SPECTRUM OF PRESSURE FLUCTUATIONS
IN A TURBULENT FLOW

by
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1.   As is known, in an incompressible liquid the pressure is connected with the velocity by correlation

$$\Delta p = - \rho \frac{\partial u_\alpha}{\partial x_\beta} \frac{\partial u_\beta}{\partial x_\alpha} \quad , \tag{1}$$

where ρ is the density of the liquid and u_α the field of velocity; by twice repeating indexes a summation of from 1 to 3 is implied. During calculation of the spectrum or the structural function of pressure according to Equation (1), it is necessary to know the two-point correlative moment of the fourth order for the gradient of velocity. In Obukhov's treatise[1] the calculation of the structural function of pressure was conducted under the supposition that the fourth correlative moments of the field of velocity had been connected with the second, as during the Gaussian determination of probability. A similar supposition, which is designated the Hypothesis of Quasi-Normality, was first introduced by M. D. Millionshchikov[2]. In the reluctance interval of wave numbers $\left(L^{-1} \ll k \ll l_\nu^{-1} \equiv \langle \epsilon \rangle^{1/4} \nu^{-3/4} \right.$, where L is the outside scale, l the inner scale of turbulence[3], ν the kinematic viscosity, and $\langle \epsilon \rangle$ the mean value of the dissipation of energy$\left.\right)$ the spectrum density of the pressure, corresponding to the structural function calculated by A. M. Obukhov[1], has the form

$$\phi_p (k) \sim \rho^2 \langle \epsilon \rangle^{4/3} k^{-7/3} \quad . \tag{2}$$

An analogous result can be obtained by taking into consideration the dimensions, if it is assumed that in the reluctance interval of wave numbers, the spectrum of pressure (more exactly, the pressure shared at a constant density) is determined by only one measured parameter $\langle \epsilon \rangle$.

Doubts have recently arisen about both the correctness of the Hypothesis of Quasi-Normality and the applicability of the above indicated simple measurements at moments of a higher order[4,5]. The feature of the intermittency of the turbulent flow had been assumed[4,5] as a basis for calculating the higher correlations of moments of the turbulent field of velocity. In connection with these, a mathematical theory of random functions of new-type intermittent random functions is being developed. This paper discusses the calculation of the spectrum of pressure, based on analogous suppositions about the intermittency of the turbulent flow.

2. To define

$$a = \frac{1}{4\pi} \frac{\partial u_\alpha}{\partial x_\beta} \frac{\partial u_\beta}{\partial x_\alpha} , \qquad (3)$$

from Equation (1) we obtain

$$\phi_p(k) = (4\pi\rho)^2 k^{-4} \phi_q(k) . \qquad (4)$$

Magnitude q, which can be designated the density of the hydrodynamic charge (with corresponding potential p/ρ), is a quadratic structure derived from the various components of the tensor of the gradient of velocity. Let's propose that magnitude q has such a statistical nature that it can be expressed as* $\left(\partial u_3/\partial x_1\right)^2$. Concerning the magnitude in[4,5], it develops an intermittent random process; corresponding calculations of the spectra of the correlative moments of velocity of the fourth and sixth orders seemed to agree with the experimental data[6,7].

An analogous supposition relative to magnitude q leads to a spectrum of the following type

$$\phi_q(k) \sim \langle\epsilon\rangle^2 \nu^{-2} k^{-1+\mu} , \qquad 0 < \mu < 1. \qquad (5)$$

Finally we have

$$\phi_p(k) \sim \rho^2 \langle\epsilon\rangle^2 \nu^{-2} k^{-5+\mu} , \qquad 0 < \mu < 1. \qquad (6)$$

Whereupon in agreement with Gurvich and Zubkovskiy's work[6], $\mu \approx 0.4$.

There was a substantial difference between the spectra in Equations (2) and (6), both of which had been derived under different suppositions. Here the experiment is the deciding factor. In contrast to other authors[6,7] we note that upon determination of the spectrum of pressure the inevitable instrument neutralization occurs after the nonlinear operation, which generates the flow. In connection with this the immediate determinations of the spectrum of pressure in a turbulent flow present special interest.

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* Mathematically this is expressed by the requirement that the determined scalars of the spectral tensor of magnitude $\left(\partial u_i/\partial x_k\right) \left(\partial u_j/\partial x_l\right)$ had an identical wave number dependence in the reluctance interval.

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Calculation of the pressure spectrum in a turbulent flow, assuming the flow to be generated by a random intermittent function. The spectrum obtained on the basis of this assumption is found to differ greatly from a spectrum obtained previously on the basis of Millionshchikov's Hypothesis of Quasi-Normality.

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