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A SEMI-IMPLICIT NUMERICAL METHOD FOR TREATING THE TIME TRANSIENT GAS LUBRICATION EQUATION

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A SEMI-IMPLICIT NUMERICAL METHOD FOR TREATING THE TIME TRANSIENT GAS LUBRICATION EQUATION

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ABSTRACT

This report presents a numerical method for treating the time transient Reynolds equation which has the numerical stability properties of implicit schemes and the speed of execution of explicit methods. These advantages make this method quite suitable for both steady-state and transient calculations.

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INTRODUCTION

The research presented in this paper has been motivated by two classes of problems commonly encountered in gas lubrication technology:

1) Dynamic response and stability calculations for gas bearing systems are best solved by time transient integration of Reynolds equation in combination with the appropriate dynamic equations of motion of the system components (Ref. 1).

2) Steady-state data for extreme operating conditions such as high values of the bearing number Λ and eccentricity ratio ϵ are often obtained with great numerical difficulty by steady-state methods (Refs. 2 and 3) but can be easily obtained by time transient calculations starting from physically reasonable initial conditions and diffusing to the final answer.

Two basic approaches can be used in the time integration of Reynolds equation; they are commonly called the "explicit" and implicit" methods (Ref. 1).

By the explicit method the time derivative of pressure is evaluated from the rest of Reynolds equation taken at time T. The value of the pressure at time $T + \triangle T$ is then obtained by direct extrapolation.

By the implicit method both the value of pressure in the time diffusion term and the highest space derivatives of pressure are taken at time $T + \triangle T$ and solved in terms of the rest of the terms of Reynolds equation taken at time T.

In the explicit method each grid value of pressure at time $T + \triangle T$ is obtained by one explicit algebraic equation.

In the implicit method, due to the inclusion of derivatives of the pressure at time $T + \triangle T$, a system of algebraic equations connecting all grid pressures must be solved. Consequently, integration over one time step requires much more computation time by the implicit method than by the explicit one.

However, the numerical stability characteristics of the explicit method severely limit the size of the time step that can be utilized, while the stability characteristics of the implicit method are much better and allow much larger values of the time step ΔT.

In conclusion, the explicit method is of fast execution per time step but is limited to short time interval while the implicit method is of slow execution per time step but can utilize a long time interval.

Since it is in the interest of most problems to integrate over a certain time interval, it is obvious that a technique which combines the speed of execution of the explicit method and the numerical stability of the implicit method is quite desirable.

In what follows, one such method is exposed.

TREATMENT OF REYNOLDS EQUATION

Consider the following form of Reynolds equation:

$$\Lambda \frac{\partial \psi}{\partial T} = \left[H\psi \frac{\partial^2 \psi}{\partial X^2} - \psi^2 \frac{\partial^2 H}{\partial X^2} + H \left(\frac{\partial \psi}{\partial X}\right)^2 - \psi \frac{\partial \psi}{\partial X} \frac{\partial H}{\partial X} - \Lambda \frac{\partial \psi}{\partial X} + \right.$$

$$\left. + H\psi \frac{\partial^2 \psi}{\partial Y^2} - \psi^2 \frac{\partial^2 H}{\partial Y^2} + H \left(\frac{\partial \psi}{\partial Y}\right)^2 - \psi \frac{\partial \psi}{\partial Y} \frac{\partial H}{\partial Y} \right] \tag{1}$$

where all variables are dimensionless and X is the direction of motion.

Let the quantity $H\psi$ be arbitrarily split in the following manner.

$$H\psi = (H\psi)_{0x} + (H\psi)_{1x} = (H\psi)_{0y} + (H\psi)_{1y} \tag{2}$$

and

$$\frac{\partial \psi}{\partial T} = \frac{\psi(T + \Delta T) - \psi(T)}{\Delta T} \tag{3}$$

Equation (1) can now be written as:

$$\frac{\Lambda}{\Delta T} \psi(T + \Delta T) = (H\psi)_{0x} \frac{\partial^2 \psi(T + \Delta T)}{\partial X^2} + (H\psi)_{0y} \frac{\partial^2 \psi(T + \Delta T)}{\partial Y^2} +$$

$$+ \left[(H\psi)_{1x} \frac{\partial^2 \psi}{\partial X^2} - \psi^2 \frac{\partial^2 H}{\partial X^2} + H \left(\frac{\partial \psi}{\partial X}\right)^2 - \psi \frac{\partial \psi}{\partial X} \frac{\partial H}{\partial X} - \Lambda \frac{\partial \psi}{\partial X} + \right.$$

$$\left. + \frac{\Lambda}{\Delta T} \psi + (H\psi)_{1y} \frac{\partial^2 \psi}{\partial Y^2} - \psi^2 \frac{\partial^2 H}{\partial Y^2} + H \left(\frac{\partial \psi}{\partial Y}\right)^2 - \psi \frac{\partial \psi}{\partial Y} \frac{\partial H}{\partial Y} \right]_T \tag{4}$$

Equation (4) is a general formulation which contains both the explicit method by setting

$$(H\psi)_{0x} = (H\psi)_{0y} = 0$$

and the implicit method by setting

$$(H\psi)_{1x} = (H\psi)_{1y} = 0.$$

By numerically approximating all derivatives of ψ in equation (4) by central differences and considering that the ψ distribution at time T is known, the numerical problem reduces to

$$\left[\, L \,\right] \left\{ \psi(T + \Delta T) \right\} = \left\{ R \right\} \tag{5}$$

The solution of equation (5) is

$$\psi(T + \Delta T) = \left[\, L \,\right]^{-1} \left\{ R \right\} \tag{6}$$

For the explicit method $\left[L \right]$ is diagonal with constant members $\frac{\Lambda}{\Delta T}$. Its inverse is also diagonal with constant members $\frac{\Delta T}{\Lambda}$ and equation (6) is of simple numerical execution.

For the implicit case $\left[L \right]$ is not diagonal and its members vary with ψ from time step to time step. Therefore the inverse $\left[L \right]^{-1}$ must be obtained at each time step at the expense of calculation time. (It should be pointed out that the columnwise inversion method of references 2 and 3 is applicable to the solution of equation (5) thus reducing the inversion time considerably but not to the level of the explicit method).

A great improvement over this situation is afforded by the choice

$$(\psi H)_{0x} \text{ and } (\psi H)_{0y} \text{ independent of T.} \tag{7}$$

Indeed, equation (4) has not suffered in accuracy since

$$(\psi H)_{1x} = \psi H - (\psi H)_{0x} \text{ and } (\psi H)_{1y} = \psi H - (\psi H)_{0y}$$

compensate for any difference between the true ψH distribution at time T and the $(\psi H)_{0x}$ and $(\psi H)_{0y}$ distributions, and the operator $\left[L \right]$ and its inverse $\left[L \right]^{-1}$ do not change with time. Therefore all changes in ψ from time step to time step are reflected in changes of the members of the right hand side $\left\{ R \right\}$.

The fact that the inversion $\left[L\right]^{-1}$ is not performed at each time step reduces the amount of computation to the level of the explicit method.

The necessary condition for the usefulness of the above proposed semi-implicit method is that it be numerically more stable than the explicit method.

NUMERICAL STABILITY ANALYSIS

General treatment:

Let equation (4) be written with the aid of the following definitions

$$F_1 = \frac{\partial}{\partial\psi}\left(\frac{\partial\psi}{\partial T}\right) \tag{8}$$

$$F_2 = \frac{\partial}{\partial\left(\frac{\partial\psi}{\partial X}\right)}\left(\frac{\partial\psi}{\partial T}\right) \tag{9}$$

$$F_3 = \frac{\partial}{\partial\left(\frac{\partial^2\psi}{\partial X^2}\right)}\left(\frac{\partial\psi}{\partial T}\right) = F_{30} + F_{31} = \frac{1}{\Lambda}\left[(\psi H)_{0x} + (\psi H)_{1x}\right] \tag{10}$$

$$F_4 = \frac{\partial}{\partial\left(\frac{\partial\psi}{\partial Y}\right)}\left(\frac{\partial\psi}{\partial T}\right) \tag{11}$$

$$F_5 = \frac{\partial}{\partial\left(\frac{\partial^2\psi}{\partial Y^2}\right)}\left(\frac{\partial\psi}{\partial T}\right) = F_{50} + F_{51} = \frac{1}{\Lambda}\left[(\psi H)_{0y} + (\psi H)_{1y}\right] \tag{12}$$

By standard central difference formulae, equation (4) becomes

$$\begin{aligned}
\frac{\psi^{j+1}_{ik} - \psi^{j}_{ik}}{\Delta T} &= F_1\,\psi^{j}_{i,k} + \frac{F_2}{2\Delta X}\left(\psi^{j}_{i+1,k} - \psi^{j}_{i-1,k}\right) + \\
&+ \frac{F_4}{2\Delta Y}\left(\psi^{j}_{i,k+1} - \psi^{j}_{i,k-1}\right) + \frac{F_{30}}{(\Delta X)^2}\left(\psi^{j+1}_{i+1,k} - 2\psi^{j+1}_{i,k} + \psi^{j+1}_{i-1,k}\right) + \\
&+ \frac{F_{31}}{(\Delta X)^2}\left(\psi^{j}_{i+1,k} - 2\psi^{j}_{i,k} + \psi^{j}_{i-1,k}\right) + \frac{F_{50}}{(\Delta Y)^2}\left(\psi^{j+1}_{i,k+1} - 2\psi^{j+1}_{i,k} + \psi^{j+1}_{i,k-1}\right) + \\
&+ \frac{F_{51}}{(\Delta Y)^2}\left(\psi^{j}_{i,k+1} - 2\psi^{j}_{i,k} + \psi^{j}_{i,k-1}\right)
\end{aligned} \tag{13}$$

Where

a) The F's are evaluated somewhere in the interval according to the mean value theorem and are considered to vary much slower than the solution components causing numerical instability.

b) i is the grid index in the X direction
k is the grid index in the Y direction
j is the time grid index

Letting

$$\psi^{j}_{i,k} = \Psi^{j}_{i,k} + \epsilon^{j}_{i,k} \tag{14}$$

where $\Psi^{j}_{i,k}$ are the exact solution to equations (13) and $\epsilon^{j}_{i,k}$ the deviations of the actual solution $\psi^{j}_{i,k}$ from it,equations (13) can be transformed into equations for the deviations $\epsilon^{j}_{i,k}$. Such equations are identical in form to equations (13) with $\epsilon^{j}_{i,k}$ in place of $\psi^{j}_{i,k}$ and shall not be written. (This is due to the linearity of equation (13)).

The equations for $\epsilon^{j}_{i,k}$ can be satisfied by

$$\epsilon(X,Y,T) = \sum_{n=1}^{N} A_n \, e^{a_n T + \sqrt{-1}\,(b_n X + c_n Y)} \tag{15}$$

where every term represents a solution.

Substituting one typical term of (15) into the equations for $\epsilon^{j}_{i,k}$ and dividing by

$$\frac{e^{a_n T + \sqrt{-1}\,(b_n X + c_n Y)}}{\Delta T}$$ yields:

$$e^{a_n \Delta T} - 1 = F_1 \Delta T +$$

$$+ \frac{F_2 \Delta T}{2\Delta X}\left[e^{\sqrt{-1}\, b_n \Delta X} - e^{-\sqrt{-1}\, b_n \Delta X}\right] + \frac{F_4 \Delta T}{2\Delta Y}\left[e^{\sqrt{-1}\, c_n \Delta Y} - e^{-\sqrt{-1}\, c_n \Delta Y}\right] +$$

$$+ \frac{F_{30} \Delta T}{(\Delta X)^2}\left[e^{a_n \Delta T + \sqrt{-1}\, b_n \Delta X} - 2e^{a_n \Delta T} + e^{a_n \Delta T - \sqrt{-1}\, b_n \Delta X}\right] +$$

$$+ \frac{F_{31} \Delta T}{(\Delta X)^2}\left[e^{\sqrt{-1}\, b_n \Delta X} - 2 + e^{-\sqrt{-1}\, b_n \Delta X}\right] +$$

$$+ \frac{F_{50} \Delta T}{(\Delta Y)^2}\left[e^{a_n \Delta T + \sqrt{-1}\, {}_n \Delta Y} - 2e^{a_n \Delta T} + e^{a_n \Delta T - \sqrt{-1}\, c_n \Delta Y}\right] +$$

$$+ \frac{F_{51} \Delta T}{(\Delta Y)^2}\left[e^{\sqrt{-1}\, c_n \Delta Y} - 2 + e^{-\sqrt{-1}\, c_n \Delta Y}\right] \tag{16}$$

Collecting terms and using the definition of complex exponentials

$$- e^{a_n \Delta T}\left\{-1 - 4 F_{30} \frac{\Delta T}{(\Delta X)^2} \sin^2 \frac{b_n \Delta X}{2} - 4F_{50} \frac{\Delta T}{(\Delta Y)^2} \sin^2 \frac{c_n \Delta Y}{2}\right\} =$$

$$= 1 + F_1 \Delta T + F_2 \frac{\Delta T}{\Delta X} \sqrt{-1} \sin b_n \Delta X + F_4 \frac{\Delta T}{\Delta Y} \sqrt{-1} \sin c_n \Delta Y +$$

$$- 4F_{31} \frac{\Delta T}{(\Delta X)^2} \sin^2 \frac{b_n \Delta X}{2} - 4F_{51} \frac{\Delta T}{(\Delta Y)^2} \sin^2 \frac{c_n \Delta Y}{2} \tag{17}$$

$e^{a_n \Delta T}$ is the error growth ratio in a time step ΔT. A sufficient condition for numerical stability is that

$$\left| e^{a_n \Delta T} \right| \leq 1 \tag{18}$$

or

$$\left| \frac{+\ 1 - 4\left[F_{31}\dfrac{\Delta T}{(\Delta X)^2}\sin^2\dfrac{b_n \Delta X}{2} + F_{51}\dfrac{\Delta T}{(\Delta Y)^2}\sin^2\dfrac{c_n \Delta Y}{2}\right] + 0(\Delta T)}{+\ 1 + 4\left[F_{30}\dfrac{\Delta T}{(\Delta X)^2}\sin^2\dfrac{b_n \Delta X}{2} + F_{50}\dfrac{\Delta T}{(\Delta Y)^2}\sin^2\dfrac{c_n \Delta Y}{2}\right] + 0(\Delta T)} \right| \leq 1 \tag{19}$$

Writing equation (19) as

$$\left| \frac{1 - 4A}{1 + 4B} \right| < 1 \tag{20}$$

and using the fact that $B > 0$, the stability limits are given by

$$- B < A < 1/2 + B \tag{21}$$

But, since $A + B > 0$, the left limit is always satisfied and it must be that

$$A - B < 1/2$$

or

$$\frac{1}{\Delta T} > \frac{2}{\Lambda}\left[\frac{(\psi H)_{1x} - (\psi H)_{0x}}{(\Delta X)^2} + \frac{(\psi H)_{1y} - (\psi H)_{0y}}{(\Delta Y)^2}\right] \tag{22}$$

Obviously, when the quantity in parenthesis is negative the numerical stability condition is always satisfied. Condition (22) yields

a) the well known explicit stability limit

$$\frac{1}{\Delta T} > \frac{2}{\Lambda}\left[\frac{1}{(\Delta X)^2} + \frac{1}{(\Delta Y)^2}\right] (\psi H)$$

when

$$(\psi H)_{0x} = (\psi H)_{0y} = 0,$$

b) the unconditional stability of the implicit method

when

$$(\psi H)_{1x} = (\psi H)_{1y} = 0.$$

Simplified criteria

1) Equal treatment in both directions

Let

$(\psi H)_{1x} = (\psi H)_{1y} = (\psi H)_{1}$ and

$(\psi H)_{0x} = (\psi H)_{0y} = (\psi H)_{0}$.

Then

$$\frac{1}{\Delta T} > \frac{2}{\Lambda} \left[\frac{1}{(\Delta X)^2} + \frac{1}{(\Delta Y)^2} \right] \left[(\psi H)_1 - (\psi H)_0 \right] \tag{23}$$

The simplest way to use this criterion is to have

$$(\psi H)_1 < (\psi H)_0 \tag{24}$$

Thus by selecting $(\psi H)_0$ to be the highest thinkable value of ψH (easily done especially with bearings possessing a "leading edge") it is possible to keep the correction $(\psi H)_1$ always negative and the process always numerically stable.

The question of numerical accuracy remains unchanged, and in true dynamic transient calculations it is important to bear in mind that very large values of the time interval ΔT give erroneous results especially in self-excited whirl threshold evaluations. For the case of steady-state calculations transient accuracy is not important, and since the accuracy of the steady-state answer is not affected by the selection of time interval ΔT, this selection should be made on the basis of expediency. Indeed the rapidity with which the transient settled to the steady-state pressure distribution depends on the time step ΔT. For small values of ΔT the true transient is followed and larger values of ΔT mean longer times to reach steady-state. Typically the pressure at each point is approached asymptotically from one side. For larger values of ΔT the true transient is not followed, the solution tends to the steady-state more rapidly overshooting it and then oscillating about it. As the value of ΔT is raised further the overshoot increases and the number of time steps spent in oscillations increases. Beyond a certain value of ΔT the oscillations become so violent that the solution could be called numerically unstable. The numerical stability analysis presented above does not predict this limit because its validity is limited to small values of the errors while in this case the errors even after one time step are very large.

In conclusion, for the most economical evaluation of steady-state pressure profiles the optimum value of ΔT lies in the region where it produces an incorrect transient which just overshoots the steady-state solution. This depends on the actual geometry under investigation and on the range of running conditions. A few numerical experiments are useful in determining the most economical production value of ΔT.

2) Implicit in one direction only. It may be of great interest to use

$$(\psi H)_{0y} = 0$$

$$(\psi H)_{1y} = \psi H$$

so that the only implicit operator is

$$(\psi H)_{0x} \frac{\partial^2\psi}{\partial x^2} - \frac{\Lambda}{\Delta T} \psi \quad . \tag{25}$$

Approximating this operator by three point central difference formulae it becomes a three point operator (rather than five point as in the case of $(\psi H)_{0y} \neq 0$).

This affords the great advantage that each X-row of points forms an independent three-diagonal problem. If i denotes an X-row (that is, i runs in the Y direction), the problem can be represented as

$$\sum_j \left[A_{k,j}\right]_i \psi_{i,j} = R_{i,k}; \quad i = 1, M$$

where each matrix $\left[A\right]_i$ is tridiagonal and therefore of extremely easy inversion.

The storage problem is also greatly relieved in comparison with the full "columnwise influence coefficients" method.

Analysis of the numerical stability of this particular technique starting from the general formula (22) yields

$$\frac{1}{\Delta T} > \frac{2}{\Lambda} \frac{\psi H}{(\Delta Y)^2} \tag{26}$$

which is the numerical stability limit for one dimensional explicit methods. Therefore, the one direction which is not treated implicitly falls back into the explicit method limitations. However, it should be noticed that

$$\frac{1}{(\Delta Y)^2} < \frac{1}{(\Delta Y)^2} + \frac{1}{(\Delta X)^2}$$

and, by selecting Y so that

$$\Delta Y > \Delta X$$

improvements over the explicit method in the allowable value of ΔT by factors of 4 or 5 are easily obtained. Indeed if

$$\Delta Y = \alpha \Delta X,$$

the improvement over the explicit method stability limit is given by

$$\Delta T = (1 + \alpha^2) \; \Delta T_{(explicit)}$$

NUMERICAL EXAMPLE

The results for a numerical example are shown in Figure 1. The case presented is the solution for a one dimensional slider bearing whose particular geometry is shown on the figure. The data plotted are load/steady state load versus the iteration number, with ΔT showing the parametric study. For all cases the initial pressure distribution is set equal to the ambient pressure. From these data, the effect of changing the value of ΔT can be readily seen. For small values of ΔT, the load approaches the steady state solution from one side behaving as a normal physical transient. For the higher values of ΔT, the load overshoots the steady state result and oscillates about it, decaying into the steady state. The higher the value of ΔT, the more violent the oscillation appeared. It should be mentioned here that for this particular analysis, the threshold of stability for the explicit method is $\Delta T < .001$.

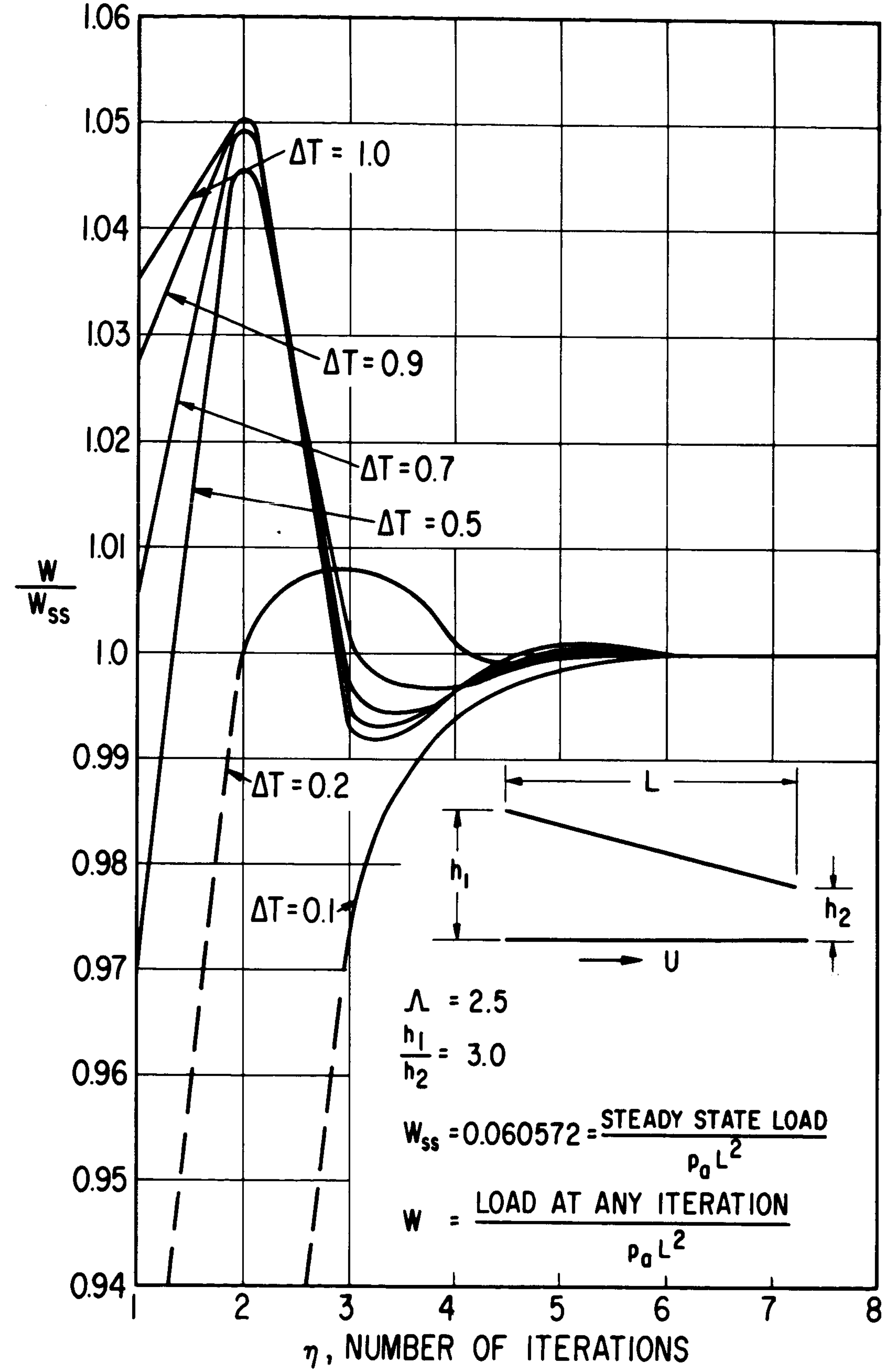
1.06
1.05
1.04
1.03
1.02
1.01
1.0
0.99
0.98
0.97
0.96
0.95
0.94
W / Wss
ΔT = 1.0
ΔT = 0.9
ΔT = 0.7
ΔT = 0.5
ΔT = 0.2
ΔT = 0.1
L
h1
h2
U
Λ = 2.5
h1/h2 = 3.0
Wss = 0.060572 = STEADY STATE LOAD / pa L2
W = LOAD AT ANY ITERATION / pa L2
1
2
3
4
5
6
7
8
η, NUMBER OF ITERATIONS

FIGURE 1

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NOMENCLATURE

c = reference clearance

H = h/c

h = local clearance

L = reference length

P = p/p_a

p = local pressure

p_a = ambient pressure

T = $\frac{tU}{2L}$

t = time

U = effective surface speed

X = $\frac{x}{L}$

x,y = dimensional coordinates on bearing surface

Y = $\frac{y}{L}$

Λ = $6\mu UL/(p_a c^2)$

μ = absolute viscosity

ψ = PH

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This report presents a numerical method for treating the time transient Reynolds equation which has the numerical stability properties of implicit schemes and the speed of execution of explicit method. These advantages make this method quite suitable for both steady-state and transient calculations.

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