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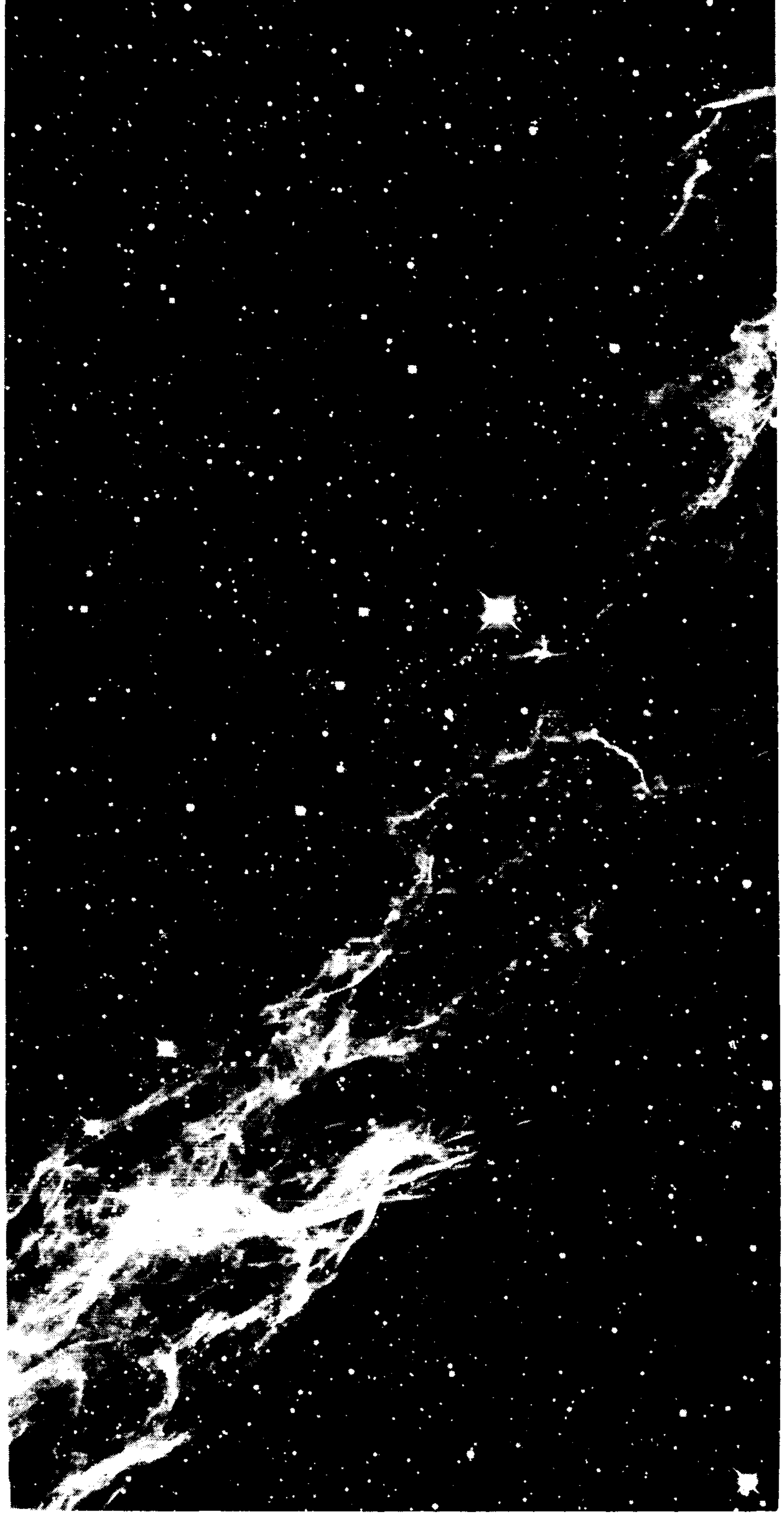

Report No. S-3

TELEMETRY COMMUNICATIONS GUIDELINES

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TELEMETRY COMMUNICATIONS GUIDELINES

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SUMMARY

This report considers the information transfer capability of spacecraft telemetry communication systems. Its purpose is to provide the advanced mission planner with general telemetry guidelines which will apply to space missions up to the 1975-1980 time frame. The first section of the report considers the major limitations to a communication system, together with the present and projected performance capabilities for various sub-systems, such as antennas, transmitters, and receivers. With these physical constraints, a set of generalized telemetry communication guideline curves are provided for determining the attainable data rates transmitted from a spacecraft, as a function of spacecraft transmitter power; the spacecraft and ground antenna sizes; and distance from the Earth to the spacecraft. The theoretical maximum data rate improvement which can be effected by means of error coding techniques are discussed. They are shown to offer improvements up to a factor of 6.5.

The section of the report which covers a minimum weight configuration for spacecraft indicates how the weight constraints of the spacecraft's telemetry equipment may be optimized with respect to the antenna size and transmitter power. For a

specified communications distance and transmission rate, the required antenna size, transmitter power, and optimum weights, are obtainable directly from the resultant curves. The specific weights assigned to the various subsystems are based on the present technology. The specific weight for attitude control propellant used for maintaining precise pointing of high gain antennas was not included as part of the analysis. The resultant curves nonetheless represent a first approximation for determining the design requirements of spacecraft telemetry systems.

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TELEMETRY COMMUNICATIONS GUIDELINE

1. INTRODUCTION

There is clear evidence that the ability to send exploratory spacecraft from the Earth to remote portions of the solar system has exceeded the present capability to communicate with these vehicles except in a marginal sense. This situation is one which could unfortunately become characteristic of nearly all future space missions. It arises from the fact that an outbound spacecraft, once sent on its way, can continue along its flight path with very little energy expended, while the required communications power must increase as the square of the range. For this reason, continuous effort is being devoted to increasing the capabilities of spacecraft telemetry, tracking, and command systems.

The purpose of this report is to provide the advanced mission planner with general telemetry guidelines. These guidelines provide an estimate of the data transmission capability as a function of the distance of the spacecraft from the Earth, the spacecraft transmitter power level, and the sizes of spacecraft and ground antennas. The data is intended to apply to advanced missions up to the 1975-1980 time frame, and therefore may predict a capability which exceeds that presently being obtained. It is not possible to specifically identify those techniques which actually will be developed and used to improve telemetry capabilities over the next 10 to 15 years. However, the theoretical maximum transmission rate capability is discussed for error control coding techniques as an indication of the upper bound on the anticipated improvements.

These guidelines also indicate the division of weight between the spacecraft transmitter and the spacecraft antenna to achieve a minimum total communications weight. This has been included merely as a means for identifying the communications subsystem requirements in planning advanced space missions.

2. TELEMETRY SYSTEM CONSTRAINTS

In acquiring experimental and scientific data from deep space missions, the physical constraints imposed upon the design of the communication system must be considered. There are five major limitations to a communication system: (1) the maximum available transmitter power, (2) the size and characteristics of the transmitting and receiving antennas, (3) the free space propagation loss, (4) the sensitivity of a receiving system, and (5) the method of modulation.

Spacecraft transmitters presently are limited in power due to spacecraft weight and volume restrictions. Early developments of space-borne transmitters provided an output power of only 3 watts, while present developments indicate a capability of 50 watts (Voyager). It is contemplated that by 1980 spacecraft transmitters may have a one kilowatt capability (Lee and Mullin 1965) with a 500 lb/kw specific weight.

The spacecraft antenna size primarily depends upon the weight and physical constraints of the spacecraft and launch vehicle. However large parabolic antennas require the pointing accuracy and stability of the spacecraft's attitude control system to be extremely precise. Figure 1 shows the accuracy requirements at 2300 MHz to keep the pointing loss to within 1 db. Thus an increase in antenna size directly increases the overall weight of the structure and also necessitates an additional increase due to the complexity of the stabilization and pointing system. Parabolic spacecraft antenna diameter probably will not greatly exceed about 8 ft in diameter in the foreseeable future.

The size of the ground-based antenna is one of the more critical elements in a deep space communication system. The size of the ground antenna appears to be limited to the 210' DSN (Deep Space Network) type of antenna recently constructed at the Goldstone Site. This is due primarily to structural limitations, i.e., the antenna must maintain its pointing accuracy and slew rates under adverse environmental conditions. The cost of manufacturing also tends to constrain the ground antenna size. The cost of large parabolic antennas varies approximately as the 2.7 power of the diameter (Balakrishnan 1963).

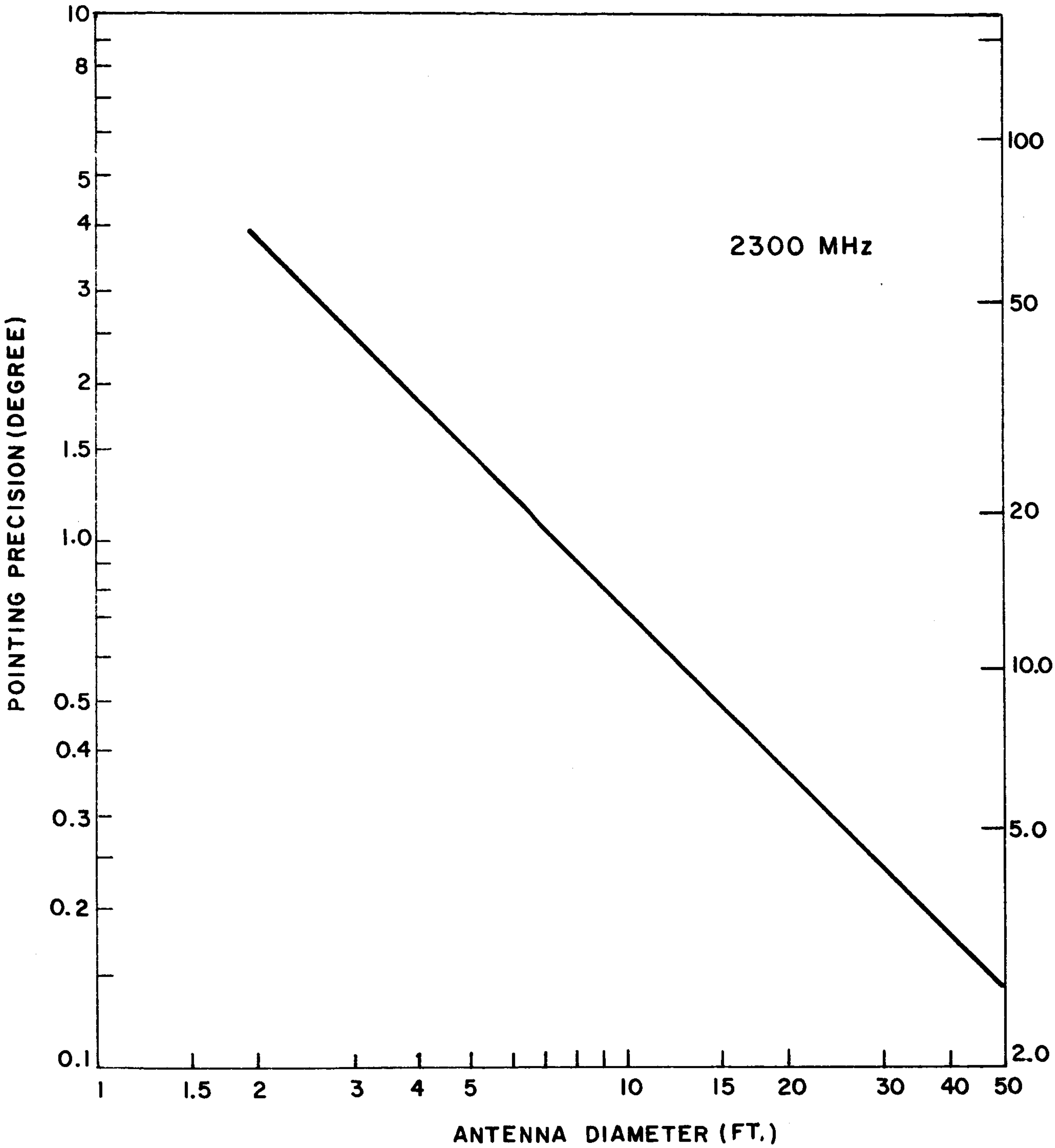


FIGURE I. POINTING ACCURACY WITHIN I DB AS A FUNCTION OF ANTENNA SIZE.

The free space propagation loss is dictated by the operating frequency and the distance between the transmitter and receiver. Current DSN operating frequencies between 2100 MHz and 2300 MHz have been chosen on the basis of minimum galactic and atmospheric noise temperatures (Thatcher 1964). However, transmission at millimeter wavelengths is under consideration for future application. Laser communications also may be anticipated.

Communications performance also is dictated by the sensitivity of the receiving system, which is the effective noise power in the receiver. This noise, with which the signal competes, may be minimized by reducing either the information bandwidth or the effective noise temperature which is the result of antenna noise and the receiver's internal noise. Present effective system noise temperatures using the 210 ft DSIF antenna (receiving mode only) with a helium cooled maser amplifier have been reduced to 25°K (Renzetti 1966). Trends indicate that this figure may be lowered to 20°K in the future.

Probably, modulation has been investigated more thoroughly than any other method for improving the transmission data rate capability. One means by which digital systems are compared is the modulation efficiency, defined as the number of cycles of bandwidth in which there must exist a unity signal-to-noise ratio per bit of transmitted information. For a given radiated power, a decrease in the number of cycles where a unity signal-to-noise ratio is required, increases the transmission rate at a given distance, or the effective distance of a transmission at a given bit rate. At the present time, deep space telemetry systems use uncoded coherent PSK (phase-shift-keying) modulation, which, for an average bit error probability of 10^{-3} yields a modulation efficiency of 6.8 cycles per bit. A figure of 3 cycles per bit can be attained by orthogonal and biorthogonal coding techniques as suggested by Viterbi (1960). These codes, theoretically at least, offer a significant improvement in performance, although the receiver detection equipment is more complex. Initial experimental results indicate that an improvement of only 1 db has been obtained over an uncoded system (Gilchriest 1966).

3. TELEMETRY COMMUNICATION GUIDELINE

To note the effect that the previously mentioned performance limitations have upon determining the maximum rate for the telemetry link, it is necessary to consider the fundamental communication equation in logarithmic notation:

$$P_R = P_S + G_T + G_R - L_S - L_M - L_N \tag{1}$$

where

P_R = received sideband power (dbm)*

P_S = transmitted sideband power (dbm)

G_T = transmitting antenna gain (db)

G_R = receiving antenna gain (db)

L_S = space loss (db)

L_M = miscellaneous losses (db)

*dbm is used to indicate units of milliwatts.

L_N = system negative tolerances (db).

As derived in Appendix A, the bit rate H is related to the received power as:

$$H = P_T + G_T + G_R - \emptyset_K - L_S - L_M - E/N/B - K - L_N \qquad (2)$$

where H is the bit rate in db, P_T is the transmitted power, E/N/B is the received signal energy per bit per unit noise per unit bandwidth, $\emptyset_K$ is the system noise spectral density in dbm/cps, and K is the modulation loss.

It is assumed that both 85 ft and 210 ft antennas will be used as part of the DSN. Also, to enhance further the capability of the receiving system in tracking and acquiring data at the planetary ranges, a helium-cooled maser amplifier is assumed in combination with the 210 ft and 85 ft antennas, and provides system noise spectral densities $\emptyset_K$ of -183.2 dbm/cps and -181.2 dbm/cps, respectively.

The present discussion considers the use of an uncoded coherent PSK system. Minimizing the required signal energy per bit for a fixed error probability is important since the system essentially is power-limited rather than bandwidth-limited.* The relationship between the bit error probability P_e, the received signal energy per bit E, and the noise (Gaussian) power per unit bandwidth N/B (Lawton 1958), is

$$P_e = 1/2\ (1\text{-erf}\ \sqrt{E/N/B}),$$

where E = ST is the received signal power times the bit period.

*Appendix A shows the relationship between the transmission rate H and the theoretical maximum rate C_M when the bandwidth is unlimited.

Normally, for telemetry data, the acceptable word error probability P_w is on the order of 1×10^{-2} or one percent. For a 7 bit word, this is equivalent to a bit error probability $P_e = 1.4 \times 10^{-3}$. As shown in Figure 2, this gives an E/N/B of 6.5 db.

The incoming RF signal essentially is divided into two components - that part of the carrier power which is converted into sideband information energy, and the remaining unmodulated RF carrier power. Therefore, to insure proper RF tracking beyond the actual data acquisition capability, the subcarrier threshold is normally 4 db higher than the RF receiver threshold. It is assumed that the transmitting system is 30 percent efficient (transmitted power to raw power), and the sideband power is assumed to consume 40 percent of the transmitter power, the balance being invested in the carrier. Therefore, each watt of transmitter power (P_T) or 0.4 watt of sideband power (P_S) requires 3.3 watts of raw power P.

To summarize the given constraints and assumptions:

Receiving antenna gain (85' DSIF) G_R	= 53 db
Receiving antenna gain (210' DSIF) G_R	= 61 db
Noise spectral density for a system noise temperature of 55°K (85' antenna)	= -181.2 dbm/cps
Miscellaneous losses L_M	= 3.5 db
System negative tolerances L_N	= 8 db
Subcarrier receiver threshold E/N/B	= 6.5 db
Frequency (S-band)	= 2300 MHz

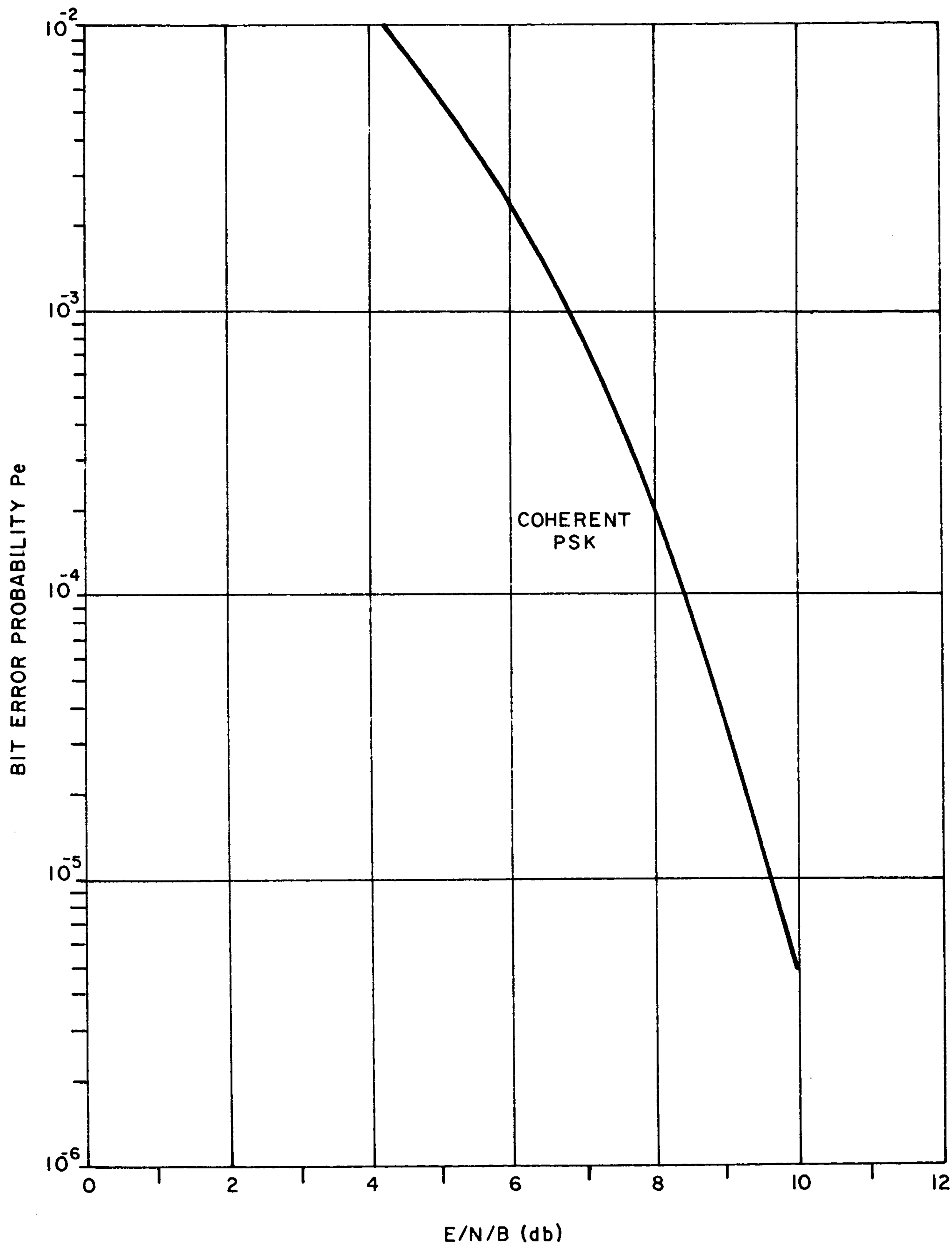


FIGURE 2. COHERENT PSK PERFORMANCE CURVE

Modulation loss K	= 4 db
Bit error probability (telemetry) P_e	= 1.4×10^{-3}
Transmitting efficiency (transmitted to raw power)	= 30%

Based on these values, the bit rate equation (2) becomes

$H(db) = P_T - L_S + 222.2$ for the 210' antenna, and

$H(db) = P_T - L_S + 212.2$ for the 85' antenna,

where $L_S(db) = 263 + 20 \log_{10} R(AU)$.

The transmission rate equations are plotted in Figure 3 as a function of the transmitted power, antenna gain product ($P_T G_T$) and the communication distance R from the Earth. The bit rate scale is calibrated for both the 210' and 85' DSIF receiving antennas.

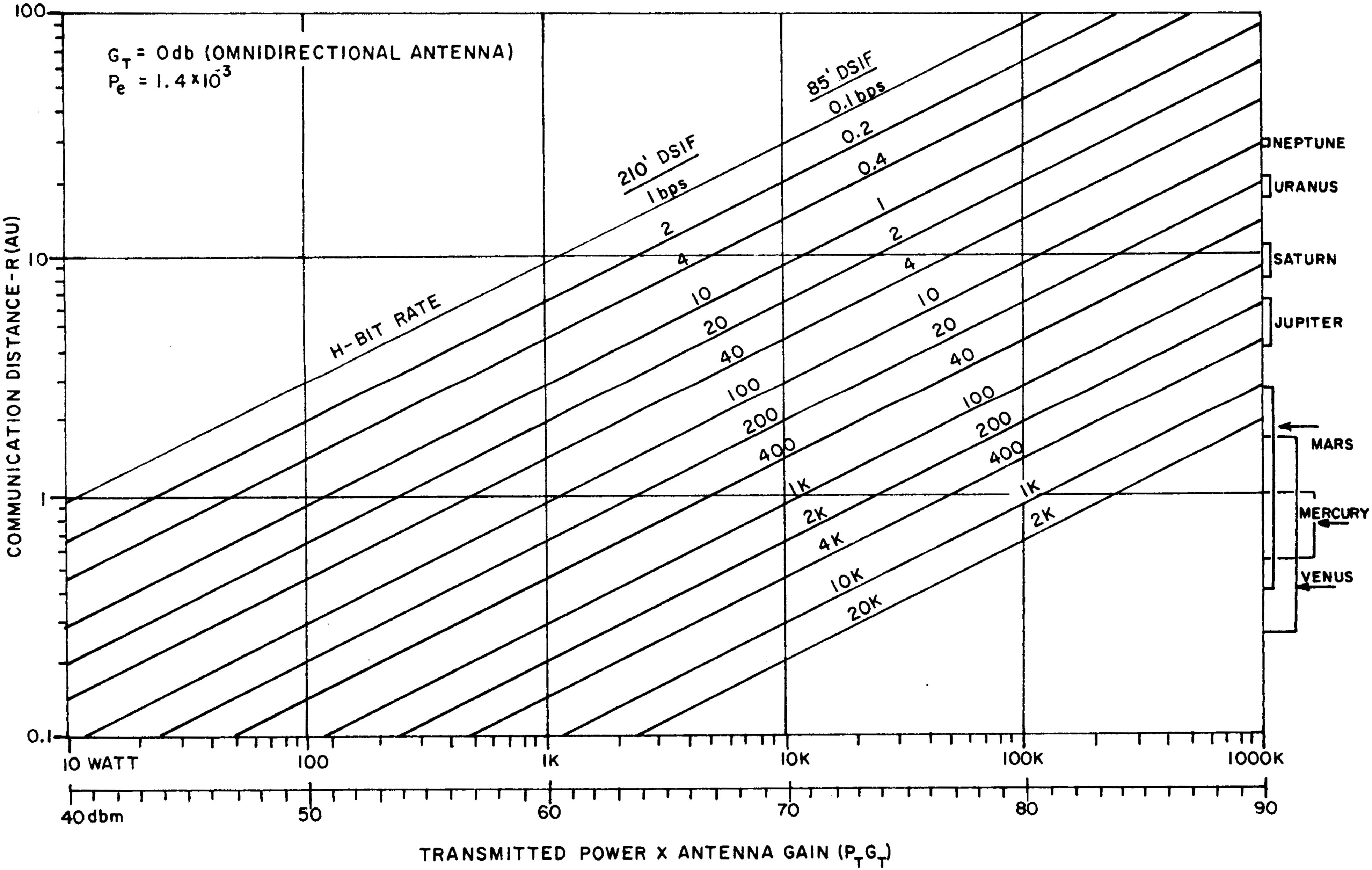
Figure 4 shows the gain G(db) improvement (at 2300 MHz) over the omnidirectional spacecraft antenna, resulting from the use of a directional antenna of diameter D ft.

The following four cases are presented as examples of the use of the communications chart.

Case I:

Given	Determine
Distance	Transmitted power
Spacecraft antenna	Raw Power
Bit rate	
DSIF antenna	

FIGURE 3: TELEMETRY COMMUNICATIONS CURVES
S-BAND (2300 MHz)



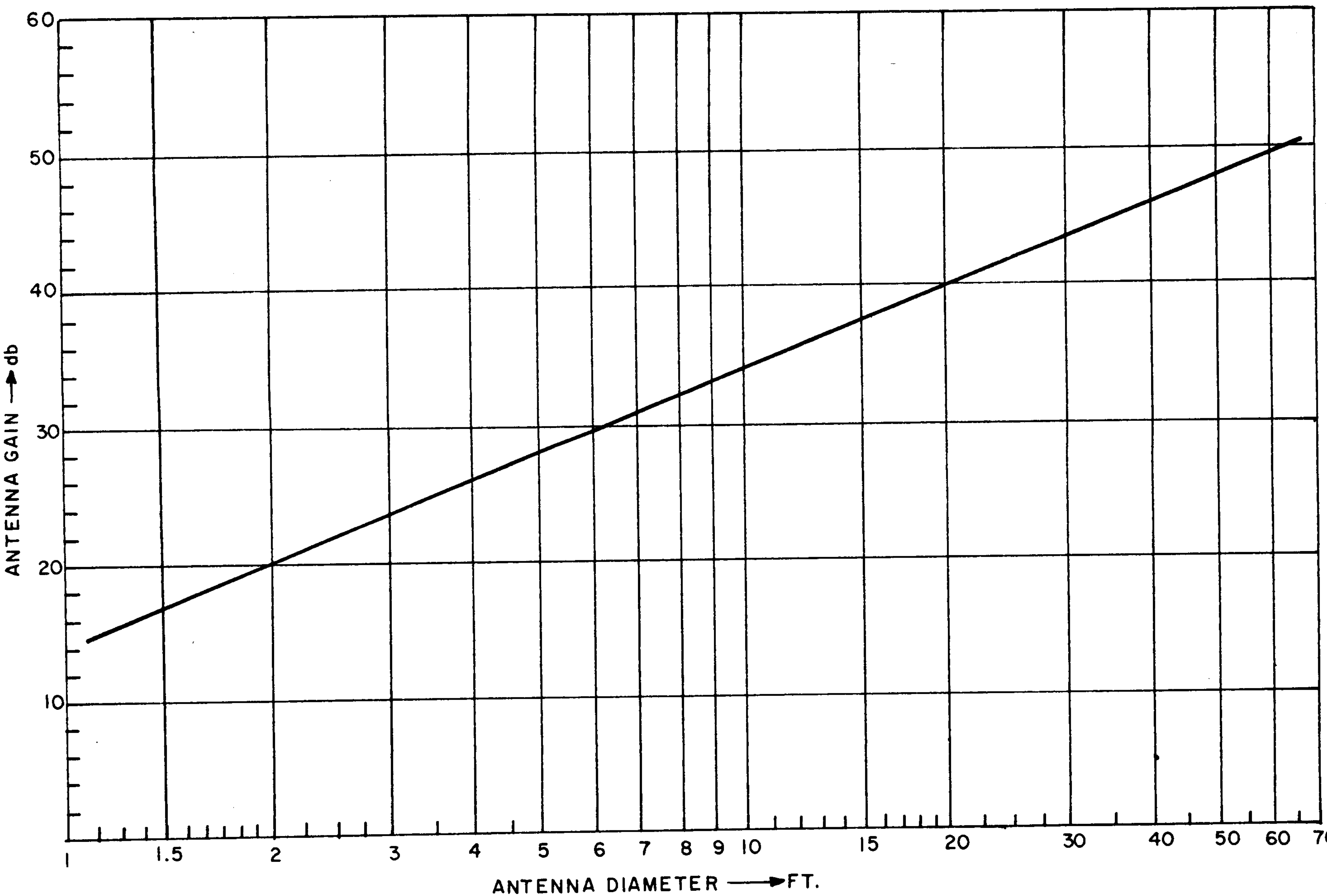


FIGURE 4. ANTENNA GAIN VS. DIAMETER AT S-BAND (2300 MHz)

Example 1:

A communications distance of 20 AU (Uranus), a bit rate of 10 bps and an omnidirectional spacecraft antenna (0 db gain) are specified. Using Figure 3, the 20 AU line intersects the 210', 10 bps line at 50 kw (77 dbm). Thus 50 kw is the required transmitter power.

Example 2:

The required power in Example 1 is clearly excessive. If a 6 ft diameter antenna is used instead of the omnidirectional spacecraft antenna, the improvement in gain G(db) is obtained from Figure 4. It is the 29 db for a 6' dish. The corresponding reduction in required power can be calculated using the dbm power scale on Figure 3. The power-gain product as above is 77 dbm. Subtracting the 29 db antenna gain from this leaves 48 dbm and the corresponding required power would be 63 watts. Similarly for the 85' DSIF, the required power with a 6' dish is 630 watts.

<u>Case II:</u>

<u>Given</u>	<u>Determine</u>
Distance	Bit rate
Spacecraft antenna	
Transmitter or raw power	
DSIF antenna	

Example 1:

A communications distance of 0.92 AU, an omnidirectional spacecraft antenna having a gain of 0 db and a transmitter power of 100 watts or 50 dbm are specified. Using Figure 3, the 0.92 AU line intersects the power-gain level of 100 watts at a bit rate of 10 bps for the 210' DSIF or 1.0 bps for the 85' DSN. These are the respective permissible bit rates.

Example 2:

If a directional antenna is used instead of the omnidirectional spacecraft antenna, an improvement can be obtained in the bit rate. For a 2' dish, Figure 4 gives a gain of 20 db. The equivalent power-gain used in Figure 3 should now be 50 dbm (100 watts) plus the gain of 20 db, i.e., 70 dbm. The intersection of 0.92 AU with 70 dbm gives a maximum bit rate of 1000 bps for the 210' DSIF or 100 bps for the 85' DSN.

Case III:

Given	Determine
Distance Bit rate Transmitter or raw power DSIF antenna	Spacecraft antenna

Example:

A communications distance of 0.92 AU, a data rate of 1000 bps, and a transmitter power of 10 watts, are specified. The intersection of the 0.92 AU line with 1000 bps (210' DSIF) in Figure 3 shows that 70 dbm (10 kw) is the required power-gain level. The specified power is 10 watts or 40 dbm and therefore the antenna must provide the additional 30 db of gain. From Figure 4, the antenna required to provide this gain is a 6.5' dish. Similarly using the 85' DSIF, a 20' dish would be required.

Case IV:

Given	Determine
Transmitter or raw power	Maximum distance
Bit rate	
Spacecraft antenna	
DSIF antenna	

Example 1:

A transmitter power of 20 watts, a bit rate of 10 bps, and an omni-directional spacecraft antenna are specified. In Figure 3, the 20 watt or 43 dbm transmitter power-antenna gain ($P_T G_T$) line intersects the 10 bps line at 0.41 AU for the 210' DSIF and at 0.13 AU for the 85' DSIF. These are the maximum transmission distances.

Example 2:

The transmission distance can be increased by the use of a directional spacecraft antenna. For a 6' diameter dish, the gain from Figure 4 is 29 db. This can be added to the transmitting power 43 dbm (20 watts) in Figure 3 to deduce the improvement. The effective power-gain is now 72 dbm and this line intersects the 10 bps line at 12 AU for the 210' DSIF and at 3.7 AU for the 85' DSIF.

4. THEORETICAL TRANSMISSION RATE CAPABILITY USING CODING TECHNIQUE

The preceding section presented telemetry communication curves for use in determining the transmission rate as a function of power, antenna gain, and communications distance. As one of the imposed constraints, it was assumed that the information to be transmitted would be uncoded; this implies that message errors would not be detectable. An effective means for increasing the information transfer rate can be achieved through the use of error-control coding. Since all error detection and correction codes add redundancy to the signal information, their use would appear to be costly in terms of the amount of information transmitted per unit bandwidth. However, since most telemetry systems are more limited in transmitter power than in bandwidth, the use of coding can increase the effective transmission rate for a given transmitter power level, antenna gain, and receiver sensitivity.

For the purpose of mission planning, it is of interest to show the theoretical maximum transmission rates that can be achieved with coding and to compare these to actual rates attainable using uncoded transmissions.

As derived in Appendix A, the difference between the theoretical maximum transmission rate or channel capacity and the actual rate (uncoded), appears as

$$C_M(\mathrm{db}) - H(\mathrm{db}) \approx 1.6 + ST/N/B \quad . \qquad \text{(A14)}$$

Using the value for 6.5 db for the subcarrier receiver threshold ST/N/B as assumed in Section 3 provides a difference of 8.1 db or a factor of 6-1/2. To show the effect that this result has in increasing the transmission capabilities, Case II, Example 1 of Section 3 again is cited.

Case II:

Given	Determine
Distance	Bit rate
Spacecraft antenna	
Transmitter power	
DSIF antenna	

Example 1:

A communications distance of 0.92 AU, an omnidirectional spacecraft antenna having a gain of 0 db, and a transmitter power of 100 watts or 50 dbm are specified. Using Figure 3, the 0.92 AU line intersects the power-gain level of 100 watts or 50 dbm at a bit rate of 10 bps for the 210' DSIF antenna or 1 bps for the 85' DSIF antenna. These are the maximum bit rates for the uncoded system. For a coded system, the theoretical maximum rates attainable are 65 bps and 6.5 bps for the 210' and 85' DSIF antennas, respectively. Conversely, a coded system can effect a reduction in the required transmitter power for a given bit rate, communications distance, and DSIF antenna.

5. MINIMUM WEIGHT SPACECRAFT TELEMETRY SYSTEM

One of the prime considerations in the design of a space communications system is weight and size. Often, a design that is optimal with respect to communication efficiency may be inconsistent with the physical constraints of the spacecraft.

For a required transmitter power-antenna gain ($P_T G_T$) product, there exists an optimum transmitter power and antenna gain which minimizes the weight of the total transmitting system.

In this context, the system weight comprises the weight of the transmitter, the proportion of the power supply required by the transmitter, and the antenna. Since the system weight does not include the weight of attitude control propellant, the use of a large parabolic antenna (requiring precise pointing) could alter the basic assumptions described above.

The basic criteria of minimum weight as applied here do not reflect the fact that the present state-of-the-art capability of producing a high power (50-100 watts) transmitter may far exceed the capability, in an operational sense, of using a large ($\gg$ 8') spacecraft antenna. The mission planner therefore must judiciously trade off, for each prescribed mission, between power and antenna gain and still satisfy the overall transmitting system weight requirements.

The resultant curves, however, do provide a first approximation for estimating the design requirements of a deep space telemetry system, and can be revised to accommodate changes in DSN performance and subsystem specific weights as technology advances.

As derived in Appendix B, the minimum transmitting system weight, transmitted power, antenna diameter, and antenna gain are functions of the respective specific weights and the transmitter power-gain product ($P_T G_T$). They appear as

$$W_{MIN}(\text{lbs}) = \left[\frac{4P_T G_T K_1 (K_2/\epsilon + K_3)}{K_5}\right]^{1/2} \qquad \text{(B11b)}$$

$$P_T(\text{watts}) = \left[\frac{P_T G_T K_1}{K_5(K_2/\epsilon + K_3)}\right]^{1/2} \quad \text{(B13)}$$

$$D(\text{ft}) = \left[\frac{P_T G_T (K_2/\epsilon + K_3)}{K_1 \; K_5}\right]^{1/4} \quad \text{(B14)}$$

$$G_T = \left[\frac{P_T G_T K_5 (K_2/\epsilon + K_3)}{K_1}\right]^{1/2} \quad \text{(B15)}$$

Also, the minimum individual weights are derived as

$$W_A(\text{lbs}) = W_p(\text{lbs}) = \frac{W_{MIN}(\text{lbs})}{2} \quad \text{(B16)}$$

The following assumptions are used for determining the values of these constants.

Specific weight of the antenna (parabolic type with aluminized-phenolic honeycomb and fiberglass skin)	$K_1 = 0.25$ lb/ft^2
Specific weight of power supply (RTG source with shielding)	$K_2 = 1.0$ lb/watt
Transmitting efficiency (transmitted to raw power)	$\epsilon = 30\%$
Specific weight of the RF transmitter	$K_3 = 0.06$ lb/watt
Specific antenna gain (2300 MHz)	$K_5 = 30/\text{ft}^2$

Substitution of the constants into the preceding equations yield

$$W_{MIN}(\text{lbs}) = \left[\frac{P_T G_T}{8.4}\right]^{1/2}$$

$$P_T(\text{watts}) = \left[\frac{P_T G_T}{430}\right]^{1/2}$$

$$D(ft) = \left[\frac{P_T G_T}{2.10}\right]^{1/4}$$

$$G_T = \left[430\ P_T G_T\right]^{1/2}$$

$$W_A(lbs) = \left[W_p(lbs) = \frac{P_T G_T}{33.6}\right]^{1/2}$$

These minimum weight parameters are plotted in Figure 5 as a function of the power gain product ($P_T G_T$). The basic telemetry curve shown in Figure 3 is repeated in Figure 5 to simplify the use of the minimum weight plots. Since a power supply specific weight of 1 lb/watt has been assumed, corresponding approximately to solar panels at about 1 to 2 AU or an RTG unit, only communications distances in excess of 1 AU have been indicated.

The following cases are provided as examples of the use of the minimum weight configuration curves of Figure 5.

Case I:

Given	Determine
Distance	Transmitted power
Bit rate	Spacecraft antenna diameter
DSIF antenna diameter	Minimum system weights

Example 1:

A communications distance of 20 AU (Uranus), a bit rate of 10 bps using the 210' DSIF antenna are specified. As shown in Figure 5, the 20 AU line intersects the 210', 10 bps line at a power-gain level of 50 kw or 77 dbm. The intersection of

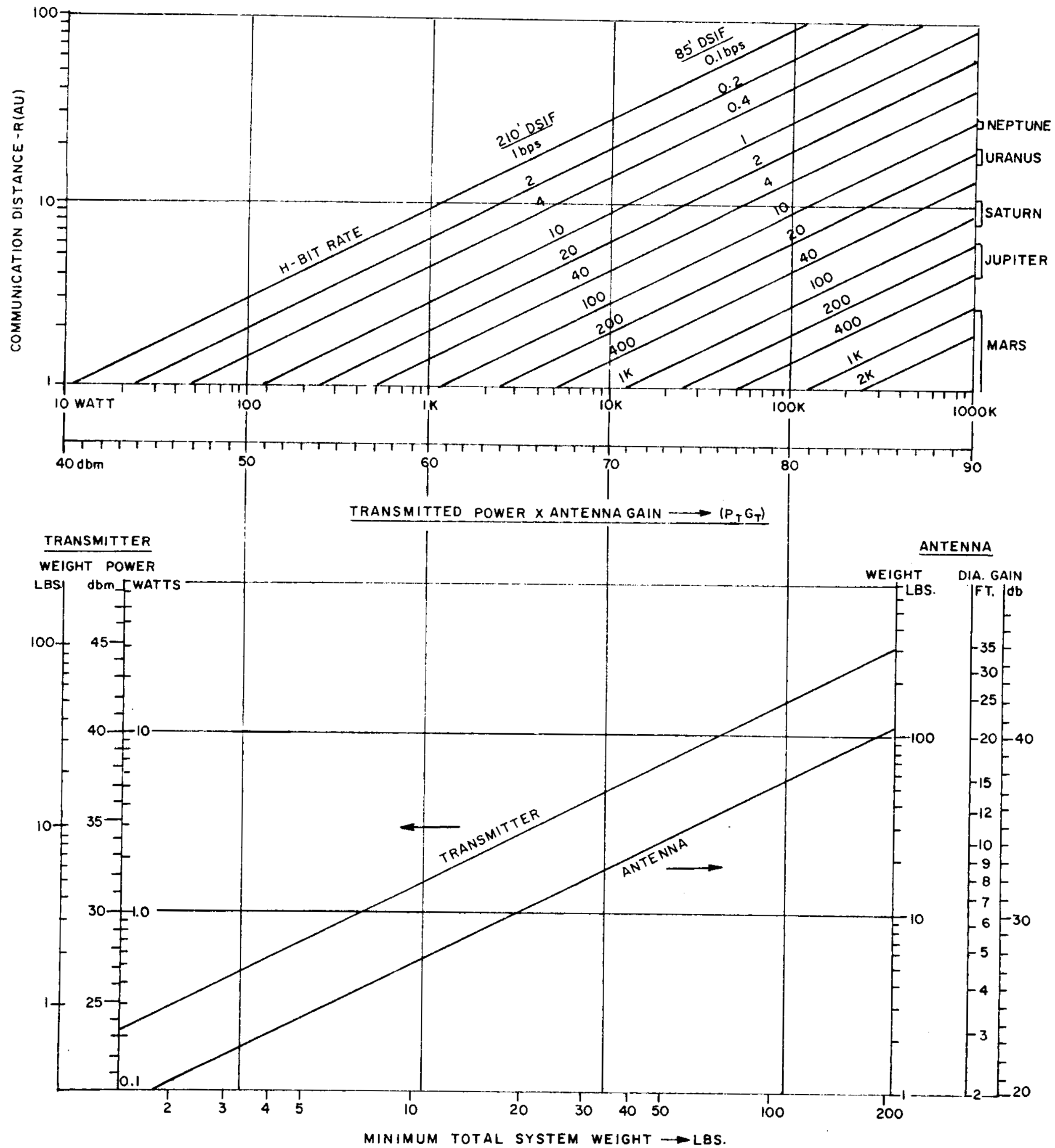


FIGURE 5. MINIMUM WEIGHT CONFIGURATION FOR SPACECRAFT COMMUNICATION SYSTEM

this level (77 dbm) with the transmitter line on the optimum curve provides the required transmitter power and transmitter weight on the left-hand scales. These are approximately 12.0 watts and 40 pounds, respectively. Correspondingly, the intersection of 77 dbm with the antenna line produces an antenna diameter and weight of approximately 13 feet and 40 pounds, respectively from the right-hand scales. Thus, the total minimum weight is 80 pounds.

Example 2:

For the prescribed transmitted power-gain ($P_T G_T$) product of 77 dbm or 50 kw of Example 1, suppose the largest size spacecraft antenna that may be used is 8', corresponding to an antenna gain of 32 db, and an antenna weight of 16 lb. In this instance, the required transmitter power is 77 - 32 = 45 dbm. By referring to the left-hand scales this corresponds to a power of 32 watts and weight of 110 lb. Therefore, it can be seen that the composite weight of 126 pounds for the total transmitting system is approximately 50% heavier than the optimum system of Example 1.

6. CONCLUSIONS

The communications guidelines are presented to assist advanced mission planners in assessing the approximate requirements of spacecraft telemetry systems. They are intended to apply up to the 1980 time frame and hence may predict a capability slightly in excess of that presently being realized. No attempt has been made here to guide the future development of communications systems, but rather, simply to predict it.

Appendix A

DERIVATION OF TRANSMISSION RATE EQUATION

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For deep space communications, free space propagational losses hold for the signal energy transfer between transmitter and receiver. The equation governing this loss is

$$L_S(\text{db}) = 195.97 + 20 \log_{10} (fR) \qquad \text{(A1)}$$

where f is the transmitting frequency in megacycles per second and R is the path distance in astronomical units (AU).

To note the effect this constraint has upon determining the maximum information rate for a telemetry link, it becomes necessary to consider the fundamental communication equation:

$$P_R = P_S + G_T + G_R - L_S - L_M - L_N \qquad \text{(A2)}$$

where

P_R = received sideband power (dbm)

P_S = transmitted sideband power (dbm)

G_T = transmitting antenna gain (db)

G_R = receiving antenna gain (db)

L_S = space loss (db)

L_M = miscellaneous losses (db)

L_N = system negative tolerances (db).

Since the receiver power is:

$$P_R = S/N/B + \emptyset_K \tag{A3}$$

where S/N/B = normalized receiver SNR (db-cps), and

$\emptyset_K$ = receiving system noise spectral density (dbm/cps);

then the resultant normalized SNR is:

$$S/N/B = P_S + G_T + G_R - \emptyset_K - L_S - L_M - L_N \; . \tag{A4}$$

Since the signal energy is equal to the signal power times the bit period,

$$E/N/B = ST/N/B \; . \tag{A5}$$

Using the logarithmic notation, (A5) may be represented as

$$E/N/B = S/N/B + T \text{ db} \tag{A6}$$

or

$$H = S/N/B - ST/N/B \tag{A7}$$

where H = bit rate (db). Combining equations (A4) and (A7) results in

$$H = P_S + G_T + G_R - \emptyset_K - L_S - L_M - ST/N/B - L_N \; . \tag{A8}$$

In terms of the transmitted power P_T,

$$H = P_T + G_T + G_R - \emptyset_K - L_S - L_M - ST/N/B - K - L_N \tag{A9}$$

where $K(\text{db}) = 10 \log_{10} \left(\frac{P_T}{P_S}\right)$ is the modulation loss and is normally specified as 4 db. The theoretical maximum rate C (channel capacity) at which information can be transmitted is readily determined from Shannon's formula (Shannon 1948) where

$$C = B \log_2 \left(1 + \frac{P_R}{B\ \phi_K}\right) \text{ bits/second.} \qquad (A10)$$

As the bandwidth B increases without limit, i.e., $P_R/B\ \phi_K \ll 1$, the channel capacity C has as its upper bound

$$C \leqslant 1.44\ P_R/\phi_K \text{ bits/second.} \qquad (A11)$$

Expressed in logarithmic notation and using the maximum value for C, (A11) becomes

$$C_M(\text{db}) = 1.6 + P_R - \phi_K \quad . \qquad (A12)$$

Substituting (A2) into (A12) yields

$$C_M(\text{db}) = 1.6 + P_S + G_T + G_R - L_S - L_M - \phi_K - L_N \ . \quad (A13)$$

The difference between the channel capacity and the actual transmission rate is then determined by the substitution of (A8) into (A13) which results in

$$C_M(\text{db}) - H(\text{db}) = 1.6 + ST/N/B \ . \qquad (A14)$$

The above equation shows that by lowering the value of ST/N/B through the use of improved coding methods, the theoretical maximum transmission rate can be approached.

Appendix B

MINIMUM TRANSMITTER SYSTEM WEIGHT

Appendix B

MINIMUM TRANSMITTER SYSTEM WEIGHT

For a given transmitted power-gain product ($P_T G_T$), there exists an optimum transmitter power and antenna gain from a point of view of minimizing the weight of the total transmitting system (Davies and Weaver 1959). In this context, the transmitting system includes the spacecraft antenna, transmitter, and the proportion of the power supply needed to drive the transmitter.

The weight of the transmitting antenna, W_A, is given by the relation

$$W_A = K_1 D^2 \tag{B1}$$

where D is the antenna diameter and K_1 is a constant dependent upon the type of antenna used and may range from 0.2 lb/ft^2 to as high as 2 lb/ft^2.

The transmitted power can be expressed as a function of the transmitter weight plus the weight of the power source. The total transmitter-source weight W_p is given by

$$W_p = P_T \, (K_2/\epsilon + K_3) \tag{B2}$$

where P_T is the transmitted power, K_2 is the specific weight of the power supply (on the order of 1 lb/watt), ϵ is the efficiency of conversion of raw power into transmitted power (assumed to be 30%), and K_3 is the specific weight of the RF transmitter (approximately 0.6 lb/watt).

The transmitted power P_T is related to the transmitter sideband power P_S as

$$P_T = K_4 P_S \tag{B3}$$

where K_4 is the modulation loss and is usually specified as 4 db.

The antenna gain G_T is related to its diameter D by

$$G_T = K_5 D^2 \tag{B4}$$

where K_5 is the specific antenna gain and depends on the wavelength at the operating frequency. At 2300 MHz its value is approximately 30.

The communications link requires a certain transmitter power-gain product ($P_T G_T$) which depends on the following from equation (A8) in Appendix A.

$$P_T G_T = \frac{K_4 \phi_K H \; L_S L_N L_M \; E/N/B}{G_R} \tag{B5}$$

where

ϕ_K is the noise spectral density

H is the bit rate

L_N is the system negative tolerances

L_M is the miscellaneous losses

E/N/B is the subcarrier receiver threshold

L_S is the space propagational loss

and G_R is the receiving antenna gain.

From equations (B1) and (B2), the total weight of the spacecraft's transmitting system is

$$W = W_A + W_p = K_1 D^2 + (K_2/\epsilon + K_3)\, P_T \; . \qquad \text{(B6)}$$

By substitution of (B4) and (B5) into (B6), the total weight may be expressed as a function of the antenna diameter

$$W = \frac{K_7}{D^2} + K_1 D^2 \qquad \text{(B7)}$$

where

$$K_7 = \frac{P_T G_T \, (K_2/\epsilon + K_3)}{K_5} \; . \qquad \text{(B8)}$$

The minimum weight W_{MIN} can be determined by differentiating (B7) with respect to the antenna diameter, i.e.,

$$\frac{dW}{dD} = \frac{-2K_7}{D^3} + 2K_1 \, D = 0 \qquad \text{(B9)}$$

from which

$$D_{MIN} = (K_7/K_1)^{1/4} \; . \qquad \text{(B10)}$$

Substitution of equation (B10) into (B7) yields

$$W_{MIN} = (4K_7K_1)^{1/2} \qquad \text{(B11a)}$$

$$= \left[\frac{4P_T G_T K_1 (K_2/\epsilon + K_3)}{K_5}\right]^{1/2} \; . \qquad \text{(B11b)}$$

Since the minimum weight comprises the weight of the antenna plus the weight of the transmitter-source, each may be shown individually as

$$W_A(\mathrm{lb}) = \frac{K_1 G_T}{K_5} \qquad \text{(B12)}$$

$$W_p(\mathrm{lb}) = P_T(K_2/\epsilon + K_3) \ . \qquad \text{(B2)}$$

In combination with the minimum weight, the antenna diameter D, the optimum transmitter power P_T, and antenna gain, also may be determined. By suitable substitution, these result in

$$P_T(\mathrm{watts}) = \left[\frac{P_T G_T K_1}{K_5 (K_2/\epsilon + K_3)}\right]^{1/2} \qquad \text{(B13)}$$

and

$$D(\mathrm{ft}) = \left[\frac{P_T G_T (K_2/\epsilon + K_3)}{K_1 K_5}\right]^{1/4} \qquad \text{(B14)}$$

$$G_T = \left[\frac{P_T G_T K_5 \ (K_2/\epsilon + K_3)}{K_1}\right]^{1/2} \ . \qquad \text{(B15)}$$

The individual weights are obtained by substitution of (B15) into (B12) and (B13) into (B2), respectively, which yield

$$W_A(\mathrm{lb}) = W_p(\mathrm{lb}) = \frac{W_{MIN}(\mathrm{lb})}{2} \ . \qquad \text{(B16)}$$

Alternatively, the equations may be expressed in logarithmic notation as

$$W_{MIN}(\mathrm{db\text{-}lb}) = \frac{1}{2}\left[P_T(\mathrm{dbm}) + G_T(\mathrm{db}) - 39.2 \ \frac{\mathrm{dbm}}{\mathrm{db\text{-}lb}}\right]$$

$$P_T(\mathrm{dbm}) = \frac{1}{2}\left[P_T(\mathrm{dbm}) + G_T(\mathrm{db}) + 3.8 \ \mathrm{db}\right]$$

$$D(\mathrm{db\text{-}ft}) = \frac{1}{4}\left[P_T(\mathrm{dbm}) + G_T(\mathrm{db}) - 33.2 \ \frac{\mathrm{db}}{\mathrm{db\text{-}ft}}\right]$$

$$G_T(\text{db}) = \frac{1}{2}\left[P_T(\text{dbm}) + G_T(\text{db}) - 3.8\ \frac{\text{dbm}}{\text{db}}\right]$$

$$W_A(\text{db-lb}) = W_p(\text{db-lb}) = \frac{1}{2}\left[P_T(\text{dbm}) + G_T(\text{db}) - 45.2\ \frac{\text{dbm}}{\text{db-lb}}\right].$$

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