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AN ACCURACY AND SPEED COMPARISON OF THE VINTI AND BROUWER ORBIT PREDICTION METHODS

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ABSTRACT

The Vinti orbit prediction method, developed as part of the tracking and prediction program for artificial earth satellites, was compared with the Brouwer orbit prediction method, where both methods considered the first four zonal harmonics of the earth's gravitational field. Several sets of inertial coordinates and velocities for both real and hypothetical satellites were used to produce one hundred revolutions of each orbit. Prediction accuracy was investigated by comparing the in-track position error committed by each of the two methods every ten revolutions. Supplementary information for this study was taken from comprehensive reports by Arsenault, Enright, and Purcell of the Aeronutronic Division of the Ford Motor Company, and A. G. Lubowe of the Bell Telephone Laboratories. In every case, it was found that the Vinti method produced considerably less in-track error, while operating at a computational speed comparable to that of the Brouwer method. It is seen that when one attempts to augment the Brouwer method so that it can carry accuracy of the order of the Vinti theory, then the computational speeds of the two methods are no longer comparable, but the Vinti program is estimated to be faster by at least a factor of eight. Simultaneously, the computer storage requirements for the Vinti program are estimated to be much less than those of the Brouwer program.

AN ACCURACY AND SPEED COMPARISON OF THE VINTI AND BROUWER ORBIT PREDICTION METHODS

INTRODUCTION

Vinti (Reference 1) has found a gravitational potential, which generalized by means of a metric-preserving transformation of the associated Cartesian system preserves separability of the problem of orbital motion when the potential coefficients $J_{2,1}$ and J_3 are taken into account. This solution accounts for all of the second and third zonal harmonics, and approximately seventy per cent of the fourth zonal harmonic. All secular terms are correct through order $J_2{}^3$, while the periodic terms are correct through order $J_2{}^2$. By the nature of this solution, both the secular and periodic terms may be calculated to any higher order of accuracy. This leads to a computing procedure for obtaining the position and velocity coordinates of a drag-free satellite from a knowledge of its initial conditions (Reference 2).

An accurate orbit prediction method is a necessary requirement for a high-quality satellite tracking procedure. The quality of the results actually obtained in the field with the complete tracking and prediction program might be considered adequate verification of the accuracy of the prediction program. However, to make realistic judgments about which procedures to use, one must know, as exactly as possible, the accuracy of each component of previous systems. Consequently, it is a matter of considerable importance to see how the Vinti method compares with the Brouwer system presently in use.

In two recent publications (References 3, 4), the authors tabulated all available literature on drag-free satellite orbit prediction. After a detailed numerical analysis, the accuracy of these prediction methods was compared against a highly accurate numerical integration scheme for different sets of initial conditions and the errors tabulated graphically. Included in this examination were the Brouwer method (Reference 5) and the Izsak method (Reference 6), the latter being a second-order solution to the Vinti potential problem, but with the coefficient of the earth's third zonal harmonic equal to zero. Therefore, we could compare the predictions of the Vinti orbit prediction method against the numerical integration scheme and then compare the errors against those of Brouwer and Izsak. As a further consistency check, a similar comparison was made between the Vinti and Brouwer methods using data obtained from the Relay-II and ANNA-1B satellites.

STATEMENT OF THE PROBLEM

The Brouwer method is a solution in canonical variables using the von Zeipel technique. This theory has singularities for circular or equatorial orbits and orbits at the "critical inclination." The periodic terms of this method are accurate to order J_2, while the secular terms are accurate to order J_2^2. The Izsak method possesses a singularity at the poles, but is accurate through order J_2^2 for both periodic and secular terms.

The first four runs by Lubowe (Reference 4) were selected, and the corresponding in-track position error is given by the expression

$$\underline{V} \cdot \Delta\underline{r} = \frac{(X_N - X_A)\dot{X}_N + (Y_N - Y_A)\dot{Y}_N + (Z_N - Z_A)\dot{Z}_N}{\sqrt{\dot{X}_N^2 + \dot{Y}_N^2 + \dot{Z}_N^2}},$$

where the subscripts N denote the numerical integration component and the subscripts A, those of the analytical orbit prediction method. The in-track position error, being a most revealing quantity for the evaluation of long-range prediction accuracy, was investigated for each of the four runs above, and then for the separate cases of the Relay-II and ANNA-1B satellites. The endpoint (100 revolutions) in-track error for each run and each method is tabulated in Table I, which also lists the initial conditions for each run. The growth of these

Table I

In-track Position Error (kilometers) with Ford-Aeronutronic Data.

Method	Run 1 (after 100 rev.)	Run 2 (after 100 rev.)	Run 3 (after 100 rev.)	Run 4 (after 100 rev.)
Vinti	5.62	2.40	1.21	3.20
Izsak	6.50	1.50	4.70	6.45
Brouwer	19.20	19.90	12.00	3.40

Initial conditions for Table I: Perigee height = 500 nautical miles, e = 0.1, $\Omega = \omega = M = 0$, for runs 1-4; i = 2°, 30°, 60°, and 88° for runs 1-4, respectively.

errors for successive revolutions is shown in Figs. I - IV, which also demonstrate the errors during selected revolutions. Values of the constants used in all cases were: $J_2 = 1082.42 \times 10^{-6}$, $J_3 = -2.5 \times 10^{-6}$, $J_4 = -1.85 \times 10^{-6}$, one earth equatorial radius (canonical unit of length) = 6378.150 kilometers, one canonical unit of time = 13.4470523 minutes, and $\mu = 1$ (earth radii)^3/(canonical unit of time)^2. With these values of J_2 and J_3, the Izsak method used a value of J_4 given by $J_4 = -J_2^2$, and the Vinti method used a value of J_4 given by the expression, $J_4 = -J_2^2 + (J_3^2/J_2)$.

DISCUSSION OF RESULTS

In general, the in-track position errors with the Vinti orbit prediction method are seen to be considerably less than with either the Izsak or Brouwer methods. In fact, except for the 88° inclination case, the in-track position errors for the Izsak method are considerably less than those of the Brouwer scheme, and the Izsak method uses a potential which does not contain the earth's third zonal harmonic. Considering the endpoint errors of Table I, the Brouwer errors are 1.06 to 9.92 times as great as those with the Vinti method. For the 2°, 60°, and 88° cases, the Izsak errors are 1.16 to 3.88 times as great as those of Vinti, while for the 30° case, the Vinti error is 1.6 times that of Izsak.

The results of comparing the Vinti and Brouwer methods for the Relay-II and ANNA-1B satellites are displayed in Tables II-A and II-B respectively, for every ten revolutions of the satellite. The in-track position errors are plotted in Figure V for Relay-II and in Figure VI for ANNA-1B. For Relay-II, the Vinti orbit prediction method gives an in-track position error of 0.0021 kilometer at the zeroth revolution, and only 0.75 kilometer after the one-hundredth revolution. The Brouwer method, on the other hand, shows an error of 0.0023 kilometer at the zeroth revolution, and an error of 13.48 kilometers at the one-hundredth revolution. This represents an ephemeris of thirteen and one-half days. Consequently, the in-track position error for the Brouwer method is nearly eighteen times greater than that of the Vinti method after one hundred revolutions of the Relay-II satellite. For the case of satellite ANNA-1B, the Vinti method gives an error of 0.0009 kilometer initially, and a 3.10-kilometer error after one hundred revolutions of the orbit. The Brouwer method produces an error of 0.0021 kilometer at the start and approximately 6.10 kilometers after one hundred revolutions. Here then, the in-track position error for Brouwer is nearly twice that of Vinti at the one-hundredth revolution of the ANNA-1B satellite. This represents an ephemeris of seven and one-half days.

It must be mentioned that in order to start the Brouwer method on an equal basis with the Vinti method, it was first necessary to apply an iterative procedure

Table II-A
In-track Position Error (kilometers) with Relay-II

Number of Revolutions →	0	10	20	30	40	50	60	70	80	90	100
Method ↓											
Vinti	0.0021	0.08	0.16	0.18	0.26	0.46	0.40	0.45	0.60	0.79	0.75
Brouwer	0.0023	1.57	3.06	4.59	6.15	7.26	8.71	10.16	11.65	12.14	13.48

Table II-B
In-track Position Error (kilometers) with ANNA-1B

Number of Revolutions →	0	10	20	30	40	50	60	70	80	90	100
Method ↓											
Vinti	-0.0009	-0.33	-0.64	-0.91	-1.25	-1.60	-1.93	-2.27	-2.62	-2.78	-3.10
Brouwer	0.0021	-0.68	-1.37	-1.75	-2.38	-3.01	-3.66	-4.34	-4.81	-5.45	-6.10

Initial conditions for Table II:

A. Relay-II (position and velocity vector components, in units of the earth's equatorial radius and equatorial radii per canonical unit of time): $x = +.86773200$, $y = +1.0052368$, $z = -.14256217$, $\dot{x} = -.54766917$, $\dot{y} = +.38465985$, $\dot{z} = -.69095995$. These correspond to an initial perigee height of 1298 statute miles, a period of 194.7 minutes, eccentricity $e = 0.23918$, inclination $i = 46.0°$, and semi-major axis $a = 1.7448$ (units of earth's equatorial radius).

B. ANNA-1B (position and velocity vector components, in units of the earth's equatorial radius and equatorial radii per canonical unit of time): $x = -.90991164$, $y = -.52800270$, $z = +.51832880$, $\dot{x} = +.088089779$, $\dot{y} = -.71307686$, $\dot{z} = -.58298870$. These correspond to an initial perigee height of 670 statute miles, a period of 107.8 minutes, eccentricity $e = 0.00622$, inclination $i = 50.1°$, and semi-major axis $a = 1.1764$ (units of earth's equatorial radius).

(Reference 7) using the Brouwer orbit generator routine with the osculating elements obtained from the given inertial coordinates. In this way, a mean set of orbital elements corresponding to the given inertial coordinates was found for Brouwer's satellite theory. As a check, these mean elements reproduced the given inertial coordinates and velocities at the initial time. This iterative procedure is not necessary for the Vinti theory, since the so-called mean elements are produced automatically during the factorization process leading to the separation of the Hamilton-Jacobi equation. On the other hand, if the Brouwer method attempts to use the osculating elements directly, then very serious errors are introduced. The results of such an application are shown in Tables III-A and III-B. The Brouwer orbit generator no longer reproduces the given inertial coordinates at the initial time, but gives a large in-track error as shown for Relay-II and ANNA-1B. This is significant for the following reason. If the given inertial insertion coordinates are not known accurately enough, then an attempt to find the mean set of elements by a least-squares fitting process over many revolutions of the orbit could conceivably fail. In addition to this initial inaccuracy, the osculating elements introduce further errors in the computed coordinates, which, if sufficiently far removed from the actual coordinates of the satellite, will prevent convergence of the least-squares process. Tables III-A and III-B show the rapid error growth for osculating elements. Although, as mentioned above, there exist routines for determining an equivalent mean set initially if needed, these same routines fail if the eccentricity is sufficiently small. It is not certain whether they will work for high eccentricities. In fact, the above-mentioned problem not only is a possibility, but can become quite real as in the case of the S-55 micrometeor-impact satellite, where the Brouwer method failed to obtain the orbit.

The results of this Vinti-Brouwer comparison study are consistent with those of the Ford-Aeronutronic and the Bell Laboratories studies, which show that for drag-free satellites under varying initial conditions, the Brouwer orbit prediction method is considerably less accurate than the Vinti method. Since this is a major component in the overall orbit tracking program, then, depending on the characteristics of the orbit, one may be forced to perform orbit corrections more frequently with the Brouwer method. This would entail the utilization of more computer time in the process.

Comparisons of actual time of computations were made using an IBM 7094 Model I electronic digital computer, with the Vinti method programmed in the FORTRAN II language and the Brouwer method in both FORTRAN II and the language known as MYSTIC. Ten-day ephemerides in minute intervals were generated by both methods using the Relay-II initial conditions. The Vinti method produced the 14,400 sets of Cartesian inertial position and velocity

Table III-A

In-track Position Error (kilometers) with Relay-II
Using Osculating Elements in the Brouwer Method*

Number of Revolutions	0	10	20	30	40	50	60	70	80	90	100
In-track Position Error	-9.58	906.06	1767.56	2626.46	3457.56	4007.94	4727.36	5412.79	6062.40	6258.20	6814.88

Table III-B

In-track Position Error (kilometers) with ANNA-1B
Using Osculating Elements in the Brouwer Method*

Number of Revolutions	0	10	20	30	40	50	60	70	80	90	100
In-track Position Error	-10.51	156.22	322.98	493.65	659.42	824.98	990.25	1155.18	1321.44	1484.56	1646.97

*These in-track position errors are not necessarily indicative of the performance of the Brouwer method of orbit prediction in actual practice, but instead reflect certain handicaps of the method under special circumstances, as indicated in the text under "Discussion of Results".

components in 425 seconds of computer operating time, while the Brouwer method in FORTRAN II produced this same output in 360 seconds of computer time. Both produced BCD tape containing the output data. The MYSTIC version of the Brouwer method took 19 minutes and 5 seconds to produce these same ephemerides. This gives approximately 2033 prediction points per minute of machine time for the Vinti method, 2400 points per minute for the Brouwer-FORTRAN II method, and about 755 points per minute for the Brouwer-MYSTIC method.

It must be emphasized that the Brouwer method is able to generate data at speeds comparable to the Vinti method, because it does not carry the accuracy of the Vinti program. Recall that the Brouwer theory (Reference 5) carries secular terms through order J_2^2 and periodic terms through order J_2 only, while the Vinti theory takes the periodic terms through order J_2^2 and secular terms through order J_2^3 . In order to obtain this increased accuracy, it is necessary to supplement the Brouwer theory with the results of Kozai (Reference 8). The latter develops expressions for the perturbations through second order in J_2 for periodic terms, and third order in J_2 for the secular terms, which is no small task. The equations are extremely lengthy, and, to the knowledge of the authors, have not been applied in practice to date. One might try to estimate the increase in time of computation for the Brouwer-Kozai combined program by counting the number of additional terms given by the Kozai treatment and multiplying this by the execution time on the IBM 7094 Model I computer. It is felt that the Brouwer-Kozai method would produce only approximately 300 points per minute of computer operation instead of 2400 as given for the Brouwer theory alone. However, there is an additional important difference aside from the speed of computation. In Kozai's solution, in order to simplify the calculation, it was necessary to combine the elements a, e, l, and g into expressions for the radius vector r, and the argument of latitude u. Because of this, one can determine only the inertial Cartesian position coordinates directly, and not the velocities. As is well known, both position and velocity coordinates are not only important but necessary in satellite astronomy work.

Both the Brouwer and Kozai methods suffer from the appearance of small divisors and singularities in certain of the elements. These problems arise for inclinations near the critical inclination, and for eccentricities and inclinations which are small. These latter two difficulties can be partially relieved by the adoption of other variables in the theoretical development (Reference 9) and by use of special additional programs in the computational process. Since the Vinti method is a solution by the separation of the Hamiltonian, it contains no such singularities or difficulties due to small divisors.

The storage requirements are somewhat more difficult to compare. On the IBM 7094 computer, the Vinti orbit prediction routine utilizes approximately 1000 core storage locations, while estimates are that the Brouwer routine uses approximately 2000 locations. If one were to supply higher-order accuracy to the Brouwer solution by adding the work of Kozai, then it is estimated that the storage requirements would increase to about 12,000 locations. This is an important consideration, since free locations can be utilized for other portions of the overall orbit prediction and tracking program.

As a final consistency check, an orbit differential correction was performed on a one-week arc of the Relay-II satellite using 50 observations (100 equations of condition covering the period of January 21-28, 1964) with the Vinti and Brouwer methods. Both programs used only the first four zonal harmonics, and a standard deviation of fit acceptance criterion for the observational residuals of seven sigma. The results were as follows. Vinti converged to a standard deviation of fit of 0.369 mills and Brouwer to 0.335 mills. Using the mean elements obtained for both theories from this differential correction, each method was allowed to predict ahead for two weeks (100 revolutions), and two comparisons were made. (1) The size of the residuals for the week outside the arc of correction was noted, and the number recorded. There were 335 observations in this week of prediction (670 residuals). The Vinti residuals were smaller (better agreement with observation) in 395 cases, the Brouwer residuals smaller in 259 cases, with 16 residuals approximately equal in magnitude. (2) The in-track position error for each method was computed every ten revolutions using a double-precision Cowell numerical integration, to which was fed the corresponding epoch Cartesian inertial coordinates of the mean elements obtained by differential correction. At the end of 100 revolutions (February 4, 1964), the Vinti error was 5.16 kilometers, and the Brouwer error was 22.64 kilometers. Extended to three weeks (155 revolutions), the Vinti error was 6.16 kilometers, and the Brouwer error was 28.07 kilometers. This accounts for the larger residuals produced by Brouwer in the first test above.*

Most important, the results of this report indicate that the Vinti method is a higher-quality orbit system component than the Brouwer method. Consequently, it would serve as a much better starting point or reference orbit for work in both satellite orbit prediction and tracking, and also scientific satellite geodesy.

*The data for this consistency check provide the basis for a forthcoming NASA technical report.

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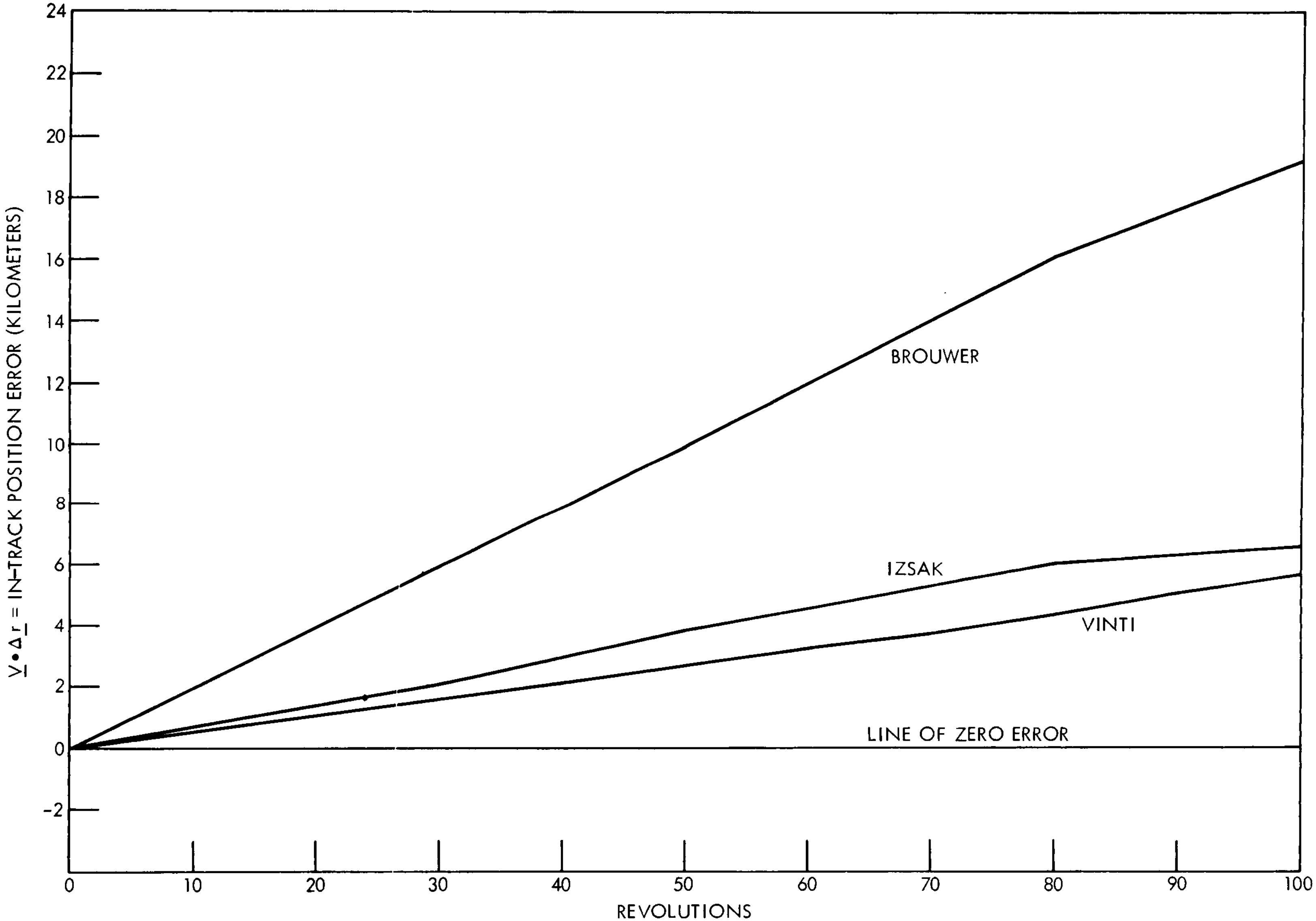

Figure 1 - In-track position error (e =0.1, i =2°, perigee height =500 nautical miles)

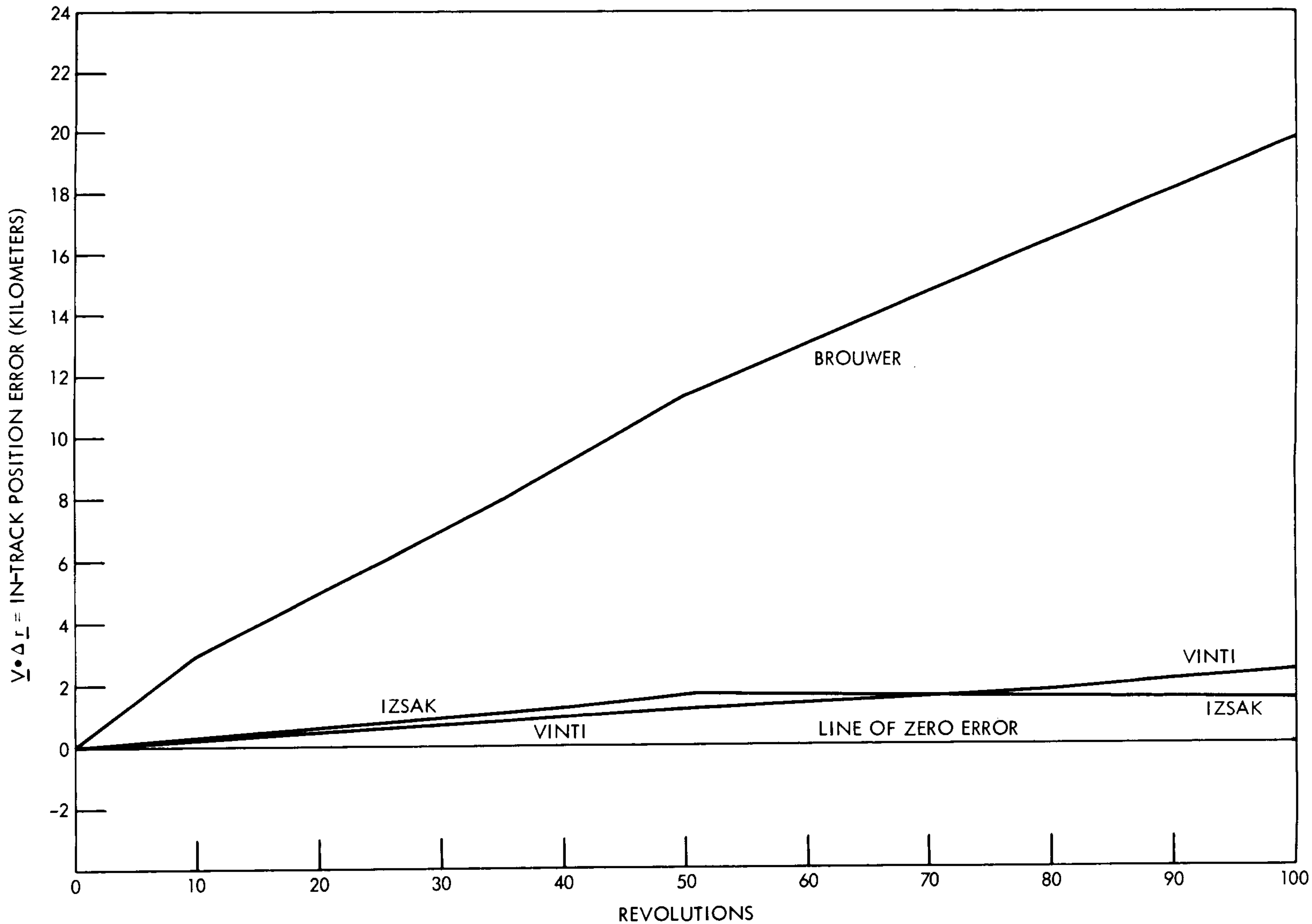

Figure 11 - In-track position error (e =0.1, i =30°, perigee height =500 nautical miles)

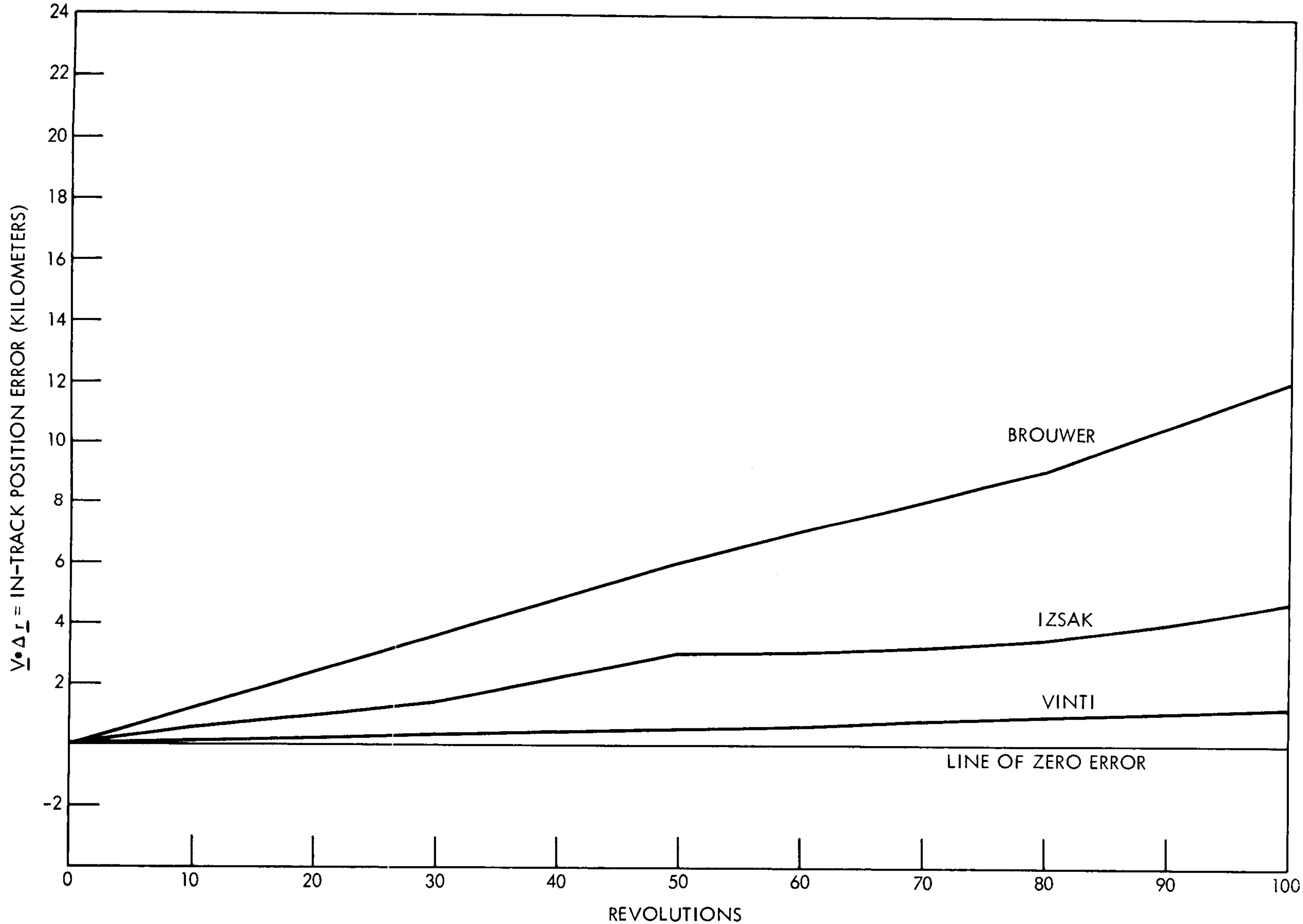

Figure III - In-track position error (e =0.1, i =60°, perigee height =500 nautical miles)

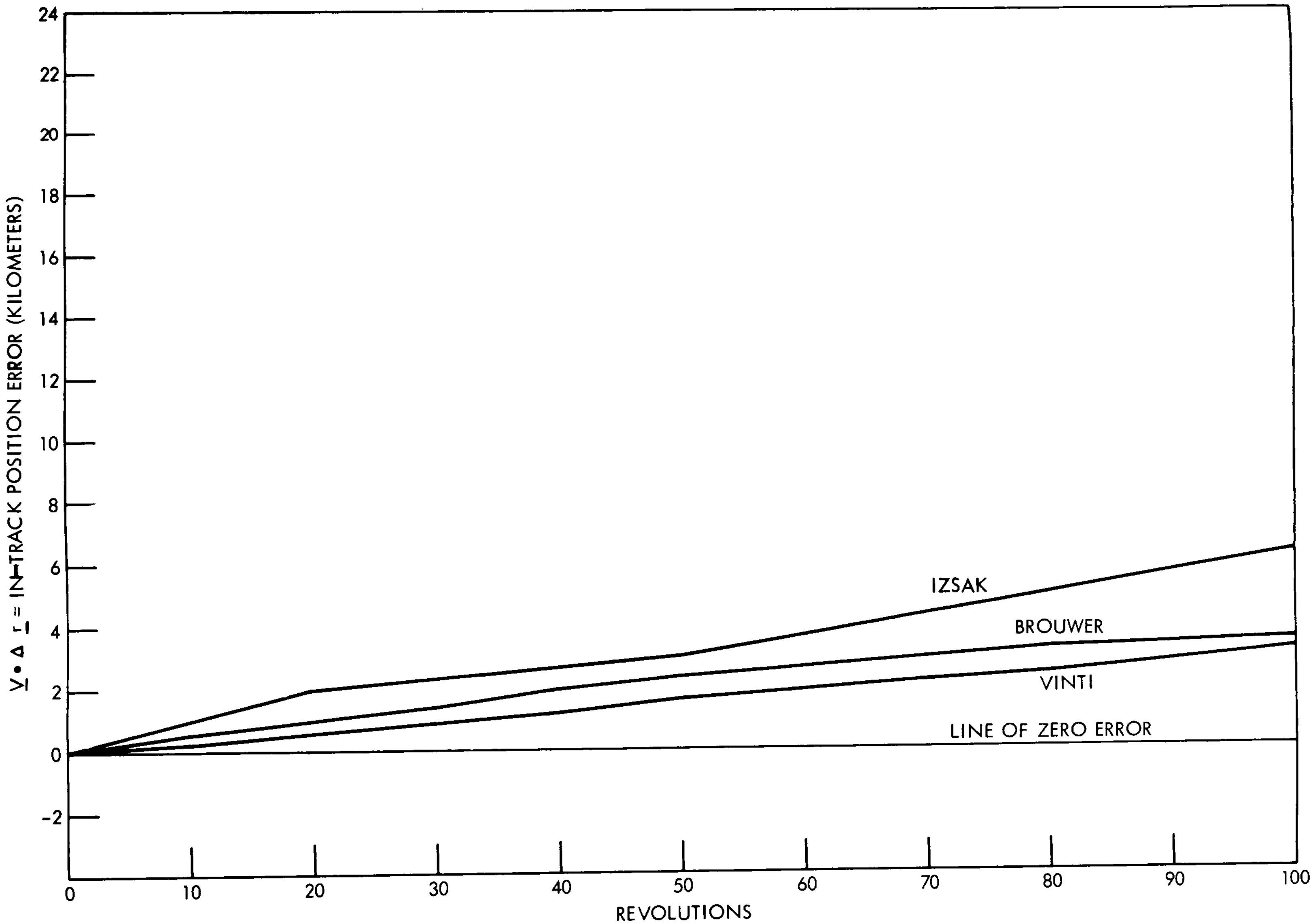

Figure IV - In-track position error (e =0.1, i =88°, perigee height = 500 nautical miles)

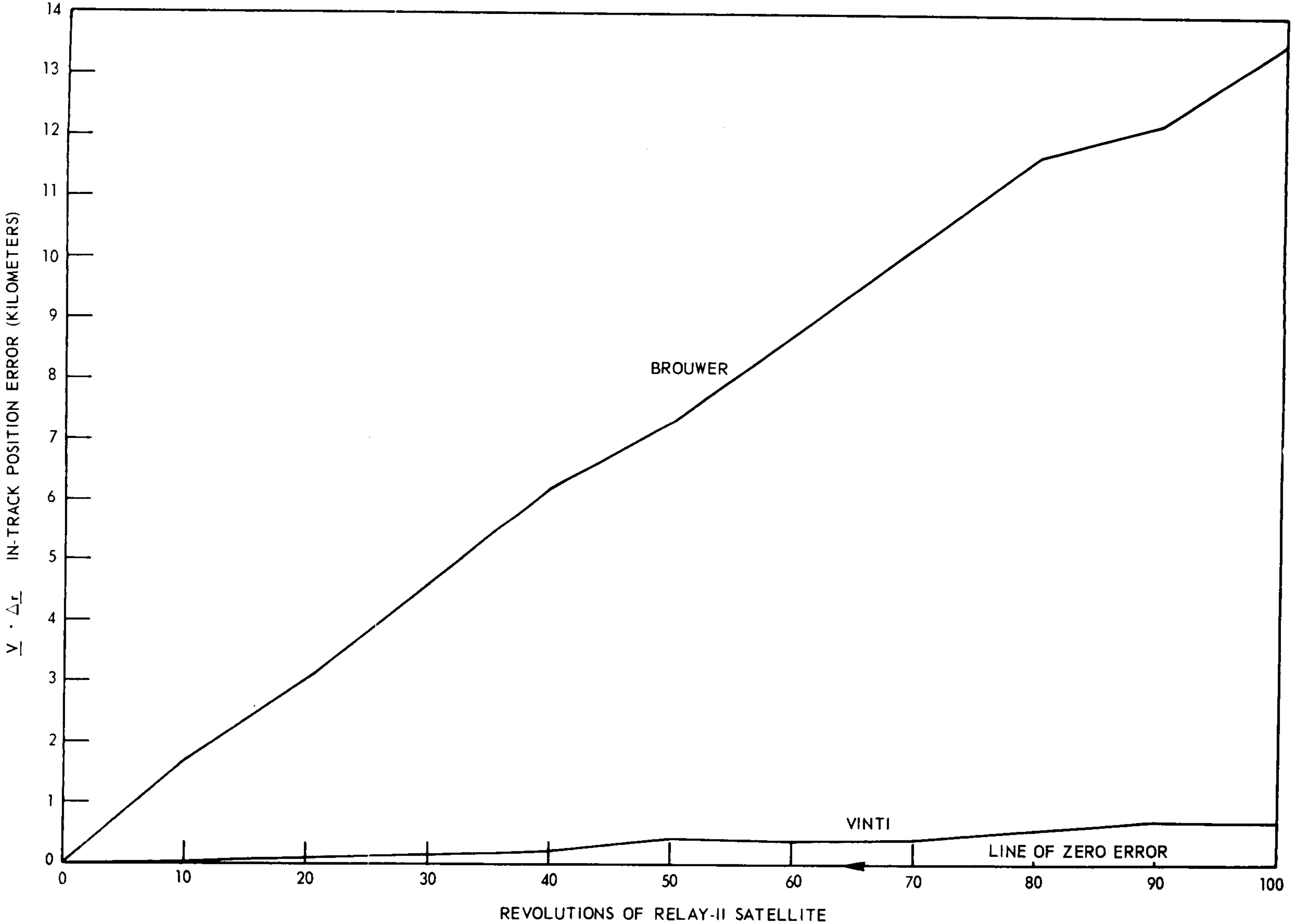

Figure V—In-track position error (e = 0.24, i = 46°, perigee height = 1128 nautical miles)

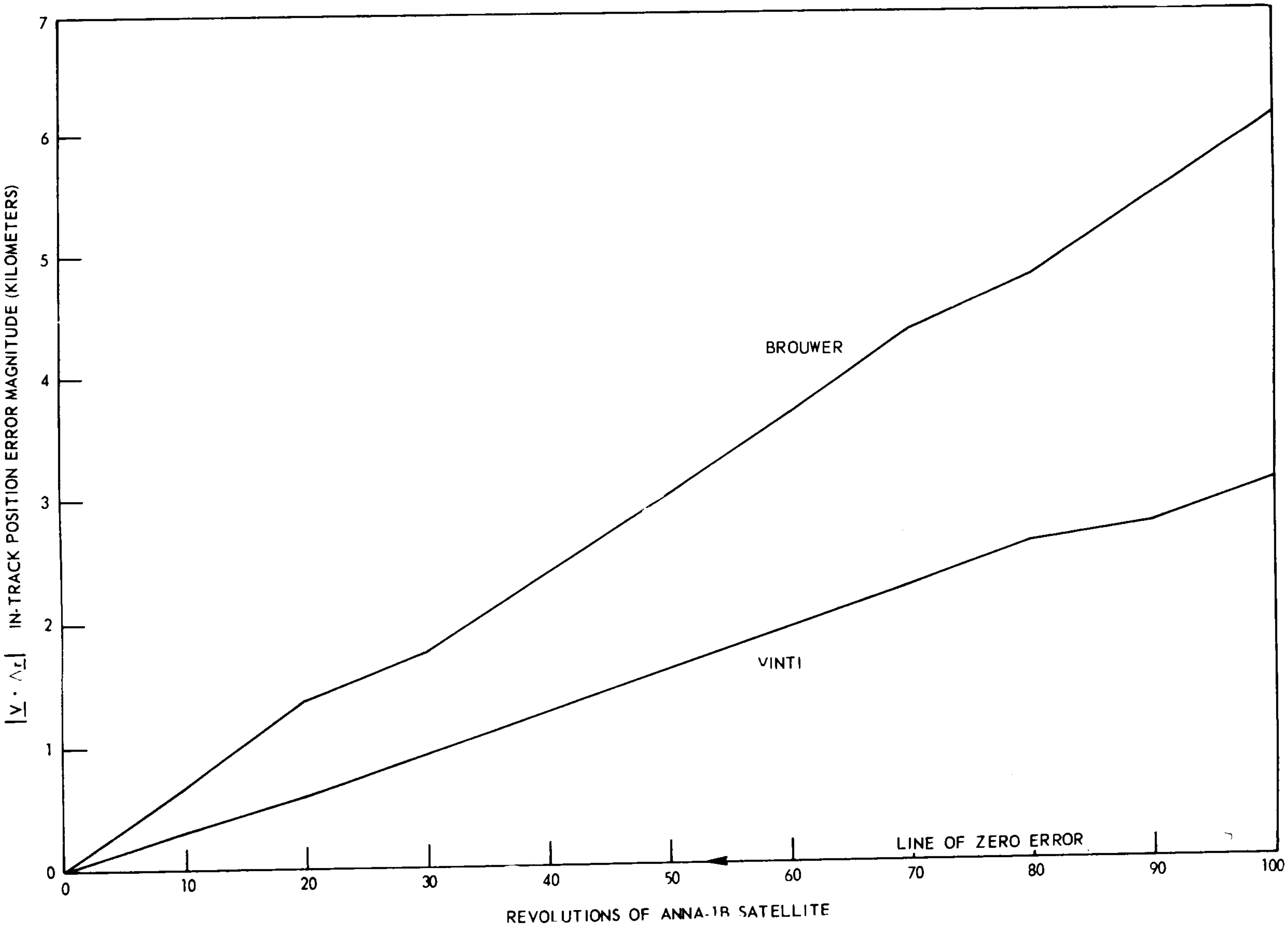

Figure VI—Magnitude of in-track position error ($e = 0.006$, $i = 50°$, perigee height = 582 nautical miles)