X-643-67-510

NASA TM X-55976

ITERATIVE INTERPOLATION AND NUMERICAL DIFFERENTIATION USING DIVIDED DIFFERENCES

E. R. LANCASTER

OCTOBER 1967

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

ITERATIVE INTERPOLATION AND NUMERICAL DIFFERENTIATION USING DIVIDED DIFFERENCES

E. R. Lancaster

October 1967

Goddard Space Flight Center
Greenbelt, Maryland

ITERATIVE INTERPOLATION AND NUMERICAL DIFFERENTIATION USING DIVIDED DIFFERENCES

E. R. Lancaster

ABSTRACT

An iterative method of interpolation based on divided differences is developed which is about three times as fast as the iterative methods of Aitken and Neville. The method is extended to numerical differentiation, with similar improvements over Hunter's extension of the Aitken and Neville methods.

October 1967

ITERATIVE INTERPOLATION AND NUMERICAL DIFFERENTIATION USING DIVIDED DIFFERENCES

Given values of a function f for x_0, x_1, x_2 ..., divided differences are computed from

$$d_{io} = f(x_i); \quad i = 0, 1, 2, \ldots$$

$$d_{ij} = \frac{d_{i,j-1} - d_{i-1,j-1}}{x_i - x_{i-j}} \; ; \; i = 1, 2, 3, \ldots; \quad j = 1, 2, \ldots, i$$

We can arrange these values in a table as follows:

x_0	d_{00}				
x_1	d_{10}	d_{11}			
x_2	d_{20}	d_{21}	d_{22}		
x_3	d_{30}	d_{31}	d_{32}	d_{33}	
x_4	d_{40}	d_{41}	d_{42}	d_{43}	d_{44}
. . .	. . .	. . .	. . .	. . .	. . .

Letting $d_j = d_{jj}$, the polynomial which takes on the values $f(x_k)$ at x_k for $k = 0, 1, 2, \ldots, i$ is given by

$$y_i(x) = \sum_{j=0}^{i} d_j \, p_j \, (x),$$

$$p_0(x) = 1,$$

$$p_j(x) = (x-x_0)\,(x-x_1)\ldots(x-x_{j-1}); \quad j = 1,\ 2,\ \ldots,\ i.$$

This can be written

$$y_i(x) = y_{i-1}(x) + d_i \, p_i(x), \ i = 1,\ 2,\ 3,\ \ldots$$

$$p_i(x) = (x-x_{i-1})p_{i-1}(x), \ i = 1,\ 2,\ 3,\ \ldots$$

$$y_0(x) = d_0, \ p_0(x) = 1$$

This is a convenient iterative method for interpolating f at x. It is approximately three times as fast as the methods of Aitken [1] and Neville [2] .

We rewrite $y_i(x)$ and $p_i(x)$ in the forms

$$p_i(x) = \sum_{j=0}^{i} b_{ij} \, x^j,$$

$$y_i(x) = \sum_{j=0}^{i} c_{ij} \, x^j,$$

and tabulate the coefficients as follows:

$$\begin{array}{lllll}
b_{00} & & & & \\
b_{10} & b_{11} & & & \\
b_{20} & b_{21} & b_{22} & & \\
b_{30} & b_{31} & b_{32} & b_{33} & \\
b_{40} & b_{41} & b_{42} & b_{43} & b_{44} \\
\cdots & \cdots & \cdots & \cdots & \cdots
\end{array}$$

$b_{ii} = 1;\ i = 0, 1, 2, \ldots$

$b_{io} = -b_{i-1,0}\, x_{i-1};\ i = 1, 2, 3, \ldots$

$b_{ij} = b_{i-1,\, j-1} - b_{i-1,j}\, x_{i-1};\ i = 1, 2, 3, \ldots;\ j = 1, 2, \ldots, i-1$

$$\begin{array}{lllll}
c_{00} & & & & \\
c_{10} & c_{11} & & & \\
c_{20} & c_{21} & c_{22} & & \\
c_{30} & c_{31} & c_{32} & c_{33} & \\
c_{40} & c_{41} & c_{42} & c_{43} & c_{44} \\
\cdots & \cdots & \cdots & \cdots & \cdots
\end{array}$$

$c_{ii} = d_i;\ i = 0, 1, 2, \ldots$

$c_{ij} = c_{i-1,j} + d_i b_{ij};\ i = 1, 2, 3, \ldots;\ j = 0, 1, \ldots, i-1$

The ith row of the c-table gives the coefficients of the polynomial $\sum_{j=0}^{i} c_{ij} x^j$ with values $f(x_k)$ at $x = x_k$; $k = 0, 1, \ldots, i$. If the origin is placed at the point of interpolation, then c_{io} is the interpolated value based on the points $(x_k, f(x_k))$; $k = 0, 1, \ldots, i$. $m! \, c_{im}$ is an approximation of the mth derivative at $x = 0$, i.e., $f^{(m)}(0)$, based on the same points. If approximations of $f^{(j)}(0)$ are not desired for $j > m$, then it is not necessary to compute b_{ij} and c_{ij} for $j > m$.

This method of iterative numerical differentiation is considerably faster than that advocated by Hunter [3], which is similar to the methods of Aitken and Neville for iterative interpolation.

The above algorithm can also be used to solve a set of Vandermonde equations or to invert a Vandermonde matrix.

The iterated divided difference method of this paper was programmed by C. R. Herron and applied to the example given by Hunter [3]. The results were identical.

REFERENCES

1. Aitken, A. C., On Interpolation by Iteration of Proportional Parts, without the Use of Differences. Proc. Edinburgh Math. Soc., Series 2, Vol. 3 (1932), 56-76.

2. Neville, E. H., Iterative Interpolation. J. Indian Math. Soc., 20 (1934), 87-120.

3. Hunter, D. B., An Iterative Method of Numerical Differentiation. The Computer Journal 3 (Jan., 1961), 270-271.