SATELLITE NAVIGATION STUDIES

~~Sixth~~ Quarterly Progress Report
1 April ~~Through~~ 30 June 1967

Y. Morita
F. Zwas
D. Colling

August 1967

Prepared Under Contract No. NASr-54-(10) by
Willow Run Laboratories
Institute of Science and Technology
The University of Michigan
Ann Arbor, Michigan

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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NOTICES

Sponsorship. The work reported herein was conducted by the Willow Run Laboratories of The University of Michigan's Institute of Science and Technology for the National Aeronautics and Space Administration under Contract No. NASr-54(10). Contracts and grants to The University of Michigan for the support of sponsored research are administered through the Office of the Vice-President for Research.

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FOREWORD

This report was prepared by The University of Michigan on National Aeronautics and Space Administration Task Order Contract NASr-54(10), Navigation Satellite Studies. The work was administered under the direction of Mr. Eugene Ehrlich, Chief, Navigation and Traffic Control, Space Applications Program Office, National Aeronautics and Space Administration.

The work reported covers the quarter 1 April through 30 June 1967. The work was performed by the Navigation and Control Systems Group of the Radio Science Laboratory, one of the Willow Run Laboratories of The University of Michigan's Institute of Science and Technology.

The originator's report number is 7657-12-P.

SUMMARY

This progress report covers the period from 1 April 1967 through 30 June 1967. A primary objective of this NASA contract is to examine the feasibility of various means of fixing aircraft and ship positions through the use of satellites. A major effort on this contract has been to generate a general position-error analysis suitable for examining satellite navigation systems through the use of digital computer models.

This report describes efforts during the current quarter regarding aircraft positional fixes using the VHF channels on the currently operational satellite ATS-1 and the soon to be launched satellite ATS-C. The digital computer model is being modified to include VHF propagation effects, and the model will be used to predict expected aircraft position fixes. In addition, equipment characteristics for use aboard aircraft are being delineated.

No effort was expended on the interferometer measurement schemes in this quarter.

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INTRODUCTION

This progress report, covering the period from 1 April 1967 through 30 June 1967, presents the results obtained from the work conducted under NASA Contract NAS4-54(10). A primary objective of this contract on satellite navigation studies is to examine the feasability of various means of fixing aircraft and ship positions through the use of satellites. To this end, the previous year's work generated a general position-error analysis suitable for examining hyperbolic and range position-fixing systems. Several systems were examined.

During the current quarter, the effort was shifted to the task of adapting the model to determine how well aircraft position fixes could be obtained when ranges are measured to two geostationary satellites. In particular, the Applications Technology Satellites (ATS), ATS-1 and ATS-C, both equipped with VHF transponders, were considered. Aircraft positions are to be determined from the satellite positions, range measurements to the satellites, and the aircraft altitude.

An experiment conducted with these satellites can take many forms in terms of equipment on the ground and in the aircraft. To minimize the time required to set up an experiment and to minimize cost, it would be desirable to use existing facilities and equipment to the greatest practical extent.

Any practical experiment will consist in either of two forms: one in which range measurements to the satellites are made aboard the aircraft and one in which range measurements are made at a ground station (or stations); the ranges measured are those from the aircraft to each satellite and from the ground station to each satellite.

Position-fix errors arise from imprecise knowledge of satellite positions, aircraft-altitude errors, and range-measurement errors. But for geostationary satellites, their range positions can be well predicted and any error due to satellite motion can be minimized. Also, previous calculations on the computer model have shown the effect of aircraft-altitude error on position fixes is small, that is to say, and altimeter error is well within the tolerable bounds allowable in determining position fix. The major effect then in calculating aircraft-position errors is caused by the range measurements between the satellites and the aircraft. One cannot, however, disregard the other errors including those mentioned above and any timing errors arising in satellite or aircraft transponders.

Several error sources exist in measuring range. Precise time measurements form the basis for accurate range measurements. Variations in transponder delays, whether on board satellites or aircraft and any deviations from straight lines in propagation paths will cause errors in range measurement. Propagation errors will be the most significant ones at VHF. Refraction of the radio waves, both in the ionosphere and atmosphere, will cause a propagation path to a satellite to lengthen, with the ionosphere causing the greater increase in path length.

Refraction effects are being modeled simply. If sufficient statistical data were available on the refractive index of the ionosphere on a diurnal, seasonal, and geographical basis, it would be possible to obtain a distribution of range errors for given times and for given penetration angles of the ionosphere by the radio paths.

A few calculations with the model have yielded values which indicate path lengthening of about 1000m. The lengthening can be attributed almost completely to the ionosphere, especially for the range measured to an aircraft flying at high altitudes.

Note that a model such as this can be made to reduce the magnitude of the range-measurement error. But in determining aircraft-position fixes, the same model cannot be used to determine and then to correct for errors in range measurements arising from propagation conditions. Consequently, one has the choice of specifying a set of conditions as those actually existing and then calculating corrections with respect to that set of conditions, or one can examine and use data on range-measurement errors obtained from an experiment. The former course is being pursued at present; efforts will be made to obtain the requisite information from the ranging experiments being made with the ATS-1. Also, suggestions will be made as to how the effects of propagation conditions might be minimized during the course of the position-fixing experiment.

The following section describes the propagation model as it presently is constituted.

TROPOSPHERIC AND IONOSPHERIC EFFECTS ON RADIO RANGE MEASUREMENTS

Errors in the measurement of distance made with radio equipment are caused in part by the refractive nature of the troposphere and ionosphere. The velocity of propagation of an electromagnetic wave is a function of the refractive index over the propagation path, and the path itself is distorted from a straight line by refractive bending of the wave front. Errors thus incurred in measuring range by assuming straight-line propagation at a velocity of light in a vacuum are called radio range errors. These errors are of considerable importance in determining the accuracy of navigational systems which employ measurements of range, range differences, range rate, or any combination of these qualities.

The actual value of the range error occurring over a particular propagation path is a function of the structure of the index of refraction at the particular time that the range measurement is made. The refractive-index structure is always to a great extent an unknown quantity; however, it has a spatial and a time variation. A refractive model that is usually employed is one in which the actual refractive index is approximated with a structure which is a function only of height above the surface of a spherical earth. With such a model, ray tracings can be performed and the average effects of the refractive-index range errors can be analyzed.

In the section that follows, an approximate method will be described to calculate the range error due to tropospheric and ionospheric refraction.

2.1. COMPUTATIONAL METHOD

The method presented here is based on that of Weisbrod and Anderson.* This method is particularly simply and can be applied to both tropospheric and ionospheric bending. The only assumptions are that the refractive-index gradient is only in the vertical plane, that the index of refraction can be approximated by a number of linear segments, and that the thickness of these steps is small compared to the earth's radius. In this method, there is no limitation on the shape of the profile or the angle of elevation. These assumptions are justifiable for the problem under consideration.

2.2. RANGE ERROR IN THE TROPOSPHERE

The signal retardation $d\tau$ caused by a layer of thickness $d\rho$ (fig. 1) is given by

$$d\tau = \left(\frac{1}{v} - \frac{1}{c}\right)\csc \beta \, d\rho$$

or

$$d\tau = (n-1) \frac{\csc \beta}{c} d\rho$$

where c and v are the signal velocities in free space and the medium, respectively, and $n = \frac{c}{v}$. The range error is given by

$$\Delta r_{jk} = \int_{\rho_j}^{\rho_k} c \, d\tau \quad \int_{\rho_j}^{\rho_k} (n-1)\csc \beta \, d\rho \tag{1}$$

---

*S. Weisbrod and L. J. Anderson, "Simple Methods for Computing Tropospheric and Ionospheric Refractive Effects on Radio Waves," Proc. I.R.E., Vol. 47, October 1959, pp. 1770-1777.

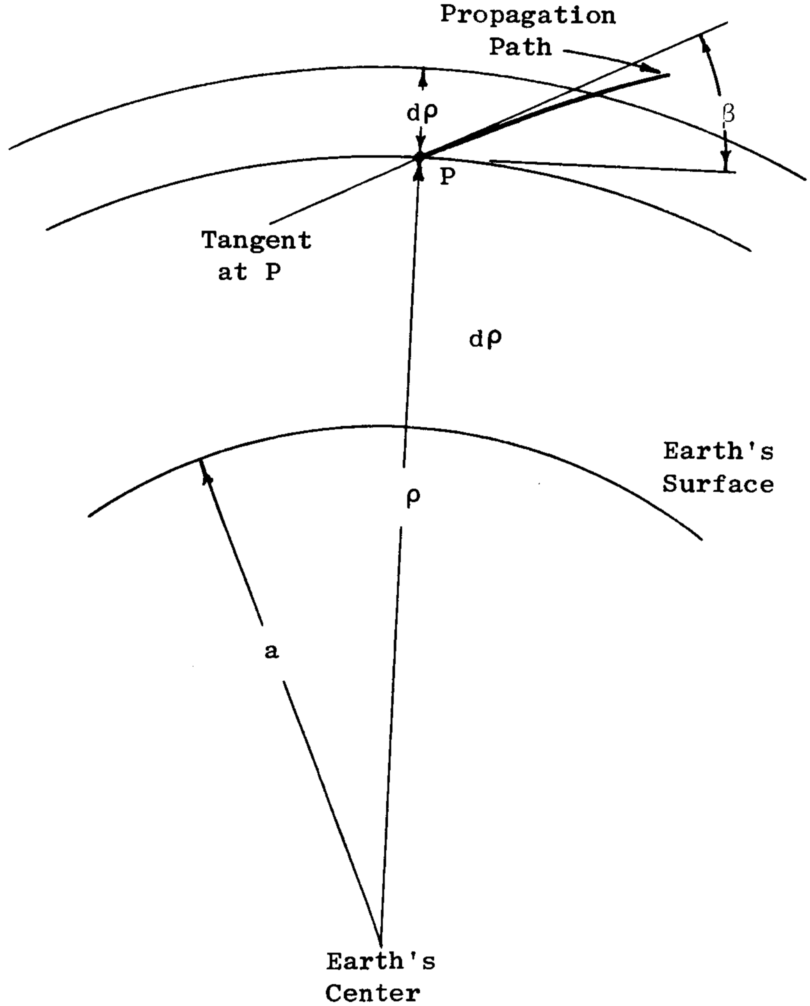

FIGURE 1. GEOMETRY OF REFRACTION BY AN INFINITESIMAL LAYER

By defining

$$N = (n-1) \times 10^6 \tag{2}$$

one can rewrite the above equation as

$$\Delta r_{jk} = \int_{\rho_j}^{\rho_k} \frac{N \times 10^{-6}}{\sin \beta} \, d\rho \tag{3}$$

This integral can be approximated by the expression

$$\Delta r_{jk} = \frac{(N_k + N_j)\ (\rho_k - \rho_j) \times 10^{-6}}{\sin \beta_k + \sin \beta_j} \tag{4}$$

Comparison of the exact solution with (4) indicates an error of a few hundredths of one percent for the case of the tropospheric propagation. In the ionosphere the errors are larger, but sufficiently small to justify the use of the approximate expression. The total error is given by

$$\Delta r = \sum_{k=o}^{n-1} \frac{(N_{k+1} + N_k)\ (\rho_{k+1} - \rho_k) \times 10^{-6}}{(\sin \beta_{k+1} + \sin \beta_k)}$$

If the ray departs from the earth's surface with an elevation angle α_o, Snell's law for spherical stratification is

$$n_o a \cos \alpha_o = n\rho \cos \beta = \text{constant}$$

where n_o = surface index of refraction

a = earth's radius

ρ = $a + h$

n = index of refraction at specified height

From Snell's law,

$$\sin \beta = \frac{n_o a}{n\rho} \left[\left(\frac{n\rho}{n_o a} \right)^2 - \cos^2 \alpha_o \right]^{1/2} \qquad (5)$$

2.3. RANGE ERROR IN THE IONOSPHERE

It can be shown that the index of refraction in the ionosphere, excluding the effect of the magnetic field and collisions, is given by

$$n = 1 - \frac{Ne\ e2}{m\epsilon o \omega^2}$$

or

$$n = 1 - b\ \frac{Ne}{\omega 2}$$

where $b = \frac{e2}{m\epsilon o}$

Ne = electron density per cubic meter

m = electron charge

ϵo = permittivity of free space

ω = angular frequency of the incident signal

To compute the ionospheric error, one can use equation 4, provided that the absolute value of N is used. Thus,

$$\Delta r = \sum_{k=o}^{n-1} \frac{\left(\left| N_{k+1} \right| + \left| N_k \right| \right) \left(\rho_{k+1} - \rho_k \right) \times 10^{-6}}{\left(\sin \beta_{k+1} + \sin \beta_k \right)} \qquad (6)$$

By combining the two errors, one obtains the total range error for oneway passage through the troposphere and the ionosphere.

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