: B is a vector whose first two components are the satellite's International Number, third component is the number of bulletins, fourth component is number of angles, and component 5 thru n are bulletin numbers.

A is a matrix of angles.

Output: Z is a matrix, whose first column is bulletin numbers, second column is angle Ai, third and fourth columns are sin Ai and cos Ai respectively, fifth column is Ai+1, and sixth and seventh columns are sin Ai+1 and cos Ai+1 respectively.



1 of 2


18
PRINT Z
FORM MATRIX OF A_{15} SIN A_{15} COS A_{15} A_{13} SIN A_{13} COS A_{13}
SECOND HEADING
FORM Z
PRINT Z
27
27
FORM MATRIX OF A_{14} SIN A_{14} COS A_{14} A_{16} SIN A_{16} COS A_{16}
THIRD HEADING
FORM Z
PRINT Z
RETURN

2 of 2

```
      ∇NTRIGROUT[⎕]∇
    ∇ Z←B NTRIGROUT A
[1]   Z←(B[3],7)ρ0
[2]   F←(B[3],6)ρ0
[3]   ((ρB)α2)/B
[4]   K←1
[5]   R←1
[6]   F[R;1]←A[R;K]
[7]   F[R;2]← 1 1 SINCOS A[R;K]
[8]   F[R;3]← 0 1 SINCOS A[R;K]
[9]   F[R;4]←A[R;K+1]
[10]  F[R;5]← 1 1 SINCOS A[R;K+1]
[11]  F[R;6]← 0 1 SINCOS A[R;K+1]
[12]  →(B[3]≥R←R+1)/6
[13]  →(K≠1)/21
[14]  '

                BLTN          A2           SINA2          COSA2           A1          SINA1
        COSA1'
[15]  R←1
[16]  Z[R;]←B[R+4],F[R;]
[17]  →(B[3]≥R←R+1)/16
[18]  Z
[19]  K←K+2
[20]  →5
[21]  →(K≠3)/29
[22]  '

                BLTN          A15          SINA15         COSA15          A13         SINA13
        COSA13'
[23]  R←1
[24]  Z[R;]←B[R+4],F[R;]
[25]  →(B[3]≥R←R+1)/24
[26]  Z
[27]  K←K+2
[28]  →5
[29]  →(K≠5)/35
[30]  '

                BLTN          A14          SINA14         COSA14          A16         SINA16
       COSA16'
[31]  R←1
[32]  Z[R;]←B[R+4],F[R;]
[33]  →(B[3]≥R←R+1)/32
[34]  →0
[35]  'IMPLEMENTATION OF K INCORRECT'
[36]  →0
    ∇
```

B NTRIGROUT A
1964 301

BLTN	*A2*	*SINA2*	*COSA2*	*A1*	*SINA1*	*COSA1*
42	134.5812	0.7122564009	¯0.7019193857	53.6172	0.8050719021	0.5931772382
44	102.9265	0.9746578397	¯0.223700932	21.7911	0.3712236086	0.9285435067
45	91.8914	0.9994551786	¯0.03300516222	9.8366	0.1708389341	0.9852989715
46	73.6024	0.9593258074	0.2823012747	¯8.2938	¯0.1442491234	0.9895414041
47	53.4343	0.8031742588	0.5957441678	¯28.6469	¯0.4794103836	0.8775908438
48	25.8583	0.4361469769	0.8998754479	¯56.6123	¯0.8349660174	0.5503015111
49	0.0755	0.001317723204	0.9999991366	¯82.5055	¯0.9914573852	0.13043102
50	¯14.1104	¯0.2437910545	0.9698277862	263.0556	¯0.9926639447	¯0.1209061165
51	¯27.4651	¯0.4612082363	0.8872919297	291.5981	¯0.9297886978	0.3680937231
52	¯46.5842	¯0.7263851717	0.6872878488	229.388	¯0.7591349926	¯0.6509332279
53	¯62.2016	¯0.8845939995	0.4663619427	214.1932	¯0.561985219	¯0.8271472764
54	¯81.0001	¯0.9876886147	0.1564327416	195.3669	¯0.2649991129	¯0.9642486631
55	263.9557	¯0.9944407755	¯0.1052973774	179.5463	0.007918476081	¯0.9999686511
56	249.864	¯0.9388781448	¯0.3442496798	165.4586	0.2510794908	¯0.9679664781
57	229.7032	¯0.7627044517	¯0.6467471867	145.1242	0.5717994201	¯0.8203934583
58	217.4389	¯0.6079150586	¯0.7940020688	132.6771	0.7351855842	¯0.6778658874
59	183.4432	¯0.06005901175	¯0.9981948236	99.0378	0.9875849216	¯0.1570860433
61	146.6025	0.5504443176	¯0.8348718803	60.6895	0.8719795736	0.4895422635
62	135.8983	0.6959341056	¯0.7181056505	49.8371	0.764213813	0.6449629851
63	127.7754	0.7904180907	¯0.6125677477	41.5232	0.6629232615	0.7486873538
64	122.5641	0.8427898097	¯0.5382428263	36.6103	0.5963691916	0.8027102782

BLTN	*A15*	*SINA15*	*COSA15*	*A13*	*SINA13*	*COSA13*
42	271.1624	¯0.9997942111	0.02028631552	107.2344	0.9551006558	¯0.2962815414
44	205.853	¯0.4360637343	¯0.8999157887	43.5822	0.6893945353	0.7243860707
45	183.7828	¯0.06597436079	¯0.9978213138	19.6732	0.336654853	0.9416281236
46	147.2048	0.5416377944	¯0.8406119813	¯16.5876	¯0.2854809615	0.9583843879
47	106.8686	0.9569727619	¯0.290177786	¯57.2938	¯0.8414523161	0.5403313827
48	51.7166	0.7849559019	0.619551641	¯113.2246	¯0.918966119	¯0.3943365086
49	0.151	0.00263544412	0.9999965316	¯165.011	¯0.2586335973	¯0.965975505
50	¯28.2208	¯0.4728706768	0.881131845	526.1112	0.2400382962	¯0.970763426
51	¯54.9302	¯0.8184526815	0.5745739391	583.1962	¯0.6844987525	¯0.7290140344
52	¯93.1684	¯0.9984713947	¯0.05527083245	458.776	0.9882923796	¯0.1525718651
53	¯124.4032	¯0.8250819418	¯0.5650130906	428.3864	0.9296890811	0.3683452417
54	¯162.002	¯0.3089837982	¯0.9510673092	390.7338	0.5110500707	0.8595509411
55	527.9114	0.2094240223	¯0.9778249275	359.0926	¯0.01583645565	0.9998745961
56	499.728	0.6464170035	¯0.7629843129	330.9172	¯0.48607306	0.8739181798
57	459.4064	0.9865539151	¯0.1634361529	290.2484	¯0.9382010068	0.3460908605
58	434.8778	0.9653716267	0.2608785756	265.3542	¯0.9967144476	¯0.08099568466
59	366.8864	0.11990118	0.9927858281	198.0756	¯0.3102716167	¯0.950647957
61	293.205	¯0.9191009617	0.3940221214	121.379	0.853741699	¯0.5206967593
62	271.7966	¯0.9995084202	0.03135144709	99.6742	0.9857792411	¯0.1680455056
63	255.5508	¯0.9683692601	¯0.249521521	83.0464	0.9926445179	0.1210655073
64	245.1282	¯0.9072511336	¯0.420589335	73.2206	0.9574233605	0.2886875875

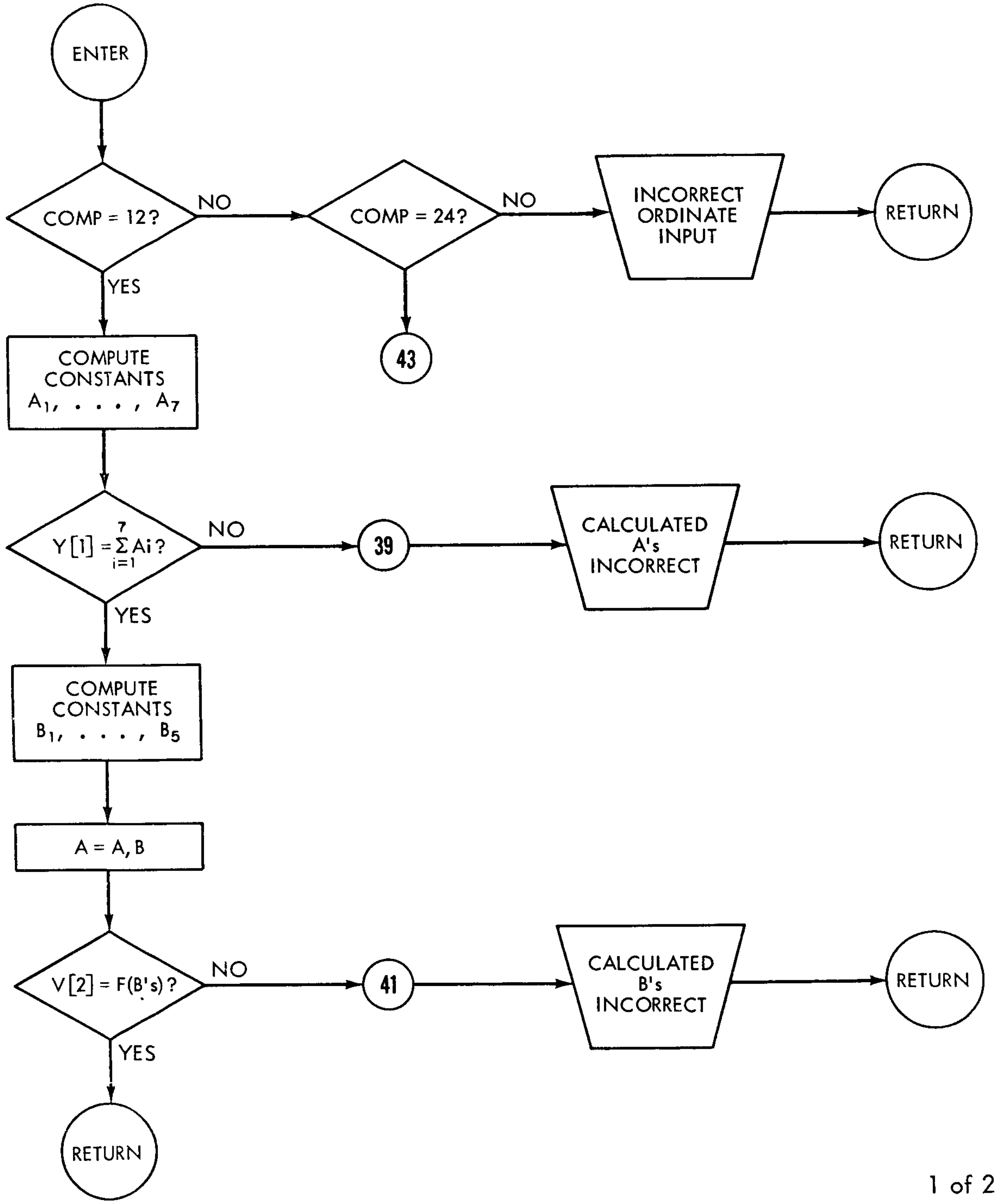
BLTN	*A14*	*SINA14*	*COSA14*	*A16*	*SINA16*	*COSA16*
42	190.1984	¯0.1770572568	¯0.9842005555	¯7.8731	¯0.1369794933	0.9905738828
44	124.7176	0.8219691307	¯0.5695320461	¯73.0116	¯0.9563639364	0.2921780889
45	101.728	0.9791235977	¯0.2032658104	¯94.1624	¯0.9973623199	¯0.07258369999
46	65.3086	0.9085708912	0.4177307076	229.101	¯0.7558648963	¯0.6547276249
47	24.7874	0.4192524457	0.9078697016	188.9498	¯0.1555690371	¯0.9878250233
48	¯30.754	¯0.5113530878	0.8593707137	134.1872	0.7170663392	¯0.6970049281
49	¯82.43	¯0.9912846526	0.1317373731	82.732	0.9919652526	0.1265106106
50	248.9452	¯0.9332372472	¯0.3592607026	54.6132	0.8152612294	0.5790933699
51	222.133	¯0.670853858	¯0.7415895803	331.9934	¯0.4695732727	0.8828935083
52	183.2038	¯0.05588772397	¯0.9984370552	¯9.5406	¯0.1657464504	0.9861684024
53	151.9916	0.4696010094	¯0.8828787558	¯40.798	¯0.6533941829	0.7570178637
54	114.3668	0.9109228839	¯0.4125766694	¯78.3672	¯0.9794599829	0.2016386644
55	83.502	0.993575804	0.1131685316	252.3208	¯0.952771798	¯0.3036871997
56	55.3226	0.8223685247	0.568955194	224.1334	¯0.6963313049	¯0.7177205017
57	14.8274	0.2559080845	0.9667011255	183.9854	¯0.06950227415	¯0.9975817884
58	350.116	¯0.1716539996	0.9851573017	159.6396	0.3479241642	¯0.937522687
59	282.481	¯0.9763677329	0.2161158509	91.2918	0.9997458452	¯0.02254425317
61	207.292	¯0.4585254803	¯0.8886812645	19.118	0.3275147498	0.944846073
62	185.7354	¯0.09993452248	¯0.9949940121	¯2.1422	¯0.03737973299	0.9993011304
63	169.2986	0.1856906257	¯0.982608263	¯18.197	¯0.3122851799	0.9499884107
64	159.1744	0.3555246141	¯0.9346669252	¯28.918	¯0.4835573996	0.8753126562

III. HARMONIC ANALYSIS

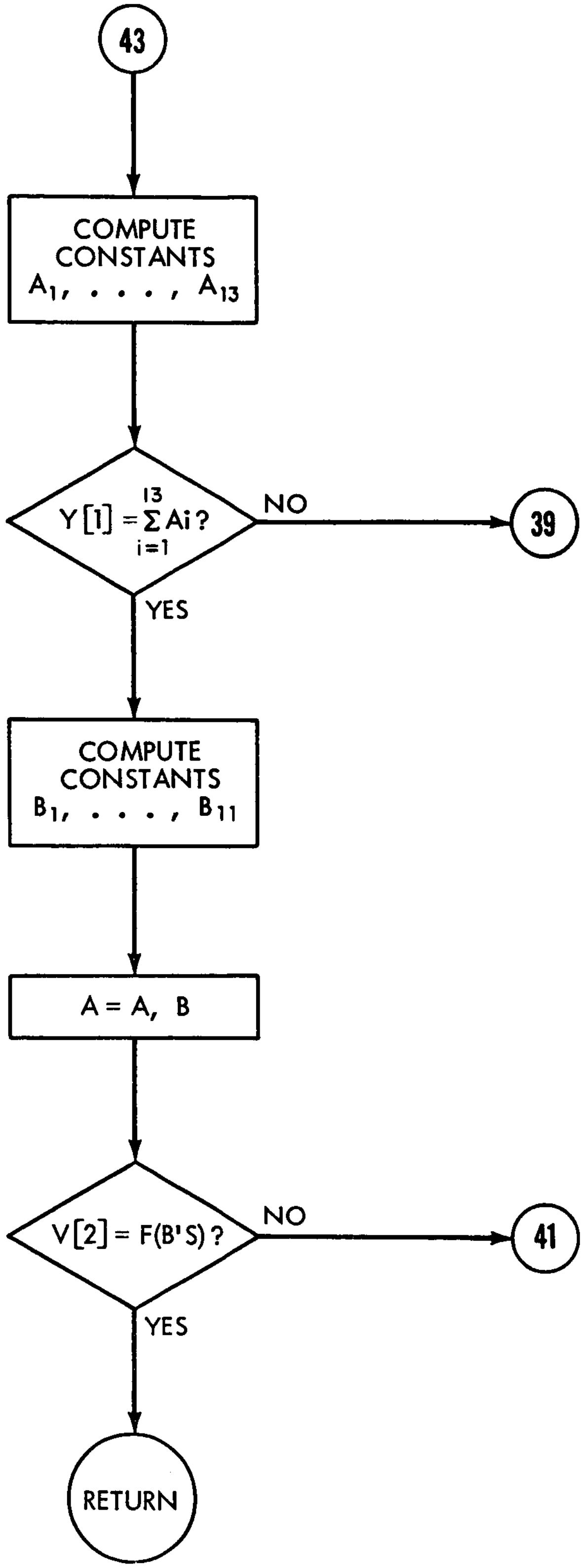
The algorithm used here is presented in reference 3. The number of ordinates, i.e., the number of divisions in the interval, can be either 12 or 24.

Harmonic Analysis - Comp Harm Y

Input: Comp = 12 or 24 designating number of ordinates
Y = 12 or 24 component vector



1 of 2


43
COMPUTE CONSTANTS $A_1, \ldots, A_{13}$
$Y[1] = \sum_{i=1}^{13} Ai$?
NO
39
YES
COMPUTE CONSTANTS $B_1, \ldots, B_{11}$
A = A, B
V[2] = F(B'S) ?
NO
41
YES
RETURN

```
      ∆
      ∇HARM[⎕]∇
    ∇ A←COMP HARM Y
[1]   →(COMP=12)/5
[2]   →(COMP=24)/43
[3]   'INCORRECT ORDINATE INPUT'
[4]   →0
[5]   A←7⍴0
[6]   B←5⍴0
[7]   J←Y[1 2 3 4 5 6]
[8]   K←Y[7 12 11 10 9 8]
[9]   U←J+K
[10]  V←J-K
[11]  C←U[1 2 3]
[12]  D←U[4 6 5]
[13]  R←C+D
[14]  S←C-D
[15]  E←V[2 3]
[16]  F←V[6 5]
[17]  P←E+F
[18]  Q←E-F
[19]  L←R[2]+R[3]
[20]  M←R[2]-R[3]
[21]  G←Q[1]+Q[2]
[22]  H←Q[1]-Q[2]
[23]  A[1]←(1÷12)×(R[1]+L)
[24]  A[2]←(1÷6)×(V[1]+(0.5×(3*0.5)×S[2])+(0.5×S[3]))
[25]  A[3]←(1÷6)×(S[1]+0.5×M)
[26]  A[4]←(1÷6)×(V[1]-S[3])
[27]  A[5]←(1÷6)×(R[1]-0.5×L)
[28]  A[6]←(1÷6)×(V[1]+(¯0.5×(3*0.5)×S[2])+0.5×S[3])
[29]  A[7]←(1÷12)×(S[1]-M)
[30]  →((|(Y[1]-(+/A)))>5E¯6)/39
[31]  B[1]←(1÷6)×(V[4]+(0.5×P[1])+(0.5×(3*0.5)×P[2]))
[32]  B[2]←(1÷12)×(3*0.5)×G
[33]  B[3]←(1÷6)×(P[1]-V[4])
[34]  B[4]←(1÷12)×(3*0.5)×H
[35]  B[5]←(1÷6)×(V[4]+(0.5×P[1])+(¯0.5×(3*0.5)×P[2]))
[36]  A←A,B
[37]  →((|(V[2]-(B[1]+B[5]+(2×B[3])+(3*0.5)×(B[2]+B[4]))))/41
[38]  →0
[39]  'CALCULATED A S INCORRECT'
[40]  →0
[41]  'CALCULATED B S INCORRECT'
[42]  →0
[43]  A←13⍴0
[44]  B←11⍴0
[45]  J←Y[1 2 3 4 5 6 7 8 9 10 11 12]
[46]  K←Y[13 24 23 22 21 20 19 18 17 16 15 14]
[47]  U←J+K
[48]  V←J-K
[49]  C1←U[1 2 3 4 5 6]
[50]  D1←U[7 12 11 10 9 8]
[51]  R←C1+D1
[52]  S←C1-D1
[53]  E1←V[2 3 4 5 6]
[54]  F1←V[12 11 10 9 8]
[55]  P←E1+F1
[56]  Q←E1-F1
[57]  R1←R[1 2 3]
[58]  R2←R[4 6 5]
[59]  L←R1+R2
[60]  M←R1-R2
[61]  Q1←Q[1 2]
[62]  Q2←Q[5 4]
[63]  G←Q1+Q2
[64]  H←Q1-Q2
[65]  E←L[2]+L[3]
[66]  F←L[2]-L[3]
[67]  C←H[1]+H[2]
[68]  D←H[1]-H[2]
[69]  CO← 0 1 SINCOS 15
[70]  SI← 1 1 SINCOS 15
[71]  A[1]←(1÷24)×(L[1]+E)
[72]  A[2]←(1÷12)×(V[1]+(CO×S[2])+((3*0.5)×0.5×S[3])+((1÷2*0.5)×S[4])+(0.5×S[5])+(SI×S[6]))
[73]  A[3]←(1÷12)×(S[1]+(0.5×(3*0.5)×M[2])+(0.5×M[3]))
[74]  A[4]←(1÷12)×(V[1]+((1÷2*0.5)×(S[2]+(-S[4])+(-S[6])))+(-S[5]))
[75]  A[5]←(1÷12)×(M[1]+0.5×F)
[76]  A[6]←(1÷12)×(V[1]+(SI×S[2])+(¯0.5×(3*0.5)×S[3])+(-(1÷2*0.5)×S[4])+(0.5×S[5])+(CO×S[6]))
[77]  A[7]←(1÷12)×(S[1]-M[3])
[78]  A[8]←(1÷12)×(V[1]+(-SI×S[2])+(¯0.5×(3*0.5)×S[3])+((1÷2*0.5)×S[4])+(0.5×S[5])+(-CO×S[6]))
[79]  A[9]←(1÷12)×(L[1]-(0.5×F))
[80]  A[10]←(1÷12)×(V[1]+(-(1÷2*0.5)×(S[2]+(-S[4])+(-S[6])))+(-S[5]))
[81]  A[11]←(1÷12)×(S[1]+(¯0.5×(3*0.5)×M[2])+(0.5×M[3]))
[82]  A[12]←(1÷12)×(V[1]+(-CO×S[2])+(0.5×(3*0.5)×S[3])+(-(1÷2*0.5)×S[4])+(0.5×S[5])+(-SI×S[6]))
[83]  A[13]←(1÷24)×(M[1]+(-F))
[84]  →((|(Y[1]-(+/A)))>5E¯6)/39
[85]  B[1]←(1÷12)×((SI×P[1])+(0.5×P[2])+((1÷2*0.5)×P[3])+(0.5×(3*0.5)×P[4])+(CO×P[5])+V[7])
[86]  B[2]←(1÷12)×((0.5×G[1])+(0.5×(3*0.5)×G[2])+Q[3])
[87]  B[3]←(1÷12)×(P[2]+(-V[7])+((1÷2*0.5)×(P[1]+P[3]+(-P[5]))))
[88]  B[4]←(1÷24)×(3*0.5)×C
[89]  B[5]←(1÷12)×((CO×P[1])+(0.5×P[2])+(-(1÷2*0.5)×P[3])+(¯0.5×(3*0.5)×P[4])+(SI×P[5])+V[7])
[90]  B[6]←(1÷12)×(G[1]+(-Q[3]))
[91]  B[7]←(1÷12)×((CO×P[1])+(¯0.5×P[2])+(-(1÷2*0.5)×P[3])+(0.5×(3*0.5)×P[4])+(SI×P[5])+(-V[7]))
[92]  B[8]←(1÷24)×(3*0.5)×D
[93]  B[9]←(1÷12)×(V[7]+(-P[2])+((1÷2*0.5)×(P[1]+P[3]+(-P[5]))))
[94]  B[10]←(1÷12)×((0.5×G[1])+(¯0.5×(3*0.5)×G[2])+Q[3])
[95]  B[11]←(1÷12)×((SI×P[1])+(¯0.5×P[2])+((1÷2*0.5)×P[3])+(¯0.5×(3*0.5)×P[4])+(CO×P[5])+(-V[7]))
[96]  A←A,B
[97]  →((|(V[2]-(((2×SI)×(B[1]+B[11]))+(B[2]+B[10])+((2*0.5)×(B[3]+B[9]))+((3*0.5)×(B[4]+B[8]))+((2×CO)×(B[
      5]+B[7]))+(2×B[6]))))>5E¯6)/41
[98]  →0
    ∇
```

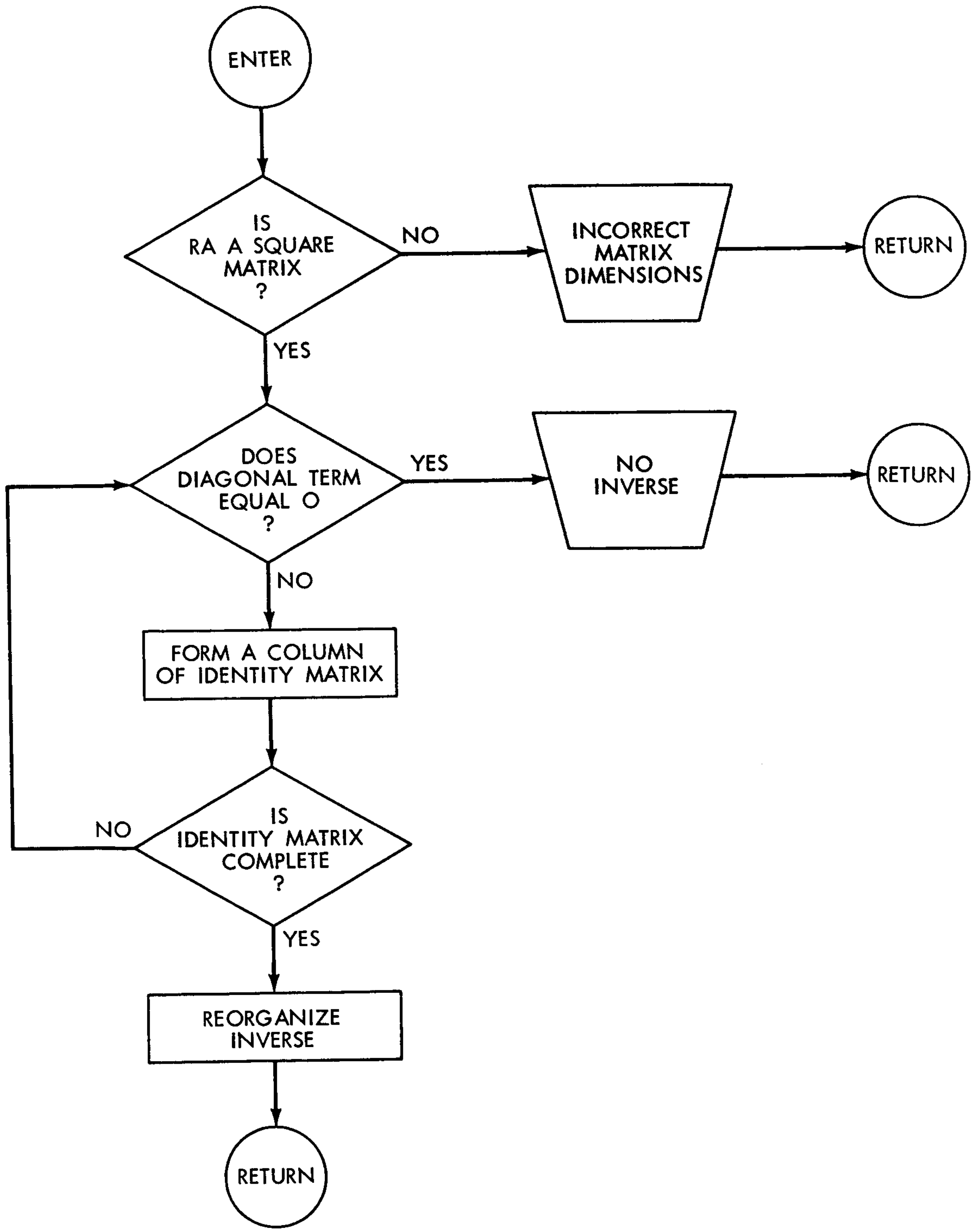
IV. MATRIX OPERATIONS

At the time of this writing the matrix operations for APL had not been implemented. These programs, 1. INV (Matrix Inverse) and 2. TRANS (Matrix Transpose), show applications with existing operations. The Matrix Inverse should be especially noted for its conciseness when compared with other languages.

RB ⟵ Inv RA

Input: RA, square matrix

Output: RB, inverse of RA found by calculations in place

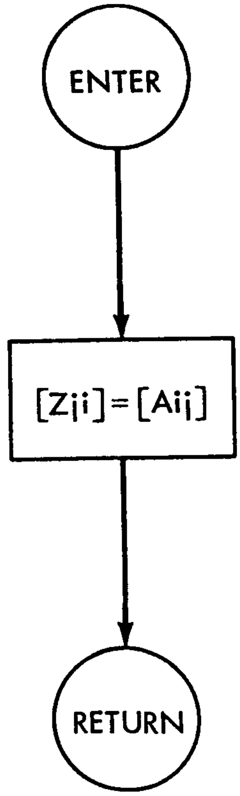


```
        ∇INV[⎕]∇
      ∇ RB←INV RA;RK;RS;RP;RI
[1]     →((2=⍴⍴RA)∧=/⍴RA)⍴4
[2]     'INCORRECT MATRIX DIMENSIONS'
[3]     →0
[4]     RK←⌊/⍴RA
[5]     RS←RK
[6]     RP←⍳RK
[7]     RA←RA[;(⍳RS),1]
[8]     RA[;1+RS]←RS⍺1
[9]     RI←(|RA[⍳RK;1])⍳⌈/|RA[⍳RK;1]
[10]    RP[1,RI]←RP[RI,1]
[11]    RA[1,RI;⍳RS]←RA[RI,1;⍳RS]
[12]    →((5E¯10)>|RA[1;1])⍴21
[13]    RA[1;]←RA[1;]÷RA[1;1]
[14]    RA←RA-((~RS⍺1)×RA[;1])∘.×RA[1;]
[15]    RA←RA[1+RS|⍳RS;(1+⍳RS),1]
[16]    RP←RP[1+RS|⍳RS]
[17]    RK←RK-1
[18]    →(RK>0)⍴8
[19]    RB←RA[;RP⍳⍳RS]
[20]    →0
[21]    'NO INVERSE'
[22]    →0
      ∇
```

$Z \longleftarrow$ Trans A

Input: A, any dimensioned matrix

Output: Z, transpose of A



```
      ∇TRANS[⎕]∇
    ∇ Z←TRANS A;I;J
[1]   I←ρA
[2]   Z←I[2 1]ρ0
[3]   J←1
[4]   Z[J;]←A[;J]
[5]   →(I[2]≥J←J+1)/4
    ∇

      A←5 6ρι30

      A

  1   2   3   4   5   6
  7   8   9  10  11  12
 13  14  15  16  17  18
 19  20  21  22  23  24
 25  26  27  28  29  30

      TRANS A

  1   7  13  19  25
  2   8  14  20  26
  3   9  15  21  27
  4  10  16  22  28
  5  11  17  23  29
  6  12  18  24  30
```

V. APPENDIX

On A Method of Computing Tan^{-1} x for Double Precision

The evaluation of Arctangent x, in this method, is accomplished by a combination of "table look-up" and evaluation of a polynomial series. This eliminates the need for a subroutine for $x < 1$ and another for $x > 1$. The subroutine has stored a table of arguments and their corresponding angles (see table 1). The "table look-up" part consists of obtaining the nearest stored angle whose argument, x is smaller in magnitude than the given argument. The subroutine then evaluates the argument of the angle subtended between the nearest table angle and the desired angle. The resulting argument is sent through a "table look-up" until the generated argument becomes smaller than the smallest non-zero argument in the table. The final generated argument is then inserted in the polynomial series. The value of the arc tangent is the sum of the values obtained by the "table look-up" process and by the polynomial series.

Example

Compute $\theta = \text{Arctan } x$ given $\tan\theta$, $x = 1.7320508$

1. The closest value to x in the stored table (see table 1) is

   $x = 1.000$ giving a $\theta_1 = .7853\ldots$ radians.

2. Now from the relationship $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha \cdot \tan\beta}$

   one has $\tan(\theta - \theta_1) = \dfrac{1.732058 - 1}{1 + 1.7320508} = .26794919$

   so $x_1 = .267949\ldots$

3. Repeat step 1 using x_1 to find $\theta_2 = .24497\ldots$

4. Repeat step 2 to find $x_2 = .01684\ldots$

   Now x_2 is smaller than any non-zero table entry so x_2 is inserted in the polynomial series to yield $.01682\ldots = \theta_3$.

5. θ then is equal to $\theta_1 + \theta_2 + \theta_3 = 1.04719\ldots$

Table 1

Tan θ, x	θ*, Arc tan x
1	.78539816 . . .
.725	.62730819227 . . .
.5	.463647609 . . .
1/3	.321750554397 . . .
.25	.244978663 . . .
.1	.099668652491 . . .
0	0

*Note that 16 figures will be required to obtain the desired accuracy.

Polynomial Series

$$x(c_0 - x^2 (c_1 - x^2 (c_2 - x^2 (c_3 - x^2 (c_4 - x^2 (c_5 - c_6 x^2))))))$$

with

$$c_0 = 1,\ c_1 = -\frac{1}{3},\ c_2 = \frac{1}{5},\ c_3 = -\frac{1}{7},\ c_4 = \frac{1}{9},\ c_5 = -\frac{1}{11},\ c_6 = +\frac{1}{13}$$

The polynomial is "nested" to avoid the effects of multiplying by a power of x greater than 2 in any one operation.

This method has been found to be accurate in a subroutine using up to 20 significant figures in increments of 4 significant figures and for a range of $\sim$25.2 radians, i.e., the approximate equivalent of 4 complete cycles.

VI. REFERENCES

1. Iverson, Kenneth E., A Programming Language. John Wiley & Sons, Inc., 1962.

2. Hastings, Approximations for Digital Computers.

3. Milne, William E., Numerical Calculus, Princeton University Press, 1949.