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ON A GENERALIZATION OF THE DEVELOPMENT OF THE DISTURBING FUNCTION

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ABSTRACT

The previous developments of the disturbing function of a first order general planetary theory involved commonly only one disturbing planet. They were performed by considering the inclination of the orbital plane of the disturbed planet on that of the disturbing planet or vice versa, that is to say by considering the mutual inclination and by referring the longitudes to the longitude of the ascending node of the disturbed or of the disturbing planet. In the present paper, we perform a more general development of the disturbing function by considering n planets instead of two ($n > 2$), by referring the inclination of each of those n planets to a common fixed plane, by referring the longitudes to a common origin and by reducing the Fourier series of the principal part of the disturbing function to the sum of its $n(n-1)(p+1)/2$ first terms, the positive integer p being unspecified. The development of the principal part and that of the indirect part of the disturbing function are performed up to the fourth powers of eccentricities and the sines of inclinations and they could be easily extended to the eight powers required for the building of a complete first order general planetary theory.

ON A GENERALIZATION OF THE DEVELOPMENT OF THE DISTURBING FUNCTION

INTRODUCTION

The building of a first order general planetary theory through Von Zeipel's method and, more precisely, its first step dealing with the elimination of the short period terms requires a suitable development of the principal and of the indirect part of the disturbing function according to the powers of eccentricities and inclinations and according to the cosines of the multiples of the mean longitudes, the longitudes of the nodes and the longitudes of the perihelia. In the case of only one disturbing planet, those developments are usually performed by referring the orbital plane of the disturbed planet to the orbital plane of the disturbing planet or vice versa, and by considering the inclination of the former on the latter that is to say the mutual inclination, the longitudes being calculated from the longitude of the ascending node of the disturbed or of the disturbing planet. Such developments are those of LeVerrier[1] and Newcomb[2] and their effective calculation up to the third powers of eccentricities and mutual inclination are recalled by Brouwer and Clemence[3] who include also in the principal part of the disturbing function those of the terms of order four with respect to the eccentricities and mutual inclination which arise from its secular part. As points out Marsden in his thesis, "This procedure is useless when one is dealing with more than two bodies at once"[4] and it is then much better to refer the inclination of each planet to a common fixed plane, the longitudes being calculated from a common origin. According to Marsden's remark, "the increase in complexity when one transfers to a general coordinate system is considerable but not unmanagable." We managed this development, both for the principal part and for the indirect part of the disturbing function, up to the fourth powers of eccentricities and inclinations. We considered n planets that is to say one disturbed planet and $n - 1$ disturbing planets. We calculated the indirect part of the disturbing function through Newcomb operators and the principal part of the disturbing function through Newcomb operators and Laplace coefficients. We reduced the Fourier series of each of the $n(n-1)/2$ terms of the principal part of the disturbing function to the sum of its $p + 1$ first terms. In doing so, we generalized a previous result of Andoyer[5] who indicated, in the case of only two planets, a development of the disturbing function according to the powers of the eccentricities and to the powers of twice the sines of the semi inclinations, the latter being referred to a common fixed plane.

NOTATIONS AND PRELIMINARY CALCULATIONS

P_1 disturbed planet referred to the Sun S,

P_2 disturbing planet referred to the center of mass of S and P_1,

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P_n disturbing planet referred to the center of mass of S, $P_1, \ldots, P_{n-1}$,

m_0 mass of S,

σ small parameter of the order of the masses of $P_1, \ldots, P_n$,

$\beta_1, \cdots, \beta_n$ finite numerical coefficients,

$\beta_1 \sigma$ mass of P_1,

$\cdot$
$\cdot$
$\cdot$

$\beta_n \sigma$ mass of P_n,

r_{02} distance between S and P_2,

$\cdot$
$\cdot$
$\cdot$

r_{0n} distance between S and P_n,

r_2 distance between P_2 and the center of mass of S and P_1,

$\cdot$
$\cdot$
$\cdot$

r_n distance between P_n and the center of mass of S, $P_1, \ldots, P_{n-1}$,

r_{12} distance between P_1 and P_2,

$\cdot$
$\cdot$
$\cdot$

$r_{n-1,n}$ distance between P_{n-1} and P_n,

a_1 semimajor axis of the osculating ellipse of P_1,

$\cdot$
$\cdot$
$\cdot$

a_n semimajor axis of the osculating ellipse of P_n,

k^2 constant of gravitation.

The Hamiltonian F of the $6n$ canonical equations of the n planets ($n \geq 2$) is:

$$F = \frac{k^2 m_0}{2} \sum_{i=1}^{n} \frac{\beta_i}{a_i} + k^2 m_0 \sum_{i=2}^{n} \beta_i \left(\frac{1}{r_{0i}} - \frac{1}{r_i}\right) + \sigma k^2 \sum_{\substack{i \neq j \\ 1 \leq i < j \leq n}} \frac{\beta_i \beta_j}{r_{i,j}} .$$

We assume that each of the $n(n-1)/2$ ratios $r_1/r_2, \ldots, r_{n-1}/r_n$ is smaller than one and we develop F in a Taylor series of σ according to the formula

$$F(\sigma) = F(0) + \sigma F'(0) + \frac{\sigma^2}{2} F''(0) + \cdots$$

We reduce F to the sum $F(0) + \sigma F'(0)$ and we put $F(0) = F_0$, $\sigma F'(0) = F_1$. We have:

$$F_0 = \frac{k^2 m_0}{2} \sum_{i=1}^{n} \frac{\beta_i}{a_i} ,$$

$$F_1 = \sigma \left(\frac{-k^2}{2} \sum_{i=1}^{n} \frac{\beta_i^2}{a_i} - k^2 \sum_{\substack{u \neq v \\ 1 \leq u < v \leq n}} \beta_u \beta_v \frac{r_u}{r_v^2} \cos \theta_{u,v} + k^2 \sum_{\substack{u \neq v \\ 1 \leq u < v \leq n}} \frac{\beta_u \beta_v}{r_v} \frac{1}{\sqrt{1 - 2 \frac{r_u}{r_v} \cos \theta_{u,v} + \frac{r_u^2}{r_v^2}}} \right),$$

$\theta_{u,v}$ being the angle of the vectors $\vec{r}_u$ and $\vec{r}_v$.

We restrict ourselves to the consideration of the two expressions

$$-\sigma k^2 \sum_{\substack{u \neq v \\ 1 \leq u < v \leq n}} \beta_u \beta_v \frac{r_u}{r_v^2} \cos \theta_{u,v}, \quad \sigma k^2 \sum_{\substack{u \neq v \\ 1 \leq u < v \leq n}} \frac{\beta_u \beta_v}{r_v} \frac{1}{\sqrt{1 - 2 \frac{r_u}{r_v} \cos \theta_{u,v} + \frac{r_u^2}{r_v^2}}} .$$

The first one is the indirect part of the disturbing function and we call if $(F_1)_I$; the second one is the principal part of the disturbing function and we call it $(F_1)_P$. We calculate separately $(F_1)_I$ and $(F_1)_P$.

CALCULATION OF $(F_1)_P$

1° We put

$$\alpha_{1,2} = \frac{a_1}{a_2}, \cdots, \alpha_{n-1,n} = \frac{a_{n-1}}{a_n}$$

and

$$D_{1,2} = \alpha_{1,2} \frac{d}{d\alpha_{1,2}}, \cdots, D_{n-1,n} = \alpha_{n-1,n} \frac{d}{d\alpha_{n-1,n}} .$$

We call I_i the inclination of the orbital plane of P_i on a common fixed plane of reference and we put $\sin I_i = \gamma_i$, $(i = 1, 2, \ldots, n)$. We call e_i the eccentricity of the osculating ellipse of P_i and we introduce the variables $\lambda_i, \bar{\omega}_i, \Omega_i$ which are connected to the mean longitude ℓ_i the longitude g_i of the perihelia and the longitude h_i of the ascending node of P_i through the equalities: $\lambda_i = \ell_i + g_i + h_i$, $\bar{\omega}_i = g_i + h_i$, $\Omega_i = h_i$. We call $b_s^{(j,1,2)}, \cdots, b_s^{(j,n-1,n)}$ the Laplace coefficients defined by the equalities.

$$b_s^{(j,u,v)} = \frac{2}{\pi} \int_0^{\pi} (1 - 2\alpha_{u,v} \cos \theta_{u,v} + \alpha_{u,v}^2)^{-s} \cos j\,\theta_{u,v}\, d\theta_{u,v}$$

with

$$s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \cdots; \qquad j = 0, 1, 2, \cdots; \qquad (u, v) = (1, 2), \cdots, (n-1, n) .$$

We have

$$(F_1)_P = \sigma k^2 \quad \left[\sum_{\substack{u \neq v \\ 1 \leq u < v \leq n}} \frac{\beta_u \, \beta_v}{a_v} \quad \sum_{j=0}^{p} \quad \left[\left\{ b_{1/2}^{(j,u,v)} \right. \right. \right.$$

$$+ e_u^2 \left(- j^2 + \frac{1}{4} D_{u,v} + \frac{1}{4} D_{u,v}^2 \right) \; b_{1/2}^{(j,u,v)}$$

$$+ e_v^2 \left(- j^2 + \frac{1}{4} D_{u,v} + \frac{1}{4} D_{u,v}^2 \right) \; b_{1/2}^{(j,u,v)}$$

$$+ e_u^4 \left(- \frac{9}{64} j^2 + \frac{1}{4} j^4 + \frac{1}{32} D_{u,v} + \left(- \frac{1}{64} - \frac{1}{8} j^2 \right) \; D_{u,v}^2 - \frac{1}{32} D_{u,v}^3 + \frac{1}{64} D_{u,v}^4 \right) \; b_{1/2}^{(j,u,v)}$$

$$+ e_u^2 \, e_v^2 \left(j^4 - \frac{1}{2} j^2 D_{u,v} + \left(\frac{1}{16} - \frac{1}{2} j^2 \right) \; D_{u,v}^2 + \frac{1}{8} D_{u,v}^3 + \frac{1}{16} D_{u,v}^4 \right) \; b_{1/2}^{(j,u,v)}$$

$$+ e_v^4 \left(- \frac{17}{64} j^2 + \frac{1}{4} j^4 + \left(\frac{3}{32} - \frac{1}{4} j^2 \right) \; D_{u,v} + \left(\frac{11}{64} - \frac{1}{8} j^2 \right) \; D_{u,v}^2 + \frac{3}{32} D_{u,v}^3 + \frac{1}{64} D_{u,v}^4 \right) \; b_{1/2}^{(j,u,v)}$$

$$+ \left(- \gamma_u^2 - \gamma_v^2 - \frac{1}{4} \gamma_u^4 - \frac{1}{4} \gamma_v^4 \right) \; \frac{\alpha_{u,v}}{8} \; (b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)})$$

$$+ (\gamma_u^2 \, e_u^2 + \gamma_v^2 \, e_u^2) \left(\frac{1}{2} j^2 - \frac{1}{8} D_{u,v} - \frac{1}{8} D_{u,v}^2 \right) \; \frac{\alpha_{u,v}}{4} \; (b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)})$$

$$+ (\gamma_u^2 \, e_v^2 + \gamma_v^2 \, e_v^2) \left(\frac{1}{2} j^2 - \frac{1}{8} D_{u,v} - \frac{1}{8} D_{u,v}^2 \right) \; \frac{\alpha_{u,v}}{4} \; (b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)})$$

$$+ \gamma_u^2 \, \gamma_v^2 \, \frac{\alpha_{u,v}}{32} \; (b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)})$$

$$+ (\gamma_u^4 + \gamma_v^4 + 5 \, \gamma_u^2 \, \gamma_v^2) \; \frac{3}{32} \; \alpha_{u,v}^2 \; b_{5/2}^{(j,u,v)}$$

$$+ \left(\frac{1}{2} \gamma_u^4 + \frac{1}{2} \gamma_v^4 + \gamma_u^2 \, \gamma_v^2 \right) \frac{3}{64} \; \alpha_{u,v}^2 \; (b_{5/2}^{(j-2,u,v)} + b_{5/2}^{(j+2,u,v)}) \Big\} \; \cos(j\lambda_u - j\lambda_v)$$

$$+ \left\{ e_u \left(j - \frac{1}{2} D_{u,v} \right) \; b_{1/2}^{(j,u,v)} \right.$$

$$+ e_u^3 \left(-\frac{1}{8} j - \frac{5}{8} j^2 - \frac{1}{2} j^3 + \left(\frac{3}{16} + \frac{5}{16} j + \frac{1}{4} j^2 \right) D_{u,v} + \left(\frac{1}{16} + \frac{1}{8} j \right) D_{u,v}^2 - \frac{1}{16} D_{u,v}^3 \right) b_{1/2}^{(j,u,v)}$$

$$+ e_u e_v^2 \left(-j^3 + \left(\frac{1}{4} j + \frac{1}{2} j^2 \right) D_{u,v} + \left(-\frac{1}{8} + \frac{1}{4} j \right) D_{u,v}^2 - \frac{1}{8} D_{u,v}^3 \right) b_{1/2}^{(j,u,v)}$$

$$+ (\gamma_u^2 e_u + \gamma_v^2 e_v) \left(-\frac{1}{2} j + \frac{1}{4} D_{u,v} \right) \frac{\alpha_{u,v}}{4} \left(b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)} \right) \Big\} \quad \cos((j+1)\lambda_u - j\lambda_v - \bar{\omega}_u))$$

$$+ \Big\{ e_u \left(-j - \frac{1}{2} D_{u,v} \right) b_{1/2}^{(j,u,v)}$$

$$+ e_u^3 \left(\frac{1}{8} j - \frac{5}{8} j^2 + \frac{1}{2} j^3 + \left(\frac{3}{16} - \frac{5}{16} j + \frac{1}{4} j^2 \right) D_{u,v} + \left(\frac{1}{16} - \frac{1}{8} j \right) D_{u,v}^2 - \frac{1}{16} D_{u,v}^3 \right) b_{1/2}^{(j,u,v)}$$

$$+ e_u e_v^2 \left(j^3 + \left(-\frac{1}{4} j + \frac{1}{2} j^2 \right) D_{u,v} + \left(-\frac{1}{8} - \frac{1}{4} j \right) D_{u,v}^2 - \frac{1}{8} D_{u,v}^3 \right) b_{1/2}^{(j,u,v)}$$

$$+ (\gamma_u^2 e_u + \gamma_v^2 e_u) \left(\frac{1}{2} j + \frac{1}{4} D_{u,v} \right) \frac{\alpha_{u,v}}{4} \left(b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)} \right) \Big\} \cos((j-1)\lambda_u - j\lambda_v + \bar{\omega}_u)$$

$$+ \Big\{ e_v \left(\frac{1}{2} - j + \frac{1}{2} D_{u,v} \right) b_{1/2}^{(j,u,v)}$$

$$+ e_u^2 e_v \left(-\frac{1}{2} j^2 + j^3 + \left(\frac{1}{8} - \frac{1}{4} j - \frac{1}{2} j^2 \right) D_{u,v} + \left(\frac{1}{4} - \frac{1}{4} j \right) D_{u,v}^2 + \frac{1}{8} D_{u,v}^3 \right) b_{1/2}^{(j,u,v)}$$

$$+ e_v^3 \left(-\frac{1}{16} + \frac{5}{16} j - \frac{7}{8} j^2 + \frac{1}{2} j^3 + \left(\frac{1}{8} + \frac{1}{16} j - \frac{1}{4} j^2 \right) D_{u,v} + \left(\frac{1}{4} - \frac{1}{8} j \right) D_{u,v}^2 + \frac{1}{16} D_{u,v}^3 \right) b_{1/2}^{(j,u,v)}$$

$$+ (\gamma_u^2 e_v + \gamma_v^2 e_v) \left(-\frac{1}{4} + \frac{1}{2} j - \frac{1}{4} D_{u,v} \right) \frac{\alpha_{u,v}}{4} \left(b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)} \right) \Big\} \cos(j\lambda_u - (j-1)\lambda_v - \bar{\omega}_v)$$

$$+ \Big\{ e_v \left(\frac{1}{2} + j + \frac{1}{2} D_{u,v} \right) b_{1/2}^{(j,u,v)}$$

$$+ e_u^2 e_v \left(-\frac{1}{2} j^2 - j^3 + \left(\frac{1}{8} + \frac{1}{4} j - \frac{1}{2} j^2 \right) D_{u,v} + \left(\frac{1}{4} + \frac{1}{4} j \right) D_{u,v}^2 + \frac{1}{8} D_{u,v}^3 \right) b_{1/2}^{(j,u,v)}$$

$$+ e_v^3 \left(-\frac{1}{16} - \frac{5}{16} j - \frac{7}{8} j^2 - \frac{1}{2} j^3 + \left(\frac{1}{8} - \frac{1}{16} j - \frac{1}{4} j^2 \right) D_{u,v} + \left(\frac{1}{4} + \frac{1}{8} j \right) D_{u,v}^2 + \frac{1}{16} D_{u,v}^3 \right) b_{1/2}^{(j,u,v)}$$

$$+ (\gamma_u^2 e_v + \gamma_v^2 e_v) \left(-\frac{1}{4} - \frac{1}{2} j - \frac{1}{4} D_{u,v}\right) \frac{\alpha_{u,v}}{4} (b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)})\Big\} \cos(j\lambda_u - (j+1)\lambda_v + \bar{\omega}_v)$$

$$+ \Big\{ e_u^2 \left(\frac{5}{8} j + \frac{1}{2} j^2 + \left(-\frac{1}{2} j - \frac{3}{8}\right) D_{u,v} + \frac{1}{8} D_{u,v}^2\right) b_{1/2}^{(j,u,v)}$$

$$+ e_u^4 \left(-\frac{11}{48} j - \frac{2}{3} j^2 - \frac{5}{8} j^3 - \frac{1}{6} j^4 + \left(\frac{11}{48} + \frac{47}{96} j + \frac{1}{2} j^2 + \frac{1}{6} j^3\right) D_{u,v} + \left(-\frac{1}{96} + \frac{1}{32} j\right) D_{u,v}^2\right.$$

$$\left. + \left(-\frac{1}{16} - \frac{1}{24} j\right) D_{u,v}^3 + \frac{1}{96} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$+ e_u^2 e_v^2 \left(-\frac{5}{8} j^3 - \frac{1}{2} j^4 + \left(\frac{5}{32} j + \frac{1}{2} j^2 + \frac{1}{2} j^3\right) D_{u,v} + \left(-\frac{3}{32} + \frac{1}{32} j\right) D_{u,v}^2 + \left(-\frac{1}{16} - \frac{1}{8} j\right) D_{u,v}^3\right.$$

$$\left. + \frac{1}{32} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$+ (\gamma_u^2 e_u^2 + \gamma_v^2 e_u^2) \left(-\frac{5}{16} j - \frac{1}{4} j^2 + \left(\frac{3}{16} + \frac{1}{4} j\right) D_{u,v} - \frac{1}{16} D_{u,v}^2\right) \frac{\alpha_{u,v}}{4} \left(b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)}\right)\Big\}$$

$$\times \cos((j+2)\lambda_u - j\lambda_v - 2\bar{\omega}_u)$$

$$+ \Big\{ e_u^2 \left(-\frac{5}{8} j + \frac{1}{2} j^2 + \left(\frac{1}{2} j - \frac{3}{8}\right) D_{u,v} + \frac{1}{8} D_{u,v}^2\right) b_{1/2}^{(j,u,v)}$$

$$+ e_u^4 \left(\frac{11}{48} j - \frac{2}{3} j^2 + \frac{5}{8} j^3 - \frac{1}{6} j^4 + \left(\frac{11}{48} - \frac{47}{96} j + \frac{1}{2} j^2 - \frac{1}{6} j^3\right) D_{u,v} + \left(-\frac{1}{96} - \frac{1}{32} j\right) D_{u,v}^2\right.$$

$$\left. + \left(-\frac{1}{16} + \frac{1}{24} j\right) D_{u,v}^3 + \frac{1}{96} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$+ e_u^2 e_v^2 \left(\frac{5}{8} j^3 - \frac{1}{2} j^4 + \left(-\frac{5}{32} j + \frac{1}{2} j^2 - \frac{1}{2} j^3\right) D_{u,v} + \left(-\frac{3}{32} - \frac{1}{32} j\right) D_{u,v}^2 + \left(-\frac{1}{16} + \frac{1}{8} j\right) D_{u,v}^3\right.$$

$$\left. + \frac{1}{32} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$+ (\gamma_u^2 e_u^2 + \gamma_v^2 e_u^2) \left(\frac{5}{16} j - \frac{1}{4} j^2 + \left(\frac{3}{16} - \frac{1}{4} j\right) D_{u,v} - \frac{1}{16} D_{u,v}^2\right) \frac{\alpha_{u,v}}{4} (b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)})\Big\}$$

$$\times \cos(j-2)\lambda_u - j\lambda_v + 2\bar{\omega}_u)$$

$$+ \Big\{ e_u e_v \left(\frac{1}{2} j - j^2 + \left(-\frac{1}{4} + j\right) D_{u,v} - \frac{1}{4} D_{u,v}^2\right) b_{1/2}^{(j,u,v)}$$

$$+ e_u^3 e_v \left(-\frac{1}{16} j - \frac{3}{16} j^2 - \frac{3}{8} j^3 + \frac{1}{2} j^4 + \left(\frac{3}{32} - \frac{1}{2} j^2 - \frac{1}{2} j^3\right) D_{u,v} + \left(\frac{1}{8} + \frac{5}{32} j\right) D_{u,v}^2 + \frac{1}{8} j D_{u,v}^3\right.$$

$$- \frac{1}{32} D_{u,v}^4 \Big) \; b_{1/2}^{(j,u,v)}$$

$$+ e_u e_v^3 \left(- \frac{1}{16} j + \frac{5}{16} j^2 - \frac{7}{8} j^3 + \frac{1}{2} j^4 + \left(\frac{1}{32} - \frac{1}{32} j + \frac{1}{2} j^2 - \frac{1}{2} j^3 \right) D_{u,v} + \left(- \frac{1}{16} + \frac{7}{32} j \right) D_{u,v}^2 \right.$$

$$\left. + \left(- \frac{1}{8} + \frac{1}{8} j \right) D_{u,v}^3 - \frac{1}{32} D_{u,v}^4 \right) b_{1/2}^{(j,u,v)}$$

$$+ (\gamma_u^2 e_u e_v + \gamma_v^2 e_u e_v) \left(- \frac{1}{4} j + \frac{1}{2} j^2 + \left(\frac{1}{8} - \frac{1}{2} j \right) D_{u,v} + \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} \left(b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)} \right) \Big\}$$

$$\times \cos \left((j+1) \lambda_u - (j-1) \lambda_v - \bar{\omega}_u - \bar{\omega}_v \right)$$

$$+ \Big\{ e_u e_v \left(\frac{1}{2} j + j^2 - \frac{1}{4} D_{u,v} - \frac{1}{4} D_{u,v}^2 \right) b_{1/2}^{(j,u,v)}$$

$$+ e_u^3 e_v \left(- \frac{1}{16} j - \frac{7}{16} j^2 - \frac{7}{8} j^3 - \frac{1}{2} j^4 + \left(\frac{3}{32} + \frac{9}{32} j + \frac{1}{8} j^2 \right) D_{u,v} + \left(\frac{1}{8} + \frac{9}{32} j + \frac{1}{4} j^2 \right) D_{u,v}^2 \right.$$

$$\left. - \frac{1}{32} D_{u,v}^4 \right) b_{1/2}^{(j,u,v)}$$

$$+ e_u e_v^3 \left(- \frac{1}{16} j - \frac{5}{16} j^2 - \frac{7}{8} j^3 - \frac{1}{2} j^4 + \left(\frac{1}{32} + \frac{9}{32} j + \frac{3}{8} j^2 \right) D_{u,v} + \left(- \frac{1}{16} + \frac{9}{32} j + \frac{1}{4} j^2 \right) D_{u,v}^2 \right.$$

$$\left. - \frac{1}{8} D_{u,v}^3 - \frac{1}{32} D_{u,v}^4 \right) b_{1/2}^{(j,u,v)}$$

$$+ (\gamma_u^2 e_u e_v + \gamma_v^2 e_u e_v) \left(- \frac{1}{4} j - \frac{1}{2} j^2 + \frac{1}{8} D_{u,v} + \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} \left(b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)} \right) \Big\}$$

$$\times \cos \left((j+1) \lambda_u - (j+1) \lambda_v - \bar{\omega}_u + \bar{\omega}_v \right)$$

$$+ \Big\{ e_u e_v \left(- \frac{1}{2} j + j^2 - \frac{1}{4} D_{u,v} - \frac{1}{4} D_{u,v}^2 \right) b_{1/2}^{(j,u,v)}$$

$$+ e_u^3 e_v \left(\frac{1}{16} j - \frac{7}{16} j^2 + \frac{7}{8} j^3 - \frac{1}{2} j^4 + \left(\frac{3}{32} - \frac{9}{32} j + \frac{1}{8} j^2 \right) D_{u,v} + \left(\frac{1}{8} - \frac{9}{32} j + \frac{1}{4} j^2 \right) D_{u,v}^2 \right.$$

$$\left. - \frac{1}{32} D_{u,v}^4 \right) b_{1/2}^{(j,u,v)}$$

$$+ e_u e_v^3 \left(\frac{1}{16} j - \frac{5}{16} j^2 + \frac{7}{8} j^3 - \frac{1}{2} j^4 + \left(\frac{1}{32} - \frac{9}{32} j + \frac{3}{8} j^2\right) D_{u,v} + \left(-\frac{1}{16} - \frac{9}{32} j + \frac{1}{4} j^2\right) D_{u,v}^2\right.$$

$$\left. - \frac{1}{8} D_{u,v}^3 - \frac{1}{32} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$+ (\gamma_u^2 e_u e_v + \gamma_v^2 e_u e_v) \left(\frac{1}{4} j - \frac{1}{2} j^2 + \frac{1}{8} D_{u,v} + \frac{1}{8} D_{u,v}^2\right) \frac{\alpha_{u,v}}{4} (b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)})\Big\}$$

$$\times \cos((j-1)\lambda_u - (j-1)\lambda_v + \bar{\omega}_u - \bar{\omega}_v)$$

$$+ \Big\{ e_u e_v \left(-\frac{1}{2} j - j^2 + \left(-\frac{1}{4} - j\right) D_{u,v} - \frac{1}{4} D_{u,v}^2\right) b_{1/2}^{(j,u,v)}$$

$$+ e_u^3 e_v \left(\frac{1}{16} j - \frac{3}{16} j^2 - \frac{3}{8} j^3 + \frac{1}{2} j^4 + \left(\frac{3}{32} + \frac{3}{32} j - \frac{1}{2} j^2 + \frac{1}{2} j^3\right) D_{u,v} + \left(\frac{1}{8} - \frac{5}{32} j\right) D_{u,v}^2\right.$$

$$\left. - \frac{1}{8} j D_{u,v}^3 - \frac{1}{32} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$+ e_u e_v^3 \left(\frac{1}{16} j + \frac{5}{16} j^2 + \frac{7}{8} j^3 + \frac{1}{2} j^4 + \left(\frac{1}{32} + \frac{1}{32} j + \frac{1}{2} j^2 + \frac{1}{2} j^3\right) D_{u,v} + \left(-\frac{1}{16} - \frac{7}{32} j\right) D_{u,v}^2\right.$$

$$\left. + \left(-\frac{1}{8} - \frac{1}{8} j\right) D_{u,v}^3 - \frac{1}{32} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$+ (\gamma_u^2 e_u e_v + \gamma_v^2 e_u e_v) \left(\frac{1}{4} j + \frac{1}{2} j^2 + \left(\frac{1}{8} + \frac{1}{2} j\right) D_{u,v} + \frac{1}{8} D_{u,v}^2\right) \frac{\alpha_{u,v}}{4} (b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)})\Big\}$$

$$\times \cos((j-1)\lambda_u - (j+1)\lambda_v + \bar{\omega}_u + \bar{\omega}_v)$$

$$+ \Big\{ e_v^2 \left(\frac{1}{2} - \frac{9}{8} j + \frac{1}{2} j^2 + \left(\frac{5}{8} - \frac{1}{2} j\right) D_{u,v} + \frac{1}{8} D_{u,v}^2\right) b_{1/2}^{(j,u,v)}$$

$$+ e_u^2 e_v^2 \left(-\frac{1}{2} j^2 + \frac{9}{8} j^3 - \frac{1}{2} j^4 + \left(\frac{1}{8} - \frac{9}{32} j - \frac{1}{2} j^2 + \frac{1}{2} j^3\right) D_{u,v} + \left(\frac{9}{32} - \frac{13}{32} j\right) D_{u,v}^2\right.$$

$$\left. + \left(\frac{3}{16} - \frac{1}{8} j\right) D_{u,v}^3 + \frac{1}{32} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$+ e_v^4 \left(-\frac{1}{6} + \frac{31}{48} j - \frac{7}{6} j^2 + \frac{19}{24} j^3 - \frac{1}{6} j^4 + \left(-\frac{1}{48} + \frac{29}{96} j - \frac{1}{2} j^2 + \frac{1}{6} j^3 \right) D_{u,v} \right.$$

$$\left. + \left(\frac{23}{96} - \frac{5}{32} j \right) D_{u,v}^2 + \left(\frac{5}{48} - \frac{1}{24} j \right) D_{u,v}^3 + \frac{1}{96} D_{u,v}^4 \right) b_{1/2}^{(j,u,v)}$$

$$\left. + (\gamma_u^2 e_v^2 + \gamma_v^2 e_v^2) \left(-\frac{1}{4} + \frac{9}{16} j - \frac{1}{4} j^2 + \left(-\frac{5}{16} + \frac{1}{4} j \right) D_{u,v} - \frac{1}{16} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} \left(b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)} \right) \right\}$$

$$\times \cos \left(j \lambda_u - (j-2) \lambda_v - 2\bar{\omega}_v \right)$$

$$+ \left\{ e_v^2 \left(\frac{1}{2} + \frac{9}{8} j + \frac{1}{2} j^2 + \left(\frac{5}{8} + \frac{1}{2} j \right) D_{u,v} + \frac{1}{8} D_{u,v}^2 \right) b_{1/2}^{(j,u,v)} \right.$$

$$+ e_u^2 e_v^2 \left(-\frac{1}{2} j^2 - \frac{9}{8} j^3 - \frac{1}{2} j^4 + \left(\frac{1}{8} + \frac{9}{32} j - \frac{1}{2} j^2 - \frac{1}{2} j^3 \right) D_{u,v} + \left(\frac{9}{32} + \frac{13}{32} j \right) D_{u,v}^2 \right.$$

$$\left. + \left(\frac{3}{16} + \frac{1}{8} j \right) D_{u,v}^3 + \frac{1}{32} D_{u,v}^4 \right) b_{1/2}^{(j,u,v)}$$

$$+ e_v^4 \left(-\frac{1}{6} - \frac{31}{48} j - \frac{7}{6} j^2 - \frac{19}{24} j^3 - \frac{1}{6} j^4 + \left(-\frac{1}{48} - \frac{29}{96} j - \frac{1}{2} j^2 - \frac{1}{6} j^3 \right) D_{u,v} \right.$$

$$\left. + \left(\frac{23}{96} + \frac{5}{32} j \right) D_{u,v}^2 + \left(\frac{5}{48} + \frac{1}{24} j \right) D_{u,v}^3 + \frac{1}{96} D_{u,v}^4 \right) b_{1/2}^{(j,u,v)}$$

$$\left. + (\gamma_u^2 e_v^2 + \gamma_v^2 e_v^2) \left(-\frac{1}{4} - \frac{9}{16} j - \frac{1}{4} j^2 + \left(-\frac{5}{16} - \frac{1}{4} j \right) D_{u,v} - \frac{1}{16} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} \left(b_{3/2}^{(j-1,u,v)} + b_{3/2}^{(j+1,u,v)} \right) \right\}$$

$$\times \cos \left(j \lambda_u - (j+2) \lambda_v + 2\bar{\omega}_v \right)$$

$$+ e_u^3 \left(\frac{13}{24} j + \frac{5}{8} j^2 + \frac{1}{6} j^3 + \left(-\frac{17}{48} - \frac{11}{16} j - \frac{1}{4} j^2 \right) D_{u,v} + \left(\frac{3}{16} + \frac{1}{8} j \right) D_{u,v}^2 - \frac{1}{48} D_{u,v}^3 \right) b_{1/2}^{(j,u,v)}$$

$$\times \cos \left((j+3) \lambda_u - j \lambda_v - 3\bar{\omega}_u \right)$$

$$+ e_u^3 \left(-\frac{13}{24} j + \frac{5}{8} j^2 - \frac{1}{6} j^3 + \left(-\frac{17}{48} + \frac{11}{16} j - \frac{1}{4} j^2 \right) D_{u,v} + \left(\frac{3}{16} - \frac{1}{8} j \right) D_{u,v}^2 - \frac{1}{48} D_{u,v}^3 \right) b_{1/2}^{(j,u,v)}$$

$$\times \cos (j-3) \lambda_u - j \lambda_v + 3\bar{\omega}_u)$$

$$+e_u^2 e_v \left(\frac{5}{16} j - \frac{3}{8} j^2 - \frac{1}{2} j^3 + \left(-\frac{3}{16} + \frac{7}{16} j + \frac{3}{4} j^2\right) D_{u,v} + \left(-\frac{1}{8} - \frac{3}{8} j\right) D_{u,v}^2 + \frac{1}{16} D_{u,v}^3\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos((j+2)\lambda_u - (j-1)\lambda_v - 2\bar{\omega}_u - \bar{\omega}_v)$$

$$+e_u^2 e_v \left(-\frac{5}{16} j - \frac{3}{8} j^2 + \frac{1}{2} j^3 + \left(-\frac{3}{16} - \frac{7}{16} j + \frac{3}{4} j^2\right) D_{u,v} + \left(-\frac{1}{8} + \frac{3}{8} j\right) D_{u,v}^2 + \frac{1}{16} D_{u,v}^3\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos((j-2)\lambda_u - (j+1)\lambda_v + 2\bar{\omega}_u + \bar{\omega}_v)$$

$$+e_u^2 e_v \left(\frac{5}{16} j + \frac{7}{8} j^2 + \frac{1}{2} j^3 + \left(-\frac{3}{16} - \frac{5}{16} j - \frac{1}{4} j^2\right) D_{u,v} + \left(-\frac{1}{8} - \frac{1}{8} j\right) D_{u,v}^2 + \frac{1}{16} D_{u,v}^3\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos((j+2)\lambda_u - (j+1)\lambda_v - 2\bar{\omega}_u + \bar{\omega}_v)$$

$$+e_u^2 e_v \left(-\frac{5}{16} j + \frac{7}{8} j^2 - \frac{1}{2} j^3 + \left(-\frac{3}{16} + \frac{5}{16} j - \frac{1}{4} j^2\right) D_{u,v} + \left(-\frac{1}{8} + \frac{1}{8} j\right) D_{u,v}^2 + \frac{1}{16} D_{u,v}^3\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos((j-2)\lambda_u - (j-1)\lambda_v + 2\bar{\omega}_u - \bar{\omega}_v)$$

$$+e_u e_v^2 \left(\frac{1}{2} j - \frac{9}{8} j^2 + \frac{1}{2} j^3 + \left(-\frac{1}{4} + \frac{19}{16} j - \frac{3}{4} j^2\right) D_{u,v} + \left(-\frac{5}{16} + \frac{3}{8} j\right) D_{u,v}^2 - \frac{1}{16} D_{u,v}^3\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos((j+1)\lambda_u - (j-2)\lambda_v - \bar{\omega}_u - 2\bar{\omega}_v)$$

$$+e_u e_v^2 \left(\frac{1}{2} j + \frac{9}{8} j^2 + \frac{1}{2} j^3 + \left(-\frac{1}{4} + \frac{1}{16} j + \frac{1}{4} j^2\right) D_{u,v} + \left(-\frac{5}{16} - \frac{1}{8} j\right) D_{u,v}^2 - \frac{1}{16} D_{u,v}^3\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos((j+1)\lambda_u - (j+2)\lambda_v - \bar{\omega}_u + 2\bar{\omega}_v)$$

$$+e_u e_v^2 \left(-\frac{1}{2} j + \frac{9}{8} j^2 - \frac{1}{2} j^3 + \left(-\frac{1}{4} - \frac{1}{16} j + \frac{1}{4} j^2\right) D_{u,v} + \left(-\frac{5}{16} + \frac{1}{8} j\right) D_{u,v}^2 - \frac{1}{16} D_{u,v}^3\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos((j-1)\lambda_u - (j-2)\lambda_v + \bar{\omega}_u - 2\bar{\omega}_v)$$

$$+ e_u e_v^2 \left(-\frac{1}{2} j - \frac{9}{8} j^2 - \frac{1}{2} j^3 + \left(-\frac{1}{4} - \frac{19}{16} j - \frac{3}{4} j^2\right) D_{u,v} + \left(-\frac{5}{16} - \frac{3}{8} j\right) D_{u,v}^2 - \frac{1}{16} D_{u,v}^3\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos((j-1)\lambda_u - (j+2)\lambda_v + \bar{\omega}_u + 2\bar{\omega}_v)$$

$$+ e_v^3 \left(\frac{27}{48} - \frac{65}{48} j + \frac{7}{8} j^2 - \frac{1}{6} j^3 + \left(\frac{19}{24} - \frac{15}{16} j + \frac{1}{4} j^2\right) D_{u,v} + \left(\frac{1}{4} - \frac{1}{8} j\right) D_{u,v}^2 + \frac{1}{48} D_{u,v}^3\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos(j\lambda_u - (j-3)\lambda_v - 3\bar{\omega}_v)$$

$$+ e_v^3 \left(\frac{27}{48} + \frac{65}{48} j + \frac{7}{8} j^2 + \frac{1}{6} j^3 + \left(\frac{19}{24} + \frac{15}{16} j + \frac{1}{4} j^2\right) D_{u,v} + \left(\frac{1}{4} + \frac{1}{8} j\right) D_{u,v}^2 + \frac{1}{48} D_{u,v}^3\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos(j\lambda_u - (j+3)\lambda_v + 3\bar{\omega}_v)$$

$$+ e_u^4 \left(\frac{103}{192} j + \frac{283}{384} j^2 + \frac{5}{16} j^3 + \frac{1}{24} j^4 + \left(-\frac{71}{192} - \frac{55}{64} j - \frac{1}{2} j^2 - \frac{1}{12} j^3\right) D_{u,v}\right.$$

$$\left. + \left(\frac{95}{384} + \frac{17}{64} j + \frac{1}{16} j^2\right) D_{u,v}^2 + \left(-\frac{3}{64} - \frac{1}{48} j\right) D_{u,v}^3 + \frac{1}{384} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos((j+4)\lambda_u - j\lambda_v - 4\bar{\omega}_u)$$

$$+ e_u^4 \left(-\frac{103}{192} j + \frac{283}{384} j^2 - \frac{5}{16} j^3 + \frac{1}{24} j^4 + \left(-\frac{71}{192} + \frac{55}{64} j - \frac{1}{2} j^2 + \frac{1}{12} j^3\right) D_{u,v}\right.$$

$$\left. + \left(\frac{95}{384} - \frac{17}{64} j + \frac{1}{16} j^2\right) D_{u,v}^2 + \left(-\frac{3}{64} + \frac{1}{48} j\right) D_{u,v}^3 + \frac{1}{384} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos((j-4)\lambda_u - j\lambda_v + 4\bar{\omega}_u)$$

$$+ e_u^3 e_v \left(\frac{13}{48} j - \frac{11}{48} j^2 - \frac{13}{24} j^3 - \frac{1}{6} j^4 + \left(-\frac{17}{96} + \frac{27}{96} j + \frac{7}{8} j^2 + \frac{1}{3} j^3\right) D_{u,v}\right.$$

$$\left. + \left(-\frac{1}{12} - \frac{15}{32} j - \frac{1}{4} j^2\right) D_{u,v}^2 + \left(\frac{1}{12} + \frac{1}{12} j\right) D_{u,v}^3 - \frac{1}{96} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos((j+3)\lambda_u - (j-1)\lambda_v - 3\bar{\omega}_u - \bar{\omega}_v)$$

$$+ e_u^3 e_v \left(-\frac{13}{48} j - \frac{11}{48} j^2 + \frac{13}{24} j^3 - \frac{1}{6} j^4 + \left(-\frac{17}{96} - \frac{27}{96} j + \frac{7}{8} j^2 - \frac{1}{3} j^3 \right) D_{u,v} \right.$$

$$\left. + \left(-\frac{1}{12} + \frac{15}{32} j - \frac{1}{4} j^2 \right) D^2_{u,v} + \left(\frac{1}{12} - \frac{1}{12} j \right) D^3_{u,v} - \frac{1}{96} D^4_{u,v} \right) b_{1/2}^{(j,u,v)}$$

$$\times \cos((j-3)\lambda_u - (j+1)\lambda_v + 3\bar{\omega}_u + \bar{\omega}_v)$$

$$+ e_u^3 e_v \left(\frac{13}{48} j + \frac{41}{48} j^2 + \frac{17}{24} j^3 + \frac{1}{6} j^4 + \left(-\frac{17}{96} - \frac{41}{96} j - \frac{1}{2} j^2 - \frac{1}{6} j^3 \right) D_{u,v} \right.$$

$$\left. + \left(-\frac{1}{12} - \frac{3}{32} j \right) D^2_{u,v} + \left(\frac{1}{12} + \frac{1}{24} j \right) D^3_{u,v} - \frac{1}{96} D^4_{u,v} \right) b_{1/2}^{(j,u,v)}$$

$$\times \cos((j+3)\lambda_u - (j+1)\lambda_v - 3\bar{\omega}_u + \bar{\omega}_v)$$

$$+ e_u^3 e_v \left(-\frac{13}{48} j + \frac{41}{48} j^2 - \frac{17}{24} j^3 + \frac{1}{6} j^4 + \left(-\frac{17}{96} + \frac{41}{96} j - \frac{1}{2} j^2 + \frac{1}{6} j^3 \right) D_{u,v} \right.$$

$$\left. + \left(-\frac{1}{12} + \frac{3}{32} j \right) D^2_{u,v} + \left(\frac{1}{12} - \frac{1}{24} j \right) D^3_{u,v} - \frac{1}{96} D^4_{u,v} \right) b_{1/2}^{(j,u,v)}$$

$$\times \cos(j-3)\lambda_u - (j-1)\lambda_v + 3\bar{\omega}_u - \bar{\omega}_v)$$

$$+ e_u^2 e_v^2 \left(\frac{5}{16} j - \frac{29}{64} j^2 - \frac{1}{4} j^3 + \frac{1}{4} j^4 + \left(-\frac{3}{16} + \frac{9}{16} j + \frac{3}{8} j^2 - \frac{1}{2} j^3 \right) D_{u,v} \right.$$

$$\left. + \left(-\frac{11}{64} - \frac{3}{16} j + \frac{3}{8} j^2 \right) D^2_{u,v} + \left(\frac{1}{32} - \frac{1}{8} j \right) D^3_{u,v} + \frac{1}{64} D^4_{u,v} \right) b_{1/2}^{(j,u,v)}$$

$$\times \cos((j+2)\lambda_u - (j-2)\lambda_v - 2\bar{\omega}_u - 2\bar{\omega}_v)$$

$$+ e_u^2 e_v^2 \left(-\frac{5}{16} j - \frac{29}{64} j^2 + \frac{1}{4} j^3 + \frac{1}{4} j^4 + \left(-\frac{3}{16} - \frac{9}{16} j + \frac{3}{8} j^2 + \frac{1}{2} j^3 \right) D_{u,v} \right.$$

$$\left. + \left(-\frac{11}{64} + \frac{3}{16} j + \frac{3}{8} j^2 \right) D^2_{u,v} + \left(\frac{1}{32} + \frac{1}{8} j \right) D^3_{u,v} + \frac{1}{64} D^4_{u,v} \right) b_{1/2}^{(j,u,v)}$$

$$\times \cos((j-2)\lambda_u - (j+2)\lambda_v + 2\bar{\omega}_u + 2\bar{\omega}_v)$$

$$+ e_u^2 e_v^2 \left(\frac{5}{16} j + \frac{61}{64} j^2 + \frac{7}{8} j^3 + \frac{1}{4} j^4 + \left(-\frac{3}{16} - \frac{9}{32} j - \frac{1}{8} j^2\right) D_{u,v}\right.$$

$$\left. + \left(-\frac{11}{64} - \frac{9}{32} j - \frac{1}{8} j^2\right) D_{u,v}^2 + \frac{1}{32} D_{u,v}^3 + \frac{1}{64} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos\left((j + 2) \lambda_u - (j + 2) \lambda_v - 2 \bar{\omega}_u + 2 \bar{\omega}_v\right)$$

$$+ e_u^2 e_v^2 \left(-\frac{5}{16} j + \frac{61}{64} j^2 - \frac{7}{8} j^3 + \frac{1}{4} j^4 + \left(-\frac{3}{16} + \frac{9}{32} j - \frac{1}{8} j^2\right) D_{u,v}\right.$$

$$\left. + \left(-\frac{11}{64} + \frac{9}{32} j - \frac{1}{8} j^2\right) D_{u,v}^2 + \frac{1}{32} D_{u,v}^3 + \frac{1}{64} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos\left((j - 2) \lambda_u - (j - 2) \lambda_v + 2 \bar{\omega}_u - 2 \bar{\omega}_v\right)$$

$$+ e_u e_v^3 \left(\frac{27}{48} j - \frac{65}{48} j^2 + \frac{7}{8} j^3 - \frac{1}{6} j^4 + \left(-\frac{27}{96} + \frac{47}{32} j - \frac{11}{8} j^2 + \frac{1}{3} j^3\right) D_{u,v}\right.$$

$$\left. + \left(-\frac{19}{48} + \frac{23}{32} j - \frac{1}{4} j^2\right) D_{u,v}^2 + \left(-\frac{1}{8} + \frac{1}{12} j\right) D_{u,v}^3 - \frac{1}{96} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos\left((j + 1) \lambda_u - (j - 3) \lambda_v - \bar{\omega}_u - 3 \bar{\omega}_v\right)$$

$$+ e_u e_v^3 \left(-\frac{27}{48} j - \frac{65}{48} j^2 - \frac{7}{8} j^3 - \frac{1}{6} j^4 + \left(-\frac{27}{96} - \frac{47}{32} j - \frac{11}{8} j^2 - \frac{1}{3} j^3\right) D_{u,v}\right.$$

$$\left. + \left(-\frac{19}{48} - \frac{23}{32} j - \frac{1}{4} j^2\right) D_{u,v}^2 + \left(-\frac{1}{8} - \frac{1}{12} j\right) D_{u,v}^3 - \frac{1}{96} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos\left((j - 1) \lambda_u - (j + 3) \lambda_v + \bar{\omega}_u + 3 \bar{\omega}_v\right)$$

$$+ e_u e_v^3 \left(\frac{27}{48} j + \frac{65}{48} j^2 + \frac{7}{8} j^3 + \frac{1}{6} j^4 + \left(-\frac{27}{96} + \frac{11}{96} j + \frac{1}{2} j^2 + \frac{1}{6} j^3\right) D_{u,v}\right.$$

$$\left. + \left(-\frac{19}{48} - \frac{7}{32} j\right) D_{u,v}^2 + \left(-\frac{1}{8} - \frac{1}{24} j\right) D_{u,v}^3 - \frac{1}{96} D_{u,v}^4\right) b_{1/2}^{(j,u,v)}$$

$$\times \cos\left((j + 1) \lambda_u - (j + 3) \lambda_v - \bar{\omega}_u + 3 \bar{\omega}_v\right)$$

$$+ e_u e_v^3 \left(-\frac{27}{48} j + \frac{65}{48} j^2 - \frac{7}{8} j^3 + \frac{1}{6} j^4 + \left(-\frac{27}{96} - \frac{11}{96} j + \frac{1}{2} j^2 - \frac{1}{6} j^3 \right) D_{u,v} \right.$$

$$\left. + \left(-\frac{19}{48} + \frac{7}{32} j \right) D^2_{u,v} + \left(-\frac{1}{8} + \frac{1}{24} j \right) D^3_{u,v} - \frac{1}{96} D^4_{u,v} \right) b^{(j,u,v)}_{1/2}$$

$$\times \cos\left((j-1)\lambda_u - (j-3)\lambda_v + \overline{\omega}_u - 3\overline{\omega}_v\right)$$

$$+ e_v^4 \left(\frac{256}{384} - \frac{323}{192} j + \frac{499}{384} j^2 - \frac{19}{48} j^3 + \frac{1}{24} j^4 + \left(\frac{65}{64} - \frac{93}{64} j + \frac{5}{8} j^2 - \frac{1}{12} j^3 \right) D_{u,v} \right.$$

$$\left. + \left(\frac{155}{384} - \frac{21}{64} j + \frac{1}{16} j^2 \right) D^2_{u,v} + \left(\frac{11}{192} - \frac{1}{48} j \right) D^3_{u,v} + \frac{1}{384} D^4_{u,v} \right) b^{(j,u,v)}_{1/2}$$

$$\times \cos\left(j\lambda_u - (j-4)\lambda_v - 4\overline{\omega}_v\right)$$

$$+ e_v^4 \left(\frac{256}{384} + \frac{323}{192} j + \frac{499}{384} j^2 + \frac{19}{48} j^3 + \frac{1}{24} j^4 + \left(\frac{65}{64} + \frac{93}{64} j + \frac{5}{8} j^2 + \frac{1}{12} j^3 \right) D_{u,v} \right.$$

$$\left. + \left(\frac{155}{384} + \frac{21}{64} j + \frac{1}{16} j^2 \right) D^2_{u,v} + \left(\frac{11}{192} + \frac{1}{48} j \right) D^3_{u,v} + \frac{1}{384} D^4_{u,v} \right) b^{(j,u,v)}_{1/2}$$

$$\times \cos\left(j\lambda_u - (j+4)\lambda_v + 4\overline{\omega}_v\right)$$

$$+ \left\{ \left(\frac{1}{4} \gamma_u \gamma_v + \frac{1}{32} \gamma_u \gamma_v^3 + \frac{1}{32} \gamma_u^3 \gamma_v \right) \alpha_{u,v}\, b^{(j-1,u,v)}_{3/2} \right.$$

$$+ \gamma_u \gamma_v e_u^2 \left(-\frac{1}{2} j^2 + \frac{1}{8} D_{u,v} + \frac{1}{8} D^2_{u,v} \right) \frac{\alpha_{u,v}}{2} b^{(j-1,u,v)}_{3/2}$$

$$+ \gamma_u \gamma_v e_v^2 \left(-\frac{1}{2} j^2 + \frac{1}{8} D_{u,v} + \frac{1}{8} D^2_{u,v} \right) \frac{\alpha_{u,v}}{2} b^{(j-1,u,v)}_{3/2}$$

$$+ (-\gamma_u \gamma_v^3 - \gamma_v \gamma_u^3) \frac{\alpha_{u,v}}{32} b^{(j-1,u,v)}_{3/2}$$

$$+ (-\gamma_u^3 \gamma_v - \gamma_u \gamma_v^3) \frac{3}{16} \alpha^2_{u,v}\, b^{(j,u,v)}_{5/2}$$

$$\left. + (-\gamma_u^3 \gamma_v - \gamma_u \gamma_v^3) \frac{3}{32} \alpha^2_{u,v}\, b^{(j-2,u,v)}_{5/2} \right\} \cos\left(j\lambda_u - j\lambda_v - \Omega_u + \Omega_v\right)$$

$$+\left\{\left(\frac{1}{4}\gamma_u\,\gamma_v+\frac{1}{32}\,\gamma_u\,\gamma_v^3+\frac{1}{32}\,\gamma_u^3\,\gamma_v\right)\,\alpha_{u,v}\,b_{3/2}^{(j+1,u,v)}\right.$$

$$+\gamma_u\,\gamma_v\,e_u^2\,\left(-\frac{1}{2}\,j^2+\frac{1}{8}\,D_{u,v}+\frac{1}{8}\,D_{u,v}^2\right)\,\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j+1,u,v)}$$

$$+\gamma_u\,\gamma_v\,e_v^2\,\left(-\frac{1}{2}\,j^2+\frac{1}{8}\,D_{u,v}+\frac{1}{8}\,D_{u,v}^2\right)\,\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j+1,u,v)}$$

$$+(-\gamma_u\,\gamma_v^3-\gamma_v\,\gamma_u^3)\,\frac{\alpha_{u,v}}{32}\,b_{3/2}^{(j+1,u,v)}$$

$$+(-\gamma_u^3\,\gamma_v-\gamma_u\,\gamma_v^3)\,\frac{3}{16}\,\alpha_{u,v}^2\,b_{5/2}^{(j,u,v)}$$

$$\left.+(-\gamma_u^3\,\gamma_v-\gamma_u\,\gamma_v^3)\,\frac{3}{32}\,\alpha_{u,v}^2\,b_{5/2}^{(j+2,u,v)}\right\}\,\cos\,(j\,\lambda_u-j\,\lambda_v+\Omega_u-\Omega_v)$$

$$+\gamma_u\,\gamma_v\,e_u\left(\frac{1}{2}\,j-\frac{1}{4}\,D_{u,v}\right)\,\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j-1,u,v)}\,\cos\,((j+1)\,\lambda_u-j\,\lambda_v-\bar{\omega}_u-\Omega_u+\Omega_v)$$

$$+\gamma_u\,\gamma_v\,e_u\,\left(-\frac{1}{2}\,j-\frac{1}{4}\,D_{u,v}\right)\,\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j+1,u,v)}\,\cos\,((j-1)\,\lambda_u-j\,\lambda_v+\bar{\omega}_u+\Omega_u-\Omega_v)$$

$$+\gamma_u\,\gamma_v\,e_u\left(\frac{1}{2}\,j-\frac{1}{4}D_{u,v}\right)\,\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j+1,u,v)}\,\cos\big((j+1)\,\lambda_u-j\,\lambda_v-\bar{\omega}_u+\Omega_u-\Omega_v\big)$$

$$+\gamma_u\,\gamma_v\,e_u\,\left(-\frac{1}{2}\,j-\frac{1}{4}D_{u,v}\right)\,\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j-1,u,v)}\,\cos\big((j-1)\,\lambda_u-j\,\lambda_v+\bar{\omega}_u-\Omega_u+\Omega_v\big)$$

$$+\gamma_u\,\gamma_v\,e_v\,\left(\frac{1}{4}-\frac{1}{2}j+\frac{1}{4}D_{u,v}\right)\,\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j-1,u,v)}\,\cos\,(j\,\lambda_u-(j-1)\,\lambda_v-\bar{\omega}_v-\Omega_u+\Omega_v)$$

$$+\gamma_u\,\gamma_v\,e_v\,\left(\frac{1}{4}+\frac{1}{2}j+\frac{1}{4}D_{u,v}\right)\,\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j+1,u,v)}\,\cos\,(j\,\lambda_u-(j+1)\,\lambda_v+\bar{\omega}_v+\Omega_u-\Omega_v)$$

$$+\gamma_u\,\gamma_v\,e_v\,\left(\frac{1}{4}-\frac{1}{2}j+\frac{1}{4}D_{u,v}\right)\,\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j+1,u,v)}\,\cos\,(j\,\lambda_u-(j-1)\,\lambda_v-\bar{\omega}_v+\Omega_u-\Omega_v)$$

$$+\gamma_u\,\gamma_v\,e_v\,\left(\frac{1}{4}+\frac{1}{2}j+\frac{1}{4}D_{u,v}\right)\,\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j-1,u,v)}\,\cos\,(j\,\lambda_u-(j+1)\,\lambda_v+\bar{\omega}_v-\Omega_u+\Omega_v)$$

$$+ \gamma_u \gamma_v e_u^2 \left(\frac{5}{16} j + \frac{1}{4} j^2 + \left(-\frac{3}{16} - \frac{1}{4} j \right) D_{u,v} + \frac{1}{16} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j-1,u,v)}$$

$$\cos((j+2)\lambda_u - j\lambda_v - 2\bar{\omega}_u - \Omega_u + \Omega_v)$$

$$+ \gamma_u \gamma_v e_u^2 \left(-\frac{5}{16} j + \frac{1}{4} j^2 + \left(-\frac{3}{16} + \frac{1}{4} j \right) D_{u,v} + \frac{1}{16} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j+1,u,v)}$$

$$\cos((j-2)\lambda_u - j\lambda_v + 2\bar{\omega}_u + \Omega_u - \Omega_v)$$

$$+ \gamma_u \gamma_v e_u^2 \left(\frac{5}{16} j + \frac{1}{4} j^2 + \left(-\frac{3}{16} - \frac{1}{4} j \right) D_{u,v} + \frac{1}{16} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j+1,u,v)}$$

$$\cos((j+2)\lambda_u - j\lambda_v - 2\bar{\omega}_u + \Omega_u - \Omega_v)$$

$$+ \gamma_u \gamma_v e_u^2 \left(-\frac{5}{16} j + \frac{1}{4} j^2 + \left(-\frac{3}{16} + \frac{1}{4} j \right) D_{u,v} + \frac{1}{16} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j-1,u,v)}$$

$$\cos((j-2)\lambda_u - j\lambda_v + 2\bar{\omega}_u - \Omega_u + \Omega_v)$$

$$+ \gamma_u \gamma_v e_u e_v \left(\frac{1}{4} j - \frac{1}{2} j^2 + \left(-\frac{1}{8} + \frac{1}{2} j \right) D_{u,v} - \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j-1,u,v)}$$

$$\cos((j+1)\lambda_u - (j-1)\lambda_v - \bar{\omega}_u - \bar{\omega}_v - \Omega_u + \Omega_v)$$

$$+ \gamma_u \gamma_v e_u e_v \left(-\frac{1}{4} j - \frac{1}{2} j^2 + \left(-\frac{1}{8} - \frac{1}{2} j \right) D_{u,v} - \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j+1,u,v)}$$

$$\cos((j-1)\lambda_u - (j+1)\lambda_v + \bar{\omega}_u + \bar{\omega}_v + \Omega_u - \Omega_v)$$

$$+ \gamma_u \gamma_v e_u e_v \left(\frac{1}{4} j - \frac{1}{2} j^2 + \left(-\frac{1}{8} + \frac{1}{2} j \right) D_{u,v} - \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j+1,u,v)}$$

$$\cos((j+1)\lambda_u - (j-1)\lambda_v - \bar{\omega}_u - \bar{\omega}_v + \Omega_u - \Omega_v)$$

$$+ \gamma_u \gamma_v e_u e_v \left(-\frac{1}{4} j - \frac{1}{2} j^2 + \left(-\frac{1}{8} - \frac{1}{2} j\right) D_{u,v} - \frac{1}{8} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b^{(j-1,u,v)}_{3/2}$$

$$\cos\left((j-1)\lambda_u - (j+1)\lambda_v + \bar{\omega}_u + \bar{\omega}_v - \Omega_u + \Omega_v\right)$$

$$+ \gamma_u \gamma_v e_u e_v \left(\frac{1}{4} j + \frac{1}{2} j^2 - \frac{1}{8} D_{u,v} - \frac{1}{8} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b^{(j-1,u,v)}_{3/2}$$

$$\cos\left((j+1)\lambda_u - (j+1)\lambda_v - \bar{\omega}_u + \bar{\omega}_v - \Omega_u + \Omega_v\right)$$

$$+ \gamma_u \gamma_v e_u e_v \left(-\frac{1}{4} j + \frac{1}{2} j^2 - \frac{1}{8} D_{u,v} - \frac{1}{8} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b^{(j+1,u,v)}_{3/2}$$

$$\cos\left((j-1)\lambda_u - (j-1)\lambda_v + \bar{\omega}_u - \bar{\omega}_v + \Omega_u - \Omega_v\right)$$

$$+ \gamma_u \gamma_v e_u e_v \left(\frac{1}{4} j + \frac{1}{2} j^2 - \frac{1}{8} D_{u,v} - \frac{1}{8} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b^{(j+1,u,v)}_{3/2}$$

$$\cos\left((j+1)\lambda_u - (j+1)\lambda_v - \bar{\omega}_u + \bar{\omega}_v + \Omega_u - \Omega_v\right)$$

$$+ \gamma_u \gamma_v e_u e_v \left(-\frac{1}{4} j + \frac{1}{2} j^2 - \frac{1}{8} D_{u,v} - \frac{1}{8} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b^{(j-1,u,v)}_{3/2}$$

$$\cos\left((j-1)\lambda_u - (j-1)\lambda_v + \bar{\omega}_u - \bar{\omega}_v - \Omega_u + \Omega_v\right)$$

$$+ \gamma_u \gamma_v e_v^2 \left(\frac{1}{4} - \frac{9}{16} j + \frac{1}{4} j^2 + \left(\frac{5}{16} - \frac{1}{4} j\right) D_{u,v} + \frac{1}{16} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b^{(j-1,u,v)}_{3/2}$$

$$\cos\left(j\lambda_u - (j-2)\lambda_v - 2\bar{\omega}_v - \Omega_u + \Omega_v\right)$$

$$+ \gamma_u \gamma_v e_v^2 \left(\frac{1}{4} + \frac{9}{16} j + \frac{1}{4} j^2 + \left(\frac{5}{16} + \frac{1}{4} j\right) D_{u,v} + \frac{1}{16} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b^{(j+1,u,v)}_{3/2}$$

$$\cos\left(j\lambda_u - (j+2)\lambda_v + 2\bar{\omega}_v + \Omega_u - \Omega_v\right)$$

$$+ \gamma_u \gamma_v e_v^2 \left(\frac{1}{4} - \frac{9}{16} j + \frac{1}{4} j^2 + \left(\frac{5}{16} - \frac{1}{4} j\right) D_{u,v} + \frac{1}{16} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b^{(j+1,u,v)}_{3/2}$$

$$\cos\left(j\lambda_u - (j-2)\lambda_v - 2\bar{\omega}_v + \Omega_u - \Omega_v\right)$$

$$+ \gamma_u \gamma_v e_v^2 \left(\frac{1}{4} + \frac{9}{16} j + \frac{1}{4} j^2 + \left(\frac{5}{16} + \frac{1}{4} j \right) D_{u,v} + \frac{1}{16} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j-1,u,v)}$$

$$\cos (j\lambda_u - (j+2)\lambda_v + 2\bar{\omega}_v - \Omega_u + \Omega_v)$$

$$+ \gamma_u^2 \gamma_v^2 \left(\frac{1}{32} \alpha_{u,v} b_{3/2}^{(j-1,u,v)} + \frac{3}{32} \alpha_{u,v}^2 b_{5/2}^{(j-2,u,v)} + \frac{3}{64} \alpha_{u,v}^2 b_{5/2}^{(j,u,v)} \right) \cos (j\lambda_u - j\lambda_v - 2\Omega_u + 2\Omega_v)$$

$$+ \gamma_u^2 \gamma_v^2 \left(\frac{1}{32} \alpha_{u,v} b_{3/2}^{(j+1,u,v)} + \frac{3}{32} \alpha_{u,v}^2 b_{5/2}^{(j+2,u,v)} + \frac{3}{64} \alpha_{u,v}^2 b_{5/2}^{(j,u,v)} \right) \cos (j\lambda_u - j\lambda_v + 2\Omega_u - 2\Omega_v)$$

$$+ \left\{ \left(\frac{1}{4} \gamma_v^2 + \frac{1}{16} \gamma_v^4 \right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j,u,v)} \right.$$

$$+ \gamma_v^2 e_u^2 \left(-\frac{1}{2} - j - \frac{1}{2} j^2 + \frac{1}{8} D_{u,v} + \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)}$$

$$+ \gamma_v^2 e_v^2 \left(-\frac{1}{2} + j - \frac{1}{2} j^2 + \frac{1}{8} D_{u,v} + \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)}$$

$$- \gamma_u^2 \gamma_v^2 \frac{\alpha_{u,v}}{32} b_{3/2}^{(j,u,v)}$$

$$+ (-\gamma_u^2 \gamma_v^2 - \gamma_v^4) \frac{3}{64} \alpha_{u,v}^2 b_{5/2}^{(j-1,u,v)}$$

$$\left. + (-5\gamma_u^2 \gamma_v^2 - \gamma_v^4) \frac{3}{64} \alpha_{u,v}^2 b_{5/2}^{(j+1,u,v)} \right\} \cos (j+1)\lambda_u - (j-1)\lambda_v - 2\Omega_v)$$

$$+ \left\{ \left(\frac{1}{4} \gamma_v^2 + \frac{1}{16} \gamma_v^4 \right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j,u,v)} \right.$$

$$+ \gamma_v^2 e_u^2 \left(-\frac{1}{2} - j - \frac{1}{2} j^2 + \frac{1}{8} D_{u,v} + \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)}$$

$$+ \gamma_v^2 e_v^2 \left(-\frac{1}{2} + j - \frac{1}{2} j^2 + \frac{1}{8} D_{u,v} + \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)}$$

$$- \gamma_u^2 \gamma_v^2 \frac{\alpha_{u,v}}{32} b_{3/2}^{(j,u,v)}$$

$$+ (-\gamma_u^2 \gamma_v^2 - \gamma_v^4) \frac{3}{64} \alpha_{u,v}^2 b_{5/2}^{(j+1,u,v)}$$

$$+ (-5\gamma_u^2 \gamma_v^2 - \gamma_v^4) \frac{3}{64} \alpha_{u,v}^2 b_{5/2}^{(j-1,u,v)} \Big\} \cos((j-1)\lambda_u - (j+1)\lambda_v + 2\Omega_v)$$

$$+ \gamma_v^2 e_u \left(\frac{1}{2} + \frac{1}{2} j - \frac{1}{4} D_{u,v}\right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)} \cos((j+2)\lambda_u - (j-1)\lambda_v - \bar{\omega}_u - 2\Omega_v)$$

$$+ \gamma_v^2 e_u \left(-\frac{1}{2} - \frac{1}{2} j - \frac{1}{4} D_{u,v}\right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)} \cos((j-2)\lambda_u - (j+1)\lambda_v + \bar{\omega}_u + 2\Omega_v)$$

$$+ \gamma_v^2 e_u \left(\frac{1}{2} + \frac{1}{2} j - \frac{1}{4} D_{u,v}\right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)} \cos(j\lambda_u - (j+1)\lambda_v - \bar{\omega}_u + 2\Omega_v)$$

$$+ \gamma_v^2 e_u \left(-\frac{1}{2} - \frac{1}{2} j - \frac{1}{4} D_{u,v}\right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)} \cos(j\lambda_u - (j-1)\lambda_v + \bar{\omega}_u - 2\Omega_v)$$

$$+ \gamma_v^2 e_v \left(\frac{3}{4} - \frac{1}{2} j + \frac{1}{4} D_{u,v}\right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)} \cos((j+1)\lambda_u - (j-2)\lambda_v - \bar{\omega}_v - 2\Omega_v)$$

$$+ \gamma_v^2 e_v \left(-\frac{1}{4} + \frac{1}{2} j + \frac{1}{4} D_{u,v}\right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)} \cos((j-1)\lambda_u - (j+2)\lambda_v + \bar{\omega}_v + 2\Omega_v)$$

$$+ \gamma_v^2 e_v \left(\frac{3}{4} - \frac{1}{2} j + \frac{1}{4} D_{u,v}\right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)} \cos((j-1)\lambda_u - j\lambda_v - \bar{\omega}_v + 2\Omega_v)$$

$$+ \gamma_v^2 e_v \left(-\frac{1}{4} + \frac{1}{2} j + \frac{1}{4} D_{u,v}\right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)} \cos((j+1)\lambda_u - j\lambda_v + \bar{\omega}_v - 2\Omega_v)$$

$$+ \gamma_v^2 e_u^2 \left(\frac{9}{16} + \frac{13}{16} j + \frac{1}{4} j^2 + \left(-\frac{7}{16} - \frac{1}{4} j\right) D_{u,v} + \frac{1}{16} D_{u,v}^2\right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)}$$

$$\cos((j+3)\lambda_u - (j-1)\lambda_v - 2\bar{\omega}_u - 2\Omega_v)$$

$$+ \gamma_v^2 e_u^2 \left(-\frac{1}{16} + \frac{3}{16} j + \frac{1}{4} j^2 + \left(\frac{1}{16} + \frac{1}{4} j\right) D_{u,v} + \frac{1}{16} D_{u,v}^2\right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)}$$

$$\cos((j-3)\lambda_u - (j+1)\lambda_v + 2\bar{\omega}_u + 2\Omega_v)$$

$$+\gamma_v^2 e_u^2 \left(\frac{9}{16}+\frac{13}{16}j+\frac{1}{4}j^2+\left(-\frac{7}{16}-\frac{1}{4}j\right)D_{u,v}+\frac{1}{16}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}$$

$$\cos((j+1)\lambda_u-(j+1)\lambda_v-2\bar{\omega}_u+2\Omega_v)$$

$$+\gamma_v^2 e_u^2 \left(-\frac{1}{16}+\frac{3}{16}j+\frac{1}{4}j^2+\left(\frac{1}{16}+\frac{1}{4}j\right)D_{u,v}+\frac{1}{16}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}$$

$$\cos((j-1)\lambda_u-(j-1)\lambda_v+2\bar{\omega}_u-2\Omega_v)$$

$$+\gamma_v^2 e_u e_v \left(\frac{3}{4}+\frac{1}{4}j-\frac{1}{2}j^2+\left(-\frac{1}{8}+\frac{1}{2}j\right)D_{u,v}-\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}$$

$$\cos((j+2)\lambda_u-(j-2)\lambda_v-\bar{\omega}_u-\bar{\omega}_v-2\Omega_v$$

$$+\gamma_v^2 e_u e_v \left(\frac{1}{4}-\frac{1}{4}j-\frac{1}{2}j^2+\left(-\frac{1}{8}-\frac{1}{2}j\right)D_{u,v}-\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}$$

$$\cos((j-2)\lambda_u-(j+2)\lambda_v+\bar{\omega}_u+\bar{\omega}_v+2\Omega_v)$$

$$+\gamma_v^2 e_u e_v \left(\frac{3}{4}+\frac{1}{4}j-\frac{1}{2}j^2+\left(-\frac{1}{8}+\frac{1}{2}j\right)D_{u,v}-\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos(j\lambda_u-j\lambda_v-\bar{\omega}_u-\bar{\omega}_v+2\Omega_v)$$

$$+\gamma_v^2 e_u e_v \left(\frac{1}{4}-\frac{1}{4}j-\frac{1}{2}j^2+\left(-\frac{1}{8}-\frac{1}{2}j\right)D_{u,v}-\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos(j\lambda_u-j\lambda_v+\bar{\omega}_u+\bar{\omega}_v-2\Omega_v)$$

$$+\gamma_v^2 e_u e_v \left(-\frac{1}{4}+\frac{1}{4}j+\frac{1}{2}j^2+\frac{3}{8}D_{u,v}-\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos((j+2)\lambda_u-j\lambda_v-\bar{\omega}_u+\bar{\omega}_v-2\Omega_v)$$

$$+\gamma_v^2 e_u e_v \left(-\frac{3}{4}-\frac{1}{4}j+\frac{1}{2}j^2-\frac{5}{8}D_{u,v}-\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos((j-2)\lambda_u-j\lambda_v+\bar{\omega}_u-\bar{\omega}_v+2\Omega_v)$$

$$+\gamma_v^2\, e_u\, e_v \left(-\frac{1}{4}+\frac{1}{4}j+\frac{1}{2}j^2+\frac{3}{8}D_{u,v}-\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos\left(j\lambda_u-(j+2)\lambda_v-\bar{\omega}_u+\bar{\omega}_v+2\Omega_v\right)$$

$$+\gamma_v^2\, e_u\, e_v \left(-\frac{3}{4}-\frac{1}{4}j+\frac{1}{2}j^2-\frac{5}{8}D_{u,v}-\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos\left(j\lambda_u-(j-2)\lambda_v+\bar{\omega}_u-\bar{\omega}_v-2\Omega_v\right)$$

$$+\gamma_v^2\, e_v^2 \left(\frac{17}{16}-\frac{17}{16}j+\frac{1}{4}j^2+\left(\frac{9}{16}-\frac{1}{4}j\right)D_{u,v}+\frac{1}{16}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}$$

$$\cos\left((j+1)\lambda_u-(j-3)\lambda_v-2\bar{\omega}_v-2\Omega_v\right)$$

$$+\gamma_v^2\, e_v^2 \left(-\frac{1}{16}+\frac{1}{16}j+\frac{1}{4}j^2+\left(\frac{1}{16}+\frac{1}{4}j\right)D_{u,v}+\frac{1}{16}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}$$

$$\cos\left((j-1)\lambda_u-(j+3)\lambda_v+2\bar{\omega}_v+2\Omega_v\right)$$

$$+\gamma_v^2\, e_v^2 \left(\frac{17}{16}-\frac{17}{16}j+\frac{1}{4}j^2+\left(\frac{9}{16}-\frac{1}{4}j\right)D_{u,v}+\frac{1}{16}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}$$

$$\cos\left((j-1)\lambda_u-(j-1)\lambda_v-2\bar{\omega}_v+2\Omega_v\right)$$

$$+\gamma_v^2\, e_v^2 \left(-\frac{1}{16}+\frac{1}{16}j+\frac{1}{4}j^2+\left(\frac{1}{16}+\frac{1}{4}j\right)D_{u,v}+\frac{1}{16}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}$$

$$\cos\left((j+1)\lambda_u-(j+1)\lambda_v+2\bar{\omega}_v-2\Omega_v\right)$$

$$+\left(\frac{1}{4}\gamma_u^2+\frac{1}{16}\gamma_u^4\right)\frac{\alpha_{u,v}}{2}b_{3/2}^{(j,u,v)}\cos\left((j+1)\lambda_u-(j-1)\lambda_v-2\Omega_u\right)$$

$$+\left(\frac{1}{4}\gamma_u^2+\frac{1}{16}\gamma_u^4\right)\frac{\alpha_{u,v}}{2}b_{3/2}^{(j,u,v)}\cos\left((j-1)\lambda_u-(j+1)\lambda_v+2\Omega_u\right)$$

$$+\gamma_u^2\, e_u \left(\frac{1}{2}+\frac{1}{2}j-\frac{1}{4}D_{u,v}\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos\left((j+2)\lambda_u-(j-1)\lambda_v-\bar{\omega}_u-2\Omega_v\right)$$

$$+\gamma_u^2 e_u \left(-\frac{1}{2}-\frac{1}{2}j-\frac{1}{4}D_{u,v}\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos((j-2)\lambda_u-(j+1)\lambda_v+\bar{\omega}_u+2\Omega_u)$$

$$+\gamma_u^2 e_u \left(\frac{1}{2}+\frac{1}{2}j-\frac{1}{4}D_{u,v}\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos(j\lambda_u-(j+1)\lambda_v-\bar{\omega}_u+2\Omega_u)$$

$$+\gamma_u^2 e_u \left(-\frac{1}{2}-\frac{1}{2}j-\frac{1}{4}D_{u,v}\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos(j\lambda_u-(j-1)\lambda_v+\bar{\omega}_u-2\Omega_u)$$

$$+\gamma_u^2 e_v \left(\frac{3}{4}-\frac{1}{2}j+\frac{1}{4}D_{u,v}\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos((j+1)\lambda_u-(j-2)\lambda_v-\bar{\omega}_v-2\Omega_u)$$

$$+\gamma_u^2 e_v \left(-\frac{1}{4}+\frac{1}{2}j+\frac{1}{4}D_{u,v}\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos((j-1)\lambda_u-(j+2)\lambda_v+\bar{\omega}_v+2\Omega_u)$$

$$+\gamma_u^2 e_v \left(\frac{3}{4}-\frac{1}{2}j+\frac{1}{4}D_{u,v}\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos((j-1)\lambda_u-j\lambda_v-\bar{\omega}_v+2\Omega_u)$$

$$+\gamma_u^2 e_v \left(-\frac{1}{4}+\frac{1}{2}j+\frac{1}{4}D_{u,v}\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos((j+1)\lambda_u-j\lambda_v+\bar{\omega}_v-2\Omega_u)$$

$$+\left\{\gamma_u^2 e_u^2\left(-\frac{1}{2}-j-\frac{1}{2}j^2+\frac{1}{8}D_{u,v}+\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\right.$$

$$+\gamma_u^2 e_v^2\left(-\frac{1}{2}+j-\frac{1}{2}j^2+\frac{1}{8}D_{u,v}+\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}$$

$$-\gamma_u^2\gamma_v^2\frac{\alpha_{u,v}}{32}b_{3/2}^{(j,u,v)}$$

$$+\left(-\gamma_u^4-\gamma_u^2\gamma_v^2\right)\frac{3}{64}\alpha_{u,v}^2 b_{5/2}^{(j+1,u,v)}$$

$$\left.+(-\gamma_u^4-5\gamma_u^2\gamma_v^2)\frac{3}{64}\alpha_{u,v}^2 b_{5/2}^{(j-1,u,v)}\right\}\cos((j+1)\lambda_u-(j-1)\lambda_v-2\Omega_u)$$

$$+\left\{\gamma_u^2 e_u^2\left(-\frac{1}{2}-j-\frac{1}{2}j^2+\frac{1}{8}D_{u,v}+\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\right.$$

$$+\gamma_u^2 e_v^2\left(-\frac{1}{2}+j-\frac{1}{2}j^2+\frac{1}{8}D_{u,v}+\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}$$

$$- \gamma_u^2 \gamma_v^2 \frac{\alpha_{u,v}}{32} b_{3/2}^{(j,u,v)}$$

$$+ (-\gamma_u^4 - \gamma_u^2 \gamma_v^2) \frac{3}{64} \alpha_{u,v}^2 b_{5/2}^{(j-1,u,v)}$$

$$+ (-\gamma_u^4 - 5 \gamma_u^2 \gamma_v^2) \frac{3}{64} \alpha_{u,v}^2 b_{5/2}^{(j+1,u,v)} \Big\} \cos((j-1)\lambda_u - (j+1)\lambda_v + 2\Omega_u)$$

$$+ \gamma_u^2 e_u^2 \left(\frac{9}{16} + \frac{13}{16} j + \frac{1}{4} j^2 + \left(-\frac{7}{16} - \frac{1}{4} j \right) D_{u,v} + \frac{1}{16} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)}$$

$$\cos((j+3)\lambda_u - (j-1)\lambda_v - 2\bar{\omega}_u - 2\Omega_u)$$

$$+ \gamma_u^2 e_u^2 \left(-\frac{1}{16} + \frac{3}{16} j + \frac{1}{4} j^2 + \left(\frac{1}{16} + \frac{1}{4} j \right) D_{u,v} + \frac{1}{16} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)}$$

$$\cos((j-3)\lambda_u - (j+1)\lambda_v + 2\bar{\omega}_u + 2\Omega_u)$$

$$+ \gamma_u^2 e_u^2 \left(\frac{9}{16} + \frac{13}{16} j + \frac{1}{4} j^2 + \left(-\frac{7}{16} - \frac{1}{4} j \right) D_{u,v} + \frac{1}{16} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)}$$

$$\cos((j+1)\lambda_u - (j+1)\lambda_v - 2\bar{\omega}_u + 2\Omega_u)$$

$$+ \gamma_u^2 e_u^2 \left(-\frac{1}{16} + \frac{3}{16} j + \frac{1}{4} j^2 + \left(\frac{1}{16} + \frac{1}{4} j \right) D_{u,v} + \frac{1}{16} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)}$$

$$\cos((j-1)\lambda_u - (j-1)\lambda_v + 2\bar{\omega}_u - 2\Omega_u)$$

$$+ \gamma_u^2 e_u e_v \left(\frac{3}{4} + \frac{1}{4} j - \frac{1}{2} j^2 + \left(-\frac{1}{8} + \frac{1}{2} j \right) D_{u,v} - \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)}$$

$$\cos((j+2)\lambda_u - (j-2)\lambda_v - \bar{\omega}_u - \bar{\omega}_v - 2\Omega_u)$$

$$+ \gamma_u^2 e_u e_v \left(\frac{1}{4} - \frac{1}{4} j - \frac{1}{2} j^2 + \left(-\frac{1}{8} - \frac{1}{2} j \right) D_{u,v} - \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)}$$

$$\cos((j-2)\lambda_u - (j+2)\lambda_v + \bar{\omega}_u + \bar{\omega}_v + 2\Omega_u)$$

$$+\gamma_u^2 e_u e_v \left(\frac{3}{4}+\frac{1}{4}j-\frac{1}{2}j^2+\left(-\frac{1}{8}+\frac{1}{2}j\right)D_{u,v}-\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos\left(j\lambda_u-j\lambda_v-\bar\omega_u-\bar\omega_v+2\Omega_u\right)$$

$$+\gamma_u^2 e_u e_v \left(\frac{1}{4}-\frac{1}{4}j-\frac{1}{2}j^2+\left(-\frac{1}{8}-\frac{1}{2}j\right)D_{u,v}-\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos\left(j\lambda_u-j\lambda_v+\bar\omega_u+\bar\omega_v-2\Omega_u\right)$$

$$+\gamma_u^2 e_u e_v \left(-\frac{1}{4}+\frac{1}{4}j+\frac{1}{2}j^2+\frac{3}{8}D_{u,v}-\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos\left((j+2)\lambda_u-j\lambda_v-\bar\omega_u+\bar\omega_v-2\Omega_u\right)$$

$$+\gamma_u^2 e_u e_v \left(-\frac{3}{4}-\frac{1}{4}j+\frac{1}{2}j^2-\frac{5}{8}D_{u,v}-\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos\left((j-2)\lambda_u-j\lambda_v+\bar\omega_u-\bar\omega_v+2\Omega_u\right)$$

$$+\gamma_u^2 e_u e_v \left(-\frac{1}{4}+\frac{1}{4}j+\frac{1}{2}j^2+\frac{3}{8}D_{u,v}-\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos\left(j\lambda_u-(j+2)\lambda_v-\bar\omega_u+\bar\omega_v+2\Omega_u\right)$$

$$+\gamma_u^2 e_u e_v \left(-\frac{3}{4}-\frac{1}{4}j+\frac{1}{2}j^2-\frac{5}{8}D_{u,v}-\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}\cos\left(j\lambda_u-(j-2)\lambda_u+\bar\omega_u-\bar\omega_v-2\Omega_u\right)$$

$$+\gamma_u^2 e_v^2 \left(\frac{17}{16}-\frac{17}{16}j+\frac{1}{4}j^2+\left(\frac{9}{16}-\frac{1}{4}j\right)D_{u,v}+\frac{1}{16}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}$$

$$\cos\left((j+1)\lambda_u-(j-3)\lambda_v-2\bar\omega_v-2\Omega_u\right)$$

$$+\gamma_u^2 e_v^2 \left(-\frac{1}{16}+\frac{1}{16}j+\frac{1}{4}j^2+\left(\frac{1}{16}+\frac{1}{4}j\right)D_{u,v}+\frac{1}{16}D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}b_{3/2}^{(j,u,v)}$$

$$\cos\left((j-1)\lambda_u-(j+3)\lambda_v+2\bar\omega_v+2\Omega_u\right)$$

$$+\gamma_u^2 e_v^2 \left(\frac{17}{16} - \frac{17}{16} j + \frac{1}{4} j^2 + \left(\frac{9}{16} - \frac{1}{4} j \right) D_{u,v} + \frac{1}{16} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)}$$

$$\cos\left((j-1)\lambda_u - (j-1)\lambda_v - 2\bar{\omega}_v + 2\Omega_u\right)$$

$$+\gamma_u^2 e_v^2 \left(-\frac{1}{16} + \frac{1}{16} j + \frac{1}{4} j^2 + \left(\frac{1}{16} + \frac{1}{4} j \right) D_{u,v} + \frac{1}{16} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{4} b_{3/2}^{(j,u,v)}$$

$$\cos\left((j+1)\lambda_u - (j+1)\lambda_v + 2\bar{\omega}_v - 2\Omega_u\right)$$

$$+\left\{ \left(-\frac{1}{4}\gamma_u \gamma_v - \frac{1}{32}\gamma_u \gamma_v^3 - \frac{1}{32}\gamma_u^3 \gamma_v \right) \alpha_{u,v} b_{3/2}^{(j,u,v)} \right.$$

$$+\gamma_u \gamma_v e_u^2 \left(\frac{1}{2} + j + \frac{1}{2} j^2 - \frac{1}{8} D_{u,v} - \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j,u,v)}$$

$$+\gamma_u \gamma_v e_v^2 \left(\frac{1}{2} + j + \frac{1}{2} j^2 - \frac{1}{8} D_{u,v} - \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j,u,v)}$$

$$+(\gamma_u \gamma_v^3 + \gamma_v \gamma_u^3) \frac{\alpha_{u,v}}{32} b_{3/2}^{(j,u,v)}$$

$$+(\gamma_u^3 \gamma_v + 2\gamma_u \gamma_v^3) \frac{3}{32} \alpha_{u,v}^2 b_{5/2}^{(j-1,u,v)}$$

$$\left. +(2\gamma_u^3 \gamma_v + \gamma_u \gamma_v^3) \frac{3}{32} \alpha_{u,v}^2 b_{5/2}^{(j+1,u,v)} \right\} \cos\left((j+1)\lambda_u - (j-1)\lambda_v - \Omega_u - \Omega_v\right)$$

$$+\left\{ \left(-\frac{1}{4}\gamma_u \gamma_v - \frac{1}{32}\gamma_u \gamma_v^3 - \frac{1}{32}\gamma_u^3 \gamma_v \right) \alpha_{u,v} b_{3/2}^{(j,u,v)} \right.$$

$$+\gamma_u \gamma_v e_u^2 \left(\frac{1}{2} + j + \frac{1}{2} j^2 - \frac{1}{8} D_{u,v} - \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j,u,v)}$$

$$+\gamma_u \gamma_v e_v^2 \left(\frac{1}{2} + j + \frac{1}{2} j^2 - \frac{1}{8} D_{u,v} - \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j,u,v)}$$

$$+(\gamma_u \gamma_v^3 + \gamma_v \gamma_u^3) \frac{\alpha_{u,v}}{32} b_{3/2}^{(j,u,v)}$$

$$+ (\gamma_u^3\ \gamma_v + 2\,\gamma_u\ \gamma_v^3)\ \frac{3}{32}\,\alpha_{u,v}^2\ b_{5/2}^{(j+1,u,v)}$$

$$\left. + (2\,\gamma_u^3\ \gamma_v + \gamma_u\ \gamma_v^3)\ \frac{3}{32}\ \alpha_{u,v}^2\ b_{5/2}^{(j-1,u,v)} \right\}\ \cos\left((j-1)\lambda_u - (j+1)\,\lambda_v + \Omega_u + \Omega_v\right)$$

$$+ \gamma_u\,\gamma_v\ e_u\ \left(-\frac{1}{2} - \frac{1}{2}\,j + \frac{1}{4}D_{u,v}\right)\ \frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)} \cos\left((j+2)\,\lambda_u - (j-1)\,\lambda_v - \bar{\omega}_u - \Omega_u - \Omega_v\right)$$

$$+ \gamma_u\,\gamma_v\ e_u\left(\frac{1}{2} + \frac{1}{2}\,j + \frac{1}{4}D_{u,v}\right)\ \frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)} \cos\left((j-2)\lambda_u - (j+1)\,\lambda_v + \bar{\omega}_u + \Omega_u + \Omega_v\right)$$

$$+ \gamma_u\,\gamma_v\ e_u\ \left(-\frac{1}{2} - \frac{1}{2}\,j + \frac{1}{4}D_{u,v}\right)\ \frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)} \cos\left(j\,\lambda_u - (j+1)\,\lambda_v - \bar{\omega}_u + \Omega_u + \Omega_v\right)$$

$$+ \gamma_u\,\gamma_v\ e_u\left(\frac{1}{2} + \frac{1}{2}\,j + \frac{1}{4}D_{u,v}\right)\ \frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)} \cos\left(j\lambda_u - (j-1)\,\lambda_v + \bar{\omega}_u - \Omega_u - \Omega_v\right)$$

$$+ \gamma_u\,\gamma_v\ e_v\ \left(-\frac{3}{4} + \frac{1}{2}\,j - \frac{1}{4}D_{u,v}\right)\ \frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)} \cos\left((j+1)\,\lambda_u - (j-2)\,\lambda_v - \bar{\omega}_v - \Omega_u - \Omega_v\right)$$

$$+ \gamma_u\,\gamma_v\ e_v\ \left(\frac{1}{4} - \frac{1}{2}\,j - \frac{1}{4}D_{u,v}\right)\ \frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)} \cos\left((j-1)\,\lambda_u - (j+2)\,\lambda_v + \bar{\omega}_v + \Omega_u + \Omega_v\right)$$

$$+ \gamma_u\,\gamma_v\ e_v\ \left(-\frac{3}{4} + \frac{1}{2}\,j - \frac{1}{4}D_{u,v}\right)\ \frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)} \cos\left((j-1)\lambda_u - j\,\lambda_v - \bar{\omega}_v + \Omega_u + \Omega_v\right)$$

$$+ \gamma_u\,\gamma_v\ e_v\ \left(\frac{1}{4} - \frac{1}{2}\,j - \frac{1}{4}D_{u,v}\right)\ \frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)} \cos\left((j+1)\,\lambda_u - j\,\lambda_v + \bar{\omega}_v - \Omega_u - \Omega_v\right)$$

$$+ \gamma_u\,\gamma_v\ e_u^2\ \left(-\frac{9}{16} - \frac{13}{16}\,j - \frac{1}{4}\,j^2 + \left(\frac{7}{16} + \frac{1}{4}\,j\right)\ D_{u,v} - \frac{1}{16}\ D_{u,v}^2\right)\ \frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)}$$

$$\cos\left((j+3)\,\lambda_u - (j-1)\,\lambda_v - 2\,\bar{\omega}_u - \Omega_u - \Omega_v\right)$$

$$+ \gamma_u\,\gamma_v\ e_u^2\left(\frac{1}{16} - \frac{3}{16}\,j - \frac{1}{4}\,j^2 + \left(-\frac{1}{16} - \frac{1}{4}\,j\right)\ D_{u,v} - \frac{1}{16}\ D_{u,v}^2\right)\ \frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)}$$

$$\cos\left((j-3)\lambda_u - (j+1)\,\lambda_v + 2\,\bar{\omega}_u + \Omega_u + \Omega_v\right)$$

$$+\gamma_u\,\gamma_v\,e_u^2\left(-\frac{9}{16}-\frac{13}{16}j-\frac{1}{4}j^2+\left(\frac{7}{16}+\frac{1}{4}j\right)D_{u,v}-\frac{1}{16}D_{u,v}^2\right)\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)}$$

$$\cos\left((j+1)\lambda_u-(j+1)\lambda_v-2\,\overline{\omega}_u+\Omega_u+\Omega_v\right)$$

$$+\gamma_u\,\gamma_v\,e_u^2\left(\frac{1}{16}-\frac{3}{16}j-\frac{1}{4}j^2+\left(-\frac{1}{16}-\frac{1}{4}j\right)D_{u,v}-\frac{1}{16}D_{u,v}^2\right)\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)}$$

$$\cos\left((j-1)\lambda_u-(j-1)\lambda_v+2\overline{\omega}_u-\Omega_u-\Omega_v\right)$$

$$+\gamma_u\,\gamma_v\,e_u\,e_v\left(-\frac{3}{4}-\frac{1}{4}j+\frac{1}{2}j^2+\left(\frac{1}{8}-\frac{1}{2}j\right)D_{u,v}+\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)}$$

$$\cos\left((j+2)\lambda_u-(j-2)\lambda_v-\overline{\omega}_u-\overline{\omega}_v-\Omega_u-\Omega_v\right)$$

$$+\gamma_u\,\gamma_v\,e_u\,e_v\left(-\frac{1}{4}+\frac{1}{4}j+\frac{1}{2}j^2+\left(\frac{1}{8}+\frac{1}{2}j\right)D_{u,v}+\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)}$$

$$\cos\left((j-2)\lambda_u-(j+2)\lambda_v+\overline{\omega}_u+\overline{\omega}_v+\Omega_u+\Omega_v\right)$$

$$+\gamma_u\,\gamma_v\,e_u\,e_v\left(-\frac{3}{4}-\frac{1}{4}j+\frac{1}{2}j^2+\left(\frac{1}{8}-\frac{1}{2}j\right)D_{u,v}+\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)}$$

$$\cos\left(j\,\lambda_u-j\,\lambda_v-\overline{\omega}_u-\overline{\omega}_v+\Omega_u+\Omega_v\right)$$

$$+\gamma_u\,\gamma_v\,e_u\,e_v\left(-\frac{1}{4}+\frac{1}{4}j+\frac{1}{2}j^2+\left(\frac{1}{8}+\frac{1}{2}j\right)D_{u,v}+\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)}$$

$$\cos\left(j\,\lambda_u-j\,\lambda_v+\overline{\omega}_u+\overline{\omega}_v-\Omega_u-\Omega_v\right)$$

$$+\gamma_u\,\gamma_v\,e_u\,e_v\left(\frac{1}{4}-\frac{1}{4}j-\frac{1}{2}j^2-\frac{3}{8}D_{u,v}+\frac{1}{8}D_{u,v}^2\right)\frac{\alpha_{u,v}}{2}\,b_{3/2}^{(j,u,v)}$$

$$\cos\left((j+2)\lambda_u-j\,\lambda_v-\overline{\omega}_u+\overline{\omega}_v-\Omega_u-\Omega_v\right)$$

$$+\gamma_u \gamma_v e_u e_v \left(\frac{3}{4}+\frac{1}{4} j - \frac{1}{2} j^2 + \frac{5}{16} D_{u,v} + \frac{1}{8} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j,u,v)}$$

$$\cos\left((j-2)\lambda_u - j\lambda_v + \overline{\omega}_u - \overline{\omega}_v + \Omega_u + \Omega_v\right)$$

$$+\gamma_u \gamma_v e_u e_v \left(\frac{1}{4}-\frac{1}{4} j - \frac{1}{2} j^2 - \frac{3}{8} D_{u,v} + \frac{1}{8} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j,u,v)}$$

$$\cos\left(j\lambda_u - (j+2)\lambda_v - \overline{\omega}_u + \overline{\omega}_v + \Omega_u + \Omega_v\right)$$

$$+\gamma_u \gamma_v e_u e_v \left(\frac{3}{4}+\frac{1}{4} j - \frac{1}{2} j^2 + \frac{5}{16} D_{u,v} + \frac{1}{8} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j,u,v)}$$

$$\cos\left(j\lambda_u - (j-2)\lambda_v + \overline{\omega}_u - \overline{\omega}_v - \Omega_u - \Omega_v\right)$$

$$+\gamma_u \gamma_v e_v^2 \left(-\frac{17}{16}+\frac{17}{16} j - \frac{1}{4} j^2 + \left(-\frac{9}{16}+\frac{1}{4} j\right) D_{u,v} - \frac{1}{16} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j,u,v)}$$

$$\cos\left((j+1)\lambda_u - (j-3)\lambda_v - 2\overline{\omega}_v - \Omega_u - \Omega_v\right)$$

$$+\gamma_u \gamma_v e_v^2 \left(\frac{1}{16}-\frac{1}{16} j - \frac{1}{4} j^2 + \left(-\frac{1}{16}-\frac{1}{4} j\right) D_{u,v} - \frac{1}{16} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j,u,v)}$$

$$\cos\left((j-1)\lambda_u - (j+3)\lambda_v + 2\overline{\omega}_v + \Omega_u + \Omega_v\right)$$

$$+\gamma_u \gamma_v e_v^2 \left(-\frac{17}{16}+\frac{17}{16} j - \frac{1}{4} j^2 + \left(-\frac{9}{16}+\frac{1}{4} j\right) D_{u,v} - \frac{1}{16} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j,u,v)}$$

$$\cos\left((j-1)\lambda_u - (j-1)\lambda_v - 2\overline{\omega}_v + \Omega_u + \Omega_v\right)$$

$$+\gamma_u \gamma_v e_v^2 \left(\frac{1}{16}-\frac{1}{16} j - \frac{1}{4} j^2 + \left(-\frac{1}{16}-\frac{1}{4} j\right) D_{u,v} - \frac{1}{16} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b_{3/2}^{(j,u,v)}$$

$$\cos\left((j+1)\lambda_u - (j+1)\lambda_v + 2\overline{\omega}_u - \Omega_u - \Omega_v\right)$$

$$+\gamma_u \gamma_v^3 \frac{3}{32} \alpha_{u,v}^2 b_{5/2}^{(j+1,u,v)} \cos((j+1)\lambda_u - (j-1)\lambda_v + \Omega_u - 3\Omega_v)$$

$$+\gamma_u \gamma_v^3 \frac{3}{32} \alpha_{u,v}^2 b_{5/2}^{(j-1,u,v)} \cos((j-1)\lambda_u - (j+1)\lambda_v - \Omega_u + 3\Omega_v)$$

$$+\gamma_u^3 \gamma_v \frac{3}{32} \alpha_{u,v}^2 b_{5/2}^{(j-1,u,v)} \cos((j+1)\lambda_u - (j-1)\lambda_v - 3\Omega_u + \Omega_v)$$

$$+\gamma_u^3 \gamma_v \frac{3}{32} \alpha_{u,v}^2 b_{5/2}^{(j+1,u,v)} \cos((j-1)\lambda_u - (j+1)\lambda_v + 3\Omega_u - \Omega_v)$$

$$+\gamma_v^4 \frac{3}{128} \alpha_{u,v}^2 b_{5/2}^{(j,u,v)} \cos((j+2)\lambda_u - (j-2)\lambda_v - 4\Omega_v)$$

$$+\gamma_v^4 \frac{3}{128} \alpha_{u,v}^2 b_{5/2}^{(j,u,v)} \cos((j-2)\lambda_u - (j+2)\lambda_v + 4\Omega_v)$$

$$+\gamma_u^4 \frac{3}{128} \alpha_{u,v}^2 b_{5/2}^{(j,u,v)} \cos((j+2)\lambda_u - (j-2)\lambda_v - 4\Omega_u)$$

$$+\gamma_u^4 \frac{3}{128} \alpha_{u,v}^2 b_{5/2}^{(j,u,v)} \cos((j-2)\lambda_u - (j+2)\lambda_v + 4\Omega_u)$$

$$+\gamma_u^2 \gamma_v^2 \frac{9}{64} \alpha_{u,v}^2 b_{5/2}^{(j,u,v)} \cos((j+2)\lambda_u - (j-2)\lambda_v - 2\Omega_u - 2\Omega_v)$$

$$+\gamma_u^2 \gamma_v^2 \frac{9}{64} \alpha_{u,v}^2 b_{5/2}^{(j,u,v)} \cos((j-2)\lambda_u - (j+2)\lambda_v + 2\Omega_u + 2\Omega_v)$$

$$-\gamma_u \gamma_v^3 \frac{3}{32} \alpha_{u,v}^2 b_{5/2}^{(j,u,v)} \cos((j+2)\lambda_u - (j-2)\lambda_v - \Omega_u - 3\Omega_v)$$

$$-\gamma_u \gamma_v^3 \frac{3}{32} \alpha_{u,v}^2 b_{5/2}^{(j,u,v)} \cos((j-2)\lambda_u - (j+2)\lambda_v + \Omega_u + 3\Omega_v)$$

$$-\gamma_u^3 \gamma_v \frac{3}{32} \alpha_{u,v}^2 b_{5/2}^{(j,u,v)} \cos((j+2)\lambda_u - (j-2)\lambda_v - 3\Omega_u - \Omega_v)$$

$$-\gamma_u^3 \gamma_v \frac{3}{32} \alpha_{u,v}^2 b_{5/2}^{(j,u,v)} \cos((j-2)\lambda_u - (j+2)\lambda_v + 3\Omega_u + \Omega_v)\Big]\Bigg] \qquad (1).$$

2° from (1), we obtain at once the new Hamiltonian $(F_1')_P$ which results from the elimination of the short period terms of $(F_1)_P$ and which is connected to $(F_1)_P$, according to Von Zeipel's method, through the equality:

$$(F_1)_P(a_1, \dots, a_n; e_1, \dots, e_n; \gamma_1, \dots, \gamma_n; \lambda_1, \dots, \lambda_n; \overline{\omega}_1, \dots, \overline{\omega}_n; \Omega_1, \dots \Omega_n)$$

$$= (F_1')_P (a_1', \dots, a_n'; e_1', \dots, e_n'; \gamma_1', \dots, \gamma_n'; \overline{\omega}_1', \dots, \overline{\omega}_n'; \Omega_1', \dots, \Omega_n')$$

the accented letters $a_1', \dots, \Omega_n'$ being the new variables which correspond respectively to the old variables $a_1, \dots, \Omega_n$.

We have:

$$(F_1')_P = \sigma k^2 \Bigg[\sum_{\substack{u \neq v \\ 1 \leq u < v \leq n}} \frac{\beta_u \beta_v}{a_v'} \Bigg[\Bigg\{ b_{1/2}^{(o,u,v)}$$

$$+ e_u'^2 \left(\frac{1}{4} D_{u,v} + \frac{1}{4} D_{u,v}^2 \right) b_{1/2}^{(o,u,v)}$$

$$+ e_v'^2 \left(\frac{1}{4} D_{u,v} + \frac{1}{4} D_{u,v}^2 \right) b_{1/2}^{(o,u,v)}$$

$$+ e_u'^4 \left(\frac{1}{32} D_{u,v} - \frac{1}{64} D_{u,v}^2 - \frac{1}{32} D_{u,v}^3 + \frac{1}{64} D_{u,v}^4 \right) b_{1/2}^{(o,u,v)}$$

$$+ e_u'^2 e_v'^2 \left(\frac{1}{16} D_{u,v}^2 + \frac{1}{8} D_{u,v}^3 + \frac{1}{16} D_{u,v}^4 \right) b_{1/2}^{(o,u,v)}$$

$$+ e_v'^4 \left(\frac{3}{32} D_{u,v} + \frac{11}{64} D_{u,v}^2 + \frac{3}{32} D_{u,v}^3 + \frac{1}{64} D_{u,v}^4 \right) b_{1/2}^{(o,u,v)}$$

$$+ \left(-\gamma_u'^2 - \gamma_v'^2 - \frac{1}{4} \gamma_u'^4 - \frac{1}{4} \gamma_v'^4 \right) \frac{\alpha_{u,v}}{4} \; b_{3/2}^{(1,u,v)}$$

$$+ (-\gamma_u'^2 e_u'^2 - \gamma_v'^2 e_u'^2) \left(\frac{1}{8} D_{u,v} + \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{2} \; b_{3/2}^{(1,u,v)}$$

$$+ (-\gamma_u'^2 e_v'^2 - \gamma_v'^2 e_v'^2) \left(\frac{1}{8} D_{u,v} + \frac{1}{8} D_{u,v}^2 \right) \frac{\alpha_{u,v}}{2} \; b_{3/2}^{(1,u,v)}$$

$$+ \gamma_u'^2 \gamma_v'^2 \frac{\alpha_{u,v}}{16} b_{3/2}^{(1,u,v)}$$

$$+ (\gamma_u'^4 + \gamma_v'^4 + 5\,\gamma_u'^2\,\gamma_v'^2)\ \frac{3}{32}\ \alpha_{u,v}^2\ b_{5/2}^{(o,u,v)}$$

$$+\left(\frac{1}{2}\gamma_u'^4 + \frac{1}{2}\ \gamma_v'^4 + \gamma_u'^2\,\gamma_v'^2\right)\frac{3}{32}\ \alpha_{u,v}^2\ b_{5/2}^{(2,u,v)}\Bigg\}$$

$$+\ \Bigg\{e_u'\ e_v'\ \left(1 - \frac{1}{2}\ D_{u,v} - \frac{1}{2}\ D_{u,v}^2\right)\ b_{1/2}^{(1,u,v)}$$

$$+ e_u'^3\ e_v'\ \left(-\frac{1}{8}\ D_{u,v} + \frac{3}{16}\ D_{u,v}^2 - \frac{1}{16}\ D_{u,v}^4\right) b_{1/2}^{(1,u,v)}$$

$$+ e_u'\,e_v'^3\left(\frac{1}{4} + \frac{1}{4}\ D_{u,v} - \frac{3}{16}\ D_{u,v}^2 - \frac{1}{4}\ D_{u,v}^3 - \frac{1}{16}\ D_{u,v}^4\right) b_{1/2}^{(1,u,v)}$$

$$+ (\gamma_u'^2\ e_u'\ e_v' + \gamma_v'^2\,e_u'\,e_v')\ \left(-\frac{1}{2} + \frac{1}{4}\,D_{u,v} + \frac{1}{4}\ D_{u,v}^2\right)\frac{\alpha_{u,v}}{4}\ \left(b_{3/2}^{(2,u,v)} + b_{3/2}^{(o,u,v)}\right)\Bigg\}\ \cos(\bar{\omega}_u' - \bar{\omega}_v')$$

$$+ e_u'^2\ e_v'^2\left(\frac{3}{8} - \frac{1}{4}\ D_{u,v} - \frac{7}{32}\ D_{u,v}^2 + \frac{1}{16}\,D_{u,v}^3 + \frac{1}{32}\ D_{u,v}^4\right)\ b_{1/2}^{(2,u,v)}\ \cos\,(2\,\bar{\omega}_u' - 2\,\bar{\omega}_v')$$

$$+\Bigg\{\left(\frac{1}{2}\ \gamma_u'\ \gamma_v' + \frac{1}{16}\ \gamma_u'\,\gamma_v'^3 + \frac{1}{16}\ \gamma_u'^3\ \gamma_v'\right)\ \alpha_{u,v}\ b_{3/2}^{(1,u,v)}$$

$$+ \gamma_u'\ \gamma_v'\ e_u'^2\left(\frac{1}{4}\ D_{u,v} + \frac{1}{4}\ D_{u,v}^2\right)\ \frac{\alpha_{u,v}}{2}\ b_{3/2}^{(1,u,v)}$$

$$+\ \gamma_u'\ \gamma_v'\ e_v'^2\ \left(\frac{1}{4}\ D_{u,v} + \frac{1}{4}\ D_{u,v}^2\right)\ \frac{\alpha_{u,v}}{2}\ b_{3/2}^{(1,u,v)}$$

$$+ (-\gamma_u'\,\gamma_v'^3 - \gamma_v'\ \gamma_u'^3)\ \frac{\alpha_{u,v}}{16}\ b_{3/2}^{(1,u,v)}$$

$$+ (-\,\gamma_u'^3\,\gamma_v' - \gamma_u'\,\gamma_v'^3)\ \frac{3}{8}\ \alpha_{u,v}^2\ b_{5/2}^{(o,u,v)}$$

$$+\ (-\,\gamma_u'^3\,\gamma_v' - \gamma_u'\ \gamma_v'^3)\ \frac{3}{16}\ \alpha_{u,v}^2\ b_{5/2}^{(2,u,v)}\Bigg\}\ \cos\,(\Omega_u' - \Omega_v')$$

$$+\gamma'_u \gamma'_v e'_u e'_v \left(\frac{1}{2} - \frac{1}{4} D_{u,v} - \frac{1}{4} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b^{(2,u,v)}_{3/2} \cos(\bar{\omega}'_u - \bar{\omega}'_v + \Omega'_u - \Omega'_v)$$

$$+\gamma'_u \gamma'_v e'_u e'_v \left(\frac{1}{2} - \frac{1}{4} D_{u,v} - \frac{1}{4} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b^{(o,u,v)}_{3/2} \cos(\bar{\omega}'_u - \bar{\omega}'_v - \Omega'_u + \Omega'_v)$$

$$+\gamma'^2_u \gamma'^2_v \left(\frac{1}{16} \alpha_{u,v} b^{(1,u,v)}_{3/2} + \frac{3}{16} \alpha^2_{u,v} b^{(2,u,v)}_{5/2} + \frac{3}{32} \alpha^2_{u,v} b^{(o,u,v)}_{5/2}\right) \cos(2\Omega'_u - 2\Omega'_v)$$

$$+\gamma'^2_v e'^2_u \left(\frac{3}{8} + \frac{1}{8} D_{u,v} + \frac{1}{8} D^2_{u,v}\right) \frac{\alpha_{u,v}}{4} b^{(1,u,v)}_{3/2} \cos(2\bar{\omega}'_u - 2\Omega'_v)$$

$$+\gamma'^2_v e'_u e'_v \left(1 - \frac{1}{4} D_{u,v} - \frac{1}{4} D^2_{u,v}\right) \frac{\alpha_{u,v}}{4} b^{(o,u,v)}_{3/2} \cos(\bar{\omega}'_u + \bar{\omega}'_v - 2\Omega'_v)$$

$$+\gamma'^2_v e'^2_v \left(\frac{3}{8} + \frac{1}{8} D_{u,v} + \frac{1}{8} D^2_{u,v}\right) \frac{\alpha_{u,v}}{4} b^{(1,u,v)}_{3/2} \cos(2\bar{\omega}'_v - 2\Omega'_v)$$

$$+\gamma'^2_u e'^2_u \left(\frac{3}{8} + \frac{1}{8} D_{u,v} + \frac{1}{8} D^2_{u,v}\right) \frac{\alpha_{u,v}}{4} b^{(1,u,v)}_{3/2} \cos(2\bar{\omega}'_u - 2\Omega'_u)$$

$$+\gamma'^2_u e'_u e'_v \left(1 - \frac{1}{4} D_{u,v} - \frac{1}{4} D^2_{u,v}\right) \frac{\alpha_{u,v}}{4} b^{(o,u,v)}_{3/2} \cos(\bar{\omega}'_u + \bar{\omega}'_v - 2\Omega'_u)$$

$$+\gamma'^2_u e'^2_v \left(\frac{3}{8} + \frac{1}{8} D_{u,v} + \frac{1}{8} D^2_{u,v}\right) \frac{\alpha_{u,v}}{4} b^{(1,u,v)}_{3/2} \cos(2\bar{\omega}'_v - 2\Omega'_u)$$

$$+\gamma'_u \gamma'_v e'^2_u \left(-\frac{3}{8} - \frac{1}{8} D_{u,v} - \frac{1}{8} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b^{(1,u,v)}_{3/2} \cos(2\bar{\omega}'_u - \Omega'_u - \Omega'_v)$$

$$+\gamma'_u \gamma'_v e'_u e'_v \left(-1 + \frac{1}{4} D_{u,v} + \frac{1}{4} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b^{(o,u,v)}_{3/2} \cos(\bar{\omega}'_u + \bar{\omega}'_v - \Omega'_u - \Omega'_v)$$

$$+\gamma'_u \gamma'_v e'^2_v \left(-\frac{3}{8} - \frac{1}{8} D_{u,v} - \frac{1}{8} D^2_{u,v}\right) \frac{\alpha_{u,v}}{2} b^{(1,u,v)}_{3/2} \cos(2\bar{\omega}'_v - \Omega'_u - \Omega'_v)\Bigg]\Bigg] \qquad (2).$$

3° Each term

$$\sigma k^2 \frac{\beta_u \beta_v}{r_v} \frac{1}{\sqrt{1 - 2\frac{r_u}{r_v}\cos\theta_{u,v} + \frac{r^2_u}{r^2_v}}}$$

of $(F_1)_P$ has been obtained by applying the operator

$$\Theta = \left(\frac{r_u}{a_u}\right)^{D_{u,v}} \left(\frac{r_v}{a_v}\right)^{1-D_{u,v}} \exp \sqrt{-1}\, p(f_u - \ell_u) \exp \sqrt{-1}\, q(f_v - \ell_v)$$

to the corresponding term

$$\sigma k^2 \frac{\beta_u \beta_v}{a_v} \frac{1}{\sqrt{1 - 2\alpha_{u,v} \cos \theta_{u,v} + \alpha_{u,v}^2}}$$

of an $(F_1)_P$ in which the orbits would be circular, f_u being the true orbital longitude of P_u, f_v the true orbital longitude of P_v, p and q relative integers respectively equal to j and $-j$ for a term of class zero in the Newcomb sense, to $j+1$ and $-j+1$ for a term of class one in the Newcomb sense, to $j+2$ and $-j+2$ for a term of class two in the Newcomb sense, . . . Let us call "circular $(F_1)_P$" such an $(F_1)_P$. In (1), each of the $69(n-1)n(p+1)/2$ first terms arises from the corresponding term of the circular $(F_1)_P$ which is of class zero in the Newcomb sense; each of the $84(n-1)n(p+1)/2$ next terms arises from the corresponding term of the circular $(F_1)_P$ which is of class one in the Newcomb sense; each of the $10(n-1)n(p+1)/2$ last terms arises from the corresponding term of the circular $(F_1)_P$ which is of class two in the Newcomb sense. In (2), each of the $7(n-1)n(p+1)/2$ first terms arises from the corresponding term of the circular $(F'_1)_P$ which is of class zero in the Newcomb sense; each of the $9(n-1)n(p+1)/2$ last terms arises from the corresponding term of the circular $(F'_1)_P$ which is of class one in the Newcomb sense. No terms of class two in the Newcomb sense appears in the circular $(F'_1)_P$ in so far as we neglect the powers of eccentricities and inclinations higher than the fourth.

CALCULATION OF $(F_1)_I$

We have:

$$(F_1)_I = -\sigma k^2 \left[\sum_{\substack{u \neq v \\ 1 \leq u < v \leq n}} \beta_u \beta_v \frac{\alpha_{u,v}}{a_v} \left[\left(1 - \frac{1}{4}\gamma_u^2 - \frac{1}{4}\gamma_v^2 - \frac{1}{16}\gamma_u^4 - \frac{1}{16}\gamma_v^4 + \frac{1}{16}\gamma_u^2\gamma_v^2 + e_u^2\left(-\frac{1}{2} + \frac{1}{8}\gamma_u^2 + \frac{1}{8}\gamma_v^2\right)\right.\right.\right.$$

$$\left. + e_v^2\left(-\frac{1}{2} + \frac{1}{8}\gamma_u^2 + \frac{1}{8}\gamma_v^2\right) - \frac{1}{64}e_u^4 + \frac{1}{4}e_u^2 e_v^2 - \frac{1}{64}e_v^4\right) \cos(\lambda_u - \lambda_v)$$

$$+ \left(e_u\left(\frac{1}{2} - \frac{1}{8}\gamma_u^2 - \frac{1}{8}\gamma_v^2\right) - \frac{3}{8}e_u^3 - \frac{1}{4}e_u e_v^2\right) \cos(2\lambda_u - \lambda_v - \bar{\omega}_u)$$

$$+ \left(e_u\left(-\frac{3}{2} + \frac{3}{8}\gamma_u^2 + \frac{3}{8}\gamma_v^2\right) + \frac{3}{4}e_u e_v^2\right) \cos(-\lambda_v + \bar{\omega}_u)$$

$$+ \left(e_v\left(2 - \frac{1}{2}\gamma_u^2 - \frac{1}{2}\gamma_v^2\right) - e_u^2 e_v - \frac{3}{2}e_v^3\right) \cos(\lambda_u - 2\lambda_v + \bar{\omega}_v)$$

$$+ \left(e_u^2 \left(\frac{3}{8} - \frac{3}{32}\gamma_u^2 - \frac{3}{32}\gamma_v^2 \right) - \frac{3}{8} e_u^4 - \frac{3}{16} e_u^2 e_v^2 \right) \cos (3\lambda_u - \lambda_v - 2\bar{\omega}_u)$$

$$+ \left(e_u^2 \left(\frac{1}{8} - \frac{1}{32}\gamma_u^2 - \frac{1}{32}\gamma_v^2 \right) + \frac{1}{24} e_u^4 - \frac{1}{16} e_u^2 e_v^2 \right) \cos (-\lambda_u - \lambda_v + 2\bar{\omega}_u)$$

$$+ \left(e_u e_v \left(1 - \frac{1}{4}\gamma_u^2 - \frac{1}{4}\gamma_v^2 \right) - \frac{3}{4} e_u^3 e_v - \frac{3}{4} e_u e_v^3 \right) \cos (2\lambda_u - 2\lambda_v - \bar{\omega}_u + \bar{\omega}_v)$$

$$+ \left(e_u e_v \left(-3 + \frac{3}{4}\gamma_u^2 + \frac{3}{4}\gamma_v^2 \right) + \frac{9}{4} e_u e_v^3 \right) \cos (-2\lambda_v + \bar{\omega}_u + \bar{\omega}_v)$$

$$+ \left(e_v^2 \left(\frac{1}{8} - \frac{1}{32}\gamma_u^2 - \frac{1}{32}\gamma_v^2 \right) - \frac{1}{16} e_u^2 e_v^2 + \frac{1}{24} e_v^4 \right) \cos (\lambda_u + \lambda_v - 2\bar{\omega}_v)$$

$$+ \left(e_v^2 \left(\frac{27}{8} - \frac{27}{32}\gamma_u^2 - \frac{27}{32}\gamma_v^2 \right) - \frac{27}{16} e_u^2 e_v^2 - \frac{27}{8} e_v^4 \right) \cos (\lambda_u - 3\lambda_v + 2\bar{\omega}_v)$$

$$+ \frac{1}{3} e_u^3 \cos (4\lambda_u - \lambda_v - 3\bar{\omega}_u)$$

$$+ \frac{1}{24} e_u^3 \cos (-2\lambda_u - \lambda_v + 3\bar{\omega}_u)$$

$$+ \frac{3}{4} e_u^2 e_v \cos (3\lambda_u - 2\lambda_v - 2\bar{\omega}_u + \bar{\omega}_v)$$

$$+ \frac{1}{4} e_u^2 e_v \cos (-\lambda_u - 2\lambda_v + 2\bar{\omega}_u + \bar{\omega}_v)$$

$$+ \frac{1}{16} e_u e_v^2 \cos (2\lambda_u + \lambda_v - \bar{\omega}_u - 2\bar{\omega}_v)$$

$$+ \frac{27}{16} e_u e_v^2 \cos (2\lambda_u - 3\lambda_v - \bar{\omega}_u + 2\bar{\omega}_v)$$

$$- \frac{3}{16} e_u e_v^2 \cos (\lambda_v + \bar{\omega}_u - 2\bar{\omega}_v)$$

$$- \frac{81}{16} e_u e_v^2 \cos (-3\lambda_v + \bar{\omega}_u + 2\bar{\omega}_v)$$

$$+ \frac{1}{6} e_v^3 \cos(\lambda_u + 2\lambda_v - 3\bar{\omega}_v)$$

$$+ \frac{16}{3} e_v^3 \cos(\lambda_u - 4\lambda_v + 3\bar{\omega}_v)$$

$$+ \frac{125}{384} e_u^4 \cos(5\lambda_u - \lambda_v - 4\bar{\omega}_u)$$

$$+ \frac{3}{128} e_u^4 \cos(-3\lambda_u - \lambda_v + 4\bar{\omega}_u)$$

$$+ \frac{2}{3} e_u^3 e_v \cos(4\lambda_u - 2\lambda_v - 3\bar{\omega}_u + \bar{\omega}_v)$$

$$+ \frac{1}{12} e_u^3 e_v \cos(-2\lambda_u - 2\lambda_v + 3\bar{\omega}_u + \bar{\omega}_v)$$

$$+ \frac{3}{64} e_u^2 e_v^2 \cos(3\lambda_u + \lambda_v - 2\bar{\omega}_u - 2\bar{\omega}_v)$$

$$+ \frac{81}{64} e_u^2 e_v^2 \cos(3\lambda_u - 3\lambda_v - 2\bar{\omega}_u + 2\bar{\omega}_v)$$

$$+ \frac{1}{64} e_u^2 e_v^2 \cos(-\lambda_u + \lambda_v + 2\bar{\omega}_u - 2\bar{\omega}_v)$$

$$+ \frac{27}{64} e_u^2 e_v^2 \cos(-\lambda_u - 3\lambda_v + 2\bar{\omega}_u + 2\bar{\omega}_v)$$

$$+ \frac{1}{12} e_u e_v^3 \cos(2\lambda_u + 2\lambda_v - \bar{\omega}_u - 3\bar{\omega}_v)$$

$$+ \frac{8}{3} e_u e_v^3 \cos(2\lambda_u - 4\lambda_v - \bar{\omega}_u + 3\bar{\omega}_v)$$

$$- \frac{1}{4} e_u e_v^3 \cos(2\lambda_v + \bar{\omega}_u - 3\bar{\omega}_v)$$

$$+ 8 e_u e_v^3 \cos(-4\lambda_v + \bar{\omega}_u + 3\bar{\omega}_v)$$

$$+ \frac{27}{128} e_v^4 \cos(\lambda_u + 3\lambda_v - 4\bar{\omega}_v)$$

$$+ \frac{3125}{384} e_v^4 \cos(\lambda_u - 5\lambda_v + 4\bar{\omega}_v)$$

$$+ \left(\frac{1}{4}\gamma_v^2 + \frac{1}{16}\gamma_v^4 - \frac{1}{16}\gamma_u^2\gamma_v^2 - \frac{1}{8}e_u^2\gamma_v^2 - \frac{1}{8}e_v^2\gamma_v^2\right) \cos(\lambda_u + \lambda_v - 2\Omega_v)$$

$$+ \frac{1}{8} e_u \gamma_v^2 \cos(2\lambda_u + \lambda_v - \bar{\omega}_u - 2\Omega_v)$$

$$- \frac{3}{8} e_u \gamma_v^2 \cos(\lambda_v + \bar{\omega}_u - 2\Omega_v)$$

$$+ \frac{1}{2} e_v \gamma_v^2 \cos(\lambda_u + 2\lambda_v - \bar{\omega}_v - 2\Omega_v)$$

$$+ \frac{3}{32} e_u^2 \gamma_v^2 \cos(3\lambda_u + \lambda_v - 2\bar{\omega}_u - 2\Omega_v)$$

$$+ \frac{1}{32} e_u^2 \gamma_v^2 \cos(-\lambda_u + \lambda_v + 2\bar{\omega}_u - 2\Omega_v)$$

$$+ \frac{1}{4} e_u e_v \gamma_v^2 \cos(2\lambda_u + 2\lambda_v - \bar{\omega}_u - \bar{\omega}_v - 2\Omega_v)$$

$$- \frac{3}{4} e_u e_v \gamma_v^2 \cos(2\lambda_v + \bar{\omega}_u - \bar{\omega}_v - 2\Omega_v)$$

$$+ \frac{27}{32} e_v^2 \gamma_v^2 \cos(\lambda_u + 3\lambda_v - 2\bar{\omega}_v - 2\Omega_v)$$

$$+ \frac{1}{32} e_v^2 \gamma_v^2 \cos(\lambda_u - \lambda_v + 2\bar{\omega}_v - 2\Omega_v)$$

$$+ \left(\frac{1}{4}\gamma_u^2 + \frac{1}{16}\gamma_u^4 - \frac{1}{16}\gamma_u^2\gamma_v^2 - \frac{1}{8}e_u^2\gamma_u^2 - \frac{1}{8}e_v^2\gamma_u^2\right) \cos(\lambda_u + \lambda_v - 2\Omega_u)$$

$$+ \frac{1}{8} e_u \gamma_u^2 \cos(2\lambda_u + \lambda_v - \bar{\omega}_u - 2\Omega_u)$$

$$- \frac{3}{8} e_u \gamma_u^2 \cos(\lambda_v + \bar{\omega}_u - 2\,\Omega_u)$$

$$+ \frac{1}{2} e_v \gamma_u^2 \cos(\lambda_u + 2\,\lambda_v - \bar{\omega}_v - 2\,\Omega_u)$$

$$+ \frac{3}{32} e_u^2 \gamma_u^2 \cos(3\,\lambda_u + \lambda_v - 2\,\bar{\omega}_u - 2\,\Omega_u)$$

$$+ \frac{1}{32} e_u^2 \gamma_u^2 \cos(-\lambda_u + \lambda_v + 2\,\bar{\omega}_u - 2\,\Omega_u)$$

$$+ \frac{1}{4} e_u e_v \gamma_u^2 \cos(2\,\lambda_u + 2\,\lambda_v - \bar{\omega}_u - \bar{\omega}_v - 2\,\Omega_u)$$

$$- \frac{3}{4} e_u e_v \gamma_u^2 \cos(2\,\lambda_v + \bar{\omega}_u - \bar{\omega}_v - 2\,\Omega_u)$$

$$+ \frac{27}{32} e_v^2 \gamma_u^2 \cos(\lambda_u + 3\,\lambda_v - 2\,\bar{\omega}_v - 2\,\Omega_u)$$

$$+ \frac{1}{32} e_v^2 \gamma_u^2 \cos(\lambda_u - \lambda_v + 2\,\bar{\omega}_v - 2\,\Omega_u)$$

$$+ \left(\frac{1}{2} \gamma_u \gamma_v - \frac{1}{4} e_u^2 \gamma_u \gamma_v - \frac{1}{4} e_v^2 \gamma_u \gamma_v\right) \cos(\lambda_u - \lambda_v - \Omega_u + \Omega_v)$$

$$+ \frac{1}{4} e_u \gamma_u \gamma_v \cos(2\,\lambda_u - \lambda_v - \bar{\omega}_u - \Omega_u + \Omega_v)$$

$$- \frac{3}{4} e_u \gamma_u \gamma_v \cos(-\lambda_v + \bar{\omega}_u - \Omega_u + \Omega_v)$$

$$+ e_v \gamma_u \gamma_v \cos(\lambda_u - 2\,\lambda_v + \bar{\omega}_v - \Omega_u + \Omega_v)$$

$$+ \frac{3}{16} e_u^2 \gamma_u \gamma_v \cos(3\,\lambda_u - \lambda_v - 2\,\bar{\omega}_u - \Omega_u + \Omega_v)$$

$$+ \frac{1}{16} e_u^2 \gamma_u \gamma_v \cos(-\lambda_u - \lambda_v + 2\,\bar{\omega}_u - \Omega_u + \Omega_v)$$

$$+ \frac{1}{2} e_u e_v \gamma_u \gamma_v \cos(2\lambda_u - 2\lambda_v - \bar{\omega}_u + \bar{\omega}_v - \Omega_u + \Omega_v)$$

$$- \frac{3}{2} e_u e_v \gamma_u \gamma_v \cos(-2\lambda_v + \bar{\omega}_u + \bar{\omega}_v - \Omega_u + \Omega_v)$$

$$+ \frac{1}{16} e_v^2 \gamma_u \gamma_v \cos(\lambda_u + \lambda_v - 2\bar{\omega}_v - \Omega_u + \Omega_v)$$

$$+ \frac{27}{16} e_v^2 \gamma_u \gamma_v \cos(\lambda_u - 3\lambda_v + 2\bar{\omega}_v - \Omega_u + \Omega_v)$$

$$+ \left(-\frac{1}{2} \gamma_u \gamma_v + \frac{1}{4} e_u^2 \gamma_u \gamma_v + \frac{1}{4} e_v^2 \gamma_u \gamma_v\right) \cos(\lambda_u + \lambda_v - \Omega_u - \Omega_v)$$

$$- \frac{1}{4} e_u \gamma_u \gamma_v \cos(2\lambda_u + \lambda_v - \bar{\omega}_u - \Omega_u - \Omega_v)$$

$$+ \frac{3}{4} e_u \gamma_u \gamma_v \cos(\lambda_v + \bar{\omega}_u - \Omega_u - \Omega_v)$$

$$- e_v \gamma_u \gamma_v \cos(\lambda_u + 2\lambda_v - \bar{\omega}_v - \Omega_u - \Omega_v)$$

$$- \frac{3}{16} e_u^2 \gamma_u \gamma_v \cos(3\lambda_u + \lambda_v - 2\bar{\omega}_u - \Omega_u - \Omega_v)$$

$$- \frac{1}{16} e_u^2 \gamma_u \gamma_v \cos(-\lambda_u + \lambda_v + 2\bar{\omega}_u - \Omega_u - \Omega_v)$$

$$- \frac{1}{2} e_u e_v \gamma_u \gamma_v \cos(2\lambda_u + 2\lambda_v - \bar{\omega}_u - \bar{\omega}_v - \Omega_u - \Omega_v)$$

$$+ \frac{3}{2} e_u e_v \gamma_u \gamma_v \cos(2\lambda_v + \bar{\omega}_u - \bar{\omega}_v - \Omega_u - \Omega_v)$$

$$- \frac{27}{16} e_v^2 \gamma_u \gamma_v \cos(\lambda_u + 3\lambda_v - 2\bar{\omega}_v - \Omega_u - \Omega_v)$$

$$- \frac{1}{16} e_v^2 \gamma_u \gamma_v \cos(\lambda_u - \lambda_v + 2\bar{\omega}_v - \Omega_u - \Omega_v)$$

$$\left. \left. + \frac{1}{16} \gamma_u^2 \gamma_v^2 \cos(\lambda_u - \lambda_v - 2\Omega_u + 2\Omega_v) \right] \right] \qquad (3).$$

Each term

$$-\sigma k^2 \beta_u \beta_v \frac{r_u}{r_v^2} \cos \theta_{u,v}$$

of $(F_1)_I$ has been obtained by applying the operator Θ to the corresponding term

$$-\sigma k^2 \beta_u \beta_v \frac{\alpha_{u,v}}{a_v} \cos \theta_{u,v}$$

of the circular $(F_1)_I$, the integers p and q which appear in Θ verifying successively each of the four equalities

$$p = -q = 1 \quad -p = q = 1, \quad p = q = 1 \quad -p = -q = 1.$$

CONCLUSION

1. In considering n planets instead of two, their inclinations with respect to a common fixed plane instead of their mutual inclinations, in referring their longitudes to a common origin instead of referring them to the longitude of the ascending node of the disturbed or of the disturbing planet and in reducing the Fourier series of the principal part of the disturbing function to the sum of its $(n-1)\,n\,(p+1)/2$ first terms, the value of the positive integer p being not specified, we obtained an expression of the disturbing function much more general than the previous ones. We point out, by the way, that in each argument, the sum of the coefficients of the longitudes is equal to zero and that the sum of the coefficients of the λ_i's corresponding to the mean longitudes ℓ_i $(i = 1, 2, \ldots, n)$ is equal to the smallest power of the eccentricities and the sines of inclinations which appear in the coefficient of its cosine, which means that the D'Alembert's rule is verified.

2. The only direction in which our expression of the disturbing function can be generalized deals with the powers of the eccentricities and the sines of the inclinations. An extension of our calculation up to the eight powers of the eccentricities and the sines of inclinations which is the precision required in order to build a complete first order general planetary theory which could be compared with the previous theories and up to the twelve powers of the eccentricities and the sines of inclinations which is the precision required in order to go efficiently beyond such a theory could be easily carried out through the way we indicated, the only difficulty dealing with the length of the calculation itself.

3. The values of the integers n and p depend upon the set of planets we consider and no general rule may be formulated concerning them, each particular case involving its own system of values of n and p. In our solar system and present knowledge of the big planets, n cannot exceed 9. As for p, it is so much the more larger than the ratios $\alpha_{u,v}$ $(u, v = 1, 2, \ldots, u, v = n-1, n)$ of the semi major axis are closer to 1 and it can reach a very high value when the $\alpha_{u,v}$'s are very close to 1. A literal development of our expression of the disturbing function may be obtained, through the way of harmonic analysis, by considering numerical values of the ratios $\alpha_{u,v}$ at the very beginning, the coefficients being therefore functions of the eccentricities and sines of inclinations and it could lend to a generalization of the tables for the development of the disturbing function as they were previously settled by Brown and Shook[6] and Brown and Brouwer.[7]

4. We have now at hand all the elements in order to perform with the highest degree of accuracy, in the restricted frame of Newton's law, the elimination of the short period terms of a first order general planetary theory through Von Zeipel's method. In order to perform completely this elimination which will enlarge the results of our two previous papers [8,9], it would be however necessary to include the relativity effect and the asteroidal effect. We plan to investigate, later on, this matter.

REFERENCES

1. LeVerrier, U.J., "Développement de la fonction qui sert de base au calcul des perturbations des mouvements des planetès," Mémoires, Annales de l'Observatoire de Paris, vol. 1, pp258-330, Mallet Bachelier, Paris, 1855.

2. Newcomb, S., "A development of the perturbative function in cosines of multiples of the mean anomalies and of angles between the perihelia and common node and in powers of the eccentricities and mutual inclination," Astronomical Papers of the American Ephemeris and Nautical Almanac, vol. 5, pp. 1-48, 1895.

3. Brouwer, D. and Clemence, G.M., "Methods of Celestial Mechanics," Chapt. 15, §6, pp. 490-494, Academic Press, New York, 598 pp, 1961.

4. Marsden, B.G., "The Motions of the Galilean Satellites of Jupiter," Chapt. 14, page 79, thesis, Yale University, 189 pp, 1966.

5. Andoyer, H., "Cours de Mécanique Céleste," tome 1, Chapt. 13, pp. 403-438, Gauthier Villars, Paris, 1923.

6. Brown, E.W., and Shook, C.A., "Planetary Theory", Appendix A, University Press, Cambridge, England, 302 pp, 1933.

7. Brown, E.W., and Brouwer, D., "Tables for the development of the Disturbing Function," University Press, Cambridge, England, 1933 and "Trans Yale Univ. Observatory," 6, Pt. 5, 1932.

8. Meffroy, J., "On Von Zeipel's Method in General Planetary Theory," Special Report n°229, Chapt. 2, pp. 8-13, Smithsonian Astrophysical Observatory, Cambridge, Mass. 80 pp. 1966.

9. Meffroy, J., "On the elimination of the short period terms of a first order general planetary theory through Von Zeipel's method," Document X-641-67-318, Goddard Space Flight Center, Greenbelt, Md., 22 pp., June 1967.

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