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MATHEMATICAL RELATIONSHIPS

OF THE MFOD ANTENNA AXES

Prepared by I. M. Salzberg

I. M. Salzberg
Head, Advanced Techniques Office
Manned Flight Engineering Branch

Released by Dale W. Call

D. Call
Head, Manned Flight Engineering Branch

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1. INTRODUCTION

There are three different primary coordinate systems used for tracking equipment in this country: Azimuth-Elevation (figure 1); X-Y (figures 2 and 3); Hour Angle-Declination (figure 4). To complete a space fix, the third orthogonal coordinate is slant range when used.

Each of these coordinate systems have particular advantages and disadvantages when used for tracking orbital and translunar vehicles. For convenience these have been tabulated in table 1.

At many of the remote Manned Space Flight Network Tracking Sites, equipment utilizing two different coordinate systems has been installed in relatively close proximity. To use pointing data in one coordinate system on equipment utilizing another coordinate system requires a transformation of the data from one coordinate system to the other. The purpose of this report is to develop all of the equations required to interrelate position data of the three above mentioned coordinate systems.

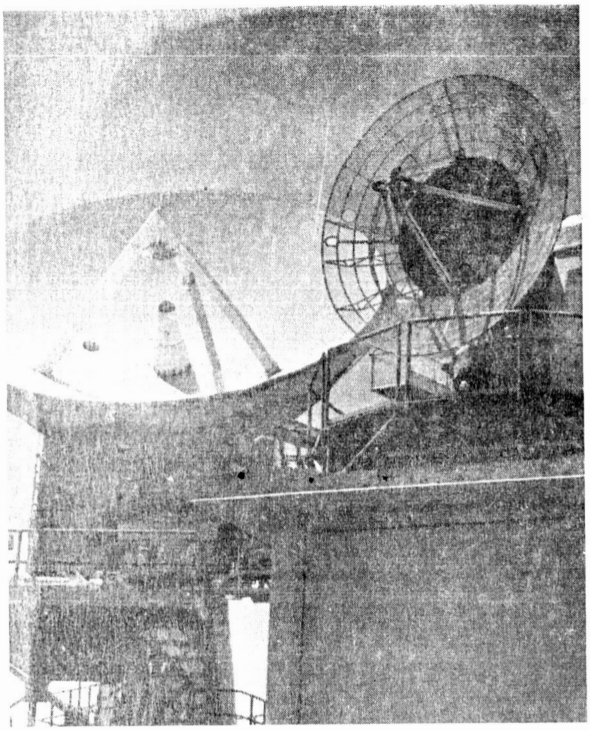

Figure 1. BDA FPS-16 and FPQ-6 Azimuth - Elevation Antenna

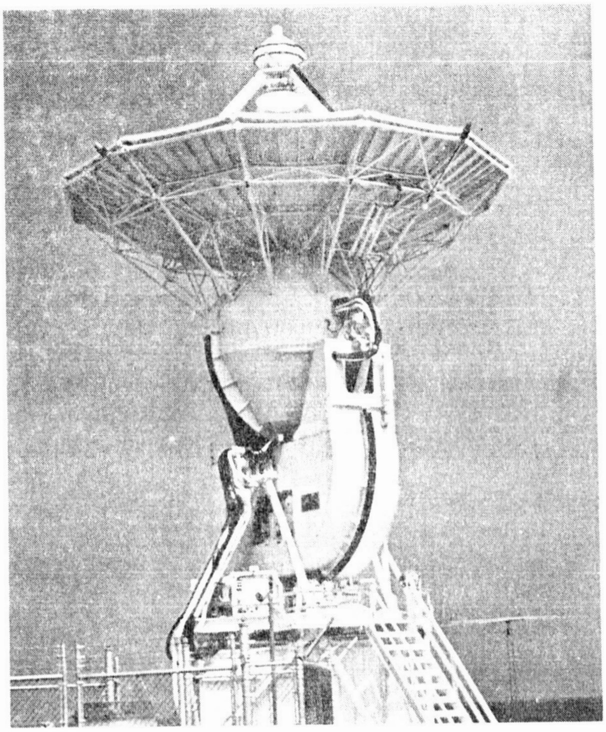

Figure 2. BDA 30-Foot USB X-Y Antenna

Figure 3. Goldstone Apollo 85-Foot USB X-Y Antenna

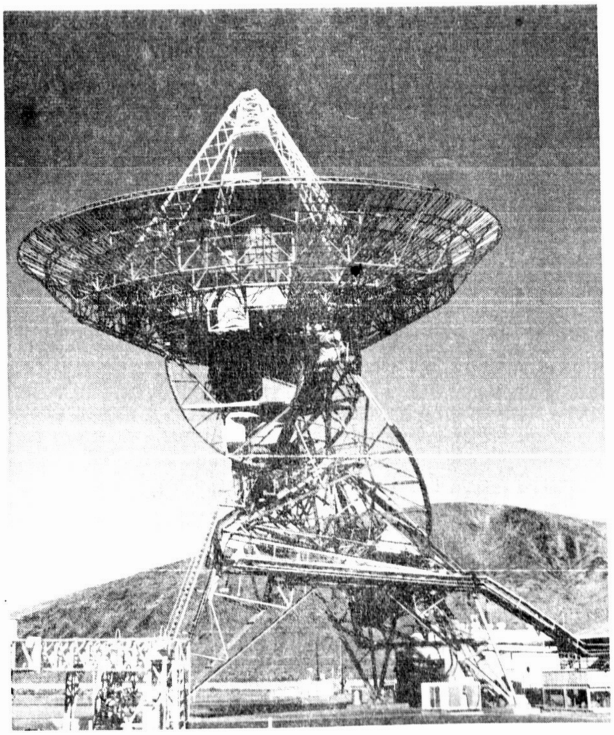

Figure 4. JPL Pioneer 85-Foot DSIF Hour Angle-Declination Antenna

Table 1. Summary of Antennae Mount Characteristics

Type of Antenna Coordinate System	Disadvantages	Advantages
AZ, EL	Maximum AZ rate occurs at EL = 90° which causes loss of track on overhead passes. Azimuth is not defined at EL = 90°	Since all points of azimuth can be covered at EL = 0°, there are no horizon acquisition problems.
X, Y	Mechanical design constraints require a small vertical separation between the X and Y axes and a limited Y axis movement. For the 30- and 85-foot USB antennas this prevents tracking in a small area (keyhole) about the horizon as shown in figure 5. This causes some horizon acquisition problem about the north-south axis for 30-foot antennas and east-west axis for 85-foot antennae.	Complete tracking capability through the zenith. Maximum X and Y tracking rates occur in the area of the keyhole and therefore place no additional restriction on the antennae.
HR, DEC	Maximum HR occurs at DEC = ± 90° which causes loss of track on pass which go through AZ = 0 and an EL = 90-ϕ. Hour angle is not defined at DEC = 90°. For the same reason as in the X and Y mounts discussed above, a keyhole exists in the two northern quadrants as shown in figure 5 for this type of mount.	Tracking a fixed point in space becomes a matter of merely driving the Hour Angle at the counterspeed of the earth's rotational rate after setting Declination.

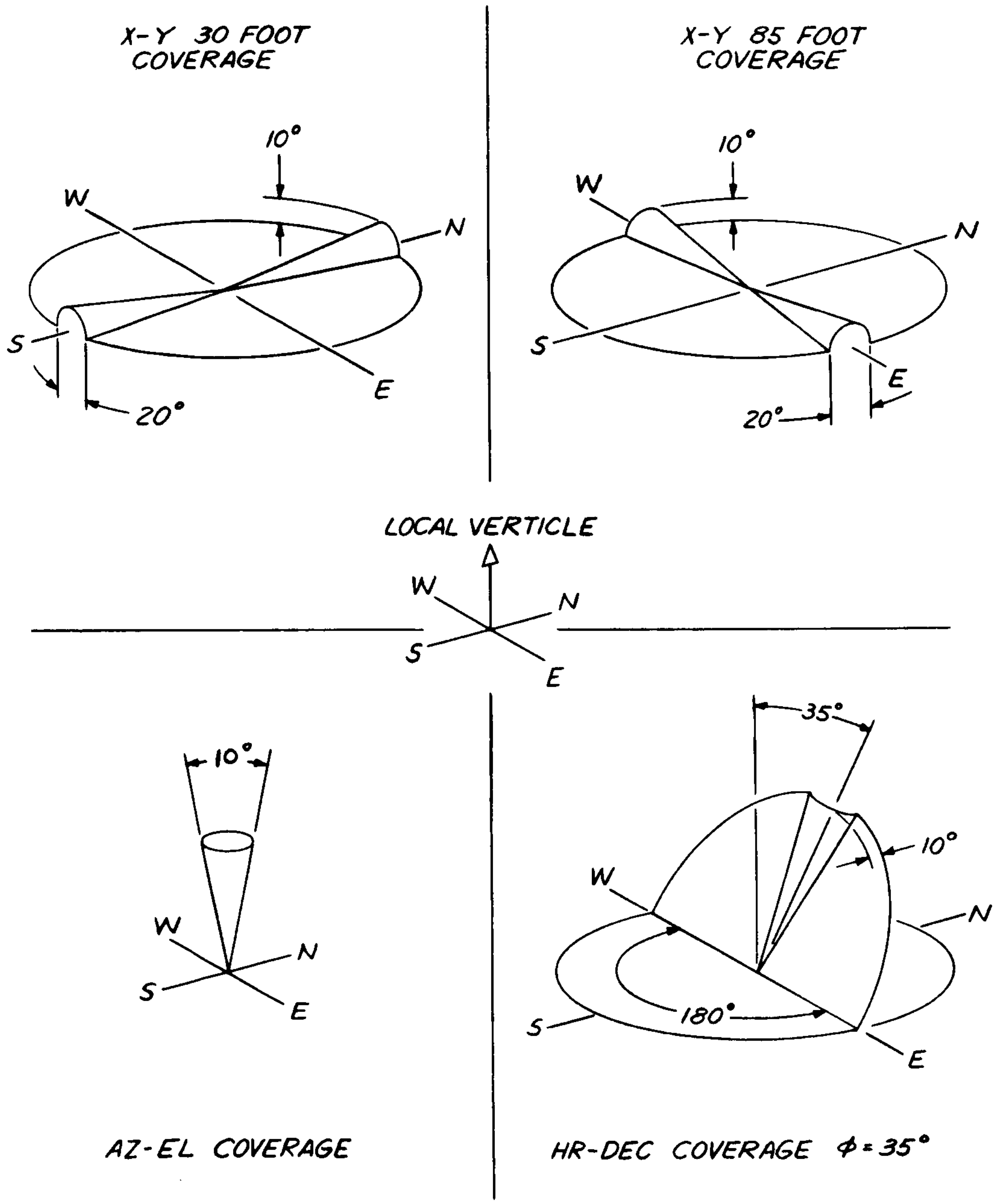

Figure 5. Coverage Area For Different Coordinate Systems

2. DEFINITIONS AND FORMULAE

Site latitude (ϕ) is positive northward	$\lvert\phi\rvert \le 90^\circ$
Hour angle (HR) is positive westward	$0^\circ \le HR < 360^\circ$
Declination (DEC) is ϕ at zenith	$\lvert DEC - \phi\rvert \le 90^\circ$
Angle X_{85} is positive southward	$\lvert X_{85}\rvert \le 90^\circ$
Angle Y_{85} is positive eastward	$\lvert Y_{85}\rvert \le 90^\circ$
Angle X_{30} is positive eastward	$\lvert X_{30}\rvert \le 90^\circ$
Angle Y_{30} is positive northward	$\lvert Y_{30}\rvert \le 90^\circ$
Azimuth (AZ) is positive clockwise	$0^\circ \le AZ < 360^\circ$
Elevation (EL) is 90° at zenith	$0^\circ \le EL \le 90^\circ$ or 180° *
Slant Range (r)	$0 \le r$

Referring to Figure 6

$-X_{85}$ is angle AOF

Y_{85} is angle FOR

X_{30} is angle AOB

Y_{30} is angle BOR

AZ is angle EOD

EL is angle ROD

-HR is angle AOB when $\phi = 0^\circ$

DEC is angle BOR when $\phi = 0^\circ$

*Dependent upon encoder readout

E, N, V are components of the range vector and can be defined in terms of the various coordinate angles by trigonometry. This is shown in matrix equation 1.

$$\begin{bmatrix} E_r \\ N_r \\ V_r \end{bmatrix} = r \begin{bmatrix} \cos EL \sin AZ \\ \cos EL \cos AZ \\ \sin EL \end{bmatrix} = r \begin{bmatrix} \cos Y_{30} \sin X_{30} \\ \sin Y_{30} \\ \cos Y_{30} \cos X_{30} \end{bmatrix} = r \begin{bmatrix} \sin Y_{85} \\ -\cos Y_{85} \sin X_{85} \\ \cos Y_{85} \cos X_{85} \end{bmatrix} \quad (1)$$

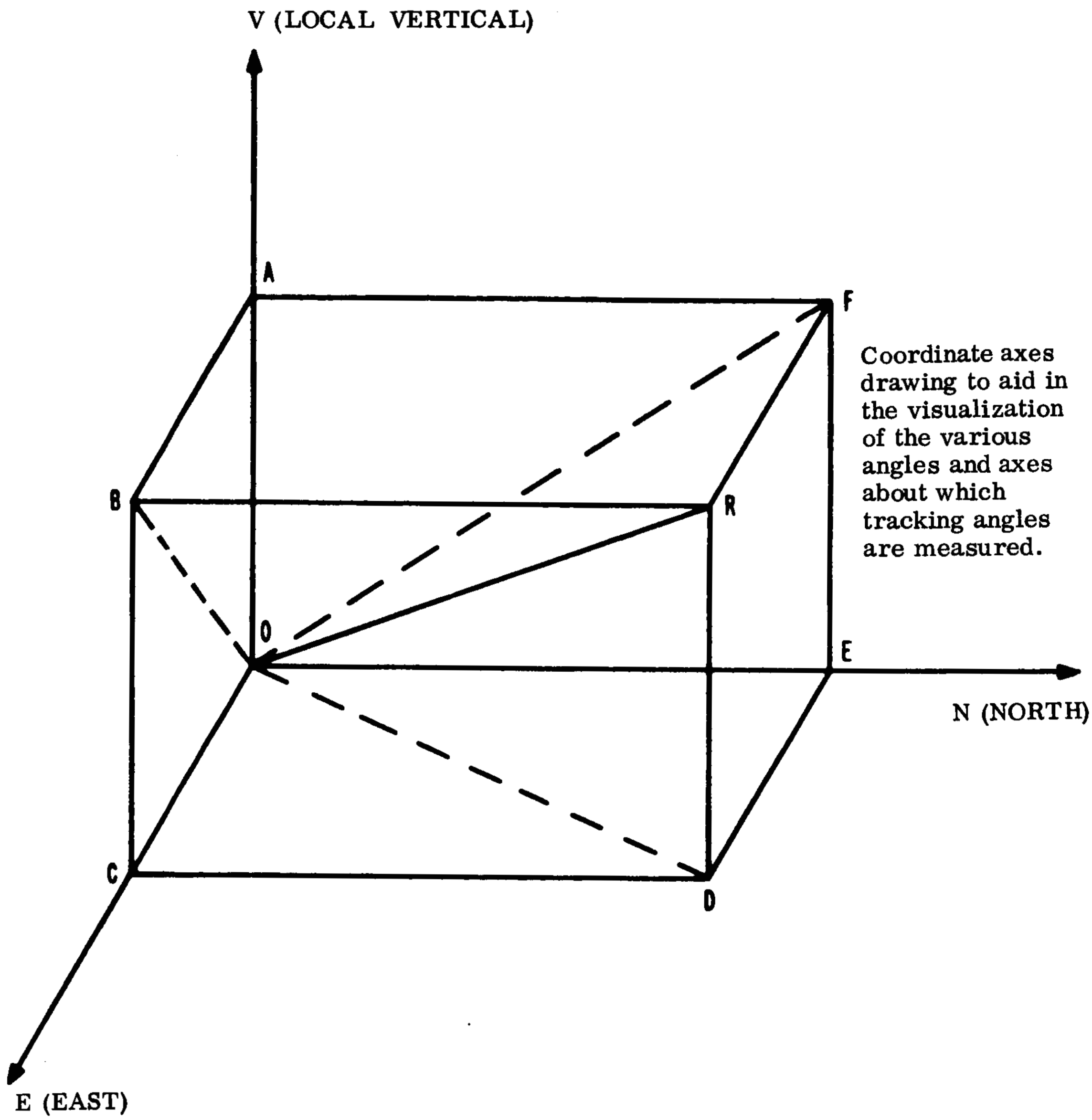

Figure 6. Arbitrary E, N, V Coordinate System

To describe Hour angle and Declination for all latitudes requires a coordinate system which maintains one axis parallel to the north-south axis of the earth. Thus, if the E, N, V coordinate system of **Figure 6** were to be rotated counter-clockwise about the E axis (when looking toward the origin), an amount, ϕ, the resulting coordinate system E', N', V' would allow Hour angle and Declination to be described.

That is:

$$\begin{bmatrix} E' \\ N' \\ V' \end{bmatrix} = R_{E_{ccw}}(\phi) \begin{bmatrix} E \\ N \\ V \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} E \\ N \\ V \end{bmatrix} = \begin{bmatrix} E \\ N\cos\phi + V\sin\phi \\ -N\sin\phi + V\cos\phi \end{bmatrix}$$

Since, by trigonometry,

$$\begin{bmatrix} E'_r \\ N'_r \\ V'_r \end{bmatrix} = r \begin{bmatrix} -\cos DEC \ \sin HR \\ \sin DEC \\ \cos DEC \ \cos HR \end{bmatrix}$$

then,

$$\begin{bmatrix} E_r \\ N_r\cos\phi + V_r\sin\phi \\ -N_r\sin\phi + V_r\cos\phi \end{bmatrix} = r \begin{bmatrix} -\cos DEC \sin HR \\ \sin DEC \\ \cos DEC \cos HR \end{bmatrix} \tag{2}$$

Conversly,

$$\begin{bmatrix} E \\ N \\ V \end{bmatrix} = R_{E_{ccw}}(-\phi) \begin{bmatrix} E' \\ N' \\ V' \end{bmatrix} = r \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} -\cos DEC \ \sin HR \\ \sin DEC \\ \cos DEC \ \cos HR \end{bmatrix}$$

or

$$\begin{bmatrix} E_r \\ N_r \\ V_r \end{bmatrix} = r \begin{bmatrix} -\cos DEC \ \sin HR \\ \cos\phi \ \sin DEC \ -\sin\phi \ \cos DEC \ \cos HR \\ \sin\phi \ \sin DEC \ +\cos\phi \ \cos DEC \ \cos HR \end{bmatrix} \tag{3}$$

From the definitions of the angles we have derived equations 1, 2 and 3 which are all that are required to develop any of the particular equations relating one coordinate system to another.

For example, to determine the equations which define Hour angle and Declination in terms of X_{30}, Y_{30}, we proceed as follows:

utilizing equation 2, we have,

$$\begin{bmatrix} E'_r \\ N'_r \\ V'_r \end{bmatrix} = r \begin{bmatrix} \cos Y_{30} \sin X_{30} \\ \sin Y_{30} \cos \phi + \cos Y_{30} \cos X_{30} \sin \phi \\ - \sin Y_{30} \sin \phi + \cos Y_{30} \cos X_{30} \cos \phi \end{bmatrix} = r \begin{bmatrix} -\cos DEC \sin HR \\ \sin DEC \\ \cos DEC \cos HR \end{bmatrix}$$

constructing the ratio of $\frac{E'_r}{V'_r}$ and setting the N'_r vectors equal we have,

$$\tan HR = \frac{\cos Y_{30} \sin X_{30}}{\cos Y_{30} \cos X_{30} \cos \phi - \sin Y_{30} \sin \phi} \qquad (4)$$

$$\sin DEC = \sin Y_{30} \cos \phi + \cos Y_{30} \cos X_{30} \sin \phi \qquad (5)$$

For a second example the relation defining X_{30} and Y_{30} as a function of X_{85}, Y_{85} is derived as follows:

utilizing equation 1, form the ratio $\frac{E_r}{V_r}$ and equate N_r vectors to obtain:

$$\tan X_{30} = \frac{\tan Y_{85}}{\cos X_{85}} \qquad (6)$$

$$\sin Y_{30} = -\cos Y_{85} \sin X_{85} \qquad (7)$$

Any of the remaining 44 equations can be easily derived in a similar manner.

3. SUMMARY OF EQUATIONS

Due to the location of manned space flight tracking equipment there is a particular interest in the several sets of equations listed below:

$$AZ, EL \longleftrightarrow X_{30}, Y_{30}$$

$$\sin Y_{30} = \cos EL \cos AZ \tag{8}$$

$$\tan X_{30} = \cot EL \sin AZ \tag{9}$$

$$\sin EL = \cos Y_{30} \cos X_{30} \tag{10}$$

$$\tan AZ = \cot Y_{30} \sin X_{30} \tag{11}$$

$$AZ, EL \longleftrightarrow X_{85}, Y_{85}$$

$$\sin Y_{85} = \cos EL \sin AZ \tag{12}$$

$$\tan X_{85} = -\cot EL \cos AZ \tag{13}$$

$$\sin EL = \cos Y_{85} \cos X_{85} \tag{14}$$

$$\tan AZ = \frac{-\tan Y_{85}}{\sin X_{85}} \tag{15}$$

$$X_{85}, Y_{85} \longleftrightarrow HR, DEC$$

$$\sin DEC = \cos Y_{85} \sin (\phi - X_{85}) \tag{16}$$

$$\tan HR = \frac{-\sin Y_{85}}{\cos Y_{85} \cos (\phi - X_{85})} \tag{17}$$

$$\sin Y_{85} = -\cos DEC \sin HR \tag{18}$$

$$\tan X_{85} = \frac{\sin\phi \cos DEC \cos HR - \cos\phi \sin DEC}{\cos\phi \cos DEC \cos HR + \sin\phi \sin DEC} \tag{19}$$

Collins Radio Corp. has developed charts, Figures 7 and 8, to help in the quick visual conversion of equations 8 thru 11 and 12 thru 15, respectively.

Somewhat similar graphs have been derived by ATO from equations 16 thru 19. Since equations 16 thru 19 are functions of latitude it was necessary to make separate charts, Figures 9, 10 and 11, to cover Goldstone, Tidbinbilla and Madrid, respectively. These figures are approximations ($\pm 2^\circ$) since the antennas are not located next to each other and approximate latitudes have been used.

30 FOOT ANTENNA SYSTEMS
RELATIONSHIP OF X-Y TO AZ-EL COORDINATES

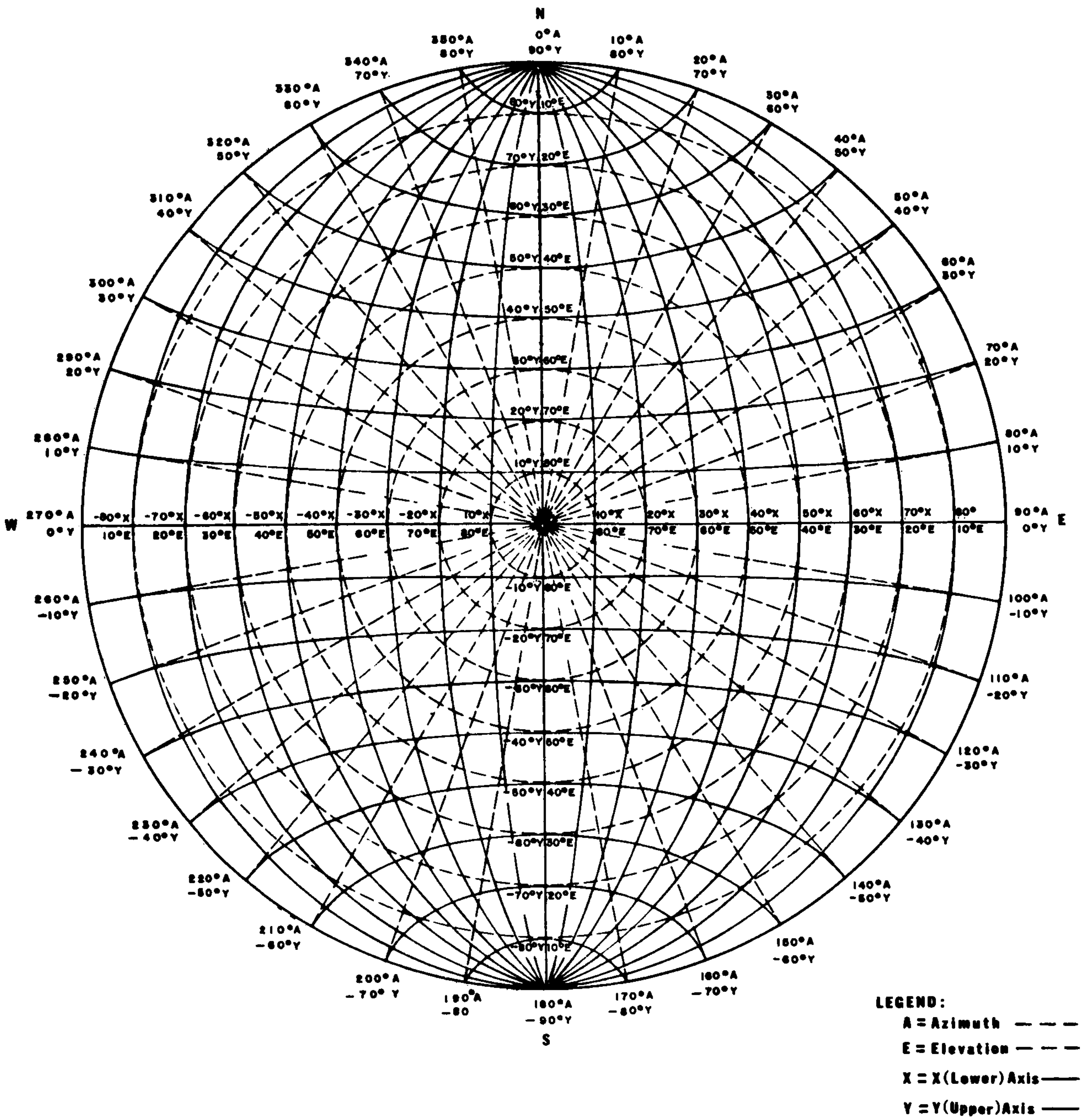

Figure 7. Collins Conversion Graph AZ-EL to X_{30} -Y_{30}

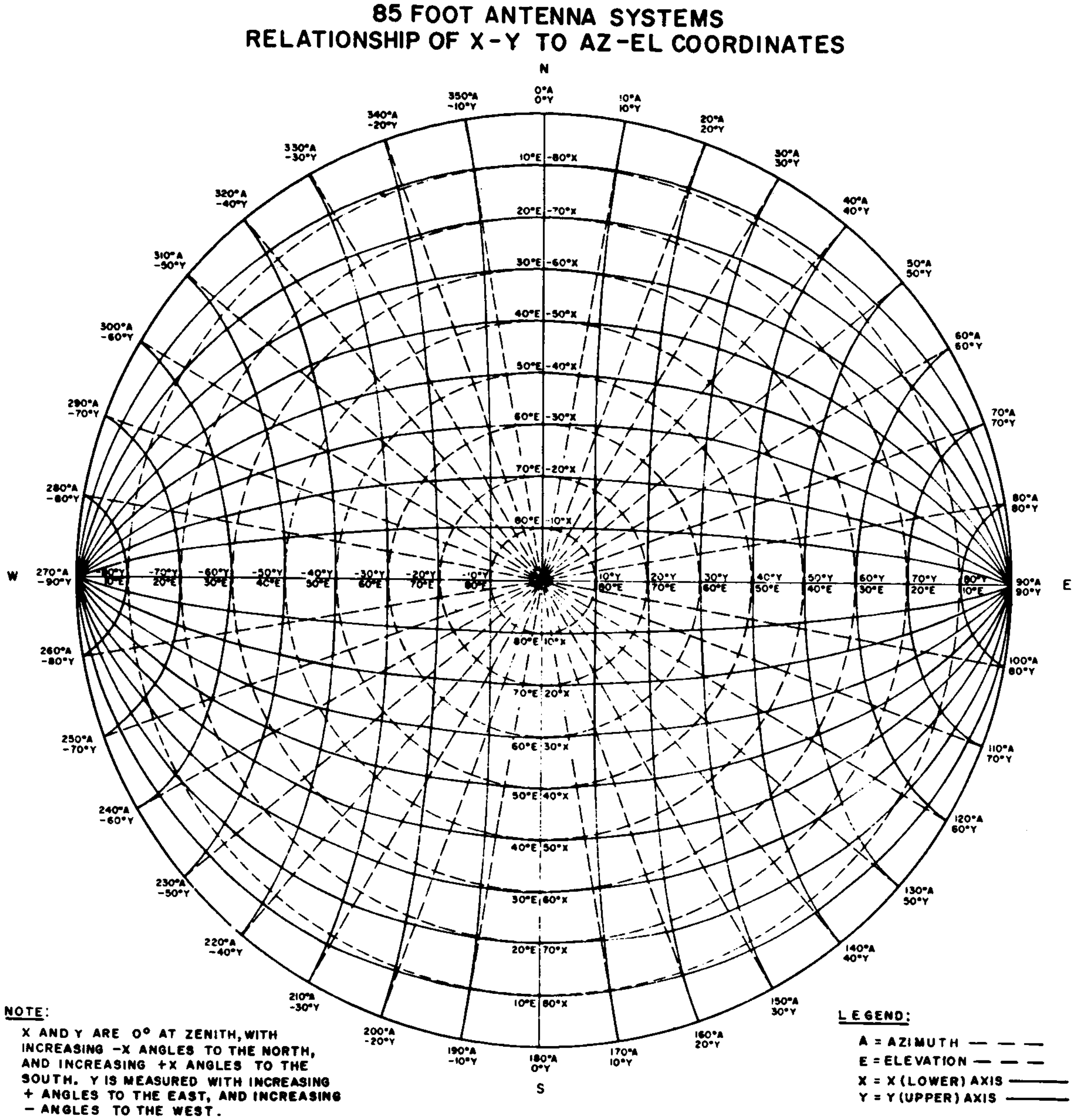

Figure 8. Collins Conversion Graph AZ-EL to X_{85}-Y_{85}

85 FOOT ANTENNA SYSTEMS
RELATIONSHIP OF X-Y TO HR-DEC COORDINATES
LATITUDE = 35° 20'

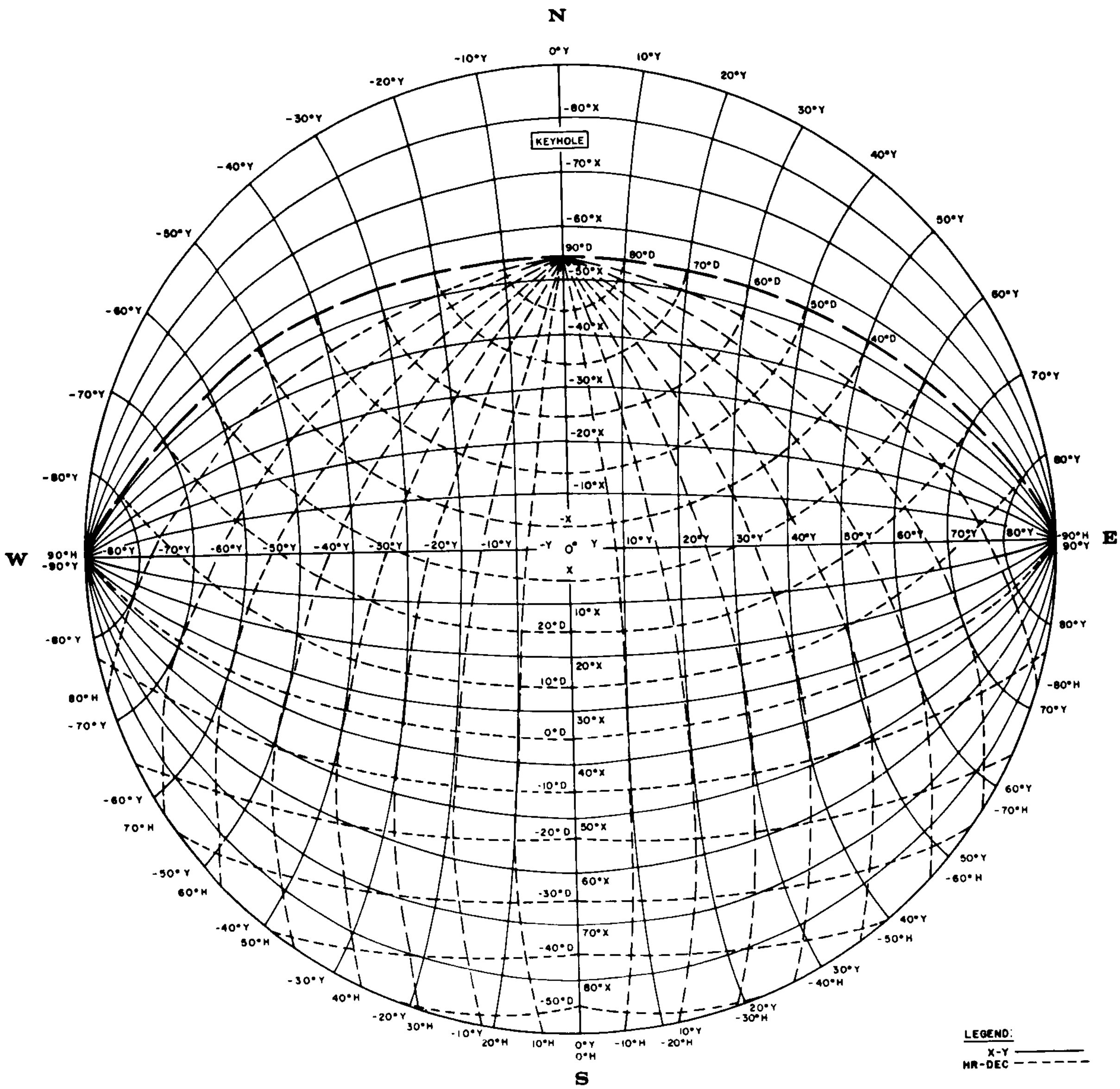

Figure 9. Conversion Graph X_{85}-Y_{85} to Hour Angle-Declination, $\phi=35^{\circ}\,20'$

85 FOOT ANTENNA SYSTEMS
RELATIONSHIP OF X-Y TO HR-DEC COORDINATES
LATITUDE= -35° 20'

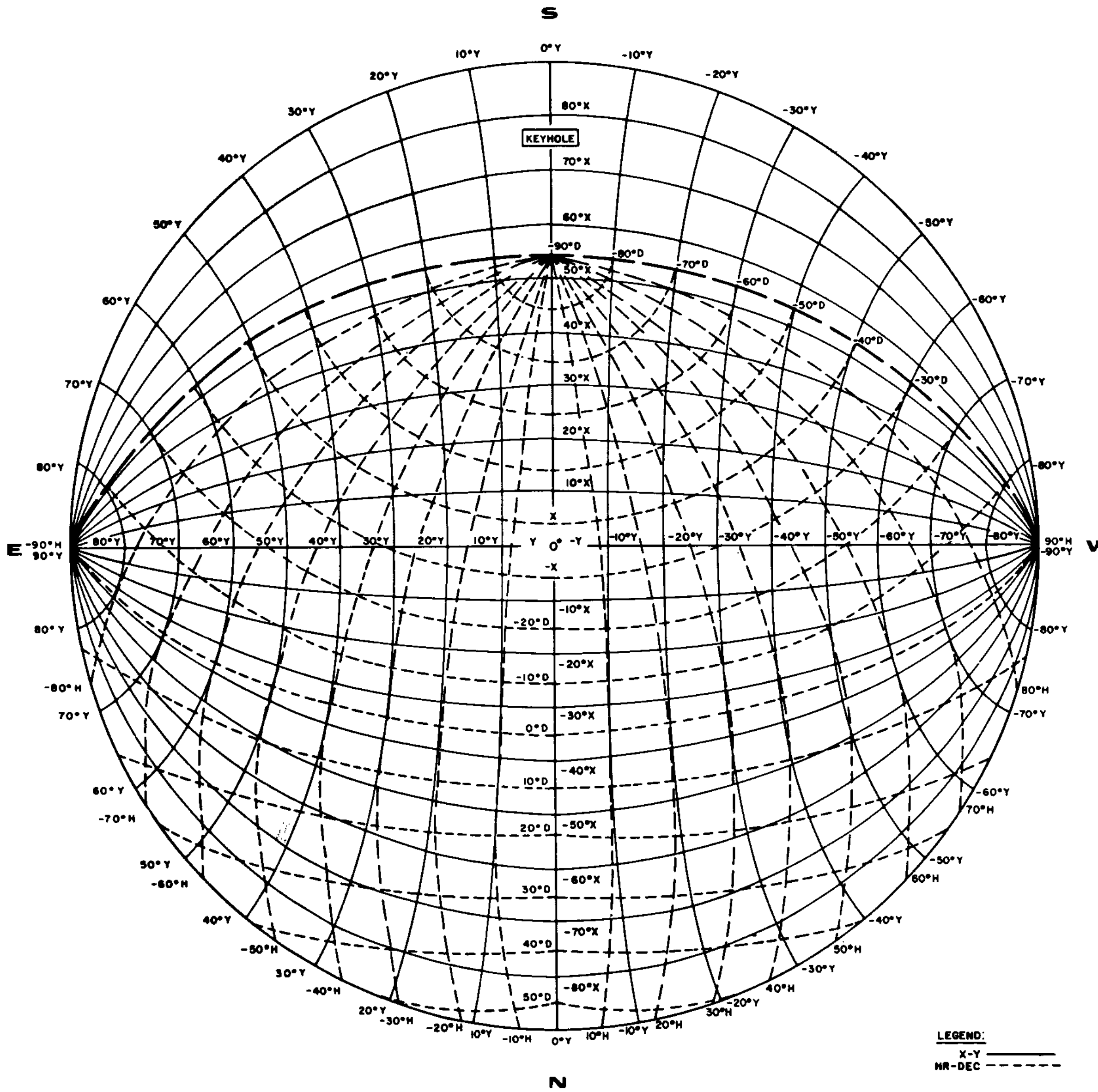

Figure 10. Conversion Graph X_{85}-Y_{85} to Hour Angle-Declination ϕ=35° 20'

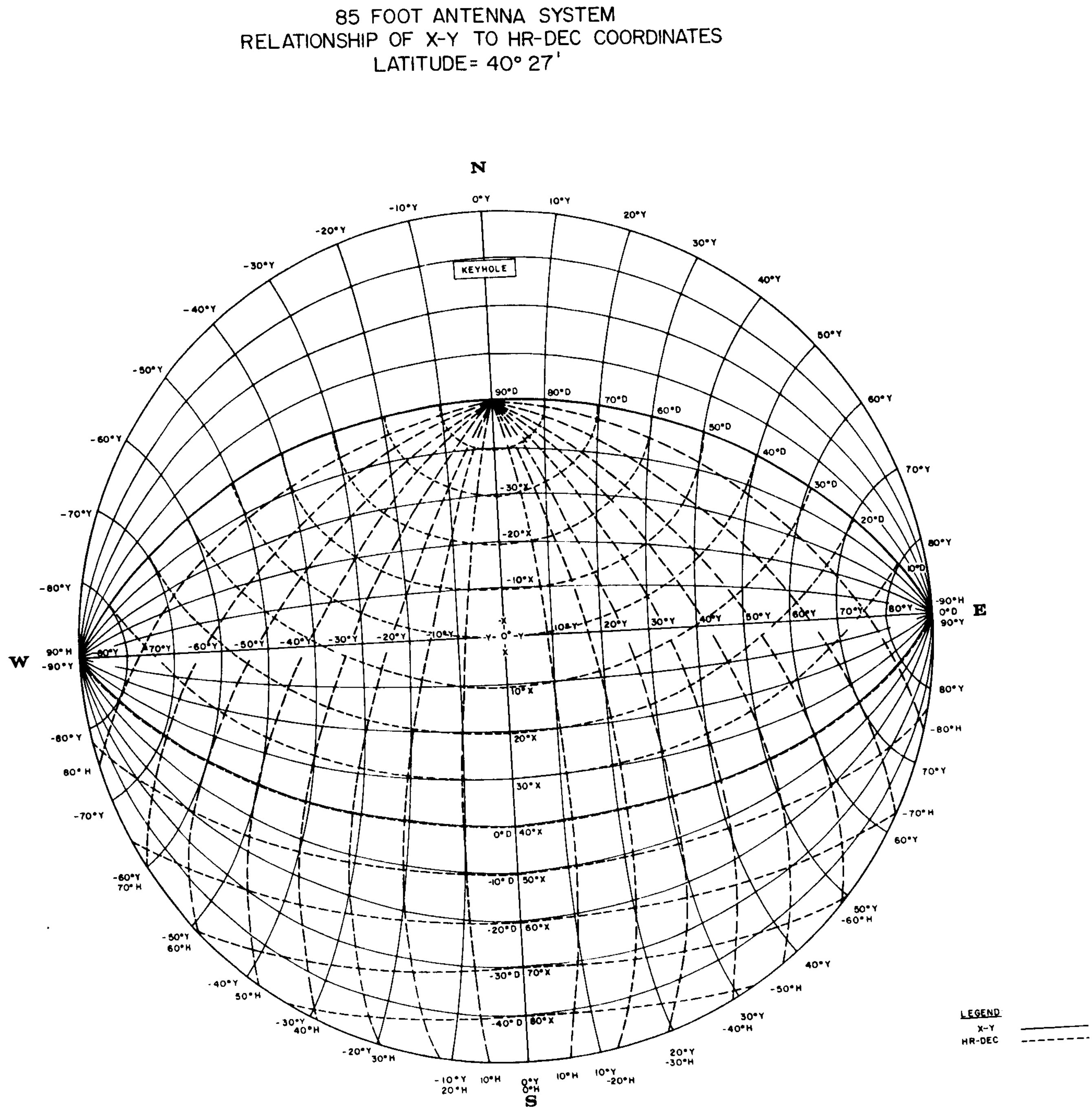

Figure 11. Conversion Graph X_{85}-Y_{85} to Hour Angle-Declination ϕ=40° 27'