AD

RSIC-565

THE REDUCTION OF THE SOLUTION OF A GAME OF PURSUIT FOR SURVIVAL PAYOFF TO THE SOLUTION OF A COUCHY PROBLEM FOR A FIRST ORDER PARTIAL DIFFERENTIAL EQUATION

by
L. A. Petrosyan

Doklady Akademii Nauk Armyanskoi SSR; 40, No. 4, 193-196 (1965)

Translated from the Russian by W. Urusky

June 1966

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

REDSTONE SCIENTIFIC INFORMATION CENTER
REDSTONE ARSENAL, ALABAMA

JOINTLY SUPPORTED BY

U.S. ARMY MISSILE COMMAND

GEORGE C. MARSHALL SPACE FLIGHT CENTER

DISCLAIMER

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other, authorized documents.

DISPOSITION

Destroy this report when it is no longer needed. Do not return it to the originator.

1 June 1966 RSIC-565

THE REDUCTION OF THE SOLUTION OF A GAME OF PURSUIT FOR SURVIVAL PAYOFF TO THE SOLUTION OF A COUCHY PROBLEM FOR A FIRST ORDER PARTIAL DIFFERENTIAL EQUATION

by
L. A. Petrosyan

Doklady Akademii Nauk Armyanskoi, SSR; 40, No. 4, 193-196 (1965)

Translated from the Russian

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Translation Branch
Redstone Scientific Information Center
Research and Development Directorate
U. S. Army Missile Command
Redstone Arsenal, Alabama 35809

Assuming that $T = [0, \infty)$. Let's examine the vector-functions set

$$\left\{\varphi\ (x_1,\ x_2,\ x_3,\ x_4,\ t) = \left[\varphi_1\ (x_1,\ x_2,\ x_3, x_4,\ t),\ \varphi_2\ (x_1,\ x_2,\ x_3,\ x_4,\ t)\right]\right\},$$

given for $R^4 \times T$ with values in R^2, and the vector-functions set

$$\left\{\psi\ (x_1,\ x_2,\ x_3,\ x_4,\ t) = \left[\psi_1\ (x_1,\ x_2,\ x_3,\ x_4,\ t),\ \psi_2\ (x_1,\ x_2,\ x_2,\ x_4,\ t)\right]\right\},$$

given for $R^4 \times T$ with values in R^2, satisfying the following conditions:

1. For any $\psi \in \{\psi\}$ and $\varphi \in \{\varphi\}$ system of equations

$$\begin{aligned}\dot{x}_1 &= \varphi_1\ (x_1,\ x_2,\ x_3,\ x_4,\ t),\\ \dot{x}_2 &= \varphi_2\ (x_1,\ x_2,\ x_3,\ x_4,\ t),\\ \dot{x}_3 &= \varphi_3\ (x_1,\ x_2,\ x_3,\ x_4,\ t),\\ \dot{x}_4 &= \varphi_4\ (x_1,\ x_2,\ x_3,\ x_4,\ t)\end{aligned} \qquad (1)$$

has a unique solution at any initial conditions

$$\xi = \left(\xi_1,\ \xi_2\right) \text{ and } \eta = \left(\eta_1,\ \eta_2\right).$$

2. $$\varphi_1^2\ (x_1,\ x_2,\ x_3,\ x_4,\ t) + \varphi_2^2\ (x_1,\ x_2,\ x_3,\ x_4,\ t) = \upsilon^2\ (x_1,\ x_2)$$

$$\psi_1^2\ (x_1,\ x_2,\ x_3,\ x_4,\ t) + \psi_2^2\ (x_1,\ x_2,\ x_3,\ x_4,\ t) = u^2\ (x_3,\ x_4,),$$

where $\upsilon\ (x_1,\ x_2)$ and $u\ (x_3,\ x_4)$ are certain given strictly positive functions. Sets $\{\psi\}$ and $\{\varphi\}$, satisfying 1-2, we will designate as E and Π correspondingly.

For any ξ, η we will determine the differential game for survival in a normal form, which we will designate conditionally as $G\left(\xi,\ \eta\right)$.

Game $G\left(\xi,\ \eta\right)$ represents an antagonistic game of two faces $\overline{P}$ and $\overline{E}$. The vector-function sets Π and E represent sets of strategies of games $\overline{P}$ and $\overline{E}$.

Each situation $(\varphi,\ \psi)$ under the initial conditions ξ, η unambigously corresponds to a specific solution of Equations (1), called the game party, which we will designate as $x\ (t)$.

In R^4 is given a certain 3-dimensional manifold M

$$x_1(t_1, t_2, t_3),\ x_2(t_1, t_2, t_3),\ x_3(t_1, t_2, t_3),\ x_4(t_1, t_2, t_3).$$

It is assigned a specific, sufficiently smooth, real function b (x) limited from below.

The function of winning in each situation (φ, ψ) is determined as follows: let x(t) be the party in situation (φ, ψ) and let

$$t_0 = \inf\left\{t : x(t) \in M\right\} \text{ and } t_0 < \infty,$$

then $K\left(\xi, \eta, \varphi, \psi\right) = b(x(t_0))$. If there is no point t, namely $x(t) \in M$ in the situation (φ, ψ), then

$$K\left(\xi, \eta, \varphi, \psi\right) = \alpha, \text{ where } \alpha < \inf_{x \in M} b(x).$$

Scarf[1] first investigated this type of game without limiting the games to two strategies per set.

Let us assume that for any initial position x there is a game value in pure strategies and that it is a constantly differentiated function of the initial position.

Lemma 1. If in the game $G\left(\xi, \eta\right)$ is a situation of equilibrium in pure strategies and the function V (x) representing the game value for survival with the initial position x, which is constantly differentiated, then it satisfies the experimental-differential equation

$$\max_{\varphi_1 \varphi_2} \min_{\psi_1 \psi_2} \left[\frac{\partial V}{\partial x_1}\varphi_1 + \frac{\partial V}{\partial x_2}\varphi_2 + \frac{\partial V}{\partial x_3}\psi_1 + \frac{\partial V}{\partial x_4}\psi_2\right] = 0, \tag{2}$$

at the boundary condition

$$V(x) = b(x), \text{ for } x \in M.$$

Using (2) and condition 2 for the strategy it is possible, utilizing the principle of indefinite factors, to reduce our extreme-differential equation to a differential equation in partial derivatives of the first order.

[1]G. E. Scarf. ON DIFFERENTIAL GAMES WITH SURVIVAL PAY-OFF. Ann. of Math. Studies No. 39, Princeton, 1957.

For calculation

$$\max_{\varphi_1\ \varphi_2} \left[\frac{\partial V}{\partial x_1} \varphi_1 + \frac{\partial V}{\partial x_2} \varphi_2 + \frac{\partial V}{\partial x_3} \psi_1^{*} + \frac{\partial V}{\partial x_4} \psi_2^{*} \right]$$

under conditions

$$\varphi_1^2 + \varphi_2^2 = v^2 (x_1,\ x_2)$$

and

$$\min_{\psi_1\ \psi_2} \left[\frac{\partial V}{\partial x_1} \varphi_1^{*} + \frac{\partial V}{\partial x_2} \varphi_2^{*} + \frac{\partial V}{\partial x_3} \psi_1 + \frac{\partial V}{\partial x_4} \psi_2 \right]$$

under condition

$$\psi_1^2 + \psi_2^2 = u^2 (x_3,\ x_4),$$

we use the Lagrange principle of indefinite factors according to which the extreme point should satisfy system

$$\begin{aligned} &\frac{\partial V}{\partial x_1} + \lambda_P 2\varphi_1^{*} = 0, \qquad && \frac{\partial V}{\partial x_3} + \lambda_E 2\psi_1^{*} = 0 \\ &\frac{\partial V}{\partial x_2} + \lambda_P 2\varphi_2^{*} = 0, \qquad && \frac{\partial V}{\partial x_4} + \lambda_E 2\psi_2^{*} = 0. \end{aligned} \tag{3}$$

From it, after excluding λ_P, λ_E, φ_2^{*}, ψ_2^{*}, we obtain

$$\varphi_1^{*} \left[\frac{\partial V}{\partial x_1} + \frac{\partial V}{\partial x_2} \cdot \frac{\partial V}{\partial x_2} \bigg/ \frac{\partial V}{\partial x_1} \right] +$$

$$+ \psi_1^{*} \left[\frac{\partial V}{\partial x_3} + \frac{\partial V}{\partial x_4} \cdot \frac{\partial V}{\partial x_4} \bigg/ \frac{\partial V}{\partial x_3} \right] = 0.$$

From 2 we will immediately obtain also the expression for optimal strategies

$$\begin{aligned} \varphi_1^{*} &= v\ (x_1,\ x_2) \cdot \frac{\partial V}{\partial x_1} \bigg/ \sqrt{\left(\frac{\partial V}{\partial x_1}\right)^2 + \left(\frac{\partial V}{\partial x_2}\right)^2}\ , \\ \psi_1^{*} &= u\ (x_3,\ x_4) \cdot \frac{\partial V}{\partial x_3} \bigg/ \sqrt{\left(\frac{\partial V}{\partial x_3}\right)^2 + \left(\frac{\partial V}{\partial x_4}\right)^2}\ . \end{aligned} \tag{4}$$

Here, for φ_1^*, the root sign is assumed to be equal to symbol $\frac{\partial V}{\partial x_1}$, and for ψ_1^* the root sign is assumed to be opposed to $\frac{\partial V}{\partial x_3}$.

If $\frac{\partial V}{\partial x_1} \cdot \frac{\partial V}{\partial x_3} \neq 0$, then

we obtain

$$\frac{\left(\frac{\partial V}{\partial x_1}\right)^2 + \left(\frac{\partial V}{\partial x_2}\right)^2}{\left(\frac{\partial V}{\partial x_3}\right)^2 + \left(\frac{\partial V}{\partial x_4}\right)^2} = \frac{u^2(x_3, x_4)}{\nu^2(x_1, x_2)} . \tag{5}$$

We come to the conclusion that the game function value V (x) should be the solution of the Couchy problem for the Equation (5) at the boundary condition $V(x)\big|_M = b(x)$.

If u and v are interpreted as velocities of games $\overline{E}$ and $\overline{P}$, then the obtained equation has an interesting theoretical-game sense.

"The greater the velocity of the game, the less the winning function at its deviation from the optimal strategy is subjected to change".

It can be shown that the solution of the Couchy problem for Equation (5) appears to be a sufficient condition of the existence of a constantly differentiated value of the game. This yields the following characteristic theorem:

Theorem. In order for the game $G(\xi, \eta)$ to have a constantly differentiated value in pure strategies, it is necessary and sufficient that the Couchy problem for the equation

$$\frac{\left(\frac{\partial V}{\partial x_1}\right)^2 + \left(\frac{\partial V}{\partial x_2}\right)^2}{\left(\frac{\partial V}{\partial x_3}\right)^2 + \left(\frac{\partial V}{\partial x_4}\right)^2} = \frac{u^2(x_3, x_4)}{\nu^2(x_1, x_2)}$$

at the boundary condition

$V(x) = b(x)$ for $x \in M$

have the solution.

The solution of the Couchy problem for a general, quasilinear equation in partial derivatives of the first order can be found by the Couchy method[2]. Here are given specific conditions for the manifold M and functions ν (x), u (x), and b (x) where this solution exists. Nevertheless, in numerous concrete problems of pursuit for survival the existence and a constant differentiation of the game function value come from theoretical-game calculations. Then, according to the theorem, a solution of the Couchy problem for the Equation (5) exists and is used for locating the game value V (x) and the optimal strategies for games $\overline{P}$ and $\overline{E}$.

[2]R. Curant. EQUATIONS WITH PARTIAL DERIVATIVES, M., 1964.

DISTRIBUTION

	No. of Copies
EXTERNAL	
Air University Library ATTN: AUL3T Maxwell Air Force Base, Alabama 36112	1
U. S. Army Electronics Proving Ground ATTN: Technical Library Fort Huachuca, Arizona	1
U. S. Naval Ordnance Test Station ATTN: Technical Library, Code 753 China Lake, California 93555	1
U. S. Naval Ordnance Laboratory ATTN: Library Corona, California 91720	1
Lawrence Radiation Laboratory ATTN: Technical Information Division P. O. Box 808 Livermore, California	1
Sandia Corporation ATTN: Technical Library P. O. Box 969 Livermore, California 94551	1
U. S. Naval Postgraduate School ATTN: Library Monterey, California 93940	1
Electronic Warfare Laboratory, USAECOM Post Office Box 205 Mountain View, California 94042	1
Jet Propulsion Laboratory ATTN: Library (TDS) 4800 Oak Grove Drive Pasadena, California 91103	2
U. S. Naval Missile Center ATTN: Technical Library, Code N3022 Point Mugu, California	1
U. S. Army Air Defense Command ATTN: ADSX Ent Air Force Base, Colorado 80912	1
Central Intelligence Agency ATTN: OCR/DD-Standard Distribution Washington, D. C. 20505	4
Harry Diamond Laboratories ATTN: Library Washington, D. C. 20438	1
Scientific & Tech. Information Div., NASA ATTN: ATS Washington, D. C. 20546	1

	No. of Copies
U. S. Atomic Energy Commission ATTN: Reports Library, Room G-017 Washington, D. C. 20545	1
U. S. Naval Research Laboratory ATTN: Code 2027 Washington, D. C. 20390	1
Weapons Systems Evaluation Group Washington, D. C. 20305	1
John F. Kennedy Space Center, NASA ATTN: KSC Library, Documents Section Kennedy Space Center, Florida 32899	2
APGC (PGBPS-12) Eglin Air Force Base, Florida 32542	1
U. S. Army CDC Infantry Agency Fort Benning, Georgia 31905	1
Argonne National Laboratory ATTN: Report Section 9700 South Cass Avenue Argonne, Illinois 60440	1
U. S. Army Weapons Command ATTN: AMSWE-RDR Rock Island, Illinois 61201	1
Rock Island Arsenal ATTN: SWERI-RDI Rock Island, Illinois 61201	1
U. S. Army Cmd. & General Staff College ATTN: Acquisitions, Library Division Fort Leavenworth, Kansas 66027	1
Combined Arms Group, USACDC ATTN: Op. Res., P and P Div. Fort Leavenworth, Kansas 66027	1
U. S. Army CDC Armor Agency Fort Knox, Kentucky 40121	1
Michoud Assembly Facility, NASA ATTN: Library, I-MICH-OSD P. O. Box 29300 New Orleans, Louisiana 70129	1
Aberdeen Proving Ground ATTN: Technical Library, Bldg. 313 Aberdeen Proving Ground, Maryland 21005	1
NASA Sci. & Tech. Information Facility ATTN: Acquisitions Branch (S-AK/DL) P. O. Box 33 College Park, Maryland 20740	5
U. S. Army Edgewood Arsenal ATTN: Librarian, Tech. Info. Div. Edgewood Arsenal, Maryland 21010	1

	No. of Copies
National Security Agency ATTN: C3/TDL Fort Meade, Maryland 20755	1
Goddard Space Flight Center, NASA ATTN: Library, Documents Section Greenbelt, Maryland 20771	1
U. S. Naval Propellant Plant ATTN: Technical Library Indian Head, Maryland 20640	1
U. S. Naval Ordnance Laboratory ATTN: Librarian, Eva Liberman Silver Spring, Maryland 20910	1
Air Force Cambridge Research Labs. L. G. Hanscom Field ATTN: CRMXLR/Stop 29 Bedford, Massachusetts 01730	1
Springfield Armory ATTN: SWESP-RE Springfield, Massachusetts 01101	1
U. S. Army Materials Research Agency ATTN: AMXMR-ATL Watertown, Massachusetts 02172	1
Strategic Air Command (OAI) Offutt Air Force Base, Nebraska 68113	1
Picatinny Arsenal, USAMUCOM ATTN: SMUPA-VA6 Dover, New Jersey 07801	1
U. S. Army Electronics Command ATTN: AMSEL-CB Fort Monmouth, New Jersey 07703	1
Sandia Corporation ATTN: Technical Library P. O. Box 5800 Albuquerque, New Mexico 87115	1
ORA(RRRT) Holloman Air Force Base, New Mexico 88330	1
Los Alamos Scientific Laboratory ATTN: Report Library P. O. Box 1663 Los Alamos, New Mexico 87544	1
White Sands Missile Range ATTN: Technical Library White Sands, New Mexico 88002	1
Rome Air Development Center (EMLAL-1) ATTN: Documents Library Griffiss Air Force Base, New York 13440	1

	No. of Copies
Brookhaven National Laboratory Technical Information Division ATTN: Classified Documents Group Upton, Long Island, New York	1
Watervliet Arsenal ATTN: SWEWV-RD Watervliet, New York 12189	1
U. S. Army Research Office (ARO-D) ATTN: CRD-AA-IP Box CM, Duke Station Durham, North Carolina	1
Lewis Research Center, NASA ATTN: Library 21000 Brookpark Road Cleveland, Ohio 44135	1
Systems Engineering Group (RTD) ATTN: SEPIR Wright-Patterson Air Force Base, Ohio 45433	1
U. S. Army Artillery & Missile School ATTN: Guided Missile Department Fort Sill, Oklahoma 73503	1
U. S. Army CDC Artillery Agency ATTN: Library Fort Sill, Oklahoma 73504	1
U. S. Army War College ATTN: Library Carlisle Barracks, Pennsylvania 17013	1
U. S. Naval Air Development Center ATTN: Technical Library Johnsville, Warminster, Pennsylvania 18974	1
Frankford Arsenal ATTN: C-2500-Library Philadelphia, Pennsylvania 19137	1
Div. of Technical Information Ext., USAEC P. O. Box 62 Oak Ridge, Tennessee	1
Oak Ridge National Laboratory ATTN: Central Files P. O. Box X Oak Ridge, Tennessee	1
Air Defense Agency, USACDC ATTN: Library Fort Bliss, Texas 79916	1
U. S. Army Air Defense School ATTN: AKBAAS-DR-R Fort Bliss, Texas 79906	1

	No. of Copies
U. S. Army CDC Nuclear Group Fort Bliss, Texas 79916	1
Manned Spacecraft Center, NASA ATTN: Technical Library, Code BM6 Houston, Texas 77058	1
Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
U. S. Army Research Office ATTN: STINFO Division 3045 Columbia Pike Arlington, Virginia 22204	1
U. S. Naval Weapons Laboratory ATTN: Technical Library Dahlgren, Virginia 22448	1
U. S. Army Engineer Res. & Dev. Labs. ATTN: Scientific & Technical Info. Br. Fort Belvoir, Virginia 22060	2
Langley Research Center, NASA ATTN: Library, MS-185 Hampton, Virginia 23365	1
Research Analysis Corporation ATTN: Library McLean, Virginia 22101	1
U. S. Army Tank Automotive Center ATTN: SMOTA-RTS.1 Warren, Michigan 48090	1
Hughes Aircraft Company Electronic Properties Information Center Florence Ave. & Teale St. Culver City, California	1
Foreign Technology Division ATTN: Library Wright-Patterson Air Force Base, Ohio 45400	1
Clearinghouse for Federal Scientific and Technical Information U. S. Department of Commerce Springfield, Virginia 22151	1
Foreign Science & Technology Center, USAMC ATTN: Mr. Shapiro Washington, D. C. 20315	3

	No. of Copies
INTERNAL	
Headquarters U. S. Army Missile Command Redstone Arsenal, Alabama	
ATTN: AMSMI-D	1
AMSMI-XE, Mr. Lowers	1
AMSMI-XS, Dr. Carter	1
AMSMI-Y	1
AMSMI-R, Mr. McDaniel	1
AMSMI-RAP	1
AMSMI-RBLD	10
USACDC-LnO	1
AMSMI-RBT	8
AMSMI-RB, Mr. Croxton	1
National Aeronautics & Space Administration Marshall Space Flight Center Huntsville, Alabama	
ATTN: MS-T, Mr. Wiggins	5
R-AERO-G, Mr. Carter	1

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Redstone Scientific Information Center Research and Development Directorate U. S. Army Missile Command, Redstone Arsenal, Alabama 35809	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP N/A

3. REPORT TITLE
THE REDUCTION OF THE SOLUTION OF A GAME OF PURSUIT FOR SURVIVAL PAYOFF TO THE SOLUTION OF A COUCHY PROBLEM FOR A FIRST ORDER PARTIAL DIFFERENTIAL EQUATION
Doklady Akademii Nauk Armyanskoi SSR: 40, No. 4, 193-196 (1965)

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Translated from the Russian by W. Urusky

5. AUTHOR(S) (Last name, first name, initial)

Petrosyan, L. A.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
1 June 1966	8	2

8a. CONTRACT OR GRANT NO. N/A b. PROJECT NO. N/A c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) RSIC-565 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) AD ____________

10. AVAILABILITY/LIMITATION NOTICES

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
None	Same as No. 1

13. ABSTRACT

The solution of the Couchy problem for a general, quasilinear equation in partial derivatives of the first order can be found by the Couchy method. Given are the specific conditions for the manifold M and functions ν (x), u (x), and b (x) where this solution exists. In numerous concrete problems of pursuit for survival the existence and a constant differentiation of the game function value come from theoretical-game calculations. According to the theorem, a solution of the Couchy problem for Equation (5) exists and is used for locating the game value V (x) and the optimal strategies for games $\overline{\text{P}}$ and $\overline{\text{E}}$.

DD FORM 1 JAN 64 1473

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Couchy problem						
Game of pursuit						
Indefinite factors						
Lagrange principle						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through
_______________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through
_______________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through
_______________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.